

F.Y. B. Tech SEM-II
Applied Mathematics-II

Practice Problems
Linear Differential Equation with Constant Coefficient
(Module 1: Sub-module 1.3 & 1.4)

Solve the following.

1. $\frac{d^3 y}{d x^3} - 5 \frac{d^2 y}{d x^2} + 8 \frac{d y}{d x} - 4 y = 0$ [Ans: $y = (c_1 + c_2 x)e^{2x} + c_3 e^x$]

2. $\frac{d^3 y}{d x^3} - 6 \frac{d^2 y}{d x^2} + 11 \frac{d y}{d x} - 6 y = 0$ [Ans: $y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$]

3. $\frac{d^4 y}{d x^4} + k^4 y = 0$

[Ans:

$y = e^{(k/\sqrt{2})x} [c_1 \cos(k/\sqrt{2})x + c_2 \sin(k/\sqrt{2})x] + e^{-(k/\sqrt{2})x} [c_3 \cos(k/\sqrt{2})x + c_4 \sin(k/\sqrt{2})x]$]

4. $\frac{d^4 y}{d x^4} + 6 \frac{d^2 y}{d x^2} + 9 y = 0$ Ans: $y = (c_1 + c_2 x) \cos \sqrt{3} x + (c_3 + c_4 x) \sin \sqrt{3} x$

5. $\frac{d^4 y}{d x^4} + 2 \frac{d^2 y}{d x^2} + y = 0$

6. $\frac{d^4 y}{d x^4} + y = 0$

[Ans: $y = e^{(x/\sqrt{2})} [c_1 \cos(x/\sqrt{2}) + c_2 \sin(x/\sqrt{2})] + e^{-(x/\sqrt{2})} [c_3 \cos(x/\sqrt{2}) + c_4 \sin(x/\sqrt{2})]$]

7. $(D^3 - D^2 + D - 1)y = 0$

8. $(D^3 - 3D^2 + 4)y = 0$

9. $(D^2 - 1)(D - 1)^2 y = 0$

10. $(D^4 + 8D^2 + 16)y = 0$ [Ans: $y = (c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x$]

11. $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0$ [Ans: $y = e^x [(c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x]$]

12. $(D^2 + 2D + 1)y = 0$

13. Solve $(D^2 - 2D + 1)y = e^x + 1$

14. Solve $\frac{d^2 y}{d x^2} + 3 \frac{d y}{d x} + 2y = e^{-x}$

15. Solve $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = e^x + 1$

16. Solve $\frac{d^2 y}{d x^2} - (a + b) \frac{d y}{d x} + ab y = e^{ax} + e^{bx}$

17. Solve $\frac{d^3 y}{d x^3} - 4 \frac{d y}{d x} = 2 \cos h^2 2x$

18. Solve $(2D + 1)^2 y = 4e^{-\frac{x}{2}}$

19. Solve $(D^4 + 1)y = \cos h \ 4x \sin h \ 3x$

20. Solve $\frac{d^2 y}{dx^2} + y = \sin x \sin 2x$

21. Solve $(D^3 + D^2 + D + 1)y = \sin^2 x$

22. Solve $(D^4 + 10D^2 + 9)y = 96 \sin 2x \cos x$

23. Solve $(D^3 + 2D^2 + D)y = x^2 + x$

24. Solve $(D^2 - D - 2)y = 2 \log x + \frac{1}{x} + \frac{1}{x^2}$

25. Solve $\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = \sin(e^x)$

26. Solve $\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x$

27. Solve $(D^2 - 4)y = x \sin hx$

Answer: $\left[y = c_1 e^{2x} + c_2 e^{-2x} - \frac{x}{3} \sin hx - \frac{2}{9} \cos hx \right]$

28. Solve $(D^2 - 1)y = x^2 \sin 3x$

Answer: $\left[y = c_1 e^x + c_2 e^{-x} - \frac{1}{10} \left\{ x^2 \sin 3x + \frac{6}{5} x \cos x - \frac{13}{25} \sin 3x \right\} \right]$

29. Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$

Answer: $\left[y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x - \frac{1}{48} (x^4 - 9x^2) \cos x + \frac{x^3}{12} \sin x \right]$

30. Solve $(D^2 + 1)y = x^2 \sin 2x$

31. Solve $(D^2 - 1)y = e^x \sin 3x$

Answer: $\left[y = c_1 e^x + c_2 e^{-x} - \frac{e^x}{117} (6 \cos 3x + 9 \sin 3x) \right]$

32. Solve $([D^3 - D^2 - D + 1])y = \cos hx \sin x$

Answer: $\left[y = (c_1 + c_2 x)e^x + c_3 e^{-x} + \frac{1}{10} e^x (\cos x - 2 \sin x) - \frac{e^{-x}}{50} (3 \cos x - 4 \sin x) \right]$

33. Solve $(D^2 - 1)y = x \sin hx$

Answer: $\left[y = c_1 e^x + c_2 e^{-x} + \frac{x^2}{4} \cos hx - \frac{x}{4} \sin hx \right]$

Hint Put $\left[\sin hx = \frac{e^x - e^{-x}}{2}, \cos hx = \frac{e^x + e^{-x}}{2} \right]$

34. Solve $(D^3 - 3D^2 + 3D - 1)y = x e^x + e^x$

Answer: $\left[y = (c_1 + c_2 x + c_3 x^2)e^x + e^x \frac{x^4}{24} + e^x \frac{x^3}{6} \right]$

35. Solve $(D^2 - 2D + 1)y = \frac{3e^x}{x^2}$

Answer: $[y = (c_1 + c_2 x)e^x - 3e^x \log x]$

36. Solve $(D^2 + 3D + 2)y = e^{-2x} + e^x \cos 2x$

37. Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos 3x$

38. Solve $(D^2 - 3D + 2)y = 2e^x \sin\left(\frac{x}{2}\right)$

39. Solve $(D^3 - 7D - 6)y = e^{2x}(x + 1)$

40. Solve $(D^3 - 2D + 4)y = 3x^2 - 5x + 2$

Answer: $\left[y = c_1 e^{-2x} + e^x(c_2 \cos x + c_3 \sin x) + \frac{1}{4}[3x^2 - 2x + 1] \right]$

Hint: $D^3 - 2D + 4 = D^3 + 2D^2 - 2D^2 - 4D + 2D + 4 = 0 = (D + 2)(D^2 - 2D + 2) = 0$
 $= D = -2, 1 \pm i$

41. Solve $\frac{d^3y}{dt^3} + \frac{dy}{dt} = \cos t + t^3 + 3$

Answer: $\left[y = c_1 + c_2 \cos t + c_3 \sin t + \frac{1}{2}[-t \cos t + \sin t] + \frac{t^3}{3} + t \right]$

42. Solve $(D^2 + 4D + 4)y = x^2$ Answer: $[y = (c_1 + c_2 x)e^{-2x} + \frac{1}{4}[x^2 - 2x + \frac{3}{2}]]$

43. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = x - 2x^2$

Answer: $\left[y = e^{-x}(c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x) + \frac{1}{3}\left(-\frac{10}{9} + \frac{11}{3}x - 2x^2\right) \right]$

44. Solve $(D^3 - D^2 + 6D)y = x^2 + \sin x$

Answer: $\left[y = c_1 + c_2 e^{-2x} + c_3 e^{-3x} - \frac{1}{6}\left(\frac{x^3}{3} - \frac{x^2}{6} + \frac{7x}{18}\right) + \frac{1}{50}(\sin x + 7 \cos x) \right]$

45. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x \cos\left(\frac{x}{2}\right)$ [Answer : $\therefore y = c_1 e^x + c_2 e^{2x} + \frac{8}{5}e^x \left[-2 \sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)\right]$]