* Equivalent force system:-

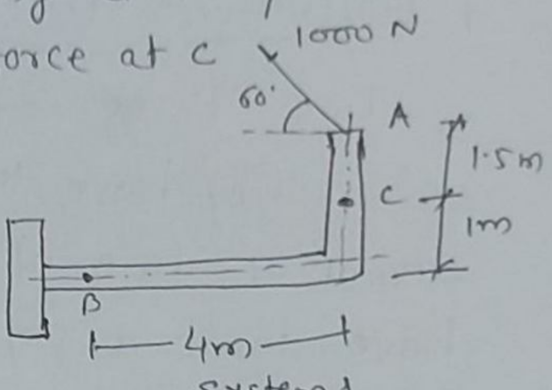
Two force systems are said to be equivalent when they have the same resultant in magnitude direct^n & line of action i.e. two force system must have same x & y components of resultant & same moment @ any pt. in the plane.

$$\therefore (R_x)_1 = (R_x)_2 \qquad (\Sigma F_x)_1 = (\Sigma F_x)_2$$
$$(R_y)_1 = (R_y)_2 \qquad (\Sigma F_y)_1 = (\Sigma F_y)_2$$
$$(M)_1 = (M_2) \qquad (\Sigma M)_1 = (\Sigma M)_2$$

A non concurrent force system or a single force can be replaced by

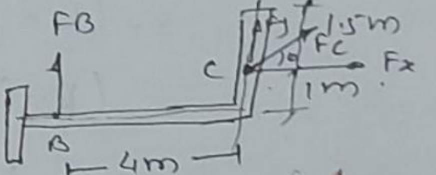
i. Two parallel forces.
ii. Two or three non-parallel forces.

* Replace the 1000 N force acting at A by two forces a vertical force at B & a force at C



system 1

Assume force at B is to be upward & at C force is is in 1st quadrant.



$(\Sigma f_x)_1 = (\Sigma f_x)_2$

$\cancel{(Rx)_1 = (Rx)_2}$

$-1000 \cos 60 = fx$

$Fx = -500 N = 500 N (\leftarrow)$

$\cancel{(Ry)_1 = (Ry)_2}$ $(\Sigma Fy)_1 = (\Sigma Fy)_2$

$100 \sin 60 = FB + Fy$ — ①

$(Mc)_1 = (Mc)_2$

$1000 \cos 60 \times 1.5 = -FB \times 4$

$FB = -187.5 N. = 187.5 N (\downarrow)$

Put FB ①

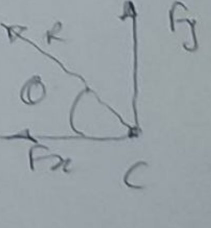
$\therefore Fy = 274.1 N \uparrow$

$R = \sqrt{Fx^2 + Fy^2} = \sqrt{(-500)^2 + (274.1)^2}$

$R = 570.2 N.$

$\theta = \tan^{-1}(Fy/Fx)$

$\theta = 28.73°$



(14)

* Find the resultant of following force system & also find the equivalent force & couple at pt. A of the same force system

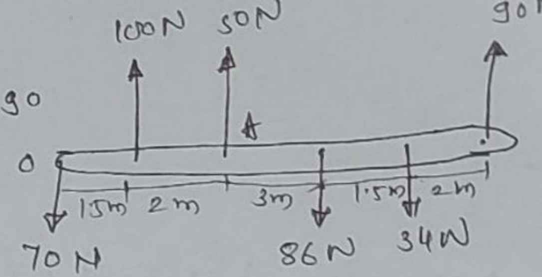
$\Sigma F_x = 0$

$\Sigma F_y = -70 + 100 + 50 - 86 - 34 + 90$

$= 50$ N.

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$= 50$ N $(\uparrow)$



$\Sigma M_o = 100 \times 1.5 + 50 \times 3.5 - 86 \times 6.5 - 34 \times 8 + 90 \times 10$

$= 394$ N-m $(\circlearrowleft)$

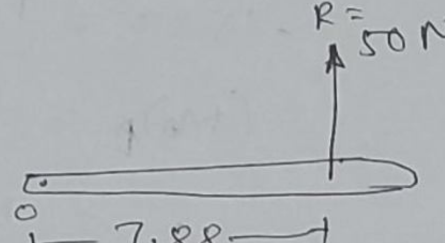
Using varignon's thm

why upward

$\Sigma M_o = R \times d$

$394 = 50 \times d$

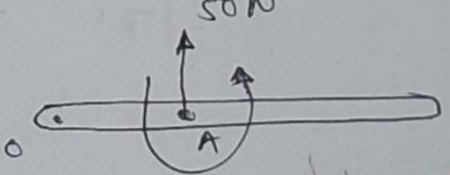
$d = 7.88$ m.



couple at A :-

$\Sigma M_A = 70 \times 3.5 - 100 \times 2 - 86 \times 3 - 34 \times 4.5 + 90 \times 6.5$

$\Sigma M_A = 219$ N-m $(\circlearrowleft)$

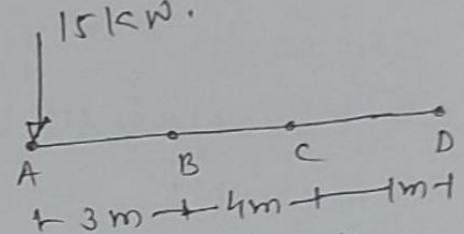


R(7.88-3.5)

A

Dec-2011 (05 marks)

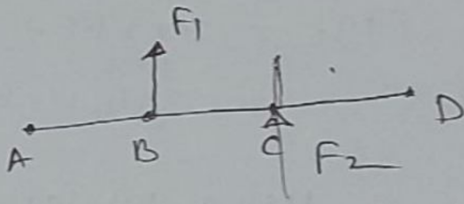
* Resolve 15 kN force acting at A into two Parallel componant at B & C.



system 1

→

Assuming force at B is (↑)
& C at (↑)



system 2

$(F_y)_1 = (F_y)_2$

$-15\times10^3 = F_1 + F_2$ — (1)

$(M_A)_1 = (M_A)_2$

$0 = F_1\times 3 + F_2\times 7$

$3F_1 = -F_2\times 7$

$F_1 = -7/3\, F_2$

$-15\times10^3 = -7/3\, F_2 + F_2$

$-15\times10^3 = -4/3\, F_2$

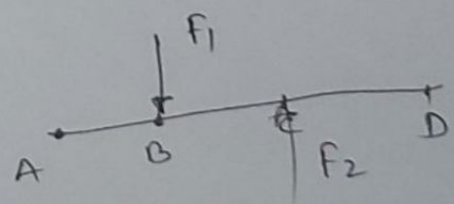
$F_2 = +11.25\,kN$ (↑)

~~= 11.25 kN (↓)~~

$-15\times10^3 = +11.25\times10^3 + F_1$

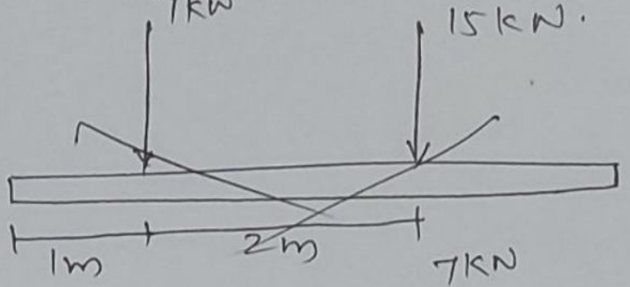
$F_1 = -26.25\,kN$ ~~(↑)~~

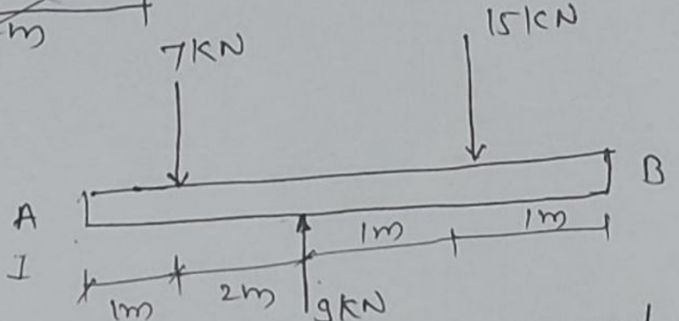
$= 26.25\,kN$ (↓)



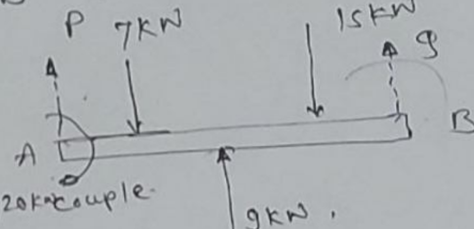
May 2012 (5 marks) 18 15

* The resultant of the three forces shown in fig & other two forces P & Q acting at A & B is a couple of magnitude 120 kNm clockwise. Determine the force P & Q.





Assume force at A & B are upward.



$\Sigma F_y = 0$

~~$P + Q - 7 - 15 + 9 = 0$~~

~~$P + Q = 13$ — (1)~~

(+ve) −ve

$\Sigma M_B = -120$

$15\times1 - 9\times2 + 7\times4 + P\times5 = -120$

$\boxed{P = 29\text{ kN } (\uparrow)}$

M_A

$-120 = -7\times1 + 9\times3 - 15\times4 + Q\times5$

$-120 = -7 + 27 - 60 + 5Q$

$Q = -16\text{ kN}$

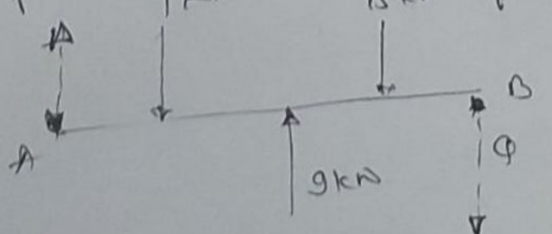
$= 16\text{ kN } (\downarrow)$

$P + (-16) = 13$

$P = 13 + 16$

$P = 29\text{ kN}$

$= 29\text{ kN } (\uparrow)$

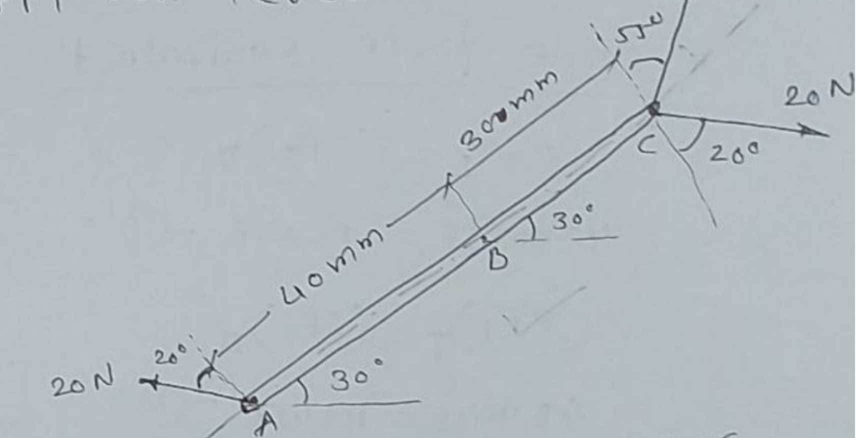


19

* Three control forces acting on lever

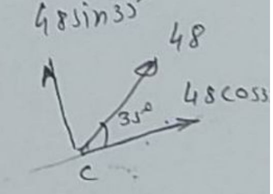
(a) Replace the three forces with an equivalent force couple at B.

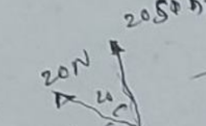
(b) Det. the single force which is equivalent to the force-couple system of part-a & specify its pt. of $appl^n$. on lever.

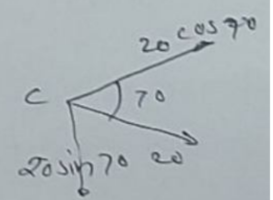


→ force - couple system at. B.

~~Σfx =~~







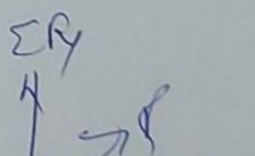
$\Sigma f_x = -\cancel{20\cos 70} + 48\cos 35 + \cancel{20\cos 70}$

$= 39.32\ N.$

$\Sigma f_y = \cancel{20\sin 70} + 48\sin 35 - \cancel{20\sin 70}$

$= \cancel{48\ N}\ 27.53\ N.$

$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2} = 48\ N.$



$\theta = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right)$

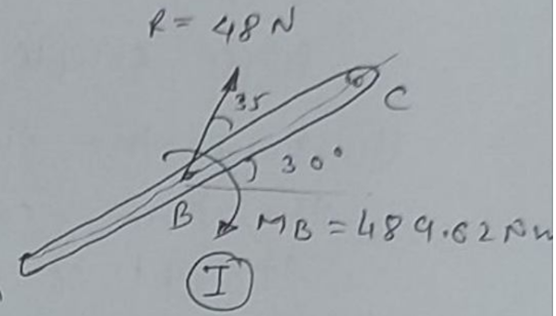
$= 35°$ with lever AC
or $65°$ with horizontal.

$M_B = (48\sin 35 \times 30) - 20\sin 70 \times 40 - 20\sin 70 \times 30$

$= -482.62\ N.mm$

$= 489.62\ N.mm$

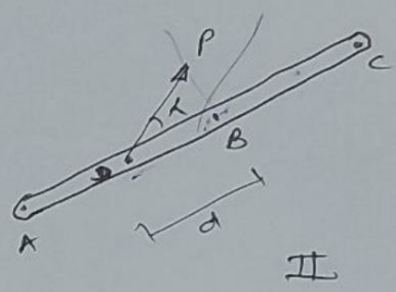
force - couple equivalent system



* single force equivalent :-

$$(\Sigma F_x)_I = (\Sigma F_x)_{II}$$

$$48 \cos 35 = P \cos \alpha \quad (1)$$



$$(\Sigma F_y)_I = (\Sigma F_y)_{II}$$

$$48 \sin 35 = P \sin \alpha \quad (2)$$

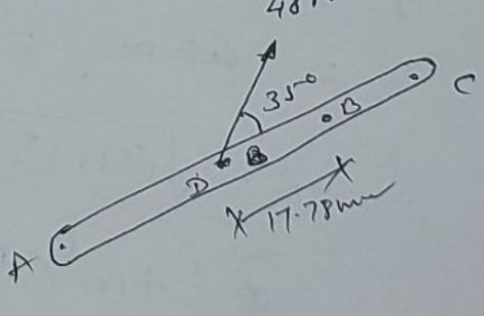
solving (1) & (2)

$\alpha = 35°$ & $P = 48$ N.

$$(M_I)_B = (M_{II})_B$$

$$-48962 = -P \sin \alpha \times d$$

$$\boxed{d = 17.78 \text{ mm.}}$$

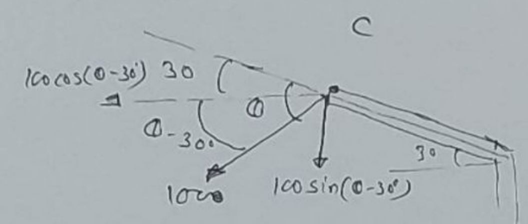


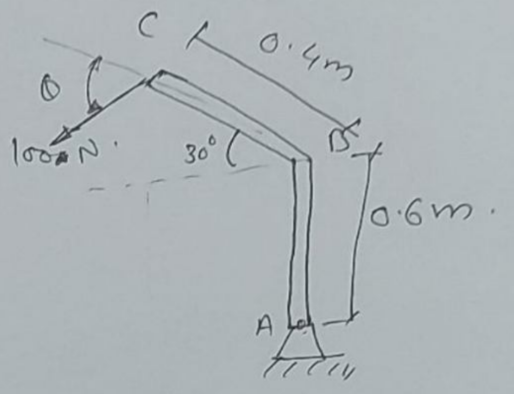
* find angle θ for which moment of force of 100 N @ A is max^{m}. find max^{m} moment. (20)

→

$\Sigma M_A =$

$= \cancel{100 \sin(}$





$$\Sigma M_A = 100\sin(\theta - 30) \times 0.4\cos 30°$$
$$+ 100\cos(\theta - 30)\,(0.6 + 0.4\sin 30)$$

$$= 34.64 \sin(\theta - 30°) + 80\cos(\theta - 30°) \quad — ①$$

M_A to be max^{m} $\quad \dfrac{dM_A}{d\theta} = 0$

$$\frac{d(\text{eq}^n ①)}{d\theta} = 0$$

$$34.64\cos(\theta - 30) - 80\sin(\theta - 30°) = 0$$

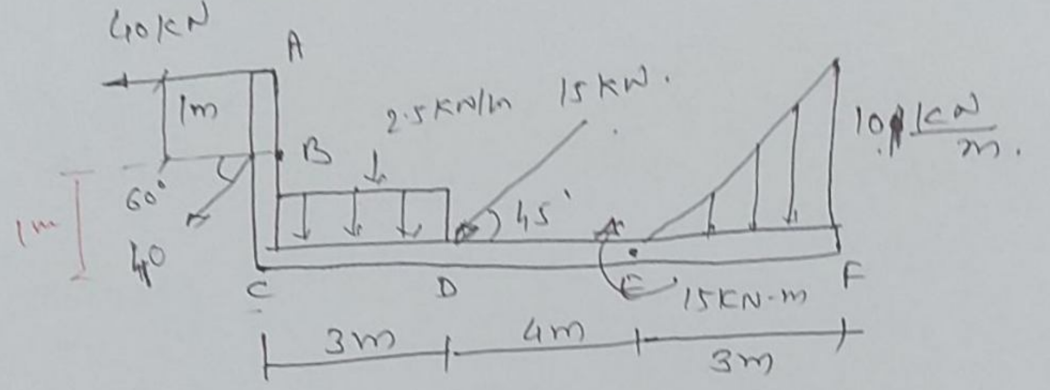
$$\therefore\ 34.64\cos(\theta - 30°) = 80\sin(\theta - 30°)$$

$$\frac{\sin(\theta - 30)}{\cos(\theta - 30)} = \frac{34.64}{80}$$

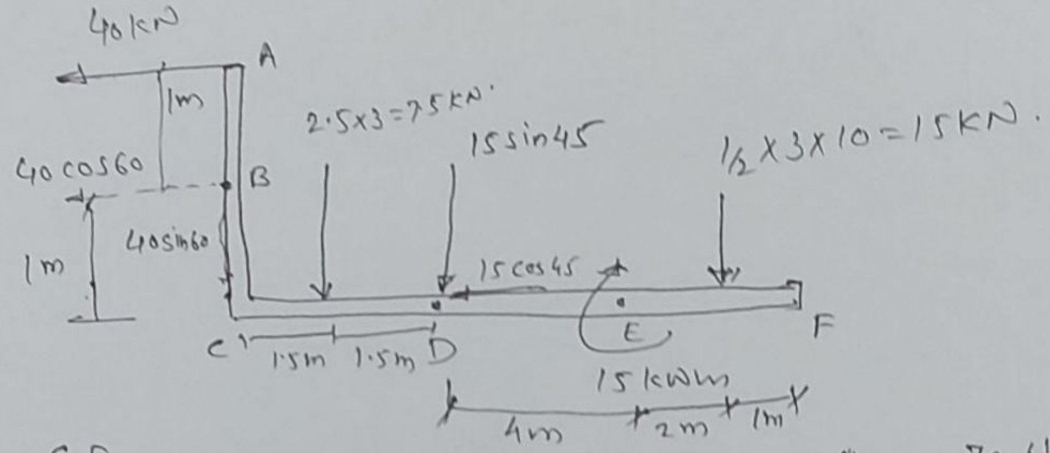
$$\theta - 30 = \tan^{-1}(0.433)$$
$$= 23.41$$
$$\theta = 53.41°$$

$$\therefore\ M_{A\,max} = 34.64\sin 23.41 + 80\cos(23.41)$$
$$= 87.18\ \text{N-m}$$

* Replace the force system as shown in fig w.r.t. C



$\longrightarrow$



$\Sigma F_x = -40 - 40\cos 60 - 15\cos 45 = -70.61\,kN = 70.61\,kN(\leftarrow)$

$\Sigma F_y = -40\sin 60 - 7.5 - 15\sin 45 - 15 = -67.75\,kN = 67.75\,kN$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 97.86\,kN.$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

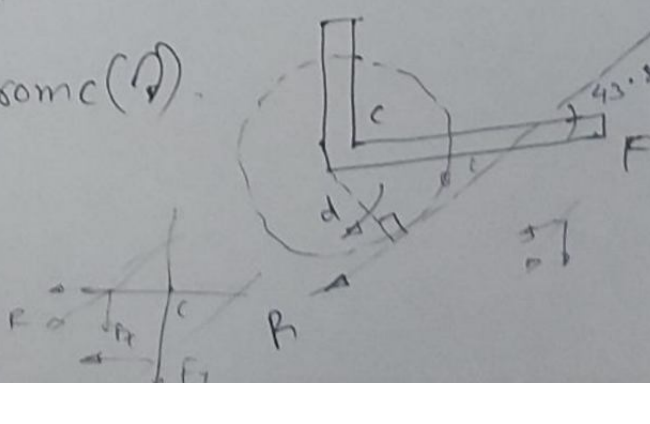
$$= 43.81^\circ$$

Varignon th^m at C

$R \times d = 40\times 2 + 40\cos 60\times 1 - 7.5\times 1.5 - 15\sin 45\times 3 - 15 - 15\times 9$

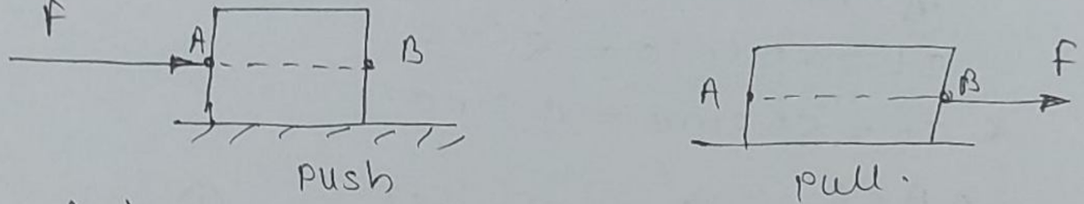
$97.86 \times d = -93.07$

$d = 0.95\,m$ from C



* principle of Transmissibility of force :-

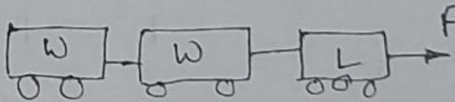
" condⁿ of eqm or uniform motion of rigid body will remain unchanged if pt. of applⁿ of a force acting on rigidy body is transmitted to act at any other pt. along its line of actⁿ.



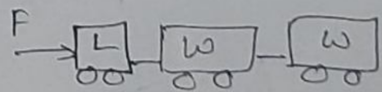
A force ~~force~~ ~~of~~ F acts on body at A, can be replaced by same force F at ~~the~~ pt. B provided pt A & B lies on same line of actⁿ of force. Though nature changes from push to pull but external effect remains unchanged due to principle of transmissibility of force.

e.g. locomotive pulling wagons W to right by exerting force F from front.

This force gets transmitted to all wagons & move forward

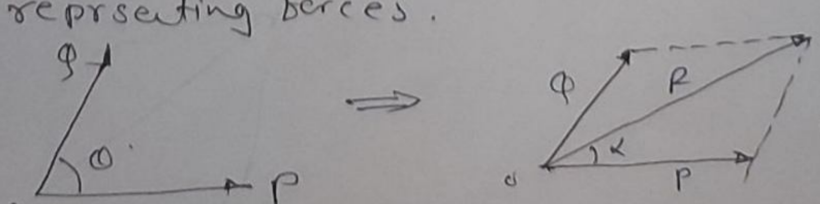


The same effect is observed if Locomotive pushes the wagons from behind



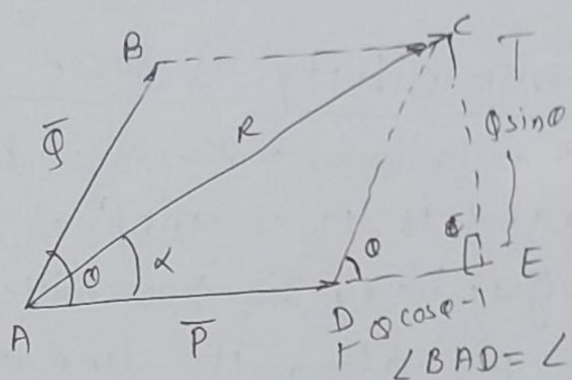
* Law of Parallelogram of forces :-

if two forces acting simultaneously on a body at pt. are represented in magnitude & directⁿ by two adjacent sides of ||alogram then their resultant is represented in mag. & directⁿ by the diagonal of ||alogram which passes thro' the pt. of intersection of two sides represeting forces.



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$



$AB = CD = Q$

$AD = P$ (magn of $\bar{P}$)

$AC = R$ (magni. of $\bar{R}$)

$\angle BAD = \angle CDE = \theta$

ΔCDE

$DE = CD\cos\theta = Q\cos\theta$

& $CE = CD\sin\theta = Q\sin\theta$.

ΔACE

$$AC^2 = AE^2 + CE^2 \quad - (1)$$

But $AE = AD + DE = P + Q\cos\theta$. $CE = Q\sin\theta$ & $AC = R$.

$\therefore$ (1) $\Rightarrow$

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$= P^2 + 2PQ\cos\theta + Q^2(\cos^2\theta + \sin^2\theta)$$

$$R^2 = P^2 + 2PQ\cos\theta + Q^2$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}.$$

ΔACE

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

This law is used to find resultant of two vector.