Assignment - 1 CHANDAN UPPULURI

1)

(A) Given: X : D - dimensional vector

Each component of X follows uniform distribution $(0,1)$.

So the nearest edge distance is,

$$D = 1/2 - \max_{i=1 \dots d} |1/2 - x_i|$$

The definition of Independence Says that,

$$P(x_1 \in A_1, x_2 \in A_2 \dots x_d \in A_d) = P(x_1 \in A_1) \times P(x_2 \in A_2) \times \dots P(x_d \in A_d).$$

So,

$$P(D \geq a) = P(x_1 > a) \times P(x_2 > a) \times P(x_3 > a) \dots P(x_d > a)$$

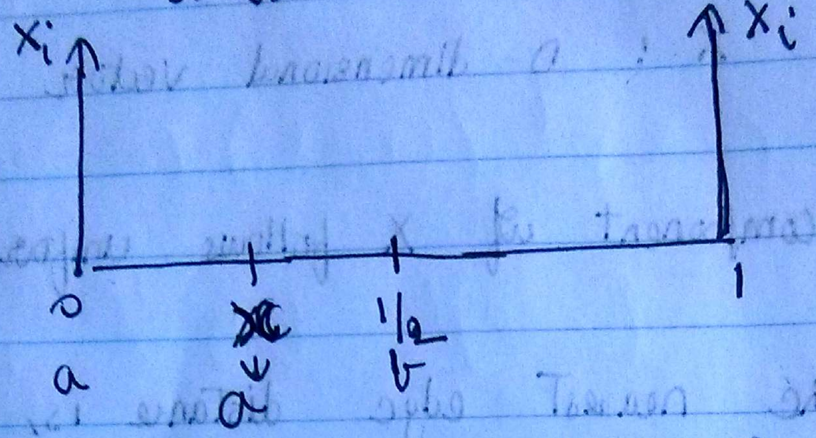
Hence,

$$P(x_i > a) = 1 - P(x_i \leq a)$$
$$= 1 - \frac{x-a}{b-a} \quad \left(\text{as } x \in (0, 1/2)\right)$$

For a uniform distribution, cumilative distributive function is $\frac{x-a}{b-a}$ for $(a<x<b)$

here, a is the ~~distance from the edge~~ one of ; edge

b is the distance ~~from~~ to the center. 1/2



$$\therefore P(x_i > a) = 1 - \left(\frac{a-0}{1/2 - 0}\right)$$

$$= (1 - 2a)$$

$$\therefore P(x_i > a) = (1 - 2a)$$

$$\therefore P(D > a) = (1-2a) \times (1-2a) \cdots (1-2a)$$

$$= (1-2a)^d \quad \text{(For } d \text{ variables)}$$

(B) $P(D < \delta) \geq 0.9$ Find d ? (no. of dimension)

$\Rightarrow 1 - P(D > \delta) \geq 0.9$

$\Rightarrow 1 - (1 - 2\delta)^d \geq 0.9$

$(1 - 2\delta)^d \leq 1 - 0.9$

$$\Rightarrow (1-2\delta)^d \le 0.1$$

$$\log (1-2\delta)^d \le \log(0.1)$$

$$d \log(1-2\delta) \le \log(0.1)$$

$$d \le \frac{\log(0.1)}{\log(1-2\delta)}$$

$$\therefore d \le \frac{\log(0.1)}{\log(1-2\delta)}$$ for the given probability condition

this is the number of maximum dimensions.

and $\delta \in (0, 1/2)$