



①	vertex	degree
	a	4
	b	7
	c	4
	d	4
	e	1
	f	4

② No. we cannot have a graph of three vertices with 1, 3, 5 degrees as this contradicts Handshake Lemma.
Sum of degrees of vertices = $2 \times \text{no. of edges}$
 $1 + 3 + 5 = 2 \times \text{no. of edges}$
 $\therefore \text{no. of edges} = 4.5$
This cannot be possible.

③ 4 regular graph, no. of edges = 12
degree of every vertex in the graph = 4
No. of vertices = p
Let n be the no. of vertices in the graph
using Handshake Lemma,
 $\sum \text{deg}(v) = 2e$
 $n \times 4 = 2 \times 12$
 $n = \frac{2 \times 12}{4}$
 $n = 6$
Total number of vertices in the graph = 6

4

(a)

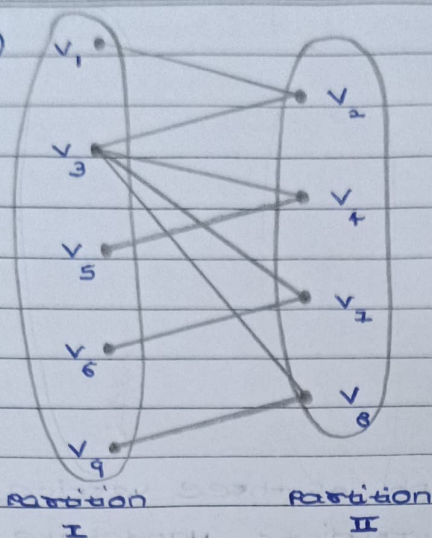
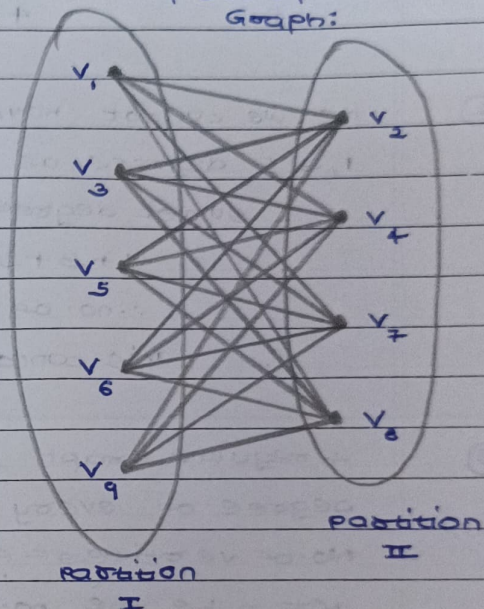


Figure (a) is Bipartite.

12 more edges are required to make it a complete bipartite graph.

Complete Bipartite Graph:



(b)

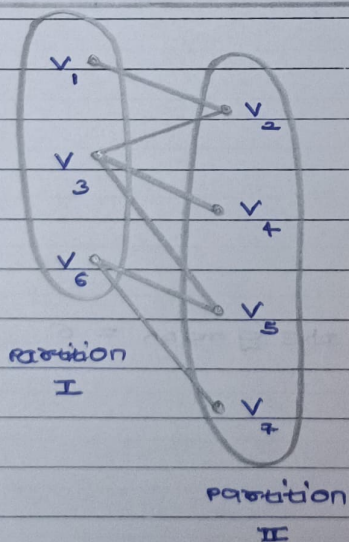
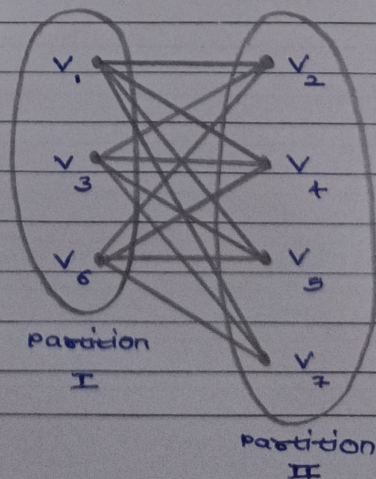


Figure (b) is Bipartite.

6 more edges are required to make it a complete bipartite graph.



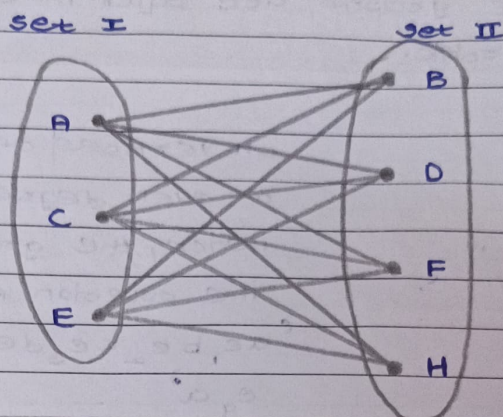


5)

$K_{3,4}$

No. of vertices in set I = 3 (Suppose A, C, E)

No. of vertices in set II = 4 (Suppose B, D, F, H)



6)

a)

Graph 1

Graph 2

• 8 vertices

• 8 vertices

• 10 edges

• 10 edges

• 4 (2 degree)

• 4 (2 degree)

vertices

vertices

• 4 (3 degree)

• 4 (3 degree)

vertices

vertices

G_1	\rightarrow	G_2
1	\rightarrow	a
2	\rightarrow	c
3	\rightarrow	b
4	\rightarrow	d
5	\rightarrow	e
6	\rightarrow	g
7	\rightarrow	f
8	\rightarrow	h

There exists a bijective function between Graph 1 and Graph 2. No. of vertices, no. of edges, degree of vertex of both the graphs are equal. Therefore, both the graphs are isomorphic.

b)

Graph 1

Graph 2

• 5 vertices

• 5 vertices

• 8 edges

• 8 edges

• 4 (3 degree)

• 4 (3 degree)

vertices

vertices

• 1 (4 degree)

• 1 (4 degree)

vertex

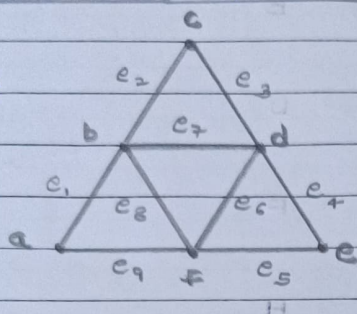
vertex

G_1	\rightarrow	G_2
1	\rightarrow	b
2	\rightarrow	c
3	\rightarrow	d
4	\rightarrow	e
5	\rightarrow	a

There exists a bijective function between Graph 1 and Graph 2. No. of vertices, no. of edges, degree of vertex of both the graphs are equal. Therefore, both the graphs are isomorphic.

7)

a)



All vertices of the graph are of even degree.

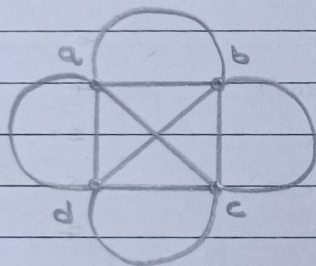
Hence, the graph is Eulerian.

The Eulerian trail is -

$a \rightarrow e_1 \rightarrow b \rightarrow e_2 \rightarrow c \rightarrow e_3 \rightarrow d \rightarrow e_4 \rightarrow e \rightarrow e_5 \rightarrow f \rightarrow e_6 \rightarrow d \rightarrow e_7 \rightarrow b \rightarrow e_8 \rightarrow a \rightarrow e_9 \rightarrow a$

All edges are covered, not repeated and the Eulerian graph (trail) starts and ends with the same vertex.

b)



4 vertices of the graph are of odd degree.

Hence, the graph is not Eulerian.