Engineering Mechanics

Module 1

Syllabus:

1.1 System of Coplanar forces: resultant of concurrent forces, parallel forces, non-concurrent non parallel system of forces, moment of force about a point, couples, Varignon's theorem, Principle of transmissibility of forces

1.2 Resultant of forces in Space--- (Refer class ppt and worksheet 2)

CHARACTERISTICS OF A FORCE

In order to determine the effects of a force, acting on a body, we must know the following

1. Magnitude of the force (i.e., 100 N, 50 N, 20 kN, 5 kN, etc.)

2. The direction of the line, along which the force acts (i.e., along OX, OY, at 30° North or East etc.). It is also known as line of action of the force.

3. Nature of the force (i.e., whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.

4. The point at which (or through which) the force acts on the body.

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

It states, "If a force acts at any point on a †rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body."

SYSTEM OF FORCES

When two or more forces act on a body, they are called to form a system of forces. Following systems of forces are important from the subject point of view

1. Coplanar forces. The forces, whose lines of action lie on the same plane, are known as coplanar forces.

2. Collinear forces. The forces, whose lines of action lie on the same line, are known as collinear forces.

3. Concurrent forces. The forces, which meet at one point, are known as concurrent forces.The concurrent forces may or may not be collinear.

4. Coplanar concurrent forces. The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.

5. Coplanar non-concurrent forces. The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.

6. Non-coplanar concurrent forces. The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

7. Non-coplanar non-concurrent forces. The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

RESULTANT FORCE

If a number of forces, P, Q, R ... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them i.e., which would produce the same effect as produced by all the given forces. This single force is called resultant force and the given forces R ...etc. are called component forces.

COMPOSITION OF FORCES

The process of finding out the resultant force, of a number of given forces, is called composition of forces or compounding of forces.

METHODS FOR THE RESULTANT FORCE

Though there are many methods for finding out the resultant force

1. Analytical method. 2. Method of resolution.

ANALYTICAL METHOD FOR RESULTANT FORCE

The resultant force, of a given system of forces, may be found out analytically by the following

1. Parallelogram law of forces. 2. Method of resolution.

PARALLELOGRAM LAW OF FORCES

It states, “If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.”

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

and $$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

Note

1. If $\theta = 0$ *i.e.*, when the forces act along the same line, then

$$R = F_1 + F_2 \qquad \text{...(Since } \cos 0° = 1)$$

2. If $\theta = 90°$ *i.e.*, when the forces act at right angle, then

$$\theta = R = \sqrt{F_1^2 + F_2^2} \qquad \text{...(Since } \cos 90° = 0)$$

3. If $\theta = 180°$ *i.e.*, when the forces act along the same straight line but in opposite directions, then $R = F_1 - F_2$ ...(Since $\cos 180° = -1$)

   In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal *i.e.*, when $F_1 = F_2 = F$ then

$$R = \sqrt{F^2 + F^2 + 2F^2 \cos\theta} = \sqrt{2F^2(1 + \cos\theta)}$$

$$= \sqrt{2F^2 \times 2\cos^2\left(\frac{\theta}{2}\right)} \qquad \text{...}\left[\because\ 1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$

$$= \sqrt{4F^2 \cos^2\left(\frac{\theta}{2}\right)} = 2F\cos\left(\frac{\theta}{2}\right)$$

RESOLUTION OF A FORCE

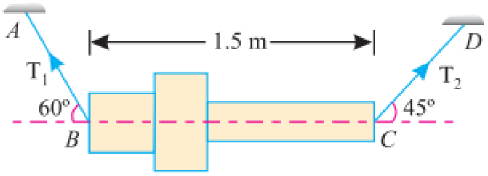
The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors.

PRINCIPLE OF RESOLUTION

It states, “The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction.”

In general, the forces are resolved in the vertical and horizontal directions.

Q. A machine component 1.5 m long and weight 1000 N is supported by two ropes AB and CD as shown in figure. Calculate the tensions T1 and T2 in the ropes AB and CD.



Given: Weight of the component = 1000 N Resolving the forces horizontally (i.e., along BC) and equating the same,

Resolving the forces horizontally (*i.e.*, along *BC*) and equating the same,

$$T_1 \cos 60° = T_2 \cos 45°$$

$$\therefore \quad T_1 = \frac{\cos 45°}{\cos 60°} \times T_2 = \frac{0.707}{0.5} \times T_2 = 1.414\, T_2 \qquad \text{...}(i)$$

and now resolving the forces vertically,

$$T_1 \sin 60° + T_2 \sin 45° = 1000$$

$$(1.414\, T_2)\, 0.866 + T_2 \times 0.707 = 1000$$

$$1.93\, T_2 = 1000$$

$$\therefore \quad T_2 = \frac{1000}{1.93} = 518.1 \text{ N} \quad \textbf{Ans.}$$

and $$T_1 = 1.414 \times 518.1 = 732.6 \text{ N} \quad \textbf{Ans.}$$

METHOD OF RESOLUTION FOR THE RESULTANT FORCE

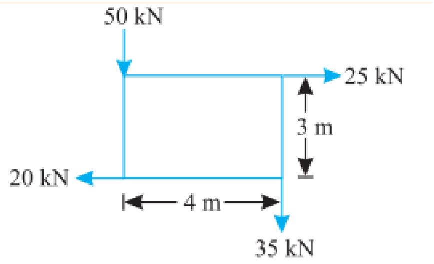
1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., ∑H).
2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (i.e., ∑V).
3. The resultant R of the given forces will be given by the equation:

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

Q. A system of forces are acting at the corners of a rectangular block as shown in figure. Determine the magnitude and direction of the resultant force.



Magnitude of the resultant force

Resolving forces horizontally,

$$\Sigma H = 25 - 20 = 5 \text{ kN}$$

and now resolving the forces vertically

$$\Sigma V = (-50) + (-35) = -85 \text{ kN}$$

$\therefore$ Magnitude of the resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(5)^2 + (-85)^2} = 85.15 \text{ kN} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

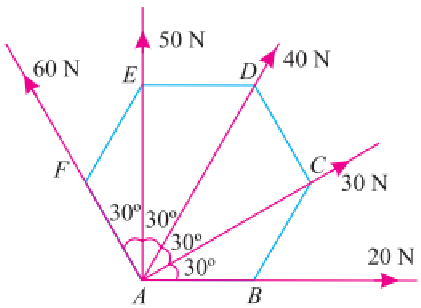
We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-85}{5} = -17 \quad \text{or} \quad \theta = 86.6°$$

Since ΣH is positive and ΣV is negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force

$$= 360° - 86.6° = 273.4° \quad \textbf{Ans.}$$

Q. The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.



Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$
$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60 (-0.5) \text{ N}$$
$$= 36.0 \text{ N} \quad ...(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to *AB*),

$$\Sigma V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$
$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$
$$= 151.6 \text{ N} \quad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

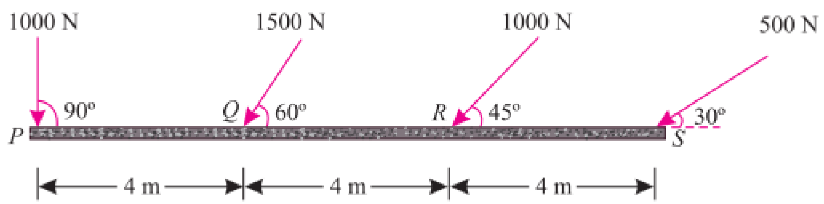
Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, *AB*).

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Q. A horizontal line PQRS is 12 m long, where PQ = QR = RS = 4 m. Forces of 1000 N, 1500 N, 1000 N and 500 N act at P, Q, R and S respectively with downward direction. The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant force.



Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30° \text{ N}$$
$$= (1000 \times 0) + (1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866) \text{ N}$$
$$= 1890 \text{ N} \quad ...(i)$$

and now resolving all the forces vertically,

$$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30° \text{ N}$$
$$= (1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5) \text{ N}$$
$$= 3256 \text{ N} \quad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with *PS*.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8° \quad \textbf{Ans.}$$

Since both the values of ΣH and ΣV are +ve. therefore resultant lies between 0° and 90°.
Position of the resultant force

Let x = Distance between P and the line of action of the resultant force.

Now taking moments* of the vertical components of the forces and the resultant force about P, and equating the same,

$$3256\,x = (1000 \times 0) + (1500 \times 0.866)\,4 + (1000 \times 0.707)8 + (500 \times 0.5)12$$
$$= 13\,852$$

$$\therefore \quad x = \frac{13\,852}{3256} = 4.25 \text{ m} \quad \textbf{Ans.}$$

LAWS FOR THE RESULTANT FORCE

The resultant force, of a given system of forces, may also be found out by the following laws

1. Triangle law of forces. 2. Polygon law of forces.

TRIANGLE LAW OF FORCES

It states, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."

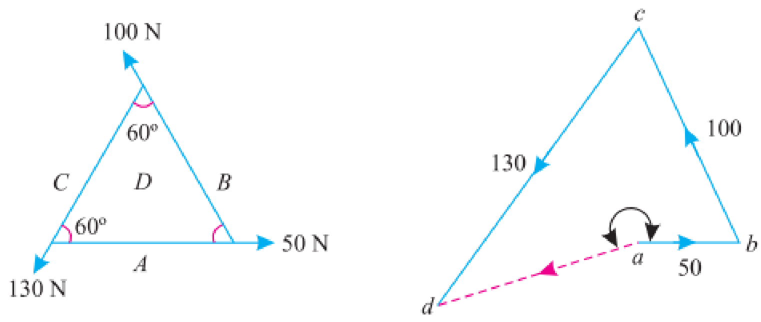
POLYGON LAW OF FORCES

It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."
GRAPHICAL (VECTOR) METHOD FOR THE RESULTANT FORCE

It is another name for finding out the magnitude and direction of the resultant force by the polygon law of forces.

1. Construction of space diagram (position diagram). It means the construction of a diagram showing the various forces (or loads) along with their magnitude and lines of action.
2. Use of Bow's notations. All the forces in the space diagram are named by using the Bow's notations. It is a convenient method in which every force (or load) is named by two capital letters, placed on its either side in the space diagram.
3. Construction of vector diagram (force diagram). It means the construction of a diagram starting from a convenient point and then go on adding all the forces vectorially one by one (keeping in view the directions of the forces) to some suitable scale.
4. Now the closing side of the polygon, taken in opposite order, will give the magnitude of the resultant force (to the scale) and its direction.

Q. A particle is acted upon by three forces equal to 50 N, 100 N and 130 N, along the three sides of an equilateral triangle, taken in order. Find graphically the magnitude and direction of the resultant force.



(*a*) Space diagram (*b*) Vector diagram

Select some suitable point *a* and draw *ab* equal to 50 N to some suitable scale and parallel to the 50 N force of the space diagram.

Through *b*, draw *bc* equal to 100 N to the scale and parallel to the 100 N force of the space diagram.

Similarly through *c*, draw *cd* equal to 130 N to the scale and parallel to the 130 N force of the space diagram.

Join *ad*, which gives the magnitude as well as direction of the resultant force.

By measurement, we find the magnitude of the resultant force is equal to 70 N and acting at an angle of 200° with *ab*. **Ans.**

MOMENT OF A FORCE

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically, moment,

$$M = P \times l$$

where P = Force acting on the body, and

l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

Graphical Representation of Moment

Consider a force P represented, in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out, as shown in Fig. 3.1. From O, draw OC perpendicular to AB. Join OA and OB.

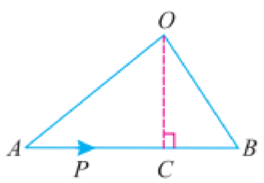


Fig. 3.1. Representation of moment

Now moment of the force P about O

$$= P \times OC = AB \times OC$$

But $AB \times OC$ is equal to twice the area of triangle ABO.

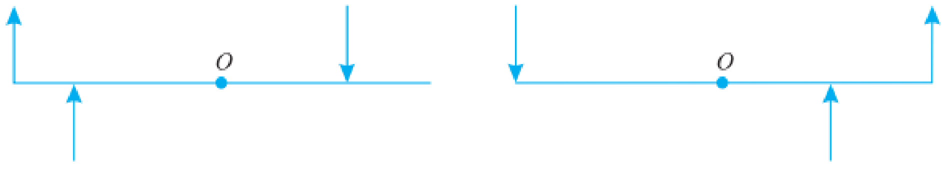
Thus the moment of a force, about any point, is equal to twice the area of the triangle, whose base is the line to some scale representing the force and whose vertex is the point about which the moment is taken.

Units of Moment

N-m, kN-m, N-mm

TYPES OF MOMENTS

1. Clockwise moments. 2. Anticlockwise moments.



(*a*) Clockwise moments (*b*) Anticlockwise moments

VARIGNON'S PRINCIPLE OF MOMENTS (OR LAW OF MOMENTS)

It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."

APPLICATIONS OF MOMENTS

Though the moments have a number of applications, in the field of Engineering science, yet the following are important from the subject point of view:

1. Position of the resultant force 2. Levers.

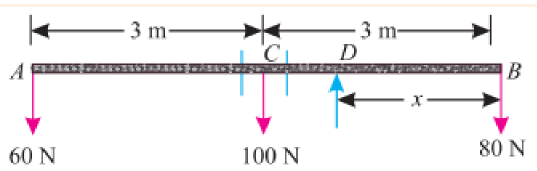
POSITION OF THE RESULTANT FORCE BY MOMENTS

It is also known as analytical method for the resultant force. The position of a resultant force may be found out by moments as discussed below:

1. First of all, find out the magnitude and direction of the resultant force by the method of resolution as discussed earlier in chapter 'Composition and Resolution of Forces'.

2. Now equate the moment of the resultant force with the algebraic sum of moments of the given system of forces about any point. This may also be found out by equating the sum of clockwise moments and that of the anticlockwise moments about the point, through which the resultant force will pass.

Q. A uniform beam AB of weight 100 N and 6 m long had two bodies of weights 60 N and 80 N suspended from its two ends.



Let $\quad x$ = Distance between B and the point where the beam should be supported.

We know that for the beam to rest horizontally, the moments of the weights should be equal.

Now taking moments of the weights about D and equating the same,

$$80x = 60(6 - x) + 100(3 - x)$$

$$= 360 - 60x + 300 - 100x = 660 - 160x$$

$$240x = 660$$

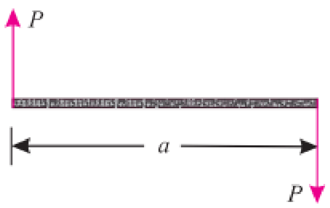
or $\quad x = \dfrac{660}{240} = 2.75 \text{ m}$ **Ans.**

COUPLE

A pair of two equal and unlike parallel forces (i.e. forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.

ARM OF A COUPLE

The perpendicular distance between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple



MOMENT OF A COUPLE

The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically:

Moment of a couple $= P \times a$

where $\quad P$ = Magnitude of the force, and

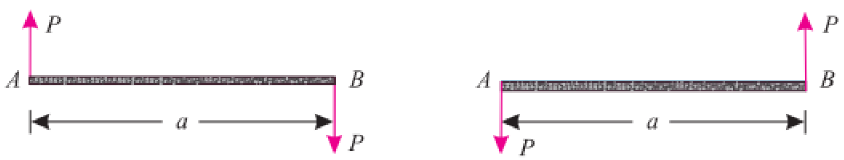
a = Arm of the couple.

Classifications of Couple

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts :

1. Clockwise couple, and 2. Anticlockwise couple.

Clockwise Couple-A couple whose tendency is to rotate a body, on which it acts, in a clockwise direction is known as a clockwise couple.



(*a*) Clockwise couple (*b*) Anticlockwise couple

Anticlockwise Couple-A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple.

Characteristics of Couple

A couple (whether clockwise or anticlockwise) has the following characteristics :

1. The algebraic sum of the forces, constituting the couple, is zero.
2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
4. Any no. of coplaner couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.