K J SOMAIYA COLLEGE OF ENGINEERING, MUMBAI-77
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ACCircuits

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3.2 TERMS RELATED TO ALTERNATING QUANTITIES

[May 2015]

Waveform A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Figure 3.3 shows a few waveforms.

Cycle One complete set of positive and negative values of an alternating quantity is termed a cycle.

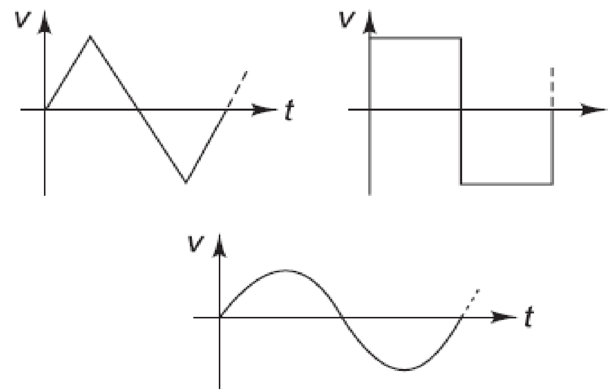


Fig. 3.3 *Alternating waveforms*

Frequency The number of cycles per second of an alternating quantity is known as its frequency. It is denoted by f and is measured in hertz (Hz) or cycles per second (c/s).

Time Period The time taken by an alternating quantity to complete one cycle is called its time period. It is denoted by T and is measured in seconds.

$$T = \frac{1}{f}$$

Amplitude The maximum positive or negative value of an alternating quantity is called the amplitude.

Phase The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.

Phase Difference This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum value may be different in magnitude.

A leading alternating quantity is one which reaches its maximum or zero value earlier compared to the other quantity.

A lagging alternating quantity is one which attains its maximum or zero value later than the other quantity.

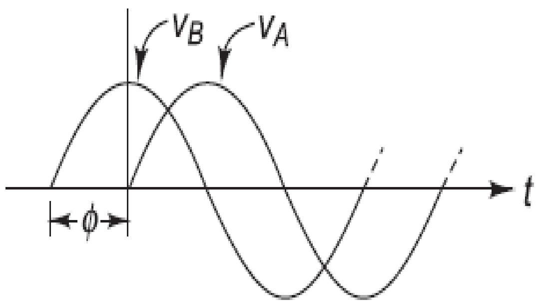


Fig. 3.4 *Phase difference*

A plus (+) sign, when used in connection with the phase difference, denotes 'lead' whereas a minus (–) sign denotes 'lag'.

$$v_A = V_m \sin \omega t$$

$$v_B = V_m \sin (\omega t + \phi)$$

Here, the quantity B leads A by a phase angle ϕ.

ROOT MEAN SQUARE (RMS) OR EFFECTIVE VALUE

Let I be the value of the direct current that while flowing through the same resistance does the same amount of work in the same time t. Then

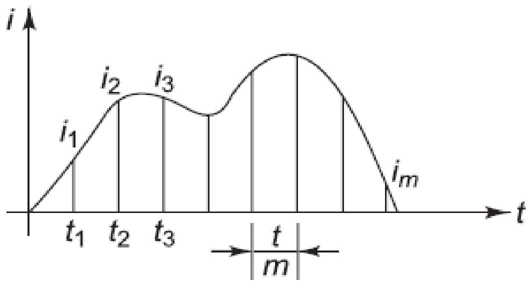


Fig. 3.5 *Mid-ordinate method*

$$I^2 R t = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m} \times R t$$

$$I^2 = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}$$

Hence, rms value of the alternating current is given by

$$I_{\text{rms}} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}} = \sqrt{\text{Mean value of } (i)^2}$$

3.3.1 RMS Value of Sinusoidal Waveform

$$v = V_m \sin \theta \qquad 0 < \theta < 2\pi$$

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2 \theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2 \theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2} - 0 - 0 + 0\right]}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$= 0.707\, V_m$$

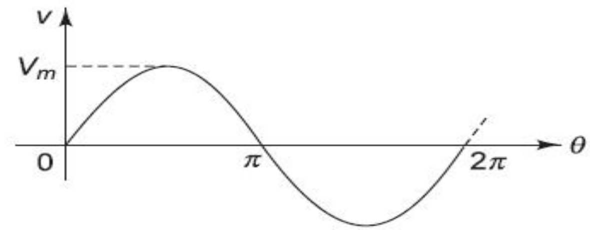


Fig. 3.6 *Sinusoidal waveform*

Crest or Peak or Amplitude Factor It is defined as the ratio of maximum value to rms value of the given quantity.

$$\text{Peak factor } (k_p) = \frac{\text{Maximum value}}{\text{rms value}}$$

3.4.1 Average Value of Sinusoidal Waveform [Dec 2013]

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

Since this is a symmetrical waveform, the average value is calculated over half the cycle.

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\,d\theta$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin\theta\,d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1]$$

$$= \frac{2V_m}{\pi}$$

$$= 0.637\,V_m$$

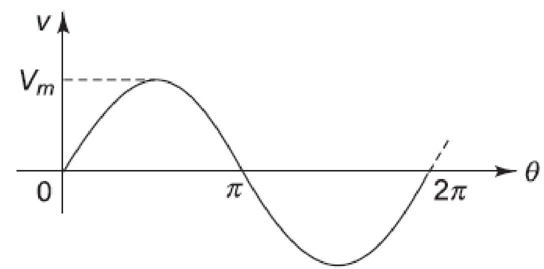


Fig. 3.7 *Sinusoidal waveform*

Form Factor It is defined as the ratio of rms value to the average value of the given quantity.

$$\text{Form factor } (k_f) = \frac{\text{rms value}}{\text{Average value}}$$

BEHAVIOUR OF A PURE RESISTOR IN AN AC CIRCUIT

Consider a pure resistor R connected across an alternating voltage source v as shown in Fig. 4.1. Let the alternating voltage be $v = V_m \sin \omega t$.

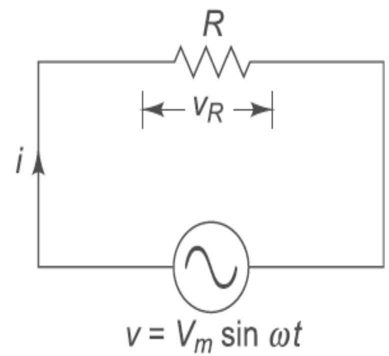


Fig. 4.1 *Purely resistive circuit*

Current The alternating current i is given by

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \left(\because I_m = \frac{V_m}{R} \right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current is in phase with the voltage in a purely resistive circuit.

Waveforms The voltage and current waveforms are shown in Fig. 4.2.

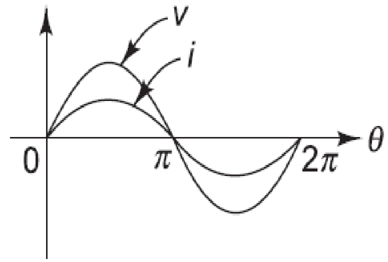


Fig. 4.2 *Waveforms*

Phasor Diagram The phasor diagram is shown in Fig. 4.3. The voltage and current phasors are drawn in phase and there is no phase difference.

Fig. 4.3 *Phasor diagram*

Impedance It is the resistance offered to the flow of current in an ac circuit. In a purely resistive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$$

Phase Difference Since the voltage and current are in phase with each other, the phase difference is zero.

$$\phi = 0°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{Power factor} = \cos\phi = \cos(0°) = 1$$

Power Instantaneous power p is given by

$$p = vi$$
$$= V_m \sin \omega t \, I_m \sin \omega t$$
$$= V_m I_m \sin^2 \omega t$$
$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$
$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

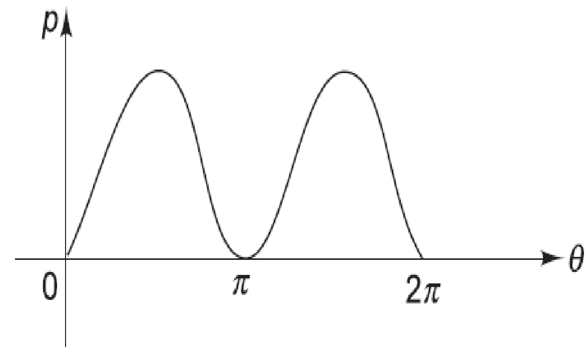


Fig. 4.4 *Power waveform*

The power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$. The frequency of the fluctuating power is twice the applied voltage frequency and its average value over one complete cycle is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

Thus, power in a purely resistive circuit is equal to the product of rms values of voltage and current.

4.2 BEHAVIOUR OF A PURE INDUCTOR IN AN AC CIRCUIT

Consider a pure inductor L connected across an alternating voltage v as shown in Fig. 4.5. Let the alternating voltage be $v = V_m \sin \omega t$.

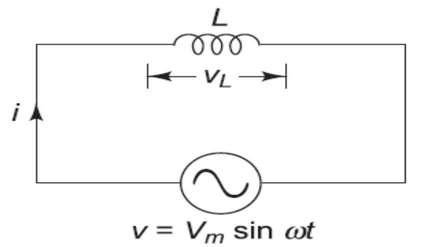


Fig. 4.5 *Purely inductive circuit*

Current The alternating current i is given by

$$i = \frac{1}{L}\int v\,dt$$

$$= \frac{1}{L}\int V_m \sin \omega t\, dt$$

$$= \frac{V_m}{\omega L}(-\cos \omega t)$$

$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \cdots \left(I_m = \frac{V_m}{\omega L}\right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current lags behind the voltage by 90° in a purely inductive circuit.

Waveforms The voltage and current waveforms are shown in Fig. 4.6.

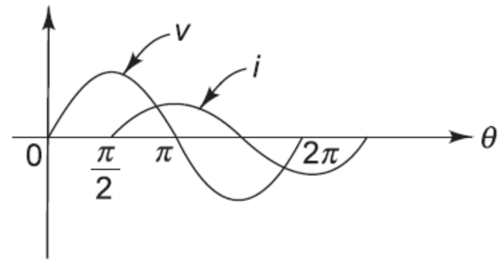


Fig. 4.6 *Waveforms*

Phasor Diagram The phasor diagram is shown in Fig. 4.7. Here, voltage $\bar{V}$ is chosen as reference phasor. Current $\bar{I}$ is drawn such that it lags behind $\bar{V}$ by 90°.

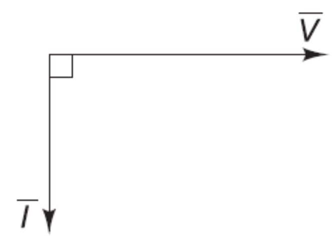


Fig. 4.7 *Phasor diagram*

Impedance In a purely inductive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$$

The quantity ωL is called inductive reactance, is denoted by X_L and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_L = 0$

Thus, an inductor acts as a short circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$$\phi = 90°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

4.3 BEHAVIOUR OF A PURE CAPACITOR IN AN AC CIRCUIT

[Dec 2013]

Consider a pure capacitor C connected across an alternating voltage v as shown in Fig. 4.9. Let the alternating voltage be $v = V_m \sin \omega t$.

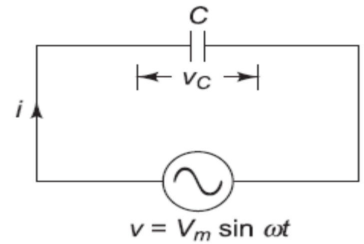


Fig. 4.9 *Purely capacitive circuit*

Current The alternating current i is given by

$$i = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}(V_m \sin \omega t)$$

$$= \omega C V_m \cos \omega t$$

$$= \omega C V_m \sin(\omega t + 90°)$$

$$= I_m \sin(\omega t + 90°) \quad \ldots (I_m = \omega C V_m)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current leads the voltage by 90° in a purely capacitive circuit.

BEHAVIOUR OF A PURE CAPACITOR IN AN AC CIRCUIT

Waveforms The voltage and current waveforms are shown in Fig. 4.10.

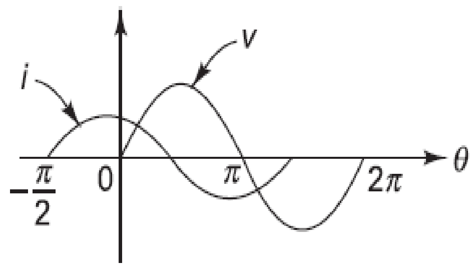


Fig. 4.10 *Waveforms*

Phasor Diagram The phasor diagram is shown in Fig. 4.11. Here, voltage $\bar{V}$ is chosen as reference phasor. Current $\bar{I}$ is drawn such that it leads $\bar{V}$ by 90°.

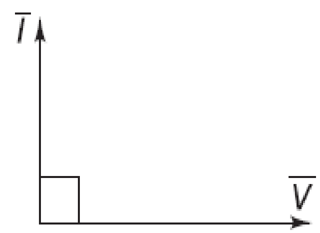


Fig. 4.11 *Phasor diagram*

Impedance In a purely capacitive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

The quantity $\frac{1}{\omega C}$ is called capacitive reactance, is denoted by X_C and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_C = \infty$

Thus, the capacitor acts as an open circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$\phi = 90°$

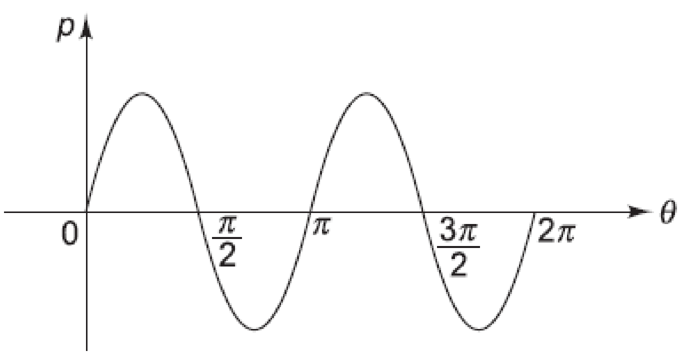


Fig. 4.12 *Power waveform*

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power Instantaneous power p is given by

$$p = vi$$
$$= V_m \sin\omega t \; I_m \sin(\omega t + 90)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely capacitive circuit is zero.

When power is positive, i.e., voltage increases across the plates of capacitor, energy is supplied form source to build up the electrostatic field between the plates of capacitor and the capacitor is energized. When power is negative, i.e., voltage decreases, the collapsing electrostatic field returns the stored energy to the source. This circulating power is called as reactive power.

Example 1

An ac circuit consists of a pure resistance of 10 ohms and is connected across an ac supply of 230 V, 50 Hz. Calculate (i) current, (ii) power consumed, (iii) power factor, and (iv) write down the equations for voltage and current.

Solution $R = 10\ \Omega$

$V = 230$ V

$f = 50$ Hz

$$v = V_m \sin \omega t$$
$$i = I_m \sin \omega t$$

$$V_m = \sqrt{2}\ V$$
$$I_m = \sqrt{2}\ I$$

Example 1

An ac circuit consists of a pure resistance of 10 ohms and is connected across an ac supply of 230 V, 50 Hz. Calculate (i) current, (ii) power consumed, (iii) power factor, and (iv) write down the equations for voltage and current.

Solution

$$R = 10\ \Omega$$
$$V = 230\text{ V}$$
$$f = 50\text{ Hz}$$

(i) Current

$$I = \frac{V}{R} = \frac{230}{10} = 23\text{ A}$$

(ii) Power consumed

$$P = VI = 230 \times 23 = 5290\text{ W}$$

(iii) Power factor

Since the voltage and current are in phase with each other, $\phi = 0°$

$$\text{pf} = \cos\phi = \cos(0°) = 1$$

(iv) Voltage and current equations

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 230 = 325.27\text{ V}$$
$$I_m = \sqrt{2}\,I = \sqrt{2} \times 23 = 32.53\text{ A}$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s}$$
$$v = V_m \sin\omega t = 325.27 \sin 314.16\,t$$
$$i = I_m \sin\omega t = 32.53 \sin 314.16\,t$$

Example 2

A voltage of 150 V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current. **[Dec 2012]**

Solution

$$V_{rms} = 150 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$v = V_m \sin 2\pi f t$$

$$V_m = V_{rms}\sqrt{2}$$

$$i = I_m \sin(2\pi f t - 90°)$$

$$I_m = \frac{V_m}{X_L}$$

$$X_L = 2\pi f L$$

Example 2

A voltage of 150 V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current. **[Dec 2012]**

Solution

$$V_{\text{rms}} = 150 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$L = 0.2 \text{ H}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$$

$$V_m = V_{\text{rms}}\sqrt{2} = 150\sqrt{2} = 212.13 \text{ V}$$

$$I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = 3.38 \text{ A}$$

$$v = V_m \sin 2\pi f t = 212.13 \sin 2\pi \times 50 \times t = 212.13 \sin 100\pi t$$

$$i = I_m \sin(2\pi f t - 90°) = 3.38 \sin(100\pi t - 90°)$$

Example 5

A capacitor has a capacitance of 30 microfarads which is connected across a 230 V, 50 Hz supply. Find (i) capacitive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

Solution $C = 30\ \mu\text{F}$

$V = 230\ \text{V}$

$f = 50\ \text{Hz}$

$$v = V_m \sin 2\pi f t \qquad V_m = \sqrt{2}\ V \qquad I_m = \sqrt{2}\ I$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \qquad I = \frac{V}{X_C} \qquad X_C = \frac{1}{2\pi f C}$$

(i) Capacitive reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}} = 106.1\ \Omega$$

(ii) rms value of current

$$I = \frac{V}{X_C} = \frac{230}{106.1} = 2.17\ \text{A}$$

(iii) Power

Since the current leads the voltage by 90° in purely capacitive circuit, $\phi = 90°$

$$P = VI\cos\phi = 230 \times 2.17 \times \cos(90°) = 0$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

(v) Equations for voltage and current

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 230 = 325.27\ \text{V}$$

$$I_m = \sqrt{2}\,I = \sqrt{2} \times 2.17 = 3.07\ \text{A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16\ \text{rad/s}$$

$$v = V_m \sin\omega t = 325.27 \sin 314.16\, t$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = 3.07 \sin\left(314.16\, t + \frac{\pi}{2}\right)$$

4.4 SERIES R-L CIRCUIT

Figure 4.13 shows a pure resistor R connected in series with a pure inductor L across an alternating voltage v.

Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90°.

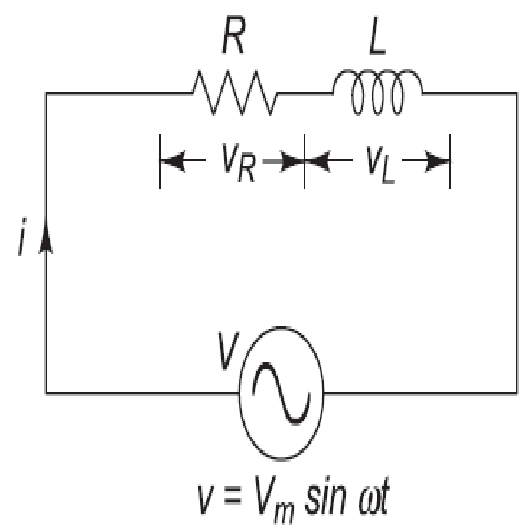


Fig. 4.13 Series R-L circuit

Phasor Diagram

Steps for drawing phasor diagram

1. Since the same current flows through series circuit, $\bar{I}$ is taken as reference phasor.
2. Draw $\bar{V}_R$ in phase with $\bar{I}$.
3. Draw $\bar{V}_L$ such that it leads $\bar{I}$ by 90°.
4. Add $\bar{V}_R$ and $\bar{V}_L$ by triangle law of vector addition such that

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

5. Mark the angle between $\bar{I}$ and $\bar{V}$ as ϕ.

The phasor diagram is shown in Fig. 4.14.

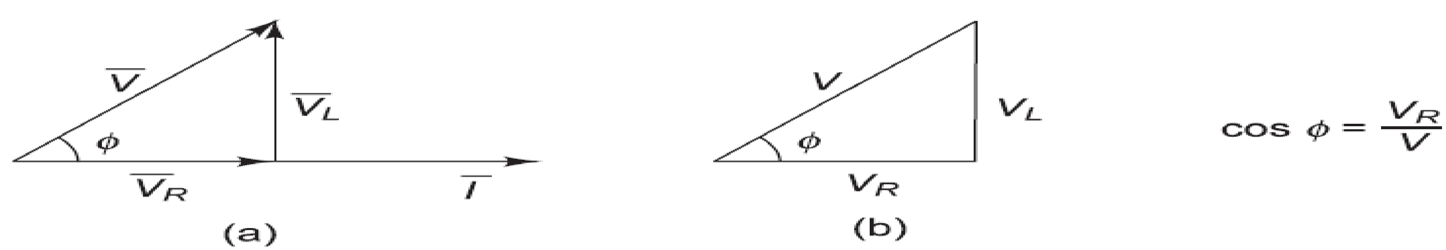
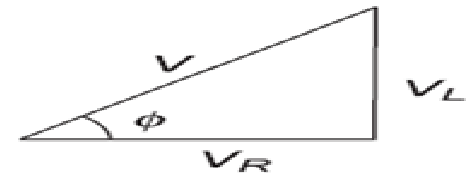


Fig. 4.14 *(a) Phasor diagram (b) Voltage triangle*

It is clear from phasor diagram that current $\bar{I}$ lags behind applied voltage $\bar{V}$ by an angle $\phi\ (0° < \phi < 90°)$



(b)

Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_L = R\bar{I} + jX_L\,\bar{I} = (R + jX_L)\,\bar{I}$$

$$\frac{\bar{V}}{\bar{I}} = R + jX_L = \bar{Z}$$

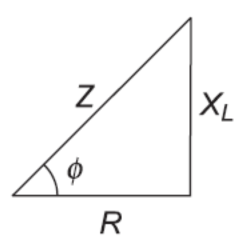
$$\bar{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The quantity Z is called the *complex impedance* of the R-L circuit.

Impedance Triangle The impedance triangle is shown in Fig. 4.15.



$$\cos\phi = \frac{R}{Z}$$

Fig. 4.15 *Impedance triangle*

Current From the phasor diagram, it is clear that the current I lags behind the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin (\omega t - \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Waveforms The voltage and current waveforms are shown in Fig. 4.16.

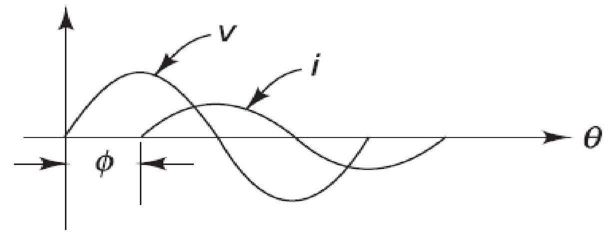


Fig. 4.16 *Waveforms*

Power Instantaneous power p is given by

$$p = v\,i$$

$$= V_m \sin \omega t \, I_m \sin (\omega t - \phi)$$

$$= V_m I_m \sin \omega t \sin (\omega t - \phi)$$

$$= V_m I_m \left[\frac{\cos \phi - \cos (2\omega t - \phi)}{2} \right]$$

$$= \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos (2\omega t - \phi)$$

Thus, power consists of a constant part $\frac{V_m I_m}{2}$ cos ϕ and a fluctuating part $\frac{V_m I_m}{2}$ cos $(2\omega t - \phi)$. The frequency of the fluctuating part is twice the applied voltage frequency and its average value over one complete cyce is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi = VI \cos \phi$$

Thus, power is dependent upon the in-phase component of the current. The average power is also called *active power* and is measured in watts.

We know that a pure inductor and capacitor consume no power because all the power received from the source in a half cycle is returned to the source in the next half cycle. This circulating power is called *reactive power*. It is a product of the voltage and reactive component of the current, i.e., $I \sin \phi$ and is measured in VAR (volt–ampere-reactive).

Reactive power $Q = VI \sin \phi$.

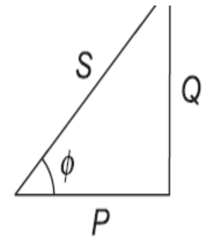
The product of voltage and current is known as *apparent power* (S) and is measured in volt–ampere (VA).

$$S = \sqrt{P^2 + Q^2}$$

$$P = VI\cos\phi = ZII\frac{R}{Z} = I^2R$$

$$Q = VI\sin\phi = ZII\frac{X_L}{Z} = I^2X_L$$

$$S = VI = ZII = I^2Z$$



$$\cos\phi = \frac{P}{S}$$

Fig. 4.17 *Power triangle*

The power triangle is shown in Fig. 4.17.

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

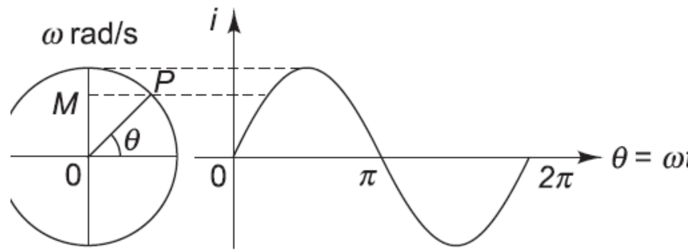
From impedance triangle, $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an R-L series circuit, the power factor is lagging in nature since the current lags behind the voltage by an angle ϕ.

PHASOR REPRESENTATIONS OF ALTERNATING QUANTITIES

$$OM = OP \sin \omega t$$
$$= I_m \sin \omega t = i$$



(a) (b)

3.6 MATHEMATICAL REPRESENTATIONS OF PHASORS

A phasor can be represented in four forms.

(i) Rectangular form

$$\bar{V} = X \pm jY$$

Magnitude of phasor, $V = \sqrt{X^2 + Y^2}$

Phase angle $\phi = \tan^{-1}\left(\frac{Y}{X}\right)$

(ii) Trigonometric form

$$\bar{V} = V(\cos\phi \pm j\sin\phi)$$

(iii) Exponential form

$$\bar{V} = V e^{\pm j\phi}$$

(iv) Polar form

$$\bar{V} = V \angle \pm \phi$$

Example 1

Two sinusoidal currents are given as

$$i_1 = 10\sqrt{2}\,\sin \omega t,\; i_2 = 20\sqrt{2}\,\sin(\omega t + 60°).$$

Find the expression for the sum of these currents.

Solution

$$i_1 = 10\sqrt{2}\,\sin \omega t$$

$$i_2 = 20\sqrt{2}\,\sin(\omega t + 60°)$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{10\sqrt{2}}{\sqrt{2}}\angle 0° = 10\angle 0°$$

$$\bar{I}_2 = \frac{20\sqrt{2}}{\sqrt{2}}\angle 60° = 20\angle 60°$$

$$
\begin{aligned}
\bar{I} &= \bar{I}_1 + \bar{I}_2 \\
&= 10\angle 0^\circ + 20\angle 60^\circ \\
&= 26.46\ \angle 40.89^\circ \\
i &= 26.46\sqrt{2}\ \sin\,(\omega t + 40.89^\circ) \\
&= 37.42\ \sin\,(\omega t + 40.89^\circ)
\end{aligned}
$$

Example 1

An alternating voltage of 80 + j60 V is applied to a circuit and the current flowing is 4 – j2 A. Find the (i) impedance, (ii) phase angle, (iii) power factor, and (iv) power consumed.

Solution

$$\bar{V} = 80 + j60 \text{ V}$$

$$\bar{I} = 4 - j2 \text{ A}$$

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{80 + j60}{4 - j2} = \frac{100\angle 36.87^\circ}{4.47\angle -26.56^\circ} = 22.37\ \angle 63.43^\circ\ \Omega$$

$$Z = 22.37\ \Omega$$

(ii) Phase angle

$$\phi = 63.43^\circ$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(63.43^\circ) = 0.447 \text{ (lagging)}$$

(iv) Power consumed

$$P = VI\cos\phi = 100 \times 4.47 \times 0.447 = 199.81 \text{ W}$$

Example 1

An alternating voltage of 80 + j 60 V is applied to a circuit and the current flowing is 4 – j 2 A. Find the (i) impedance, (ii) phase angle, (iii) power factor, and (iv) power consumed.

Solution

$$\bar{V} = 80 + j60 \text{ V}$$

$$\bar{I} = 4 - j2 \text{ A}$$

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{80 + j60}{4 - j2} = \frac{100\angle 36.87^\circ}{4.47\angle -26.56^\circ} = 22.37\ \angle 63.43^\circ\ \Omega$$

$$Z = 22.37\ \Omega$$

(ii) Phase angle

$$\phi = 63.43^\circ$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(63.43^\circ) = 0.447 \text{ (lagging)}$$

(iv) Power consumed

$$P = VI\cos\phi = 100 \times 4.47 \times 0.447 = 199.81 \text{ W}$$

Example 2

The voltage and current in a circuit are given by $\bar{V} = 150\ \angle 30°\ V$ and $\bar{I} = 2\ \angle -15°\ A$. If the circuit works on a 50 Hz supply, determines impedance, resistance, reactance, power factor and power loss considering the circuit as a simple series circuit.

Solution

$$\bar{V} = 150\ \angle 30°\ \text{V}$$

$$\bar{I} = 2\angle -15°\ \text{A}$$

$$f = 50\ \text{Hz}$$

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{150\angle 30°}{2\angle -15°} = 75\ \angle 45°\ \Omega = 53.03 + j53.03\ \Omega$$

$$Z = 75\ \Omega$$

(ii) Resistance

$$R = 53.03\ \Omega$$

(iii) Reactance

$$X = 53.03\ \Omega$$

(iv) Power factor

$$\phi = 45°$$

$$\text{pf} = \cos\phi = \cos(45°) = 0.707\ \text{(lagging)}$$

(v) Power loss

$$P = VI\cos\phi = 150 \times 2 \times 0.707 = 212.1\ \text{W}$$

Example 3

An rms voltage of 100 ∠0° V is applied to a series combination of Z_1 and Z_2 when Z_1 = 20 ∠30° Ω. The effective voltage drop across Z_1 is known to be 40 ∠–30° V. Find the reactive component of Z_2.

Solution

$$\bar{V} = 100\angle 0^\circ \text{ V}$$

$$\bar{Z}_1 = 20\angle 30^\circ\ \Omega$$

$$\bar{V}_1 = 40\angle -30^\circ \text{ V}$$

$$\bar{I} = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{40\angle -30^\circ}{20\angle 30^\circ} = 2\angle -60^\circ \text{ A}$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{100\angle 0^\circ}{2\angle -60^\circ} = 50\angle 60^\circ = 25 + j43.3\ \Omega$$

$$\bar{Z}_1 = 20\angle 30^\circ = 17.32 + j10\ \Omega$$

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$$

$$\bar{Z}_2 = \bar{Z} - \bar{Z}_1 = 25 + j43.3 - 17.32 - j10 = 7.68 + j33.3\ \Omega$$

Reactive component of $\bar{Z}_2 = 33.3\ \Omega$

Example 4

A voltage v(t) = 177 sin (314t + 10°) is applied to a circuit. It causes a steady-state current to flow, which is described by i(t) = 14.14 sin (314t – 20°). Determine the power factor and average power delivered to the circuit.

Solution

$$v(t) = 177 \sin (314t + 10°)$$

$$i(t) = 14.14 \sin (314t - 20°)$$

(i) Power factor

Current $i(t)$ lags behind voltage $v(t)$ by 30°.

$$\phi = 30°$$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (lagging)}$$

(ii) Average power

$$P = VI\cos\phi = \frac{177}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \times 0.866 = 1083.7 \text{ W}$$

Example 6

In a series circuit containing resistance and inductance, the current and voltage are expressed as $i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right)$ and $v(t) = 20\sin\left(314t + \frac{5\pi}{6}\right)$. (i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance and power factor? (iii) What is the average power drawn by the circuit?

Solution

$$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right)$$

$$v(t) = 20\sin\left(314t + \frac{5\pi}{6}\right)$$

(i) Impedance

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{20}{5} = 4\ \Omega$$

(ii) Power factor, resistance and inductance

Current $i(t)$ lags behind voltage $v(t)$ by an angle $\phi = 150° - 120° = 30°$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (lagging)}$$

$$\bar{Z} = 4\angle 30° = 3.464 + j2\ \Omega$$

$$R = 3.464\ \Omega$$

$$X_L = 2\ \Omega$$

$$X_L = \omega L$$

$$2 = 314 \times L$$

$$L = 6.37 \text{ mH}$$

(iii) Average power

$$P = VI\cos\phi = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3 \text{ W}$$

Example 15

A resistor of 25 Ω is connected in series with a choke coil. The series combination when connected across a 250 V, 50 Hz supply, draws a current of 4 A which lags behind the voltage by 65°. Calculate (i) resistance and inductance of the coil, (ii) total power, (iii) power consumed by resistance, and (iv) power consumed by choke coil. **[May 2014]**

Solution

$$R = 25\ \Omega$$
$$V = 250\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$I = 4\ \text{A}$$
$$\phi = 65°$$

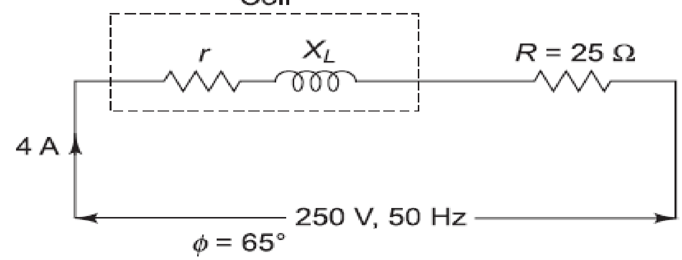


Fig. 4.22

(i) Resistance and inductance of the coil

$$Z = \frac{V}{I} = \frac{250}{4} = 62.5\ \Omega$$

$$\bar{Z} = Z\angle\phi = 62.5\ \angle 65° = 26.41 + j56.64\ \Omega$$

But $$\bar{Z} = (R + r) + jX_L$$

$$X_L = 56.64\ \Omega$$
$$R + r = 26.41$$
$$r = 26.41 - 25 = 1.41\ \Omega$$
$$X_L = 2\pi f L$$
$$56.64 = 2\pi \times 50 \times L$$
$$L = 0.18\ \text{H}$$

(ii) Total power

$$P = I^2 (R + r) = (4)^2 \times 26.41 = 422.56 \text{ W}$$

(iii) Power consumed by resistance

$$P_R = I^2 R = (4)^2 \times 25 = 400 \text{ W}$$

(iv) Power consumed by choke coil

$$P_{\text{coil}} = I^2 r = (4)^2 \times 1.41 = 22.56 \text{ W}$$

Figure 4.32 shows a pure resistor R connected in series with a pure capacitor C across an alternating voltage v.

Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

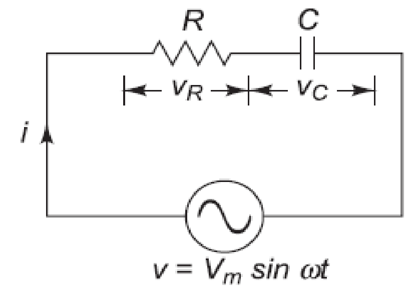


Fig. 4.32 *Series R-C circuit*

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

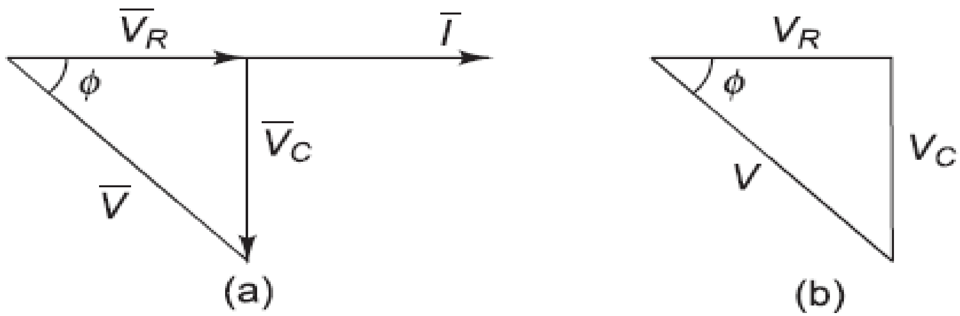
$$\bar{V} = \bar{V}_R + \bar{V}_C$$

Phasor Diagram

Steps for drawing phasor diagram

1. Since the same current flows through series circuit, $\bar{I}$ is taken as reference phasor.
2. Draw $\bar{V}_R$ in phase with $\bar{I}$.
3. Draw $\bar{V}_C$ such that it lags behind $\bar{I}$ by 90°.
4. Add $\bar{V}_R$ and $\bar{V}_C$ by triangle law of addition such that
$$\bar{V} = \bar{V}_R + \bar{V}_C$$
5. Mark the angle $\bar{I}$ and $\bar{V}$ as ϕ.

The phasor diagram is shown in Fig. 4.33. It is clear from phasor diagram that current $\bar{I}$ leads applied voltage $\bar{V}$ by an angle $\phi\ (0° < \phi < 90°)$.



$$\cos\phi = \frac{V_R}{V}$$

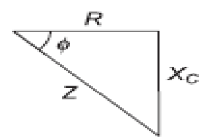
Fig. 4.33 *(a) Phasor diagram (b) Voltage triangle*

Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_C$$
$$= R\bar{I} - jX_C\bar{I}$$
$$= (R - jX_C)\bar{I}$$
$$\frac{\bar{V}}{\bar{I}} = R - jX_C = \bar{Z}$$
$$\bar{Z} = Z\angle -\phi$$
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

The quantity $\bar{Z}$ is called the *complex impedance* of the *R-C* circuit.

Impedance Triangle The impedance triangle is shown in Fig. 4.34.



$$\cos\phi = \frac{R}{Z}$$

Fig. 4.34 *Impedance triangle*

Current From the phasor diagram, it is clear that the current I leads the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t + \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Waveforms The voltage and current waveforms are shown in Fig. 4.35.

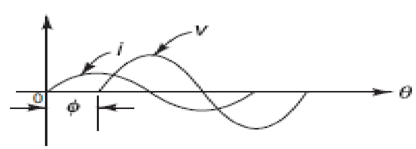


Fig. 4.35 *Waveforms*

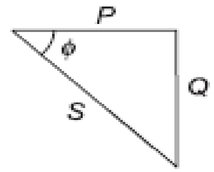
Power

Active power $P = VI\cos\phi = I^2R$

Reactive power $Q = VI\sin\phi = I^2X_C$

Apparent power $S = VI = I^2Z$

Power Triangle The power triangle is shown in Fig. 4.36.



$$\cos\phi = \frac{P}{S}$$

Fig. 4.36 _Power triangle_

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an R-C series circuit, the power factor is leading in nature since the current leads the voltage by an angle ϕ.

Example 1

Example 1

The voltage applied to a circuit is e = 100 sin (ωt + 30°) and the current flowing in the circuit is i = 15 sin (ωt + 60°). Determine impedance, resistance, reactance, power factor and power.

Solution

$$e = 100 \sin(\omega t + 30°)$$

$$i = 15 \sin(\omega t + 60°)$$

(i) Impedance

$$\bar{E} = \frac{100}{\sqrt{2}} \angle 30° \text{ V}$$

$$\bar{I} = \frac{15}{\sqrt{2}} \angle 60° \text{ A}$$

$$\bar{Z} = \frac{\bar{E}}{\bar{I}} = \frac{\frac{100}{\sqrt{2}}\angle 30°}{\frac{15}{\sqrt{2}}\angle 60°} = 6.67 \angle -30° = 5.77 - j3.33 = R - j X_C$$

$$Z = 6.67 \ \Omega$$

(ii) Resistance

$$R = 5.77 \ \Omega$$

(iii) Reactance

$$X_C = 3.33 \ \Omega$$

(iv) Power factor

$$\text{pf} = \cos \phi = \cos (30°) = 0.866 \text{ (leading)}$$

(v) Power

$$P = EI \cos \phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.866 = 649.5 \text{ W}$$

Example 6

A voltage of 125 V at 50 Hz is applied across a non-inductive resistor connected in series with a capacitor. The current is 2.2 A. The power loss in the resistor is 96.8 W. Calculate the resistance and capacitance.

Solution

$$V = 125\text{ V}$$

$$f = 50\text{ Hz}$$

$$I = 2.2\text{ A}$$

$$P = 96.8\text{ W}$$

(i) Resistance

$$Z = \frac{V}{I} = \frac{125}{2.2} = 56.82\text{ A}$$

$$P = I^2 R$$

$$96.8 = (2.2)^2 \times R$$

$$R = 20\ \Omega$$

(ii) Capacitance

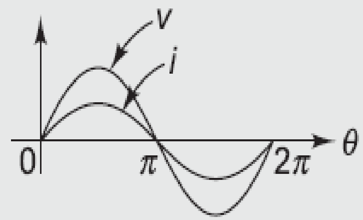
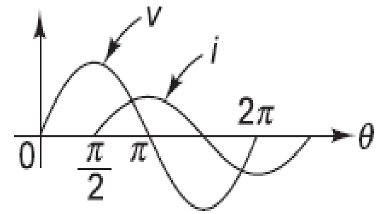
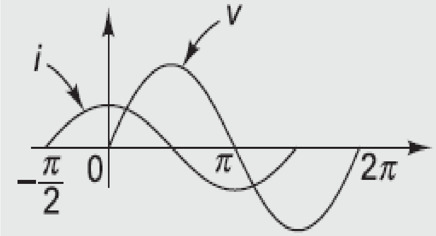
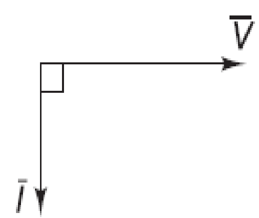
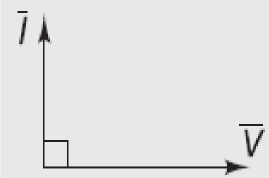
$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(56.82)^2 - (20)^2} = 53.18\ \Omega$$

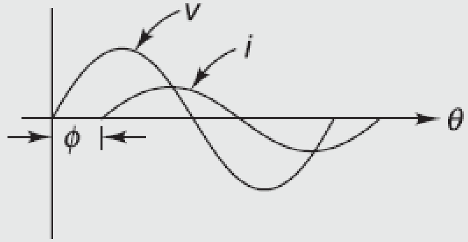
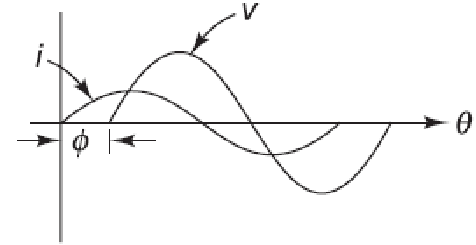
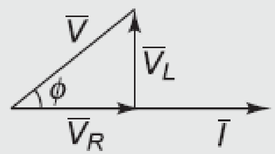
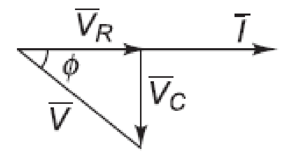
$$X_C = \frac{1}{2\pi f C}$$

$$53.18 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 59.85\ \mu\text{F}$$

Useful Formulae

	R	L	C
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin \omega t$	$I_m \sin (\omega t - 90°)$	$I_m \sin (\omega t + 90°)$
Waveform			
Phasor Diagram			
Impedance	R	$j\omega L$	$\frac{1}{j\omega C} = -j\frac{1}{\omega C}$
Phase Difference	0°	90°	90°
Power Factor	1	0	0
Power	VI	0	0

	Series R-L Circuit	*Series R-C Circuit*
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin(\omega t - \phi)$	$I_m \sin(\omega t + \phi)$
Waveform		
Phasor Diagram		
Impedance	$R + jX_L, Z\angle\phi$	$R - jX_C, Z\angle-\phi$
Phase Difference	$0° < \phi < 90°$	$0° < \phi < 90°$
Power Factor	$\frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$	$\frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$
Power	$P = VI\cos\phi = I^2R$ $Q = VI\sin\phi = I^2X_L$ $S = VI = I^2Z$	$P = VI\cos\phi = I^2R$ $Q = VI\sin\phi = I^2X_C$ $S = VI = I^2Z$

4.6 SERIES *R-L-C* CIRCUIT

Figure 4.39 shows a pure resistor R, pure inductor L and pure capacitor C connected in series across an alternating voltage v.

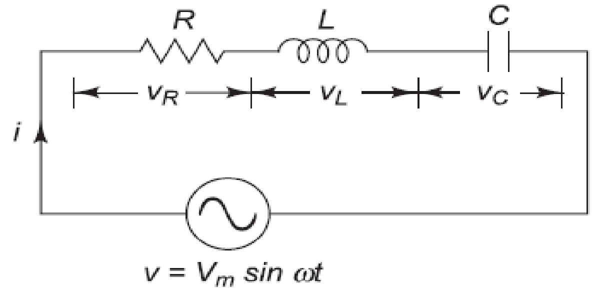


Fig. 4.39 *Series R-L-C circuit*

Let V and I be the rms values of the applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$, the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90° and the voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

Phasor Diagram Since the same current flows through R, L and C, the current I is taken as a reference phasor.

Case (i) $X_L > X_C$

The reactance X will be inductive in nature and the circuit will behave like an R-L circuit.

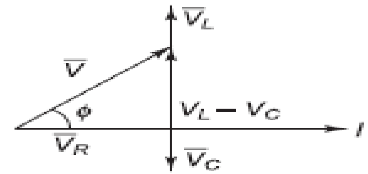


Fig. 4.40 Phasor diagram

Case (ii) $X_C > X_L$

The reactance X will be capacitive in nature and the circuit will behave like an R-C circuit.

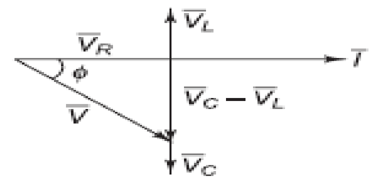


Fig. 4.41 Phasor diagram

Impedance

$$\overline{V} = \overline{V}_R + \overline{V}_L + \overline{V}_C = R\overline{I} + jX_L I - jX_C\overline{I} = [R + j\,(X_L - X_C)]\,\overline{I}$$

$$\frac{\overline{V}}{\overline{I}} = R + j\,(X_L - X_C) = \overline{Z}$$

$$\overline{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Impedance Triangles Impedance triangles are shown in Fig. 4.42.

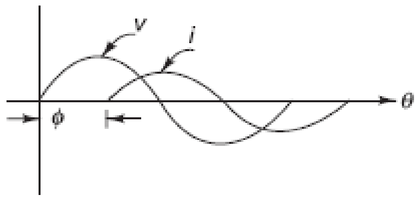
Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$

Fig. 4.42 Impedance triangles

Current Equation If the applied voltage is given by $v = V_m \sin \omega t$ then current equation will be

Waveforms The voltage and current waveforms are shown in Fig. 4.43.

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$



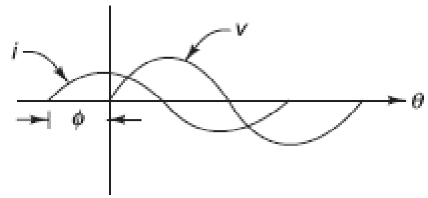


Fig. 4.43 *Waveforms*

Power

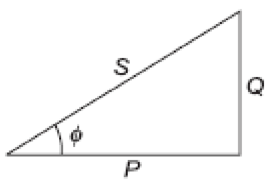
Average power $P = VI \cos \phi = I^2 R$

Reactive power $Q = VI \sin \phi = I^2 X$

Apparent power $S = VI = I^2 Z$

Power Triangles Power triangles are shown in Fig. 4.44.

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$



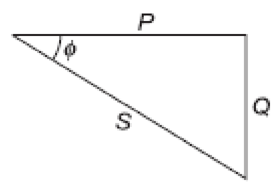


Fig. 4.44 *Power triangles*

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$\text{pf} = \cos \phi$$

$$\text{pf} = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

Example 1

A resistor of 20 Ω, inductor of 0.05 H and a capacitor of 50 μF are connected in series. A supply voltage 230 V, 50 Hz is connected across the series combination. Calculate the following: (i) impedance, (ii) current drawn by the circuit, (iii) phase difference and power factor, and (iv) active and reactive power consumed by the circuit.

$$\text{pf} = \frac{\cdots R}{V} = \frac{\cdots}{Z} = \frac{\cdots}{S}$$

Example 1

A resistor of 20 Ω, inductor of 0.05 H and a capacitor of 50 μF are connected in series. A supply voltage 230 V, 50 Hz is connected across the series combination. Calculate the following: (i) impedance, (ii) current drawn by the circuit, (iii) phase difference and power factor, and (iv) active and reactive power consumed by the circuit.

Solution

$$R = 20\ \Omega$$
$$L = 0.05\ \text{H}$$
$$C = 50\ \mu\text{F}$$
$$V = 230\ \text{V}$$
$$f = 50\ \text{Hz}$$

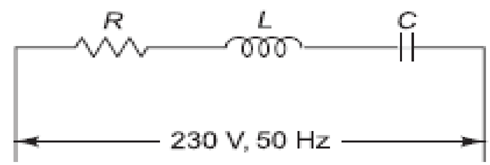


Fig. 4.45

(i) Impedance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\bar{Z} = R + jX_L - jX_C$$
$$= 20 + j15.71 - j63.66$$
$$= 51.95 \angle -67.36°\ \Omega$$
$$Z = 51.95\ \Omega$$

(ii) Phase difference

$$\phi = 67.36°$$

(iii) Current

$$I = \frac{V}{Z} = \frac{230}{51.95} = 4.43\ \text{A}$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(67.36°) = 0.385\ \text{(leading)}$$

(v) Active power

$$P = VI\cos\phi = 230 \times 4.43 \times 0.385 = 392.28\ \text{W}$$

(vi) Reactive power

$$Q = VI\sin\phi = 230 \times 4.43 \times \sin(67.36°) = 940.39\ \text{VAR}$$

$$\text{pf} = \frac{V_R}{V} = \frac{\quad}{Z} = \frac{\quad}{S}$$

Example 1

A resistor of 20 Ω, inductor of 0.05 H and a capacitor of 50 μF are connected in series. A supply voltage 230 V, 50 Hz is connected across the series combination. Calculate the following: (i) impedance, (ii) current drawn by the circuit, (iii) phase difference and power factor, and (iv) active and reactive power consumed by the circuit.

Solution

$R = 20\ \Omega$

$L = 0.05\ \text{H}$

$C = 50\ \mu\text{F}$

$V = 230\ \text{V}$

$f = 50\ \text{Hz}$

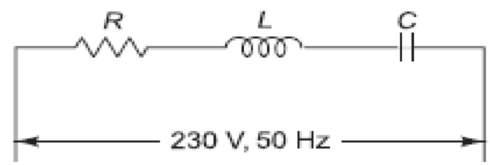


Fig. 4.45

(i) Impedance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\begin{aligned}\bar{Z} &= R + jX_L - jX_C \\ &= 20 + j15.71 - j63.66 \\ &= 51.95\ \angle -67.36^\circ\ \Omega\end{aligned}$$

$$Z = 51.95\ \Omega$$

(ii) Phase difference

$$\phi = 67.36^\circ$$

(iii) Current

$$I = \frac{V}{Z} = \frac{230}{51.95} = 4.43\ \text{A}$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(67.36^\circ) = 0.385\ \text{(leading)}$$

(v) Active power

$$P = VI\cos\phi = 230 \times 4.43 \times 0.385 = 392.28\ \text{W}$$

(vi) Reactive power

$$Q = VI\sin\phi = 230 \times 4.43 \times \sin(67.36^\circ) = 940.39\ \text{VAR}$$

Example 2

A circuit consists of a pure inductor, a pure resistor and a capacitor connected in series. When the circuit is supplied with 100 V, 50 Hz supply, the voltages across inductor and resistor are 240 V and 90 V respectively. If the circuit takes a 10 A leading current, calculate (i) value of inductance, resistance and capacitance, (ii) power factor of the circuit, and (iii) voltage across the capacitor.

Solution

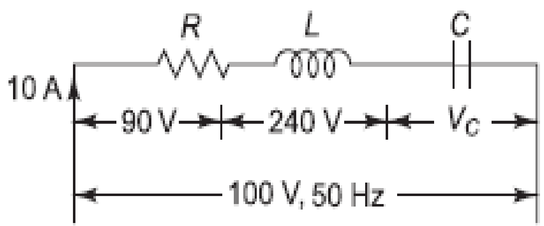


Fig. 4.46

$$V = 100 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$V_L = 240 \text{ V}$$
$$V_R = 90 \text{ V}$$
$$I = 10 \text{ A}$$

(i) Value of inductance, resistance and capacitance

$$R = \frac{V_R}{I} = \frac{90}{10} = 9\ \Omega$$

$$X_L = \frac{V_L}{I} = \frac{240}{10} = 24\ \Omega$$

$$Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$$

$$\bar{Z} = R + j\,X_L - j\,X_C = R - j(X_C - X_L)$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$10 = \sqrt{(9)^2 + (X_C - 24)^2}$$

$$X_C = 28.36\ \Omega$$

$$X_L = 2\pi f L$$

$$24 = 2\pi \times 50 \times L$$

$$L = 0.076\ \text{H}$$

$$X_C = \frac{1}{2\pi f C}$$

$$28.36 = \frac{1}{2\pi \times 50 \times C}$$

$$\text{C} = 112.24\ \mu\text{F}$$

(ii) Power factor of the circuit

$$\text{pf} = \frac{R}{Z} = \frac{9}{10} = 0.9\ \text{(leading)}$$

(iii) Voltage across the capacitor

$$V_C = X_C I = 28.36 \times 10 = 283.6\ \text{V}$$

Example 6

A coil of 3 Ω resistance and an inductance of 0.22 H is connected in series with an imperfect capacitor. When such a series circuit is connected across a 200 V, 50 Hz supply, it has been observed that their combined impedance is (3.8 + j6.4) Ω. Calculate the resistance and capacitance of the imperfect capacitor.

Solution

$$r = 3\ \Omega$$
$$L = 0.22\ \text{H}$$
$$V = 200\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$Z = 3.8 + j\,6.4\ \Omega$$

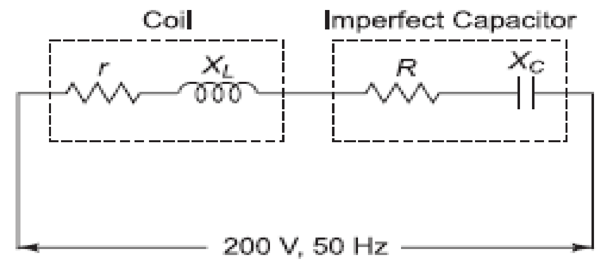


Fig. 4.48

(i) Resistance of the imperfect capacitor

$$\bar{Z} = 3.8 + j6.4\ \Omega$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 0.22 = 69.12\ \Omega$$
$$\bar{Z} = 3 + j69.12 + R - jX_C = (3 + R) + j\,(69.12 - X_C)$$
$$3 + R = 3.8$$
$$R = 0.8\ \Omega$$

(ii) Capacitance of the imperfect capacitor

$$69.12 - X_C = 6.4$$
$$X_C = 62.72\ \Omega$$
$$X_C = \frac{1}{2\pi fC}$$
$$62.72 = \frac{1}{2\pi \times 50 \times C}$$
$$C = 50.75\ \mu\text{F}$$

Example 7

An R-L-C series circuit has a current which lags the applied voltage by 45°. The voltage across the inductance has a maximum value equal to twice the maximum value of voltage across the capacitor. Voltage across the inductance is 300 sin (1000t) and R = 20 Ω. Find the value of inductance and capacitance.

Solution

$$\phi = 45°$$
$$v_L = 300 \sin(1000t)$$
$$R = 20\ \Omega$$

(i) Value of inductance

$$V_{L(\max)} = 2V_{C(\max)}$$
$$V_L = 2V_C$$
$$I X_L = 2I X_C$$
$$X_L = 2X_C$$
$$\cos\phi = \frac{R}{Z}$$
$$\cos(45°) = \frac{20}{Z}$$
$$Z = 28.28\ \Omega$$

For a series *R-L-C* circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$28.28 = \sqrt{(20)^2 + (2X_C - X_C)^2}$$
$$= \sqrt{400 + X_C^2}$$
$$X_C = 20\ \Omega$$
$$X_L = 2X_C = 40\ \Omega$$
$$X_L = \omega L$$
$$40 = 1000 \times L$$
$$L = 0.04\ \text{H}$$

(ii) Value of capacitance

$$X_C = \frac{1}{\omega C}$$
$$20 = \frac{1}{1000 \times C}$$
$$C = 50\ \mu\text{F}$$

Example 8

A coil having a power factor of 0.5 is in series with a 79.57 μF capacitor and when connected across a 50 Hz supply, the p.d. across the coil is equal to the p.d. across the capacitor. Find the resistance and inductance of the coil.

Solution

$$pf_{coil} = 0.5$$
$$C = 79.57\ \mu F$$
$$f = 50\ Hz$$
$$V_{coil} = V_C$$

(i) Resistance of the coil

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 79.57 \times 10^{-6}} = 40\ \Omega$$
$$V_{coil} = V_C$$
$$I Z_{coil} = I X_C$$
$$Z_{coil} = X_C = 40\ \Omega$$
$$pf_{coil} = \cos\phi = \frac{R}{Z_{coil}}$$
$$0.5 = \frac{R}{40}$$
$$R = 20\ \Omega$$

(ii) Inductance of the coil

$$X_L = \sqrt{Z_{coil}^2 - R^2} = \sqrt{(40)^2 - (20)^2} = 34.64\ \Omega$$
$$X_L = 2\pi f L$$
$$34.64 = 2\pi \times 50 \times L$$
$$L = 0.11\ H$$

4.7 PARALLEL AC CIRCUITS

In parallel circuits, resistor, inductor and capacitor or any combination of these elements are connected across same supply. Hence the voltage is same across each branch of the parallel ac circuit. The total current supplied to the circuit is equal to the phasor sum of the branch currents.

For the parallel ac circuit shown in Fig. 4.57,

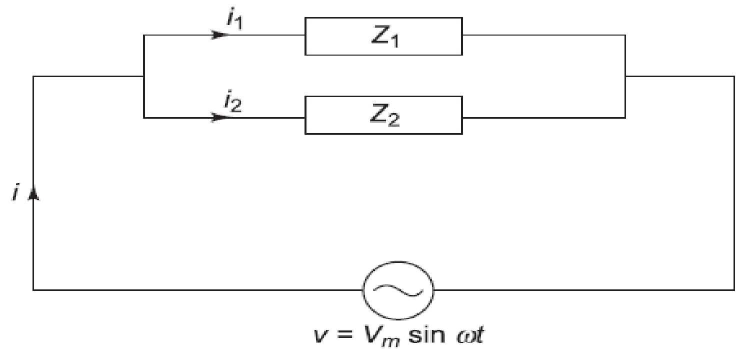


Fig. 4.57 *Parallel ac circuit*

Example 1

A resistance of 10 Ω and a pure coil of inductance 31.8 mH are connected in parallel across 200 V, 50 Hz supply. Find the total current and power factor. **[May 2015]**

Solution

$$V = 200\text{ V}$$

$$R = 10\ \Omega$$

$$L = 31.8\text{ mH}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} \approx 10\ \Omega$$

$$I_R = \frac{V}{R} = \frac{200}{10} = 20\text{ A}$$

$$\bar{I}_L = \frac{V}{jX_L} = \frac{200}{j10} = \frac{200}{10\angle 90^\circ} = 20\angle -90^\circ\text{ A}$$

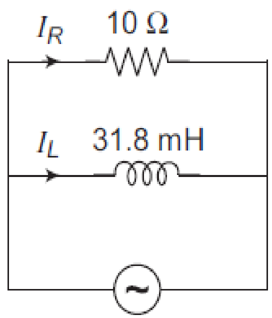


Fig. 4.60

(i) Total current

$$\bar{I} = \bar{I}_R + \bar{I}_L = 20\angle 0^\circ + 20\angle -90^\circ\text{ A} = 28.28\angle -45^\circ\text{A}$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(45^\circ) = 0.707\text{ (lagging)}$$

Example 2

A coil having a resistance of 50 Ω and an inductance of 0.02 H is connected in parallel with a capacitor of 25 μF across a single-phase 200 V, 50 Hz supply. Calculate the current in coil and capacitance. Calculate also the total current drawn, total pf and total power consumed by the circuit.

Solution

$$R = 50\ \Omega$$
$$L = 0.02\ \text{H}$$
$$C = 25\ \mu\text{F}$$
$$V = 200\ \text{V}$$
$$f = 50\ \text{Hz}$$

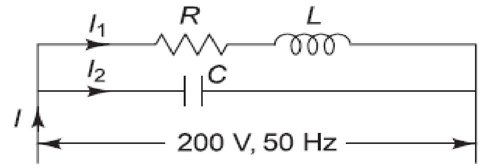


Fig. 4.61

(i) Current in coil and capacitance

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = 127.32\ \Omega$$

$$\bar{Z}_1 = R + jX_L = 50 + j\,6.28 = 50.39\ \angle 7.16°\ \Omega$$

$$\bar{Z}_2 = -jX_C = -j127.32 = 127.32\ \angle -90°\ \Omega$$

$$\bar{V} = 200\ \angle 0° = 200\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200}{50.39\angle 7.16^\circ} = 3.97\angle{-7.16^\circ}\text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200}{127.32\angle -90^\circ} = 1.57\angle 90^\circ\text{ A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 3.97\angle{-7.16^\circ} + 1.57\angle 90^\circ = 4.08\angle 15.27^\circ\text{ A}$$

(iii) Total pf

$$\text{pf} = \cos\phi = \cos(15.27^\circ) = 0.965\ (\text{lagging})$$

(iv) Total power consumed

$$P = VI\cos\phi = 200 \times 4.08 \times 0.965 = 787.44\text{ W}$$

Example 4

Calculate the branch current I_1 and I_2 for the circuit shown in Fig. 4.61.

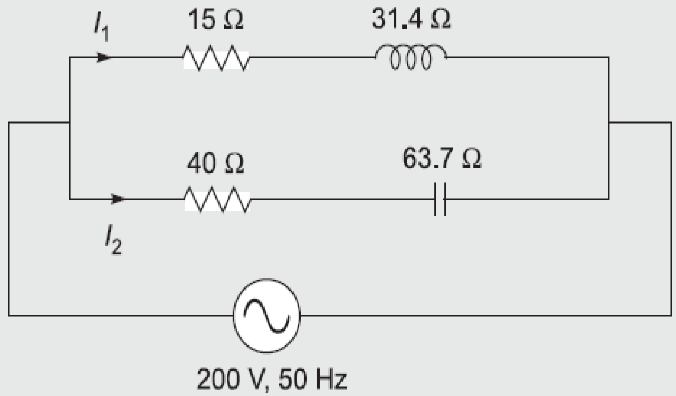


Fig. 4.63 [Dec 2012]

Solution

$$\bar{Z}_1 = 15 + j31.4 = 34.8 \angle 64.47° \ \Omega$$

$$\bar{Z}_2 = 40 - j63.7 = 75.22 \angle -57.87° \ \Omega$$

$$\bar{V} = 200 \text{ V}$$

(i) Branch current I_1

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200}{34.8 \angle 64.47°} = 5.75 \angle -64.47° \text{ A}$$

(i) Branch current I_2

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200}{75.22 \angle -57.87°} = 2.66 \angle 57.87° \text{ A}$$

4.8 SERIES RESONANCE

[Dec 2013, May 2016]

A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it. At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.

Consider the series R-L-C circuit as shown in Fig. 4.93. The impedance of the circuit is

$$\bar{Z} = R + jX_L - jX_C$$

$$= R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

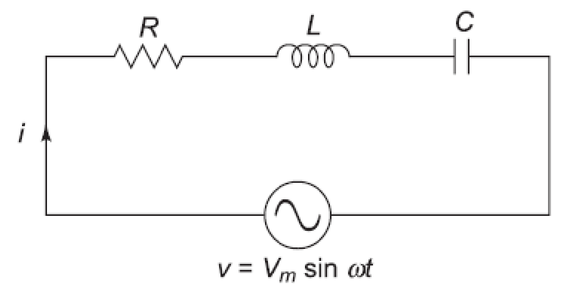


Fig. 4.93 *Series circuit*

At resonance, Z must be resistive. Therefore, the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Power Factor

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

At resonance $Z = R$

$$\text{Power factor} = \frac{R}{R} = 1$$

Current Since impedance is minimum, the current is maximum at resonance. Thus, the circuit accepts more current and as such, an R-L-C circuit under resonance is called an *acceptor circuit*.

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

Voltage At resonance,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 L\, I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L_0} = V_{C_0}$$

Phasor Diagram The phasor diagram is shown in Fig. 4.94.

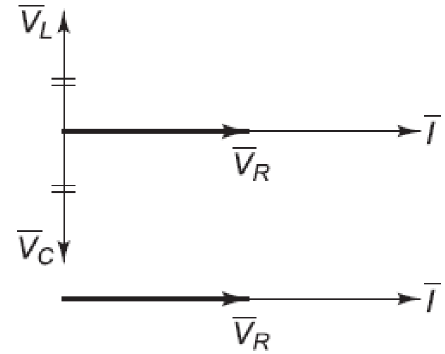


Fig. 4.94 *Phasor diagram*

Behaviour of R, L and C with Change in Frequency Resistance remains constant with the change in frequencies. Inductive reactance X_L is directly proportional to frequency f. It can be drawn as a straight line passing through the origin. Capacitive reactance X_C is inversely proportional to the frequency f. It can be drawn as a rectangular hyperbola in the fourth quadrant.

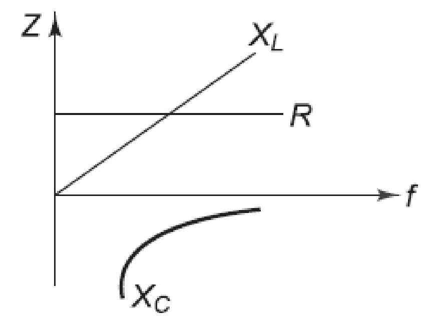


Fig. 4.95 *Behaviour of R, L and C with change in frequency*

Total impedance $\overline{Z} = R + j\,(X_L - X_C)$

(a) When $f < f_0$, impedance is capacitive and decreases up to f_0. The power factor is leading in nature.

Bandwidth For the series *R-L-C* circuit, bandwidth is defined as the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$ where P_0 is the power delivered to R at resonance. From the shape of the resonance curve, it is clear that there are two frequencies for which the power delivered to R is half the power at resonance. For this reason, these frequencies are referred as those corresponding to the half-power points. The magnitude of the current at each half-power point is the same.

Hence,
$$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$$

where the subscript 1 denotes the lower half point and the subscript 2, the higher half point. It follows then that

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

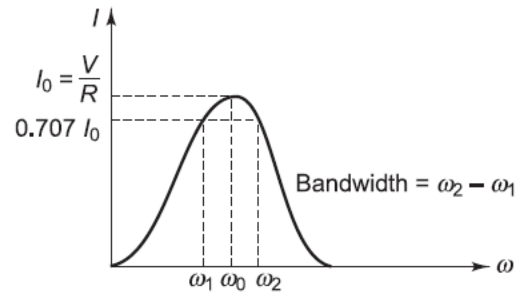


Fig. 4.97 *Resonance curve*

Accordingly, the bandwidth may be identified on the resonance curve as the range of frequencies over which the magnitude of the current is equal to or greater than 0.707 of the current at resonance. In Fig. 4.97, the bandwidth is $\omega_2 - \omega_1$.

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}$$

or $$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}$$

Quality Factor It is a measure of voltage magnification in the series resonant circuit. It is also a measure of selectivity or sharpness of the series resonant circuit.

$$Q_0 = \frac{\text{Voltage across inductor or capacitor}}{\text{Voltage at resonance}}$$

$$= \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

Substituting values of V_{L_0} and V,

$$Q_0 = \frac{I_0 X_{L_0}}{I_0 R}$$

$$= \frac{X_{L_0}}{R}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Substituting values of ω_0,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R}$$

Substituting values of ω_0,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Example 1

A series R-L-C circuit has the following parameter values: R = 10 Ω, L = 0.01 H, C = 100 μF. Compute the resonant frequency, bandwidth, and lower and upper frequencies of the bandwidth.

Solution

$$R = 10\ \Omega$$
$$L = 0.01\ \text{H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times 100\times 10^{-6}}} = 159.15\ \text{Hz}$$

(ii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi\times 0.01} = 159.15\ \text{Hz}$$

(iii) Lower frequency of bandwidth

$$f_1 = f_0 - \frac{BW}{2} = 159.15 - \frac{159.15}{2} = 79.58\ \text{Hz}$$

(iv) Upper frequency of bandwidth

$$f_2 = f_0 + \frac{BW}{2} = 159.15 + \frac{159.15}{2} = 238.73\ \text{Hz}$$

Example 2

For a series RLC circuit having R = 10 Ω, L = 0.01 H and C = 100 μF, Find the resonant frequency, quality factor and bandwith. **[Dec 2014]**

Solution

$$R = 10\ \Omega$$
$$L = 0.01\ \text{H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times100\times10^{-6}}} = 159.15\ \text{Hz}$$

(ii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.01}{100\times10^{-6}}} = 1$$

(iii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi\times0.01} = 159.15\ \text{Hz}$$

Example 3

A series RLC circuit has the following parameter values: R = 10 Ω, L = 0.014 H, C = 100 μF. Compute the resonant frequency, quality factor, bandwidth, lower cut-off frequency and upper cut-off frequency. **[May 2015]**

Solution

$$R = 10\ \Omega$$
$$L = 0.014\ \text{H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.014 \times 100 \times 10^{-6}}} = 134.51\,\text{kHz}$$

(ii) Qualify factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.014}{100 \times 10^{-6}}} = 1.18$$

(iii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.014} = 113.68\ \text{Hz}$$

(iv) Lower cut-off frequency (f_1)

$$f_1 = f_0 - \frac{BW}{2} = 134.51 - \frac{113.68}{2} = 77.67\ \text{Hz}$$

(v) Upper cut-off frequency (f_2)

$$f_2 = f_0 + \frac{BW}{2} = 134.51 + \frac{113.68}{2} = 191.35\ \text{Hz}$$