Should geophysicists use the gravity disturbance or the anomaly?

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SUMMARY
The gravity anomaly is defined as the difference between the Earth's gravity on the geoid and the normal gravity on the reference ellipsoid. Because these quantities are not at the same point, the anomaly contains centrifugal accelerations and cannot be considered a harmonic function. The gravity disturbance is the difference between gravity and normal gravity at the same point. Consequently, the centrifugal effects can be neglected and the disturbance can be considered a harmonic function. This is the premise behind most potential-field data processing techniques (e.g., upward/downward continuation). Unlike the anomaly, the disturbance is due solely to the gravitational effects of geologic sources, making it the most appropriate for geophysical purposes. Use of the gravity anomaly in geophysics carries with it the implicit assumption that it is a good approximation for the gravity disturbance. However, bear in mind that the difference between the gravity disturbance and the free-air anomaly can be larger than 10 mGal worldwide. In fact, we argue that the assumptions made during gravity forward and inverse modeling imply that the quantity being modelled is the disturbance, not the anomaly.

Key words: Geopotential theory – Gravity anomalies and Earth structure – Reference systems – Numerical modelling.

1 INTRODUCTION

The *gravity vector* is the sum of the gravitational and centrifugal accelerations felt by a body at rest on the Earth's surface. Its intensity is what geoscientists call *gravity* (Heiskanen & Moritz 1967; Hofmann-Wellenhof & Moritz 2005). When measured on moving platforms (e.g., airplanes, helicopters, marine vessels), there are additional non-gravitational accelerations due to the motion of the vehicle, such as the Coriolis acceleration and high-frequency vibrations (Glennie et al. 2000; Nabighian et al. 2005; Baumann et al. 2012). Geodesists use gravity measurements to estimate the Geoid (Li & Götze 2001) whereas geophysicists use gravity to estimate the Earth's internal density distribution. Hence, geophysicists are usually only interested in the gravitational component of the observed gravity because that is what that reflects the Earth's internal density distribution.

The first step toward isolating the gravitational component is to remove the effects of vehicle motion, Earth tides, instrumental drift, and barometric pressure changes, among others. If these effects are properly removed, the resultant observations are considered to be solely the sum of the centrifugal acceleration due to the Earth's rotation and the gravitational acceleration produced by the internal density distribution of the whole Earth. The isolation of this particular gravitational component, and its subsequent use for estimating density distributions related to geological structures in subsurface, are the main goals of applied gravimetry (Blakely 1996). The computation of the gravitational effect produced by the geological structures is known in geophysics as *gravity modelling*. Not to be confused with *gravity field modelling*, which is commonly used in geodesy to denote the mathematical characterization of the gravity field in local, regional, or global scales (e.g., spherical harmonic analysis).

We present a discussion of whether the gravity disturbance or the gravity anomaly should be used in geophysical applications. The concepts discussed here are well-established in the literature. However, the debate around this theoretical issue has been carried about from a more geodetic than a geophysical point of view (LaFehr 1991; Chapin 1996; Li & Götze 2001; Fairhead et al. 2003; Hackney & Featherstone 2003; Hinze et al. 2005). We will approach the topic by comparing the geophysical interpretations of the disturbance and the anomaly. Our reasoning suggests that the gravity disturbance is more appropriated for geophysical purposes than the gravity anomaly.

2 NORMAL EARTH AND NORMAL GRAVITY

The Earth's gravity field is traditionally approximated by the gravity field of a reference ellipsoid (or level ellipsoid). This model is

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rigid and geocentric, with a minor axis b which coincides with the mean rotation axis of the Earth Z. The ellipsoid has the same total mass as the Earth (including the atmosphere) and also the same angular velocity (Heiskanen & Moritz 1967; Vaníček & Krakiwsky 1987; Hofmann-Wellenhof & Moritz 2005; Torge & Müller 2012). Its limiting surface coincides with a particular equipotential of its own gravity field. Here, we follow (Torge & Müller 2012) and call this reference ellipsoid the *normal Earth*. Analogously to the Earth, the *normal gravity vector* is the sum of the gravitational and centrifugal accelerations exerted by the normal Earth on a body at rest at a point P. The intensity of the normal gravity vector is called *normal gravity*.

It is worth noting that, although the normal Earth has the same total mass as the Earth, its internal density distribution is left unspecified. The search for geologically meaningful density distributions for the interior of the normal Earth has geophysical rather than geodetic motives (Marussi et al. 1974). In physical geodesy, a model for the normal gravity field can be arbitrarily defined with the sole purpose of keeping the difference from the actual gravity field as small as possible (Vaníček & Krakiwsky 1987). The only condition imposed on its internal density distribution is that it produces a gravity field having a particular equipotential which coincides with its limiting surface.

3 TERRESTRIAL REFERENCE SYSTEMS USED IN GRAVITY MODELLING

For geophysical gravity modelling purposes, there are three important Terrestrial Reference Systems. They rotate with the Earth and are used for describing positions and movements of objects on and close to the Earth's surface (Torge & Müller 2012).

The first is a geocentric system of Cartesian coordinates having the Z-axis coincident with the mean rotational axis of the Earth, the X-axis pointing to the Greenwich meridian, and the Y-axis completing a right-handed system (Fig. 1). This reference system is called by different names in the literature: Mean Terrestrial System (e.g., Soler 1976), Earth-fixed geocentric Cartesian system (e.g., Torge & Müller 2012) or Earth-centered Earth-fixed system (e.g., Bouman et al. 2013), for example. Here, we opted for using the term Geocentric Cartesian System (GCS).

The second is a geocentric system of geodetic coordinates (the Geocentric Geodetic System or GGS), which is defined by the reference ellipsoid used as the Normal Earth (Heiskanen & Moritz 1967; Soler 1976; Torge & Müller 2012; Bouman et al. 2013). The position of a point is defined by the *geometric height* h, the *geodetic latitude* φ, and the *longitude* λ (Fig. 1). There are also three mutually-orthogonal unit vectors defined at a given point (Soler 1976):

$$\hat{\boldsymbol{u}} = \begin{bmatrix} \cos\varphi\cos\lambda \\ \cos\varphi\sin\lambda \\ \sin\varphi \end{bmatrix} \hat{\boldsymbol{v}} = \begin{bmatrix} -\sin\varphi\cos\lambda \\ -\sin\varphi\sin\lambda \\ \cos\varphi \end{bmatrix} \hat{\boldsymbol{w}} = \begin{bmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{bmatrix}. \tag{1}$$

Equations for converting between these systems can be found in the literature (e.g., Heiskanen & Moritz 1967; Torge & Müller 2012; Bouman et al. 2013).

The third is the Topocentric Cartesian System (TCS), which is commonly used in local or regional scale geophysical studies. The origin of the TCS is located at a point P, axes x and y are parallel to the unit vectors $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{w}}$ (eq. 1), respectively, and the z-axis is opposite to the unit vector $\hat{\boldsymbol{u}}$ (Fig. 1c).
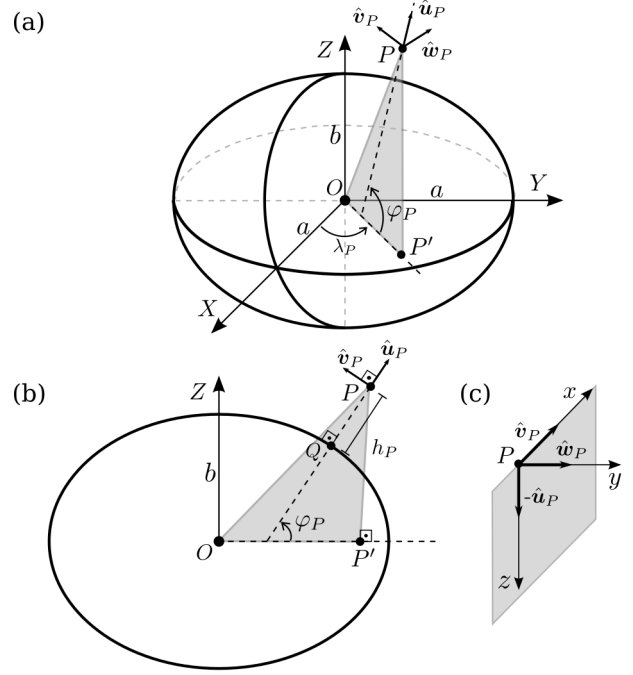

Figure 1. (a) and (b) The Geocentric Cartesian System (GCS) and the Geocentric Geodetic System (GGS). The GGS is defined by an oblate ellipsoid with semi-minor axis b and semi-major axis a. In this coordinate system, the position of a point is determined by the geometric height h, geodetic latitude φ, and longitude λ. O is the Earth's center of mass, P is an observation point, and unit vectors $\hat{\boldsymbol{u}}$, $\hat{\boldsymbol{v}}$, and $\hat{\boldsymbol{w}}$ define mutually orthogonal directions at P (eq. 1). In (b), Q is the projection of P onto the reference ellipsoid at the same latitude and longitude. (c) The topocentric Cartesian coordinate system (TCS) with origin at a point P. Axes x and y are parallel to the unit vectors $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{w}}$, respectively, and the z axis is opposite to $\hat{\boldsymbol{u}}$. The gray plane is the same shown in (a) and (b).
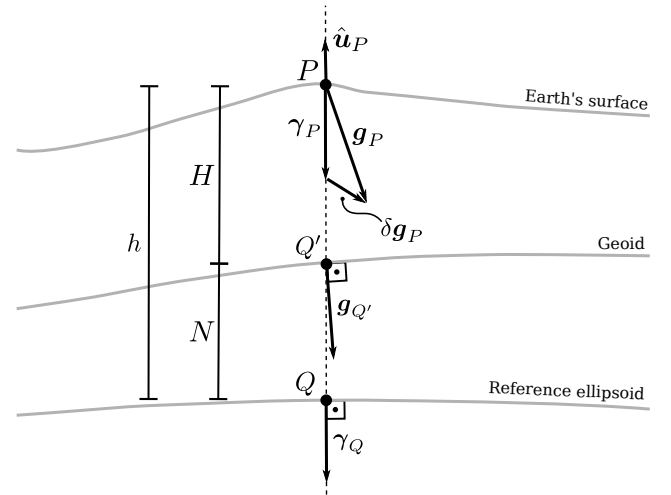

Figure 2. The gravity vector $\vec{\boldsymbol{g}}_P$, normal gravity vector $\vec{\boldsymbol{\gamma}}_P$, gravity disturbance vector $\delta\vec{\boldsymbol{g}}_P$ (eq. 2), and unit vector $\hat{\boldsymbol{u}}_P$ (eq. 1) at a point P on the surface of the Earth. The gravity vector $\vec{\boldsymbol{g}}_{Q'}$ at a point Q' on the geoid, normal gravity vector $\vec{\boldsymbol{\gamma}}_Q$ at a point Q on the reference ellipsoid, geometric height h, orthometric height H, and geoidal height N. The dashed line passing through Q, Q' and P is normal to the surface of the reference ellipsoid at Q. This figure shows a commonly used approximation in which the ellipsoid and geoid are represented as parallel surfaces so that $h \approx H + N$.

4 GRAVITY DISTURBANCE AND GRAVITY ANOMALY

Let $\vec{\gamma}_P$ and $\vec{g}_P$ and be, respectively, the *normal gravity vector* and the Earth's *gravity vector* (corrected from non-gravitational effects due to vehicle motion and time variations such as Earth tides and instrumental drift), both located at a point P. In this case, the gravity vector $\vec{g}_P$ is the gradient of the scalar *gravity potential*, which is the sum of a gravitational potential and a centrifugal potential. Similarly, the normal gravity vector $\vec{\gamma}_P$ is the gradient of the scalar *normal potential*, which is also the sum of a gravitational and a centrifugal potential. By definition, the centrifugal part of the normal potential and the gravity potential are equal.

The difference between $\vec{g}_P$ and $\vec{\gamma}_P$, which we emphasize are located at the same point P, is the *gravity disturbance vector* (Fig. 2):

$$\delta\vec{g}_P = \vec{g}_P - \vec{\gamma}_P . \tag{2}$$

Because the centrifugal parts of both the normal gravity vector and the Earth's gravity vector are equal, the gravity disturbance vector $\delta\vec{g}_P$ represents a purely gravitational effect. As a consequence of the superposition principal (Blakely 1996), this effect is caused by contrasts between the internal density distributions of the actual Earth and the normal Earth. In applied geophysics, these density differences are generally called *anomalous masses* (e.g., Hammer 1945; LaFehr 1965), *density anomalies* (e.g., Forsberg 1984), or *gravity sources* (e.g., Blakely 1996). Here, we opted for using the last term.

The difference between the magnitudes of the gravity vector $g_P = \|\vec{g}_P\|$ and the normal gravity vector $\gamma_P = \|\vec{\gamma}_P\|$, at the same point P, is the *gravity disturbance* (Heiskanen & Moritz 1967; Hofmann-Wellenhof & Moritz 2005):

$$\delta g_P = g_P - \gamma_P . \tag{3}$$

Notice that δg_P is not equivalent to the magnitude of the gravity disturbance vector $\delta\vec{g}_P$ (Barthelmes 2013; Sansò & Sideris 2013).

The *gravity anomaly vector* is the difference between the gravity vector at a point Q' on the geoid (a particular equipotential surface of the gravity potential) and the normal gravity vector at a point Q on the surface of the ellipsoid, both located at the same geodetic latitude and longitude (Fig. 2):

$$\Delta\vec{g} = \vec{g}_{Q'} - \vec{\gamma}_Q. \tag{4}$$

Similarly to the gravity disturbance (eq. 3), the *gravity anomaly* is given by

$$\Delta g = g_{Q'} - \gamma_Q, \tag{5}$$

where $g_{Q'}$ is the magnitude of the gravity vector on the geoid, and γ_Q is the magnitude of the normal gravity vector on the reference ellipsoid (Fig. 2).

The gravity anomaly is not defined at a particular point. Instead, as properly pointed out by Barthelmes (2013), it depends only on longitude and latitude and is not a function of height. Because gravity and normal gravity are located at different points, the gravity anomaly is a combination of gravitational and centrifugal effects, rather than purely gravitational effects like the gravity disturbance. Consequently, its upward continuation, for example, cannot be rigorously done because it is not a harmonic function. However, upward continuation of gravity anomalies is commonly

seen in the literature. In doing so, an implicit assumption is being made that the gravity anomaly is an approximation of the gravity disturbance, which in turn can be represented by a harmonic function and can be upward continued.

Different approximations for the gravity anomaly can be calculated depending on the corrections applied to them. These corrections are usually called *gravity reductions*. The *free-air anomaly*, for example, can be defined as (Blakely 1996; Hofmann-Wellenhof & Moritz 2005):

$$\Delta g^F = g_P - \gamma_Q - \frac{\partial\gamma}{\partial h}H_P , \tag{6}$$

where the rightmost term is called the *free-air correction*, $\partial\gamma/\partial h \approx -0.3086$ mGal/m is the derivative of the normal gravity with respect to the geometric height h, and H_P is the orthometric height (i.e., with respect to the geoid) of point P (Fig. 2). The free-air correction can be interpreted as a downward continuation of g_P to the geoid or as an upward continuation of γ_Q to a height H_P. Regardless, we can also apply the free-air correction to calculate the gravity disturbance by approximating the normal gravity γ_P at a point P on the Earth's surface from the normal gravity at the ellipsoid γ_Q:

$$\delta g_P \approx g_P - \left(\gamma_Q + \frac{\partial\gamma}{\partial h}h_P\right) . \tag{7}$$

where h_P is the geometric height of point P. However, using the free-air correction to calculate the disturbance is discouraged because an analytical expression exists for calculating γ at any point on or above the ellipsoid (Li & Götze 2001).

Using eqs. 6 and 7, we can estimate the absolute difference between the gravity disturbance and the free-air anomaly:

$$\left|\delta g_P - \Delta g^F\right| \approx \left|\frac{\partial\gamma}{\partial h}N\right| , \tag{8}$$

where $N \approx h - H$ is the geoidal height (Fig. 2). This approximation assumes that the geoid and the surface of the reference ellipsoid are close to being parallel at P and the surrounding area. Eqs 7 and 8 are commonly used in geodesy to define the *fundamental equation of physical geodesy* (Hofmann-Wellenhof & Moritz 2005).

We know empirically that N is $\approx \pm 1$ m on the oceans and reaches a maximum absolute value of ≈ 120 m (e.g., Torge & Müller 2012; Sansò & Sideris 2013). Fig. 3 shows the absolute difference between the gravity disturbance and the free-anomaly globally calculated using eq. 8 and the spherical harmonic model EIGEN-6c4 (Förste et al. 2014). In Brazil, for example, the geoidal height varies between $\approx \pm 30$ m (IBGE 2015). Consequently, the maximum absolute difference between the gravity disturbance and the free-air anomaly in Brazil, according to eq. 8, is ≈ 9.3 mGal.

5 THE EFFECT OF TOPOGRAPHIC MASSES

The *Bouguer correction* can be applied to the free-air anomaly to remove the gravitational effect of the topographic masses between the observation point P and the point on the geoid Q'. The *Bouguer anomaly* is defined in the continents as (Blakely 1996; Hofmann-Wellenhof & Moritz 2005):
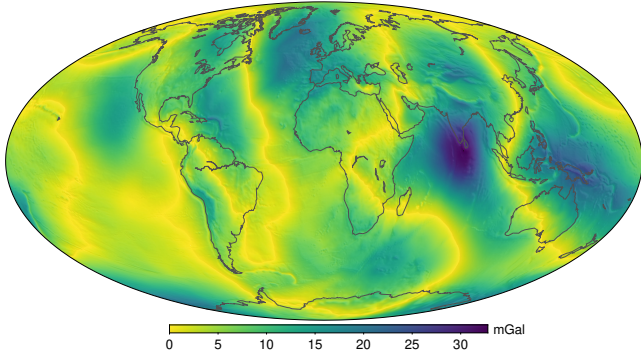
Figure 3. Absolute difference between the gravity disturbance and free-air anomaly. Calculated using eq. 8, the EIGEN-6c4 (Förste et al. 2014) spherical harmonic model, and software *Fatiando a Terra* (Uieda et al. 2013). The figure was made using GMT (Wessel et al. 2013).

$$\Delta g^B = g_P - \gamma_Q - \frac{\partial \gamma}{\partial h} H_P - 2\pi G \rho_t H_P \ , \qquad (9)$$

where ρ_t is the density of the topographic masses and G is the gravitational constant. The Bouguer correction approximates the gravitational effect that all topographic masses above the geoid exert on point P by the effect of a homogeneous, infinitely extended slab of constant density ρ_t and thickness H_P. A similar correction can be applied to the gravity disturbance (using the approximation in eq. 7) to remove the effect of topographic masses between the observation point P and a point on the ellipsoid Q. We will call this quantity the *Bouguer disturbance*:

$$\delta g_P^B \approx g_P - \left(\gamma_Q + \frac{\partial \gamma}{\partial h} h_P\right) - 2\pi G \rho_t h_P \ . \qquad (10)$$

At first glance, the only difference between eqs. 9 and 10 is the use of orthometric (H_P) versus geometric (h_P) heights. However, their geophysical interpretations are significantly different. Apart from the centrifugal effect mentioned in the previous section, the Bouguer anomaly (eq. 9) still contains the gravitational effect of topographic masses between the geoid and ellipsoid. Conversely, when calculating the Bouguer disturbance one removes the effects of an ellipsoidal reference model (the normal Earth) and the known topographic masses between the surface of the ellipsoid and observation point.

There was a certain lack of comprehension regarding the geophysical meaning of gravity anomalies until the mid 90s. As properly pointed out by Chapin (1996), "although the corrections which bring about a Bouguer gravity anomaly are well established, the reasons for doing them are not well understood. One cause of this common misunderstanding is that the subject has been poorly presented in many of the basic texts". In his seminal book, Blakely (1996) shed some light on the geophysical meaning of gravity anomalies from the perspective of applied geophysics and their relationship to the gravity sources. We complement this interpretation by stressing the fact that gravity anomalies reflect not only the gravity sources, but also a combination of gravitational and centrifugal effects. These additional, non-harmonic and undesired effects are due to the calculation of the normal gravity at a point other than that were gravity is measured.

6   GRAVITY MODELLING

We have established that the gravity disturbance is due solely to the gravitational effect of the gravity sources. Now we will discuss how this relates to some assumptions made during gravity modelling.

The gravity vector $\vec{g}_i$ (eq. 2) at a point (x_i, y_i, z_i) in the TCS (Fig. 1c) can be represented by:

$$\vec{g}_i = \vec{\gamma}_i + \delta\vec{g}_i \ . \qquad (11)$$

Furthermore, the gravity g_i can be approximated by a first order Taylor's expansion (Sansò & Sideris 2013):

$$g_i \approx \gamma_i + \hat{\vec{\gamma}}_i^\top \delta\vec{g}_i \ , \qquad (12)$$

where $^\top$ denotes transposition and $\hat{\vec{\gamma}}_i$ is a unit vector with the direction of the normal gravity vector. This approximation is valid because $\gamma_i \gg \|\delta\mathbf{g}_i\|$ at all points on or above the Earth's surface. It is known in geodesy (e.g., Sansò & Sideris 2013) and is analogous to the approximation used for total-field magnetic anomalies (e.g., Blakely 1996). In local or regional scales, the unit vector $\hat{\vec{\gamma}}_i$ may be considered constant throughout the study area and parallel to the z axis of the TCS (Fig. 1c).

Using eq. 12 and fixing γ at a point P at the origin of the TCS (Fig. 1c), the gravity disturbance can be rewritten as:

$$\delta g_i \approx \hat{\vec{\gamma}}_P^\top \delta\vec{g}_i \ . \qquad (13)$$

This equation shows that the gravity disturbance δg_i is approximately the component of the $\delta\vec{g}_i$ in the direction of the normal gravity vector $\vec{\gamma}_P$ (Hofmann-Wellenhof & Moritz 2005; Sansò & Sideris 2013). Because z-axis of the TCS is parallel to $\vec{\gamma}_P$ (Fig. 2), the gravity disturbance can be defined as the z-component of the gravitational attraction exerted by the gravity sources at the point (x_i, y_i, z_i). As a consequence, the gravity disturbance can be modelled by the harmonic function:

$$d_i = \iiint\limits_v \frac{G \, \Delta\rho(x', y', z') \, [z_i - z']}{\sqrt{[x_i - x']^2 + [y_i - y']^2 + [z_i - z']^2}} \, dv' \ , \quad (14)$$

where $\Delta\rho(x', y', z')$ is the density contrast at a point (x', y', z') within the volume v and the integration is conducted over x', y' and z'.

In practice, most instances of gravity modelling in the literature use Bouguer anomalies and function d_i (usually presented as "g_z") to represent the gravitational effect of gravity sources. Consequently, these studies are using the gravity anomaly as an approximation for the gravity disturbance. Notice that d_i does not depend on the height with respect to the geoid (orthometric height). Rather, it depends on the relative position of the gravity sources and the observation point (x_i, y_i, z_i) in the TCS (Fig. 1c). Finally, it is important to mention that gravimeters do not measure d_i but $\|\vec{g}_i\|$.

7   CONCLUSIONS

Although the gravity disturbance is well-known in geodesy, its use in applied geophysics is uncommon. For years, geophysicists have been using gravity anomalies for modelling the gravitational effects of geological bodies. Whether geophysicists should use the gravity anomaly or the gravity disturbance is of more than mere academic

interest. It is a question that underpins the very assumptions about what it is that we are actually modelling.

The gravity anomaly is the difference between gravity on the geoid and normal gravity on the reference ellipsoid. The gravity disturbance is the difference between gravity and normal gravity at the same point. Because of this, the gravity disturbance is a purely gravitational effect, whereas the gravity anomaly is a combination of gravitational and centrifugal effects. Consequently, the gravity disturbance is a harmonic function while the gravity anomaly is not. Being a harmonic function is the premise required by most techniques for processing potential-field data (e.g., upward/downward continuation, operations with the equivalent layer, conversions between gravity and magnetic data, computation of vertical derivatives). Hence, the gravity disturbance is the logical choice for representing the gravitational effects produced by the heterogeneous density distribution of the Earth.

More often than not, gravity modelling is done by computing the vertical component of the gravitational acceleration exerted by the sources. We have shown that, in doing so, one is actually computing the gravity disturbance while attempting to fit a gravity anomaly. This is a clear inconsistency. Doing so may even result in wrong interpretations of a quantitative nature. In Brazil, for example, the maximum difference between the gravity disturbance and the free-air anomaly reaches ≈ 9.3 mGal. This is approximately the maximum difference between the vertical gravitational accelerations of a sphere with depth-to-center of 700 m and one with 1550 m, assuming both spheres have a radius of 699 m and density contrast of $600 \ \text{kg} \cdot \text{m}^{-3}$.

In practice, most of these issues can be nullified during data processing by substituting orthometric heights (i.e., with respect to the geoid) with geometric heights (i.e., with respect to the ellipsoid). For modern acquisitions, this requires no extra work because GPS already provide geometric heights. The free-air correction can even be bypassed entirely by computing γ_P directly. As a bonus, the gravity disturbance is conceptually simpler to understand and doesn't require the concept of the geoid with which learners often struggle.

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REFERENCES

Barthelmes, F., 2013. Definition of functionals of the geopotential and their calculation from spherical harmonic models.

Baumann, H., Klingelé, E., & Marson, I., 2012. Absolute airborne gravimetry: a feasibility study, *Geophysical Prospecting*, **60**(2), 361–372.

Blakely, R. J., 1996. *Potential Theory in Gravity and Magnetic Applications*, Cambridge University Press.

Bouman, J., Ebbing, J., & Fuchs, M., 2013. Reference frame transformation of satellite gravity gradients and topographic mass reduction, *Journal of Geophysical Research: Solid Earth*, **118**(2), 759–774.

Chapin, D. A., 1996. The theory of the bouguer gravity anomaly: A tutorial, *The Leading Edge*, **15**(5), 361–363.

Fairhead, J. D., Green, C. M., & Blitzkow, D., 2003. The use of gps in gravity surveys, *The Leading Edge*, **22**(10), 954–959.

Forsberg, R., 1984. A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modelling, Tech. rep., DTIC Document.

Förste, C., Bruinsma, S., Abrikosov, O., Lemoine, J.-M., Marty, J. C., Flechtner, F., Balmino, G., Barthelmes, F., & Biancale, R., 2014. EIGEN-6C4 The latest combined global gravity field model including GOCE data up to degree and order 2190 of GFZ Potsdam and GRGS Toulouse, http://doi.org/10.5880/icgem.2015.1.

Glennie, C. L., Schwarz, K. P., Bruton, A. M., Forsberg, R., Olesen, A. V., & Keller, K., 2000. A comparison of stable platform and strapdown airborne gravity, *Journal of Geodesy*, **74**(5), 383–389.

Hackney, R. I. & Featherstone, W. E., 2003. Geodetic versus geophysical perspectives of the gravity anomaly, *Geophysical Journal International*, **154**(1), 35–43.

Hammer, S., 1945. Estimating ore masses in gravity prospecting, *Geophysics*, **10**(1), 50–62.

Heiskanen, W. A. & Moritz, H., 1967. *Physical Geodesy*, W.H. Freeman and Company.

Hinze, W. J., Aiken, C., Brozena, J., Coakley, B., Dater, D., Flanagan, G., Forsberg, R., Hildenbrand, T., Keller, G. R., Kellogg, J., Kucks, R., Li, X., Mainville, A., Morin, R., Pilkington, M., Plouff, D., Ravat, D., Roman, D., Urrutia-Fucugauchi, J., Véronneau, M., Webring, M., & Winester, D., 2005. New standards for reducing gravity data: The north american gravity database, *Geophysics*, **70**(4), J25–J32.

Hofmann-Wellenhof, B. & Moritz, H., 2005. *Physical Geodesy*, Springer.

IBGE, 2015. O novo modelo de ondulação geoidal do brasil: Mapgeo2015, Tech. rep., Instituto Brasileiro de Geografia e Estatística (IBGE).

LaFehr, T. R., 1965. The estimation of the total amount of anomalous mass by gauss's theorem, *Journal of Geophysical Research*, **70**(8), 1911–1919.

LaFehr, T. R., 1991. Standardization in gravity reduction, *Geophysics*, **56**(8), 1170–1178.

Li, X. & Götze, H.-J., 2001. Ellipsoid, geoid, gravity, geodesy, and geophysics, *Geophysics*, **66**(6), 1660–1668.

Marussi, A., Moritz, H., Rapp, R. H., & Vicente, R. O., 1974. Ellipsoidal density models and hydrostatic equilibrium: Interim report, *Physics of the Earth and Planetary Interiors*, **9**(1), 4–6.

Nabighian, M. N., Ander, M. E., Grauch, V. J. S., Hansen, R. O., LaFehr, T. R., Li, Y., Pearson, W. C., Peirce, J. W., Phillips, J. D., & Ruder, M. E., 2005. Historical development of the gravity method in exploration, *GEOPHYSICS*, **70**(6), 63ND–89ND.

eds Sansò, F. & Sideris, M. G., 2013. *Geoid Determination*, vol. 110 of **Lecture Notes in Earth System Sciences**, Springer Berlin Heidelberg, Berlin, Heidelberg.

Soler, T., 1976. On differential transformations between cartesian and curvilinear (geodetic) coordinates, Tech. rep., Ohio State University.

Torge, W. & Müller, J., 2012. *Geodesy*, de Gruyter, 4th edn.

Uieda, L., Oliveira Jr, V. C., & Barbosa, V. C. F., 2013. Modeling the Earth with Fatiando a Terra, in *Proceedings of the 12th Python in Science Conference*, pp. 96 – 103.

Vaníček, P. & Krakiwsky, E. J., 1987. *Geodesy: The Concepts, Second Edition*, Elsevier Science.

Wessel, P., Smith, W. H. F., Scharroo, R., Luis, J., & Wobbe, F., 2013. Generic Mapping Tools: Improved Version Released, *Eos, Transactions American Geophysical Union*, **94**(45), 409–410.