MFAI 7

Lecture-10 (27/08/24)

Resolution in propositional logic

$\Rightarrow ((A \vee P) \wedge (B \vee \neg P)) \supset A \vee B$

$(A \vee P) \wedge (B \vee \neg P) \Rightarrow A \vee B$

$\downarrow$

$A \vee P,\ B \vee \neg P \Rightarrow A \vee B$

$\downarrow$

$A \vee P,\ B \vee \neg P \Rightarrow A, B$

$\swarrow \qquad \searrow$

$A, B \vee \neg P \Rightarrow A, B$

Contradiction

$P, B \vee \neg P \Rightarrow A, B$

$\swarrow \qquad \searrow$

$P, B \Rightarrow A, B$

Contradiction

$\boxed{P, \neg P} \Rightarrow A, B$

Contradiction

$A \vee P \quad B \vee \neg P$

resolution

$A \vee B$

DNF (Disjunctive Normal form)

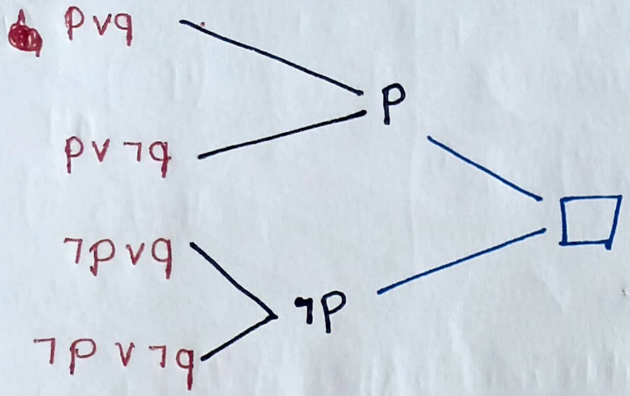
$\Rightarrow (p \vee q) \vee (\sim p \wedge \sim q)$

$\{(p \wedge q) \vee \neg p\} \wedge \{(p \vee q) \vee \neg q\}$

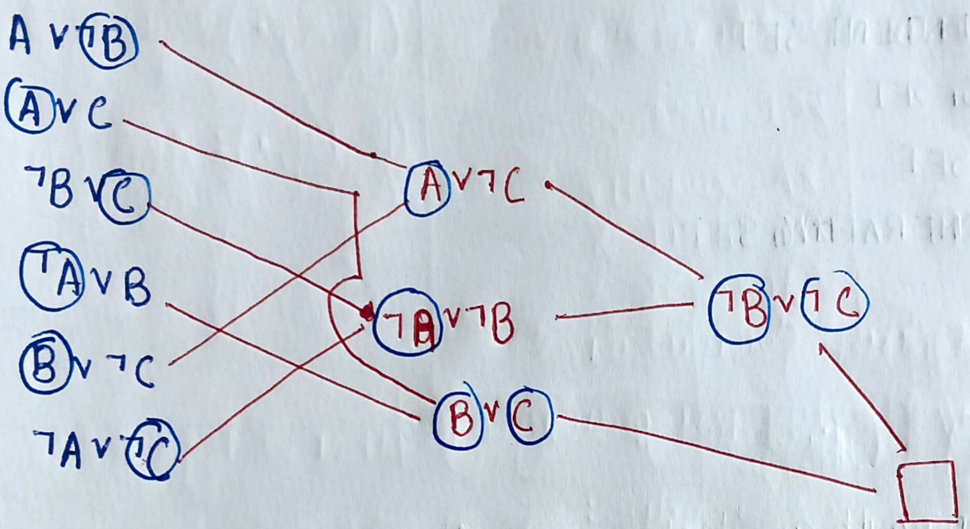
$\{\underbrace{(p \vee \neg p)}_{T} \wedge (q \vee \neg p)\} \wedge \{(p \vee \neg q) \wedge \underbrace{(q \vee \neg q)}_{T}\}$

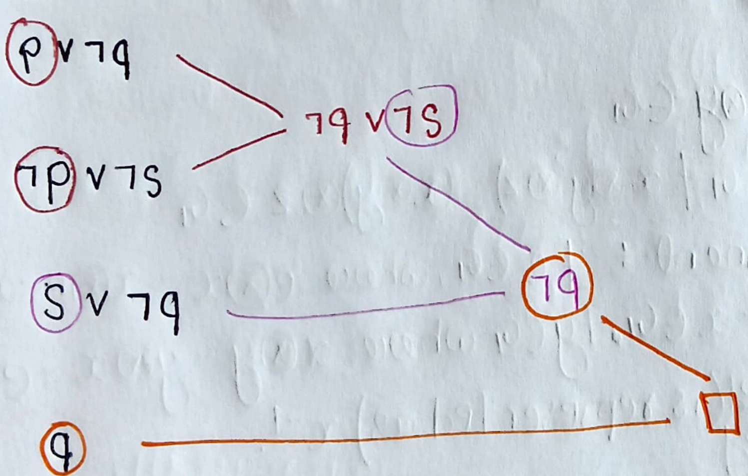
$(q \vee \neg p) \wedge (p \vee \neg q)$

CNF (Conjuctive Normal Form)



Robinson Resolution Principle





Lecture-11: MFAI (28/08/24)

- GROUP
- VECTOR SPACE
- GENERAL LINEAR ~~SPACE~~ GROUP
- LINEAR COMBINATION
- LINEAR INDEPENDENT SET
- —"— DEPENDENT SET
- GENERATING SET
- SPAN OF A SET
- MINIMAL GENERATING SET
- BASIS
- RANK

- Group

→ A set 'G' along with an algebraic operation $\circledast$ is called a group $(G, \circledast)$ if

1. Closure: $\forall x, y \in G \mid x \circledast y \in G$
2. Associative: $\forall x, y, z \in G \mid x \circledast (y \circledast z) = (x \circledast y) \circledast z \in G$
3. Neutral (Identity) Element: $\exists e \in G$, where $e \circledast x = x \circledast e = x \in G$
4. Inverse Element: $\forall x \in G, \exists y \in G$ where $x \circledast y = y \circledast x = e \in G$ then y is denoted by (is represented as) x^{-1}.

In addition if $\forall x, y \in G$, $x \circledast y = y \circledast x$ then the $(G, \circledast)$ is called commutative → Abelian Group

Example

$(\mathbb{Z},+)$: Abelian Group

$(\mathbb{Z},-)$: Not a group
- → Closure ✓
- → Associative ✗
- → Neutral ✗
- → Inverse ✗

$(\mathbb{N},+)$: Not a group
- → Closure ✓
- → Associative ✓
- → Neutral ✗
- → Inverse ✗
- → Commutative ✓

$(\mathbb{R},+)$: Abelian Group

4, 3, 5

$(4-3)-5$

$4-(3-5) = -4$

$4-(-2)$

$= 6$

$4-(3-5)$

$= 6$

$(\mathbb{R},-)$: Not a group

$(\mathbb{R},\cdot)$: Not a group

$(\mathbb{R}-\{0\},\cdot)$: Abelian Group

- **Vector space**

Let $\vec{x} \in \mathbb{R}^n$

Let $V = (\mathbb{R}^n,+)$

then V is a group

1. Closure : $\vec{x},\vec{y} \in \mathbb{R}^n$ then $\vec{x}+\vec{y} \in \mathbb{R}^n$
2. Associative : $\vec{x},\vec{y},\vec{z} \in \mathbb{R}^n$ then $(\vec{x}+\vec{y})+\vec{z} = \vec{x}+(\vec{y}+\vec{z}) \in \mathbb{R}^n$
3. Neutral : $\vec{0} \in \mathbb{R}^n$, $\vec{0}+\vec{x} = \vec{x}+\vec{0} = \vec{x} \in \mathbb{R}$
4. Inverse: $\vec{x} \in \mathbb{R}^n$, $\vec{y} = (-1)\vec{x} = -\vec{x}$

   where

   $$\vec{x}+\vec{y} = \vec{y}+\vec{x} = \vec{0}$$

In addition,

$$\forall \vec{x},\vec{y} \in \mathbb{R}^n, \vec{x}+\vec{y} = \vec{y}+\vec{x}$$

Commutative

Hence vector space $V = (\mathbb{R}^n,+)$ is a $\boxed{\text{Abelian}}$ Group.

In addition, if '$\cdot$' represents the multiplication of a scalar with a vector then

$$\forall \vec{x} \in \mathbb{R}^n,\ \alpha \cdot \vec{x} = (\overrightarrow{\alpha x}) \in \mathbb{R}^n$$

Hence, $V = (\mathbb{R}^n, +, \cdot)$

scalar with a vector

vector with a vector

General linear Group

Let $A \in \mathbb{R}^{N \times N}$

Lecture 11 (Part-B): MFA1

important!

$\boxed{\text{Prolog programming}}$

$\underline{ADD(X, 0, X)}$

$ADD(X, S(Y), S(Z)) \leftarrow ADD(X, Y, Z)$

$:- ADD(U, V, \underbrace{S(S(S(0)))}_{3})$

Find variable U, V such that $U + V = S(S(S(0)))$

$0 \rightarrow 0$
$S(0) \rightarrow 1$
$S(S(0)) \rightarrow 2$

$$\left.\begin{array}{l} 1.\ ADD(X, 0, X) \wedge \\ 2.\ ADD(X, Y, Z) \supset ADD(X, S(Y), S(Z)) \wedge \\ 3.\ \neg ADD(U, V, S(S(S(0)))) \end{array}\right\} \forall x \forall y \forall z$$

$$\equiv \{ ADD(X, 0, X) \wedge \{\neg ADD(X, Y, Z) \vee ADD(X, S(Y), S(Z))\} \wedge \neg ADD(U, V, S(S(S(0)))) \}$$

$ADD(x,0,x)$ $\neg ADD(x,y,z) \vee ADD(x, S(y), S(z))$

Unification

$y \to 0$

$x \to S(S(0))$

$\neg ADD(U, V, S(S(S(0))))$

(Unification Technique)

$U \to X$

$V \to S(y)$

$Z \to S(S(0))$

$\{\square\}$

$\neg ADD(x,y,z)$

$\downarrow$

$S(S(0))$

$U \to X \to S(S(0)) = 2$

$V \to S(y) \to S(0) = 1$

Unification:

- A $\boxed{\text{variable}}$ can be constant $\{X \to C\}$
- A $\boxed{\text{variable}}$ can be another variable $\{X \to Y\}$
- A $\boxed{\text{variable}}$ can be a function $\{X \to f(\ldots, y)\}$ and 'f' should not contain 'X'

{Refutation is computation}

$ADD(X,0,X)$ $\neg ADD(X,Y,Z) \vee ADD(X, S(Y), S(Z))$ $\neg ADD(U, V, S(S(S(0))))$

$\{\square\}$

$U \to X$

$Y \to 0$

$X \to S(S(S(0)))$

$U \to X \to S(S(S(0))) = 3$

$V \to 0 = 0$

- A function will unify with a same function $f(y) \to f(z)$

☑ Doubt: why not perform further?