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SOMAIYA VIDYAVIHAR UNIVERSITY

TUTORIAL NO. 9

using simplex method solve the following problems:

1. Maximize $z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

Sol^n: The given LPP in standard form can be written as

Max $z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

subject to $x_1 + 2x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 430$

$3x_1 + 0x_2 + 2x_3 + 0s_1 + s_2 + 0s_3 = 460$

$x_1 + 4x_2 + 0x_3 + 0s_1 + 0s_2 + s_3 = 420$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

		c_j	3	2	5	0	0	0		Min Ratio
	C_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	b	θ
x	0	s_1	1	2	1	1	0	0	430	430
	0	s_2	3	0	2	0	1	0	460	230 → s_2 leaves
	0	s_3	1	4	0	0	0	1	420	–
		z_j	0	0	0	0	0	0	0	
	x_3 enters	$c_j - z_j$	3	2	5 ↑	0	0	0		
x-y	0	s_1	-1/2	2	0	1	-1/2	0	200	100 → s_1 leaves
y	5	x_3	3/2	0	1	0	1/2	0	230	–
x	0	s_3	1	4	0	0	0	1	420	105
		z_j	15/2	0	5	0	5/2	0	1150	
	x_2 enters	$c_j - z_j$	-9/2	2 ↑	0	0	-5/2	0		
y	2	x_2	-1/4	1	0	1/2	-1/4	0	100	
	5	x_3	3/2	0	1	0	1/2	0	230	
x-4y	0	s_3	2	0	0	-2	1	1	20	
		z_j	7	2	5	1	2	0	1350	
		$c_j - z_j$	-4	0	0	-1	-2	0		

Since all $c_j - z_j$ elements are negative or less than 0, and zeros, we can say that the solution is optimal.

The optimal solution is $x_1 = 0$, $x_2 = 100$, $x_3 = 230$

and $z_{max} = 1350$

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2. Maximize $z = 10x_1 + 6x_2 + 4x_3$

subject to $x_1 + x_2 + x_3 \leq 100$

$10x_1 + 4x_2 + 5x_3 \leq 600$

$2x_1 + 2x_2 + 6x_3 \leq 300$

$x_1, x_2, x_3 \geq 0$

Sol^n: The given LPP in standard form can be written as

Max $Z = 10x_1 + 6x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$

subject to $x_1 + x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 100$

$10x_1 + 4x_2 + 5x_3 + 0s_1 + s_2 + 0s_3 = 600$

$2x_1 + 2x_2 + 6x_3 + 0s_1 + 0s_2 + s_3 = 300$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

		c_j	10	6	4	0	0	0		
	C_B	x_j	x_1	x_2	x_3	s_1	s_2	s_3	b	θ
x	0	s_1	1	1	1	1	0	0	100	100
	0	s_2	10	4	5	0	1	0	600	60 → s_2 leaves
x'	0	s_3	2	2	6	0	0	1	300	150
		z_j	0	0	0	0	0	0	0	
	x_1 enters	$c_j - z_j$	10↑	6	4	0	0	0		
$x - y$	0	s_1	0	3/5	1/2	1	-1/10	0	40	200/3 → s_1 leaves
y	10	x_1	1	2/5	1/2	0	1/10	0	60	150
$x' - 2y$	0	s_3	0	6/5	5	0	-1/5	1	180	150
		z_j	10	4	5	0	1	0	600	
	x_2 enters	$c_j - z_j$	0	2↑	-1	0	-1	0		
y	6	x_2	0	1	5/6	5/3	-1/6	0	$\frac{200}{3}$	
$x - \frac{2}{5}y$	10	x_1	1	0	1/6	-2/3	1/6	0	$\frac{100}{3}$	
$x' - \frac{6}{5}y$	0	s_3	0	0	4	-2	0	1	100	
		z_j	10	6	20/3	10/3	2/3	0	$\frac{2200}{3}$	
		$c_j - z_j$	0	0	-8/3	-10/3	-2/3	0		

Since all $c_j - z_j$ elements are negative or zeros, the solution is optimal.

The optimal solution is $x_1 = \frac{100}{3} = 33.33$,

$x_2 = \frac{200}{3} = 66.67$ and $x_3 = 0$

Max $z = \frac{2200}{3} = 733.33$

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3. Minimize $Z = x_1 - 3x_2 + 2x_3$

subject to $3x_1 - x_2 + 3x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

Sol^n: The given LPP in standard form can be written as

~~Min Z = $x_1 - 3x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3$; Max W = $-$Min Z = $-x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$~~

~~Subject to:~~

Max $W = -\text{Min } Z = -x_1 + 3x_2 - 2x_3 - 0s_1 - 0s_2 - 0s_3$

subject to: $3x_1 - x_2 + 3x_3 + s_1 + 0s_2 + 0s_3 = 7$

$-2x_1 + 4x_2 + 0x_3 + 0s_1 + s_2 + 0s_3 = 12$

$-4x_1 + 3x_2 + 8x_3 + 0s_1 + 0s_2 + s_3 = 10$

$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

		c_j	-1	3	-2	0	0	0			
	C_B	x_j	x_1	x_2	x_3	s_1	s_2	s_3	b	θ	
x	0	s_1	3	-1	3	1	0	0	7	$-$	
	0	s_2	-2	4	0	0	1	0	12	$3 \rightarrow$	s_2 leaves
x'	0	s_3	-4	3	8	0	0	1	10	$10/3$	
		z_j	0	0	0	0	0	0	0		
	x_2 enters	$c_j - z_j$	-1	$3 \uparrow$	-2	0	0	0			
$x+y$	0	s_1	$5/2$	0	3	1	$+1/4$	0	10	$4 \rightarrow$	s_1 leaves
y x	3	x_2	$-1/2$	1	0	0	$1/4$	0	3	$-$	
$x'-3y$ x'	0	s_3	$-5/2$	0	8	0	$-3/4$	1	1	$-$	
		z_j	$-3/2$	3	0	0	$3/4$	0	9		
	x_1 enters	$c_j - z_j$	$1/2 \uparrow$	0	-2	0	$-3/4$	0			
y	-1	x_1	1	0	$6/5$	$2/5$	$1/10$	0	4		
$x + \frac{y}{2}$	3	x_2	0	1	$3/5$	$1/5$	$3/10$	0	5		
$x' + \frac{5}{2}y$	0	s_3	0	0	11	1	$-1/2$	1	11		
		z_j	-1	3	$3/5$	$1/5$	$4/5$	0	11		
		$c_j - z_j$	0	0	$-13/5$	$-1/5$	$-4/5$	0			

Since all $c_j - z_j$ elements are negative or zeros, sol^n is optimal.

The optimal solution is $x_1 = 4$, $x_2 = 5$, $x_3 = 0$

Max $W = -\text{Min } Z$, Min $Z = -11$

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Show that the following LPP has degenerate solution and in simplex method illustrates that there is an optimal solution and nonoptimal solution.

4. Maximize $z = 28x_1 + 30x_2$

subject to $6x_1 + 3x_2 \le 18$

$3x_1 + x_2 \le 8$

$4x_1 + 5x_2 \le 30$

$x_1, x_2 \geq 0$

Sol^n: In standard form, Max $Z = 28x_1 + 30x_2 + 0s_1 + 0s_2 + 0s_3$

subject to $6x_1 + 3x_2 + s_1 + 0s_2 + 0s_3 = 18$

$3x_1 + x_2 + 0s_1 + s_2 + 0s_3 = 8$

$4x_1 + 5x_2 + 0s_1 + 0s_2 + s_3 = 30$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

		c_j	28	30	0	0	0		
	C_B	x_j	x_1	x_2	s_1	s_2	s_3	b	θ
	0	s_1	6	3	1	0	0	18	6 → s_1 leaves
x	0	s_2	3	1	0	1	0	8	8
x'	0	s_3	4	5	0	0	1	30	6
		z_j	0	0	0	0	0	0	
	x_2 enters	$c_j - z_j$	28	30 ↑	0	0	0		
y	30	x_2	2	1	1/3	0	0	6	
x − y	0	s_2	1	0	−1/3	1	0	2	
x' − 5y	0	s_3	−6	0	−5/3	0	1	0	
		z_j	60	30	10	0	0	180	
		$c_j - z_j$	−32	0	−10	0	0		

In the first iteration, there is a tie between s_1 and s_3 to leave the basis. This is an indication of a degenerate solution. If we arbitrarily choose s_1 as the variable that leaves the basis we get the optimal solution as

$Z_{max} = 180$, $x_1 = 0$, $x_2 = 6$

However if we choose s_2, we can see that we'd reach the same optimal solution eventually. The difference lies in the number of iterations in simplex method required to get the optimal solution.

we can also try to resolve the degeneracy,

	s_1	s_2	s_3
s_1	1	0	0
s_2	0	1	0
s_3	0	0	1

	s_1	s_2	s_3
s_1	1/3	0	0
s_2	0	1	0
s_3	0	0	1/5

Here the minimum ratio occurs for the 3rd row
$\therefore$ s_3 will leave the basis.

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Method → II

	C_B	c_j / x_j	28 / x_1	30 / x_2	0 / s_1	0 / s_2	0 / s_3	b	θ
x'	0	s_1	6	3	1	0	0	18	6
x	0	s_2	3	1	0	1	0	8	8
	0	s_3	4	5	0	0	1	30	6 → s_3 leaves
		z_j	0	0	0	0	0	0	
	x_2 enters	$c_j - z_j$	28	30 ↑	0	0	0		
$x' - 3y$	0	s_1	18/5	0	1	0	−3/5	0	0 → s_1 leaves
$x - y$	0	s_2	11/5	0	0	1	−1/5	2	10/11
y	30	x_2	4/5	1	0	0	1/5	6	15/2
		z_j	24	30	0	0	6	180	
	x_1 enters	$c_j - z_j$	4 ↑	0	0	0	−6		
y	28	x_1	1	0	5/18	0	−1/6	0	
$x - \frac{11}{5}y$	0	s_2	0	0	−11/18	1	1/6	2	
$x' - \frac{4}{5}y$	30	x_2	0	1	−2/9	0	1/3	6	
		z_j	28	30	10/9	0	16/3	180	
		$c_j - z_j$	0	0	−10/9	0	−16/3		

Therefore, we can come to the conclusion that in the first iteration, choosing which variable leaves decides whether our solution would be optimal or not. The first one being the more optimal solution and the second method being the non-optimal one.

Q.5 Using Big-M method solve the following problems:

5. Minimize $z = 3x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \geq 90$

$4x_1 + 3x_2 \geq 120$

$x_1, x_2 \geq 0$

Sol^n: Given LPP can be written in standard form as:

$-\text{Min } z = \text{Max } w = -3x_1 - 4x_2 - 0s_1 - 0s_2 - MA_1 - MA_2$

Subject to $2x_1 + 3x_2 - s_1 + A_1 + 0s_2 + 0A_2 = 90$

$4x_1 + 3x_2 - s_2 + A_2 + 0s_1 + 0A_1 = 120$

$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$

		c_j	-3	-4	0	0	$-M$	$-M$			
	C_B	x_j	x_1	x_2	s_1	s_2	A_1	A_2	b	θ	
x	$-M$	A_1	2	3	-1	0	1	0	90	45	
	$-M$	A_2	4	3	0	-1	0	1	120	$30 \rightarrow$	A_2 leaves
		z_j	$-6M$	$-6M$	M	M	$-M$	$-M$	$-210M$		
	x_1 enters	$c_j - z_j$	$\uparrow 6M-3$	$6M-4$	$-M$	$-M$	0	0			
$x - 2y$	$-M$	A_1	0	$3/2$	-1	$1/2$	1	$-1/2$	30	$20 \rightarrow$	A_1 leaves
y x	-3	x_1	1	$3/4$	0	$-1/4$	0	$1/4$	30	40	
		z_j	-3	$-\frac{3M}{2} - \frac{9}{4}$	M	$-\frac{M}{2} + \frac{3}{4}$	$-M$	$\frac{M}{2} - \frac{3}{4}$	$-30M-90$		
	x_2 enters	$c_j - z_j$	0	$\uparrow \frac{3M}{2} - \frac{7}{4}$	$-M$	$\frac{M}{2} - \frac{3}{4}$	0	$-\frac{3}{2}M + \frac{3}{4}$			
y	-4	x_2	0	1	$-2/3$	$1/3$	$2/3$	$-1/3$	20		
$x - \frac{3}{4}y$	-3	x_1	1	0	$1/2$	$-1/2$	$-1/2$	$1/2$	15		
		z_j	-3	-4	$7/6$	$1/6$	$-7/6$	$-1/6$	-125		
		$c_j - z_j$	0	0	$-7/6$	$-1/6$	$-M + \frac{7}{6}$	$-M + \frac{1}{6}$			

Since all $c_j - z_j$ elements are negative or zeros, we can conclude that the solution is optimal. The optimal solution is $\text{Max } w = -125$, $\text{Min } z = -(-125) = 125$

$x_1 = 15$, $x_2 = 20$

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6. Maximize $z = 107x_1 + x_2 + 2x_3$

subject to $14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + \frac{1}{2}x_2 + 6x_3 \leq 5$

$3x_1 - x_2 - x_3 \leq 0$

$x_1, x_2, x_3, x_4 \geq 0$

Sol^n: Standard form of the LPP is given by:

Max $z = 107x_1 + x_2 + 2x_3 + 0x_4 + 0s_2 + 0s_3$ $+0s_2 + 0s_3$

Subject to $\frac{14}{3}x_1 + \frac{1}{3}x_2 - \frac{6}{3}x_3 + x_4 = \frac{7}{3}$

$16x_1 + \frac{1}{2}x_2 + 6x_3 + 0x_4 + s_2 + 0s_3 = 5$

$3x_1 - x_2 - x_3 + 0x_4 + 0s_2 + s_3 = 0$

$x_1, x_2, x_3, x_4, s_2, s_3 \geq 0$

		c_j	107	1	2	0	0	0			
	C_B	x_j	x_1	x_2	x_3	x_4	s_2	s_3	b	θ	
x'	0	x_4	14/3	1/3	-2	1	0	0	7/3	1/2	
x	0	s_2	16	1/2	6	0	1	0	5	5/16	
	0	s_3	3	-1	-1	0	0	1	0	0 →	s_3 leaves
		z_j	0	0	0	0	0	0	0		
	x_1 enters	$c_j - z_j$	107↑	1	2	0	0	0			
$x' - \frac{14}{3}y^{x'}$	0	x_4	0	17/9	-4/9	1	0	-14/9	7/3	–	
$x - 16y$	0	s_2	0	35/6	34/3	0	1	-16/3	5	15/34 →	s_2 leaves
y^x	107	x_1	1	-1/3	-1/3	0	0	1/3	0	–	
		z_j	107	-107/3	-107/3	0	0	107/3	0		
	x_3 enters	$c_j - z_j$	0	110/3	113/3↑	0	0	-107/3			
$x' + \frac{4}{9}y^{x'}$	0	x_4	0	36/17	0	1	2/51	-30/17	43/17	43/36	
y	2	x_3	0	35/68	1	0	3/34	-8/17	15/34	6/7 →	x_3 leaves
$x + \frac{y}{3}^x$	107	x_1	1	-11/68	0	0	1/34	3/17	5/34	–	
		z_j	107	$-\frac{1107}{68}$	2	0	113/34	305/17	565/34		
	x_2 enters	$c_j - z_j$	0	$\frac{1107}{68}$↑	0	0	$-\frac{113}{34}$	$-\frac{305}{17}$			
$x' - \frac{36}{17}y$	0	x_4	0	0	$-\frac{144}{35}$	1	$-\frac{34}{105}$	6/35	5/7		
y	1	x_2	0	1	68/35	0	6/35	-32/35	6/7		
$x + \frac{11}{68}y$	107	x_1	1	0	11/35	0	2/35	1/35	2/7		

z_j		107	1	$249/7$	0	$44/7$	$15/7$	$\frac{220}{7}$	
$c_j - z_j$		0	0	$-\frac{235}{7}$	0	$-44/7$	$-15/7$		

Since all $c_j - z_j$ elements are negative or zeros, the solution obtained is optimal.

The optimal solution is max $z_j = \frac{220}{7} = 31.43$

$$x_1 = \frac{2}{7}, \quad x_2 = \frac{6}{7}, \quad x_3 = 0$$

or so it would seem.

On observation of the above tableau, we see that at the end of the second iteration we get a solution for z_{max} which is more optimal than the one we obtained.

$z = \frac{107}{3} = 35.67$ and $x_1 = 0, x_2 = 0, x_3 = 0$