CSIC576: Assignment II

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Q1: The following sequence of real numbers has been obtained sampling a signal: 5.8, 6.2, 6.2, 7.2, 7.3, 7.3, 6.5, 6.8, 6.8, 6.8, 5.5, 5.0, 5.2, 5.2, 5.8, 6.2, 6.2, 6.2, 5.9, 6.3, 5.2, 4.2, 2.8, 2.8, 2.3, 2.9, 1.8, 2.5, 2.5, 3.3, 4.1, 4.9
This signal is then quantized using the interval [0,8] and dividing it into 32 uniformly distributed levels.

- **What does the quantized sequence look like? For ease of computation, assume that you placed the level 0 at 0.25, the level 1 at 0.5, level 2 at 0.75, level 3 at 1.0 and so on. This should simplify your calculations. Round off any fractional value to the nearest integer levels**

Table 1: Quantizing the input into different levels

Original signal samples	Quantized Values	Quantization level
5.8	5.75	22
6.2	6.25	24
6.2	6.25	24
7.2	7.25	28
7.3	7.25	28
7.3	7.25	28
6.5	6.5	25
6.8	6.75	26
6.8	6.75	26
6.8	6.75	26
5.5	5.5	21
5	5	19
5.2	5.25	20
5.2	5.25	20
5.8	5.75	22
6.2	6.25	24
6.2	6.25	24
6.2	6.25	24
5.9	6	23
6.3	6.25	24
5.2	5.25	20
4.2	4.25	16
2.8	2.75	10
2.8	2.75	10
2.3	2.25	8
2.9	3	11
1.8	1.75	6
2.5	2.5	9
2.5	2.5	9
3.3	3.25	12
4.1	4	15
4.9	5	19

- **How many bits do you need to transmit it?**

Number of bits required to transmit the given 32 values of data is `32x5 = 160 bits`.

- **If you need to encode the quantized output using DPCM. Compute the successive differences between the values what is the maximum and minimum value for the difference? Assuming that this is your range, how many bits are required to encode the sequence now?**

Table 2: Quantization levels and corresponding DPCM values

Quantization level	DPCM
22	
24	2
24	0
28	4
28	0
28	0
25	-3
26	1
26	0
26	0
21	-5
19	-2
20	1
20	0
22	2
24	2
24	0
24	0
23	-1
24	1
20	-4
16	-4
10	-6
10	0
8	-2
11	3
6	-5
9	3
9	0
12	3
15	3
19	4

- Maximum value of difference `4`
- Minimum value of difference `-6`

Number of bits required to encode the DPCM sequence is calculated as follows:

- First value is encoded with `5 bits`.
- Remaining 31 values are encoded with `4 bits each`.

$\therefore$ Total number of bits required to encode the given sequence is `5+(4x31)= 129 bits`.

- **What is the compression ratio you have achieved?**

Compression ratio achieved is `129:160`.

- **Instead of transmitting the differences, you use Huffman coded values for the differences. How many bits do you need now to encode the sequence?**

Table 3: DPCM levels, corresponding Huffman code for each level and the number of bits required to encode the sequence based on the number of occurrences of the symbol

DPCM levels - Symbols	Huffman Code	Number of occurrences	Number of bits, to encode
-6	00010	1	5
-5	0010	2	8
-4	0111	2	8
-3	00011	1	5
-2	1000	2	8
-1	0000	1	4
0	11	10	20
1	001	3	9
2	010	3	9
3	101	4	12
4	1001	2	8
		Total number of bits	96

First value is transmitted with 5 bits, hence the total number of bits required to encode the given sequence is equal to `101(96+5) bits`.

- **What is the compression ratio you have achieved now?**

Compression Ratio achieved is:

- `101:160` with respect to Quantizing and transmitting
- `101:129` with respect to DPCM tramsmission

Q2: Consider two symbols, A and B, with the probability of occurrence of 0.8 and 0.2, respectively. The coding efficiency can be improved by combining N symbols at a time (called symbol blocking.). Say N = 3, so you are grouping symbols of 3 and giving them a unique code. (Assume that each symbol occurrence is independent of previous symbol occurrences).

- **How many types of different outcomes are there and what are their probabilities?**

Table 4: Outcomes and their corresponding probabilities

Distinct Outcome	Probability
AAA	0.512
AAB	0.128
ABA	0.128
ABB	0.032
BAA	0.128
BAB	0.032
BBA	0.032
BBB	0.008

- **Show the arrangement of symbols on the unit interval [0, 1] and determine the arithmetic code for the three-symbol sequence.**

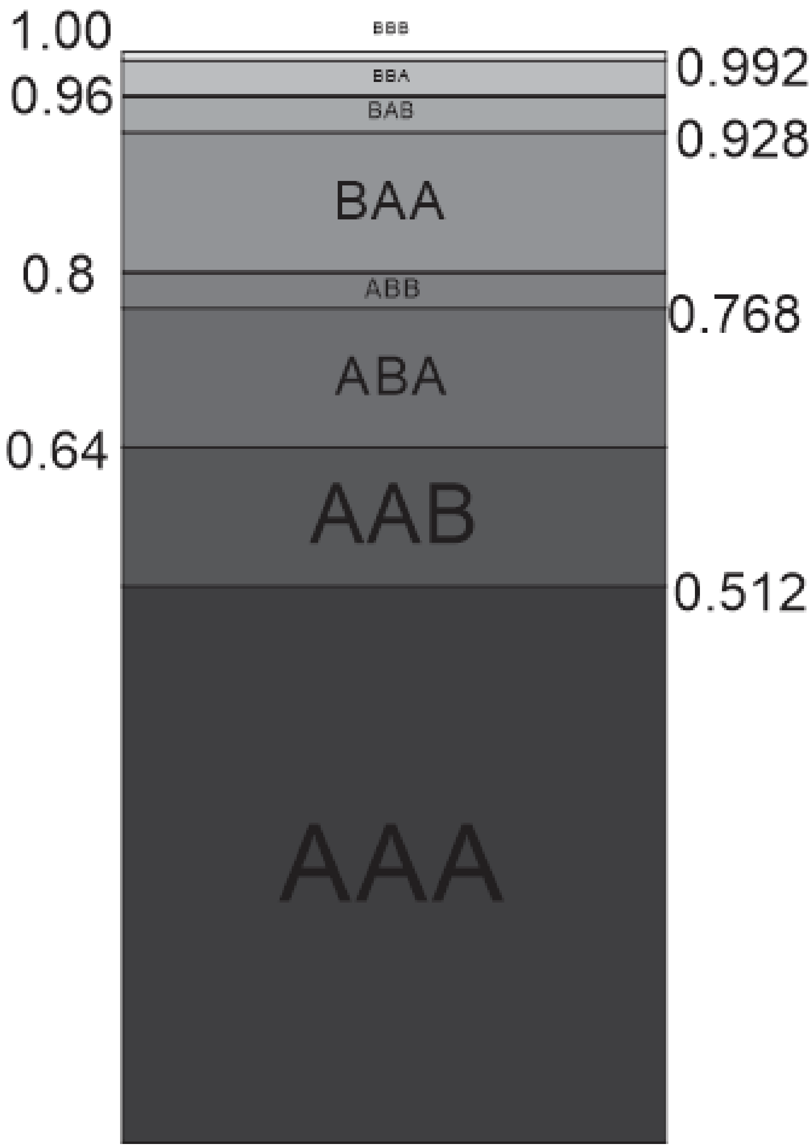

Figure 1: Arrangement of Symbols

Table 5: Outcomes and their corresponding Code word

Distinct Outcome	Code Word
AAA	01
AAB	1001
ABA	1011
ABB	11001
BAA	1101
BAB	11110
BBA	111110
BBB	1111111

- **What is the average code word length? Is it optimum?**

Table 6: Average codeword length

Code length	Probability	Product
2	0.512	1.024
4	0.128	0.512
4	0.128	0.512
5	0.032	0.16
4	0.128	0.512
5	0.032	0.16
6	0.032	0.192
7	0.008	0.056
	Average Code word length	`3.128`

No, this is not the optimum average code word length.

- **How many bits are required to code the message "ABABBAABBAAABBB"**

`4+6+5+2+7 = 24 bits` are required to encode given sequence.

- **How could you do better than the above code length?**

This can be made better by increasing the value of N to 5 and then finding the codewords for each of the 5 symbols in a group.

Q3: Vector quantization

Below set off images show the comparison between input and output images for various values of N.

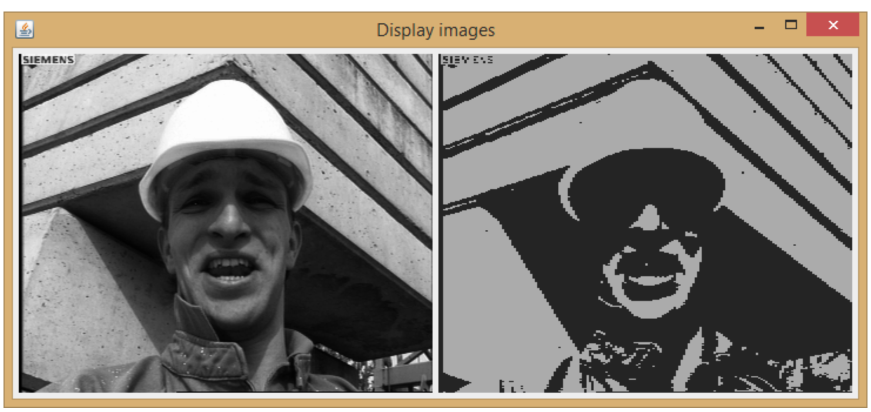

Figure 2: N = 2

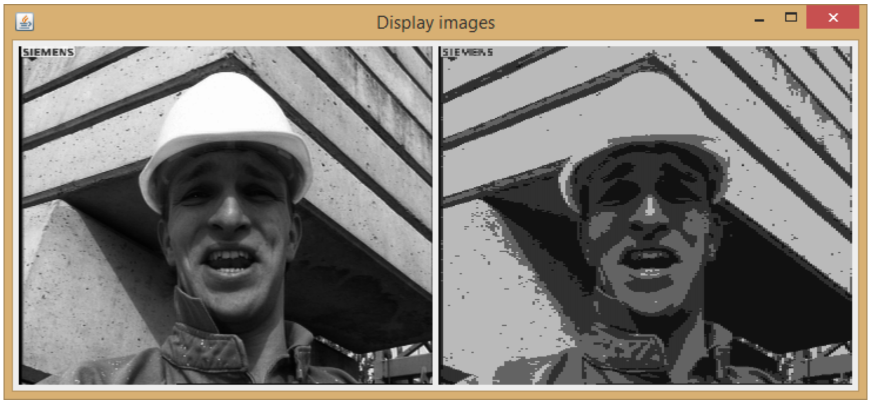

Figure 3: N = 4

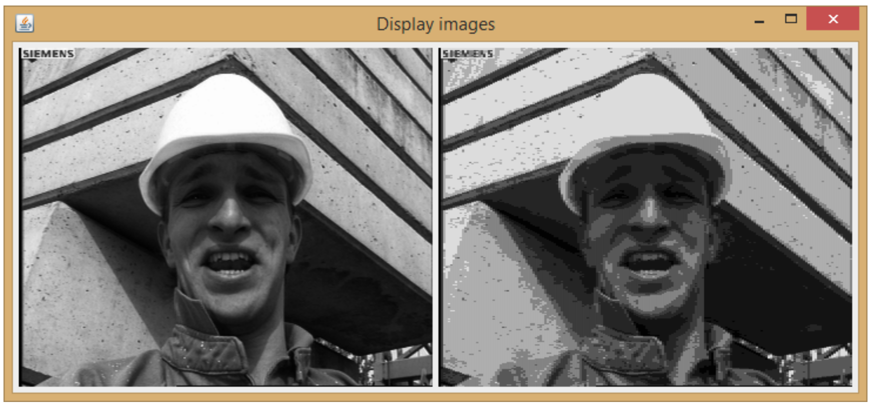

Figure 4: N = 8

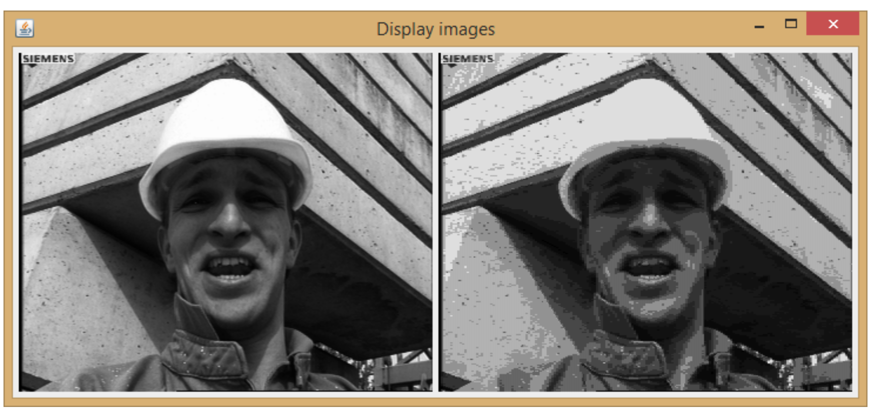

Figure 5: N = 16

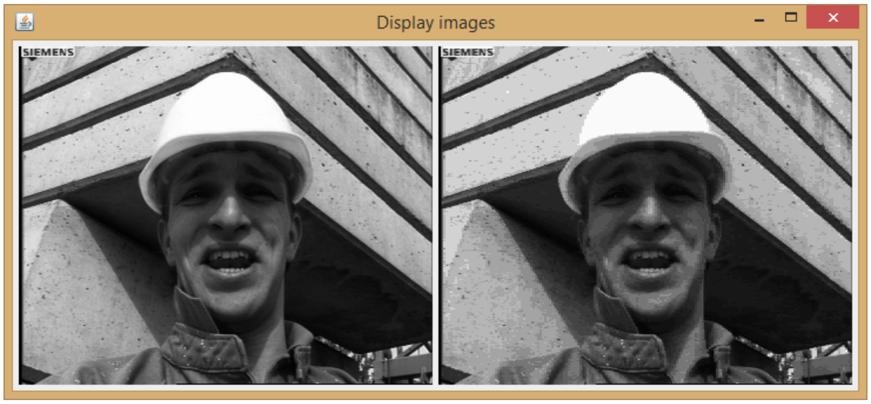

Figure 6: N = 32

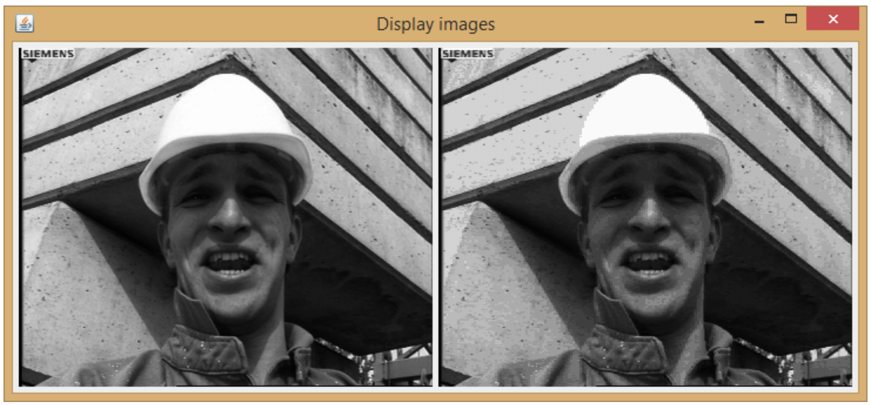

Figure 7: N = 64

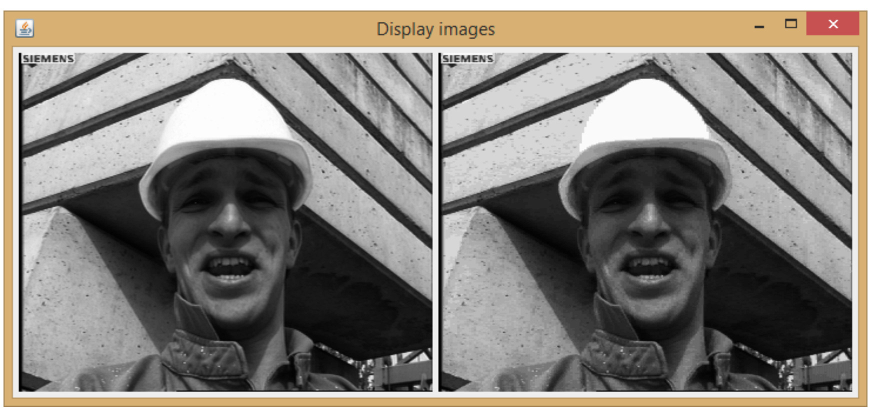

Figure 8: N = 128

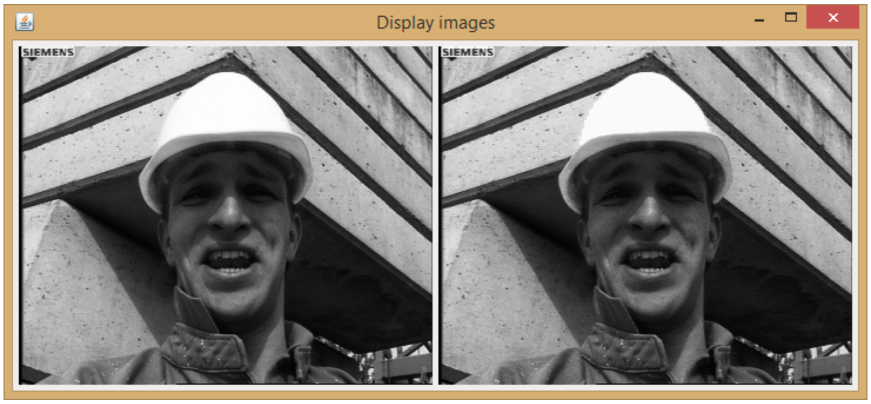

Figure 9: N = 256

Below Graph shows initial pixel distribution and distribution of pixels after vector quantization for N=16.

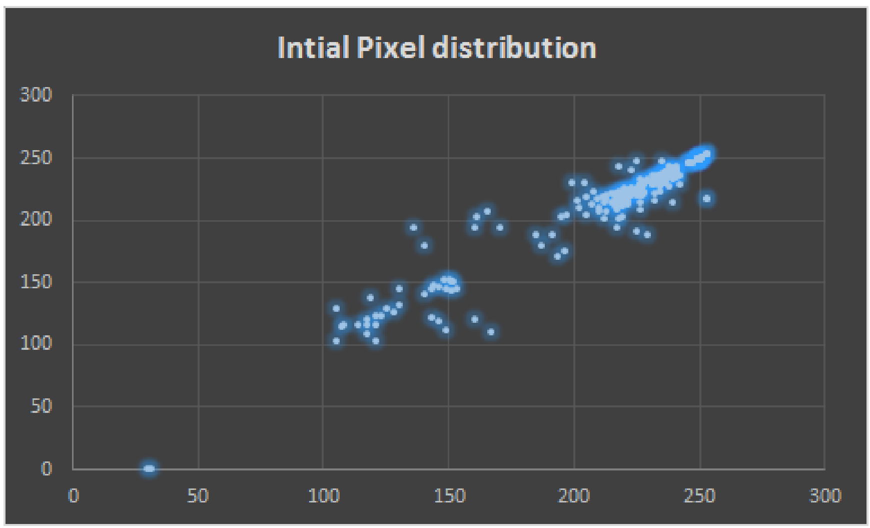

Figure 10: Initial pixel distribution

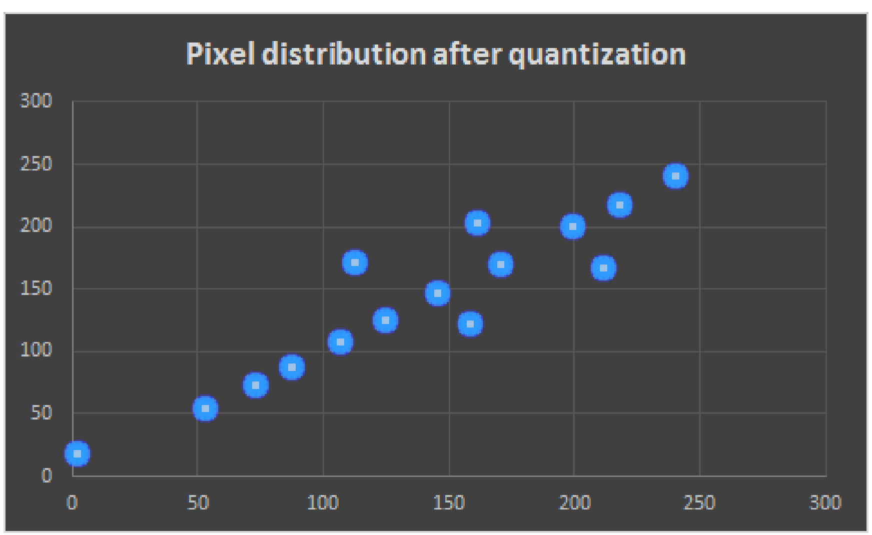

Figure 11: Pixel distribution after Vector quantization