DiS Notes (divB):- ODD 2020-2021

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Tools to reduce boolean expressions:-

a) Algebraic (using Boolean properties)

b) Visual (Karnaugh Maps)

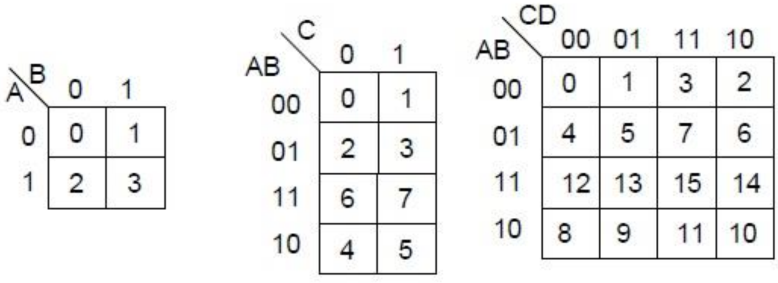
c) Tabular (Quine-McKluskey method)

Karnaugh Maps:

- Now, let us discuss about the K-Maps for 2 to 6 variables one by one.
- K-map is the best tool for minimization of five or fewer variables functions for humans.
- K-maps are graphic and require pattern-matching which is one of human's strongest abilities.
- Many believe that humans solve problems by creative pattern-matching.
- K-map is a number of squares which are labeled using reflective gray code (each code is only 1 change from an adjacent code). For a given square, the user enters 0 or 1 corresponding to the function value at the inputs represented by the labels.

Here is the location of each min-term on a Karnaugh-Map:

A B C D	Min-term,m
0 0 0 0	0
0 0 0 1	1
0 0 1 0	2
0 0 1 1	3
0 1 0 0	4
0 1 0 1	5
0 1 1 0	6
0 1 1 1	7
1 0 0 0	8
1 0 0 1	9
1 0 1 0	10
1 0 1 1	11
1 1 0 0	12
1 1 0 1	13
1 1 1 0	14
1 1 1 1	15



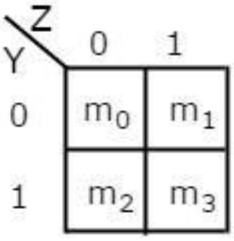
F(A,B)=AB 2-Variables

F(A,B,C) 3-Variables

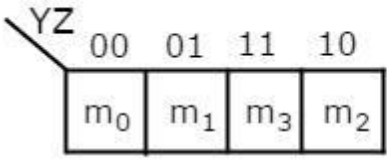
F(A,B,C,D) 4-Variables

2 Variable K-Map

The number of cells in 2 variable K-map is four, since the number of variables is two. The following figure shows **2 variable K-Map**.



or

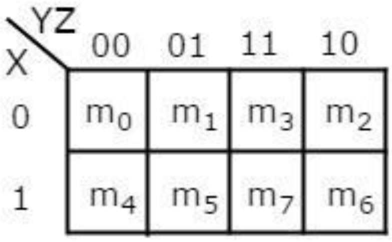


- There is only one possibility of grouping 4 adjacent min terms.

- The possible combinations of grouping 2 adjacent min terms are {(m0, m1), (m2, m3), (m0, m2) and (m1, m3)}.

3 Variable K-Map

The number of cells in 3 variable K-map is eight, since the number of variables is three. The following figure shows **3 variable K-Map**.



- There is only one possibility of grouping 8 adjacent min terms.

- The possible combinations of grouping 4 adjacent min terms are {(m0, m1, m3, m2), (m4, m5, m7, m6), (m0, m1, m4, m5), (m1, m3, m5, m7), (m3, m2, m7, m6) and (m2, m0, m6, m4)}.

- The possible combinations of grouping 2 adjacent min terms are {(m0, m1), (m1, m3), (m3, m2), (m2, m0), (m4, m5), (m5, m7), (m7, m6), (m6, m4), (m0, m4), (m1, m5), (m3, m7) and (m2, m6)}.

- If x=0, then 3 variable K-map becomes 2 variable K-map.

4 Variable K-Map
The number of cells in 4 variable K-map is sixteen, since the number of variables is four. The following figure shows **4 variable K-Map**.

WX \ YZ	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

- There is only one possibility of grouping 16 adjacent min terms.

- Let R1, R2, R3 and R4 represents the min terms of first row, second row, third row and fourth row respectively. Similarly, C1, C2, C3 and C4 represents the min terms of first column, second column, third column and fourth column respectively. The possible combinations of grouping 8 adjacent min terms are {(R1, R2), (R2, R3), (R3, R4), (R4, R1), (C1, C2), (C2, C3), (C3, C4), (C4, C1)}.

- If w=0, then 4 variable K-map becomes 3 variable K-map.

5 Variable K-Map
The number of cells in 5 variable K-map is thirty-two, since the number of variables is 5. The following figure shows **5 variable K-Map**.

V=0

WX \ YZ	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

V=1

WX \ YZ	00	01	11	10
00	m_{16}	m_{17}	m_{19}	m_{18}
01	m_{20}	m_{21}	m_{23}	m_{22}
11	m_{28}	m_{29}	m_{31}	m_{30}
10	m_{24}	m_{25}	m_{27}	m_{26}

- There is only one possibility of grouping 32 adjacent min terms.
- There are two possibilities of grouping 16 adjacent min terms. i.e., grouping of min terms from m0 to m15 and m16 to m31.
- If v=0, then 5 variable K-map becomes 4 variable K-map.
- In the above all K-maps, we used exclusively the min terms notation. Similarly, you can use exclusively the Max terms notation.

Minimization of Boolean Functions using K-Maps

- If we consider the combination of inputs for which the Boolean function is „1", then we will get the Boolean function, which is in **standard sum of products** form after simplifying the K-map.
- Similarly, if we consider the combination of inputs for which the Boolean function is „0", then we will get the Boolean function, which is in **standard product of sums** form after simplifying the K-map.

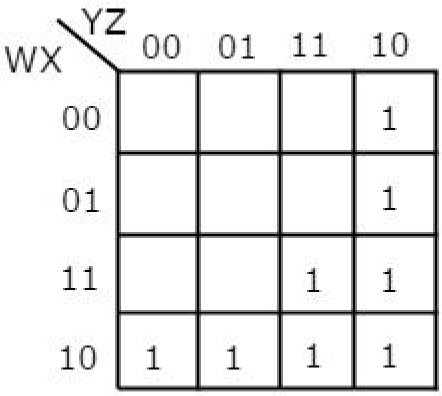
Example
Let us **simplify** the following Boolean function,

f(W, X, Y, Z)= WX’Y’ + WY + W’YZ’ using K-map.

WX’Y’Z + WX’Y’Z’ +
WX’YZ’ + WX’YZ + WXYZ’ + WXYZ +
W’X’YZ’ + W’XYZ’

m9 + m8 +
m10 + m11 + m14 + m15 +
m2 + m6

The given Boolean function is in sum of products form. It is having 4 variables W, X, Y & Z. So, we require **4 variable K-map**. The **4 variable K-map** with ones corresponding to the given product terms is shown in the following figure.



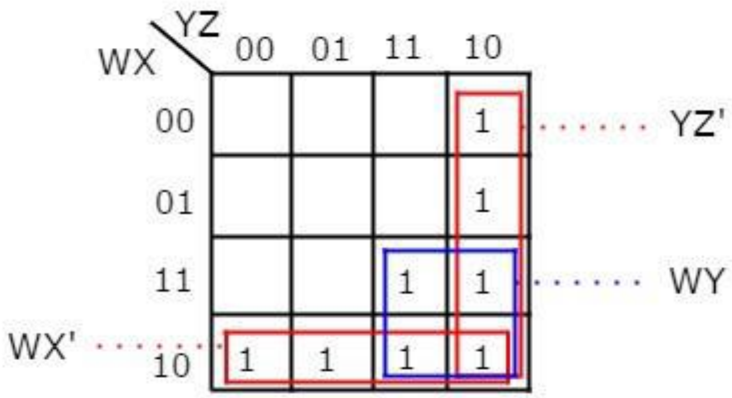
Here, 1s are placed in the following cells of K-map.

 The cells, which are common to the intersection of Row 4 and columns 1 & 2 are corresponding to the product term, **WX’Y’**.

 The cells, which are common to the intersection of Rows 3 & 4 and columns 3 & 4 are corresponding to the product term, **WY**.

 The cells, which are common to the intersection of Rows 1 & 2 and column 4 are corresponding to the product term, **W’YZ’**.

There are no possibilities of grouping either 16 adjacent ones or 8 adjacent ones. There are three possibilities of grouping 4 adjacent ones. After these three groupings, there is no single one left as ungrouped. So, we no need to check for grouping of 2 adjacent ones. The **4 variable K-map** with these three **groupings** is shown in the following figure.



Here, we got three prime implicants WX‟, WY & YZ‟

Therefore, the **simplified Boolean function** is

f= WX’ + WY + YZ’

- Example: Use K-map to minimize F(A,B,C) = $A.B.C + A.B.C + A.B.C + A.B.C$

  Solution:

  1. Use a truth table to identify all the Min-terms (Over time you can do this mentally, so it would not be necessary to draw it).

A B C	F	Min-term, m_i
0 0 0	0	0
0 0 1	1	1
0 1 0	0	2
0 1 1	1	3
1 0 0	0	4
1 0 1	1	5
1 1 0	0	6
1 1 1	1	7

  2. Fill in the K-map:
     a. Select the K-Map that matches the number of variables in your function, (3 for the Example)
     b. Draw the K-map (remember the labels are reflective Gray Code)
     c. Enter the value of the function for the corresponding min-term. If the value of the function is unspecified then enter – which means don't care.

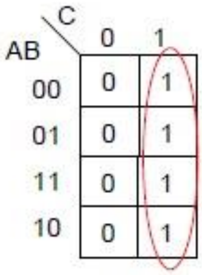
AB \ C	0	1
00	0	1
01	0	1
11	0	1
10	0	1

F(A,B,C)

  3. The next step is to group as many neighboring ones as possible. Cells with one variable is complemented are referred to as neighboring cells:
     a. Grouping adjacent min-terms (boxes) is applying the Adjacency theorem graphically, i.e.

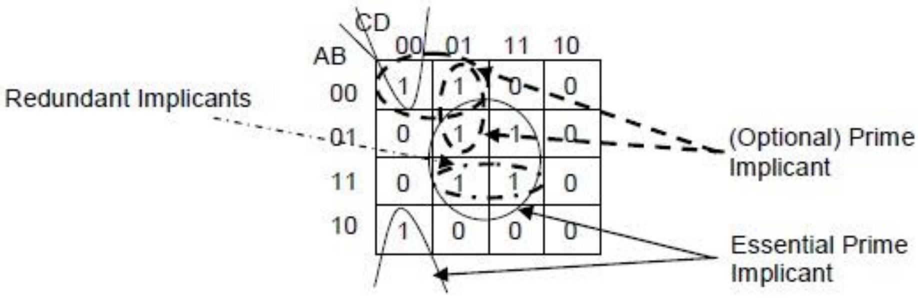
$$\overline{A}.C + A.C = C.$$

     b. The goal is to get as large a grouping of 1s as possible
        (Must form a full rectangle – cannot group diagonally)



F(A,B,C)
3-Variables

4. For each identified group, look to see which variable has a unique value. In this case, F(A,B,C) = C since F's value is not dependent on the value of A and B.



F(A,B,C,D)
4-Variables

Minimized function = $\overline{B}.\overline{C}.\overline{D} + B.D + \overline{A}.\overline{B}.\overline{C}$

- An ***Implicant*** is the product term where the function is evaluated to 1 or complemented to 0. An Implicant implies the term of the function is 1 or complemented to 0. Each square with a 1 for the function is called an implicant (p). If the complement of the function is being discussed, then 0's are called implicants (r).
  Note: To find the complement of F, apply the same rules to 0 entries in the K-map instead of 1.

- *A **Prime Implicant*** of a function is a rectangular (each side is powers of 2) group of product terms that is not completely contained in a single larger implicant.

- *An **Essential Prime Implicant*** of a function is a product term that provides the only coverage for a given min-term and must be used in the set of product terms to express a given function in minimum form.

- *An **Optional Prime Implicant*** of a function is a product term that provides an alternate covering for a given Min-term and may be used in the set of product terms to express a function in a minimum form. Some functions can be represented in a minimum form in more than one way because of optional prime implicants.

K maps

$F(x, y) = x + x'y = x + y$

$= x(y + y') + x'y$

$= xy + xy' + x'y = \Sigma m(1, 2, 3)$

x	y	$F(x, y)$	
0	0	0	$x'y'$
0	1	1	$x'y$
1	0	1	xy'
1	1	1	xy

$x \backslash y$	0	1
0	0	1
1	1	1

$x'y + xy$ x changing remove x.

$= y$. — II

$xy' + xy$ y changes, so remove y

$= x$. — I

I + II $= x + y$.

$F(a,b,c) = \prod M(0,1,2,3,7)$

Here we have Maxterms (POS)

M_0, M_1, M_2, M_3, M_7

a	b	c	F(a,b,c)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$(a+b+c) = M_0$

$M_1 = (a+b+\bar{c})$

$M_2 = (\bar{a}+\bar{b}+c)$

$M_3 = (a+\bar{b}+\bar{c})$

$M_7 = (\bar{a}+\bar{b}+\bar{c})$

a \ bc	00	01	11	10
0	0_0	0_1	0_3	0_2
1	4	5	0_7	6

a \ bc	00	01	11	10
0	0	0	0	0
1			0	

I

II

$I = \bar{a}$

$II = \bar{b}+\bar{c}$

$I \cdot II = \bar{a}(\bar{b}+\bar{c})$

$F(a,b,c) = \cancel{\Sigma}\,\Pi M(0,2,4,6)$

$\cancel{\Sigma\,\Pi M(0,2,4,6)}$

I

$a \backslash bc$	00	01	11	10
0	0_{0}	$_{1}$	$_{3}$	0_{2}
1	0_{4}	$_{5}$	$_{7}$	0_{6}

↪ wrap around! 2

We have one group of 4!

$\Rightarrow$ a changes across rows.

b changes across colums.

only c is constant $\Rightarrow$ 0.

$\therefore F(a,b,c) =$ c

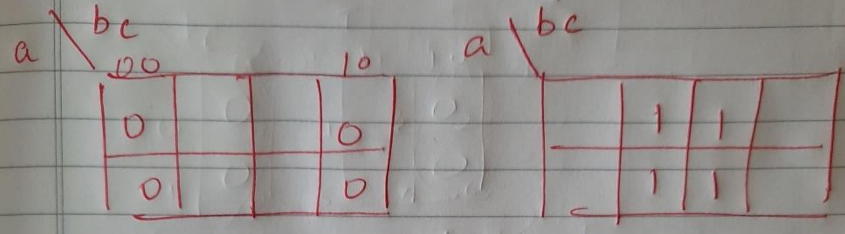
$$F(a,b,c) = \Sigma m(1,3,5,7)$$

a \ bc	00	01	11	10
0	0	1 1	1 3	2
1	4	1 5	1 7	6

a changes across rows.
b changes acros cols.
only c is left

$\therefore F(a,b,c) = c$

DID YOU NOTICE?



$\Downarrow$ $\Downarrow$

c c

~~ΠM(2,4~~,

$\Pi M(0,2,4,6) \Longleftrightarrow \Sigma m(1,3,5,7)$

$$F(x, y) = xy' + x'y = \Sigma m(1, 2)$$

x	y	$F(x,y)$	
0	0	0	$x'y'$
0	1	1	$x'y$
1	0	1	xy'
1	1	0	xy

$x'y$

$x \backslash y$	0	1
0	0	1
1	1	0

xy'

no groups

$\therefore$ no reduction

Answer is same.

$= xy' + x'y$.

$F(a,b,c) = \sum m(0,1,2,3,7)$

	a	b	c	F(a,b,c)
m_0	0	0	0	1
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	1

a \ bc	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

=

1_0	1_1	1_3	1_2
0_4	0_5	1_7	0_6

a \ bc	00	01	11	10
0	1	1	1	1
1	0	0	1	0

I

II

I = a // Both b and c change in value
∴ Both b and c are eliminated

II = bc // a changes 0 → 1

I + II = a + bc.

More examples:-

Y1(a,b,c) = ΠM(0,2,6) = **(a+c).(b'+c)**

Y2 = ΠM(3,4,7,12) =

Y3 = ΠM(0,1,2,3) = **a**

Y4 = ΠM(0,1,2,4,5,6)

Y5 = ΠM(3,4,7,12)

Y6 = ΠM(8,10,12,14,15)

Y7(a,b,c,d) = ΠM(8,9,10,11,12,13,14,15)

Y8(a,b,c,d) = ΠM(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)