# ARTIFICIAL INTELLIGENCE (AI)
# FUZZY LOGIC

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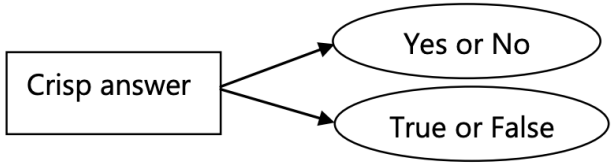
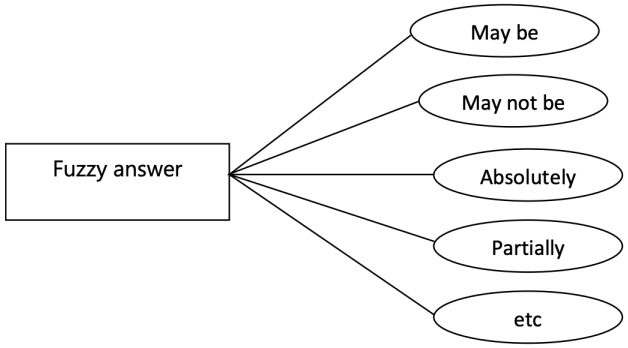
# FUZZY LOGIC

- Dictionary meaning of fuzzy is not clear, noisy etc
  - Example: Is the picture on this slide is fuzzy?

# FUZZY LOGIC





Our world is better described with “Fuzzily”!

# MEMBERSHIP FUNCTION

- Imagine you want to describe how **hot** a day is.
- Classical logic: Yes|No. Fuzzy logic: allows us to say how **hot** the day is on a scale from 0 to 1.

  - If the temperature is **10°C or below**, it's **not hot** at all, so the membership value is 0.
  - If the temperature is **30°C or above**, it's **definitely hot**, so the membership value is 1.
  - For temperatures between **10°C and 30°C**, it's somewhere in between:For 20°C, it might be **0.5** hot (partially hot).
  - For 25°C, it might be **0.75** hot.

# RULES FOR EVALUATING COMPLEX STATEMENTS

- AND condition.
  - Imagine you are deciding whether to go for a walk. You want to go if it is **warm AND not raining**.
  - If it is **70% warm** and **40% not raining**, how comfortable would you feel about going for a walk?
  - *You would feel **only 40% comfortable** because even though it's somewhat warm, there's a bigger chance of rain.*
  - *the strength of this "truth" depends on the **weaker** condition, because both have to be satisfied.*

# RULES FOR EVALUATING COMPLEX STATEMENTS

- OR condition.
  - You want to decide whether to relax. You will relax if it is either **sunny OR** you have **free time**. If it is **80% sunny** and you have **30% free time**, how much would you relax?
  - *You would likely relax **80%** because it's mostly sunny, even though you have little free time.*
  - *the strength of this "truth" depends on the stronger condition.*

# Fuzzy Logic

↝ Fuzzy set theory is a means of specifying how well an object satisfies a vague description.

Truth value is b/w 0 and 1.

# Rules for evaluating fuzzy truth, T, of a complex sentence are

(1) $T(A \wedge B) = \min(T(A), T(B))$

(2) $T(A \vee B) = \max(T(A), T(B))$

(3) $T(\neg A) = 1 - T(A)$

# Example: Cardiac Health Management

## Fuzzy Rules

1. Diet is low AND Exercise is high ⇒ Balanced
2. Diet is high OR Exercise is low ⇒ Unbalanced
3. Balanced ⇒ Risk is low
4. Unbalanced ⇒ Risk is high

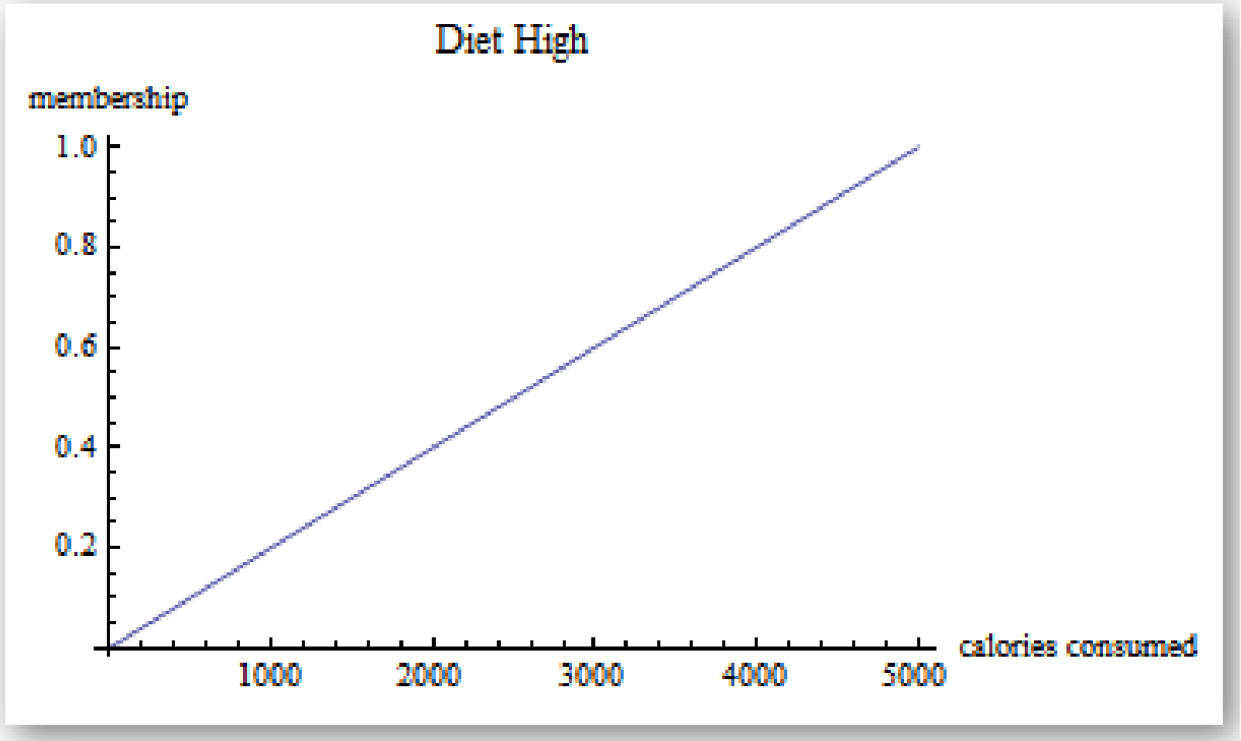
For a person it is given that:

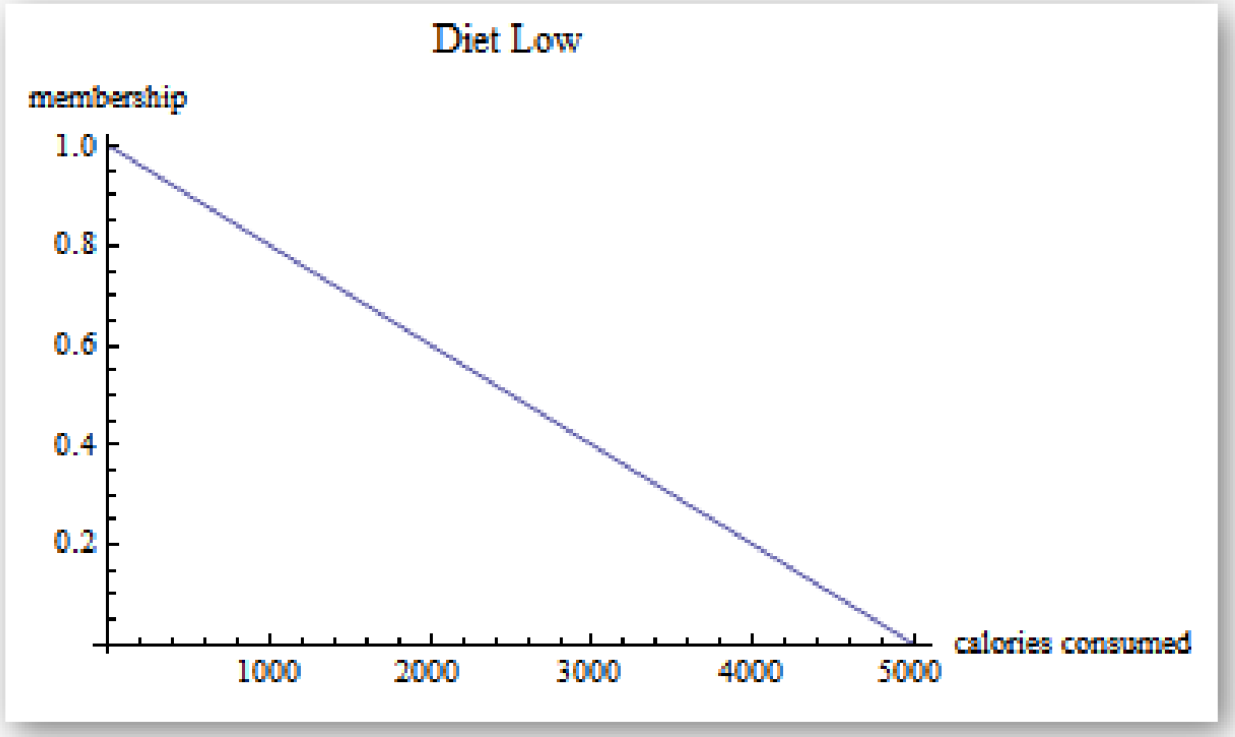
- Diet = 3000 calories per day
- Exercise = burning 1000 calories per day

**What is the risk of heart disease?**

One Sample input data

# Membership Functions



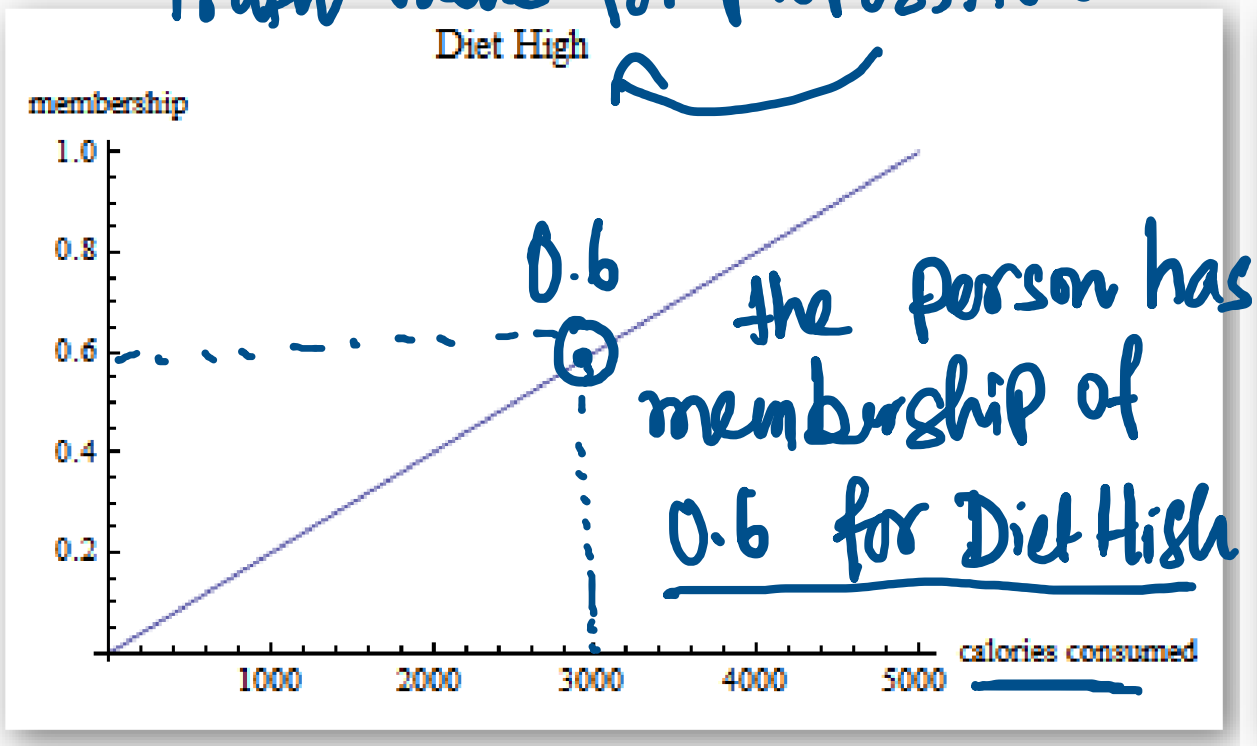



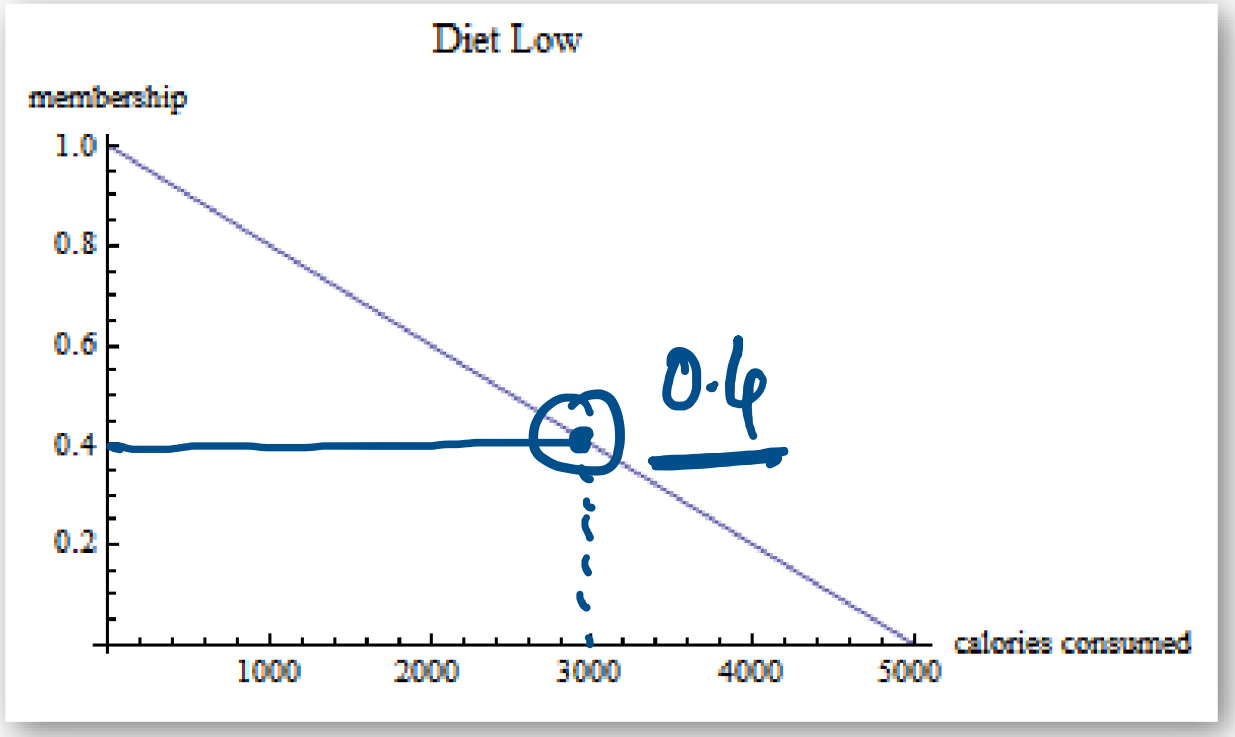

$$f_{diet\ high}(x) = \frac{1}{5000}x$$

$$f_{diet\ low}(x) = 1 - \frac{1}{5000}x$$

# Membership Functions

Truth value for proposition



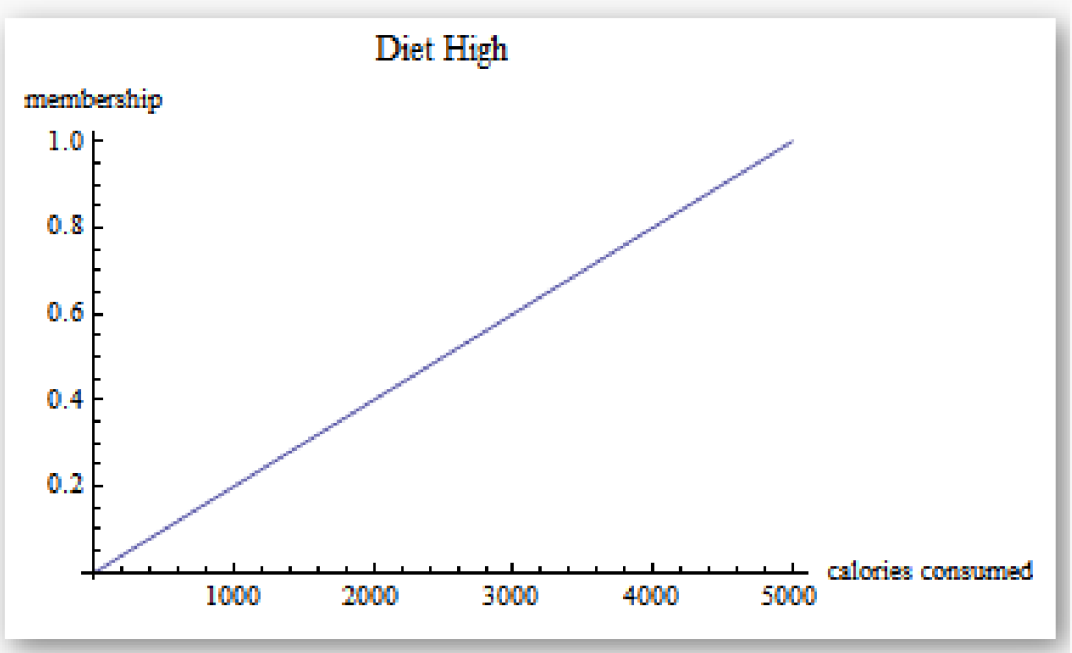



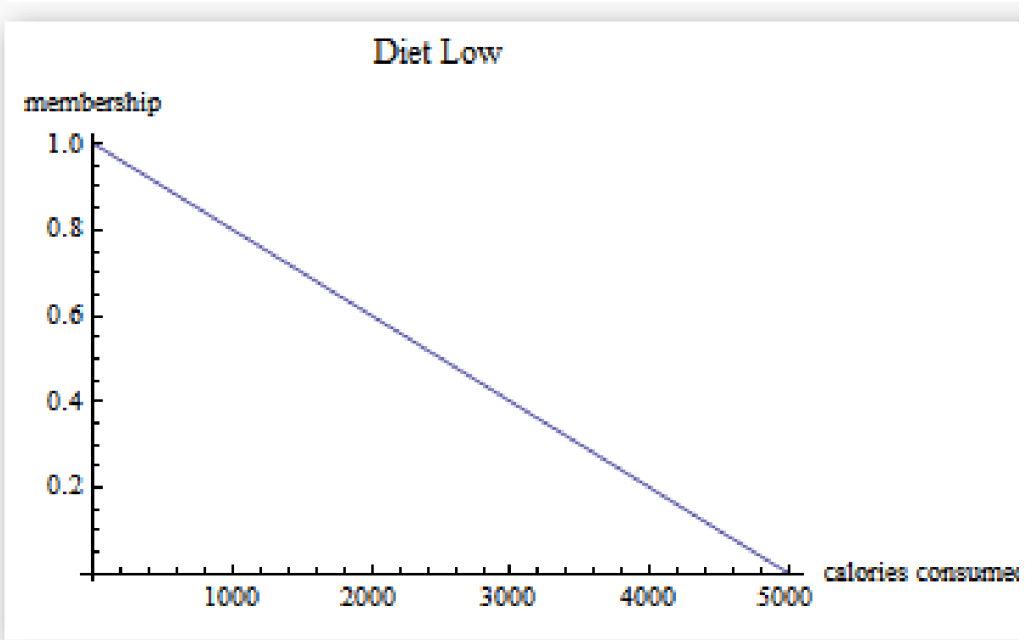

$$f_{diet\ high}(x) = \frac{1}{5000}x$$

negation of this

$$f_{diet\ low}(x) = 1 - \frac{1}{5000}x$$

# Membership Functions




$$f_{diet\ high}(x) = \frac{1}{5000}x$$

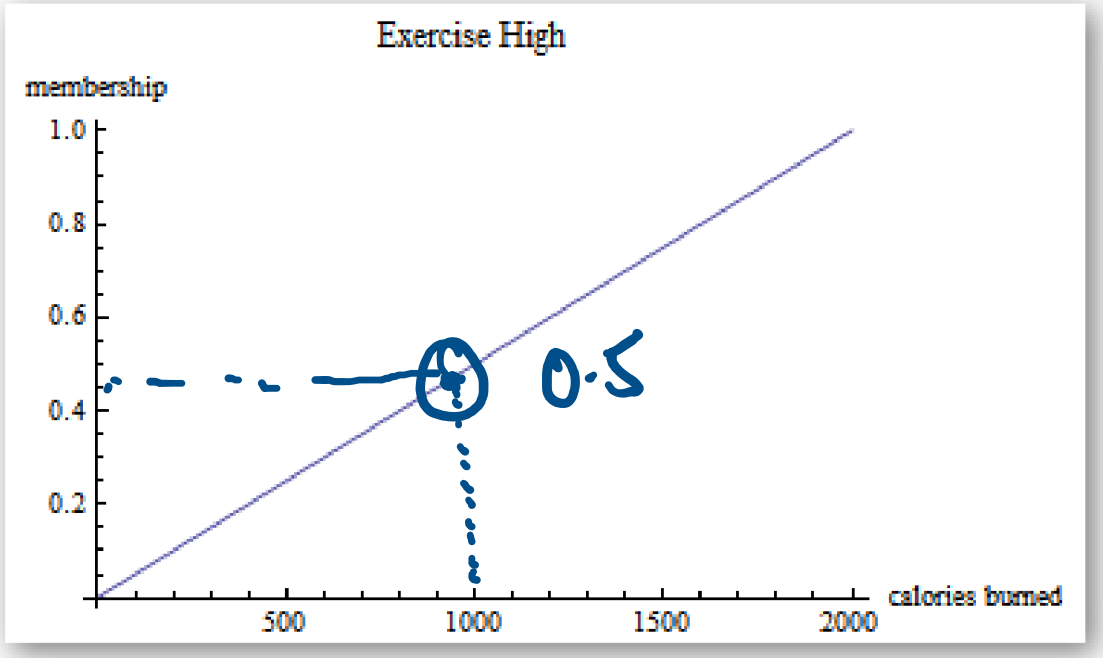
$$f_{diet\ low}(x) = 1 - \frac{1}{5000}x$$

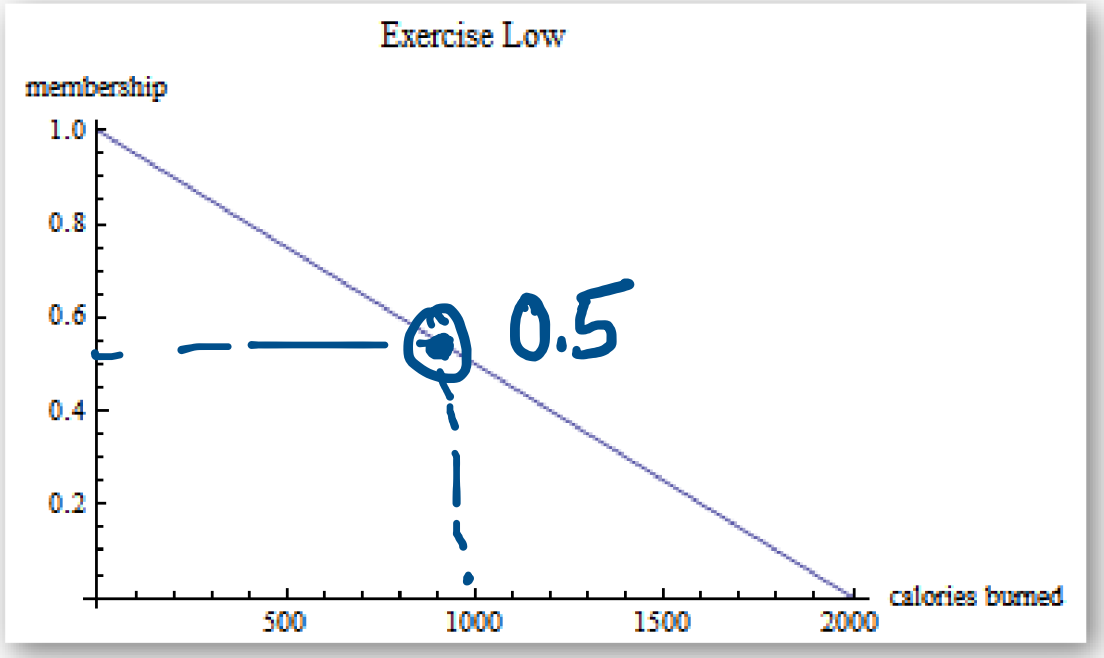
**For daily calorie intake of 3000:**

**Membership for Diet-High = 3000 / 5000 = 0.6**

**Membership for Diet-Low = 0.4**

# Membership Functions

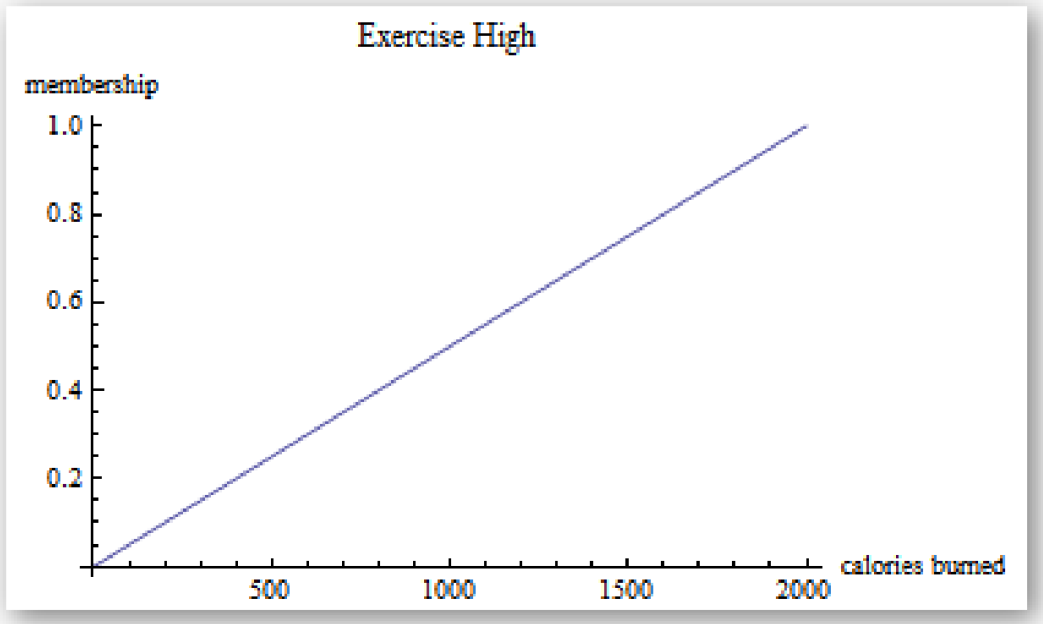


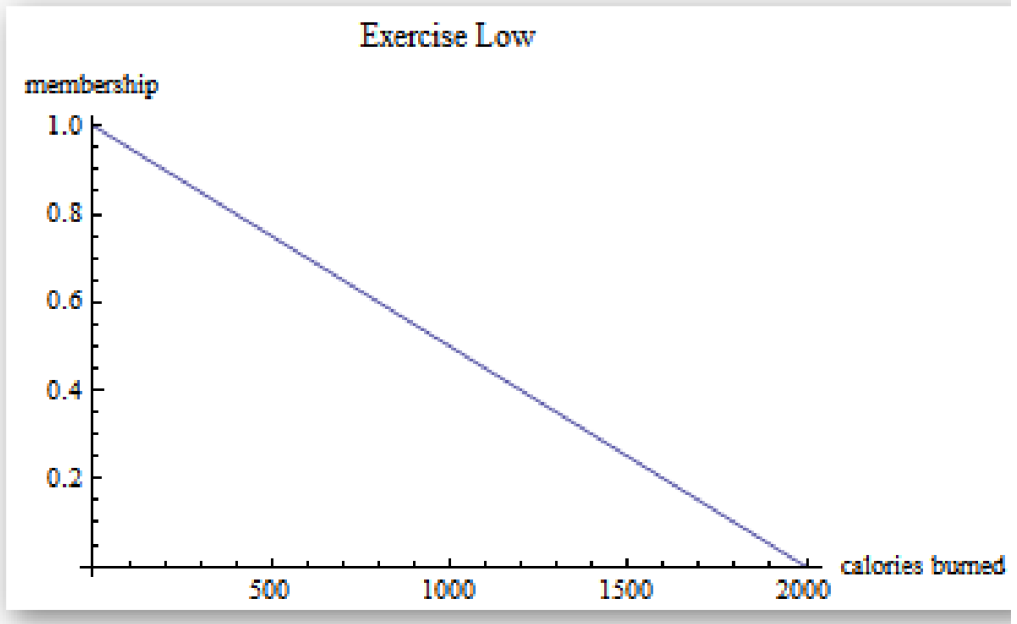




$$f_{exercise\ high}(x) = \frac{1}{2000}x$$

$$f_{exercise\ low}(x) = 1 - \frac{1}{2000}x$$

# Membership Functions




$$f_{exercise\ high}(x) = \frac{1}{2000}x$$

$$f_{exercise\ low}(x) = 1 - \frac{1}{2000}x$$

**For daily calorie burned of 1000:**

**Membership for Exercise-High = 1000 / 2000 = 0.5**

**Membership for Exercise-Low = 0.5**

# RULE EVALUATION

**Fuzzy Rules**

1. **Diet is low AND Exercise is high ⇒ Balanced**
2. **Diet is high OR Exercise is low ⇒ Unbalanced**
3. **Balanced ⇒ Risk is low**
4. **Unbalanced ⇒ Risk is high**

# Rule Evaluation

Truth( Diet-High ) = 0.6

Truth( Diet-Low ) = 0.4

Truth( Exercise-High ) = 0.5

Truth( Exercise-Low ) = 0.5

**Diet is low AND Exercise is high ⇒ Balanced**

- Truth( Balanced ) = min { Truth( Diet-Low ), Truth( Exercise-High ) } = min { 0.4, 0.5 } = 0.4

**Diet is high OR Exercise is low ⇒ Unbalanced**

- Truth( Unbalanced ) = max { Truth( Diet-High ), Truth( Exercise-Low ) } = max { 0.6, 0.5 } = 0.6

**Balanced ⇒ Risk is low**

- Truth( Risk-Low ) = Truth( Balanced ) = 0.4

**Unbalanced ⇒ Risk is high**

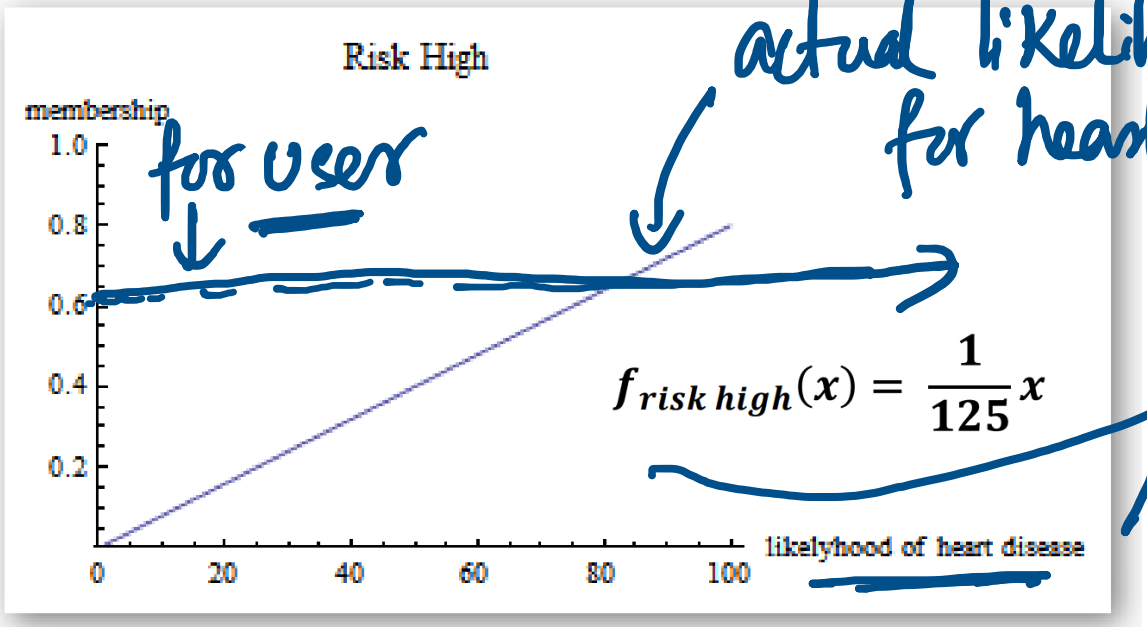
- Truth( Risk-High ) = Truth( Unbalanced ) = 0.6

Now from this Truth values, we need to get the actual Probability

↓

Next membership function

# Risk-High Evaluation




for user

actual likelihood for heart disease

Truth value (Risk-high) = 0.6

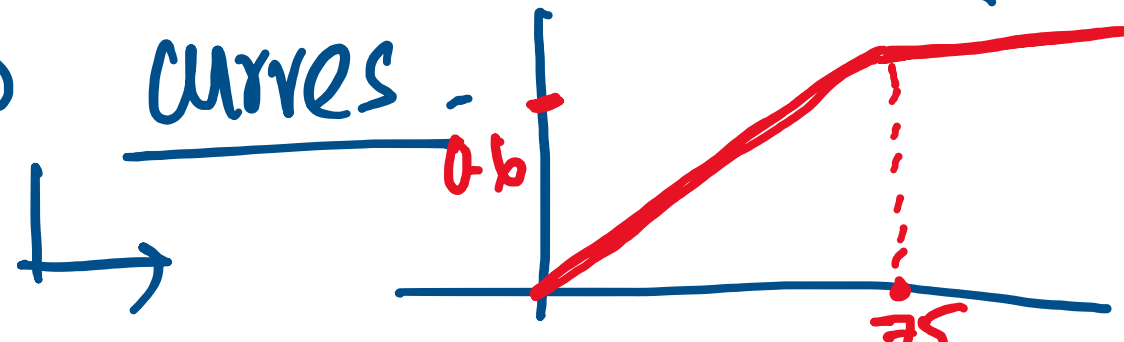
Given

So, $0.6 = \frac{x}{125}$

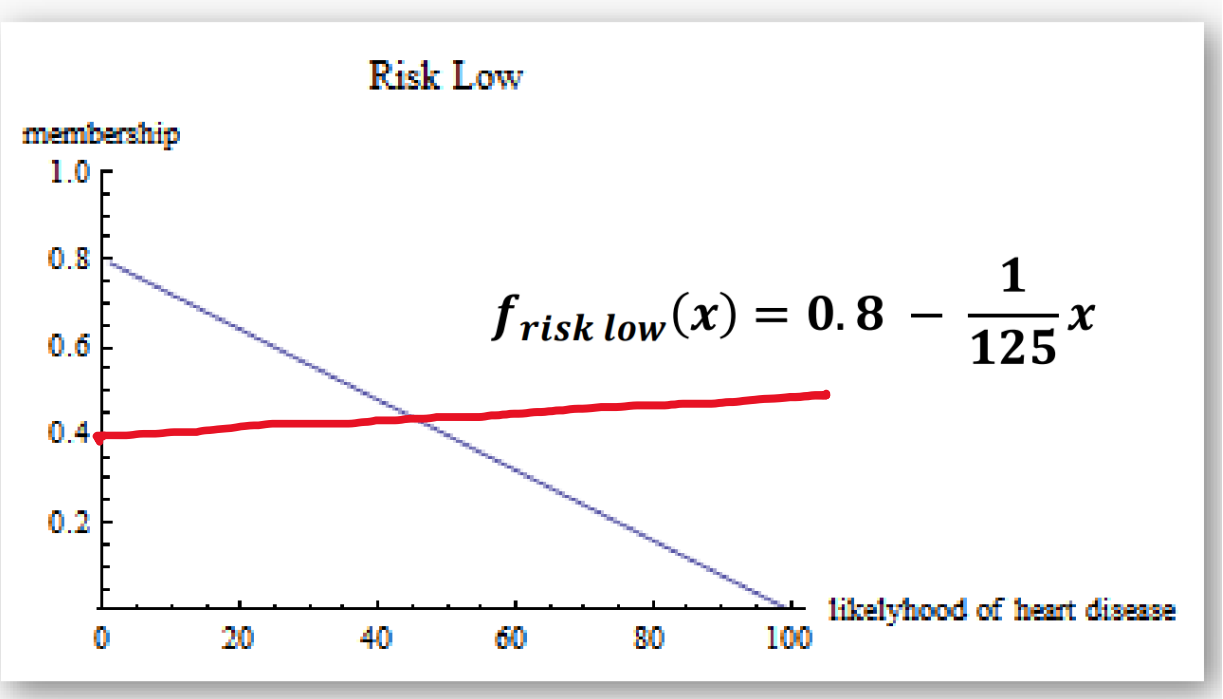
$x = 75$

Next need to see intersection of two curves.

We take MIN



# Risk-Low Evaluation

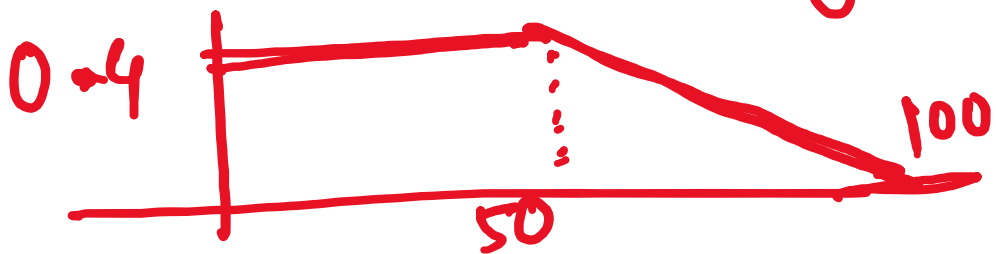



$T(\text{Risk low}) = 0.4$

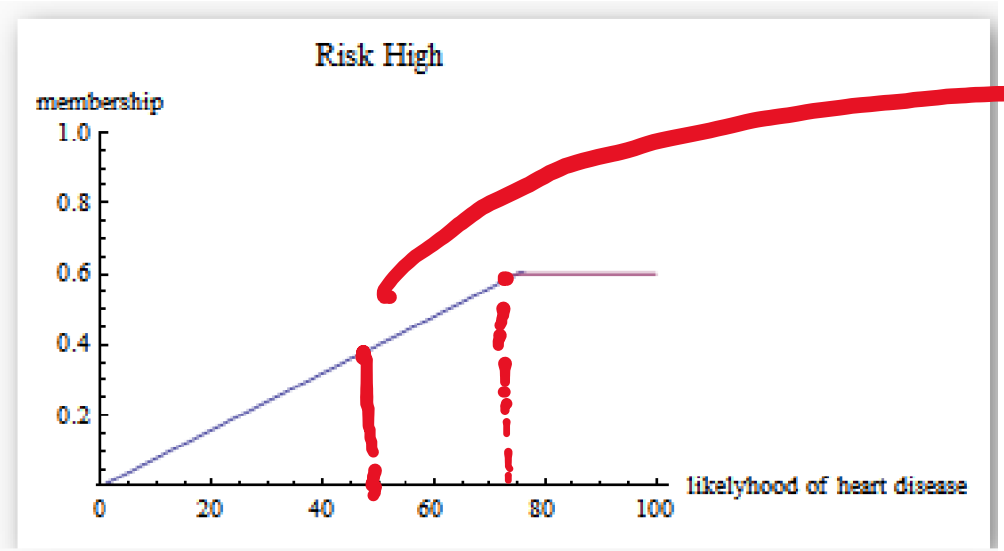
$0.4 = 0.8 - \frac{x}{125}$

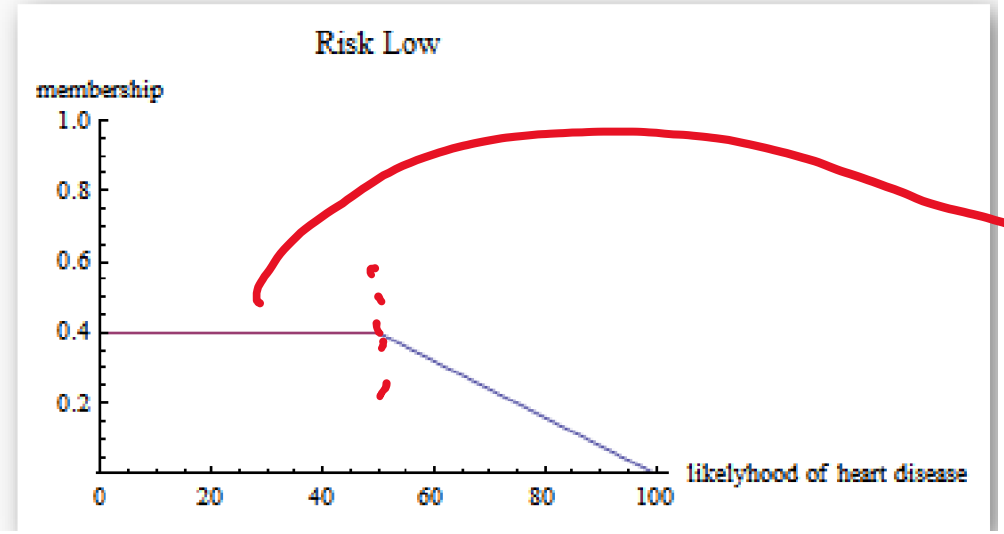
$x = 50$

Take intersection → Fuzzy logic this means MIN



# Aggregated Risk Function




Aggregated Risk

UNION

0.6

0.4

1

50

75

100

# Defuzzification

$$\int_0^{100} f_{aggregated\ risk}\, dx$$

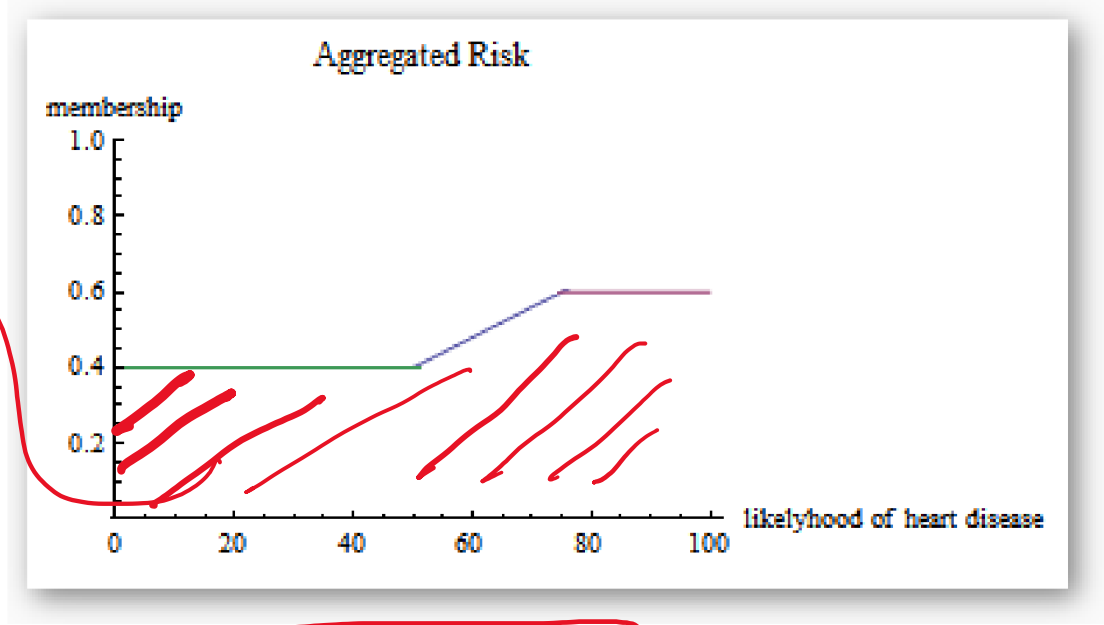
$$= \int_0^{50} 0.4\, dx + \int_{50}^{75} \frac{1}{125} x\, dx + \int_{75}^{100} 0.6\, dx$$

$$= 50 \times 0.4 + \frac{1}{125}\left[\frac{x^2}{2}\right]_{50}^{75} + 25 \times 0.6$$

$$= 20 + (75^2 - 50^2)/250 + 15$$

$$= 47.5$$

∴ the likelihood of a heart disease for the person is 47.5%




Area under this curve