Binomial distribution (BD)

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BD is also known as a Bernoulli Distribution. This distribution is used under the following conditions.

i) A trial is repeated n times where n is a finite number.

ii) Each trial results only in two ways-success or failure.

iii) These possibilities are mutually exclusive, exhaustive but not necessarily equally likely.

iv) If p & q are the probabilities of success and failure then $p + q=1$

v) The events are independent i.e., the probability p of success in each trial remains constant in all trials.

Definition: A random variable X is said to follow a binomial distribution if its pmf of X is given by

$P(X=x) = {}^{n}c_{x}\, p^{x}\, q^{n-x}$; $x=0,1,2,3\ldots\ldots n$ where $p + q =1$,

n & p are called the **parameters** of the distribution.

p- Probability of successes, q- Probability of failures in a single trial.

n- Total number of trials.

Properties:

1) Mean of Binomial distribution = $\mu = np$,

2) Variance of the binomial distribution = $\sigma^2 = npq$

3) Additive property of binomial distribution: If X_1 and X_2 are two binomial variables with parameters (n_1 , p) and (n_2 , p) then $X_1 + X_2$ is a binomial variable with parameter (n_1+n_2 , p)

EXAMPLE ON BIONAMAL DISTRIBUTION

1. Find the probability of three sixes in 5 tosses of a die.
2. A die is thrown eight times and it is required to find the probability that three will show i) exactly 2 times ii) at least seven times iii) at least once.
3. Six dice are thrown 729 times. How many times do you expect at least three dice to show five or six? [233]
4. The incidence of an occupational disease in any industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers four or more will catch the disease? [53/3125]
5. If on average rain falls on 10 days in every 30, find the probability that i) the first three days of a week will be fine and remaining wet ii) the rain will fall on at least three days of a given week. [8/2187,313/729]
6. An irregular six face die is thrown and the probability that in 20 throws it will give 5 even numbers is twice the probability that it will give 5 odd numbers. How many times in 10,000 sets of 10 throws would you expect it to give no even number? [13.65]
7. In a precision bombing attack, there is a 50% chance that any bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give more than 99% chance of destroying the target? [min.11]
8. In a binomial distribution consisting of five independent trials probabilities of one and two successes are 0.4096 and 0.2048 respectively. Find the probability of getting 3 successes and determine the mean of the distribution. [p=0.2,P(3)=0.0512, mean=1]
9. Out of 800 families with 4 children each how many would you expect to have i) 2 Boys and 2 Girls ii) at least one boy iii) no girl iv) at most 2 girls? [300,750,50,550]
10. If X is Binomial distribution with $E(X)=2$ and $Var(X)=4/3$, find the probability distribution of X and $p(X<4)$
11. The mean and variance of binomial distribution is 3 and 1.2. Find 'n', 'p' and $p(X<4)$

12. Find a binomial distribution for the following data and compare the theoretical frequencies with the actual ones:

x:	0	1	2	3	4	5
f:	2	14	20	34	22	8

[Ans.100(0.432 + 0.568)

13. Fit a Binomial distribution to the following data

X	0	1	2	3	4
F	28	62	46	10	4

[Ans. $\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{200}{15} = 1.33; p = 0.333; q = 0.667$]

14. Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys (ii) 5 girls and (iii) either 2 or 3 boys? Assuming equal probabilities for girls and boys. [Ans.(i)250 (ii) 25 (iii) 500]

15. The mean and variance of binomial distribution is 5 and 10/3. Find P(X=2)

16. Let X, Y be two independent binomial varieties with parameters $(n_1 = 6, p = 0.5) and (n_2 = 4, p = 0.5)$ respectively. Find $P(X + Y = 3)$ $and\ P(X + Y \geq 3)$ [0.1172, 0.945]

17. Let X, Y be two independent binomial varieties with parameters $(n_1 = 5, p = 0.4) and (n_2 = 7, p = 0.4)$ respectively. Find $P(X + Y \geq 3)$

18. Let X, Y be two independent binomial varieties with parameters $(n_1 = 8, p = 0.3) and (n_2 = 6, p = 0.3)$ respectively. Find $P(X + Y = 2)$

19. A communication system consists of n components, each of which functions independently with probability p. The total system will be able to function effectively if at least one-half of its components are functioning. For what value of p is a 5-component system more likely to operate effectively than 3-component system?

20. Suppose it is known that in a certain population 10% of the population is color blind. If a random sample of 25 people is drawn from this population, find the probability that (a) Five or fewer will be color blind. (b) Six or more will be color blind, (c)Between six and nine inclusive will be color blind, (d) Two, or three, or four will be color blind.

21. In a sampling of a large number of parts produced by a machine the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples how many samples would you expect to contain i) at least 3 defectives, ii) exactly 5, iii) between 7 and 15?

22. The probability of failure in physics practical examination is 20%. If 25 batches of 6 students each take the examination. In how many batches i) 4 or more, ii) exactly 3, iii) exactly 5 would pass?

23. If m things are distributed among 'a' men and 'b' women, show that the probability that the number of things receive by men is odd is $\frac{1}{2}\left[\frac{(b+a)^m-(b-a)^m}{(b+a)^m}\right]$

24. If X is a Binomial distributed with parameters n, and p, Show that $E\left(\frac{X}{n}-p\right)^2=\frac{pq}{n}$

25. The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is 1/4. What is the probability of 4 successes in 6 independent trials.

26. For special security in a certain protected area, it was decided to put three lighting bulbs on each pole if each bulb has probability p of burning out in the first 100 hours of service, calculate the probability that at least one of them is still good after 1oo hours. If p=0.3 how many bulbs would be needed on each pole to ensure 99%safety that at least one is good after 100 hours? Also find the probability that at least one of the bulbs is still working after 100 hours. [n=4]

27. Seven coins are tossed, and the number of heads is noted. The experiment is repeated 128 times and the following distribution is obtained:

No of heads	0	1	2	3	4	5	6	7	Total
Frequencies	7	6	19	35	30	23	7	1	128

Fit a binomial distribution (B.D.) assuming (i) the coin is unbiased.
(ii) the nature of the coin is not known.

28. The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 such men now at 60 at least 7 will live up to 70? [0.5138]