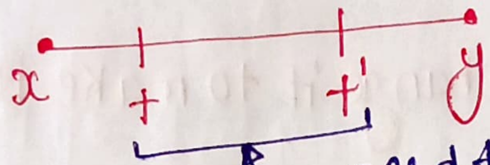
Reverse direction

Proof:

Assume $\nabla^2 f(\bar{x}) \geq 0 \ \forall \bar{x} \in S$

Take any $x, y \in S$

Proove complexity of f restricted to $L(\bar{x}, \bar{y})$



need to choose t and t' prime ??

Lecture-29 } missed
Lecture-30 }

Lecture-31

Constrained Convex Minimization

$\min_x f(x)$: f - convex function
$f: \mathbb{R}^n \to \mathbb{R}$

- find $\vec{x}$ satisfying
  $\nabla f(\vec{x}) = \vec{0}$

opposite of convex

- Every local minimum of f is a global minimum in convex

Converting to $\max f(x)$ f : Concave function opposite in concave

suffice to find x $\nabla f(\vec{x}) = \vec{0}$

Minimize $f(\vec{x})$ subject to ① $h_i(x) \leq 0$ $\forall i=1,\dots,m$

Langrage multiplier should be ≥ 0

② $g_j(x)=0$ $\forall j=1,\dots,p$

Convex set

$C = \{\bar{x} \mid \bar{x}$ satisfies ① and ② $\}$

= Set of all feasible solutions

▷ We are going to change it to make it unconstrainted

▷ C is a convex set provided each $h_i(x)$ is convex function and each $g_i(x)$ is an affine function.

Claim1:

$$\forall x \in C,\ f(x) = \max_{\substack{\vec{\alpha} \geq \vec{0},\\ \vec{\beta}}} L(\vec{x}, \vec{\alpha}, \vec{\beta})$$

Langrage multiplier

$$L(\vec{x}, \vec{\alpha}, \vec{\beta}) = f(x) + \sum_{i=1}^{m} \alpha_i h_i(x) + \sum_{j=1}^{p} \beta_i g_i(x)$$

$$= f(x) \langle \vec{\alpha} \cdot \vec{h}(\vec{x}) \rangle + \langle \vec{\beta} \cdot \vec{g}(\vec{x}) \rangle$$

$$\forall x \notin C \quad \max_{\vec{\alpha}, \vec{\beta}} L(\vec{x}, \vec{\alpha}, \vec{\beta})$$

$$\underbrace{\min_{x \in C} f(x) = \min_{x} \max_{\substack{\vec{\alpha} \geq \vec{0},\\ \vec{\beta}}} L(\vec{x}, \vec{\alpha}, \vec{\beta})}_{\text{Primal problem}}$$

$$\min_{x} \max_{\vec{\alpha} \geq \vec{0}, \vec{\beta}} L(\vec{x}, \vec{\alpha}, \vec{\beta}) \geq \max_{\vec{\alpha} \geq 0, \vec{\beta}} \boxed{\min L(\vec{x}, \vec{\alpha}, \vec{\beta})}$$

maximization problem over $\vec{\alpha}, \vec{\beta} \geq 0$

$$= \underbrace{\max_{\vec{\alpha} \geq \vec{0}, \vec{\beta}} \min_{\vec{x}} f(\vec{x}) + \vec{\alpha}\vec{h}(\vec{x}) + \vec{\beta} \cdot \vec{g}(\vec{x})}$$

Concave

(Lower bound)

Gradient becomes 0

This is known as Langrage's multiplier method.

① If 'f' is linear;

$\max_{\vec{\alpha} \geq \vec{0}, \vec{\beta}} L(\vec{x}, \vec{\alpha}, \vec{\beta})$ is a convex function of $\vec{\alpha}, \vec{\beta}$ over $\vec{\alpha} \geq 0$

② If each $f(x)$, $h_i(\vec{x})$ and $g_j(\vec{x})$ is linear

subject to $\sum_{i=1}^{n} a_{ij} x_j (\leq, =, \geq) b_i$

$i = 1, \cdots, m$

one of these three

$$f(x_1, x_2, x_3) = x_1 x_2 x_3 + e^{2x_1^2}$$

$$\Rightarrow \nabla f(x) = \boxed{x_2 x_3 + e^{2x_1^2} \cdot 4x_1 + x_1 x_3 + x_1 x_2}$$

after partial Derivative

$a = x_1 x_2$

$b = a x_3$

$c = x_1^2$

$d = e^c$

$f = b + d$

$$\frac{\partial f}{\partial b} = 1$$

$$\frac{\partial f}{\partial d} = 1$$

$$\frac{\partial f}{\partial c} = \frac{\partial f}{\partial d} \cdot \frac{\partial d}{\partial c} = e^c$$

$$\frac{\partial c}{\partial x_1} = 2x_1$$

$$\frac{\partial b}{\partial x_3} = a \; ; \; \frac{\partial b}{\partial a} = x_3$$

$$\frac{\partial f}{\partial x_3} = 1 \cdot \frac{\partial b}{\partial x_3}$$

$f : \mathbb{R}^n \rightarrow \mathbb{R}^3$

$g : \mathbb{R}^n \rightarrow \mathbb{R}$

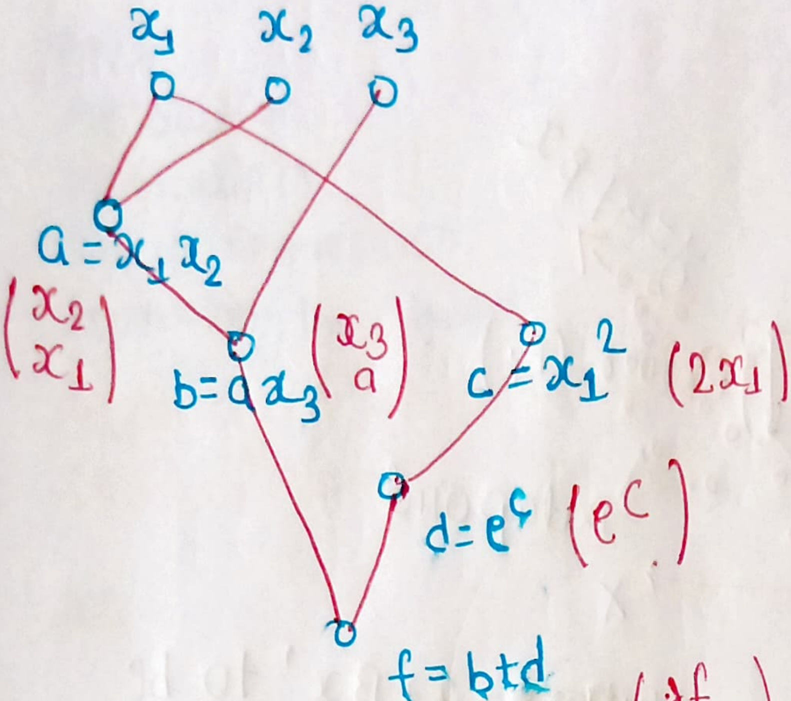
$h = g \circ f$

$= g(f(x))$

$h(x) =$

$\nabla g(f(x)) \cdot f'(x)$

$$= \nabla g(f(x)) \cdot \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$$



~~Last~~ Last Topic:

Quadratic Programming (Not noted down)

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial b} \\ \frac{\partial f}{\partial d} \end{pmatrix}$$

$b : \mathbb{R}^2 \to \mathbb{R}$

$a : \mathbb{R}^2 \to \mathbb{R}$

$b = a x_3$

$$\begin{pmatrix} \frac{\partial b}{\partial x_1} \\ \frac{\partial b}{\partial x_2} \\ \frac{\partial b}{\partial x_3} \end{pmatrix} = \begin{bmatrix} x_2 x_3 \\ x_1 x_3 \\ x_1 x_2 \end{bmatrix}$$

$b = a(x_1, x_2) \cdot x_3$

$\cancel{\nabla b = \nabla a\, x_3 + a(x_1}$

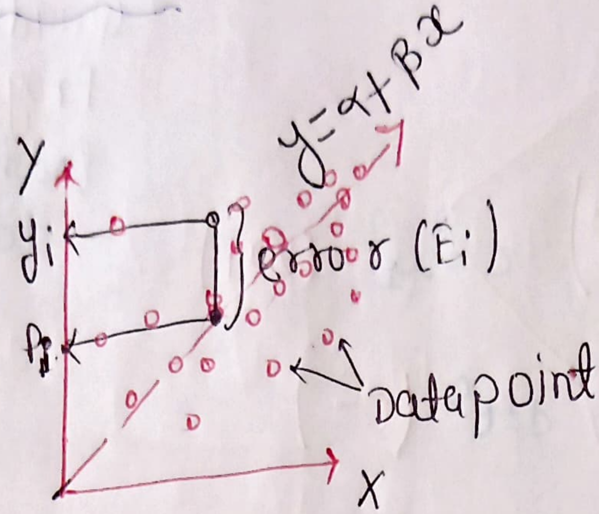
$$\frac{\partial b}{\partial x_1} = \frac{\partial a}{\partial x_1} \cdot x_3 + a \frac{\partial x_3}{\partial x_1} = x_2 x_3$$

$$\frac{\partial b}{\partial x_2} = \frac{\partial a}{\partial x_2} \cdot x_3 + a \frac{\partial x_3}{\partial x_2} = x_1 x_3$$

$$\frac{\partial b}{\partial x_3} = \frac{\partial a}{\partial x_3} \cdot x_3 + a \frac{\partial x_3}{\partial x_3} = x_1 x_2$$

L-31 (Part B)

Linear Regression



we want '$y = \alpha + \beta x$' to be the best fit line

$x = (x_1, x_2 \cdots x_n)$

$y = (y_1, y_2 \cdots y_n)$

$y_i = \alpha + \beta x_i + E_i$

random $E_i \sim N[0, \sigma^2]$

$\boxed{\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i}$ our model $\bar{y} + \hat{\beta}(x_i - \bar{x})$

Entertainment

Satapathy Sir: You don't write I will send PDF!

⚠: Important

PNF
Resolution
Unification
Linear Regression
Separating in clauses
Assignment : Gentzen's table method
(End of Month) ← Prolog