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Assignment on self-study Topics

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Batch: P3-3 Roll No.: 1601042223A
~~Experiment~~ / ~~assignment~~ / tutorial No. 07
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Q.1 Express $\tan 7\theta$ in terms of powers of $\tan\theta$.

Hence deduce: $7\tan^6\frac{\pi}{14} - 35\tan^4\frac{\pi}{14} + 21\tan^2\frac{\pi}{14} - 1 = 0$

Solⁿ: By De-Moivre's theorem,

$\cos 7\theta + i\sin 7\theta = (\cos\theta + i\sin\theta)^7 \rightarrow ①$

$(\cos\theta + i\sin\theta)^7 = (\cos\theta)^7 + 7(\cos\theta)^6(i\sin\theta) + 21(\cos\theta)^5(i\sin\theta)^2 + 35(\cos\theta)^4(i\sin\theta)^3 + 35(\cos\theta)^3(i\sin\theta)^4 + 21(\cos\theta)^2(i\sin\theta)^5 + 7(\cos\theta)(i\sin\theta)^6 + (i\sin\theta)^7$

$= \cos^7\theta + (i)7\cos^6\theta\sin\theta - 21\cos^5\theta\sin^2\theta + (-i)35\cos^4\theta\sin^3\theta + 35\cos^3\theta\sin^4\theta + (i)21\cos^2\theta\sin^5\theta - 7\cos\theta\sin^6\theta - (i)\sin^7\theta$

$\cos 7\theta + i\sin 7\theta = [\cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta] + i[7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta]$

$\rightarrow$ [From ①]

On equating real and imaginary parts, we get

$\cos 7\theta = \cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta \rightarrow ②$

$\sin 7\theta = 7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta \rightarrow ③$

$\tan 7\theta = \dfrac{\sin 7\theta}{\cos 7\theta}$

$= \dfrac{7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta}{\cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta}$ $\rightarrow$ [From ② & ③]

Dividing the numerator and denominator by $\cos^7\theta$, we get

$\tan 7\theta = \dfrac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}$

Thus, we can express $\tan 7\theta$ in terms of $\tan\theta$, its powers;

Now putting $\theta = \frac{\pi}{14}$ in the above eqⁿ we get $\tan 7\theta = \tan \cancel{7}\left(\frac{\pi}{\cancel{14}_2}\right)$

$\tan 7\theta = \infty \rightarrow$ The denominator must be zero.

$1 - 21\tan^2\frac{\pi}{14} + 35\tan^4\frac{\pi}{14} - 7\tan^6\frac{\pi}{14} = 0$

Multiplying by -1 on both sides we get,

$7\tan^6\frac{\pi}{14} - 35\tan^4\frac{\pi}{14} + 21\tan^2\frac{\pi}{14} - 1 = 0$

Hence, proved.

Q.2 show that: $2^6 \sin^4\theta \cos^3\theta = \cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta$

Soln: Let $x = \cos\theta + i\sin\theta$, $\therefore \frac{1}{x} = \cos\theta - i\sin\theta$

$x + \frac{1}{x} = 2\cos\theta$ and $x - \frac{1}{x} = 2i\sin\theta \rightarrow (1)$

$x^n = \cos n\theta + i\sin n\theta$, $\frac{1}{x^n} = \cos n\theta - i\sin n\theta$

$x^n + \frac{1}{x^n} = 2\cos n\theta$ and $x^n - \frac{1}{x^n} = 2i\sin n\theta \rightarrow (2)$

$$(2i\sin\theta)^4 (2\cos\theta)^3 = \left(x - \frac{1}{x}\right)^4 \left(x + \frac{1}{x}\right)^3 \rightarrow \text{[From (1)]}$$

$$2^7 \sin^4\theta \cos^3\theta = \left(x - \frac{1}{x}\right)^2 \left(x - \frac{1}{x}\right)^2 \left(x + \frac{1}{x}\right)^2 \left(x + \frac{1}{x}\right)$$

$$= \left(x^2 - 2 + \frac{1}{x^2}\right)\left(x^4 - 2 + \frac{1}{x^4}\right)\left(x + \frac{1}{x}\right)$$

$$= \left(x^3 + \frac{1}{x^3} - x + \left(-\frac{1}{x}\right)\right)\left(x^4 - 2 + \frac{1}{x^4}\right)$$

$$= x^7 + x - x^5 - x^3 - 2x^3 - \frac{2}{x^3} + 2x + \frac{2}{x}$$

$$+ \frac{1}{x} + \frac{1}{x^7} - \frac{1}{x^3} - \frac{1}{x^5}.$$

$$= \left(x^7 + \frac{1}{x^7}\right) - \left(x^5 + \frac{1}{x^5}\right) - 3\left(x^3 + \frac{1}{x^3}\right)$$

$$+ 3\left(x + \frac{1}{x}\right)$$

$$= (2\cos 7\theta) - (2\cos 5\theta) - 3(2\cos 3\theta)$$

$$+ 3(2\cos\theta) \rightarrow \text{[From (1) \& (2)]}$$

$$2^7 \sin^4\theta \cos^3\theta = 2[\cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta]$$

$$2^6 \sin^4\theta \cos^3\theta = \cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta$$

(∵ Dividing by 2 on both sides)

Hence, proved.

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Q.3 If $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 2 & 2 & 0 \end{bmatrix}$ verify that $A(\text{adj} A) = |A| I$.

Hence find the inverse of (adj A).

Sol^n: $|A| = 2[0-2] - 1[0-2] - 1[0-4]$

$= -4 + 2 + 4$

$|A| = 2 \neq 0$

$[A_{ij}] \rightarrow$ cofactor matrix, where $A_{ij} = (-1)^{i+j} M_{ij}$

M_{ij} is the minor of a_{ij}

$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = 0 - 2 = -2$

$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -[0-2] = 2$

$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 0 & 2 \\ 2 & 2 \end{vmatrix} = 0 - 4 = -4$

$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = -[0+2] = -2$

$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix} = 0 + 2 = 2$

$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = -[4-2] = -2$

$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3$

$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -[2+0] = -2$

$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 4 - 0 = 4$

Hence, the cofactor matrix $[A_{ij}] = \begin{bmatrix} -2 & 2 & -4 \\ -2 & 2 & -2 \\ 3 & -2 & 4 \end{bmatrix}$

$\text{adj} A = ([A_{ij}])^T$

$\text{adj} A = \begin{bmatrix} -2 & -2 & 3 \\ 2 & 2 & -2 \\ -4 & -2 & 4 \end{bmatrix}$

$$A(\text{adj}A) = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} -2 & -2 & 3 \\ 2 & 2 & -2 \\ -4 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4+2+4 & -4+2+2 & 6-2-4 \\ 0+4-4 & 0+4-2 & 0-4+4 \\ -4+4+0 & -4+4+0 & 6-4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A(\text{adj}A) = |A| I \quad (\because |A| = 2) \rightarrow (1)$

Hence, proved.

$$|\text{adj}A| = \begin{vmatrix} -2 & -2 & 3 \\ 2 & 2 & -2 \\ -4 & -2 & 4 \end{vmatrix}$$

$$= -2[8-4] + 2[8-8] + 3[-4+8]$$

$$= -2(4) + 2(0) + 3(4)$$

$$= -8 + 12$$

$|\text{adj}A| = 4 \neq 0$

Therefore, $(\text{adj}A)^{-1}$ exists.

Considering eq^n (1), $A(\text{adj}A) = |A| I$

$$\frac{A}{|A|}(\text{adj}A) = I$$

$$\therefore (\text{adj}A)^{-1} = \frac{A}{|A|}$$

$$= \frac{1}{|A|} \times A$$

$$(\text{adj}A)^{-1} = \frac{1}{2} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$

Therefore, the inverse of $(\text{adj}A) = \frac{1}{2} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 2 & 2 & 0 \end{bmatrix}$

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Q.4 Find the matrix A, if $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

Sol^n: $\underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}} \underset{m\times n}{A} \underset{2\times 2}{\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}}$

matrix A will be a (2×2) order matrix.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -3(2a+c)+5(2b+d) & 2(2a+c)-3(2b+d) \\ -3(3a+2c)+5(3b+2d) & 2(3a+2c)-3(3b+2d) \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -6a-3c+10b+5d & 4a+2c-6b-3d \\ -9a-6c+15b+10d & 6a+4c-9b-6d \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$$

$\therefore -6a+10b-3c+5d = -2 \rightarrow ①$

$4a-6b+2c-3d = 4 \rightarrow ②$

$-9a+15b-6c+10d = 3 \rightarrow ③$

$6a-9b+4c-6d = -1 \rightarrow ④$

Adding ① and ④, we get: $b+c-d = -3 \rightarrow ⑤$

Multiplying ② by 3 and adding it with ③, we get:

$3a-3b+d = 15 \rightarrow ⑥$

Multiplying ① by 2 and subtracting ③ from it, we get:

$-3a+5b = -7 \rightarrow ⑦$

Multiplying ② by 2 and subtracting ④ from it, we get:

$2a-3b = 9 \rightarrow ⑧$

Multiplying ⑦ by 2 and ⑧ by 3, adding them we get:

$b = 13$

Substituting $b=13$ in ⑧, we get: $a = 24$

Now substituting $a=24$, $b=13$ in ⑥, we get: $d = -18$

Substituting values of $b=13$, $d=-18$ in ⑤, we get: $c = -34$

Thus, the required matrix $A = \begin{bmatrix} 24 & 13 \\ -34 & -18 \end{bmatrix}$