

THE NORMAL DISTRIBUTION

1. If X is normal variate with mean 25 and standard deviation 5, find the value (i) of $X=x_1$, Such that $P(X \geq x_1) = 0.32$, (ii) of $X=x_2$, such that $P(X \leq x_2) = 0.73$, (iii) of $X=x_3$ such that $P(X \leq x_3) = 0.24$.
 $z = 0.47$ $z = 0.61$ $z = -0.71$
 [27.35, 28.05, 21.45]
2. If X is a normal variate with mean 30 and standard deviation 5. Find the probabilities that $30 < X < 35, 35 < X < 40, 25 < X < 30, 20 < X < 30, X > 33.3, X > 26.2, X < 34.7, |X - 30| > 5$
3. In an examination marks obtained by students in Mathematics, Physics and Chemistry are normally distributed with means 51, 53 and 46 with standard deviation 15, 12, 16 respectively. Find the probability of securing total marks (i) 180 or above, (ii) 90 or below. [0.1151, 0.0082]
4. The marks obtained by students in a certain examination follow a normal distribution with mean 45 and standard deviation 10. If 1000 students appeared at an examination, calculate the number of students scoring (i) less than 40 marks, (ii) more than 60 marks. [309, 67]
5. The incomes of a group of 10,000 persons were found to be normally distributed with mean Rs. 520 and standard deviation Rs. 60. Find (i) the number of persons having incomes between Rs. 400 and 550, (ii) the lowest income of the richest 500. [6687, 618.7]
6. Find the mean and the standard deviation of a normal distribution of marks in an examination where 58% of the candidates obtained marks below 75, 4% got above 80 and the rest between 75 and 80. [74.4, 3.125]
7. In a distribution exactly normal 7% of items are under 35 and 89% are under 63. What are the mean and standard deviation? [50.3, 10.33]
8. The local authorities in a certain city installed 10,000 eclectic lamps in the streets of the city. If these lamps have average life of 1000 burning hours with a standard deviation of 200 hours, what number of lamps might be expected to fail (i) in the first 800 hours, (ii) between 800 and 1200 hours? After what period of burning hours would you expect that (i) 10% of the lamps would fail, (ii) 10% of the lamps would be still burning? [1587, 6826, 744, 1256].
9. The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement after 12 months? [2386]
10. If the marks in a particular subject are assumed to follow a normal distribution with mean 40 and variance 9, how many of 1000 students get marks (i) below 35 (ii) between 43 and 46. Find also range of marks obtained by the middle 50% of the students. [48, 136, 38 & 42]
11. Determine the probability that by guess work a student can correctly answer 25 to 30 questions in a multiple choice quiz consisting of 80 questions. Assume that in each question with 4 choices, only one is correct and the students have no knowledge. [0.1196].
12. Using normal distribution find the probability of getting 55 heads in the toss of 100 fair coins. [0.04839]
13. Steel rods are manufactured to be 3 inches in diameter but they are acceptable if they are inside the limits 2.99. Inches and 3.01 inches. It is observed that 5% are rejected as oversize and 5% are rejected as undersize. Assuming that the diameters are normally distributed, find the standard deviation of the distribution. Hence calculate what would be the proportion of rejects if the permissible limits were widened to 2.985 inches and 3.015 inches. [$\mu=3, \sigma=\frac{1}{163}, 1.34\%$]
14. Pipes for tobacco are being packed in fancy plastic boxes. The length of the pipes is normally distributed with mean 5" and SD 0.1". The internal length of the boxes is 5.2 inches. What is the probability that the box would be small for the pipe? [0.0228] $z > 5.2$

15. Assuming that the diameters of 1000 brass plug taken consecutively from a machine form a normal distribution with mean 0.7515 cm. and standard deviation 0.002 cm. How many of the e plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm. [52]

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16. In an engineering examination, a student is considered to have failed, secured second class, First class and distinction, according as he scores less than 45%, between 45% and 60 %, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students get distinction. Find the percentages of students who have get first class and second class. .
[$\mu=58.15$ $\sigma=10.28$, 38, 47]

17. A set of examination marks is approximately normally distributed with mean of 75 and standard deviation of 5. If the top 5% of students get grade A and the bottom 25% get grade F, what marks is the lowest A and what marks is the highest F? [72, 83]

18. Of a large group of men, 5% are under 60 inches in height and 40% are between 60 and 65 inches. Assuming a normal distribution, find the mean height and standard deviation. [$\mu=65.42$ and $\sigma=3.29$]

$$0.45 \rightarrow z = 0.125, \quad 0.05 \rightarrow z = 1.645$$

19. The marks obtained by the students in SS, EWT and ECI in an engineering exam are normally distributed with means 53, 51 and 49 and standard deviations 11, 9 and 6 respectively. Find the probability that a student selected at random has secured a total of (i) 181 or above and (ii) 134 or less. [

$$1. \quad \mu = 153, \quad \sigma = 15.43 \quad [0.0352, 0.1094]$$

20. The marks scored by 1000 students in an exam. is normally distributed with mean 40% and SD 10 %. Find approximately (i) how many will pass if 30% is fix min? (ii) What should be the min if 750 candidates to pass? [841, 33.25]

21. Find the equation of best fit normal curve to the following data

$$x: 2 \quad 4 \quad 6 \quad 8 \quad 10$$

$$f: 1 \quad 4 \quad 6 \quad 4 \quad 1 \quad [\mu=6, \sigma=2]$$

22. The probability that an electric component will fail in less than 1200 hrs of continuous use is 0.25. Using normal approx to binomial, find the probability that among 200 such components fewer than 45 will fail in less than 1200 hrs of continuous use. [0.1841]

23. A large number of automobile batteries have average life of 24 months. If 34 % of of them have average life b/w 22&26 months & 272 of them last longer than 29 months how many were in the group tested?. [2000]

24. The marks obtained by number of students in a certain subject are approximately normal with mean 65 & variance 25. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75? [0.0667].

25. If two independent random variables X & Y have distribution N (45, 2) and N (44, 1.5). respectively. What is the probability that randomly chosen values of X and Y differ by 1.5 or more? [0.5794].

26. If two independent random variables each having the distribution N (0, 3), what is the probability that the point (X,Y) lies b/w the lines $3X+4Y=5$ and $3X+4Y=10$. [0.1193]