Lecture-16

Deterministic
- Present at certain state
- After a transition agent is (given rule) present at a certain state

[There is some uncertainity]

Reasoning under uncertainity

Agent is either block A or B or C

$\Rightarrow$ Present$(A \vee B \vee C)$

Additional Info:
Likelihood of
Block A >>
Block B
or
Block C

We cannot say probability or likehood of agent to present in a particular block

- We use probability to say about the likelihood during uncertainity

Probability Theory:

→ Gives us a quantitative way of encoding likelihood.

Axioms of Probability

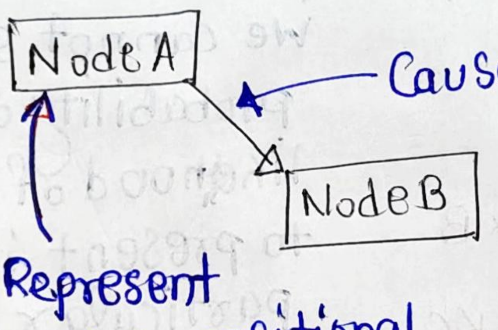
1. $0 \leq P(A) \leq 1$

2. $P(True) = 1 \rightarrow P(False) = 0$

3. $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$

$P(A) \leftarrow$ Probability of some [propositional ~~logi~~ formula 'A']

Let we have 'N' such propositions which will be used to represent the world



Causal link: A causes B

or

B is the reason for A

Represent some propositional

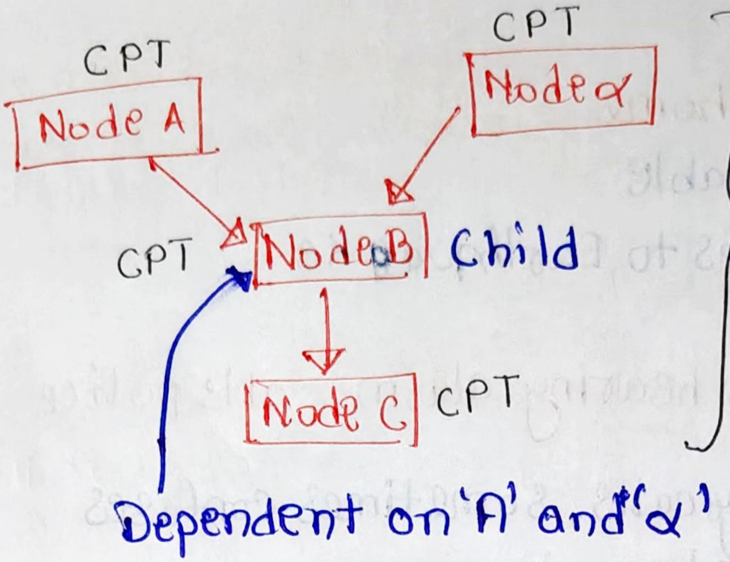
1. Node
2. Link/Edges

} structure of the domain ~~depends~~ in terms of dependencies b/w variables

3. Conditional Probability Table

↳ Some measurement or some likelihood for the child to be true

Gives the quantative value



Directed Acyclic Graph known as Bayesian Belief Network

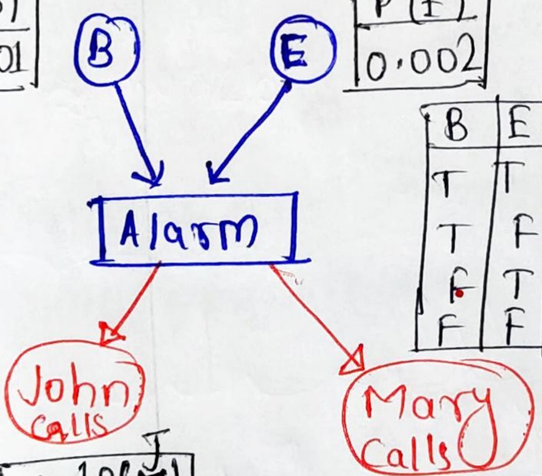
Hepatitis Example

$P(CH | VH) = 0.3$

- Burglar alarm at home
  - → Fairly Reliable
  - → Sometimes to Earthquake
- Two neighbors, on hearing alarm, calls police
  - → John always calls, sometimes confuses with telephone ring
  - → Mary calls always but sometimes miss alarm due to music loud)

P(B)
0.001

P(E)
0.002



B	E	P(A)
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	P(J)
T	0.9
F	0.05

A	P(M)
T	0.7
F	0.01

$P(\neg A \mid \neg B \wedge E)$

$= 1 - 0.29$

$= 0.71$

$P(A \wedge \neg B \wedge E)$

$= ?$

Joint Probability Distribution

we need to figure out an automated way when number of nodes increases in the network.

$x_1, x_2 \cdots x_n$

$P(x_1 . x_2 \ldots x_n)$

$= \prod_{i=1}^{n} P(x_i \mid \text{Parents}(x_i))$

Joint Probability Distribution (JPD)

⚠ Imp.

using JPD

$P(A \wedge (\neg B \wedge E))$, with $A = x_1$, $\neg B = x_2$, $E = x_3$

$= P(A \mid \neg B \wedge E) \cdot P(\neg B \wedge E)$

$= P(A \mid \neg B \wedge E) \cdot P(\neg B) \cdot P(E)$

$= 0.29 \times (1 - 0.001) \times 0.002$

$= \boxed{0.00058}$ (approx)

4. $P(A \wedge B)$

$= P(A) \cdot P(B)$

when 'A' and 'B' are independent

5. Bayes' Rule

$P(A \cap B) =$
$P(A \mid B) \cdot P(B)$

$P(A \wedge B) = P(B \mid A) \cdot P(A)$

$P(B \mid A) = \dfrac{P(A \mid B) \cdot P(B)}{P(A)}$

Q: Probability of the event:

Alarm has sounded But neither 'B' nor 'E' has occured and Both John Mary Calls

Sol:

$P(J \wedge M \wedge A \wedge \neg B \wedge \neg E)$

$= P(J \mid A) \cdot P(M \mid A) \cdot P(A \mid \neg B \wedge \neg E) \cdot P(\neg B) \cdot P(\neg E)$

$= 0.9 \times 0.7 \times 0.001 \times (1 - 0.001) \times (1 - 0.002) = \boxed{0.000628}$ (approx)

Ex.

$P(J|B) = ?$

Not the immediate parent

using Bayes' Rule

$$= \frac{P(JB)}{P(B)}$$

$$= \frac{\boxed{P(JB)}}{0.001} \rightarrow P(JBA) + P(JBA')$$

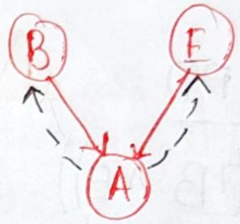
$$= P(J|BA) \cdot P(AB) + P(J|BA') \cdot P(A'B)$$

Because there is no direct dependency

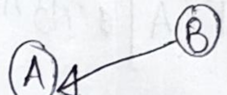
$$= P(J|A) \cdot \underbrace{P(AB)}_{P(ABE) + P(ABE')} + P(J|A') \cdot \underbrace{P(A'B)}_{P(A'BE) + P(A'BE')}$$

• Probability of several different events combinations of Bayesian network (BBN)

$P(\text{Alarm}) = P(AB'E') + P(ABE) + P(AB'E) + P(ABE')$



$= P(A|B'E') \cdot P(B'E') + P(A|BE) \cdot P(BE) + P(A|B'E) \cdot P(B'E) + P(A|BE') \cdot P(BE')$

random 

$\boxed{P(A) = P(AB) + P(AB')}$

Sanon's Rule ???

≈ 0.0025 (approx)

Probabaility of Alarm Rings

$P(\text{John Calls}) = P(JA) + P(JA')$

$= P(J|A) \cdot P(A) + P(J|A') \cdot P(A')$ (1)

Say, $P(A|J) = \dfrac{P(AJ)}{P(J)}$

Alarm will ring given the John has called

$\dfrac{P(J|A) \cdot P(A)}{P(J)} = \dfrac{0.9 \times 0.0025}{\text{Using (1)}}$

Using Bayes' Rule

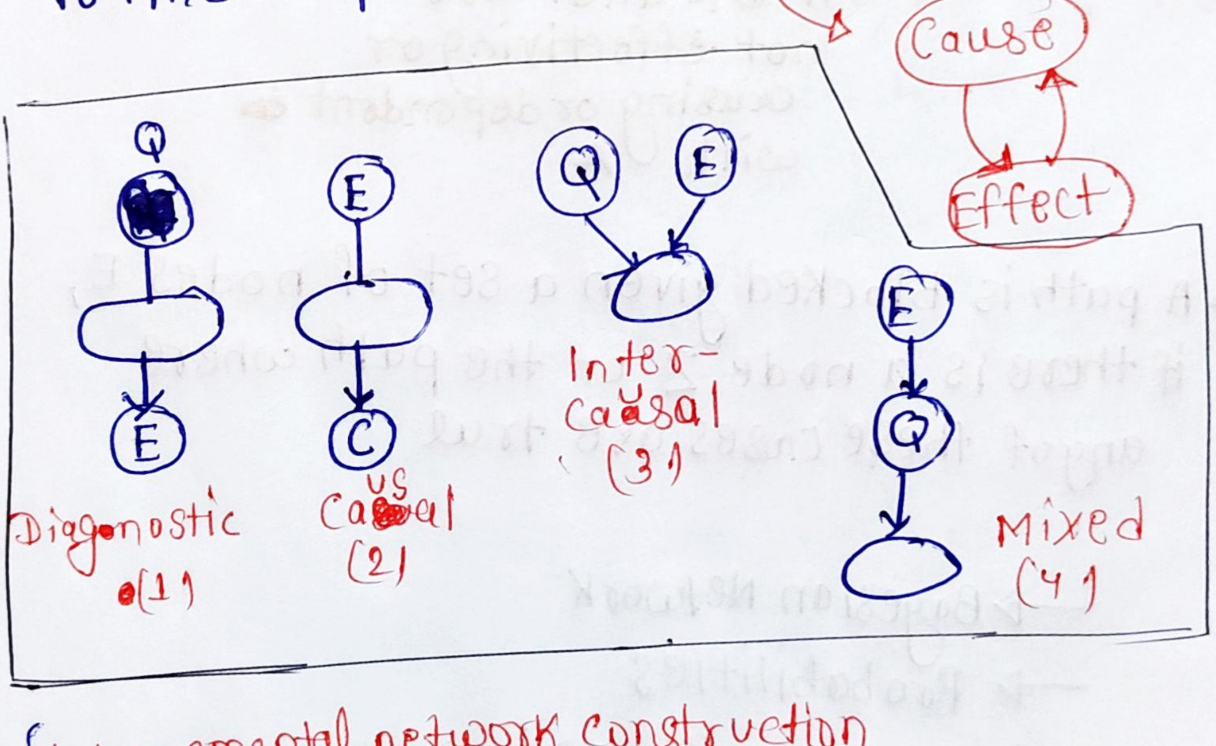
4. Mixed Inference

$P(A \mid J \wedge \neg E)$

setting $\left\{ \begin{array}{l} J = \text{True} \\ E = \text{False} \end{array} \right\}$

We have some queries → Prediction, Classification

BNN gaves us an automated way to find out probable effect and causes.



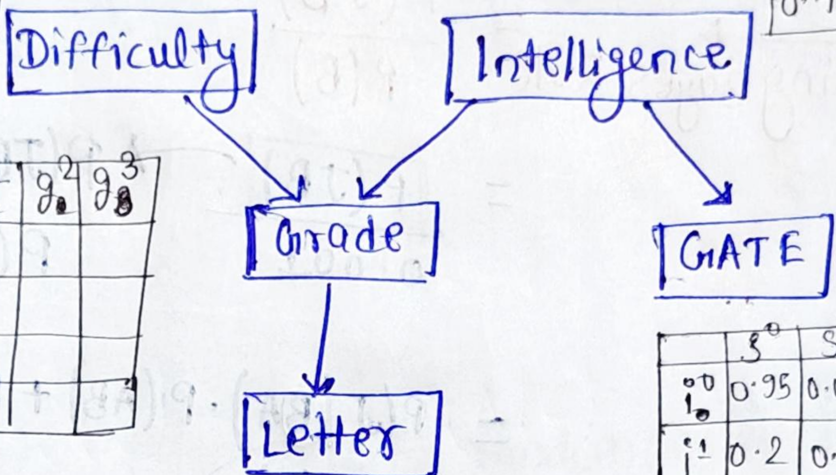
1. Incremental network construction
2. Conditional relation

Given a realworld scenario we need to find these

Gate Example

d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3



	g^1	g^2	g^3
$i^0 d^0$			
$i^0 d_1$			
$i^1 d_0$			
$i^1 d^1$			

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	i^0	i^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01

Inference from BN or BBN

Given that John calls $P(B|J) = ?$

1. Effect to Cause Inference (Diagnostic Inference)

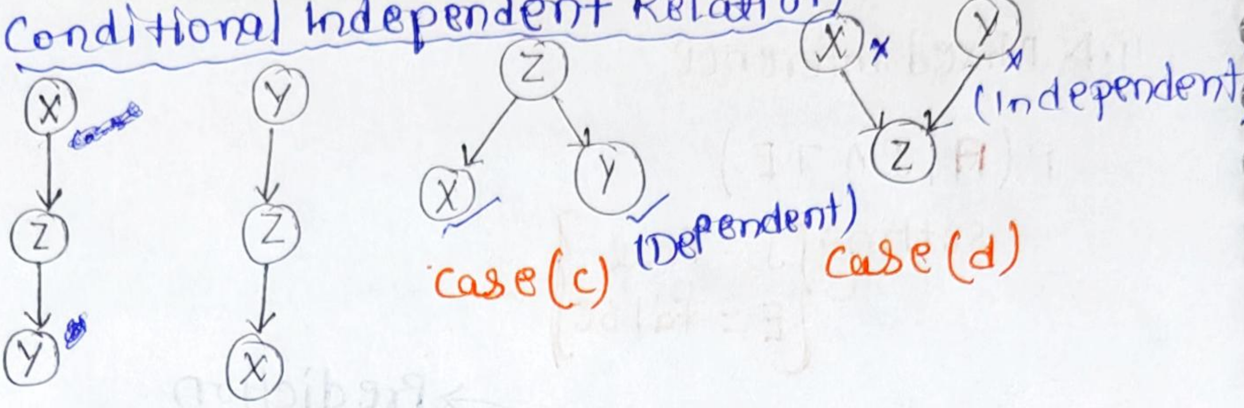
2. Opposite: Cause to inference (Causal Inference)

$P(M|B) = ?$

3. Inter-causal Inference (a) $P(B|A)$ (Between causes a common effect)

(b) Additional: $P(B|A \land E)$

Conditional Independent Relation



Case (a) Case (b) → $[Z \in E]$

$P(A|C\,DEF) \longrightarrow P(A|C)$

If D, E and F are not effectiving or causing or dependent with A

▷ A path is blocked given a set of nodes E, if there is a node Z on the path where any of these cases are true

→ Bayesian Network
→ Probabilities
→ Structure Dependency