

IIT Bhubaneswar
Mid-semester Examination

Sub: Mathematical Foundations of Computer Science (CS6L015)

Time: 2 hours

Full Marks: 60

(Answer all questions, all of equal marks)

Advice: Look before you leap!



Q1. $\{A, B\}$ means $A \vee B$

[5+5]

$\{A, B\}, \{C, D\}$ means $\{A, B\} \wedge \{C, D\}$

\sim is the negation operator

(a) Construct a resolution tree of the following set of clauses:

$\{P, Q\}, \{\sim P, Q\}, \{P, \sim Q\}, \{\sim P, \sim Q\}$

(b) Can the derived resolution DAG be converted to a Gentzen proof tree?
If so, derive one.

$\sim Q, P$

Q2. Assume all the brackets are equivalent. Using proof Theory find out if the following propositional formula is always valid.

$$(A \supset C) \supset [(B \supset C) \supset \{(A \vee B) \supset C\}]$$

Q3.

[5+5]

(a) Assume the universe of discourse (i.e. the domain) is empty set. Then what is the truth value of the following formula?

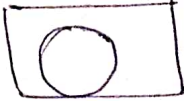
$$(\forall x. P(x)) \supset (\exists x P(x))$$

$\{ \}$
 $(P \supset Q) \cdot (\bar{P} \supset Q)$
 $P \supset Q$

(b) What is the truth value of the same formula if the domain is not empty?

(1)

$\sim \forall x P(x) \vee \exists x P(x)$
 $\sim \forall x P(x)$
 $\exists x \neg P(x) \vee \exists x P(x)$

$$\forall x. \text{People}(x) \rightarrow \text{fool} \quad \text{People}$$


Q4.

Abraham Lincoln made one of his most famous statements: "You can fool all the people some of the time and some of the people all the time, but you cannot fool all the people all the time."

Translate Lincoln's quote to first order logic.

$$\begin{aligned} & \text{fool} \quad \text{people}(x) \quad \text{time} \\ & \forall x \text{People}(x) \rightarrow \text{fool}(t, \text{People}(x)) \\ & \wedge \neg \text{fool}(t) \quad \wedge [\text{People}(y)] \end{aligned}$$

Q5. Convert the following into Prenix normal form

$$\sim \forall x [P(x) \vee \exists y \sim Q(y)] \vee [\forall z P(z) \vee \exists w \sim Q(w)]$$

Q6.

(A certain minister stops his vehicle) and (carries a sick man from the road to the hospital.)

Make a logical analysis of this statement and comment on the conduct of the health minister. Be precise.

$$P: \text{carries}$$

$$P: A \rightarrow q$$

----- paper ends -----

$$\begin{aligned} 000 & \rightarrow F \\ 001 & \rightarrow F \\ 010 & \rightarrow F \\ 011 & \rightarrow F \\ 100 & \rightarrow F \\ 101 & \rightarrow F \\ 110 & \rightarrow T \end{aligned}$$

$$\begin{aligned} 00 & \rightarrow \checkmark \\ 01 & \rightarrow \checkmark \\ 10 & \rightarrow \checkmark \\ 11 & \rightarrow \checkmark \end{aligned}$$

$$\left[\forall x \text{People}(x) \rightarrow \text{fool}(t, \text{People}(x)) \right] \wedge \neg \forall x \text{People}(x) \rightarrow \text{fool}(t, \text{People}(x))$$

$$\wedge \forall x \forall t \text{fool}(\text{People}(x), \text{time})$$