Kernel: SageMath 10.0

Name - Jitendra Karekar

Div -

Batch -

Roll No -

Laplace transform of standard Functions and piecewise functions

In [1]:

```
# Laplace tranform of a constant function f(t)= c
t,s,c=var('t,s,c')
f(t)= c
f.laplace(t, s)
show(f.laplace(t, s))
```

Out[1]: $t \mapsto \frac{c}{s}$

In [2]:

```
var('n')
var('t')
var('s')
laplace(t^n, t, s, algorithm='sympy')
```

Out[2]:

```
(gamma(n + 1)/(s*s^n), 0, re(n) > -1)
```

In [3]:

```
show(laplace(t^n, t, s, algorithm='sympy'))
```

Out[3]: $\left(\frac{\Gamma\left(n+1\right)}{s s^{n}}, 0, \mathtt{re(n)\ >\ -1}\right)$

In [4]:

```
## Laplace tranform of f(t)=sin(t)
f=sin(4*t)
f.laplace(t,s)
show(f.laplace(t,s))
```

Out[4]: $\frac{4}{s^{2}+16}$

In [5]:

```
## Laplace tranform of a piecewise defined function
t,s=var('t,s')
f = piecewise([[(0,2),1],[(2,4),t],[(4,infinity),exp(-2*t)]])
show(f)
Fs=f.laplace(t,s)
show(Fs)
```

Out[5]: $piecewise\left(\left(((0,2),1),((2,4),t),\left((4,+\infty),e^{(-2t)}\right)\right),t\right)$

$$\frac{(2s+1)e^{(-2s)}}{s^2}-\frac{e^{(-2s)}}{s}+\frac{e^{(-4s)}}{se^8+2e^8}-\frac{(4s+1)e^{(-4s)}}{s^2}+\frac{1}{s}$$

Derivative and Integral using Sage

In [8]:

```
var('t')
f = sin(t)
show(f)
show(diff(f,t))
```

Out[8]: $\sin(t)$

$\cos(t)$

In [9]:

```
var('t')
f = arctan(t)
show(f)
show(diff(f,t))
```

Out[9]: $\arctan(t)$

$$\frac{1}{t^2+1}$$

In [14]:

```
f(t) = t*sin(t)
show(f(t))
var('a')
show(f.integral(t,0,a))
```

Out[14]: $t\sin(t)$

$-a\cos(a)+\sin(a)$

Laplace transform of derivative and Integral

In [10]:

```
## Laplace tranform of 1st derivative
var('t')
f = function('f')(t)
laplace(diff(f,t),t,s)
```

Out[10]: `s*laplace(f(t), t, s) - f(0)`

In [11]:

```
show(laplace(diff(f,t),t,s))
```

Out[11]: $s\mathcal{L}(f(t),t,s)-f(0)$

Solving ODE using Laplace Transform

Example: Solve x'(t) + x(t) = cos(2t), x(0) = 2 using the Laplace transform.

In [12]:
```
show(laplace(diff(f,t,2),t,s))
```

Out[12]: $s^2 \mathcal{L}\left(f\left(t\right), t, s\right) - s f\left(0\right) - \mathrm{D}_{0}\left(f\right)\left(0\right)$

In [15]:
```
f(t) = t*sin(t)
show(f(t))
var('a')
g = f.integral(t,0,a)
show(g)
show(g.laplace(a,s))
h = g.laplace(a,s)
show(h.full_simplify())
```

Out[15]: $t \sin\left(t\right)$

$-a \cos\left(a\right) + \sin\left(a\right)$

$$-\frac{2 \, s^{2}}{{\left(s^{2} + 1\right)}^{2}} + \frac{2}{s^{2} + 1}$$

$$\frac{2}{s^{4} + 2 \, s^{2} + 1}$$

In [0]:
```
desolve_laplace(de,x,ics=[0,2])
```

In [0]:
```
show(desolve_laplace(de,x,ics=[0,2]))
```

Inverse Laplace Transforms

If L[f(t)] = F(s), then L inv [F(s)] = f(t)

Example Solve the 2nd order initial value problem

x''(t) + 2x'(t) + 2x = e^(-2t), x(0) = 0, x'(0) = 0

In [16]:
```
F(s) = 1/s^11*factorial(10)
inverse_laplace(F(s),s,t)
```

Out[16]: t^10

In [17]:
```
show(inverse_laplace(F(s),s,t))
```

Out[17]: t^{10}

In [18]:
```
F(s) = s/(s^3+s^2+s+1)
show(F(s))
show(inverse_laplace(F(s),s,t))
```

Out[18]:

$$\frac{s}{s^3 + s^2 + s + 1}$$

$$\frac{1}{2}\cos(t) - \frac{1}{2}e^{(-t)} + \frac{1}{2}\sin(t)$$

Solving ODE using Laplace Transform

Example: Solve x'(t) + x(t) = cos(2t), x(0) = 2 using the Laplace transform.

In [1]:
```
s,t = var('s,t')
x = function('x')(t)
de = diff(x,t) + x == cos(2*t)
```

In [2]:
```
desolve_laplace(de,x,ics=[0,2])
```

Out[2]:
```
1/5*cos(2*t) + 9/5*e^(-t) + 2/5*sin(2*t)
```

In [3]:
```
show(desolve_laplace(de,x,ics=[0,2]))
```

Out[3]:

$$\frac{1}{5}\cos(2t) + \frac{9}{5}e^{(-t)} + \frac{2}{5}\sin(2t)$$

Example Solve the 2nd order initial value problem

x''(t) + 2x'(t) + 2x = e^(-2t), x(0) = 0, x'(0) = 0

In [4]:
```
s,t = var('s,t')
x = function('x')(t)
de = diff(x,t,t)+2*diff(x,t)+2*x==exp(-2*t)
```

In [5]:
```
show(desolve_laplace(de,x,ics=[0,0,0]))
```

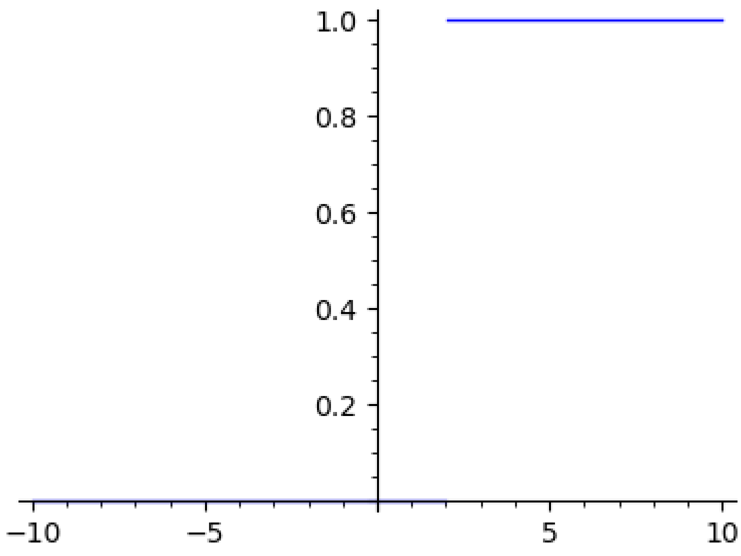
Out[5]:

$$-\frac{1}{2}(\cos(t) - \sin(t))e^{(-t)} + \frac{1}{2}e^{(-2t)}$$

Dirac_delta, unit_step and heaviside functions

In [6]:
```
plot(heaviside(t-2),-10,10,exclude =[2], figsize=4)
```

Out[6]:



In [7]:
```
## Laplace transform of heaviside function
laplace(heaviside(t), t, s) ## Not able to compute with defacult
algorithm ,'maxima'
```

Out[7]: 1/s

In [8]:
```
show(laplace(heaviside(t-2), t, s, algorithm='giac'))
```

Out[8]: $\frac{e^{(-2\,s)}}{s}$

In [9]:
```
F(s) = e^(-2*s)/s
inverse_laplace(F(s),s,t,algorithm='giac')
```

Out[9]: heaviside(t - 2)

In [10]:
```
laplace(dirac_delta(t),t,s)
```

Out[10]: 1

In [11]:
```
laplace(dirac_delta(t-2),t,s)
```

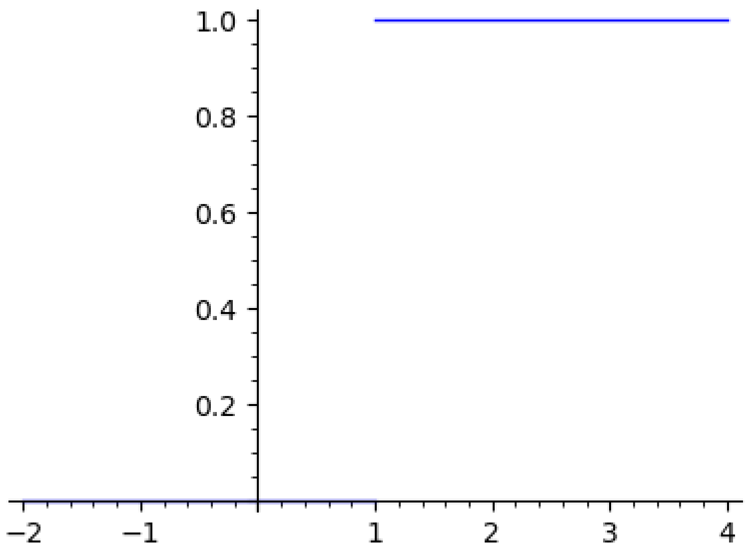
Out[11]: e^(-2*s)

In [12]:
```
F(s) = e^(-2*s)
inverse_laplace(F(s),s,t,algorithm='giac')
```

Out[12]: dirac_delta(t - 2)

In [13]:
```
plot(unit_step(t-1),-2,4,exclude = [1],figsize=4)
```

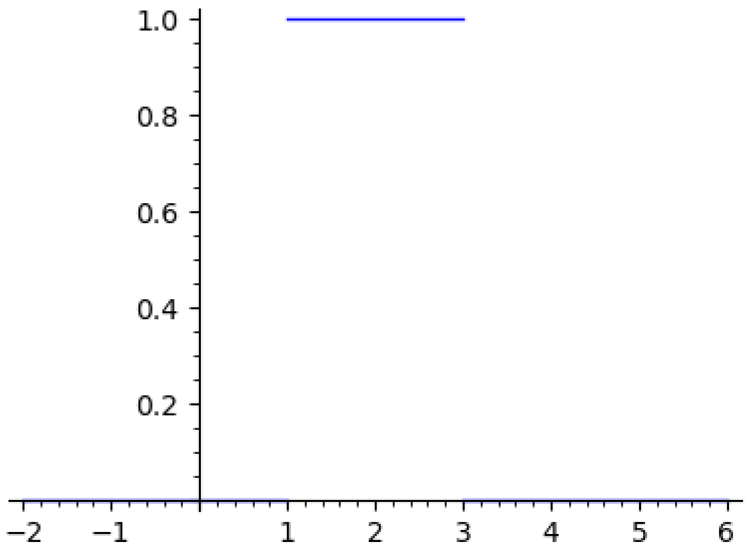
Out[13]:



In [14]:

```
plot(unit_step(t-1)-unit_step(t-3),-2,6,exclude = [1,3],figsize=4)
```

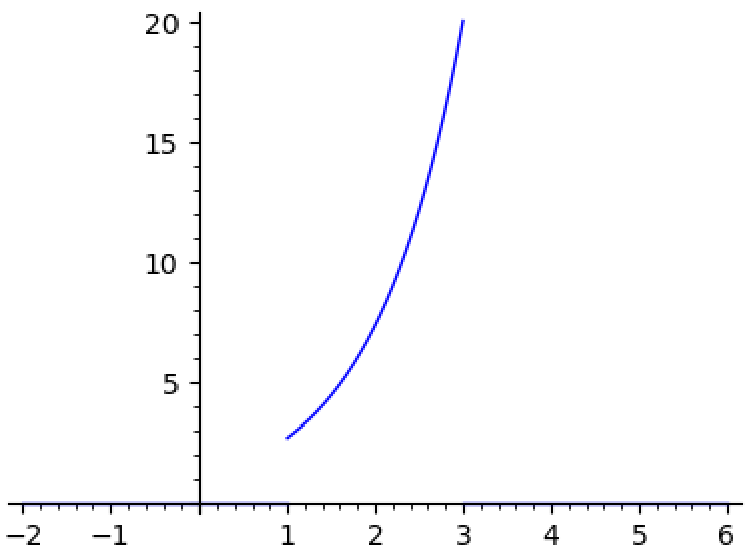
Out[14]:



In [15]:

```
plot(exp(t)*(unit_step(t-1)-unit_step(t-3)),-2,6,exclude =
[1,3],figsize=4)
```

Out[15]:



In [16]:

```
show(laplace(exp(t)*(unit_step(t-1)-unit_step(t-3)),t,s))
```

Out[16]: $\frac{e^{(-s+1)}}{s-1} - \frac{e^{(-3s+3)}}{s-1}$

In [17]:
```
heaviside(t).diff(t)
```

Out[17]: dirac_delta(t)

```
Q3 Find the laplace transform of the follwoing function using sageMath.
```

i) $t^3 \cos(t)$

ii) $-\frac{\cos(t)-1}{t^2}$

iii) $\int_0^t sint(t)cos(t)\, dx$

iv) $te^{-2\,t}H(t-1)$

Q4 Find the inverse Laplace transform of the following function using sagemath

i) $\frac{1}{(s^2+9)(s^2+1)}$

ii $\frac{11\,s^2-2\,s+5}{2\,s^3-3\,s^2-3\,s+2}$

Solve the follwing differential equation using sagemath

$$x'''(t) - 2x''(t) + 5x' = 0, x(0) = 0, x'(0) = 0, x''(0) = 1$$

In [0]: