* Resultant of parallel force system in space

Resultant of parallel force system is a single force $\bar{R}$ & it acts parallel to the line of action of forces.

Procedure :-

1. coordinates
2. Force vector $\bar{F_1}, \bar{F_2}, \bar{F_3}$ . . . .
3. Position vector $\bar{r_1}, \bar{r_2}, \bar{r_3}$ . . .
4. Moment vectors $\bar{M_1}, \bar{M_2}, \bar{M_3}$ . . . .
5. ~~Restt~~ Resultant force vector $\bar{R} = \bar{F_1} + \bar{F_2} + \bar{F_3}$
6. Resultant moment vector.

   a. sum of moment of forces @ origin ($\Sigma \bar{M_o}$)

      $\Sigma \bar{M_o} = \bar{M_1} + \bar{M_2} + \bar{M_3} + \cdots + \bar{C_1} + \bar{C_2} + \cdots$

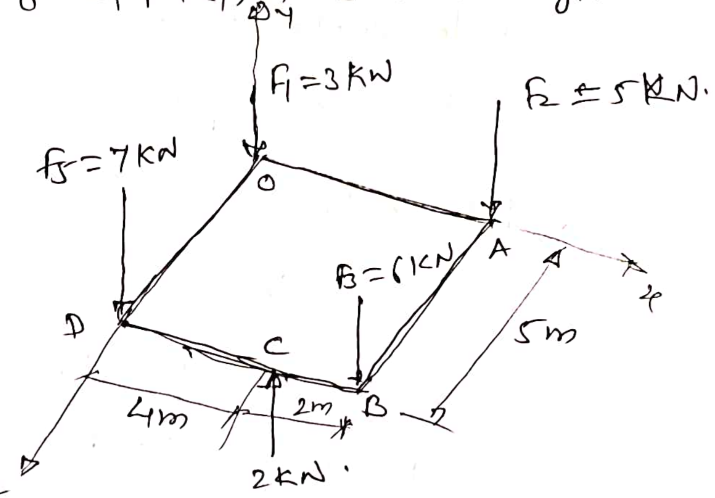
   b. moment of resultant force @ origin ($\Sigma \bar{M_R}$)

      $\bar{M_R} = \bar{r_{op}} \times \bar{R}$ P - pt. through which resultant acts

7. Apply Varignon's thm

   $\Sigma \bar{M_o} = \bar{M_R}$

* Five vertical forces are acting on a horizontal Plate shown in fig. Find the resultant of the forces & point of application w.r.t. origin



① Co-ordinates :-

O (0, 0, 0) A(6, 0, 0), B(6, 0, 5), C(4, 0, 5)

& D (0, 0, 5)

② Force vector :-

$\bar{F}_1 = -3j$, $\bar{F}_2 = -5j$, $\bar{F}_3 = -6j$

$\bar{F}_4 = 2j$ $\bar{F}_5 = -7j$

③ Position vector :-

$\bar{r}_1 = 0$, $\bar{r}_2 = \bar{OA} = 6i$, $\bar{r}_3 = \bar{OB} = 6i + 5z$

$\bar{r}_4 = \bar{OC} = 4i + 5z$

④ moment vector

$M_1 = \bar{r}_1 \times \bar{F}_1 = \begin{vmatrix} i & j & k \\ 0 & 0 & 0 \\ 0 & -3 & 0 \end{vmatrix}$, $M_2 = \bar{r}_2 \times \bar{F} = \begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix}$ $M_3 = \bar{r}_3 \times \bar{F}$ $= \begin{vmatrix} i & j & k \\ 6 & 0 & 5 \\ 0 & -6 & 0 \end{vmatrix}$

$\bar{m}_1 = 0$ $\bar{M}_2 = -30k$ $m_3 = 30i - 36k$

$M_4 = \bar{r}_4 \times \bar{F}_4$
$= \begin{vmatrix} i & j & k \\ 4 & 0 & 5 \\ 0 & 2 & 0 \end{vmatrix}$

$\bar{M}_4 = -10i + 8k$

$M_5 = \bar{r}_5 \times \bar{F}_5$
$= \begin{vmatrix} i & j & k \\ 0 & 0 & 5 \\ 0 & -7 & 0 \end{vmatrix}$

$M_5 = 35i$

(V) Resultant force vector

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 + \bar{F}_5$$
$$= -3j \mp 5j - 6j + 2j - 7j$$

$$R = -19j \ (KN)$$

(6) Resultant moment vector ($\Sigma \bar{M}_o$)

(a) moment of forces @ origin (ΣM_o)

$$\Sigma \bar{M}_o = \bar{M}_1 + \bar{M}_2 + \bar{M}_3 + \bar{M}_4 + \bar{M}_5$$
$$= 0 - 30k + 30i - 36k - 10i + 8k + 35j$$

$$\Sigma \bar{M}_o = 55i - 58\,k \quad (N\text{-}m)$$

(b) moment of resultant ($\Sigma \bar{M}_R$)

Let resultant act at pt. P (x, 0, z)

$$\bar{M}_R = \bar{r}_{OP} \times \bar{R} = \begin{vmatrix} i & j & k \\ x & 0 & z \\ 0 & -19 & 0 \end{vmatrix}$$

$$\bar{M}_R = 19z\,i - 19x\,k$$

(7) Apply Varignon thm

$$(\Sigma \bar{M}_o) = (\Sigma \bar{M}_R)$$

$$55i - 58k = 19z\,i - 19x\,k.$$

$k \rightarrow \ -19x = -58$

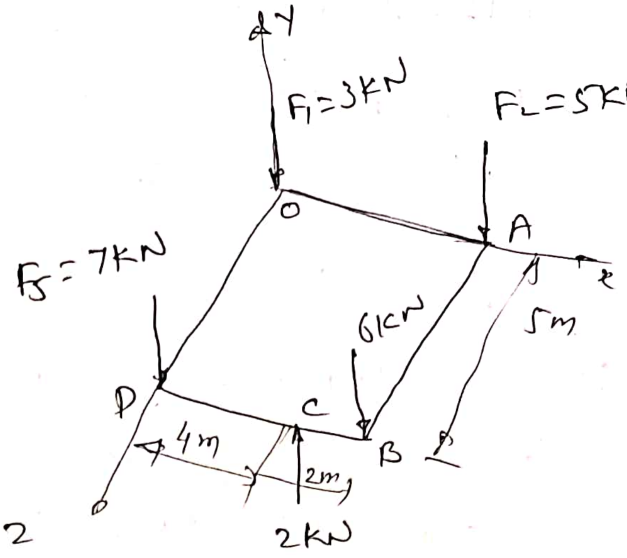
$x = 3.05\,m.$

$i \rightarrow \ 19z = 55$

$z = 2.89m$

$\bar{R} = -19j \ (KN)$ act at pt. P(3.05, 2.89m)

Method II :-



① $R = -3 - 5 - 6 + 2 - 7$

$R = -19 kN \ (\downarrow)$

② moment @ x-axis & apply varignon th^m:

$\Sigma M_x = R \times Z$

$-3\times0 + (-5)\times0 + (-6)(5) + 2\times5 + (-7)\times5 = -19 \times Z$

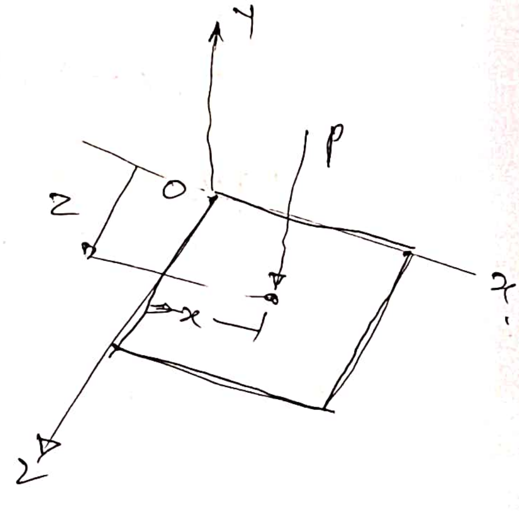
$Z = 2.89 m.$

③ moment @ z-axis & apply varignon th^m.

$\Sigma M_z = R \times x$

$(-3)\times0 + (-5)\times6 + (-6)(6) + 2\times4 + (-7)\times0 = -19\times x$

$x = 3.05 m.$



* Resultant of General force system in space

- General force system in space can't be reduced to single force. Therefore the ~~sett~~ resultant is expressed in two components.

  (1) Resultant force component.

  (2) Resultant moment (couple) component.

- Varignon's thm is not applicable to general force system.

procedure :-

(1) co-ordinates

(2) Force vector $\bar{F}_1, \bar{F}_2 + \bar{F}_3 \ldots$

(3) position Vector (w.r.t. origin or given pt)
$\bar{r}_1, \bar{r}_2, \bar{r}_3$

(4) moment vector $\bar{M}_1, \bar{M}_2, \bar{M}_3 \ldots \bar{C}_1 + \bar{C}_2$.

(5) Resultant force vector
$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 \ldots$$

(6) Resultant moment (couple) vector
$$\Sigma \bar{M} = \bar{M}_1 + \bar{M}_2 + M_3 + \ldots + C_1 + C_2$$

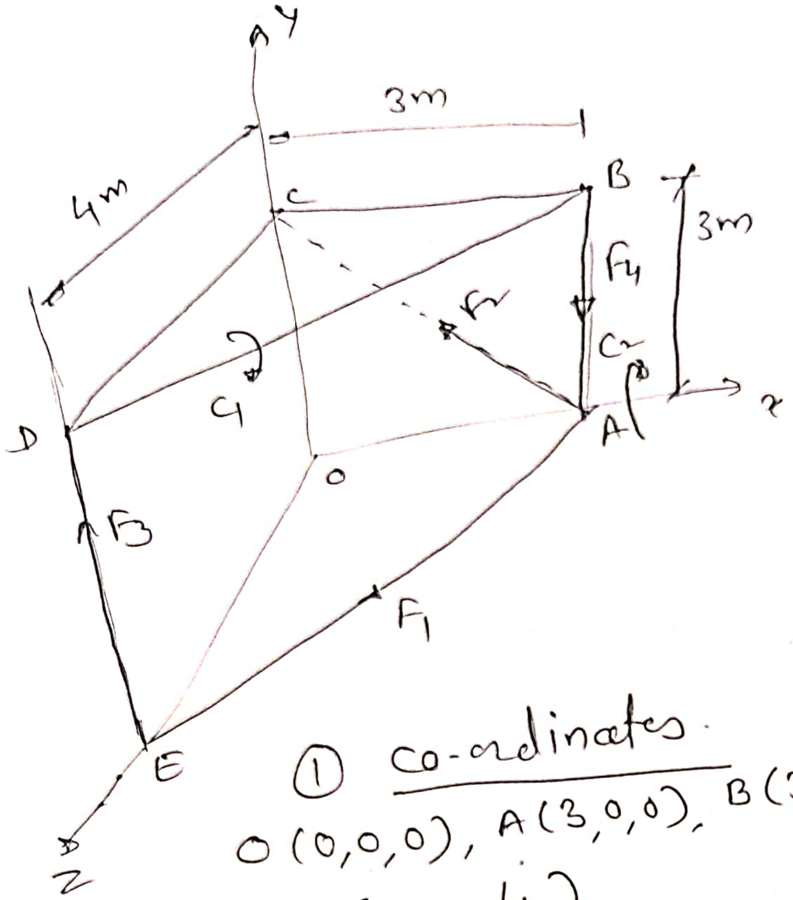
* Determine the resultant force & the resultant couple of the force system as shown. $F_1 = 100\ N$

$F_2 = 20\sqrt{2}\ N$

$F_3 = 40\ N$

$C_1 = 250\ N\text{-}m\ (D \to B)$

$C_2 = 100\ N\text{-}m.\ (A \to O)$



① co-ordinates

O (0,0,0), A (3,0,0), B (3,3,0), C (0,3,0), D (0,3,4)

E (0,0,4)

② Force vector

$$\bar{F_1} = (F_1)(\bar{e}_{AE}) = -100\left[\frac{-3i+4k}{\sqrt{9+16}}\right]$$

$$\therefore \bar{F_1} = -60i + 80k.$$

$$\bar{F_2} = (F_2)(\bar{e}_{AC}) = (20\sqrt{2})\left[\frac{-3i+3j}{\sqrt{9+9}}\right]$$

$$\bar{F_2} = -20i + 20j$$

$$\bar{F_3} = (F_3)(\bar{e}_{ED}) = 40\left[\frac{(+3-0)i + k(4-4)}{\sqrt{3^2+0^2}}\right]$$

$$= 40j$$

$$\bar{F_4} = (F_4)(e_{BA}) = \frac{\cancel{40j}}{40}\left[\frac{(3-3)i + j(0-3) + k(0-0)}{\sqrt{(-3)^2}}\right]$$

$$= -40j$$

③ Position vector

$\bar{r}_1 = OA = 3i$, $\bar{r}_2 = OA = 3i$

$\bar{r}_3 = OE = 4K$, $\bar{r}_4 = OA = 3i$

④ moment vectors :-

$$\bar{M}_1 = \bar{r}_1 \times \bar{F}_1 = \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ -60 & 0 & 80 \end{vmatrix}$$

$$\bar{M}_1 = -240j$$

$$\bar{M}_2 = \bar{r}_2 \times \bar{F}_2 = \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 20 & 20 & 0 \end{vmatrix}$$

$$\bar{M}_2 = 60K.$$

$$\bar{M}_3 = \bar{r}_3 \times \bar{F}_3 = \begin{vmatrix} i & j & k \\ 0 & 0 & 4 \\ 0 & 40 & 0 \end{vmatrix}$$

$$\bar{M}_3 = -160i$$

$$\bar{M}_4 = \bar{r}_4 \times \bar{F}_4 = \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & -40 & 0 \end{vmatrix}$$

$$M_4 = -120K.$$

$$\bar{C}_1 = C_1\,\bar{e}_{DB} = 250\left[\frac{3i-4k}{\sqrt{9+16}}\right]$$

$$\bar{C}_1 = 150i - 200k$$

$$\bar{C}_2 = (C_2)(\bar{e}_{AO}) = 100\left[\frac{-3i}{\sqrt{9}}\right]$$

$$\bar{C}_2 = -100i$$

(5) Resultant force vector :-

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$\bar{R} = (-60i+80k) + (-20i+20j) + 40j - 40j$$

$$\bar{R} = -80i + 20j + 80k.$$

(6) Resultant moment (couple) vector :-

$$\Sigma\bar{M} = \bar{M}_1 + \bar{M}_2 + \bar{M}_3 + \bar{M}_4 + \bar{C}_1 + \bar{C}_2$$

$$= -240j + 60k - 150i - 120k + 150i - 200k - 100i$$

$$\Sigma\bar{M} = -100i - 240j - 260k.$$