K. J. Somaiya College of Engineering, Mumbai-77
(A Constituent college of Somaiya Vidyavihar University)

F.Y. B. Tech SEM-II
Applied Mathematics-II

Practice Problems
Differential Equation of First Order and First Degree
(Module 1: Sub-module 1.1 & 1.2)

Solve the following.

1. $(x^3 e^x - my^2)dx + mxy\,dy = 0$
2. $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2 y\,dy = 0$
3. $(y - 2x^3)dx - x(1 - xy)dy = 0$
4. $(x^2 + y^2 + 1)dx - 2xy\,dy = 0$
5. $y(xy + e^x)dx - e^x dy = 0$
6. $(2xy^2 - y)dx + xdy = 0$
7. $(x + 2y^3)\frac{dy}{dx} = y$
8. $y(x^2 y + e^x)dx - e^x dy = 0$
9. $\frac{dy}{dx} = \frac{y}{2y\log y + y - x}$
10. $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xy\,e^{xy^2} + 3y^2\right)dy = 0$
11. $x\,dx + y\,dy = \frac{a^2(x\,dx - y\,dy)}{x^2 + y^2}$
12. $(y^3 - 3x^3 y)dy + (x^3 - 3xy^3)dx = 0$
13. $\left(x\sqrt{x^2 + y^2} - y\right)dx + \left(y\sqrt{x^2 + y^2} - x\right)dy = 0$
14. $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$
15. $(y^3 - 2yx^2)dx + (2xy^2 - x^3)dy = 0$
16. $x(x - y)dy + y^2 dx = 0$
17. $(x^3 + y^3)dx - xy^2 dy = 0$
18. $(xy + 2x^2 y^2)dx + x(xy - x^2 y^2)dy = 0$
19. $(y - xy^2)dx - (x + x^2 y)dy = 0$
20. $y(2xy + 1)dx + x(1 + 2xy - x^3 y^3)dy = 0$
21. $y(1 + xy + x^2 y^2 + x^3 y^3)dx + x(1 - xy - x^2 y^2 + x^3 y^3)dy = 0$
22. $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$
23. $\frac{dy}{dx} + \left(\frac{1-2x}{x^2}\right)y = 1$ Answer: $\left[y = x^2 + ce^{\frac{1}{x}} \cdot x^2\right]$
24. $(1 + y^2)dx = (e^{\tan^{-1} y} - x)dy$ Answer: $\left[xe^{\tan^{-1} y} = \frac{1}{2}e^{2\tan^{-1} y} + c\right]$
25. $x\,dy - (y - x)dx = 0$ Answer: $[y + x\log x = cx]$

26. $\frac{dy}{dx}+\frac{y}{1-x}=x^2-x$ Answer: $[2y=(1-x)(c^2-x^2)]$
27. $x(1-x^2)\frac{dy}{dx}+(2x^2-1)y=x^3$ Answer: $\left[y=\tan x+cx\sqrt{1-x^2}\right]$
28. $\cos^2 x\frac{dy}{dx}+y=\tan x$ Answer: $[y=\tan x-1+ce^{-\tan x}]$
29. $x(x-1)\frac{dy}{dx}-y=x^2(x-1)^2$ Answer: $\left[\frac{xy}{1-x}+\frac{x^3}{3}=c\right]$
30. $(1+x+xy^2)dy+(y+y^3)dx=0$ Answer: $[xy+\tan^{-1} y=c]$
31. $x\cos y\frac{dy}{dx}-\sin y=x\sin^2 y$ Answer: $[cosec\ y+x(\log x+c)=0]$
32. $\sec^2 y\frac{dy}{dx}+2\tan x\tan y=\sin x$ Answer: $[\sec^2 x\tan y=\sec x+c]$
33. $y\ \frac{dy}{dx}+\frac{4x}{3}-\frac{y^2}{3x}=0$ Answer: $\left[y^2x^{-\frac{2}{3}}+2x^{\frac{4}{3}}=c\right]$
34. $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ Answer: $\left[\tan y=\frac{1}{2}(x^2-1)+ce^{-x^2}\right]$
35. $\frac{dy}{dx}+(2x\tan^{-1} y-x^3)(1+y^2)=0$ Answer: $\left[\tan^{-1} y=\frac{x^2-1}{2}+ce^{-x^2}\right]$
36. $\frac{dy}{dx}+x^3\sin^2 y+x\sin 2y=x^3$ Answer: $\left[\tan y\cdot e^{x^2}=\frac{1}{2}e^{x^2}(x^2-1)+c\right]$
37. $x\cos y\frac{dy}{dx}-\sin y=x\sin^2 y$ Answer: $[cosec\ y+x(\log x+c)=0]$
38. $\sec^2 y\frac{dy}{dx}+2\tan x\tan y=\sin x$ Answer: $[\sec^2 x\tan y=\sec x+c]$
39. $y\ \frac{dy}{dx}+\frac{4x}{3}-\frac{y^2}{3x}=0$ Answer: $\left[y^2x^{-\frac{2}{3}}+2x^{\frac{4}{3}}=c\right]$
40. $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ Answer: $\left[\tan y=\frac{1}{2}(x^2-1)+ce^{-x^2}\right]$
41. $\frac{dy}{dx}+\left(2x\tan^{-1} y-x^3\right)(1+y^2)=0$ Answer: $\left[\tan^{-1} y=\frac{x^2-1}{2}+ce^{-x^2}\right]$
42. $\frac{dy}{dx}+x^3\sin^2 y+x\sin 2y=x^3$ Answer: $\left[\tan y\cdot e^{x^2}=\frac{1}{2}e^{x^2}(x^2-1)+c\right]$
43. $\frac{dy}{dx}(x^2y^3+xy)=1$ Answer: $\left[x(2-y^2)+\left(c\ x\ e^{-\frac{y^2}{2}}\right)=1\right]$
44. $\frac{dy}{dx}+y\tan x=y^2\sec x$ Answer: $\left[\frac{1}{y}\cos x=-x+c\right]$
45. $\frac{dy}{dx}+y\cos x=y^3\sin 2x$ Answer: $\left[\frac{1}{y^2}=(1+2\sin x)+c\ e^{2\sin x}\right]$
46. $\frac{dy}{dx}-xy=y^2e^{-\left(\frac{x^2}{2}\right)}\cdot\log x$ Answer: $\left[\frac{1}{y}e^{\frac{x^2}{2}}=x(1-\log x)+c\right]$
47. $x\frac{dy}{dx}+y=x^3y^c$ Answer: $\frac{1}{y^5}=\frac{5}{2}x^3+cx^5$
48. $(1-x^2)\frac{dy}{dx}+xy=y^3\sin^{-1} x$ Answer: $\left[-2\left[x\sin^{-1} x+\sqrt{1-x^2}\right]+c\right]$

49. In a circuit containing inductance L, resistance R, and voltage E, the current I is given by $L\frac{di}{dt} + Ri = E$. Find the current i at time t if at $t = 0,\ i = 0$ and L, R, E are constants.

[Ans: $i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$]

50. A constant e. m. f. E volts is applied to a circuit containing a constant resistance R ohms in series and a constant inductance L henries. The current i at any time t is given by $L\frac{di}{dt} + Ri = E$. If the initial current is zero. Show that the current builds up to half its theoretical maximum value in $t = \frac{L}{R}\log 2$.

51. The equation of an L-R circuit is given by $L\frac{di}{dt} + Ri = 10\sin t$. If $i = 0\ at\ t = 0$ express i as a function of t. [Ans: $i = \frac{10}{\sqrt{R^2 + L^2}}\left[\sin(t - \varphi) + e^{-Rt/L}\sin\varphi\right] where\ \tan\varphi = \frac{L}{R}$]

52. The change q on the plate of a condenser of capacity C charged through a resistance R by a steady voltage V satisfies the differential equation $R\frac{dq}{dt} + \frac{q}{c} = V$. If $q = 0\ at\ t = 0$, show that $q = CV\left(1 - e^{-t/RC}\right)$ Find also the current flowing into the plate.
[Ans: $i = \frac{V}{R} \cdot e^{-t/RC}$]

53. The differential equation of an electrical circuit is $R\frac{dQ}{dt} + \frac{Q}{C} = V$. If R=20 ohms , C=0.01 farad and V= $20\,e^{-5t}$ and if Q=0 when t=0, find Q in terms of t. [Ans: $Q\,e^{5t} = t$]

54. A resistance of 100 ohms and inductance of 0.5 henries are connected in series with a battery of 20 volts. Find the current at any instant if the relation between L, R, E is $L\frac{di}{dt} + Ri = E$.

[Ans: $i = 0.2\left(1 - e^{-200\,t}\right)$]

55. In a circuit of resistance R, self inductance L, the current I is given by $L\frac{di}{dt} + Ri = E\cos pt$, when E and p are constants. Find the current i at time t.

[Ans: $i = \dfrac{E}{L^2 p^2 + R^2}\left[Lp\sin pt - R\cos pt\right] + k\, e^{-Rt/L}$]

56. In a circuit of resistance R, self inductance L, the current i is given by $L\dfrac{di}{dt} + Ri = E\cos pt$, Find the current i at any time t if i=0 at t=0.

[Ans: $i = \dfrac{E}{\sqrt{L^2 p^2 + R^2}}\left[\sin(pt + \varphi) - e^{-Rt/L}\cos\varphi\right] where\ \varphi = Lp/R$]

Practice Problems
Linear Differential Equation with Constant Coefficient
(Module 1: Sub-module 1.3 & 1.4)

Solve the following.

1. $\dfrac{d^3y}{dx^3} - 5\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} - 4y = 0$ [Ans: $y = (c_1 + c_2 x)e^{2x} + c_3 e^{x}$]
2. $\dfrac{d^3y}{dx^3} - 6\dfrac{d^2y}{dx^2} + 11\dfrac{dy}{dx} - 6y = 0$ [Ans: $y = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x}$]
3. $\dfrac{d^4y}{dx^4} + k^4 y = 0$

[Ans:
$y = e^{(k/\sqrt{2})x}\left[c_1\cos(k/\sqrt{2})x + c_2\sin(k/\sqrt{2})x\right] + e^{-(k/\sqrt{2})x}\left[c_3\cos(k/\sqrt{2})x + c_4\sin(k/\sqrt{2})x\right]$]

4. $\dfrac{d^4y}{dx^4} + 6\dfrac{d^2y}{dx^2} + 9y = 0$ Ans: $y = (c_1 + c_2 x)\cos\sqrt{3}x + (c_3 + c_4 x)\sin\sqrt{3}x$]
5. $\dfrac{d^4y}{dx^4} + 2\dfrac{d^2y}{dx^2} + y = 0$
6. $\dfrac{d^4y}{dx^4} + y = 0$

[Ans: $y = e^{(x/\sqrt{2})}\left[c_1\cos(x/\sqrt{2}) + c_2\sin(x/\sqrt{2})\right] + e^{-(x/\sqrt{2})}\left[c_3\cos(x/\sqrt{2}) + c_4\sin(x/\sqrt{2})\right]$]

7. $(D^3 - D^2 + D - 1)y = 0$
8. $(D^3 - 3D^2 + 4)y = 0$
9. $(D^2 - 1)(D - 1)^2\, y = 0$
10. $(D^4 + 8D^2 + 16)y = 0$ [Ans: $y = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x$]
11. $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0$ [Ans: $y = e^{x}\left[(c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x\right]$]

12. $\left(D^2 + 2D + 1\right)y = 0$
13. Solve $(D^2 - 2D + 1)y = e^x + 1$
14. Solve $\frac{d^2y}{dx^2} + 3\ \frac{dy}{dx} + 2y = e^{-x}$
15. Solve $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = e^x + 1$
16. Solve $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + \ ab\ y = e^{ax} + e^{bx}$
17. Solve $\frac{d^3y}{dx^3} - 4\ \frac{dy}{dx} = 2\, cos\, h^2\ 2x$
18. Solve $(2D + 1)^2 y = 4e^{-\frac{x}{2}}$
19. Solve $(D^4 + 1)y = cos\, h\ \ 4x\, sin\, h\ \ 3x$
20. Solve $\frac{d^2y}{dx^2} + y = sin\, x\, sin\, 2x$
21. Solve $(D^3 + D^2 + D + 1)y = sin^2 x$
22. Solve $(D^4 + 10D^2 + 9)y = 96\, sin\, 2x\, cos\, x$
23. Solve $(D^3 + 2D^2 + D)y = x^2 + x$
24. Solve $(D^2 - D - 2)y = 2\log x + \frac{1}{x} + \frac{1}{x^2}$
25. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$
26. Solve $\frac{d^2y}{dx^2} + y = cosec\ x$
27. Solve $(D^2 - 4)y = x\sin hx$

Answer: $\left[y = c_1e^{2x} + c_2e^{-2x} - \frac{x}{3}\sin hx - \frac{2}{9}\cos hx\right]$

28. Solve $(D^2 - 1)y = x^2\sin 3x$

Answer: $\left[y = c_1e^x + c_2e^{-x} - \frac{1}{10}\left\{x^2\sin 3x + \frac{6}{5}x\cos x - \frac{13}{25}\sin 3x\ \right\}\right]$

29. Solve $(D^4 + 2D^2 + 1)y = x^2\cos x$

Answer: $\left[y = (c_1 + c_2x)\cos x + (c_3 + c_4x)\sin x - \frac{1}{48}(x^4 - 9x^2)\cos x + \frac{x^3}{12}\sin x\right]$

30. Solve $(D^2 + 1)y = x^2\sin 2x$
31. Solve $(D^2 - 1)y = e^x\sin 3x$

Answer: $\left[y = c_1e^x + c_2e^{-x} - \frac{e^x}{117}(6\cos 3x + 9\sin 3x)\right]$

32. Solve $([D^3 - D^2 - D + 1])y = \cos hx\sin x$

Answer: $\left[y = (c_1 + c_2x)e^x + c_3e^{-x} + \frac{1}{10}e^x(\cos x - 2\sin x) - \frac{e^{-x}}{50}(3\cos x - 4\sin x)\right]$

33. Solve $(D^2 - 1)y = x\sin hx$

Answer: $\left[y = c_1e^x + c_2e^{-x} + \frac{x^2}{4}\cos hx - \frac{x}{4}\sin hx\right]$

Hint Put $\left[\sin hx = \frac{e^x - e^{-x}}{2}, \cos hx = \frac{e^x + e^{-x}}{2}\right]$

34. Solve $(D^3 - 3D^2 + 3D - 1)y = x\ e^x + e^x$

Answer: $\left[y = (c_1 + c_2x + c_3x^2)e^x + e^x\frac{x^4}{24} + e^x\frac{x^3}{6}\right]$

35. Solve $(D^2 - 2D + 1)y = \frac{3e^x}{x^2}$

Answer: $[y = (c_1 + c_2x)e^x - 3e^x \log x]$

36. Solve $(D^2 + 3D + 2)y = e^{-2x} + e^x \cos 2x$

37. Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos 3x$

38. Solve $(D^2 - 3D + 2)y = 2e^x \sin\left(\frac{x}{2}\right)$

39. Solve $(D^3 - 7D - 6)y = e^{2x}(x + 1)$

40. Solve $(D^3 - 2D + 4)y = 3x^2 - 5x + 2$

Answer: $\left[y = c_1e^{-2x} + e^x(c_2\cos x + c_3\sin x) + \frac{1}{4}[3x^2 - 2x + 1]\right]$

Hint: $D^3 - 2D + 4 = D^3 + 2D^2 - 2D^2 - 4D + 2D + 4 = 0 = (D + 2)(D^2 - 2D + 2) = 0$

$$= D = -2,\ \ 1 \pm i$$

41. Solve $\frac{d^3y}{dt^3} + \frac{dy}{dt} = \cos t + t^3 + 3$

Answer: $\left[y = c_1 + c_2\cos t + c_3\sin t + \frac{1}{2}[-t\cos t + \sin t] + \frac{t^3}{3} + t\right]$

42. Solve $(D^2 + 4D + 4)y = x^2$ Answer: $[y = (c_1 + c_2x)e^{-2x} + \frac{1}{4}[x^2 - 2x + \frac{3}{2}]$

43. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = x - 2x^2$

Answer: $\left[y = e^{-x}\left(c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x\right) + \frac{1}{3}\left(-\frac{10}{9} + \frac{11}{3}x - 2x^2\right)\right]$

44. Solve $(D^3 - D^2 + 6D)y = x^2 + \sin x$

Answer: $\left[y = c_1 + c_2e^{-2x} + c_3e^{-3x} - \frac{1}{6}\left(\frac{x^3}{3} - \frac{x^2}{6} + \frac{7x}{18}\right) + \frac{1}{50}(\sin x + 7\cos x)\right]$

45. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x\cos\left(\frac{x}{2}\right)$ [Answer : $\therefore y = c_1e^x + c_2e^{2x} + \frac{8}{5}e^x\left[-2\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)\right]$]