Module 2 Kinematics of Particles & Rigid Bodies

7/10/2

Date __/__/____ Kinematics of Particle

Introduction

Engg. mechanics involves the study of both statics & dynamics. Statics is concerned with the equilibrium of bodies at rest whereas dynamics is concerned with analysis of bodies in motion.

Dynamics is divided into 2 parts.

i) Kinematics : It deals with study of the geometry of motion. Kinematics is used to relate the displacement velocity, accelerat^n & time w/o reference to the cause of motion.

ii) Kinetics : It deals with study of the relat^n existing b/w the forces acting on the body, the mass of the body & the mot^n of the body.

Types of motion

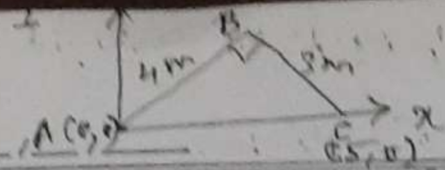
1) Rectilinear motion : When the particle is moving along a straight line it is called rectilinear motion.

2) Curvilinear motion : When the particle moves along a curved path, it is called curvilinear motion. Curved path may be circular, elliptical, parabolic, hyperbolic or any other curve.

Particle - A body with negligible dimensions is described as a particle. Physically a body can be considered as a particle

Rigid body - A body comprising infinite no. of particles whose relative position remain unchanged under the action of the force is called rigid body

Page No.

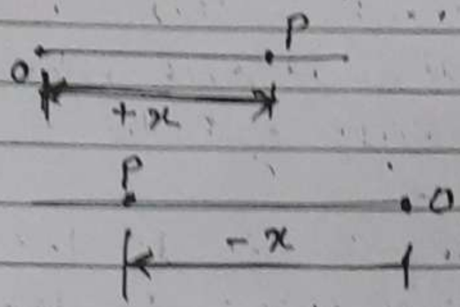


Distance = AB + BC = 4 + 3 = 7 units

Displacement = AC = $\sqrt{AB^2 + BC^2}$

$= \sqrt{4^2 + 3^2} = 5$ units along x-dir^n

Position – The location of a particle along a straight line w.r.t. reference pt. or datum represents the position of the particle.



Distance & Displacement: Distance is the actual path length covered by a moving particle or a body in a given interval of time.

Displacement: It is the shortest distance b/w initial position & final position of the particle.

Average speed & velocity: The avg. speed of a particle in a given interval of time is defined as the ratio of distance travelled to the time taken while avg. velocity is defined as the ratio of displacement to the time taken.

$$V_{avg} = \text{Avg. speed} = \frac{\Delta s}{\Delta t} \rightarrow \text{distance travelled}$$

$$\bar{v} = \text{Avg. velocity} = \frac{\Delta \vec{x}}{\Delta t} \rightarrow \text{Displacement}$$

Instantaneous speed & velocity – They are defined at a particular instant when time interval approaches to zero.

$$\text{Inst. speed}, v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$\text{Inst. velocity} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

O (0) —20m→ A (20)   Distance = OA + AB
←30m— A   = 20 + 30 = 50 m
B   Displacement = OA − AB
= 20 − 30 = −10 m

Date ___/___/_____

Avg & Instantaneous acceleration: Avg. accel^n is defined as the ratio of change in velocity $\Delta\bar{v}$ to the time interval Δt in which this change has occured

Avg. accel^n = $\bar{a}_{avg} = \frac{\Delta\bar{v}}{\Delta t}$

Inst. accel^n $\bar{a} = \lim_{\Delta t \to 0} \frac{\Delta\bar{v}}{\Delta t} = \frac{d\bar{v}}{dt}$

Uniform ~~moto~~^{n} or const velocity motion in 1-dimension

When particle undergoes same amount of displacement in equal intervals of time, then the particle is said to be moving with uniform velocity or constant velocity.

$s = vt$ or $x = vt$ or $y = vt$

Uniform acceleration motion

$v = u + at \longrightarrow$ (1)

$v^2 = u^2 + 2as \longrightarrow$ (2)

$s = ut + \frac{1}{2}at^2 \longrightarrow$ (3)

$s = s_0 + ut + \frac{1}{2}at^2 \longrightarrow$ (4) (s_0 is the distance @ $t = 0$)

Variable accel^n motion

When accel^n of particle is not constant, we use basic eq^{ns} of velocity & accel^n i.e.

i) $v = \frac{ds}{dt}$ or $\frac{dx}{dt}$ or $\frac{dy}{dt}$ $\therefore ds$ or dx or $dy = v\,dt$

ii) $a = \frac{dv}{dt}$ $\therefore dv = a\,dt$

iii) $a = \frac{v\,dv}{ds}$ or $\frac{v\,dv}{dx}$ or $\frac{v\,dv}{dy}$ $\therefore v\,dv = a\,ds$ or $a\,dx$ or $a\,dy$

2/10/22

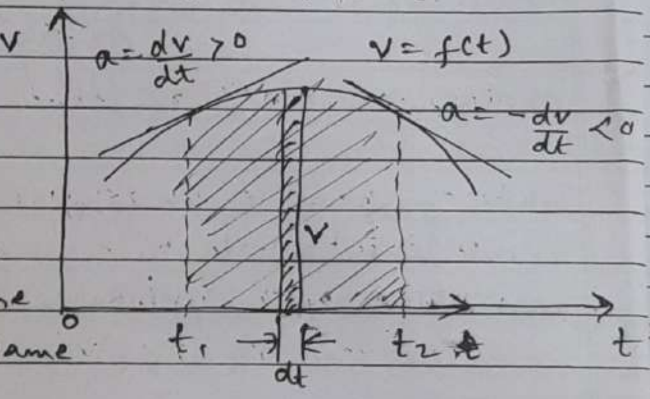
Date __/__/____

Saathi

Graphical solⁿ - Motion diagrams

Motion diagrams or motion curves are graphical representat[ion] of variation of motion parameters (accelⁿ, velocity & displacement) with time for the rectilinear motⁿ of the particle. Commonly used motion curves are

i) v-t diagram



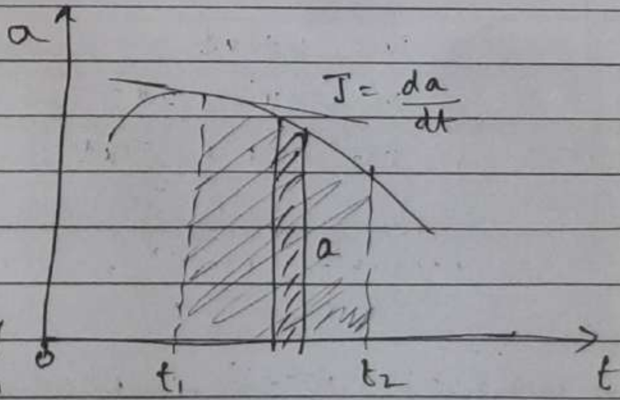
Slope of v-t diagram = Accelⁿ

$$\frac{dv}{dt} = a$$

Area under v-t diagram b/w 2 intervals = change in positⁿ of the particle in same interval

$$\text{Area } A = (x_2 - x_1) = \Delta x$$

ii) a-t diagram



Slope of a-t diagram = Jerk

$$\frac{da}{dt} = J$$

Area under a-t diagram b/w 2 intervals = Change in velocity of the particle in the same interval

$$A = (v_2 - v_1) = \Delta v$$

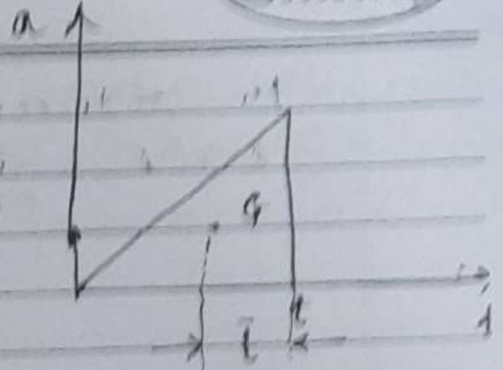
a-t diagram can be used for calculating the positⁿ of the particle @ any time t

$$x_t = x_0 + v_0 \times t + A \times \bar{t}$$

x_t = position of the particle @ time t

Page No.

Date ___/___/_____



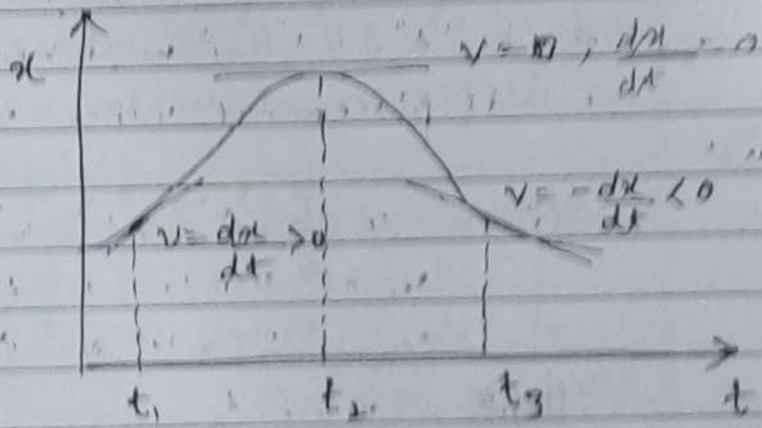
x_0 = Initial position @ $t = 0$.
v_0 = Initial velocity @ $t = 0$
A = Area of a-t diagram from 0 to t
$\bar{t}$ = Distance of centroid of area measured from time t along x-axis

positive area indicates increase in velocity & -ve area indicates decrease in velocity

(iii) x-t diagram

→ ~~In x-t diagram, position~~
slope of x-t diagram = velocity

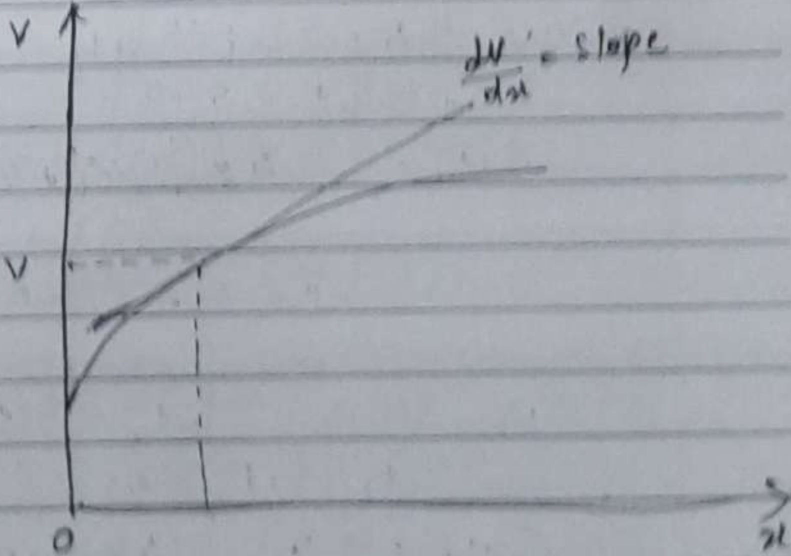
$$\frac{dx}{dt} = v$$



(iv) v-x diagram

acceln $a = v\frac{dv}{dx}$

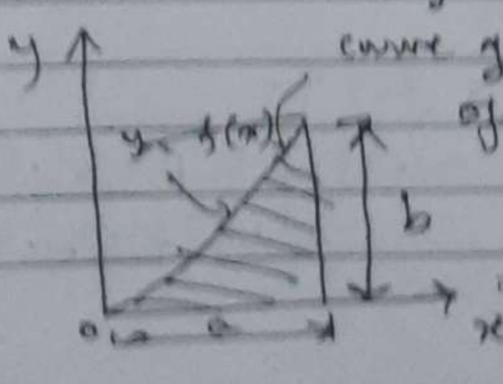
$= v \times$ (slope of v-x curve)



For inward curve, area of the curve is given by

Area $A = \frac{ab}{n+1}$

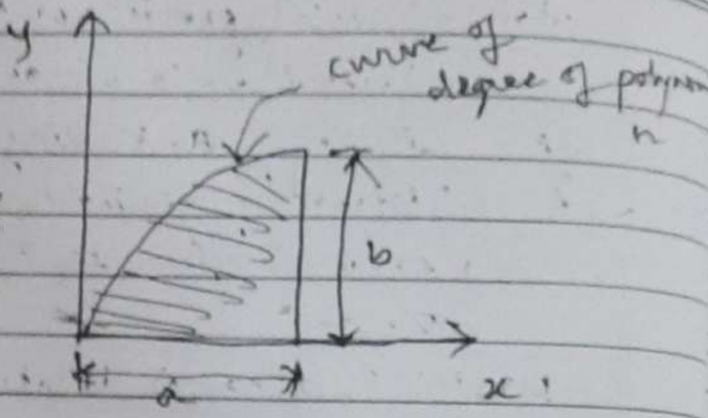
curve of degree of polynomial 'n'



Date ___/___/___

For outward curve.

Area $A = \dfrac{nab}{n+1}$



Ex 22 — A particle has a straight line motn given by eqn $x = (t^3 - 2t^2 - 4)$ m, where t is in sec. What is the cha in displacement when velocity changes from 4 m/s to 32

soln

Given $x = (t^3 - 2t^2 - 4)$ $\qquad \Delta x = ?$

$V_1 = 4$ m/s @ $t = t_1$

$V_2 = 32$ m/s @ $t = t_2$

Diff. x w.r.t. t

$\therefore v = \dfrac{dx}{dt} = 3t^2 - 4t$

$v_1 = 4 = 3t_1^2 - 4t_1$

$3t_1^2 - 4t_1 - 4 = 0$

$t_1 = \dfrac{4 \pm \sqrt{(4)^2 + 4\times3\times4}}{2\times3} = \dfrac{4 \pm \sqrt{16+48}}{6}$

$= \dfrac{4 \pm 8}{6} = 2$ sec

$V_2 = 32 = 3t_2^2 - 4t_2$

$3t_2^2 - 4t_2 - 32 = 0$

$t_2 = \dfrac{4 \pm \sqrt{16 + 4\times3\times32}}{2\times3} = \dfrac{4 \pm 20}{6} = 4$ sec

$\Delta x = x_2 - x_1$

$= (t_2^3 - t_1^3) - 2(t_2^2 - t_1^2) - (4 - 4)$

$= (4^3 - 2^3) - 2(4^2 - 2^2) = 32$ m Ans

Saathi

Date ___/___/______

25 During a test, the car moves in a straight line such that its velocity is defined by $v = 0.3(9t^2 + 2t)$ m/s. where t is in sec. Determine the position & accel^n when t = 3 sec. Given @ t = 0, s = 0

Sol^n

Given $v = 0.3(9t^2 + 2t)$

$\therefore \frac{ds}{dt} = 0.3(9t^2 + 2t)$

$ds = 0.3(9t^2 + 2t)\,dt$.

Integrating both sides

$$\int_0^s ds = \int_0^3 0.3(9t^2 + 2t)\,dt$$

$$s = 0.3\left[\frac{9t^3}{3} + \frac{2t^2}{2}\right]_0^3 = 0.3\left[3t^3 + t^2\right]_0^3$$

$$= 0.3\left[3(3)^3 + 3^2 - 0\right] = 0.3[90] = \underline{\underline{27\text{ m}}}\ \text{Ans}$$

$$\text{Accel}^n,\ a = \frac{dv}{dt} = 0.3(18t + 2)$$

$$a\Big]_{t=3\text{ sec}} = 0.3(18 \times 3 + 2) = \underline{\underline{16.8\text{ m/s}^2}}\ \text{Ans}$$

26 A sphere is fired downward into a medium with an initial speed of 27 m/s. Sphere experiences a deceler $a = -6t$ m/s^2 where t is in sec, determine the distance travelled before it comes to rest

Sol^n Here we take y to be +ve in the downward dir^n

Given, $a = -6t$

$\frac{dv}{dt} = -6t$

$dv = -6t\,dt$

Integrating both sides

Saathi

Date ___/___/_____

$$v = \frac{-6t^2}{2} + C = -3t^2 + C$$

Using B.C. @ $t = 0$, $v = 27$ m/s

$$27 = 0 + C \Rightarrow \boxed{C = 27}$$

$$\therefore v = -3t^2 + 27 \longrightarrow (1)$$

$$\frac{dy}{dt} = -3t^2 + 27$$

$$dy = (-3t^2 + 27)\,dt$$

Integrating

$$y = \frac{-3t^3}{3} + 27t + C_1 = -t^3 + 27t + C_1$$

Using B.C. @ $t = 0$, $y = 0$

$$\Rightarrow 0 = 0 + 0 + C_1 \Rightarrow \boxed{C_1 = 0}$$

$$\therefore y = -t^3 + 27t \longrightarrow (2)$$

To find the time when the sphere come to rest,

put $v = 0$ in eqn (1)

$$0 = -3t^2 + 27$$

$$\Rightarrow t = 3 \text{ sec}$$

put $t = 3$ sec in eqn (2)

$$y = -3^3 + 27 \times 3 = -27 + 81 = \underline{54\,m\,(\downarrow)} \text{ Ans.}$$

x.27 The accel^n of an oscillating particle is defined by the relation $a = -Kx$. Determine (a) the value of K such that $v = 15$ m/s when $x = 0$ & $v = 0$ when $x = 3$m (b) the speed of the particle when $x = 2$m.

Sol^n Given relation $a = -Kx$

$$\therefore \frac{v\,dv}{dx} = -Kx$$

$$v\,dv = -Kx\,dx \longrightarrow (1)$$

Page No.

Saathi

Date ___/___/_____

Using B.C. @ $x = 0$, $v = 15$ m/s

& @ $x = 3$ m, $v = 0$

Integrating eqn (1)

$$\int_{15}^{0} v\,dv = -K \int_{0}^{3} x\,dx$$

$$\left.\frac{v^2}{2}\right]_{15}^{0} = -K\left[\frac{x^2}{2}\right]_0^3$$

$$\left[0 - \frac{15^2}{2}\right] = -K\left[\frac{3^2}{2} - 0\right]$$

$$\frac{225}{\cancel{2}} = \frac{9K}{\cancel{2}} \Rightarrow K = \underline{\underline{25}} \quad \underline{\underline{\text{Ans}}}$$

Using B.C. @ $x = 3$ m, $v = 0$

& @ $x = 2$ m, $v = v$

Integrating eqn (1)

$$\int_{0}^{v} v\,dv = -K \int_{3}^{2} x\,dx$$

$$\left.\frac{v^2}{2}\right]_{0}^{v} = -25\left[\frac{x^2}{2}\right]_3^2$$

$$\frac{v^2}{\cancel{2}} = -25\left[\frac{2^2}{2} - \frac{3^2}{2}\right] = -25\left[\frac{-5}{2}\right] = \frac{125}{\cancel{2}}$$

$$v = \underline{\underline{11.18 \text{ m/s}}} \quad \underline{\underline{\text{Ans}}}$$

Q.29 The accel^n of a particle is defined by the relation $a = -60x^{-1.5}$. Knowing that particle starts from rest from a position $x = 4$ m, find the velocity of the particle when (a) $x = 0.5$ m (b) $x = 1.5$ m & the

Saathi

Date ___/___/______

(c) position of the particle when velocity is 10 m/s.

Sol^n^ :-

$$a = -60x^{-1.5}$$

$$\frac{v\,dv}{dx} = -60x^{-1.5}$$

$$v\,dv = -60x^{-1.5}\,dx$$

Integrating both sides.

$$\int v\,dv = -60\int x^{-1.5}\,dx$$

$$\frac{v^2}{2} = -60\left[\frac{x^{-1.5+1}}{-1.5+1}\right] + C = \frac{120}{\sqrt{x}} + C$$

Using B.C. @ $x = 4m$, $v = 0$

$$0 = \frac{120}{\sqrt{4}} + C \Rightarrow \boxed{C = -60}$$

$$\therefore \frac{v^2}{2} = \frac{120}{\sqrt{x}} - 60 \longrightarrow (1)$$

a] @ $x = 0.5m$, $\frac{v^2}{2} = \frac{120}{\sqrt{0.5}} - 60$

$\Rightarrow v = 14.813$ m/s Ans

b] @ $x = 1.5m$, $\frac{v^2}{2} = \frac{120}{\sqrt{1.5}} - 60$

$\Rightarrow v = 8.715$ m/s Ans.

c] @ $v = 10$ m/s, $\frac{10^2}{2} = \frac{120}{\sqrt{x}} - 60$

$\Rightarrow x = 1.19$ m Ans.

Saathi

Date ___/___/_____

The accelⁿ of a particle is defined by the relatⁿ $a = K[1 - e^{-x}]$. Knowing that, $v = 6$ m/s when $x = -2$ m & particle comes to rest @ origin, determine (a) value of K (b) velocity of the particle when $x = -1$ m

a) Given $a = K(1 - e^{-x})$

$$v\frac{dv}{dx} = K(1 - e^{-x})$$

$$v\,dv = K(1 - e^{-x})\,dx$$

Integrating

$$\int_0^6 v\,dv = \int_0^{-2} K(1 - e^{-x})\,dx$$

$$\left.\frac{v^2}{2}\right]_0^6 = K\left[x + e^{-x}\right]_0^{-2}$$

$$\frac{6^2}{2} = K\left[-2 + e^{+2} - 0 - e^{-0}\right]$$

$\underline{K = 4.1}$ $\underline{\text{Ans.}}$

b) $v\Big]_{x = -1\,m}$? , $a = 4.1(1 - e^{-x})$

$$v\frac{dv}{dx} = 4.1(1 - e^{-x})$$

$$v\,dv = 4.1(1 - e^{-x})\,dx$$

Integrating,

$$\int_0^v v\,dv = \int_0^{-1} 4.1(1 - e^{-x})\,dx$$

$$\left.\frac{v^2}{2}\right]_0^v = 4.1\left[x + e^{-x}\right]_0^{-1}$$

$$\frac{v^2}{2} = 4.1\left[-1 + e^{+1} - 0 - e^{-0}\right]$$

$$v = 2.427 \text{ m/s}$$

Page No. ___

Saathi

Date __/__/__

A rectilinear motⁿ of a car is governed by the eqⁿ $a = \left(\frac{8}{1.5v+2}\right)$ m/s², where v is in m/s. Assuming that the car starts from rest, find the time taken & distance covered by the motor car to attain the velocity of 8 m/s.

$$a = \frac{8}{1.5v+2}$$

$$\frac{dv}{dt} = \frac{8}{1.5v+2}$$

$$(1.5v+2)\,dv = 8\,dt$$

Integrating

$$1.5\frac{v^2}{2} + 2v = 8t + c$$

Using B.C. @ $t=0$, $v=0 \Rightarrow c=0$

$$\therefore \frac{1.5v^2}{2} + 2v = 8t \longrightarrow (1)$$

a) put $v = 8$ m/s in eqⁿ (1)

$$\frac{1.5 \times 8^2}{2} + 2 \times 8 = 8t$$

$$\underline{t = 8 \text{ sec.}} \quad \underline{\text{Ans}}$$

$$a = \frac{8}{1.5v+2}$$

$$\frac{v\,dv}{ds} = \frac{8}{1.5v+2}$$

$(1.5v^2 + 2v)\,dv = 8\,ds$ Integrating

$$1.5\frac{v^3}{3} + \frac{2v^2}{2} = 8s + c$$

Using B.C. @ $t=0$, $s=0 \Rightarrow \boxed{c=0}$

Page No.

Saathi

Date ___/___/___

$\Rightarrow \frac{1.5 v^3}{3} + v^2 = 8s$ —— (2)

b) put $v = 8$ m/s in eqn (2)

$\frac{1.5 \times 8^3}{3} + 8^2 = 8s$

$\therefore s = 40m$ Ans

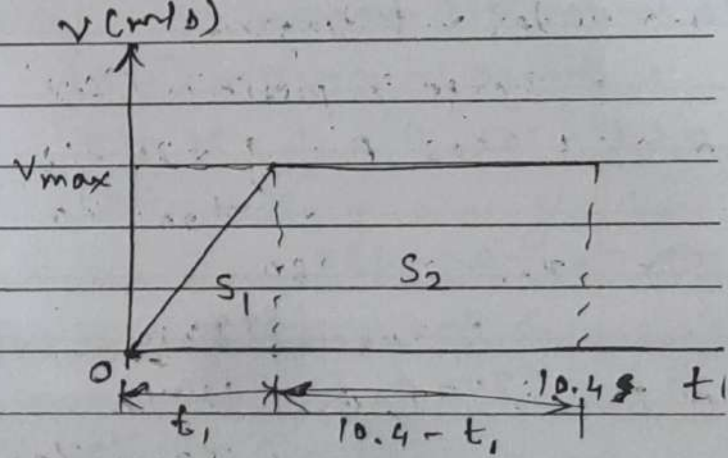
Q.39 In an asian games event of 100m run, an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4m & runs the remaining distance with that velocity. If the athlete completes the race in 10.4 sec, determine a) his initial acceln b) his max. velocity

10/22

Soln

Representing the data on v-t graph

Distance travelled = Area under v-t graph



$0 \le t \le t_1$

$A_1 = \frac{1}{2} \times V_{max} \times t_1 = 4$

$\therefore t_1 = \frac{8}{V_{max}}$ —— (1)

For $t_1 \le t \le 10.4$,

$A_2 = (10.4 - t_1) V_{max} = 96$ —— (2)

sub. t_1 from eqn (1) into eqn (2)

$\left(10.4 - \frac{8}{V_{max}}\right) \times V_{max} = 96$

$10.4 V_{max} - 8 = 96$

$\therefore V_{max} = 10$ m/s Ans.

$t_1 = \frac{8}{V_{max}} = \frac{8}{10} = 0.8$ sec

Date ___/___/______

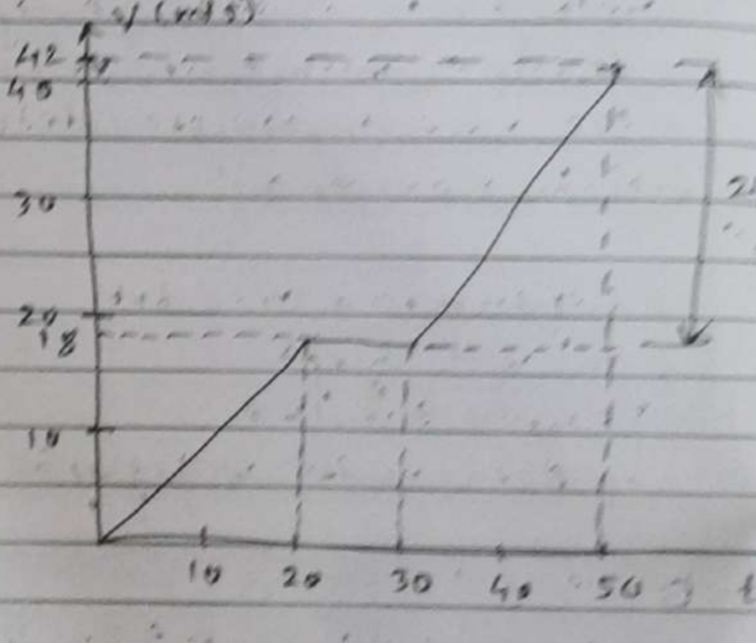
Initial accelⁿ = Slope of v-t diagram from 0 to 0.8

$a = \frac{dv}{dt} = \frac{v_{max} - v_0}{t_1 - 0} = \frac{10-0}{0.8} = \underline{12.5\ m/s^2}$

Q.42 The race car starts from rest & travels along a straight road until it reaches a speed of 42 m/s in 50 sec, shown by v-t graph. Determine the distance traveled by the race car in 50 sec. Draw x-t & a-t graph.

Ansⁿ.

Given Initial condⁿ.
@ $t = 0$, $v_0 = 0$, $x_0 = 0$



In x-t diagram
Area under v-t diagram = change in position (Δx)

In $0 \le t \le 20s$ $A_1 = \frac{1}{2} \times 20 \times 18 = x_{20} - x_0$

$\Rightarrow x_{20} - 0 = 180$

$\therefore x_{20} = 180\ m$

in $20 \le t \le 30$, $A_2 = 18 \times 10 = x_{30} - x_{20}$

$\Rightarrow 180 = x_{30} - 180$

$\therefore x_{30} = 360\ m$

For $30 \le t \le 50$, $A_3 = \frac{1}{2} \times 20 \times 24 + 18 \times 20 = x_{50} - x_{30}$

$\Rightarrow 600 = x_{50} - 360$ $\therefore x_{50} = 960\ m$

in $0 \le t \le 20s$, $a = \frac{dv}{dt} = \frac{v_{20} - v_0}{20 - 0} = \frac{18-0}{20} = 0.9\ m/s^2$

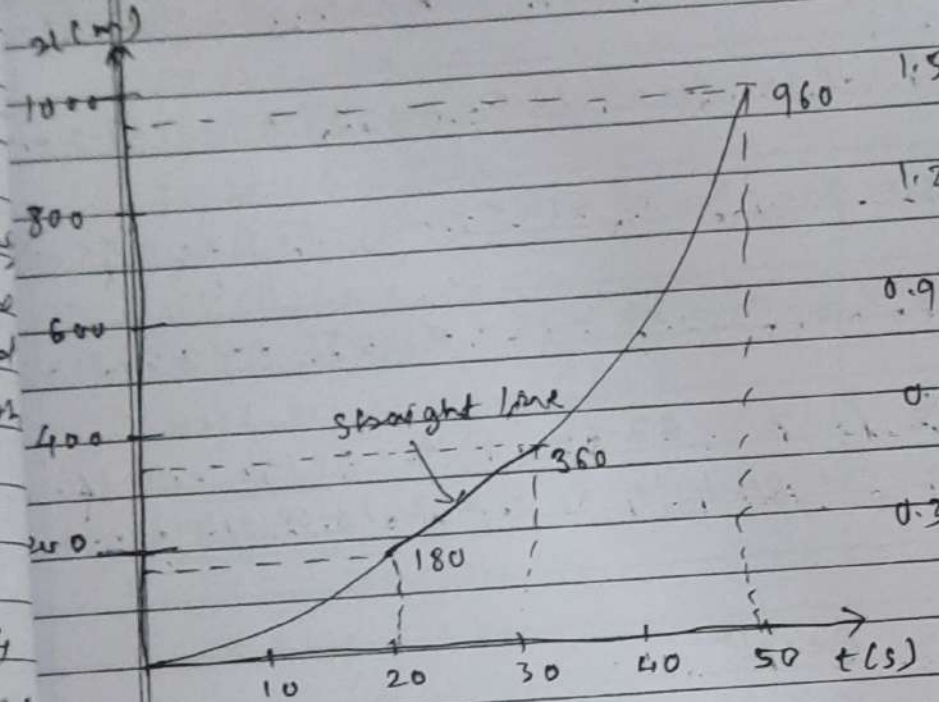
in $20 \le t \le 30s$, $a = \frac{dv}{dt} = \frac{v_{30} - v_{20}}{30 - 20} = \frac{18-18}{10} = 0$

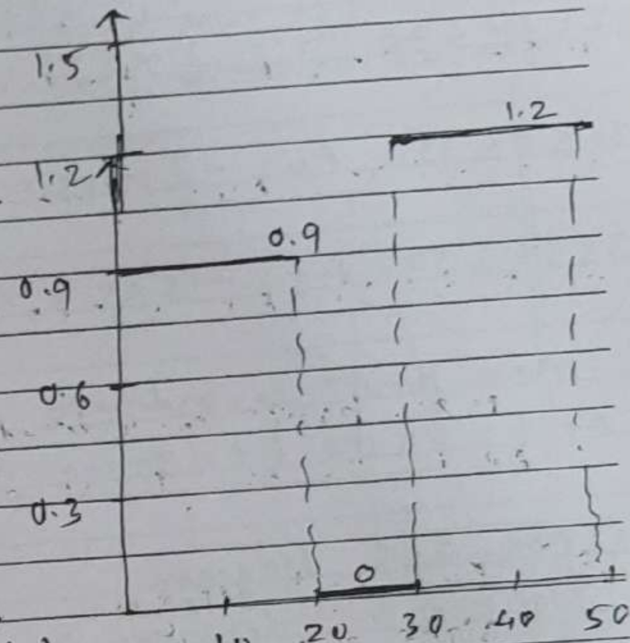
in $30 \le t \le 50$, $a = \frac{dv}{dt} = \frac{v_{50} - v_{30}}{50 - 30} = \frac{42-18}{20} = 1.2\ m/s^2$

Saathi

Date __/__/__

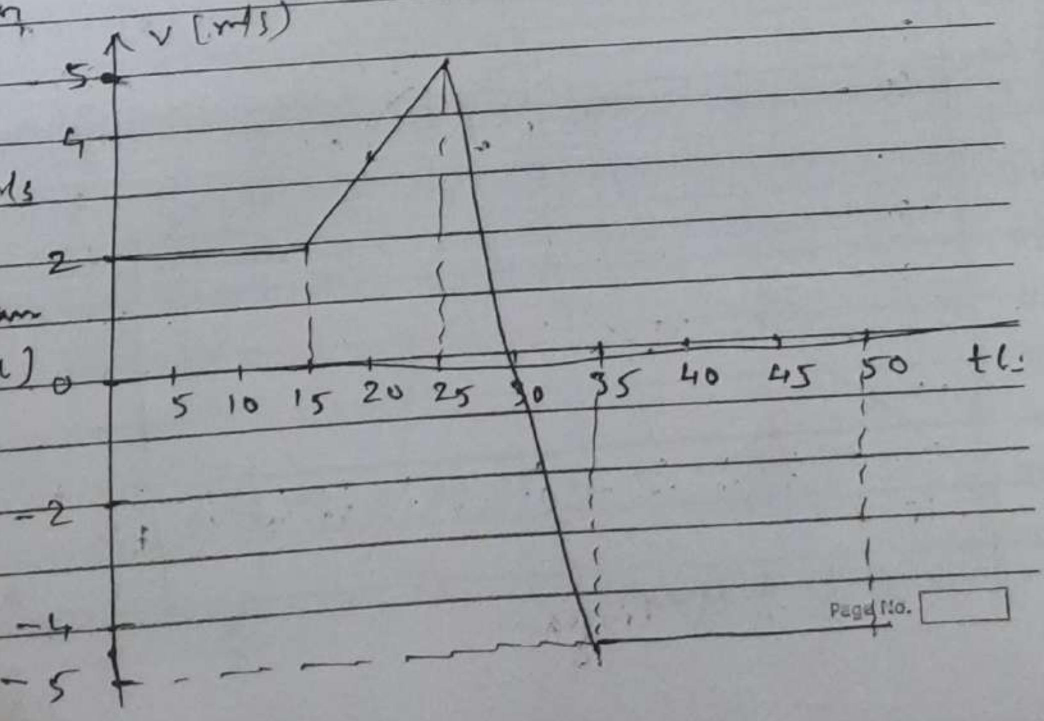
Distance travelled by race car in 50 s is $s = 960\,m$ Ans.





x - t diagram

44 The v-t diagram for a particle moving along straight line is shown in fig. Knowing that $x = -10\,m$ @ $t = 0$ for $0 < t < 50\,s$. a) plot x-t & a-t diagram b) Determine the max. value of position coordinate & the value of t for which the particle is at a distance of 55 m from the origin



30 en I.C.

$t = 0$ $x_0 = -10\,m$, $v_0 = 2\,m/s$

For x-t diagram

ea under v-t diagram

2 Change in positn (Δx)

m/s^2

Proportion

Saathi

Date __/__/__

for $0 \le t \le 15s$ $A_1 = 15 \times 2 = x_{15} - x_0 \Rightarrow 30 = x_{15} - (-10) \therefore x_{15} = 20m$

for $15s \le t \le 25s$ $A_2 = \left(\frac{2+5}{2}\right) \times 10 = x_{25} - x_{15} \Rightarrow 35 = x_{25} - 20 \therefore x_{25} = 55m$

for $25 \le t \le 30$ $A_3 = \frac{1}{2} \times 5 \times 5 = x_{30} - x_{25} \Rightarrow 12.5 = x_{30} - 55 \therefore x_{30} = 67.5m$

$30 \le t \le 35s$ $A_4 = -\frac{1}{2} \times 5 \times 5 = x_{35} - x_{30} \Rightarrow -12.5 = x_{35} - 67.5$ $x_{35} = 55m$

for $35 \le t \le 50s$ $A_5 = -15 \times 5 = x_{50} - x_{35} \Rightarrow -75 = x_{50} - 55 \therefore x_{50} = -20m$

Max. positn co-ordinate is $x = 67.5m$ @ $t = 30sec$ Ans

At $t = 25sec$ & $t = 35sec$, the particle is @ a distance of 55m Ans

b) For a-t diagram

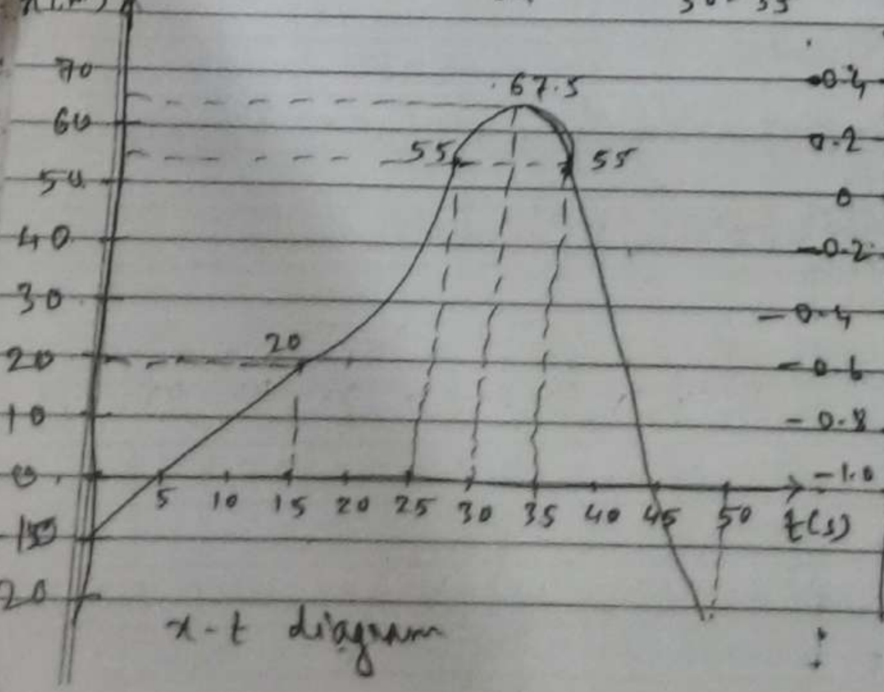
slope of v-t diagram = Acceln.

for $0 \le t \le 15s$ $a = \frac{dv}{dt} = \frac{v_{15} - v_0}{15 - 0} = \frac{2-2}{15} = 0$

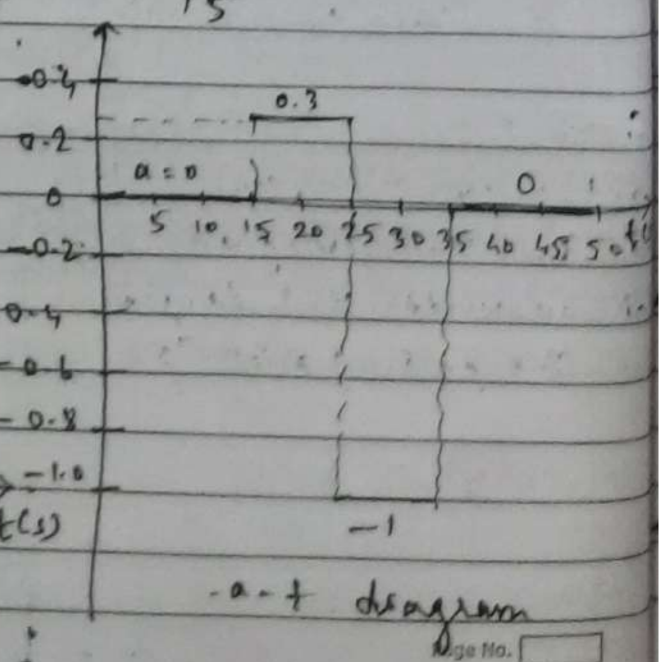
For $15 \le t \le 25s$ $a = \frac{dv}{dt} = \frac{v_{25} - v_{15}}{25 - 15} = \frac{5-2}{10} = 0.3 \, m/s^2$

$25 \le t \le 35s$ $a = \frac{dv}{dt} = \frac{v_{35} - v_{25}}{35 - 25} = \frac{-5-5}{10} = -1 \, m/s^2$

for $35 \le t \le 50s$ $a = \frac{dv}{dt} = \frac{v_{50} - v_{35}}{50 - 35} = \frac{-5-(-5)}{15} = 0$



x-t diagram

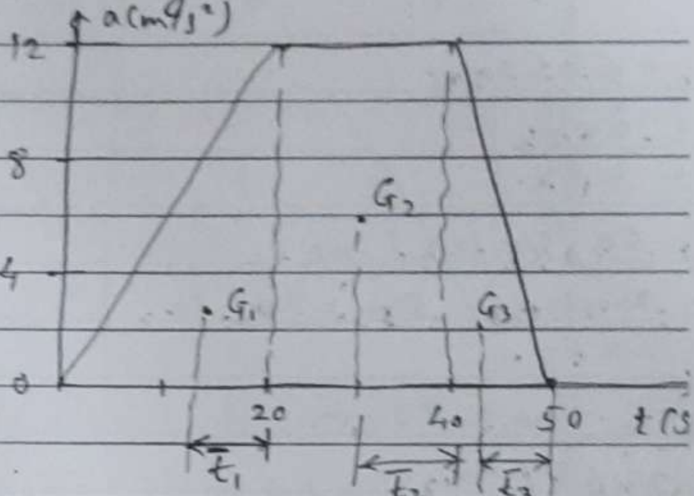


a-t diagram

Page No.

Date ___/___/___

Q.47 Fig. shows a plot of a v/s t for a particle moving along x-axis. What is the speed & distance covered by the particle after 50 sec? Find also the maximum speed & time at which the speed attained by the particle. Draw v-t & x-t diagram



I.C. @ $t = 0$, $x_0 = 0$, $v_0 = 0$

Area under a-t diagram
= change in velocity (Δv)

$0 \le t \le 20\,s$

$A_1 = \frac{1}{2} \times 20 \times 12 = 120 = v_{20} - v_0$
$= v_{20} - 0$
$\therefore v_{20} = 120$ m/s

$20 \le t \le 40\,s$ $A_2 = 20 \times 12 = 240 = v_{40} - v_{20} = v_{40} - 120 \therefore v_{40} = 360$

$40 \le t \le 50\,s$ $A_3 = \frac{1}{2} \times 10 \times 12 = 60 = v_{50} - v_{40} = v_{50} - 360 \therefore v_{50} = 420$ m/s

To find position of particle, we use
$x_t = x_0 + v_0 t + A\bar{t}$

$0 \le t \le 20\,s$, $x_{20} = x_0 + v_0 \times t_{(20-0)} + A_1 \bar{t}_1$
$= 0 + 0 + 120 \times \frac{1}{3} \times 20$

$= 800$ m

$20 \le t \le 40\,s$, $x_{40} = x_{20} + v_{20} \times t_{(40-20)} + A_2 \bar{t}_2$
$= 800 + 120 \times 20 + 240 \times \frac{1}{2} \times 20$

$= 5600$ m

$40 \le t \le 50\,s$, $x_{50} = x_{40} + v_{40} \times t_{(50-40)} + A_3 \bar{t}_3$
$= 5600 + 360 \times 10 + 60 \times \frac{2}{3} \times 10$

$= 9600$ m

Speed & distance covered by particle after 50 sec is 420 m/s & 9600 m resp. Ans

Date ___/___/___

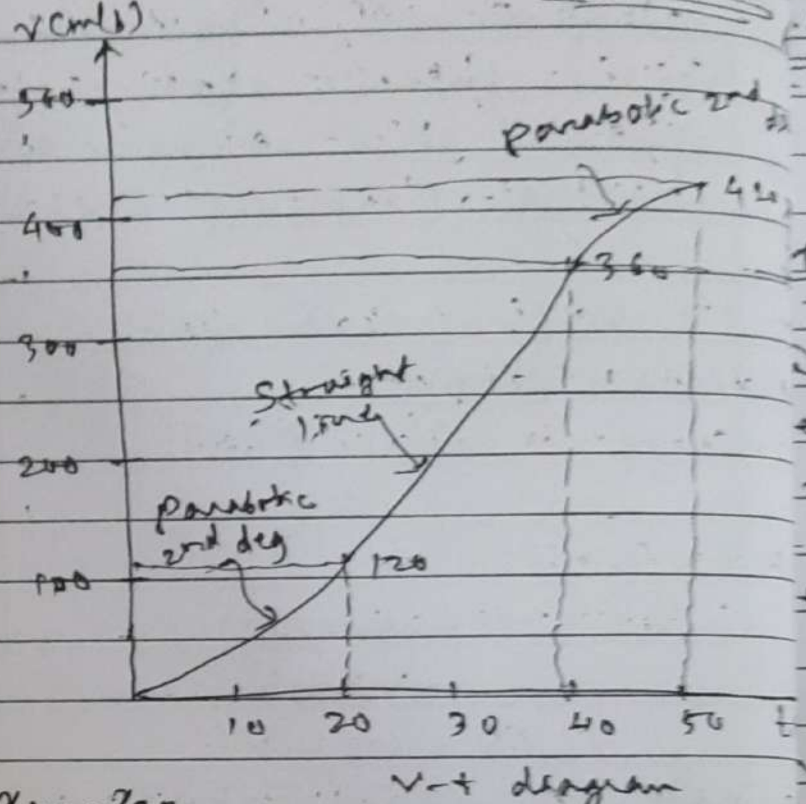
Max. speed of the particle occurs @ $t = 50s$ & $v_{max} = 4..$

Alternative soln for

x - t diagram using v-t diagram

Area under v-t diagram

= Change in position (Δx)



For $0 \le t \le 20s$,

$$A_1 = \frac{anb}{n+1}$$

$a = 20$, $b = 120$ & $n = 2$

$$A_1 = \frac{20 \times 120}{2+1} = 800 = x_{20} - x_0$$

$\therefore x_{20} = 800\ m.$

For $20 \le t \le 40s$,

$$A_2 = \left(\frac{120 + 360}{2}\right) \times 20 = 4800 = x_{40} - x_{20}$$

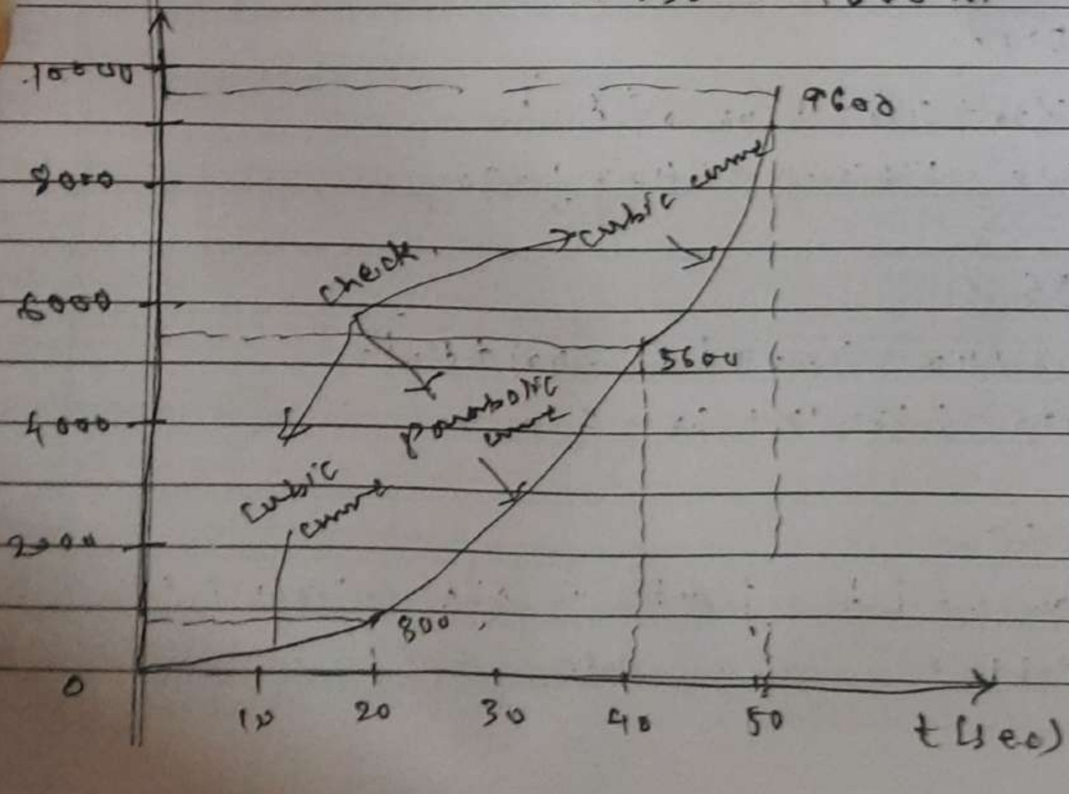
$= x_{40} - 800$ $\therefore x_{40} = 5600\ m$

For $40 \le t \le 50s$, $A_3 = 10 \times 360 + \frac{nab}{n+1}$, here $n = 2$, $a = 10$, $b = 60$

$$A_3 = 10 \times 360 + \frac{2 \times 10 \times 60}{2+1} = 4000 = x_{50} - x_{40}$$

$= x_{50} - 5600$

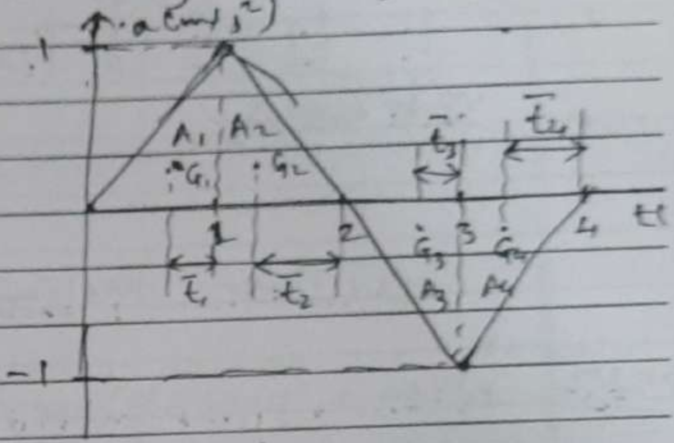
$\therefore x_{50} = 9600\ m$



Page No.

Date __/__/__

Q48 For the a-t diagram of particle shown in fig. draw v-t & x-t diagram. Also calculate the velocity @ the end of 3 sec & distance travelled in 4 sec. Assume that particle starts from rest from origin



Solⁿ

I.C. (assumed)

@ $t=0$, $x_0=0$, $v_0=0$

Area under a-t diagram = change in velocity (Δv)

In $0 \le t \le 1s$,

Area $A = \frac{1}{2} \times 1 \times 1 = v_1 - v_0$

$0.5 = v_1 - 0 \quad \therefore v_1 = 0.5$ m/s.

In $1 \le t \le 2s$, Area $A_2 = \frac{1}{2} \times 1 \times 1 = v_2 - v_1 \Rightarrow 0.5 = v_2 - 0.5$, $v_2 = 1$ m

In $2 \le t \le 3s$, $A_3 = -\frac{1}{2} \times 1 \times 1 = v_3 - v_2 \Rightarrow -0.5 = v_3 - 1$, $v_3 = 0.5$ m/s

In $3 \le t \le 4s$, $A_4 = -\frac{1}{2} \times 1 \times 1 = v_4 - v_3 \Rightarrow -0.5 = v_4 - 0.5$; $v_4 = 0$ m/s

To find positⁿ of particle from a-t diagram we use

$x_t = x_0 + v_0 t + A\bar{t}$

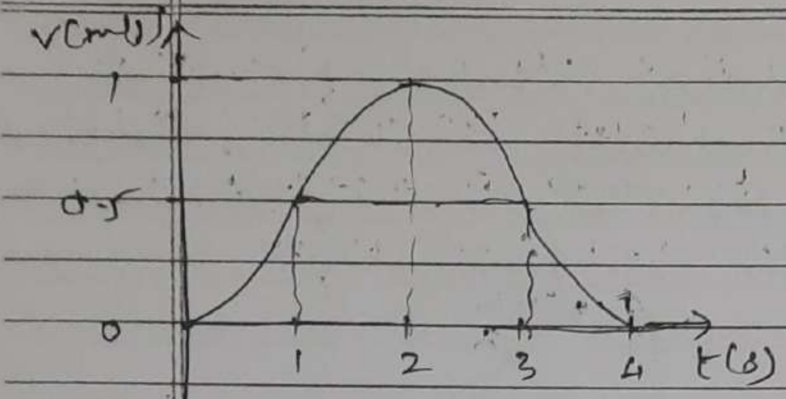
In $0 \le t \le 1s$, $x_1 = x_0 + v_0 t_1 + A_1 \bar{t}_1 = 0 + 0 + 0.5 \times \frac{1}{3} \times 1 = 0.166$ m

In $1 \le t \le 2s$, $x_2 = x_1 + v_1 t_2 + A_2 \bar{t}_2 = 0.166 + 0.5 \times 1 + 0.5 \times \frac{2}{3} \times 1 = 1$ m

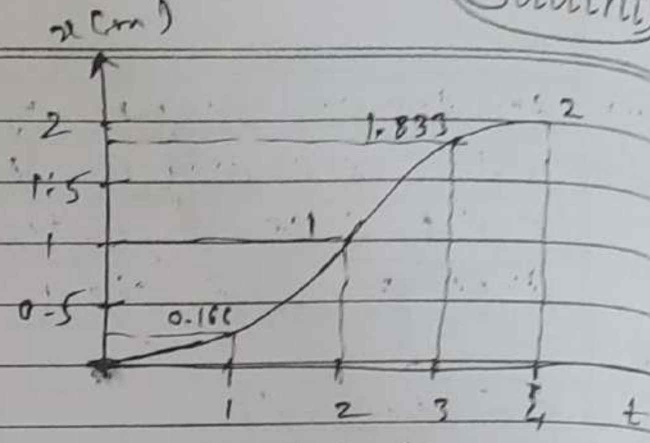
In $2 \le t \le 3s$, $x_3 = x_2 + v_2 t_3 + A_3 \bar{t}_3 = 1 + 1 \times 1 + (-0.5) \times \frac{1}{3} \times 1 = 1.833$

In $3 \le t \le 4s$, $x_4 = x_3 + v_3 t_4 + A_4 \bar{t}_4 = 1.833 + 0.5 \times 1 + (-0.5) \times \frac{2}{3} \times 1 = 2$ m

Date ___/___/_____



v-t diagram



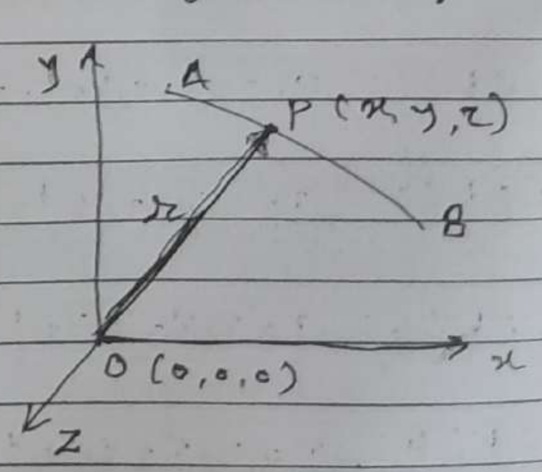
x-t diagram

Curvilinear Motion

<u>Def^n</u> When a particle moves along a curved path, the motion is called curvilinear motion. In curvilinear motion vector analysis is used to formulate the particle's position, velocity & accel^n.

1) <u>Position vector</u> - The position of particle P moving along curve AB measured from fixed pt. O will be designated by the position vector r. This vector is a f^n of time t

- The magnitude & dir^n of P.V. changes as the particle moves along the curve.



P.V. $r = OP = x\bar{i} + y\bar{j} + z\bar{k}$

Magnitude of the particle p from the origin O is

$|r| = OP = \sqrt{x^2 + y^2 + z^2}$

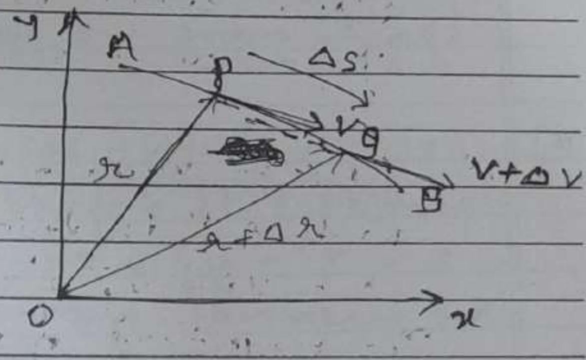
2) <u>Displacement</u> - During time interval Δt, the particle moves a distance Δs along the curve to occupy a new position Q defined by the new P.V.

Date ___ / ___ / ____

$r' = r + \Delta r$

The displacement Δr represent the change in the particle's positⁿ & is determined by vector subtract

$\therefore \Delta r = r' - r$



3) Velocity :- The avg. velocity during the interval Δt is defined as,

$$v_{avg} = \frac{\Delta r}{\Delta t}$$

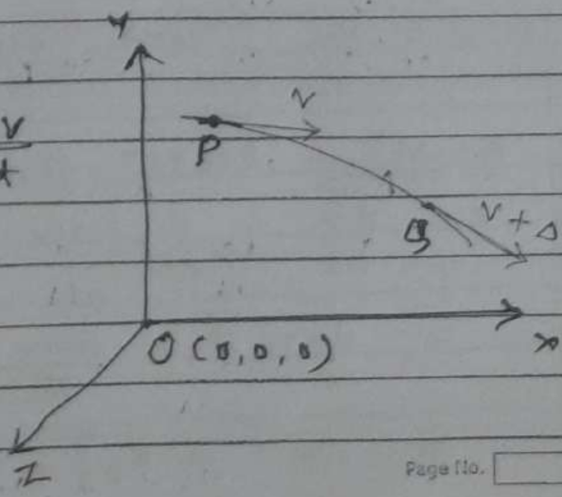
The instantaneous velocity, $v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta r} = \frac{dr}{dt}$

The dirⁿ of v is always tangent to the curve, The magnitude of v is called speed,

$$\therefore \text{speed} = |v| = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

(Doubt - Δs & Δr are same ?)

4) Acceleratⁿ - The avg. accelⁿ during the time interval Δt is $a_{avg} = \frac{\Delta v}{\Delta t}$



e instantaneous accelⁿ, $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

but $v = dr/dt$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{d^2 r}{dt^2}$$

~~$\therefore a = \frac{dv}{dt} = \frac{d^2 r}{dt^2}$~~

Date __/__/__

Co-ordinate systems in Curvilinear motn

⇒ Rectangular co-ordinates – In this system for plane curve we use x & y coordinates & for space curve we use x, y & z coordinates.

a) Position vector – Locatn of particle P is defined by P.V.

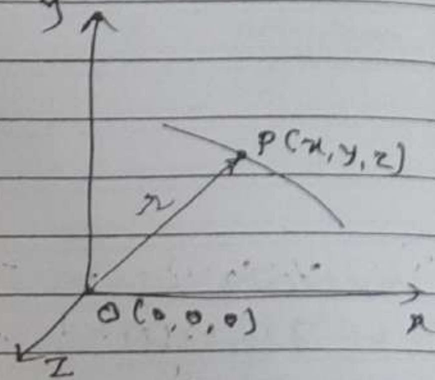
$r = x\bar{i} + y\bar{j} + z\bar{k}$

The magnitude of PV is

$|r| = r = \sqrt{x^2+y^2+z^2}$

& its dirn,

$\cos\alpha = \frac{x}{r}, \cos\beta = \frac{y}{r}$ & $\cos\gamma = \frac{z}{r}$

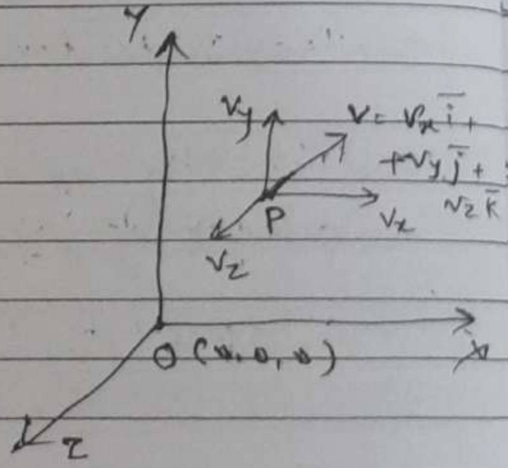


b) Velocity : The velocity of particle can be expressed as.

$v = \frac{dr}{dt} = \frac{d}{dt}(x\bar{i} + y\bar{j} + z\bar{k})$

$= \frac{dx}{dt}\bar{i} + \frac{dy}{dt}\bar{j} + \frac{dz}{dt}\bar{k}$

$= v_x\bar{i} + v_y\bar{j} + v_z\bar{k}$



magnitude of velocity is given by

$|v| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

& its dirn

$\cos\alpha = \frac{v_x}{v}$; $\cos\beta = \frac{v_y}{v}$, $\cos\gamma = \frac{v_z}{v}$

c) Acceln – Acceln of the particle can be expressed as

$a = \frac{dv}{dt} = \frac{d}{dt}(v_x\bar{i} + v_y\bar{j} + v_z\bar{k})$

$= \frac{dv_x}{dt}\bar{i} + \frac{dv_y}{dt}\bar{j} + \frac{dv_z}{dt}\bar{k}$

$= a_x\bar{i} + a_y\bar{j} + a_z\bar{k}$

Date ___/___/______ Saathi

The magnitude of accel^n is given by

$|a| = a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

& its dir^n

$\cos\alpha = \frac{a_x}{a}, \cos\beta = \frac{a_y}{a}, \cos\gamma = \frac{a_z}{a}$

For plane curve we write

P.V. $r = x\bar{i} + y\bar{j}$

V.V. $v = v_x\bar{i} + v_y\bar{j}$

a.v. $a = a_x\bar{i} + a_y\bar{j}$

2) Normal & Tangential Co-ordinate system

Plane Motion of Particle

where velocity,

$v = v e_t$, where $v = \frac{ds}{dt}$

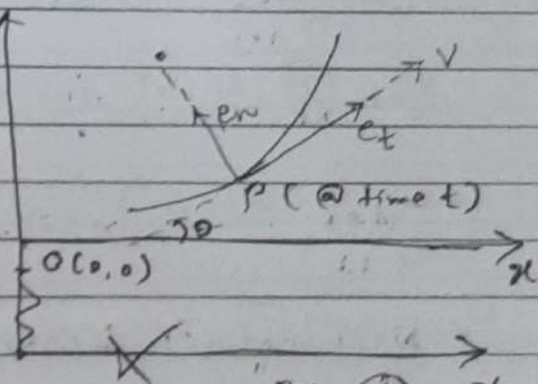


Fig. ①

the accel^n of the particle,

$a = \frac{dv}{dt} = \frac{d}{dt}(v e_t) = \frac{dv}{dt} e_t + v\frac{de_t}{dt}$ → ①

where $\frac{dv}{dt} e_t$ represents change in magnitude of velocity

$v\frac{de_t}{dt}$ represents change in dir^n of velocity

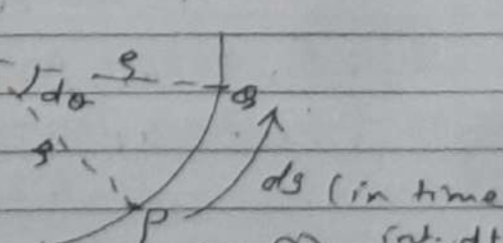


Fig. ②

to find $\frac{de_t}{dt}$

Using chain rule of differentiation

$\frac{de_t}{dt} = \frac{de_t}{d\theta} \cdot \frac{d\theta}{ds} \cdot \frac{ds}{dt}$ ..... ②

From fig. ②, $ds = \rho\, d\theta$ or $\frac{d\theta}{ds} = \frac{1}{\rho}$ → ③

Page No. [ ]

Date ___/___/___

Saathi

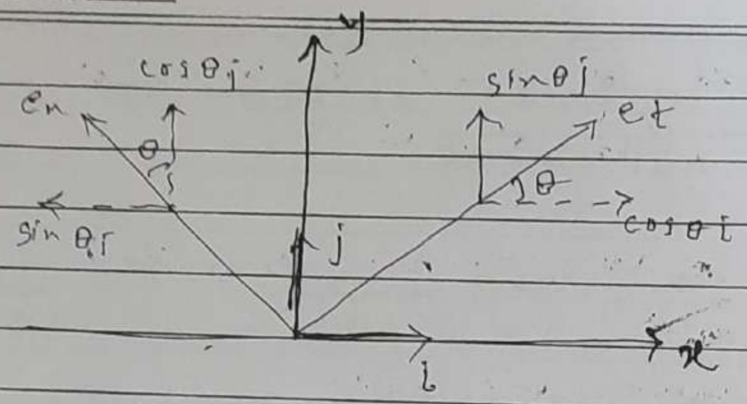


Fig. 3 Let us rep. ~~unit vector~~ in x-y co-ordinate

From fig. 3 we can write unit vector as

$e_t = \cos\theta \bar{i} + \sin\theta \bar{j}$ & $e_n = -\sin\theta \bar{i} + \cos\theta \bar{j}$

Diff. e_t w.r.t. θ

$$\frac{de_t}{d\theta} = -\sin\theta \bar{i} + \cos\theta \bar{j} = e_n$$

$$\therefore \frac{de_t}{d\theta} = e_n \longrightarrow (4)$$

Put (3) & (4) in eqn (2) :

speed

$$\frac{de_t}{dt} = e_n \times \frac{1}{\rho} \times v \quad \cdots \left\{ \because \frac{ds}{dt} = |v| = v \right\}$$

Now eqn (1) becomes

$$a = \frac{dv}{dt} = \frac{dv}{dt} e_t + v \times v \times \frac{1}{\rho} e_n = \frac{dv}{dt} e_t + \frac{v^2}{\rho} e_n$$

$$\therefore \boxed{a = a_t e_t + a_n e_n}$$

where a_t = Tangential comp. of accelⁿ = Rate of change of spee

$= \frac{dv}{dt}$

& a_n = Normal comp. of accelⁿ $= \frac{v^2}{\rho}$

& a = Total accelⁿ

= Rate of change of velocity

The mag. of total accelⁿ,

$$a = |a| = \sqrt{a_t^2 + a_n^2}$$

*
*
*
*
*

Date ___/___/___

Saathi

Dir^n of total accel^n is given by,

$\alpha = \tan^{-1}\frac{a_t}{a_n}$, where α is the angle of accel^n a w.r.t. normal accel^n

Formulaes

* Particle can move along the curve with uniform tangential accel^n. In such cases we use

$v = u + a_{tan} t$

$s = ut + \frac{1}{2} a_{tan} t^2$

& $v^2 = u^2 + 2a_{tan} s$

these eqn are scalar eqns. Here u & v are initial & final speed of the particle. s is the distance moved by the particle along the curve

* When particle moves along the curve with variable tangential accel^n, we use

$a_{tan} = \frac{d|v|}{dt} = |v|\frac{d|v|}{ds}$ where $|v|$ = speed of particle

& $|v| = \frac{ds}{dt}$ ... identical to rectilinear mot^n of particle

* Total accel^n $|a| = \sqrt{a_t^2 + a_n^2} = \sqrt{a_x^2 + a_y^2}$

* Velocity $v = |v| \cdot e_t$

* Tangential accel^n $a_t = a \cdot e_t$, where a is total accel^n

* Radius of curvature (ρ)

* i) $\rho = r$, when particle is moving along circular path

ii) $|\rho| = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ when $y = f(x)$

Date __/__/__

iii) $|\rho| = \dfrac{\left[v_x^2 + v_y^2\right]^{3/2}}{(v_x a_y - v_y a_x)}$ ~~where,~~

where $v_x = \dfrac{dx}{dt}$, $v_y = \dfrac{dy}{dt}$, $a_x = \dfrac{dv_x}{dt}$, $a_y = \dfrac{dv_y}{dt}$

Relationship b/w Rectangular components & Normal tangential components of accel^n?

From fig. 4

$a_x = a_n \sin\theta + a_t \cos\theta$

$a_y = a_n \cos\theta - a_t \sin\theta$

$a_n = a_x \sin\theta + a_y \cos\theta$

$a_t = a_x \cos\theta - a_y \sin\theta$

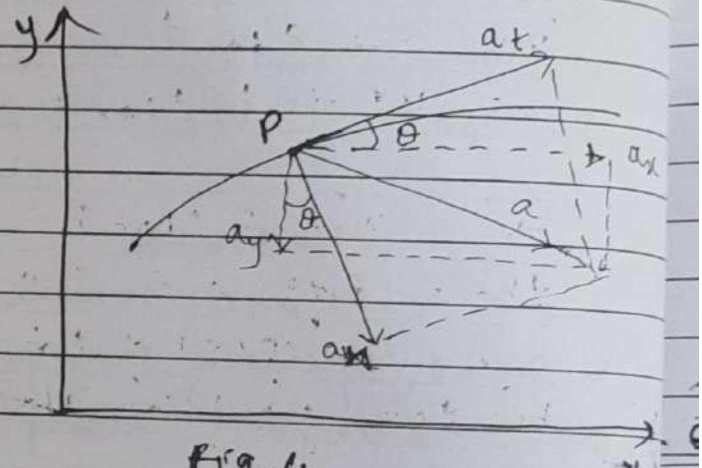


Fig. 4

Ex.57 The speed of the racing car is increasing @ a const rate from 72 kmph to 144 kmph over a distance of 200 m along a curve of radius 250 m. Determine of the magnitude of total accel^n after it has travelled 120 m

Soln Initial velocity, $u = 72 \times \dfrac{5}{18} = 20$ m/s

Final velocity, $v = 144 \times \dfrac{5}{18} = 40$ m/s

$s = 200$ m, $\rho = 250$ m

To find: accel^n, $a = ?$ when $s = 120$ m

Car is moving with uniform tang. accel^n

$\Rightarrow v^2 = u^2 + 2 a_t s$

$(40)^2 = (20)^2 + 2 a_t \times 200$

$\therefore a_t = 3$ m/s^2

Page No. ____

Saathi

Date ___/___/___

To find speed of car when $s = 120\,m$

$\Rightarrow v^2 = u^2 + 2a_t s$

$= (20)^2 + 2 \times 3 \times 120 = 1120$

$\therefore v = 33.4664\ m/s$

Normal accel^n, $a_n = \frac{v^2}{\rho} = \frac{(33.4664)^2}{250} = 4.48\ m/s^2$

Total accel^n, $a = \sqrt{a_n^2 + a_t^2} = \sqrt{(4.48)^2 + 3^2}$

$= 5.392\ m/s^2$ Ans

$\theta = \tan^{-1}\left(\frac{a_t}{a_n}\right) = \tan^{-1}\left(\frac{3}{4.48}\right) = 33.81°$ Ans

(θ is angle of accel^n a w.r.t. normal accel^n)

Q.69

~~The~~ A particle travels in a circular path whose arc distance travelled is defined by $s = (0.5t^3 + 3t)\,m$. If the total accel^n is $10\ m/s^2$ @ $t = 2s$. Find the radius of curvature.

Sol^n

Total accel^n, $a = 10\ m/s^2$ @ $t = 2\,sec$

Distance travelled, $s = (0.5t^3 + 3t)\,m$

To find ρ @ $t = 2\,sec$.

Let $s = 0.5t^3 + 3t$

$\therefore v = \frac{ds}{dt} = 1.5t^2 + 3$

$a_t = \frac{dv}{dt} = 3t$

when $t = 2\,sec$, $v = 1.5 \times (2)^2 + 3 = 9\ m/s$

$a_t = 3 \times 2 = 6\ m/s^2$

Accel^n, $a = \sqrt{a_t^2 + a_n^2}$

$10 = \sqrt{(6)^2 + a_n^2}$

$\therefore a_n = 8\ m/s^2$

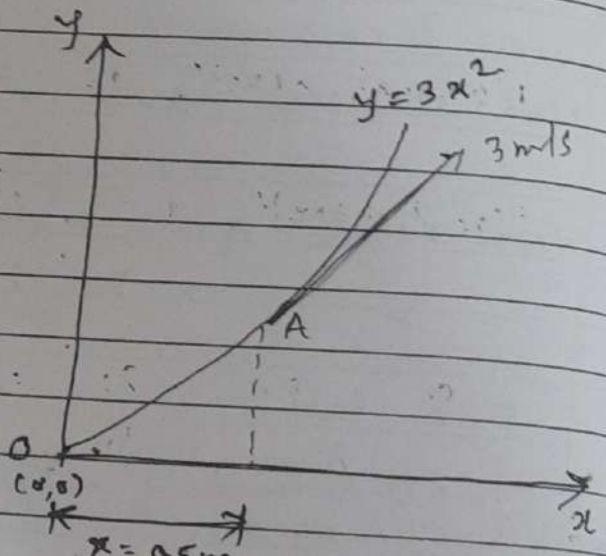
But $a_n = \frac{v^2}{\rho} \Rightarrow 8 = \frac{(9)^2}{\rho} \Rightarrow \rho = 10.125\ m$ Ans

Page No.

Date ___/___/___

Ex. 61 A particle moves with a const. speed of 3 m/s along the path shown in fig. What is the resultant acceln at a position on the path where $x = 0.5$ m? Also represent the acceln in vector form.

Soln



$v = 3$ m/s (const.)

$y = 3x^2$

Acceln, $a = \sqrt{a_t^2 + a_n^2}$ —— ①

As ~~body~~ particle is moving with const. speed, hence

$a_t = 0$ —— ②

$a_n = \frac{v^2}{\rho} = \frac{3^2}{\rho} = \frac{9}{\rho}$ —— ③

To find $\rho\Big]_{x = 0.5\,m}$

$y = 3x^2$, $\frac{dy}{dx} = 6x$; $\frac{dy}{dx}\Big]_{x=0.5} = 6 \times 0.5 = 3$

$\frac{d^2y}{dx^2}\Big]_{x=0.5} = 6$

$\because \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ ..... $y = f(x)$

$\therefore \rho\Big]_{x=0.5} = \frac{\left[1 + (3)^2\right]^{3/2}}{6} = 5.27$ m

Substitute ~~this~~ ρ in eqn ③ we get

$a_n = \frac{9}{5.27} = 1.708$ m/s^2

using eqn ① we get

$\therefore$ Resultant acceln, $a = \sqrt{0 + (1.708)^2} = 1.708$ m/s^2 Ans.

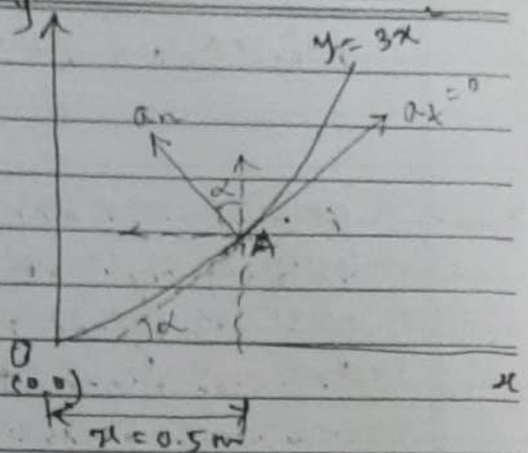
To represent acceln in vector form

Date ___/___/___

From fig:

$\tan\alpha = \left.\frac{dy}{dx}\right]_{x=0.5} = 3$

$\therefore \alpha = 71.565°$



$a = a_x\bar{i} + a_y\bar{j}$

$= (-a_n \sin\alpha)\bar{i} + (a_n\cos\alpha)\bar{j}$

$= (-1.708 \sin 71.565)\bar{i} + (1.708\cos 71.565)\bar{j}$

$= -1.62\bar{i} + 0.54\bar{j}$ Ans

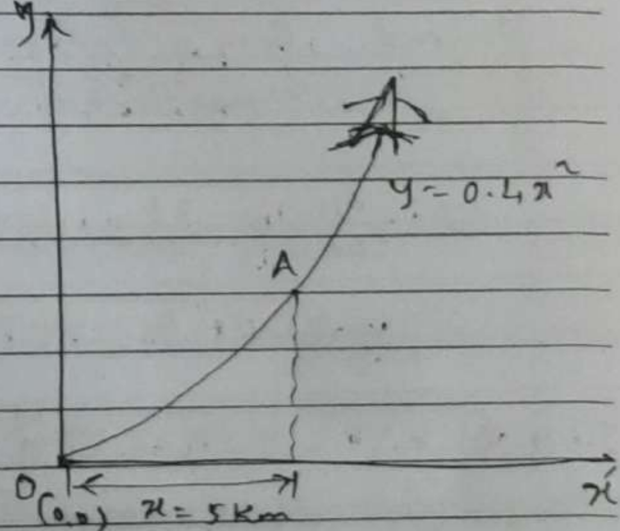
.62 A jet plane travels along the parabolic path as shown in fig. When it is at pt. A, it has a speed of 200 m/s which is increasing @ the rate of 0.8 m/s². Determine the magnitude of the accelⁿ of the plane when it is at A.

Solⁿ

$y = 0.4x^2$

$v = 200$ m/s

$a_t = 0.8$ m/s²



$a_n = \frac{v^2}{\rho}$

$y = 0.4x^2$

$\frac{dy}{dx} = 0.8x$

$\left.\frac{dy}{dx}\right]_{x=5Km} = 0.8\times5 = 4$

$\left.\frac{d^2y}{dx^2}\right]_{x=5Km} = 0.8$

$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+(4)^2\right]^{3/2}}{0.8} = 87.616$ Km $= 87616$ m

$a_n = \frac{v^2}{\rho} = \frac{(200)^2}{87616} = 0.4565$ m/s²

Date ___/___/___

Accel^n, $a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8)^2 + (0.4565)^2} = \underline{\underline{0.921\ m/s^2}}$

Ans

Q.67 A particle moving in the x-y plane with y-components of velocity $v_y = 6t$ m/s where t is in sec. The x-component of accel^n of the particle is $a_x = 3t$ m/s^2 where t is in seconds. When $t = 0$, $x = 3$ m & $y = 0$, & $v_x = 0$. Find the equation of the path of the particle. Determine the magnitude of the velocity of the particle @ the instant when $y = 10$m.

Sol^n:

~~$v_y =$~~

$v_y = 6t$ m/s

$a_x = 3t$ m/s^2

@ $t = 0$, $x = 3$ m & $y = 0$, & $v_x = 0$

To find: path of the particle ?

Magnitude of velocity, $v = ?$ @ $y = 10$m

$$v_y = \frac{dy}{dt} = 6t \text{, integrating}$$

$$y = \frac{6t^2}{2} + c_1 = 3t^2 + c_1$$

Using I.C @ $t = 0$, $y = 0$ $\Rightarrow$ $\boxed{c_1 = 0}$

$$\therefore y = 3t^2 \quad \text{——— (1)}$$

$$a_x = \frac{d^2x}{dt^2} = 3t \text{, integrating}$$

$$\frac{dx}{dt} = v_x = \frac{3t^2}{2} + c_2$$

Using I.C. @ $t = 0$, $v_x = 0$ $\Rightarrow$ $\boxed{c_2 = 0}$

$$\therefore \cancel{v_x} = \frac{dx}{dt} = v_x = \frac{3t^2}{2}$$

Date ___/___/___

Saathi

integrating again

$$x = \frac{3t^3}{6} + c_3 = \frac{t^3}{2} + c_3$$

Using IC @ $t = 0,\ x = 3$

$$3 = 0 + c_3 \Rightarrow \boxed{c_3 = 3}$$

$$\therefore x = \frac{t^3}{2} + 3 \longrightarrow (2)$$

~~From eqn~~ Eliminate t from eqn (1) & (2) to get path of par

From eqn (1) $\quad t^2 = \frac{y}{3}$

From eqn (2) $\quad x - 3 = \frac{1}{2} t^3$

or $2(x-3) = t^3$, squaring this eqn,

$4(x-3)^2 = t^6 = (t^2)^3$, put $t^2 = y/3$ in this eqn

$$4(x-3)^2 = \left(\frac{y}{3}\right)^3$$

$\Rightarrow \quad y^3 = 108(x-3)^2$ ...... Eqn of the path of particle

Ans

To find velocity, v @ $y = 10m$

Substitute $y = 10$ in eqn (1)

$10 = 3t^2$ or $t = 1.826$ sec.

$$\therefore v_x\Big]_{t=1.826} = \frac{3(1.826)^2}{2} = 5 \text{ m/s}$$

$$v_y\Big]_{t=1.826} = 6(1.826) = 10.956 \text{ m/s}$$

$$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(5)^2 + (10.956)^2} = 12.043 \text{ m/s}$$

Ans

Page No.

Saathi

Date ___/___/_____

Ex. 64 A particle moves in the x-y plane with velocity components $v_x = (8t-2)$ & $v_y = 2$ m/s. If it passes through the pt. $(x, y) = (14, 4)$ m @ $t = 2$ sec., determ. the resultant accel^n @ $t = 2$ sec. Find also the path traced by particle.

Sol^n Given $v_x = (8t-2)$ m/s

$v_y = 2$ m/s

Position of particle @ $t = 2s$; $(x, y) = (14, 4)$ m

To find: @ $t = 2s$, accel^n $a = ?$

& Path traced by particle

$v = v_x \bar{i} + v_y \bar{j} = (8t-2)\bar{i} + 2\bar{j}$ diff. w.r.t. t.

$a = \frac{dv}{dt} = 8\bar{i} = a_x \bar{i} + a_y \bar{j} \Rightarrow a_x = 8,\ a_y = 0$

$\therefore a = \sqrt{a_x^2 + a_y^2} = \sqrt{8^2 + 0^2} = 8$ m/s^2

$\therefore$ Accel^n of particle $a = 8$ m/s^2 along x-dir^n : Ans

$v_x = \frac{dx}{dt} = (8t-2)$ & $v_y = \frac{dy}{dt} = 2$, integrating

$x = 4t^2 - 2t + c_1$ & $y = 2t + c_2$

Using B.C. @ $t = 2s$, $x = 14$ & $y = 4$

$\Rightarrow 14 = 4(2)^2 - 2(2) + c_1 \Rightarrow \boxed{c_1 = 2}$

$4 = 2(2) + c_2 \Rightarrow \boxed{c_2 = 0}$

$\therefore x = 4t^2 - 2t + 2$ & $\longrightarrow$ (1)

$y = 2t \longrightarrow$ (2)

Eliminate t from eqn (1) & (2)

put $2t = y$ in eqn (1)

$\boxed{x = y^2 - y + 2}$ ..... eq^n of path of particle

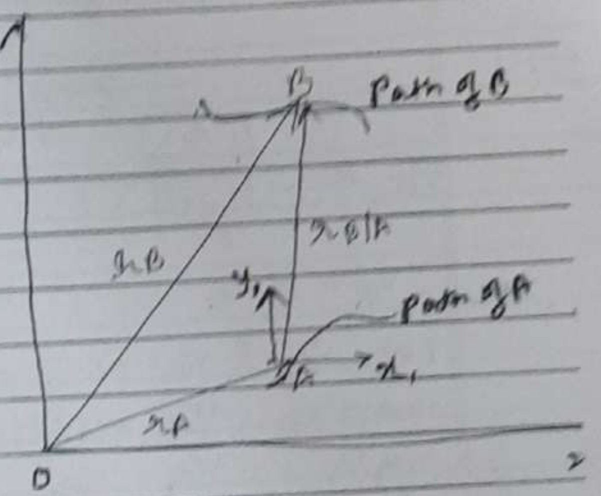
Page No.

Date __/__/__

Relative Motion

Relative motⁿ b/w two particles

Consider two particles A & B moving along the curves as shown in fig. Let r_A & r_B be P.V. of A & B w.r.t. fixed frame of reference @ O. If A observes B then position of B will be $r_{B/A}$ will be the relative P.V. of B w.r.t. A



Now by vector triangle OAB, we get

$r_B = r_A + r_{B/A} \Rightarrow r_{B/A} = r_B - r_A \longrightarrow (1)$

diff. eqn (1) w.r.t. time

$\frac{dr_B}{dt} = \frac{dr_A}{dt} + \frac{dr_{B/A}}{dt} \Rightarrow v_B = v_A + v_{B/A}$ or $v_{B/A} = v_B - v_A \longrightarrow$

diff. eqn (2) w.r.t. time

$\frac{dv_B}{dt} = \frac{dv_A}{dt} + \frac{dv_{B/A}}{dt} \Rightarrow a_B = a_A + a_{B/A}$ or $a_{B/A} = a_B - a_A \longrightarrow (3)$

These vectors can also be expressed as follows

$r_A = x_A\bar{i} + y_A\bar{j}$, $r_B = x_B\bar{i} + y_B\bar{j}$

$\therefore r_{B/A} = r_B - r_A = (x_B - x_A)\bar{i} + (y_B - y_A)\bar{j} \longrightarrow (4)$

$\therefore v_{B/A} = v_B - v_A = (v_{Bx} - v_{Ax})\bar{i} + (v_{By} - v_{Ay})\bar{j} \longrightarrow (5)$

& $a_{B/A} = a_B - a_A = (a_{Bx} - a_{Ax})\bar{i} + (a_{By} - a_{Ay})\bar{j} \longrightarrow (6)$

Relative motion concepts are usually applied in

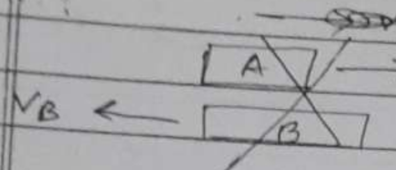
1) River boat problems
2) Rain problems
3) Aircraft wind problems.

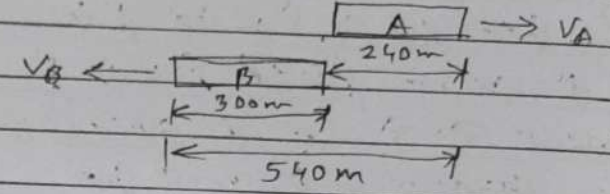
Date ___/___/___

Saathi

Ex. 98 Two trains A & B are moving on a parallel track in opp. dirn. Velocity of A is twice that of B. They take 18 seconds to pass each other. Determine their velocities given length of train A is 240 m & length of B is 300 m.

Soln.





$V_A = 2V_B$

Let V_A & V_B are velocities of two trains. Both are moving with const speed

$$V_{B/A} = V_B - V_A$$
$$= -V_B\hat{i} - 2V_B\hat{i} \quad (\because V_A = 2V_B)$$
$$= -3V_B\hat{i}$$

Relative displacement of train B w.r.t. train A

$$r_{B/A} = r_B - r_A$$
$$= -300\hat{i} - 240\hat{i}$$
$$= -540\hat{i}$$

Now $r_{B/A} = V_{B/A} \times t \quad \ldots (\because s = vt)$

$$-540\hat{i} = -3V_B\hat{i} \times t$$

$\Rightarrow$ $540 = 3V_B \times 18$

$\therefore V_B = 10\ m/s\ (\leftarrow)$ Ans

& $V_A = 2 \times 10 = 20\ m/s\ (\rightarrow)$ Ans.

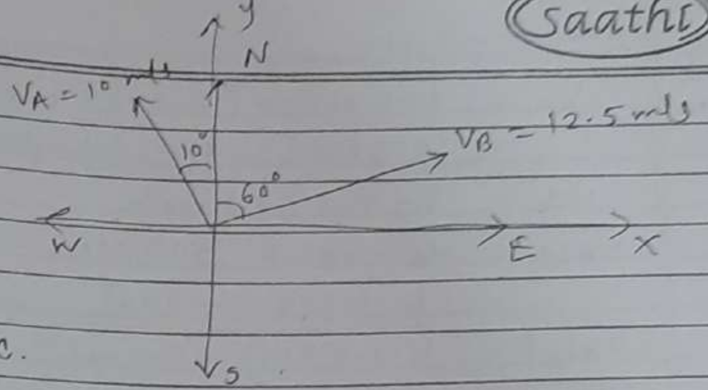
Ex. 100 Two trains leave a station in different directions @ the same instant. Train A travels @ 36 Kmph @ 10° west of north while train B travels @ 45 Kmph @ 60° east of north. Find the (i) Relative velocity of train A w.r.t. train B (ii) distance b/w the two trains after 2 mins.

103

Page No.

Date ___/___/______

Sol^n



$V_A = \frac{36 \times 5}{18} = 10$ m/s

$V_B = \frac{45 \times 5}{18} = 12.5$ m/s

t = 2 mins = 120 sec.

$V_A = -10 \sin 10\, \bar{i} + 10 \cos 10\, \bar{j}$

$= -1.736\, \bar{i} + 9.848\, \bar{j}$

$V_B = 12.5 \sin 60\, \bar{i} + 12.5 \cos 60\, \bar{j}$

$= 10.825\, \bar{i} + 6.25\, \bar{j}$

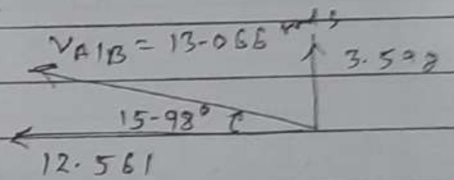
$V_{A/B} = V_A - V_B$

$= (-1.736\, \bar{i} + 9.848\, \bar{j}) - (10.825\, \bar{i} + 6.25\, \bar{j})$

$= -12.561\, \bar{i} + 3.598\, \bar{j}$

$|V_{A/B}| = \sqrt{(-12.561)^2 + (3.598)^2} = \underline{13.066 \text{ m/s}}$ Ans

$\theta = \tan^{-1}\left(\frac{3.598}{-12.561}\right) = \underline{-15.98°}$ Ans



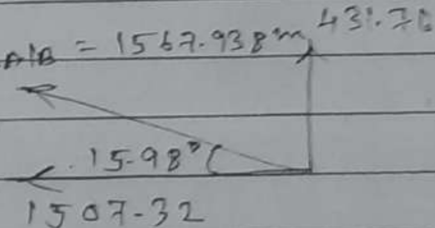
(relative displacement of train A w.r.t. B)

$r_{A/B} = V_{A/B} \times t$

$= (-12.561\, \bar{i} + 3.598\, \bar{j}) \times 120$

$= -1507.32\, \bar{i} + 431.76\, \bar{j}$

$|r_{A/B}| = \underline{1567.938 \text{ m}}$ Ans



$\theta = 15.98°$

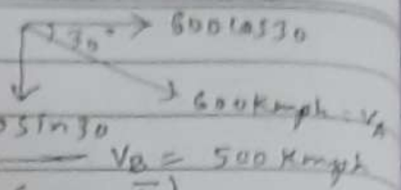
103 Planes A & B are flying @ the same altitude. If their velocities are $V_A = 600$ kmph & $V_B = 500$ kmph when the angle b/w their straight line course is 30° as shown, determine the velocity of plane A w.r.t. plane B. Also determine the distance b/w them in t = 5 min.

Date __/__/__

Soln $V_A = 600\cos 30\,\bar{i} - 600\sin 30\,\bar{j}$

$= (519.615\,\bar{i} - 300\,\bar{j})$ Kmph

$V_B = (-500\,\bar{i})$ Kmph



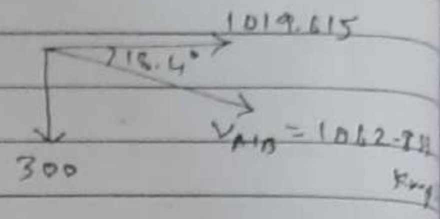
$V_{A/B} = V_A - V_B = (519.615\,\bar{i} - 300\,\bar{j}) - (-500\,\bar{i})$

$= 1019.615\,\bar{i} - 300\,\bar{j}$

$|V_{A/B}| = \sqrt{(1019.615)^2 + (-300)^2}$

$= 1062.833$ Kmph Ans

$\theta = \tan^{-1}\left(\dfrac{-300}{1019.615}\right) = -16.4°$ Ans



To find distance b/w planes after 5 mins

$S_A = V_A \times t$

$= 600 \times \dfrac{5}{60} = 50$ Km

$S_B = V_B \times t$

$= 500 \times \dfrac{5}{60} = 41.666$ Km

At $t = 5$ min

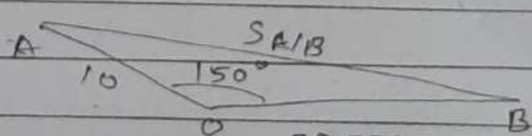
Posit^n of plane A from pt. O = $(60 - 50) = 10$ Km

Posit^n of plane B from pt. O = $80 - 41.666 = 38.334$ Km

Using cosine rule

$AB = \sqrt{(10)^2 + (38.334)^2 - 2\times 10\times 38.334\cos 150}$

$= 47.26$ Km Ans.



Ex 104 Car A is travelling east @ const. speed of 36 Kmph. As car A crosses the intersection shown in fig, car B starts from rest 35 m north of the intersection & moves south with a const acceleration of $2\ m/s^2$. Determine the position, velocity & acce^n of B relative to A, five seconds after A crosses the intersection.

For car A

$V_A = 36 \times \frac{5}{18}\,\hat{i} = 10\,\hat{i}$ (Geomat.) m/s (const)

$a_A = 0$

$\cancel{r_A}$ = P.V. of car A in 5 sec, $r_A = V_A \times t$

$r_A = 10\,\hat{i} \times 5 = 50\,\hat{i}$

For car B

Initial P.V. of car B, $r_{0B} = 35\,\hat{j}$

Initial velocity of car B, $u_B = 0$

Acceleration of car B $a_B = -2\,\hat{j}$ m/s²

At $t = 5$ sec, $V_B = u_B + a_B t = 0 - 2\hat{j} \times 5 = -10\,\hat{j}$

At $t = 5$ sec P.V. of car B, $r_B = r_{0B} + u_B t + \frac{1}{2} a_B t^2$

$$r_B = 35\,\hat{j} + 0 + \frac{1}{2}(-2\hat{j}) \times (5)^2$$

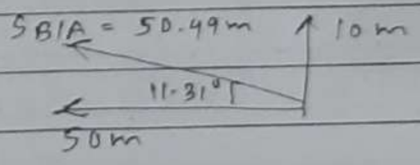
$$= 10\,\hat{j}$$

Now, $r_{B/A} = r_B - r_A$ (Relative position of B w.r.t. A)

$= 10\hat{j} - 50\hat{i}$

$\therefore$ Relative distance of B w.r.t. A, $S_{B/A} = \sqrt{(10)^2 + (-50)^2} = 50.99$ Ans

$\theta = \tan^{-1}\left(\frac{10}{-50}\right) = -11.31^\circ$ Ans

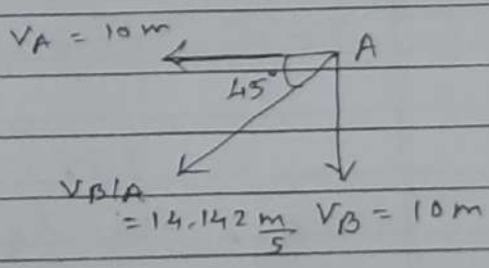


$V_{B/A} = V_B - V_A$

$= -10\hat{j} - (10\hat{i})$

Relative speed of B w.r.t. A is $|V_{B/A}| = \sqrt{(-10)^2 + (-10)^2} = 14.142$ m/s Ans

$\alpha = \tan^{-1}\left(\frac{-10}{-10}\right) = 45^\circ$ Ans



$a_{B/A} = a_B - a_A$

$= -2\hat{j} - 0$

$= -2\hat{j}$

$\therefore |a_{B/A}| = 2\,m/s^2$ (↓) Ans.

Date __/__/__

Ex. 105 Fig. shows cars A & B @ a distance of 35m. Car A moves with const. speed of 36 kmph & car B starts from rest with an accel^n of 1.5 m/s^2. Determine relative position, velocity & accel^n of car B w.r.t. car A, five seconds after car A crosses the intersection

soln

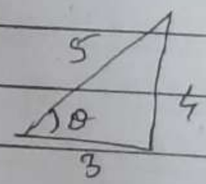
For car A @ t = 5 sec,

$v_A = 36 \times \frac{5}{18} i = 10 i$, (const.)

$r_A = v_A \times t = 10\bar{i} \times 5 = 50\bar{i}$

$a_A = 0$

For car B @ t = 5 sec



Initial p.v., $r_{oB} = 35\cos\theta\,\bar{i} + 35\sin\theta\,\bar{j}$

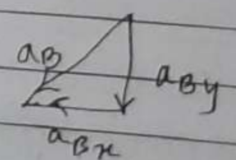
$= 35 \times \frac{3}{5}\bar{i} + 35 \times \frac{4}{5}\bar{j}$

$= 21\bar{i} + 28\bar{j}$

$u_B = 0\bar{i} + 0\bar{j}$

$a_B = -1.5\cos\theta\,\bar{i} - 1.5\sin\theta\,\bar{j}$

$= -1.5 \times \frac{3}{5}\bar{i} - 1.5 \times \frac{4}{5}\bar{j}$



$= -0.9\bar{i} - 1.2\bar{j}$

using eqn of motion

$v_B = u_B + a_B \times t = 0 + (-0.9\bar{i} - 1.2j) \times 5$

$= -4.5\bar{i} - 6\bar{j}$

P.V. of car B after 5 sec, $r_B = r_{oB} + u_B t + \frac{1}{2} a_B \times t^2$

$r_B = 21\bar{i} + 28\bar{j} + 0 + \frac{1}{2}(-0.9\bar{i} - 1.2\bar{j}) \times 5^2$

$= 9.75\bar{i} + 13\bar{j}$

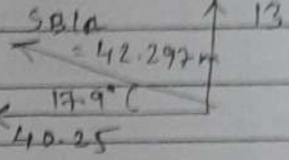
$r_{B/A} = r_B - r_A$

$= 9.75\bar{i} + 13\bar{j} - 50\bar{i}$

$= -40.25\bar{i} + 13\bar{j}$

Date ___ / ___ / _____ Saathi

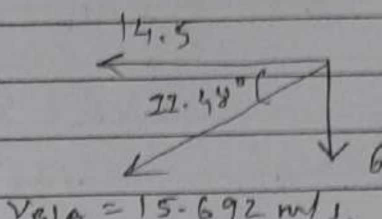
$\therefore S_{B/A} = \sqrt{(-40.25)^2 + (13)^2} = \underline{\underline{42.297\text{ m}}}$ Ans



$\theta = \tan^{-1}\left(\frac{13}{-40.25}\right) = \underline{\underline{-17.9^\circ}}$ Ans

$v_{B/A} = v_B - v_A$
$= -4.5\bar{i} - 6\bar{j} - 10\bar{i}$
$= -14.5\bar{i} - 6\bar{j}$

$|v_{B/A}| = \sqrt{(-14.5)^2 + (-6)^2}$

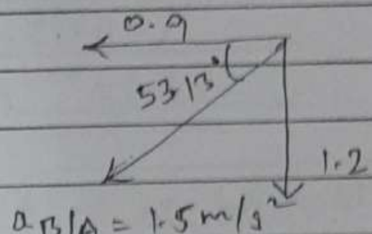


$= \underline{15.692\text{ m/s}}$ Ans

$\alpha = \tan^{-1}\left(\frac{-6}{-14.5}\right) = \underline{\underline{22.48^\circ}}$ Ans.

$a_{B/A} = a_B - a_A$
$= -0.9\bar{i} - 1.2\bar{j}$

$|a_{B/A}| = \sqrt{(-0.9)^2 + (-1.2)^2}$



$= \underline{1.5\text{ m/s}^2}$ Ans

$\beta = \tan^{-1}\left(\frac{-1.2}{-0.9}\right) = \underline{\underline{53.13^\circ}}$ Ans

Ex. 106 Automobile A is traveling along a straight highway while B is moving along a circular curve of 150 m radius. The speed of A is being increased at the rate of 1.5 m/s² & the speed of B is being decreased at the rate of 0.9 m/s². For the position shown in fig, determine the velocity of A relative to B. At this instant the speed of A is 75 kmph & speed of B is 40 kmph.

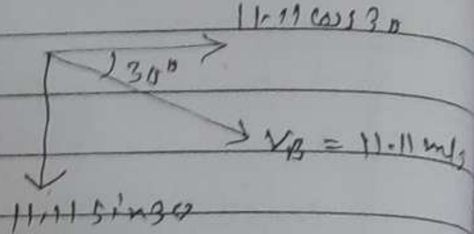
Soln

~~Speed of~~ $v_A = 75 \times \frac{5}{18} = 20.833\text{ m/s}$

$v_B = 40 \times \frac{5}{18} = 11.11\text{ m/s}$

Date __/__/__

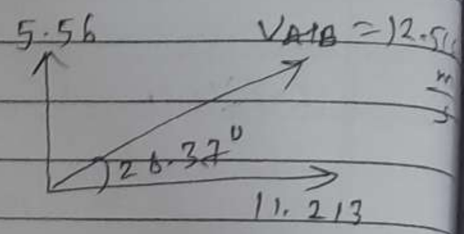
Expressing velocities in vector form we get



$V_A = 20.833\,\bar{i}$

$V_B = 11.11\cos 30\,\bar{i} - 11.11\sin 30\,\bar{j}$

$= 9.62\,\bar{i} - 5.56\,\bar{j}$



$V_{A/B} = V_A - V_B$

$= 20.833\,\bar{i} - (9.82\,\bar{i} - 5.56\,\bar{j})$

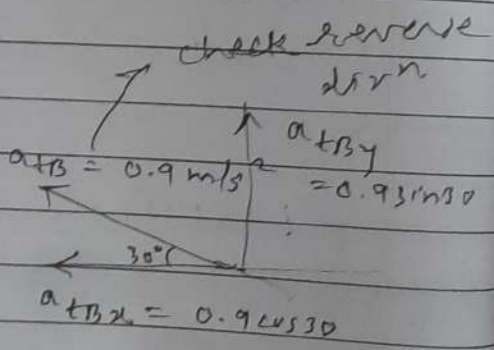
$= 11.213\,\bar{i} + 5.56\,\bar{j}$

$|V_{A/B}| = \sqrt{(11.213)^2 + (5.56)^2}$

$= \underline{12.516 \text{ m/s}}$ Ans

$\theta = \tan^{-1}\left(\frac{5.56}{11.213}\right)$

$= \underline{26.37^0}$ Ans



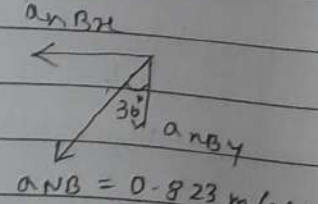
To find $a_A = 1.5\,\bar{i}$

$a_{tB} = -0.9$ m/s² (given) (Deceleration)

$a_{tB} = -0.9\cos 30\,\bar{i} + 0.9\sin 30\,\bar{j}$

$= -0.78\,\bar{i} + 0.45\,\bar{j}$

$a_{nB} = \frac{V_B^2}{R} = \frac{(11.11)^2}{150} = 0.823$ m/s²



$a_{nB} = -0.823\sin 30\,\bar{i} - 0.823\cos 30\,\bar{j}$

$= -0.411\,\bar{i} - 0.713\,\bar{j}$

$a_B = a_{tB} + a_{nB}$

$= -0.78\,\bar{i} + 0.45\,\bar{j} - 0.411\,\bar{i} - 0.713\,\bar{j}$

$= -1.191\,\bar{i} - 0.263\,\bar{j}$

$a_{A/B} = a_A - a_B$

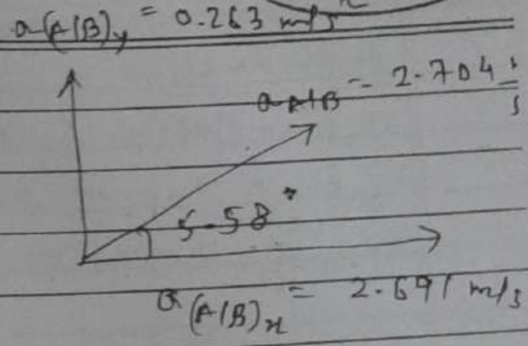
$= 1.5 - (-1.191\,\bar{i} - 0.263\,\bar{j})$

$= 2.691\,\bar{i} + 0.263\,\bar{j}$

Date ___ / ___ / ___

$$|a_{A/B}| = \sqrt{(2.691)^2 + (0.263)^2}$$

$$= \underline{\underline{2.704 \ m/s^2}} \text{ Ans.}$$



$$\beta = \tan^{-1}\left(\frac{0.263}{2.691}\right)$$

$$= \underline{\underline{5.58°}} \text{ Ans}$$

Module 2.2 Kinematics of Rigid Bodies

Date ___/___/_____

Instantaneous Centre of Rotation

Introduction – Plane motⁿ of a rigid body is a combination of translation & rotation. Such plane motion can be converted to pure rotatⁿ about an arbitrary point called instantaneous centre of rotation (ICR)

Definitⁿ of ICR

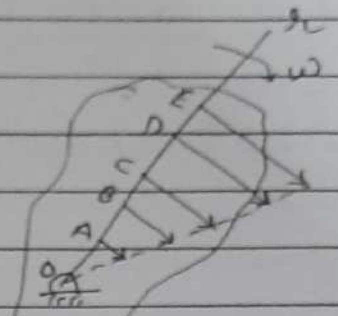


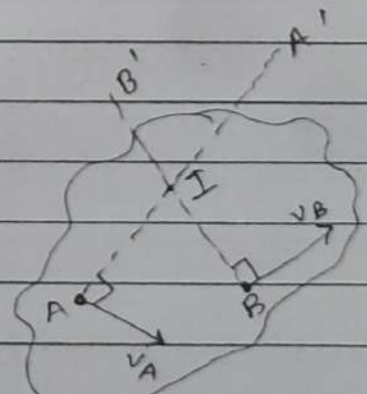
Fig. shows a plane rigid body rotates @ pt. O. Along the radial co-ordinate (or) r, it rotates with angular velocity ω

The translational velocity of some of pts on this co-ordinate are given by

$v_A = OA \times \omega$, $v_B = OB \times \omega$, $v_C = OC \times \omega$, $v_D = OD \times \omega$ & $v_E = OE \times \omega$

* The velocity v linearly varies with radial distance
* At the pt. of rotatⁿ (r = 0), linear velocity is zero
* This pt of zero velocity on a rotating body is referred as Instantaneous Centre of Rotation.

Location of ICR



* Let the translational velocities of any two pts. A & B are known
* Let us locate ICR.
* The dirⁿ of velocity is $\perp^r$ to the line joining pt. & centre of rotatⁿ
* A line AA' is $\perp^r$ to v_A & ICR must lie on this radial line
* Similarly line BB' is $\perp^r$ to v_B & ICR must line on this line.

⇒ ICR lies on pt. of intersectⁿ of these two lines.

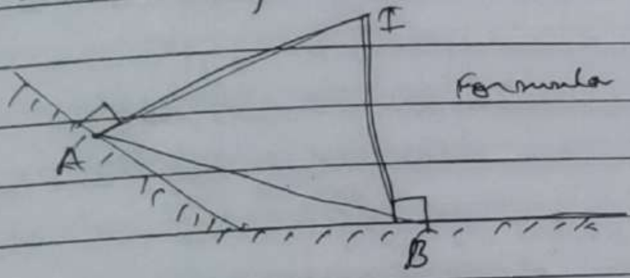
Date ___/___/_____

Characteristics of ICR

1) It is a pt. of zero velocity
2) It may lies within the body or outside the body
3) It changes from instant to instant & is not a fixed pt.
4) To locate ICR, dirⁿ of velocities of any 2 pts in the rigid body must be known
5) It's an imaginary point.

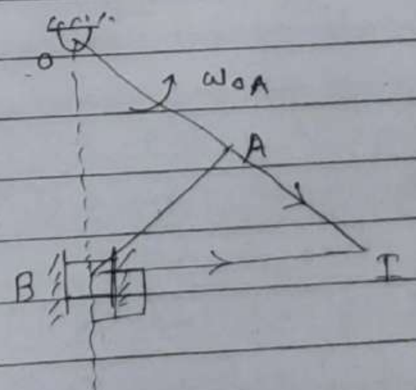
Different cases to find ICR

Case 1) When body slides on two surfaces



Formula $\omega = \frac{v_A}{IA} = \frac{v_B}{IB}$

Case 2) When one part of the body slides & another part rotates about a hinge pt.

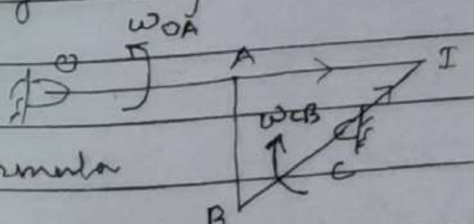


$\omega = \frac{v_A}{IA} = \frac{v_B}{IB}$ --- Formula

$v_A = OA \times \omega_{OA}$

pt. of intersect of $\perp^r$ to sliding surface & extension of rotating link

Case 3) When two links of the system rotate about 2 separate hinge pts.



ICR – pt. of intersectⁿ of extⁿ of these two links

$\omega = \frac{v_A}{IA} = \frac{v_B}{IB}$ --- Formula

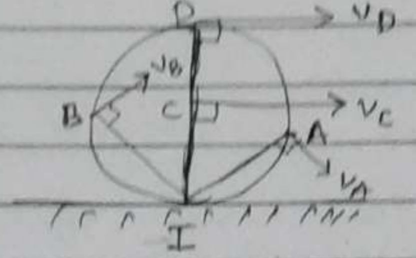
$v_A = OA \times \omega_{OA}$, $v_B = CB \times \omega_{CB}$

Page No.

Saathi

Date ___ / ___ / ______

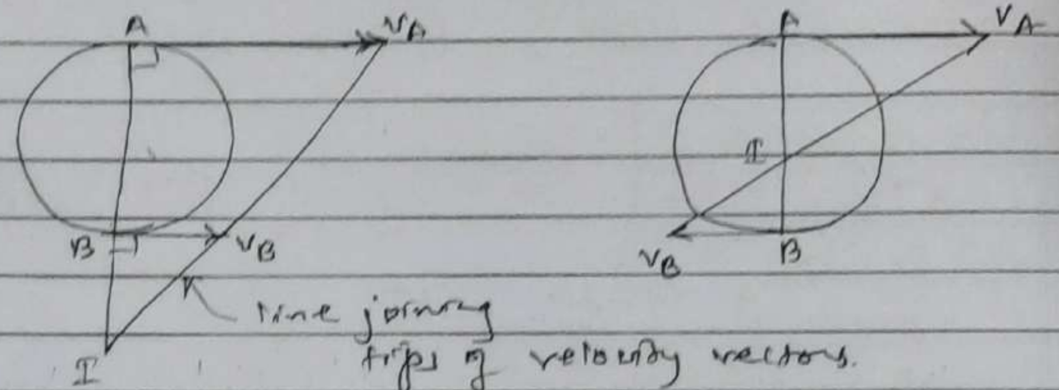
case 4) When body rolls on fixed surface



$$\omega = \frac{v_A}{IA} = \frac{v_B}{IB} = \frac{v_C}{IC} = \frac{v_D}{ID}$$

Point of contact with fixed surface becomes ICR

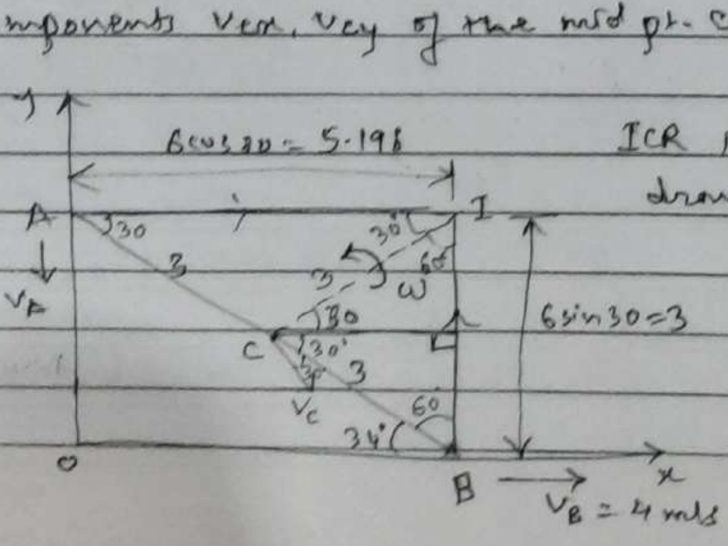
case 5) When body lies b/w 2 moving surfaces.



Formula $\omega = \frac{v_A}{IA} = \frac{v_B}{IB}$

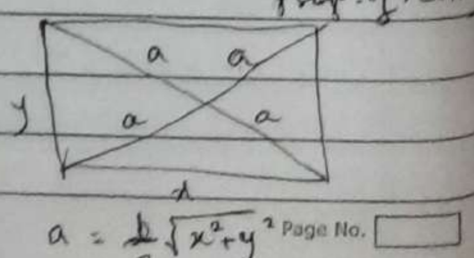
Ex. 1 Fig. shows a ladder AB = 6m resting against a vertical wall at A & horizontal ground @ B. If the end B of the ladder is pulled towards right with a const. velocity $v_B = 4$ m/s find (i) ICR of the ladder (ii) angular velocity of the ladder @ this instant (iii) velocity v_A of the end A of the ladder (iv) velocity components v_{cx}, v_{cy} of the mid pt. C of the ladder

Soln



ICR is a pt. of intersect of lines drawn $\perp^r$ to v_A & v_B Ans (i)

Prop. of rectangle



$a = \frac{1}{2}\sqrt{x^2+y^2}$

Date ___/___/___

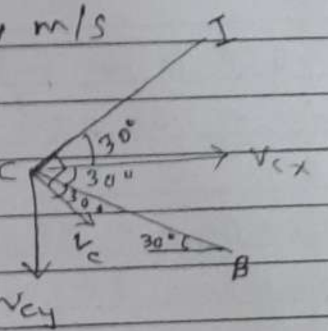
$\omega = \frac{V_A}{IA} = \frac{V_B}{IB}$

$\omega = \frac{V_A}{5.196} = \frac{4}{3} = 1.333$

$\therefore \omega = \underline{1.333 \text{ rad/s}}$ Ans (ii)

$V_A = 5.196 \times 1.333 = \underline{6.926 \text{ m/s} (\downarrow)}$ Ans (iii)

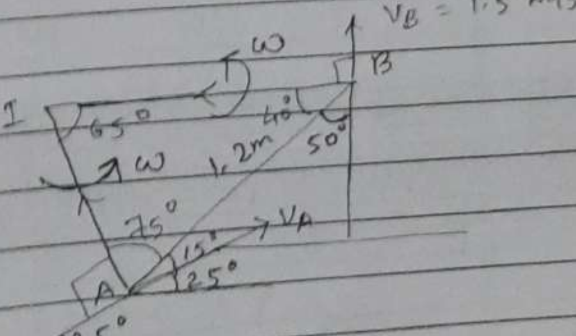
$V_C = \omega \times IC = 1.333 \times 3 = 4 \text{ m/s}$



$V_{cx} = V_c \cos 60$
$= 4 \cos 60$
$= \underline{2 \text{ m/s} (\rightarrow)}$. Ans (iv)

$V_{cy} = V_c \sin 60$
$= 4 \sin 60$
$= \underline{3.464 \text{ m/s} (\downarrow)}$ Ans (iv)

.4 Fig. shows a collar B which moves upwards with constan velocity of 1.5 m/s. At the instant when $\theta = 50°$, determin (i) the angular velocity of rod pinned @ B & freely resting @ A against 25° sloping ground & (ii) the veloci of end A of the rod.



ICR is a pt. of intersection of lines drawn ⊥ to V_A & V_B

$\omega = \frac{V_A}{IA} = \frac{V_B}{IB} \Rightarrow \frac{V_A}{IA} = \frac{1.5}{IB}$ —— (1)

From $\triangle AIB$, by sine rule

$\frac{1.2}{\sin 65} = \frac{IA}{\sin 40} = \frac{IB}{\sin 75} \Rightarrow IA = 0.851 \text{ m}$
$IB = 1.279 \text{ m}$

Page No.

Date ___/___/______

$$\omega = \frac{V_A}{0.851} = \frac{1.5}{1.279}$$

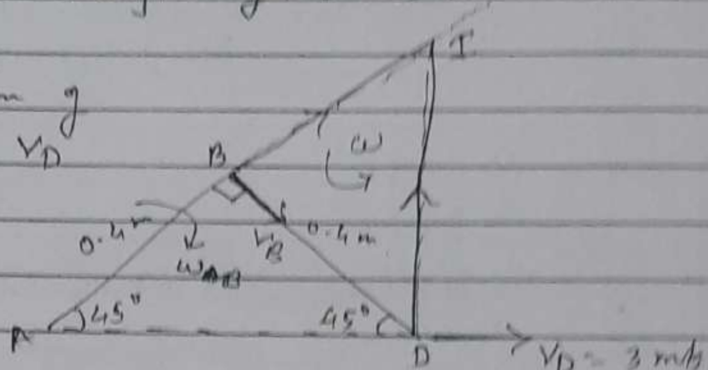
$\therefore\ \omega = 1.173$ rad/s Ans (i)

& $V_A = 0.998$ m/s $\left(\angle 25° \right)$ Ans (ii)

Ex. 6 Block D shown in fig. moves with a speed of 3 m/s. Determine angular velocity of links BD & AB & velocity of pt. B at the instant shown. Take length of AB = BD = 0.4 m

sol^n

ICR is an intersection of extension of link AB & $\perp^r$ to V_D



$V_B = AB \times \omega_{AB} = IB \times \omega$

$= 0.4\,\omega_{AB} = IB \times \omega \longrightarrow (1)$

$V_D = ID \times \omega$

$3 = ID \times \omega \longrightarrow (2)$

From $\triangle ABD$

$AD = \sqrt{0.4^2 + 0.4^2} = 0.5657$ m

From $\triangle AID$

$$\tan 45 = \frac{ID}{AD} = \frac{ID}{0.5657}$$

$$1 = \frac{ID}{0.5657} \Rightarrow ID = 0.5657$$

$AI = \sqrt{AD^2 + ID^2} = \sqrt{(0.5657)^2 + (0.5657)^2} = 0.8$ m

$\therefore\ IB = IA - AB = 0.8 - 0.4 = 0.4$ m

From (2)

$3 = 0.5657 \times \omega$ — how ?

$\therefore\ \omega = 5.303$ rad/s ↺ .... Angular vel. of link BD Ans.

$V_B = IB \times \omega = 0.4 \times 5.303 = 2.1212$ m/s Ans

$V_B = 0.4\,\omega_{AB}$

$2.1212 = 0.4\,\omega_{AB} \Rightarrow \omega_{AB} = 5.303$ rad/s ↻ Ans

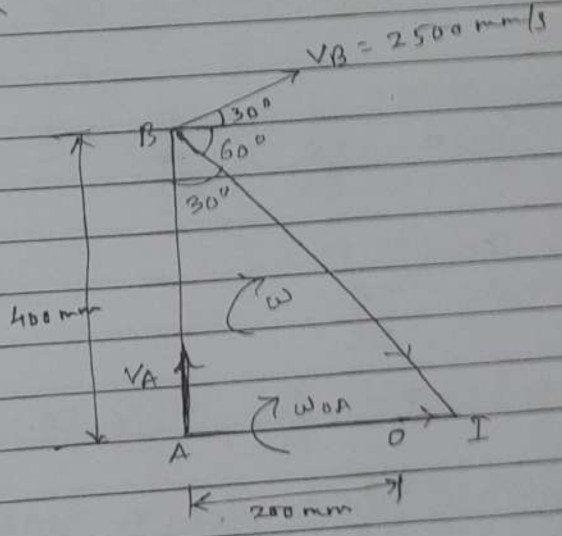
Saathi

Date ___/___/_____

Ex. 9 For the link & slider mechanism shown in fig., locate the ICR of link AB. Find also the angular vel. of link OA. Take velocity of slider @ B = 2500 mm/s

Sol^n

ICR is a pt. of intersectn of $\perp^r$ to V_A & V_B



$V_B = 2500 = IB \times \omega \rightarrow (1)$

$V_A = OA \times \omega_{OA} = IA \times \omega \rightarrow (2)$

from ΔAIB

$\cos 30 = \frac{AB}{IB} = \frac{400}{IB}$

$IB = 461.88$ mm

$\tan 30 = \frac{IA}{AB} = \frac{IA}{400}$

$IA = 230.94$ mm

from (1) $2500 = 461.88 \times \omega$

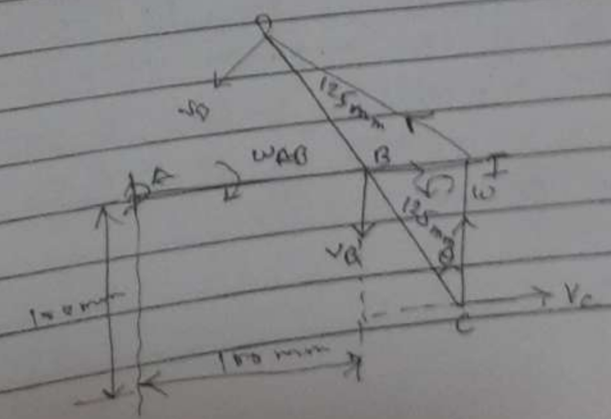
$\omega = 5.4127$ rad/s

from eqn (2) $200 \times \omega_{OA} = 230.94 \times 5.4127$

$\omega_{OA} = \underline{6.25 \text{ rad/s} (2)}$ Ans.

Ex. 10 At the position shown in fig, the crank AB has an angul velocity of 3 rad/s clockwise. Find the velocity of the slide velocity of C & pt. D at this instant.

ICR is a pt. of intersectn of $\perp^r$ to V_B & V_C



Saathi

Date ___/___/___

$v_B = AB \times \omega_{AB} = 0.1 \times 3 = 0.3$ m/s ($\downarrow$)

Also $v_B = IB \times \omega$

$0.3 = IB \times \omega$ — (1)

$v_C = IC \times \omega = 0.1 \times \omega$ — (2)

$v_D = ID \times \omega$ — (3)

from ΔIBC,

$$\cos\theta = \frac{IC}{BC} = \frac{100}{125}$$

$\therefore \theta = 36.87^\circ$

$$\sin\theta = \frac{IB}{BC}$$

$$\sin 36.87 = \frac{IB}{125} \quad \therefore IB = 75 \text{ mm} = 0.075 \text{ m}$$

Applying cosine rule to ΔICD

$$ID^2 = IC^2 + CD^2 - 2(IC)(CD)\cos\theta$$

$$= (100)^2 + (250)^2 - 2(100)(250)\cos 36.87$$

$ID = 180.278$ mm $= 0.1803$ m

From eqn (1) $0.3 = 0.075 \times \omega \Rightarrow \omega = 4$ rad/s

From eqn (2) $v_C = 0.1 \times 4 = 0.4$ m/s ($\rightarrow$) Ans

From eqn (3) $v_D = 0.1803 \times 4 = 0.7212$ m/s ($\perp^r$ to ID) Ans

Sa 12 A bar AB is 24 cm long & is hinged to a wall @ A. Another bar CD 32 cm long is connected by a pin @ B such that CB = 12 cm & B = 20 cm. At the instant shown (AB is $\perp^r$ to CD) the angular velocities of the bar are $\omega_{AB} = 4$ rad/s & $\omega_{CD} = 6$ rad/s. Determine the linear velocities of pt. C & D. Note that bar CD is for plane motion.

470.82

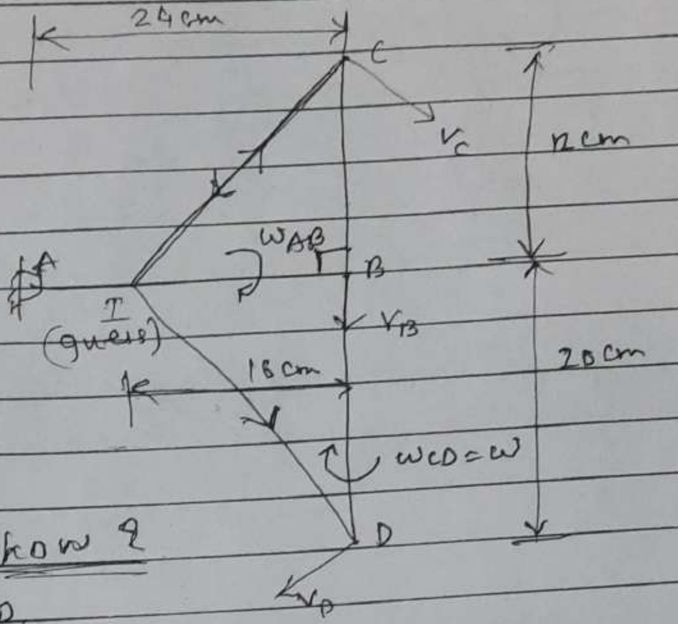
Saathi

Date ___/___/______

5^{th}

$\omega_{AB} = 4\ rad/s$

$\omega_{CD} = 6\ rad/s$



$V_B = AB \times \omega_{AB}$

$= 24 \times 4$

$= 96\ cm/s$ (↓)

∵ bar CD is in plane motion,

by concepts of ICR how?

$\omega_{CD} = \omega = \frac{v_B}{IB} = \frac{v_C}{IC} = \frac{v_D}{ID}$

$6 = \frac{96}{IB} = \frac{v_C}{IC} = \frac{v_D}{ID} \rightarrow (1)$

∴ $IB = 16\ cm$ IB must lie on AB, pt I lies to the left of B ~~along AB~~

$IC = \sqrt{IB^2 + BC^2} = \sqrt{(16)^2 + (12)^2} = 20\ cm$

$ID = \sqrt{IB^2 + BD^2} = \sqrt{(16)^2 + (20)^2} = 25.612\ cm$

From eqn (1)

$6 = \frac{v_C}{20} = \frac{v_D}{25.613}$

∴ $v_C = 120\ cm/s$ ($\perp^r$ to IC) Ans

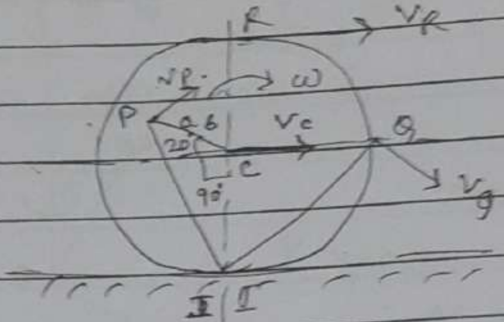
$v_D = 153.675\ cm/s$ ($\perp^r$ to ID) Ans.

Page No.

Date ___/___/___

Sn.13 A wheel of 2m dia rolls w/o slipping on a flat surface. The centre of the wheel is moving with a velocity of 4 m/s towards right. Determine the angular velocity of the wheel & velocity of P, Q & R shown on wheel.

Soln



Pt. I which is in contact with the flat surface is ICR
Let the wheel rotate with angular vel. ω

$$\therefore \omega = \frac{V_C}{IC} = \frac{V_P}{IP} = \frac{V_R}{IR} = \frac{V_Q}{IQ}$$

$$= \frac{4}{1} = \frac{V_P}{IP} = \frac{V_R}{IR} = \frac{V_Q}{IQ}$$

$$\therefore V_P = 4\,IP,\ V_R = 4\,IR,\ V_Q = 4\,IQ \quad \text{———} \ (1)$$

$\therefore \omega = 4$ rad/s (↻) Ans.

From ΔIPC, using cosine rule,

$$IP = \sqrt{(IC)^2 + (CP)^2 - 2(IC)(CP)\cos 110}$$

$$= \sqrt{(1)^2 + (0.6)^2 - 2(1)(0.6)\cos 110}$$

$$= 1.3306 \text{ m}$$

$IR = 2$m

$$IQ = \sqrt{IC^2 + CQ^2} = \sqrt{(1)^2 + (1)^2} = 1.4142 \text{ m}$$

$\therefore V_P = 4 \times 1.3306 = 5.322$ m/s ($\perp^r$ to IP) Ans

$V_R = 4 \times 2 = 8$ m/s (→) Ans

$V_Q = 4 \times 1.4142 = 5.6568$ m/s ($\perp^r$ to IQ) Ans.