# ID3 Algorithm

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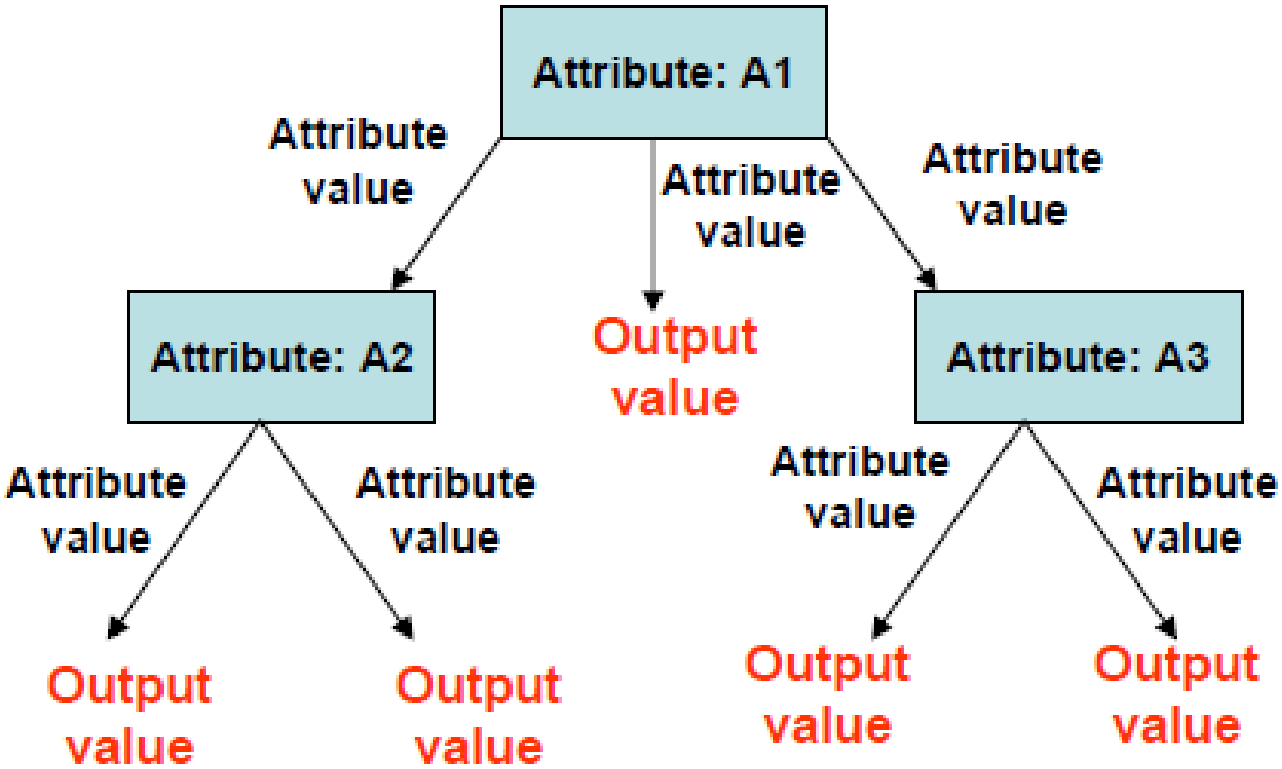
# Agenda

- Decision Trees
- What is ID3?
- Entropy
- Calculating Entropy with Code
- Information Gain
- Advantages and Disadvantages
- Example

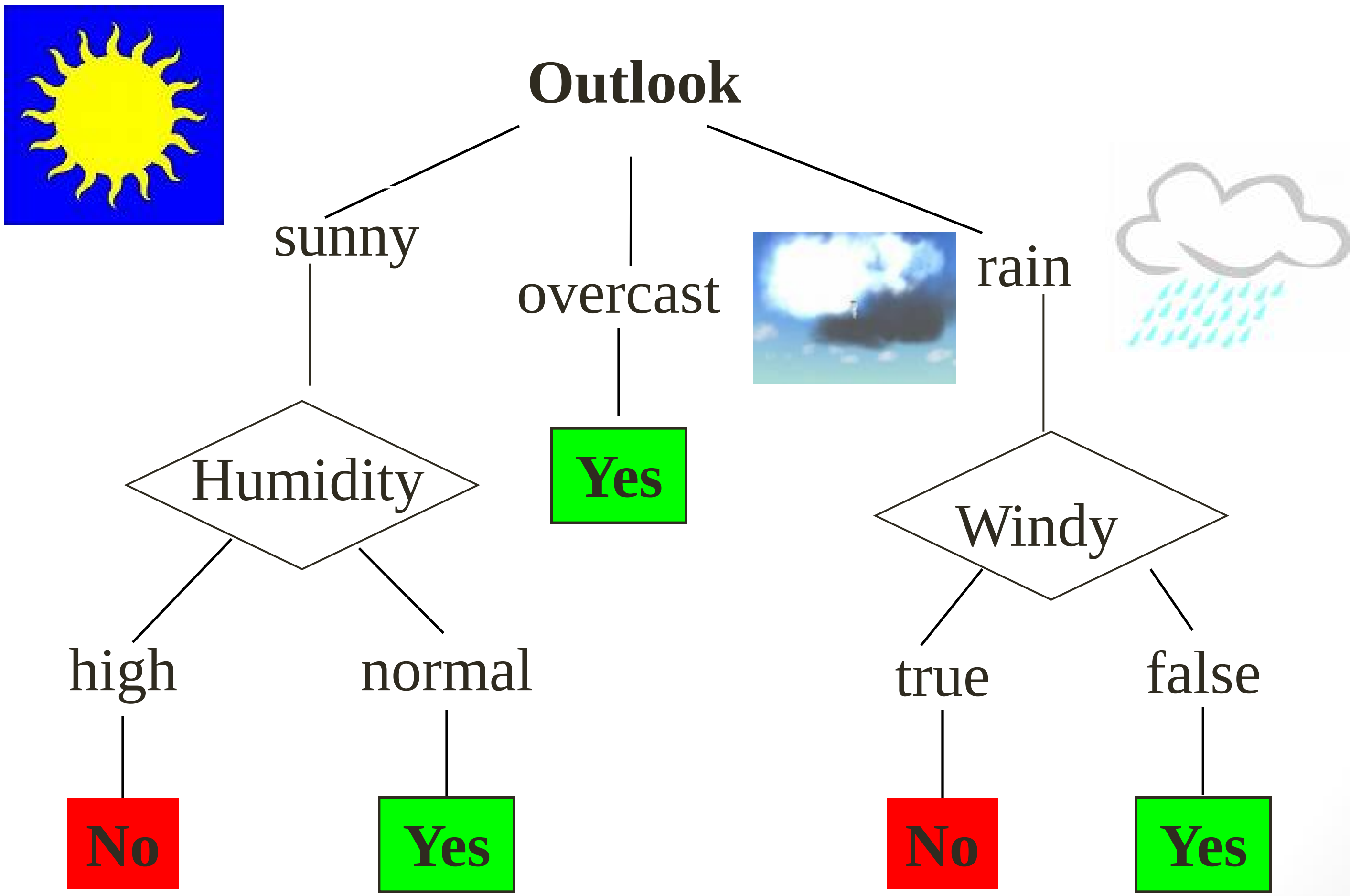
# Decision Trees

- DT learning is a method for approximating discrete value target functions, in which learned function is represented by a decision tree
- Rules (if-then-else) for classifying data using attributes.
- The tree consists of decision nodes and leaf nodes
- A decision node has two or more branches, each representing values for the attribute tested
- A leaf node attribute produces a homogeneous result (all in one class), which does not require additional classification testing.
- Most widely used approach for inductive inference

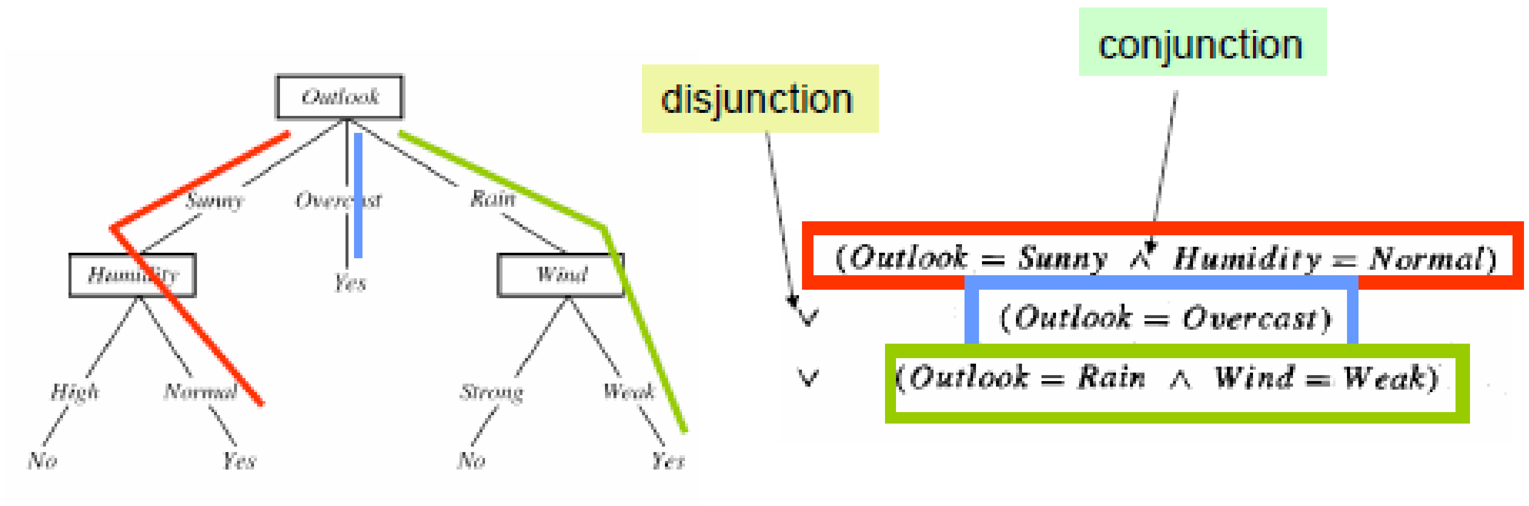
# Decision Tree Example



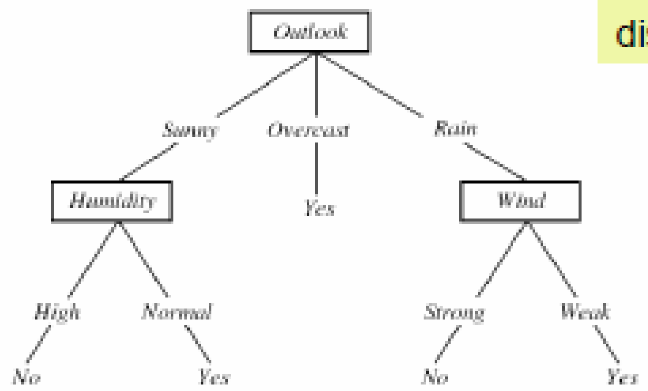
# Decision Tree Example



# Decision Tree Representation



# Decision Tree as If-then-else rule




conjunction

disjunction

$$\begin{aligned} & (Outlook = Sunny \wedge Humidity = Normal) \\ \vee\ & (Outlook = Overcast) \\ \vee\ & (Outlook = Rain \wedge Wind = Weak) \end{aligned}$$

- If (Outlook = Sunny AND humidity = Normal) then PlayTennis = Yes
- If (Outlook = Overcast) then PlayTennis = Yes
- If (Outlook = Rain AND Wind = Weak) then PlayTennis = Yes

# What is ID3?

- A mathematical algorithm for building the decision tree.
- Invented by J. Ross Quinlan in 1979.
- Uses Information Theory invented by Shannon in 1948.
- Builds the tree from the top down, with no backtracking.
- Information Gain is used to select the most useful attribute for classification.

# Entropy

$$Entropy(S) = \sum_{i=1}^{c} p_i \log_2 p_i$$

- A completely homogeneous sample has entropy of 0.
- An equally divided sample has entropy of 1.
- A formula to calculate the homogeneity of a sample.

$$\text{Entropy}(s) = -\ (p+) \log_2 (p+) - (p-) \log_2 (p-)$$

# Entropy Example

Entropy(S) =

- (9/14) $Log_2$ (9/14) - (5/14) $Log_2$ (5/14)

= 0.940

# Information Gain (IG)

- The information gain is based on the decrease in entropy after a dataset is split on an attribute.
- Which attribute creates the most homogeneous branches?
- First the entropy of the total dataset is calculated.
- The dataset is then split on the different attributes.
- The entropy for each branch is calculated. Then it is added proportionally, to get total entropy for the split.
- The resulting entropy is subtracted from the entropy before the split.
- The result is the Information Gain, or decrease in entropy.
- The attribute that yields the largest IG is chosen for the decision node.

# Information Gain (IG)

- The information gain , Gain(S,A) of an attribute A, relative to a collection of examples S, is defined as

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

where ***Values(A)*** is the set of all possible values for attribute **A**, and ***Sv,*** is the subset of **S** for which attribute **A** has value **v**, **i.e.**,

$$S_v = \{s \in S | A(s) = v\}$$

# Information Gain (IG)

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

**Entropy of S** **−** **Entropy of S after partition**

*Gain(S, A)* is the expected reduction in entropy caused by knowing the value of attribute A.

*Gain(S, A)* is the information provided about the *target &action value*, given the value of some other attribute *A*. The value of *Gain(S, A)* is the number of bits saved when encoding the target value of an arbitrary member of *S*, by knowing the value of attribute *A*.

# Information Gain (cont'd)

- A branch set with entropy of 0 is a leaf node.
- Otherwise, the branch needs further splitting to classify its dataset.
- The ID3 algorithm is run recursively on the non-leaf branches, until all data is classified.

# Advantages of using ID3

- Understandable prediction rules are created from the training data.
- Builds the fastest tree.
- Builds a short tree.
- Only need to test enough attributes until all data is classified.
- Finding leaf nodes enables test data to be pruned, reducing number of tests.
- Whole dataset is searched to create tree.

# Disadvantages of using ID3

- Data may be over-fitted or over-classified, if a small sample is tested.
- Only one attribute at a time is tested for making a decision.
- Classifying continuous data may be computationally expensive, as many trees must be generated to see where to break the continuum.

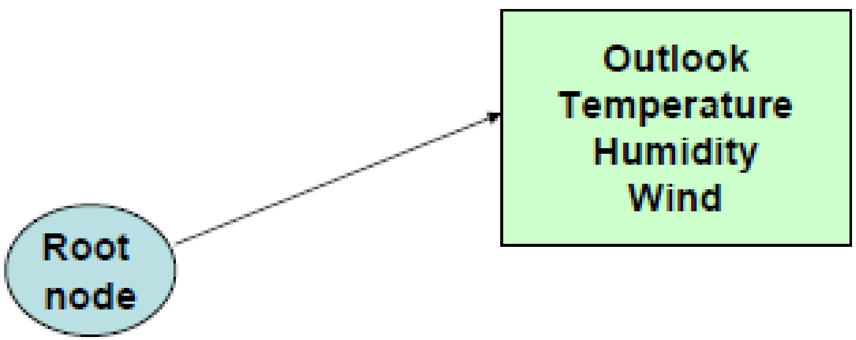
# Example: PlayTennis

| Day | *Outlook* | *Temperature* | *Humidity* | *Wind* | *PlayTennis* |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

There are 14 examples. 9 positive and 5 negative examples [9+, 5-].

The entropy of S relative to this boolean (yes/no) classification is

$$Entropy([9+, 5-]) = -(9/14)\log_2(9/14) - (5/14)\log_2(5/14)$$
$$= 0.940$$


Root node
Outlook
Temperature
Humidity
Wind


Which attribute to select ?????

| Day | Outlook | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

**Attribute:** Outlook

$Values\ (Outlook) = Sunny, Overcast, Rain$

$S = [9+, 5-]$ $\quad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{Sunny} \leftarrow [2+, 3-]$ $\quad Entropy(S_{Sunny}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = 0.971$

$S_{Overcast} \leftarrow [4+, 0-]$ $\quad Entropy(S_{Overcast}) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0$

$S_{Rain} \leftarrow [3+, 2-]$ $\quad Entropy(S_{Rain}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.971$

$$Gain\ (S, Outlook) = Entropy(S) - \sum_{v \in \{Sunny, Overcast, Rain\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Outlook)$$

$$= Entropy(S) - \frac{5}{14}Entropy(S_{Sunny}) - \frac{4}{14}Entropy(S_{Overcast})$$

$$-\frac{5}{14}Entropy(S_{Rain})$$

$$Gain(S, Outlook) = 0.94 - \frac{5}{14}0.971 - \frac{4}{14}0 - \frac{5}{14}0.971 = 0.2464$$

| Day | Outlook | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

## Attribute: Temp

$Values\ (Temp) = Hot, Mild, Cool$

$S = [9+, 5-]$ $\quad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{Hot} \leftarrow [2+, 2-]$ $\quad Entropy(S_{Hot}) = -\frac{2}{4}\log_2\frac{2}{4} - \frac{2}{4}\log_2\frac{2}{4} = 1.0$

$S_{Mild} \leftarrow [4+, 2-]$ $\quad Entropy(S_{Mild}) = -\frac{4}{6}\log_2\frac{4}{6} - \frac{2}{6}\log_2\frac{2}{6} = 0.9183$

$S_{Cool} \leftarrow [3+, 1-]$ $\quad Entropy(S_{Cool}) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.8113$

$$Gain\ (S, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Temp) = Entropy(S) - \frac{4}{14} Entropy(S_{Hot}) - \frac{6}{14} Entropy(S_{Mild}) - \frac{4}{14} Entropy(S_{Cool})$$

$$Gain(S, Temp) = 0.94 - \frac{4}{14} 1.0 - \frac{6}{14}\ 0.9183 - \frac{4}{14} 0.8113 = 0.0289$$

| Day | Outlook | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

## Attribute: Humidity

$Values\ (Humidity) = High, Normal$

$S = [9+, 5-]$ $\quad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{High} \leftarrow [3+, 4-]$ $\quad Entropy(S_{High}) = -\frac{3}{7}\log_2\frac{3}{7} - \frac{4}{7}\log_2\frac{4}{7} = 0.9852$

$S_{Normal} \leftarrow [6+, 1-]$ $\quad Entropy(S_{Normal}) = -\frac{6}{7}\log_2\frac{6}{7} - \frac{1}{7}\log_2\frac{1}{7} = 0.5916$

$$Gain\ (S, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Humidity)$$

$$= Entropy(S) - \frac{7}{14} Entropy(S_{High}) - \frac{7}{14} Entropy(S_{Normal})$$

$$Gain(S, Humidity) = 0.94 - \frac{7}{14} 0.9852 - \frac{7}{14} 0.5916 = 0.1516$$

| Day | Outlook | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

## Attribute: Wind

$Values\ (Wind) = Strong, Weak$

$S = [9+, 5-]$ $\qquad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{Strong} \leftarrow [3+, 3-]$ $\qquad Entropy(S_{Strong}) = 1.0$

$S_{Weak} \leftarrow [6+, 2-]$ $\qquad Entropy(S_{Weak}) = -\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8} = 0.8113$

$$Gain\ (S, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Wind) = Entropy(S) - \frac{6}{14} Entropy(S_{Strong}) - \frac{8}{14} Entropy(S_{Weak})$$

$$Gain(S, Wind) = 0.94 - \frac{6}{14}\ 1.0 - \frac{8}{14}\ 0.8113 = 0.0478$$

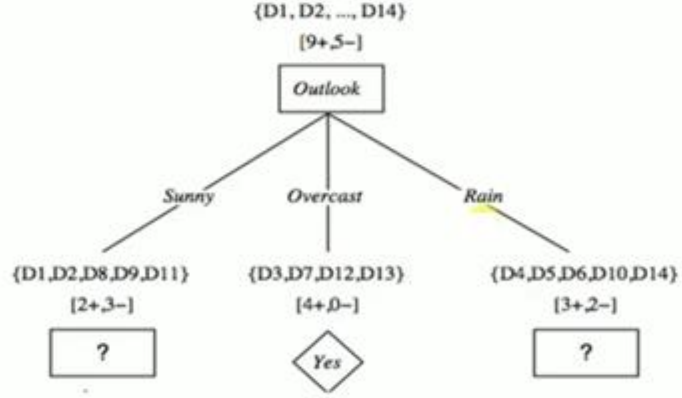
| Day | Outlook | Temp | Humidity | Wind | Play Tennis |
| --- | --- | --- | --- | --- | --- |
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

$$Gain(S, Outlook) = 0.2464$$

$$Gain(S, Temp) = 0.0289$$

$$Gain(S, Humidity) = 0.1516$$

$$Gain(S, Wind) = 0.0478$$



| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| D1 | Hot | High | Weak | No |
| D2 | Hot | High | Strong | No |
| D8 | Mild | High | Weak | No |
| D9 | Cool | Normal | Weak | Yes |
| D11 | Mild | Normal | Strong | Yes |

## Attribute: Temp

$Values\ (Temp) = Hot, Mild, Cool$

$S_{Sunny} = [2+, 3-]$ $\qquad Entropy(S_{Sunny}) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{Hot} \leftarrow [0+, 2-]$ $\qquad Entropy(S_{Hot}) = 0.0$

$S_{Mild} \leftarrow [1+, 1-]$ $\qquad Entropy(S_{Mild}) = 1.0$

$S_{Cool} \leftarrow [1+, 0-]$ $\qquad Entropy(S_{Cool}) = 0.0$

$$Gain\ (S_{Sunny}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Temp)$$

$$= Entropy(S) - \frac{2}{5}Entropy(S_{Hot}) - \frac{2}{5}Entropy(S_{Mild})$$

$$-\frac{1}{5}Entropy(S_{Cool})$$

$$Gain(S_{sunny}, Temp) = 0.97 - \frac{2}{5}0.0 - \frac{2}{5}1 - \frac{1}{5}0.0 = 0.570$$

| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| Dl | Hot | High | Weak | No |
| D2 | Hot | High | Strong | No |
| D8 | Mild | High | Weak | No |
| D9 | Cool | Normal | Weak | Yes |
| Dl1 | Mild | Normal | Strong | Yes |

## Attribute: Humidity

$Values\ (Humidity) = High, Normal$

$S_{Sunny} = [2+, 3-]$ $\quad Entropy(S) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{high} \leftarrow [0+, 3-]$ $\quad Entropy(S_{High}) = 0.0$

$S_{Normal} \leftarrow [2+, 0-]$ $\quad Entropy(S_{Normal}) = 0.0$

$$Gain\ (S_{Sunny}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Humidity) = Entropy(S) - \frac{3}{5}Entropy(S_{High}) - \frac{2}{5}Entropy(S_{Normal})$$

$$Gain(S_{sunny}, Humidity) = 0.97 - \frac{3}{5}\,0.0 - \frac{2}{5}0.0 = 0.97$$

| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| Dl | Hot | High | Weak | No |
| D2 | Hot | High | Strong | No |
| D8 | Mild | High | Weak | No |
| D9 | Cool | Normal | Weak | Yes |
| Dl1 | Mild | Normal | Strong | Yes |

## Attribute: Wind

$Values\ (Wind) = Strong, Weak$

$S_{Sunny} = [2+, 3-]$ $\quad Entropy(S) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{Strong} \leftarrow [1+, 1-]$ $\quad Entropy(S_{Strong}) = 1.0$

$S_{Weak} \leftarrow [1+, 2-]$ $\quad Entropy(S_{Weak}) = -\frac{1}{3}log_2\frac{1}{3} - \frac{2}{3}log_2\frac{2}{3} = 0.9183$

$$Gain\ (S_{Sunny}, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Wind) = Entropy(S) - \frac{2}{5}Entropy(S_{Strong}) - \frac{3}{5}Entropy(S_{Weak})$$

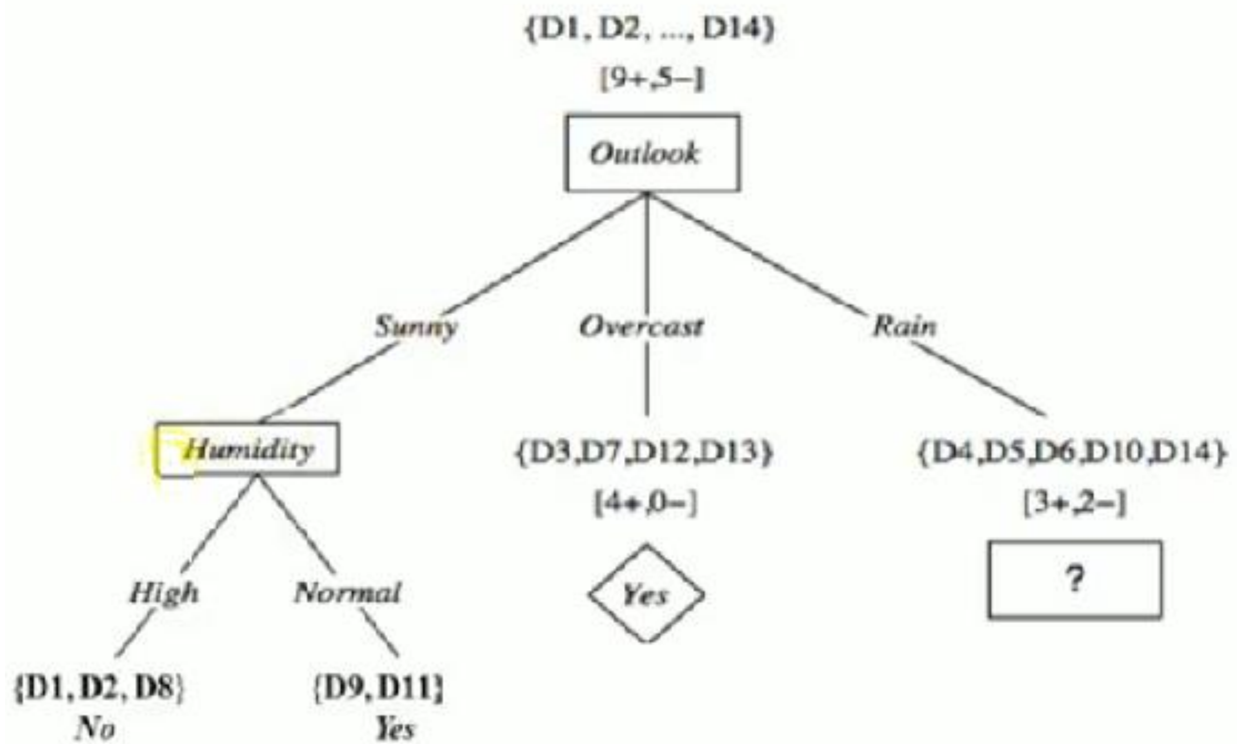
$$Gain(S_{sunny}, Wind) = 0.97 - \frac{2}{5}1.0 - \frac{3}{5}\,0.918 = 0.0192$$

| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| D1 | Hot | High | Weak | No |
| D2 | Hot | High | Strong | No |
| D8 | Mild | High | Weak | No |
| D9 | Cool | Normal | Weak | Yes |
| D11 | Mild | Normal | Strong | Yes |

$$Gain(S_{sunny}, Temp) = 0.570$$

$$Gain(S_{sunny}, Humidity) = 0.97$$

$$Gain(S_{sunny}, Wind) = 0.0192$$



| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| D4 | Mild | High | Weak | Yes |
| D5 | Cool | Normal | Weak | Yes |
| D6 | Cool | Normal | Strong | No |
| D10 | Mild | Normal | Weak | Yes |
| D14 | Mild | High | Strong | No |

## Attribute: Temp

$Values\ (Temp) = Hot, Mild, Cool$

$S_{Rain} = [3+, 2-]$ $\quad$ $Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$

$S_{Hot} \leftarrow [0+, 0-]$ $\quad$ $Entropy(S_{Hot}) = 0.0$

$S_{Mild} \leftarrow [2+, 1-]$ $\quad$ $Entropy(S_{Mild}) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$

$S_{Cool} \leftarrow [1+, 1-]$ $\quad$ $Entropy(S_{Cool}) = 1.0$

$$Gain\ (S_{Rain}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Temp)$$

$$= Entropy(S) - \frac{0}{5}Entropy(S_{Hot}) - \frac{3}{5}Entropy(S_{Mild})$$

$$-\frac{2}{5}Entropy(S_{Cool})$$

$$Gain(S_{Rain}, Temp) = 0.97 - \frac{0}{5}0.0 - \frac{3}{5}0.918 - \frac{2}{5}1.0 = 0.0192$$

| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| D4 | Mild | High | Weak | Yes |
| D5 | Cool | Normal | Weak | Yes |
| D6 | Cool | Normal | Strong | No |
| DI0 | Mild | Normal | Weak | Yes |
| DI4 | Mild | High | Strong | No |

## Attribute: Humidity

$Values\ (Humidity) = High, Normal$

$S_{Rain} = [3+, 2-]$

$Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$

$S_{High} \leftarrow [1+, 1-]$

$Entropy(S_{High}) = 1.0$

$S_{Normal} \leftarrow [2+, 1-]$

$Entropy(S_{Normal}) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$

$$Gain\ (S_{Rain}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Humidity) = Entropy(S) - \frac{2}{5}Entropy(S_{High}) - \frac{3}{5}Entropy(S_{Normal})$$

$$Gain(S_{Rain}, Humidity) = 0.97 - \frac{2}{5}\ 1.0 - \frac{3}{5}0.918 = 0.0192$$

| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| D4 | Mild | High | Weak | Yes |
| D5 | Cool | Normal | Weak | Yes |
| D6 | Cool | Normal | Strong | No |
| Dl0 | Mild | Normal | Weak | Yes |
| Dl4 | Mild | High | Strong | No |

## Attribute: Wind

$Values\ (wind) = Strong, Weak$

$S_{Rain} = [3+, 2-]$

$Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$

$S_{Strong} \leftarrow [0+, 2-]$

$Entropy(S_{Strong}) = 0.0$

$S_{Weak} \leftarrow [3+, 0-]$

$Entropy(S_{weak}) = 0.0$

$$Gain\ (S_{Rain}, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Wind) = Entropy(S) - \frac{2}{5}Entropy(S_{Strong}) - \frac{3}{5}Entropy(S_{Weak})$$

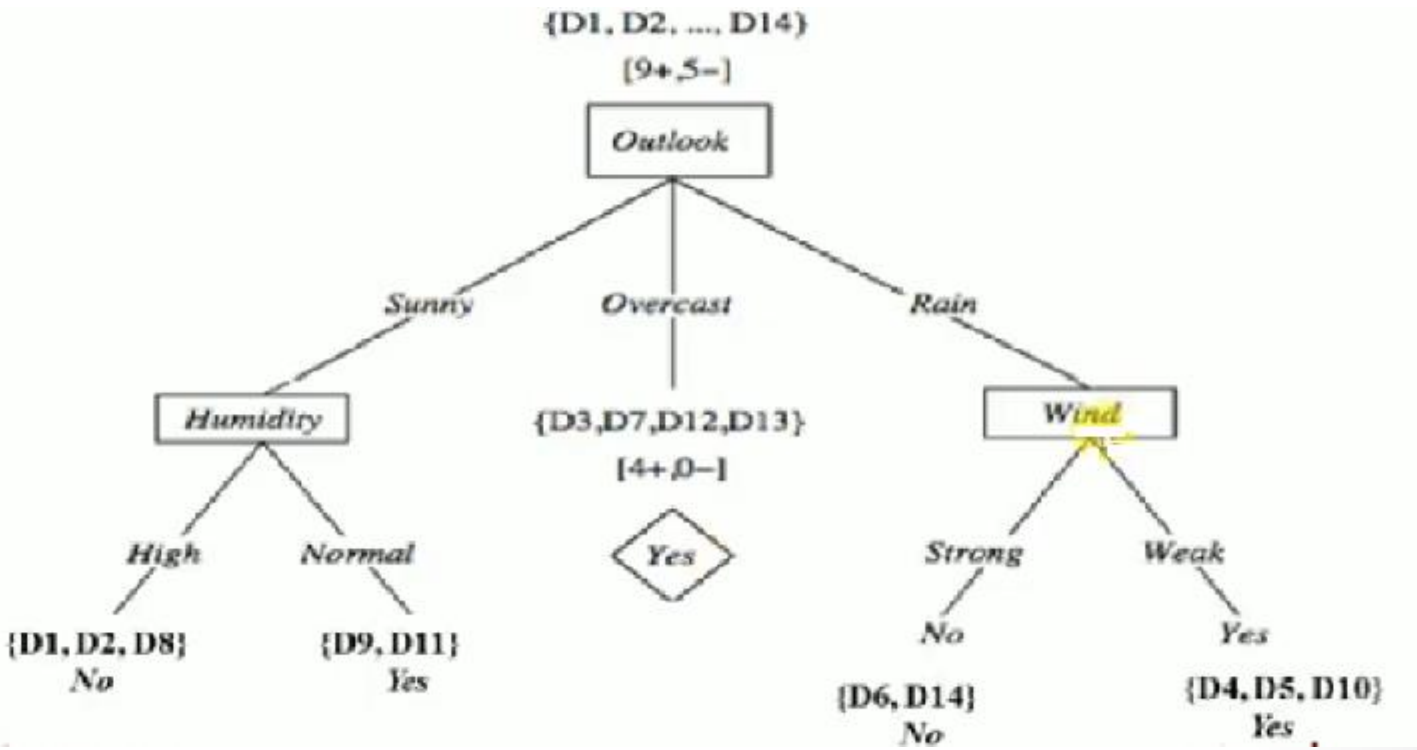
$$Gain(S_{Rain}, Wind) = 0.97 - \frac{2}{5}\ 0.0 - \frac{3}{5}0.0 = 0.97$$

| Day | Temp | Humidity | Wind | Play Tennis |
|---|---|---|---|---|
| D4 | Mild | High | Weak | Yes |
| D5 | Cool | Normal | Weak | Yes |
| D6 | Cool | Normal | Strong | No |
| DI0 | Mild | Normal | Weak | Yes |
| DI4 | Mild | High | Strong | No |

$$Gain(S_{Rain}, Temp) = 0.0192$$

$$Gain(S_{Rain}, Humidity) = 0.0192$$

$$Gain(S_{Rain}, Wind) = 0.97$$

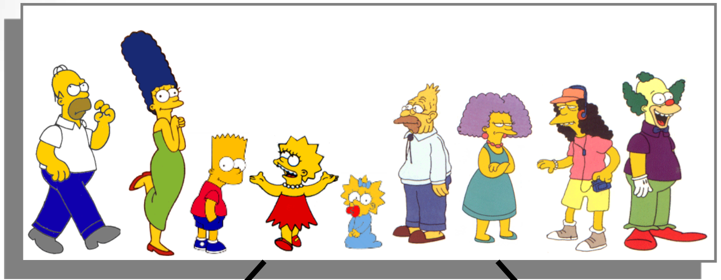


# Example: The Simpsons

| Person | Hair Length | Weight | Age | Class |
|---|---|---|---|---|
| Homer | 0” | 250 | 36 | M |
| Marge | 10” | 150 | 34 | F |
| Bart | 2” | 90 | 10 | M |
| Lisa | 6” | 78 | 8 | F |
| Maggie | 4” | 20 | 1 | F |
| Abe | 1” | 170 | 70 | M |
| Selma | 8” | 160 | 41 | F |
| Otto | 10” | 180 | 38 | M |
| Krusty | 6” | 200 | 45 | M |

| | | | | |
|---|---|---|---|---|
| Comic | 8” | 290 | 38 | ? |

$$Entropy(S) = -\frac{p}{p+n}\log_2\left(\frac{p}{p+n}\right) - \frac{n}{p+n}\log_2\left(\frac{n}{p+n}\right)$$

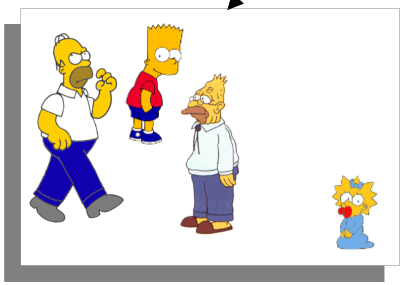


$Entropy(4\mathbf{F},5\mathbf{M})$ = -(4/9)$\log_2$(4/9) - (5/9)$\log_2$(5/9)
= **0.9911**

yes

no

Hair Length <= 5?



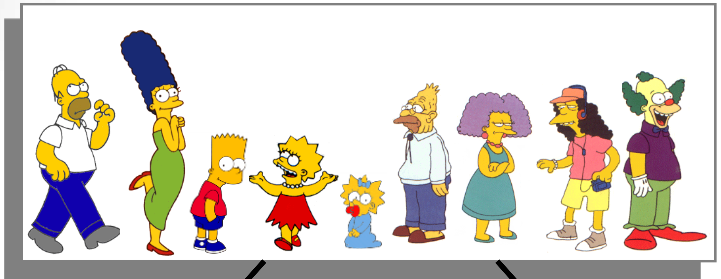
Let us try splitting on *Hair length*

$Entropy(1\mathbf{F},3\mathbf{M})$ = -(1/4)$\log_2$(1/4) - (3/4)$\log_2$(3/4)
= **0.8113**

$Entropy(3\mathbf{F},2\mathbf{M})$ = -(3/5)$\log_2$(3/5) - (2/5)$\log_2$(2/5)
= **0.9710**

$$Gain(A) = E(Current\ set) - \sum E(all\ child\ sets)$$

*Gain*(Hair Length <= 5) = **0.9911** – (4/9 * **0.8113** + 5/9 * **0.9710** ) = **0.0911**

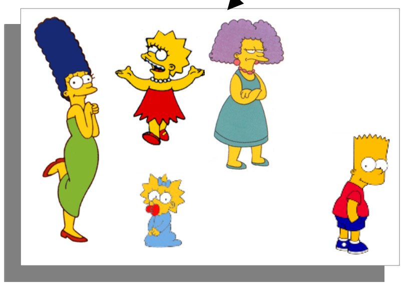


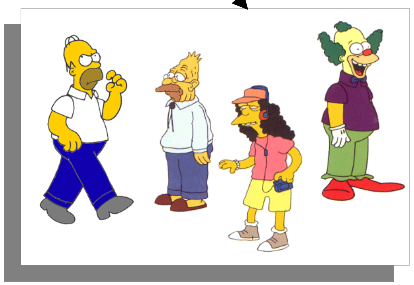
$$Entropy(S) = -\frac{p}{p+n}\log_2\left(\frac{p}{p+n}\right) - \frac{n}{p+n}\log_2\left(\frac{n}{p+n}\right)$$

*Entropy*(4**F**,5**M**) = -(4/9)$\log_2$(4/9) - (5/9)$\log_2$(5/9)
= **0.9911**

yes        no

Weight <= 160?





Let us try splitting on *Weight*

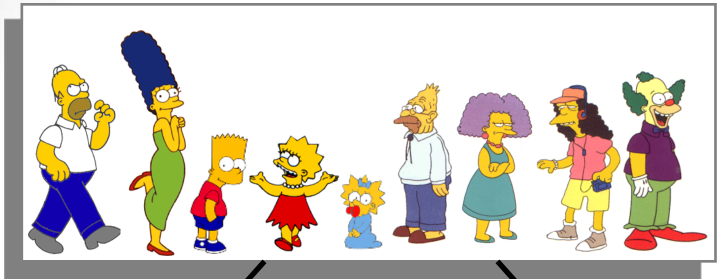
*Entropy*(4**F**,1**M**) = -(4/5)$\log_2$(4/5) - (1/5)$\log_2$(1/5)
= **0.7219**

*Entropy*(0**F**,4**M**) = -(0/4)$\log_2$(0/4) - (4/4)$\log_2$(4/4)
= ***0***

$$Gain(A) = E(Current\ set) - \sum E(all\ child\ sets)$$

*Gain*(Weight <= 160) = **0.9911** – (5/9 * **0.7219** + 4/9 * **0** ) = **0.5900**

$$Entropy(S) = -\frac{p}{p+n}\log_2\left(\frac{p}{p+n}\right) - \frac{n}{p+n}\log_2\left(\frac{n}{p+n}\right)$$



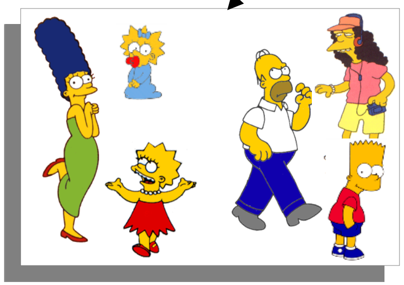
$Entropy(4\mathbf{F},5\mathbf{M}) = -(4/9)\log_2(4/9) - (5/9)\log_2(5/9)$

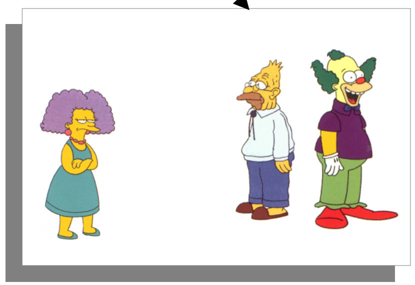
$= \mathbf{0.9911}$

yes

no

age <= 40?





Let us try splitting on *Age*

$Entropy(3\mathbf{F},3\mathbf{M}) = -(3/6)\log_2(3/6) - (3/6)\log_2(3/6)$

$= \mathbf{1}$

$Entropy(1\mathbf{F},2\mathbf{M}) = -(1/3)\log_2(1/3) - (2/3)\log_2(2/3)$

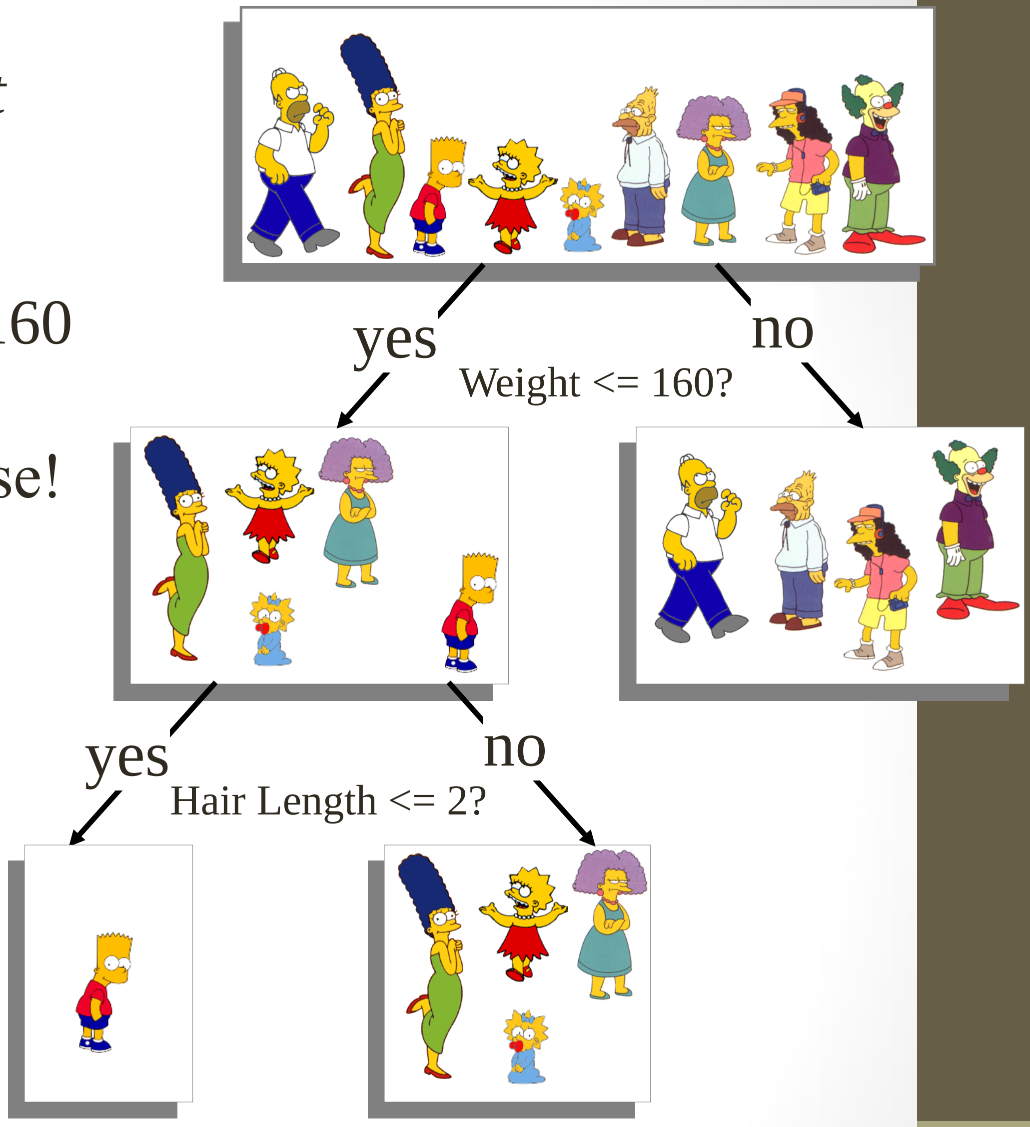
$= \mathbf{0.9183}$

$$Gain(A) = E(Current\ set) - \sum E(all\ child\ sets)$$

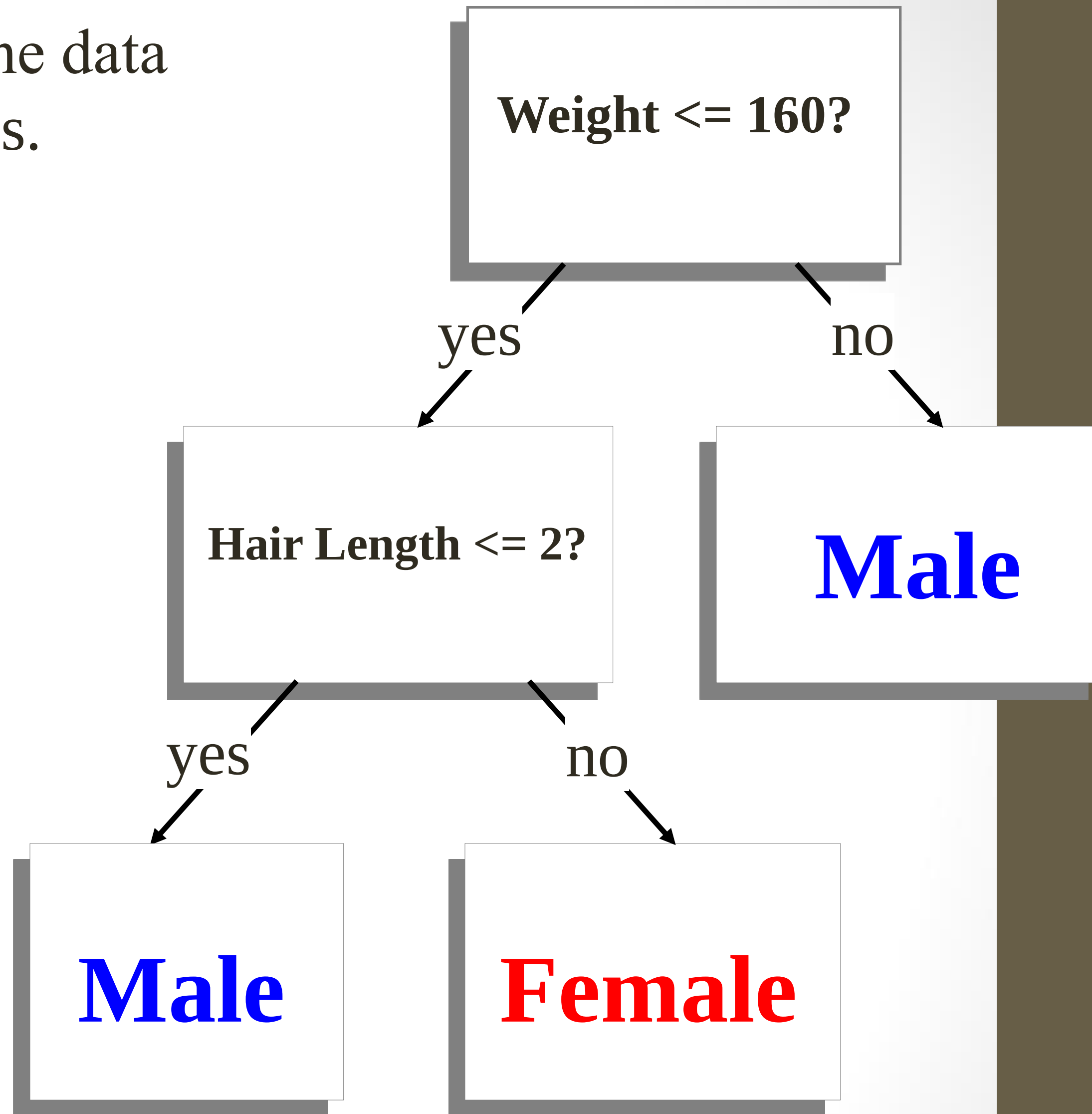
$Gain(\text{Age} <= 40) = \mathbf{0.9911} – (6/9 * \mathbf{1} + 3/9 * \mathbf{0.9183}) = \mathbf{0.0183}$

Of the 3 features we had, *Weight* was best. But while people who weigh over 160 are perfectly classified (as males), the under 160 people are not perfectly classified… So we simply recurse!

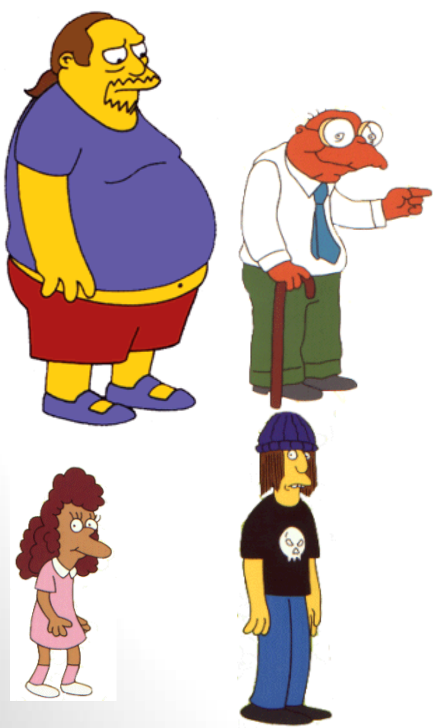
This time we find that we can split on *Hair length*, and we are done!



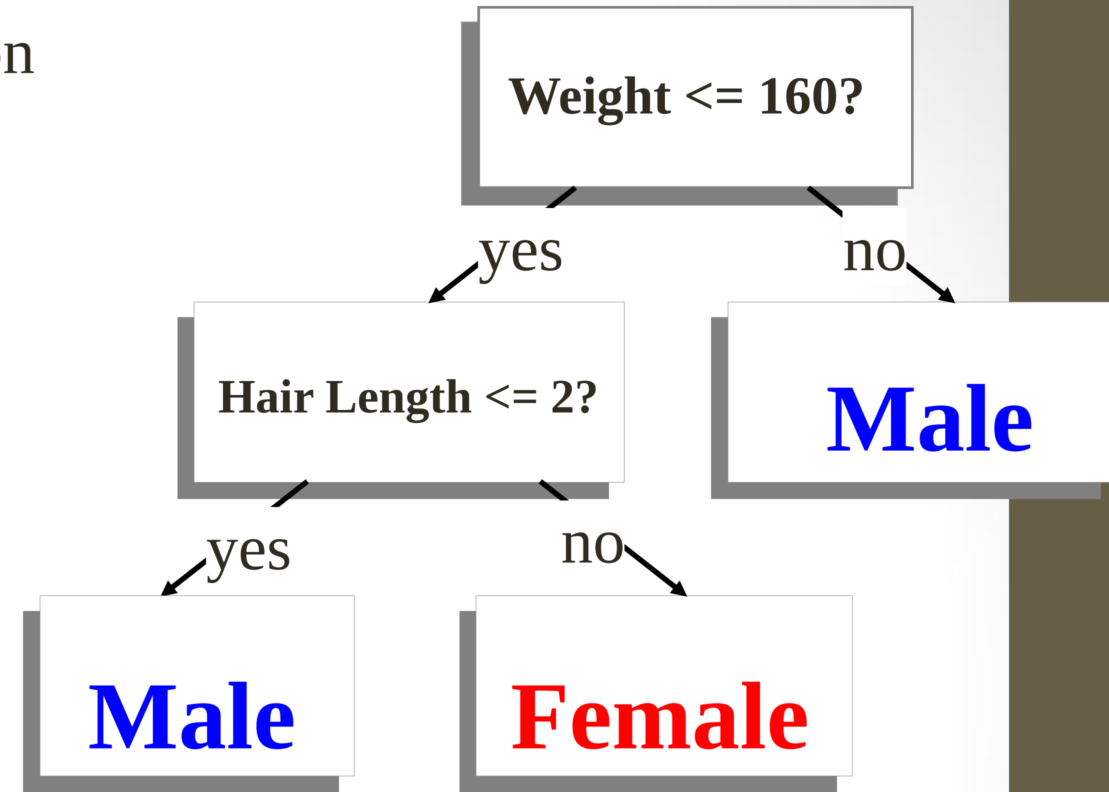
We need don’t need to keep the data around, just the test conditions.




How would these people be classified?



It is trivial to convert Decision Trees to rules…




**Rules to Classify Males/Females**

**If** *Weight* **greater than** 160, classify as **Male**
**Elseif** *Hair Length* **less than or equal** to 2, classify as **Male**
**Else** classify as **Female**