DISCRETE STRUCTURES (co3)

Relation

1. If A={1,4,5} and the relation R defined on the set A as aRb if a+b < 6 check whether the relation R is an equivalence relation
2. Define Partial Order relation and check whether R is Partial Order relation. R= {(x,y) $if\ y = x^r, r\ is\ positive\ integer\ and\ a, b \in Z\}$.
3. Show that the relation $R = \{(a, b)\ suchthat\ a - b\ is\ divisible\ by\ 5\ , a, b \in\ Z\}$ is an equivalence relation hence find all equivalence classes
4. Show that the relation $R = \{(a, b)\ suchthat\ a - b\ is\ divisible\ by\ 4\ , a, b \in\ Z\}$ is an equivalence relation hence find all equivalence classes
5. Show that the relation $R = \{(a, b)\ suchthat\ a - b\ is\ divisible\ by\ 7\ , a, b \in\ Z\}$ is an equivalence relation hence find all equivalence classes
6. Show that the relation $R = \{(a, b)\ suchthat\ 2a + 3b\ is\ divisible\ by\ 5\ , a, b \in\ Z\}$ is an equivalence relation
7. Show that the relation $R = \{(a, b)\ suchthat\ 3a + 4b\ is\ divisible\ by\ 7\ , a, b \in\ Z\}$ is an equivalence relation
8. If A={a,b,c,} and find relation R such that (i) R is reflexive , but not symmetric , not transitive (ii) R is reflexive , symmetric , but not transitive.
9. If A={a,b,c,} and find relation R such that (i) R is not reflexive , not symmetric , but transitive (ii) R is reflexive , transitive , but not symmetric
10. Draw the digraph and find matrix of relation for R$\cup$ S and R$\cap$ S if relations R & S are defined on a set $A = \{1,2,3,4,5,6\}$ as
    $R = \{(a, b)\ suchthat\ a\ devides\ b,\ \forall\ a, b \in A\ \}$
    $S = \{(a, b)\ suchthat\ a\ is\ multiple\ of\ b\ , \forall\ a, b \in A\ \}$
11. Draw the digraph for $\bar{R} \cup \bar{S}$ and $R^{-1} \cap S^{-1}$ where R & S are defined on a set A
    If $A = \{1,2,3,4\}$ as $R = \{(a, b)\ suchthat\ a < b,\ \forall\ x, y \in A\ \}$
    $\mathcal{S}$= $\{(a, b)\ suchthat\ a < b + 1, \forall\ a, b \in A\ \}$
12. Show that the relation $R = \{(a, b)\ suchthat\ 3a + 2b\ is\ divisible\ by\ 5\ , a, b \in\ Z\}$ is an equivalence relation
13. Show that the relation $R = \{(a, b)\ suchthat\ 4a + 3b\ is\ divisible\ by\ 7\ , a, b \in\ Z\}$ is an equivalence relation
14. Draw the digraph of R , find matrix of R hence check whether R is reflexive , symmetric , transitive where A = {a,b,c,d} and a relation R is defined on A as R={(a,a)(b,b)(c,c)(a,b)(b,a)(a,c)(c,b)(b,c)(d,d)(c,d)(d,c)}
15. A relation R is defined on set of integers Z as aRb if 8 divides $a - b$ Prove that R is an equivalence relation
16. If A={2,3,4,5,6} and the relation R defined on the set A as aRb if a+b < 7. (i) Draw the digraph of R (ii) find matrix of R (iii) Check whether R is reflexive , symmetric , transitive ?
17. If A={1,4,7} then write all possible partitions and corresponding equivalence relations
18. If A={a,b,c,d} then write all possible partitions and corresponding equivalence relations.
19. if relations If A={a,b,c,d} and find relation R such that (i) R is reflexive , but not symmetric , not transitive (ii) R is reflexive , symmetric , but not transitive
20. Determine whether the relation R on a set A is reflexive, symmetric, antisymmetric or transitive. A = set of all positive integers, a R b iff $|a-b| \leq 2$
21. Determine whether the relation R on a set A={}1,2,3,5} is reflexive, symmetric, antisymmetric or transitive. A = set of all positive integers, a R b iff $|a-b| \leq 4$
22. let A = {1, 2, …., 8}. Let R be the equivalence relation defined by x ≡ ymod(4) Write R as a set of ordered pairs Find the partition of A induced by R .

23. $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3,4), (4, 3), (3, 3), (4, 4)\}$. Shows that R is an equivalence relation on A hence find partition of A induced by R.
24. let A = {1, 2, 3,4}. Let R & S be an equivalence relations on A given as
$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3,4), (4, 3), (3, 3), (4, 4)\}$
$S = \{(1, 1), (2, 2), (3,1), (1, 3), (3, 3), (4, 4)\}$

find partition of A induced by $R^{-1} \cap S^{-1}$, R^{-1} , $R \cap S$

25. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, $C = \{x, y, z\}$ and
let $R = \{(1, a), (2, d), (3, a), (3, b), (3, d)\}$ be a relation from A to B and
$S = \{(b, x), (b, z), (c, y), (d, z)\}$ be a relation from B to C. Write SOR
Find Domain Range of SOR

FUNCTION:

1. Show that $f: R - \{1\} \to R - \{2\}$ such that $f(x) = \frac{2x-3}{x-1}$ is bijective
2. If $f: R - \{3\} \to R - \{0\}$ is defined **as** $(x) = \frac{1}{x-3}$. Show that $\boldsymbol{f(x)}$ is bijective and hence find $f^{-1}(x)$.
3. If f: $R - \{5\} \to R - \{0\}$ is defined as f(x) $= \frac{1}{x-5}$. Show that f(x) is bijective and hence find $f^{-1}(x)$.
4. Check whether the function $f: Z \to Z$ such that $f(x) = x^2 + x + 1$ is bijective.
5. If the functions f & g are defined as $f: R \to R$ such that $f(x) = 2 + 3x$ and $g: R \to R$ such that $g(x) = 4 - 2x$. Find $f * g(x)$ & $g * f(x)$..
6. Functions $f: R \to R$, $g: R \to R$ are defined as $f(x) = 2x + 3$, $g(x) = 3x - 4$. Find g^{-1}. f^{-1}
7. Functions $f: R \to R$, $g: R \to R$ are defined as $f(x) = 2x - 3$, $g(x) = 4 - 3x$ Solve$g^{-1}.f^{-1}(x) = 2$.
8. If f: $R - \{1\} \to R$ is defined as f(x) $= \frac{3x}{x-1}$. Show that f(x) is bijective and hence find $f^{-1}(x)$.
9. Function $f: R - \{1\} \to R - \{3\}$ is defined as $f(x) = \frac{3x-2}{x-1}$. Prove that f is bijective
10. Functions $f: R \to R$, $g: R \to R$ are defined as $f(x) = 5x + 3$, $g(x) = 1 + 3x$ then find fog , fof , gof & $gogof$
11. Functions $f: R \to R$, $g: R \to R$ are defined as $f(x) = 2x - 3$, $g(x) = 3x + 2$ then Show that $f(x), g(x)$ are bijective and hence find $f^{-1}(x), g^{-1}(x)$, gof^{-1} & $g^{-1}of$
12. Functions $f: R \to R$, $g: R \to R$ are defined as $f(x) = x - 4$, $g(x) = 6 + 7x$ then then Show that $f(x), g(x)$ are bijective and hence find $f^{-1}(x), g^{-1}(x), f^{-1}og$
13. If $f, g: R \to R$ are defined as $f(x) = 2x, g(x) = x + 4$. Show that $f(x), g(x)$ are bijective and hence find $f^{-1}(x), g^{-1}(x)$.
14. Functions $f: R \to R$, $g: N \to N$ are defined as $f(x) = x^2$, $g(x) = x^2$ Check whether the functions are injective.
15. Give function $g: N \to N$ which is injective ,but not surjective with justification
16. Give function $g: N \to N$ which is not injective ,but surjective with justification

Groups

17. $If G = \{z \text{ such that } z = e^{i\theta}\}$ Prove that G is an abelian group under usual multiplication of complex numbers.
18. Prove that $G = \{1, -1, i, -i\}$ is a group under usual multiplication of complex numbers.
19. If G is set of all nonzero real numbers and binary operation * is defined as $a * b = \frac{ab}{3}$ $, a, b$. Show that $(G,*)$ is an abelian group.
20. Prove that the set $\{ Z,*\}$ is a group where * is defined as $a * b = a + b + 2$
21. Prove that the set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ where α is real, forms a group under usual matrix multiplication.
22. Prove that the set Z_5 is a group under addition
23. Prove that the set Z_4 is a group under addition
24. Prove that third roots of unity forms a group under usual multiplication
25. $If G = \{x/\ x = 2^n \text{ ,} n \text{ is an integer}\}$ Prove that G is an abelian group under usual multiplication of real numbers Prove that $\{\begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix}$, m is an integer$\}$ forms a group under usual matrix multiplication
26. Find zero divisors and unit elements of Z_6.
27. Find zero divisors and unit elements of Z_8.
28. Find zero divisors and unit elements of Z_7.
29. If G is set of all real numbers then Show that $(G - \{-3\},*)$ is an abelian group where binary operation * is defined as $a * b = a + b + \frac{ab}{3}$.
30. If G is set of all real numbers then Show that $(G - \{-2\},*)$ is an abelian group where binary operation * is defined as $a * b = a + b + \frac{ab}{2}$.
31. Prove that $\{\begin{bmatrix} a & a \\ a & a \end{bmatrix}$, a is non zero real number$\}$ forms group under usual matrix multiplication
32. Prove that the set $\{ Z,*\}$ is a group where * is defined as $a * b = a + b - 5$
33. Prove that $\{\begin{bmatrix} a & a \\ a & a \end{bmatrix}$, a is real number$\}$ forms group under usual matrix addition
34. Prove that set Z_5-$\{\bar{0}\}$ is a abelian group under multiplication
35. Prove that set Z_7-$\{\bar{0}\}$ is a abelian group under multiplication
36. Prove that the set $\{ Z,*\}$ is a abelian group where * is defined as $a * b = a + b - 3$
37. Prove that the set $\{ Z,*\}$ is a abelian group where * is defined as $a * b = a + b - 1$
38. Prove that the set $\{ R - \{-1\},*\}$ is a abelian group where * is defined as $a * b = a + b + ab$
39. Prove that set $\{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}\}$ is a abelian group under addition modulo 10.

59 Prove that set $\{\bar{1}, \bar{3}, \bar{7}, \bar{9}\}$ is a abelian group under multiplication modulo 10

60 Prove that set $\{\bar{2}, \bar{4}, \bar{6}, \bar{8}\}$ is a abelian group under multiplication modulo 10.

Pigeonhole Principle

1. If 5 points are to be chosen in an equilateral triangle of side one unit then show that there are atleast 2 points at a distant less than half unit .
2. If 10 points are to be chosen in an equilateral triangle of side 3 units then show that there are atleast 2 points at a distant less than one unit.
3. If 7 points are to be chosen in a regular hexagon of side one unit then show that there are atleast 2 points at a distant less than one unit .
4. If 5 points are to be chosen in a square of side 2 units then show that there are atleast 2 points at a distant less than $\sqrt{2}$ units .
5. If 7 positive integers with distinct unit places are chosen then show that there are 2 positive integers whose sum is divisible by 10.

6. If 101 integers are chosen from integers 1 to 200 , then show that there are 2 integers such that one divides other.
7. If 11 integers are chosen from integers 1 to 20 , then show that there are 2 integers such that one divides other.
8. If 51 integers are chosen from integers 1 to 100 , then show that there are 2 integers such that one divides other.
9. In a group of 6 persons in which any 2 persons are either friends or enemies, then show that there are 3 persons who are either mutual friends or mutually enemies.
10. If n+1 integers are chosen from first 2n integers, then show that there are 2 integers with greatest common divisor 1