$$\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{dy}{dx}\right)^2 + y = 0$$

$$x^2 + 2x + 1 = 0.$$

$$\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{dy}{dx}\right)^2 + y = 0.$$

$\cancel{x\,dx + y = 0}$

$dx \qquad dy \qquad \frac{dy}{dx}.$

$\frac{dy}{dx}$.

$\cancel{x^2 dx} + y\,dy = 0$

$x^2 + y\frac{dy}{dx} = 0$ ✓

$\frac{dy}{dx} + y = 0.$

$y = e^{-x}$

$-e^{-x}$

$y = e^x$

$\frac{dy}{dx} = e^x \longrightarrow$

$\frac{dy}{dx} - y = 0$

$$\boxed{\frac{dy}{dx} - e^x = 0}$$ ✓

$\frac{dy}{dx} + y = 0$

$dy + y\,dx = 0$

$\int \frac{1}{y}dy + \int dx = 0$

$\log y + x = \cancel{c}$

$\log y = -x$

$y = e^{-x}$

Exact Diff. eq^{n}s.

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$\downarrow$

exact.

$\downarrow$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$N = x^2 + 2xy + y^2 - 2$

sol^n is

① $\int M\,dx + \int (\text{terms in } N \text{ free from } x)\,dy = c$

or

② $\int (\text{terms in } M \text{ free from } y)\,dx + \int N\,dy = c$

(1) $$\frac{dy}{dx} = \frac{(y+1)}{(y+2)e^y - x}$$

$$\left[(y+2)e^y - x\right] dy = (y+1)dx$$

$$(y+1)dx - \left[(y+2)e^y - x\right]dy = 0$$

$$M = y+1 \qquad N = -\left[(y+2)e^y - x\right]$$

$$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = -[0-1] = 1$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{Given D.E is exact.}$$

& Sol^n is

$$\int M\,dx + \int (\text{terms in } N \text{ free from } x)\,dy = C$$

$$\int (y+1)\,dx + \int -(y+2)e^y\,dy = C$$

$$(y+1)x - \int \underset{u}{(y+2)}\,\underset{v}{e^y}\,dy = C$$

$$(y+1)x - \left[(y+2)\,e^y - (1)\,e^y\right] = C$$

$$(y+1)x - \left[ye^y + 2e^y - e^y\right] = C$$

$$x(y+1) - \left[ye^y + e^y\right] = C$$

	$\frac{dy}{dx}$	$\int$
e^{ax}	$a e^{ax}$	$\frac{e^{ax}}{a} + c$
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + c$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + c$
x^n	$n x^{n-1}$	$\frac{x^{n+1}}{n+1} + c$
$\tan x$	$\sec^2 x$	$\log(\sec x) + c$

$\frac{\sin x}{\cos x}$

$-\int \frac{-\sin x}{\cos x}$

$= -\log(\cos x)^{-1} = \log(\sec x)$

$$(u \cdot v)' = u v' + v u'$$

$$\left(\frac{u}{v}\right)' = \frac{v u' - u v'}{v^2}$$

$$\int u \cdot v = u \int v - \int \left(\int v \cdot u' \right)$$

$$\int \underset{\downarrow}{u} \cdot v = u \underbrace{\int v}_{v_1} - u' \underbrace{\int v_1}_{v_2} + u'' \int v_2 - \text{———}$$

u has vanishing derivative

$$\int \underset{u}{x^3} \underset{v}{e^{2x}} \, dx$$

$$= (x^3)\left(\frac{e^{2x}}{2}\right) - (3x^2)\left(\frac{e^{2x}}{2^2}\right) + (6x)\left(\frac{e^{2x}}{2^3}\right) - (6)\left(\frac{e^{2x}}{2^4}\right)$$

$$x\,dx + y\,dy = \frac{a(x\,dy - y\,dx)}{x^2+y^2}$$

$$\Rightarrow \quad x\,dx + y\,dy = \frac{ax\,dy}{x^2+y^2} - \frac{ay\,dx}{x^2+y^2}$$

$$\Rightarrow \quad \left(x + \frac{ay}{x^2+y^2}\right)dx + \left(y - \frac{ax}{x^2+y^2}\right)dy$$

$$M \quad dx \quad + \quad N \quad dy$$

$$M = x + \frac{ay}{x^2+y^2} \qquad N = y - \frac{ax}{x^2+y^2}$$

$$\frac{\partial M}{\partial y} = 0 + \frac{(x^2+y^2)a - ay(2y)}{(x^2+y^2)^2}$$

$$= a\left[\frac{x^2+y^2-2y^2}{(x^2+y^2)^2}\right] = a\left[\frac{x^2-y^2}{(x^2+y^2)^2}\right]$$

$$\frac{\partial N}{\partial x} = 0 - \left[\frac{(x^2+y^2)a - ax(2x)}{(x^2+y^2)^2}\right]$$

$$= -a\left[\frac{x^2+y^2-2x^2}{(x^2+y^2)^2}\right]$$

$$= -a\left[\frac{y^2-x^2}{(x^2+y^2)^2}\right]$$

$$= a\left[\frac{x^2-y^2}{(x^2+y^2)^2}\right]$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\Rightarrow$ Given D.E is exact.

Sol^n is

$$\int M\,dx + \int (\text{terms in } N \text{ free from } x)\,dy = C$$

$$\Rightarrow \int\left(x + \frac{ay}{x^2+y^2}\right)dx + \int y\,dy = C$$

$$\Rightarrow \frac{x^2}{2} + ay\int \frac{1}{x^2+y^2}\,dx + \frac{y^2}{2} = C$$

$$\Rightarrow \frac{x^2}{2} + ay\,\frac{1}{y}\tan^{-1}\left(\frac{x}{y}\right) + \frac{y^2}{2} = C$$

$$2(1+x^2\sqrt{y})y\,dx + (x^2\sqrt{y}+2)x\,dy = 0$$

$M = 2(1+x^2\sqrt{y})y$

$= 2y + \underline{2x^2 y^{3/2}}$

$\frac{\partial M}{\partial y} = 2 + \cancel{2}x^2 \frac{3}{\cancel{2}} y^{1/2}$

$= 2 + 3x^2\sqrt{y}$

$N = (x^2\sqrt{y}+2)x$

$= x^3\sqrt{y} + 2x$

$\frac{\partial N}{\partial x} = \sqrt{y}\,3x^2 + 2$

$= 3x^2\sqrt{y} + 2$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ Given D.E. is exact.

soln is

$\int M\,dx + \int (\text{terms in } N \text{ free from } x)\,dy = c$

$\int (2y + 2x^2 y^{3/2})\,dx + \int 0\,dy = c$

$2yx + 2y^{3/2}\frac{x^3}{3} = c$

$$\left[\log(x^2+y^2) + \frac{2x^2}{x^2+y^2}\right]dx + \frac{2xy}{x^2+y^2}\,dy = 0.$$

$$M = \log(x^2+y^2) + \frac{2x^2}{x^2+y^2} \qquad N = \frac{2xy}{x^2+y^2}$$

$$\frac{\partial M}{\partial y} = \frac{1\,(2y)}{x^2+y^2} + \left[\frac{(x^2+y^2)(0) - 2x^2(2y)}{(x^2+y^2)^2}\right]$$

$$= \frac{2y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2}$$

$$= \frac{2y(x^2+y^2) - 4x^2y}{(x^2+y^2)^2}$$

$$= \frac{2yx^2 + 2y^3 - 4x^2y}{(x^2+y^2)^2}$$

$$= \frac{2y^3 - 2x^2y}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = \frac{(x^2+y^2)\,2y - 2xy\,(2x)}{(x^2+y^2)^2}$$

$$= \frac{2x^2y + 2y^3 - 4x^2y}{(x^2+y^2)^2}$$

$$= \frac{2y^3 - 2x^2y}{(x^2+y^2)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \quad \text{D.E. is exact.}$$

$$\int \frac{f'(x)}{f(x)} = \log f(x).$$

Sol^n is

$$\int (\text{terms in } M \text{ free from } y)\,dx + \int N\,dy = C.$$

$$\Rightarrow \int 0\,dx + \int \frac{2xy}{x^2+y^2}\,dy = C.$$

$$\Rightarrow x\int \frac{2y}{x^2+y^2}\,dy = C$$

$$\Rightarrow x\log(x^2+y^2) = C.$$

$$\frac{y}{x^2}\cos\left(\frac{y}{x}\right)dx - \frac{1}{x}\cos\left(\frac{y}{x}\right)dy + 2xdx = 0$$

$$\Big[\underbrace{2x + \frac{y}{x^2}\cos\left(\frac{y}{x}\right)}_{M}\Big]dx \underbrace{- \frac{1}{x}\cos\left(\frac{y}{x}\right)}_{N} dy = 0$$

$$M = 2x + \frac{y}{x^2}\cos\left(\frac{y}{x}\right) \qquad N = -\frac{1}{x}\cos\left(\frac{y}{x}\right).$$

$$\frac{\partial M}{\partial y} = 0 + \frac{1}{x^2}\left(-y\sin\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) + \cos\left(\frac{y}{x}\right)\right).$$

$$= -\frac{y}{x^3}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\frac{1}{x^2}$$

$$\frac{\partial N}{\partial x} = -\left[-\frac{1}{x}\sin\left(\frac{y}{x}\right) y\left(-\frac{1}{x^2}\right) + \cos\left(\frac{y}{x}\right)\left(-\frac{1}{x^2}\right)\right]$$

$$= -\frac{y}{x^3}\sin\left(\frac{y}{x}\right) + \frac{1}{x^2}\cos\left(\frac{y}{x}\right).$$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ D.E is exact.

$$\int M\,dx + \int(\text{terms in } N \text{ free from } x)\,dy = c$$

$$\int\left[2x + \frac{y}{x^2}\cos\left(\frac{y}{x}\right)\right]dx + \int 0\,dy = c.$$

$$\frac{\cancel{2}x^2}{\cancel{2}} + \int \frac{y}{x^2}\cos\left(\frac{y}{x}\right)dx = c.$$

$$\frac{y}{x} = t \Rightarrow -\frac{y}{x^2}dx = dt$$

$$\Rightarrow \frac{y}{x^2}dx = -dt$$

$$\Rightarrow x^2 + \int(\cos t)(-dt) = c.$$

$$\Rightarrow x^2 - \sin t = c \quad \Rightarrow x^2 - \sin\left(\frac{y}{x}\right) = c.$$

$\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0, given\ y(0)=4$

$M = 1+e^{x/y}$ $\quad N = e^{x/y} - \frac{x}{y}e^{x/y}$

$\frac{\partial M}{\partial y} = 0 + e^{x/y}\left(\frac{-x}{y^2}\right)$ $\quad \frac{\partial N}{\partial x} = e^{x/y}\left(\frac{1}{y}\right) - \frac{1}{y}\left(x\,e^{x/y}\left(\frac{1}{y}\right)+e^{x/y}\right).$

$= \cancel{\frac{1}{y}e^{x/y}} - \frac{x}{y^2}e^{x/y} - \cancel{\frac{1}{y}e^{x/y}}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

D E is exact

Sol^n is $\int M\,dx + \int(\text{terms in } N \text{ free from } x)\,dy = C$

$\int\left(1+e^{x/y}\right)dx + 0 = C$

$x + \frac{e^{x/y}}{1/y} = C \Rightarrow \underline{x + y e^{x/y} = C}$

given $y(0)=4 \Rightarrow$ at $x=0$ $y=4.$

$\therefore (0) + 4e^{0} = C \Rightarrow C = 4$

$x + y e^{x/y} = 4$