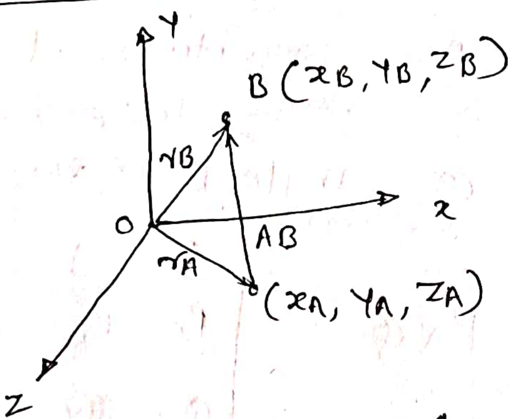
* Forces in Space

* Basics of Vectors :-



ⓐ Position vector of pt. A (x_A, y_A, z_A) w.r.t O

$$r_A = OA = x_A i + y_A j + z_A k$$

Position vector of pt. B (x_B, y_B, z_B) w.r.t O.

$$r_B = OB = x_B i + y_B j + z_B k$$

ⓑ Find Vector AB using triangle law of vector addition.

$$r_A + AB = r_B$$

$$\therefore AB = r_B - r_A$$

$$= (x_B - x_A) i + (y_B - y_A) j + (z_B - z_A) k.$$

ⓒ ∴ magnitude of Vector AB

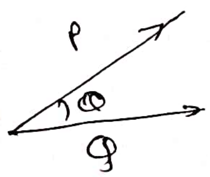
$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

ⓓ unit vector along AB $(\bar{e}_{AB})$

$$e = \frac{\text{vector AB}}{\text{magnitude of AB}} = \frac{(x_B - x_A) i + (y_B - y_A) j + (z_B - z_A) k}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

ⓔ Dot product

consider two vectors



$$\bar{P} = P_x i + P_y j + P_z k.$$

$$\bar{Q} = Q_x i + Q_y j + Q_z k.$$

$$P \cdot Q = \bar{P} \bar{Q} \cos\theta$$

where $\bar{P}$ = magni. of $\bar{P}$

$$= \sqrt{P_x^2 + P_y^2 + P_z^2}$$

θ - is angle betn vectors P & Q.

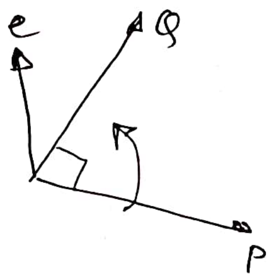
Also. $P \cdot Q = P_x Q_x + P_y Q_y + P_z Q_z$

& $\bar{Q}$ = magnitude of $\bar{Q}$

$$= \sqrt{Q_x^2 + Q_y^2 + Q_z^2}$$

(f) Cross product

$$P \times Q = \bar{P}\bar{Q} \sin\theta \cdot e$$



$\bar{P}$ - magnitude of P

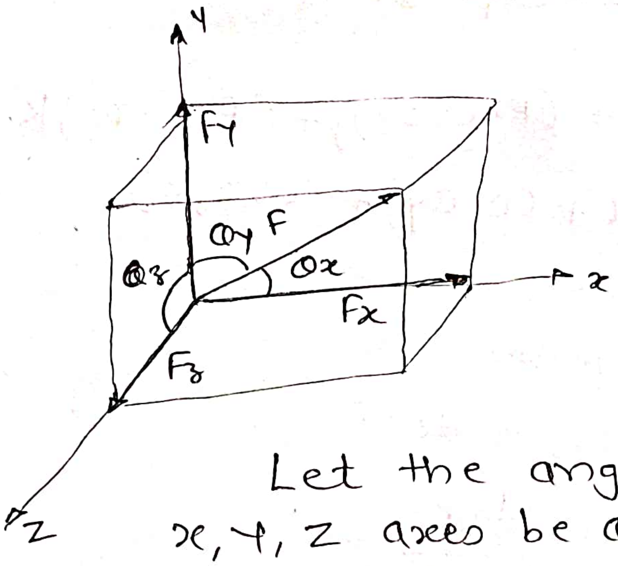
$\bar{Q}$ - magnitude of Q

θ - angle betn vecter P & Q.

e - unit vecter ⊥er to P & Q.

$$P \times Q = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

* Rectangular components of force in space



Let the angles made by force F with x, Y, Z axes be θ_x, θ_y & θ_z respectively. These angles are called as direction angles of the force.

The force F is resolved

$$F_x = F\cos\theta_x, \quad F_y = F\cos\theta_y$$

$$\& \quad F_z = F\cos\theta_z$$

$\cos\theta_x$, $\cos\theta_y$, & $\cos\theta_z$ are called direction cosines of force F

Force vector F represented by

$$\bar{F} = F_x i + F_y j + F_z k$$

i, j, k - unit vectors along x, Y & Z

Mangnitude of F is given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

$$F = (F\cos\theta_x)^2 + (F\cos\theta_y)^2 + (F\cos\theta_z)^2$$

$$1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z$$

Use above eqn, for calculating one of the angles if other two angles are known.

Force F can be express as

$$\bar{F} = (F\cos\theta_x)i + (F\cos\theta_y)j + (F\cos\theta_z)K$$

$$\bar{F} = F(\cos\theta_x i + \cos\theta_y j + \cos\theta_z K).$$

$$\bar{F} = F\cdot\bar{e} \quad \text{(dot. prodt)}$$

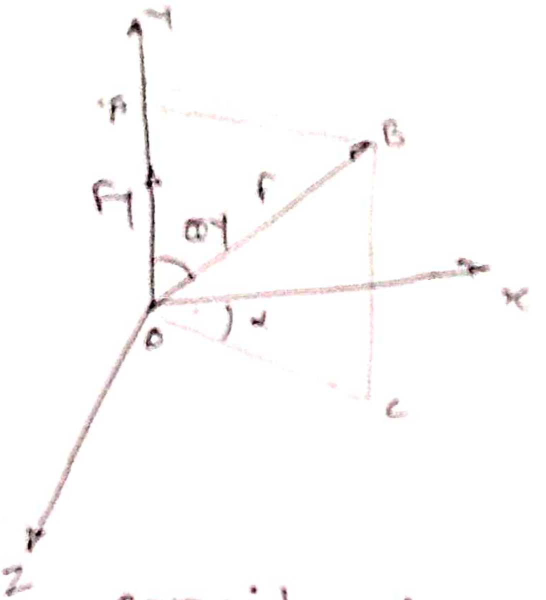
where

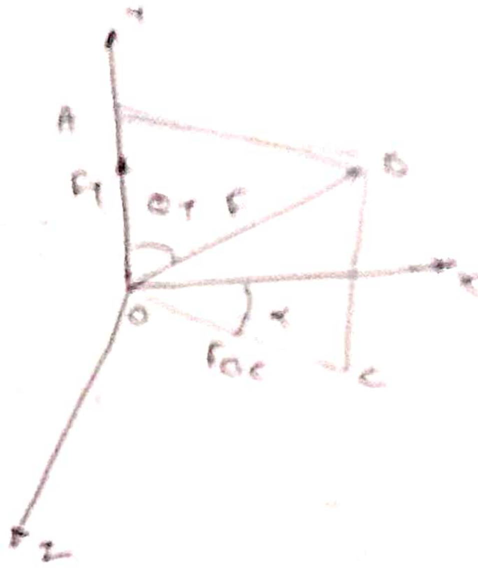
$$\bar{e} = \cos\theta_x i + \cos\theta_y j + \cos\theta_z K$$

$$\cancel{\bar{e} = \frac{\bar{F}\ \text{vector}}{F\ \text{magnitude}}}$$

$$\cancel{\bar{e} = \frac{F_x i + F_y j + F_z K}{\sqrt{F_x^2 + F_y^2 + F_z^2}}}$$

* orientation of planes :-





consider force acting at O

Y - component of force

$$F_y = F \cos\theta_y \quad -(1)$$

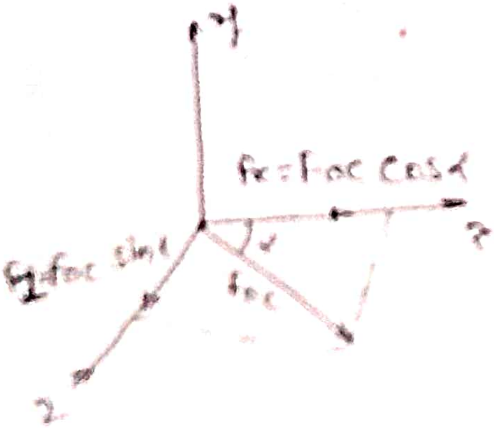
along directn OC

$$F_{OC} = F \sin\theta_y$$

Foc resolved in two rectangular components

~~force resolved in 3 rectan~~

∴ ~~force can be resolved in three~~



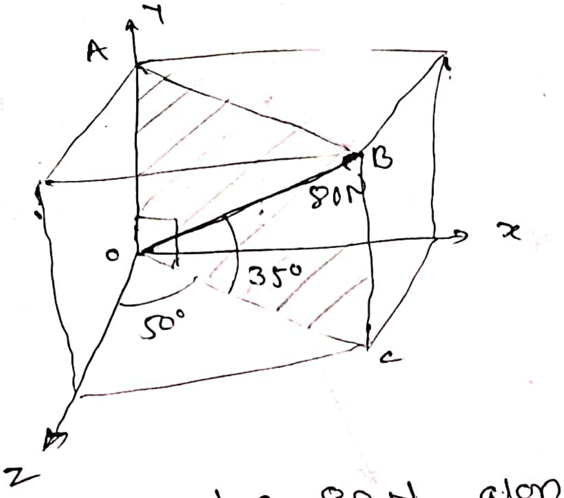
$$F_x = F_{OC} \cos\alpha = F \sin\theta_y \cdot \cos\alpha \quad -(2)$$

$$F_z = F_{OC} \cos\alpha = F \sin\theta_y \cdot \cos\alpha \quad -(3)$$

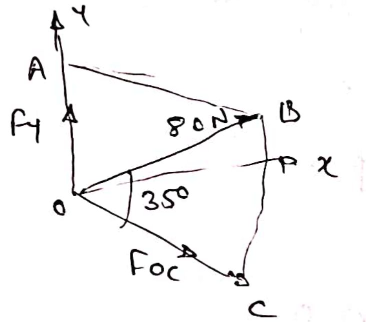
$$\cancel{F_y = F \cos\theta_y}$$

∴ force resolved in three rectangular components eqn (1), eqn (2) & eqn (3)

* Find the components of 80 N force as shown in fig.



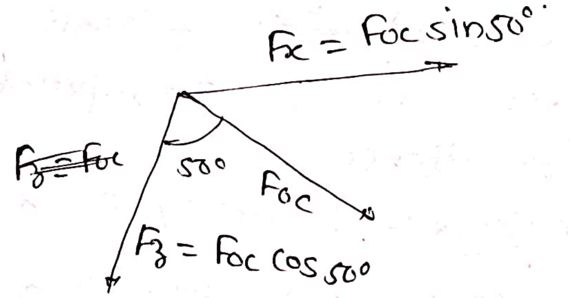
→ * resolve 80 N along OC & OA



$F_{OC} = 80 \cos 35 = 65.53 N$

$\boxed{F_y = 80 \sin 35 = 45.89 N.}$

* resolve Foc along ox & OZ



$F_x = F_{OC} \sin 50$

$= 65.35 \sin 50$

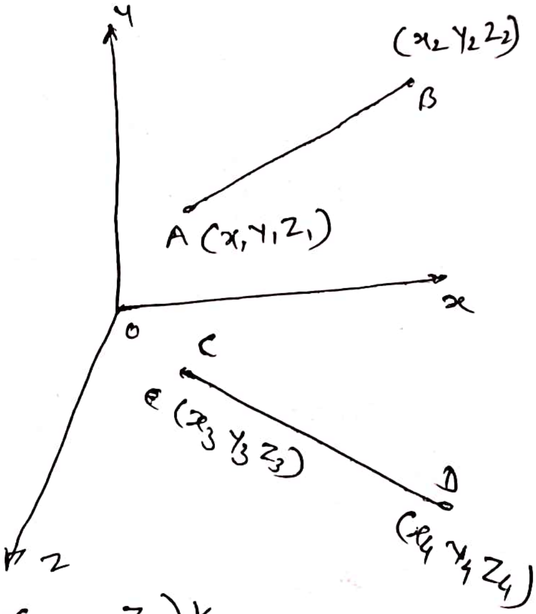
$\boxed{F_x = 50.06 N}$

$F_z = F_{OC} \cos 50°$

$= 65.35 \cos 50°$

$\boxed{F_z = 42.06 N}$

* Vector component of force along given line



① Force vector $\bar{F}$ along AB

$$\bar{F} = \bar{F}\,(\bar{e}_{AB})$$

$$\bar{F} = F_x i + F_y j + F_z k.$$

② unit vector $(\bar{e}_{CD})$ along CD

find $\bar{e}_{CD}$ ~~along~~ about line CD

$$\bar{e}_{CD} = \frac{(x_4 - x_3)\,i + (y_4 - y_3)\,j + (z_4 - z_3)\,k}{\sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2}}$$

$$\bar{e}_{CD} = x i + y j + z k.$$

③ scalar component (F_{CD})

Find the magnitude of force along given line by taking dot product $\bar{F}$ & $\bar{e}_{CD}$

$$F_{CD} = \bar{F} \cdot \bar{e}_{CD} \qquad \text{(dot. product)}$$

④ Vector component $(\bar{F}_{CD})$

find the vector component of force along given line by taking multiplication of F_{CD} & $\bar{e}_{CD}$

$$\bar{F}_{CD} = (F_{CD})\,(\bar{e}_{CD})$$

* find the direction angles for the force

$$\bar{F} = \underset{F_x}{13i} + \underset{F_y}{12j} - \underset{F_z}{6k}.$$

$$\bar{e} = \frac{\bar{F}}{F} = \frac{13i+12j-6k}{\sqrt{F_x^2+F_y^2+F_z^2}} = \frac{13i+12j-6k}{\sqrt{13^2+12^2+6^2}}$$

$$\bar{e} = 0.696i + 0.642j - 0.32k$$

we know $\bar{e} = \cos\theta_x i + \cos\theta_y j + \cos\theta_z k.$

$\cos\theta_x = 0.696$, $\cos\theta_y = 0.642$

$\boxed{\theta_x = 45.89^\circ}$ $\boxed{\theta_y = 50^\circ}$

$\cos\theta_z = -0.32$

$\boxed{\theta_z = 108.66^\circ}$

* A force F acts along AB where A (2, 1, 0) & B (3, 0, −2). The x-component of the force is 80. Find the magnitude of the force & other two components.

$A = (\underset{x_1}{2}, \underset{y_1}{1}, \underset{z_1}{0})$

→ we know

$B = (\underset{x_2}{3}, \underset{y_2}{0}, \underset{z_2}{-2})$

$$\bar{F}_{AB} = F_{AB}(\bar{e}_{AB}) \quad \text{— (1)}$$

where $\bar{F}_{AB} = F_x i + F_y j + F_z k$

$$\bar{e} = \frac{\overline{AB}}{AB} = \left[\frac{(x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)j}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}\right]$$

& F_{AB} is magnitude.

$$\bar{e} = \frac{\overline{AB}}{AB} = \left[\frac{(3-2)i - (0-1)j + (-2-0)k}{\sqrt{(3-2)^2+(0-1)^2+(-2-0)^2}}\right].$$

$$\bar{e} = \frac{i-j-2k}{\sqrt{6}}$$

(1) ⇒

$$F_x i + F_y j + F_z k = F\left[\frac{i-j-2k}{\sqrt{6}}\right]$$

But x component of $\bar{F}$ is 80 N.

$$F_x i + F_y j + F_z k = F\left(\frac{i - j - 2k}{\sqrt{6}}\right)$$

$$F_x = 80 \text{ N}$$

$$F_x = \frac{F}{\sqrt{6}}$$

$$80 = F/\sqrt{6}$$

$$\boxed{F = 195.96 \text{ N}}$$

$$F_y = -\frac{F}{\sqrt{6}} = -\frac{195.96}{\sqrt{6}} = -80 \text{ N}$$

$$F_z = -\frac{2F}{\sqrt{6}} = \frac{-2 \times 195.96}{\sqrt{6}} = -160 \text{ N.}$$

* Force of magnitude 800 N acts along AB, $A\,(\underset{x_1}{3}, \underset{y_1}{2}, \underset{z_1}{-4})$ & $B\,(\underset{x_2}{8}, \underset{y_2}{-5}, \underset{z_2}{6})$ write force vector.

→ Given $F_{AB} = 800$ N.

we know.

$$\bar{F}_{AB} = (F_{AB})(\bar{e}_{AB}) \quad -(1)$$

$$\bar{e}_{AB} = \left[\frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]$$

$$\bar{e}_{AB} = \left\{\frac{(8-3)i - (-5-2)j + [6 - (-4)]}{\sqrt{(8-3)^2 + (-5-2)^2 + [-6 - (-4)]}}\right\}$$

$$\bar{e}_{AB} = \frac{5i - 7j + 10k}{\sqrt{174}}$$

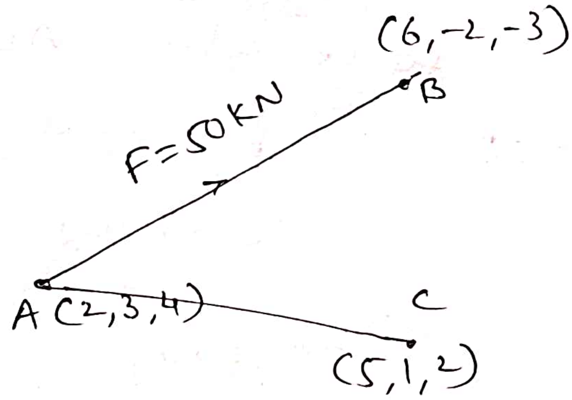
$$F_{AB} = (F_{AB})(\bar{e}_{AB})$$

$$= (800)\left(\frac{5i - 7j + 10k}{\sqrt{174}}\right)$$

$$\bar{F}_{AB} = 303.24i - 424.54j + 606.48k.$$

* Force of magnitude 50 KN is acting at pt. A(2,3,4) towards point B (6, −2, −3) m. Find the vector component of this force along line AC. Point C is (5, 1, 2) m.

→ Given F = 50 KN



(a) force vector $\bar{F}$

$$\bar{F} = (F)(\bar{e}_{AB})$$

$$= (50)\left[\frac{(6-2)i + (-2-3)j + k(-3+4)}{\sqrt{(6-2)^2 + (-2-3)^2 + (-3+4)^2}}\right]$$

$$\bar{F} = 21.08i - 26.35j - 36.89k$$

(b) unit vector $(\bar{e}_{AC})$

$$\bar{e}_{AC} = \frac{\bar{AC}}{AC} = \left[\frac{3i - 2j - 2k}{\sqrt{3^2 + 2^2 + 2^2}}\right]$$

$$\bar{e}_{AC} = 0.727i - 0.485j - 0.485k.$$

(c) scalar component (F_{AC})
magnitude of force along AC

$$F_{AC} = \bar{F} \cdot e_{AC} \quad \text{(dot prodt)}$$

$$= (21.08i - 26.35j - 36.89k) \cdot (0.727i - 0.485j - 0.485k)$$

$$F_{AC} = \cancel{49}\ 44.95 \text{ KN}.$$

(d) vector component of force along AC $(\bar{F}_{AC})$

$$\bar{F}_{AC} = (F_{AC})(\bar{e}_{AC})$$

$$= (44.95)(0.727i - 0.485j - 0.485k)$$

$$\bar{F}_{AC} = 33.44i - 22.31j - 22.31k \text{ (KN)}$$

* A force has magnitude 80 N & acts at a point P (3, 2, −1) & makes angles 60° & 45° with x & y axes. The z component of the force is positive. Find the component of the force along AB where A (−1, 1, 3) & B (0, 2, 5)

→ $F = 80\,N$, $\theta x = 60°$, $\theta y = 45°$.

we know

$$\cos^2\theta x + \cos^2\theta y + \cos^2\theta z = 1$$

on solving $\cos^2\theta z = 1$

$$\cos\theta z = \pm 0.5$$

z component is +ve

∴ $\cos\theta z$ is positive.

$$\cos\theta z = +0.5$$

$$\theta z = 60°$$

(a) Force vector $\bar{F}$

$\bar{F} = F \cdot e$ where $\bar{e}$ unit vecter

$$= 80(\cos\theta x\, i + \cos\theta y\, j + \cos\theta z\, k)$$

$$\bar{F} = 40i + 56.56j + 40k$$

(b) scalar component (FAB) $eAB = \dfrac{i+j+2k}{\sqrt{1+1+4}}$

$FAB = \bar{F} \cdot eAB$ (dot prodt)

$$= (40i + 56.56j + 40k) \cdot \left(\frac{i+j+2k}{\sqrt{6}}\right)$$

$$FAB = 76.08\,N.$$

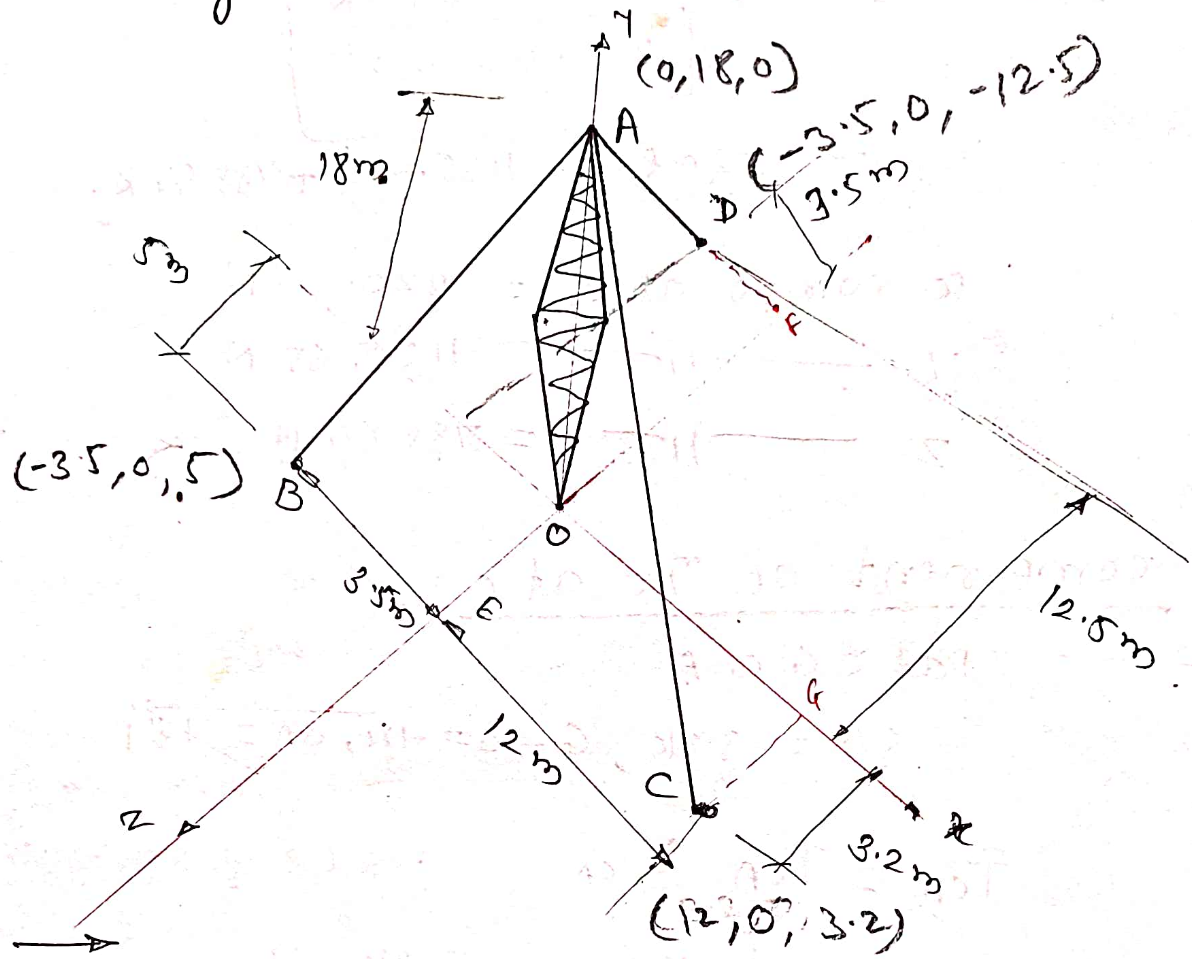
(c) Vecter component of force along AB:- ($\bar{FAC}$)

$$\overline{FAB} = (FAB)(eAB)$$

$$= (76.08)\left(\frac{i+j+2k}{\sqrt{6}}\right)$$

$$\overline{FAB} = 31.06i + 31.06j + 62.12k \text{ (N}$$

* A transmission tower is held by three wires at B, C & D. If $T_{AB} = 2000\,N$, $T_{AD} = 1400\,N$ (7) & $T_{AC} = 1600\,N$. Find compo. of forces acting at B, C & D



Force directed B − A.

Path B → E → O → A.

A (0, 18, 0) x_1, y_1, z_1

B (−3.5, 0, 5) $x_2 y_2 z_2$

B·E = 3.5 i

E·O = 5k

O·A = 18 J.

$$\bar{T}_{BA} = T_{BA} \cdot \bar{e}_{BA}$$

$$= 2000 \left[\frac{3.5i - 5k + 18J}{\sqrt{(3.5)^2 + (-5)^2 + 18^2}} \right] \qquad \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$$

$$\bar{T}_{BA} = 368.42i + 1894.74j - 526.35k$$

x compo. at B = 368.42 N.

y —— " —— = 1894.74 N.

Z —— " —— = −526.35 N.

Component at D (TAD) :- path DFOA

DF = 3.5i, $\cancel{FO = 12.5k}$,

OA = 18j

A (0, 18, 0) $x_2 y_2 z_2$

D (-3.5, 0, -12.5) $x_1 y_1 z_1$

$$\bar{T}_{DA} = T_{DA} \cdot \bar{e}$$

$$= 1400 \cdot \left[\frac{3.5i + 18j + 12.5k}{\sqrt{3.5^2 + 18^2 + 12.5^2}} \right]$$

$$\bar{T}_{DA} = 220.82\,i + 1135.65\,j + 788.64\,k.$$

x compo. at D = 220.82 N

y —"— = 1135.65 N

z —"— = 788.64 N.

Component of TAC at C :-

Pat CGOA

CG = -3.2k, G → O = -12i, OA = 18j

A (0, 18, 0) $x_2 y_2 z_2$

C (12, 0, 3.2) $x_1 y_1 z_1$

$$\bar{T}_{CA} = T_{CA} \cdot \bar{e}_{CA}$$

$$= 1600 \left[\frac{-12i + 18j - 3.2k}{\sqrt{(-3.2)^2 + 18^2 + (-12)^2}} \right]$$

$$T_{CA} = -877.91\,i + 1316.87\,j - 234.11\,k.$$

x. compo. at D = -877.91 N.

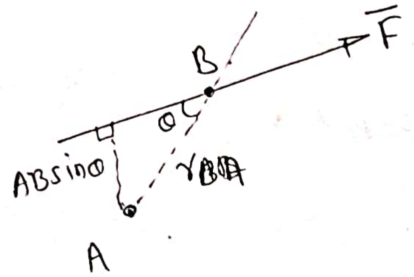
y —"— = 1316 N.

z —"— = -234.11 N.

* **moment vector** - (moment of force about point) (8)

moment :- is defined as cross product of Position vector ($\bar{r}$) & Force vector ($\bar{F}$)

$$\bar{M} = \bar{r} \times \bar{F}$$



- consider a force in space & B is any point on the line of action of force F
- consider pt. A anywhere in space.

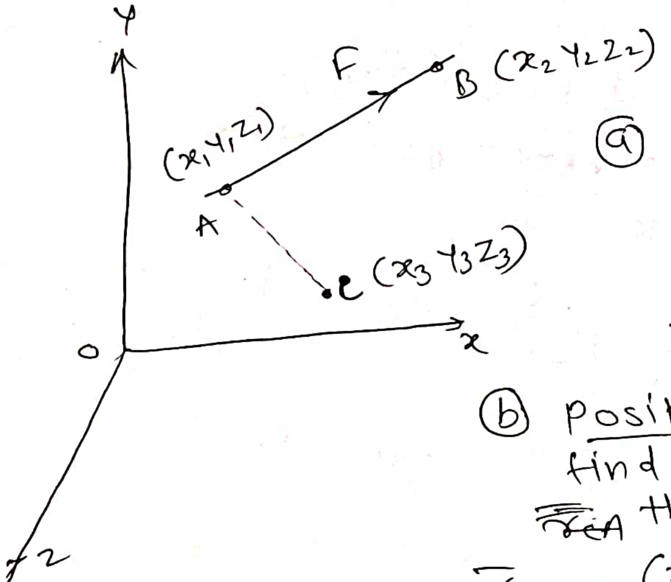
θ - angle betn $\bar{F}$ & $\bar{r}_{BA}$ (position vector)

$$M_A^F = \bar{F} \times AB\sin\theta$$
$$= \bar{F}\sin\theta \times AB = F \cdot \sin\theta \times r_{BA} \text{ (cross prodt)}$$

~~$M_A =$~~

~~$M_A = \bar{F} \times \bar{r}_{BA}$~~

$$\bar{M}_A = \bar{r}_{BA} \times \bar{F} = \begin{vmatrix} i & j & k \\ b_x - a_x & b_y - a_y & b_z - a_z \\ F_x & F_y & F_z \end{vmatrix}$$



(a) **Force vector ($\bar{F}$)**

Find force $\bar{F}$ vector.

$$\bar{F} = (F)(\bar{e}_{AB})$$
$$\bar{F} = F_x i + F_y j + F_z k$$

(b) **position vector ($\bar{r}_{CA}$)**

find $\bar{r}_{CA}$ from C to any pt. on the line of action force (A or B)

$$\bar{r}_{CA} = (x_1 - x_3)i + (y_1 - y_3)j + k(z_1 - z_3)k$$
$$\bar{r}_{CA} = xi + yj + zk.$$

(c) **moment vector ($\bar{M}_C$)**

$$\bar{M}_C = \bar{r}_{CA} \times \bar{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

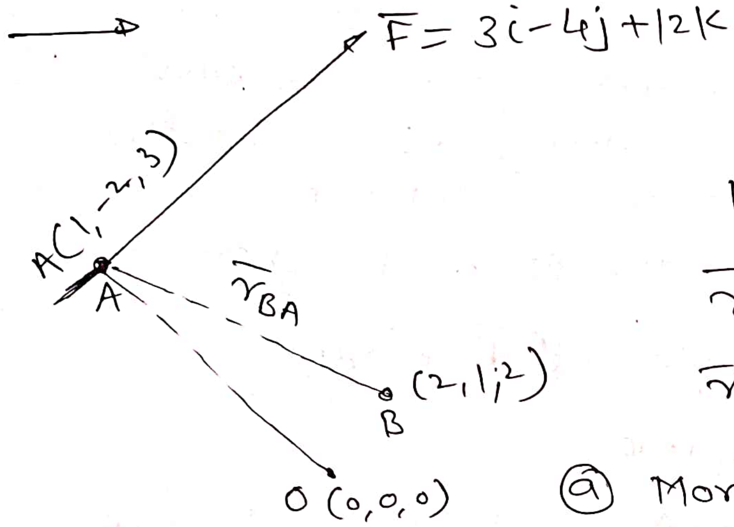
$$\bar{M}_C = M_x i + M_y j + M_z k$$

M_x, M_y & M_z are component of $\bar{M}_C$ along x, y & z respectively.

* A Force $\bar{F} = (3i - 4j + 12k)$ N acts at a point A whose co-ordinates are $(1, -2, 3)$ m. Find

(a) moment of force about origin.

(b) moment of force about pt. B $(2, 1, 2)$ m.



We know

$$M_O = \bar{r}_{OA} \times F.$$

$$\bar{r}_{OA} = (1-0)i + (-2-0)j + (3-0)k$$

$$\bar{r}_{OA} = i - 2j + 3k$$

(a) Moment vector $(\bar{M}_O)$

$$\bar{M}_O = \bar{r}_{OA} \times F = \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 3 & -4 & 12 \end{vmatrix}$$

$$\boxed{\bar{M}_O = -12i - 3j + 2k \ \text{N-m.}}$$

(b) Moment of $\bar{F}$ about pt. B $(\bar{M}_B)$

$$\bar{r}_{BA} = (1-2)i + (-2-1)j + (3-2)k$$

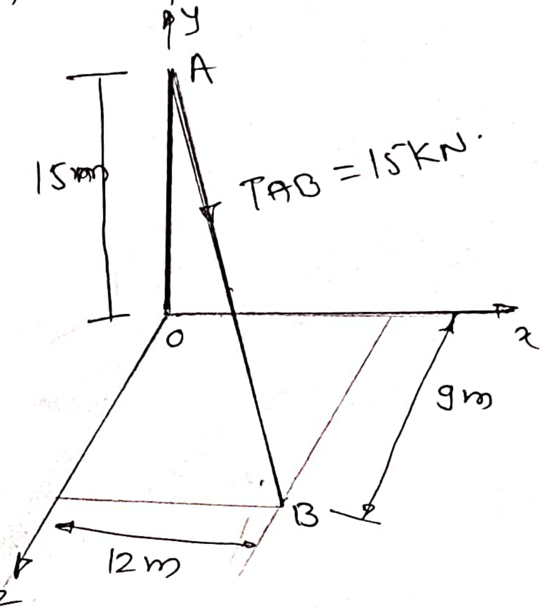
$$\bar{r}_{BA} = -i - 3j + k$$

$$\bar{M}_B = \bar{r}_{BA} \times F = \begin{vmatrix} i & j & k \\ -1 & -3 & 1 \\ 3 & -4 & 12 \end{vmatrix}$$

$$\boxed{\bar{M}_B = -32i + 15j + 13k}$$

* Tension T of magnitude 15KN is applied to the cable AB attached to the top A of the rigid mass & secured to the ground at B. Determine moment of Tension T

(a) about origin.



(1) coordinates

O (0,0,0), A (0, 15, 0), B (12, 0, 9)

(2) Force Vector ($\overline{T}_{AB}$)

$$\overline{T}_{AB} = (T_{AB})(\overline{e}_{AB})$$

$$= 15 \left[\frac{(12-0)i + (0-15)j + (5-0)k}{\sqrt{12^2 + 15^2 + 5^2}} \right]$$

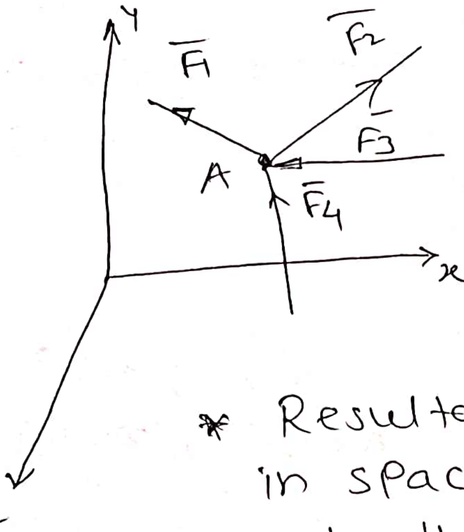
$$\overline{T}_{AB} = 9.07i - 11.33j + 6.08k \ (KN)$$

(3) Moment Vector @ origion ($\overline{M}_O$)

$$\overline{M}_O = \overline{r}_{OA} \times \overline{T}_{AB} = \begin{vmatrix} i & j & k \\ 0 & 15 & 0 \\ 9.07 & -11.33 & 6.08 \end{vmatrix}$$

$$\overline{M}_O = 102.2i - 0j - 136.05k.$$

* Resultant of concurrent force system :-



* Resultant of concurrent force system in space is single force $\bar{R}$ & it acts through pt. of concurrency.

* $\bar{F_1}, \bar{F_2}, \bar{F_3}, \bar{F_4}$ are the force vector's passing through point A.

Resultant Force vecter $\bar{R}$ = summation of all force vectors

$$\bar{R} = \bar{F_1} + \bar{F_2} + \bar{F_3} + \bar{F_4}$$

$$\bar{R} = (\Sigma F_x)i + (\Sigma F_y)j + (\Sigma F_z)k.$$

$$\bar{R} = R_x i + R_y j + R_z k.$$

Magnitude of resultant

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Direction's

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right), \quad \theta_y = \cos^{-1}(R_y/R)$$

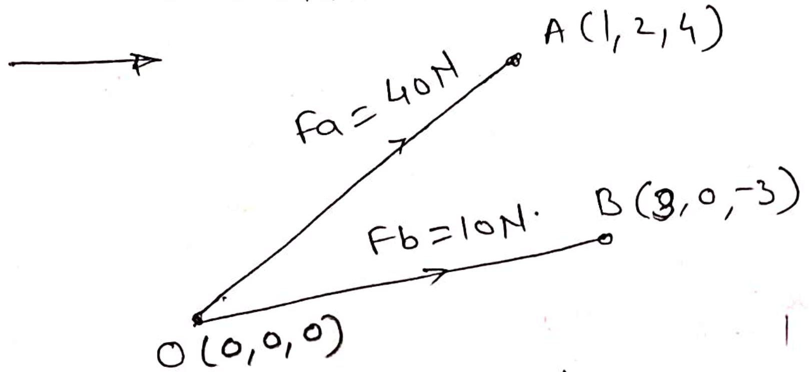
$$\theta_z = \cos^{-1}(R_z/R)$$

* if Resultant acting along x-axis then $R_x = \Sigma F_x = R$ & $R_y = \Sigma F_y = 0$, $R_z = \Sigma F_z = 0$

* if Resultant acting along y-axis $R_y = \Sigma F_y = R$ & $R_x = \Sigma F_x = 0$, $R_z = \Sigma F_z = 0$

* if resultant acting along z-axis $R_z = \Sigma F_z = R$ & $R_x = \Sigma F_x = 0$, $R_z = \Sigma F_z = 0$

* Find the magnitude & direction of their resultant.



I) Force Vecters :-

O (0, 0, 0) x_1, y_1, z_1

A (1, 2, 4) $x_2\ y_2\ z_2$

$$\bar{F}a = (Fa)(\bar{e}_{OA})$$

$$= 40 \left[\frac{i + 2j + 4k}{\sqrt{1^2 + 2^2 + 4^2}} \right] = 8.73i + 17.46j + 34.92k. \quad -①$$

$$\bar{F}b = (Fb)(\bar{e}_{OB})$$

O (0, 0, 0) x_1, y_1, z_1

B (3, 0, -3) $x_2\ y_2\ z_2$

$$= 10 \left[\frac{3i + 0j - 3k}{\sqrt{3^2 + 0^2 + 3^2}} \right]$$

$$\bar{F}b = 7.08i + 0j - 7.08k. \quad - ②$$

II) Resultant Vecter $\bar{R}$

$$\bar{R} = \bar{F}a + \bar{F}b$$

$$= (8.73i + 17.46j + 34.92k) + (7.08i - 7.08k)$$

$$= (8.73 + 7.08)i + (17.46 + 0)j + (17.46 - 7.08)k$$

$$\bar{R} = \underset{Rx}{15.81i} + \underset{Ry}{17.46j} + \underset{Rz}{10.38k}$$

III) Maggnitude of resultant

$$R = \sqrt{Rx^2 + Ry + Rz^2} = \sqrt{(15.81)^2 + (17.46)^2 + (10.38)^2}$$

$$R = 25.74\ N.$$

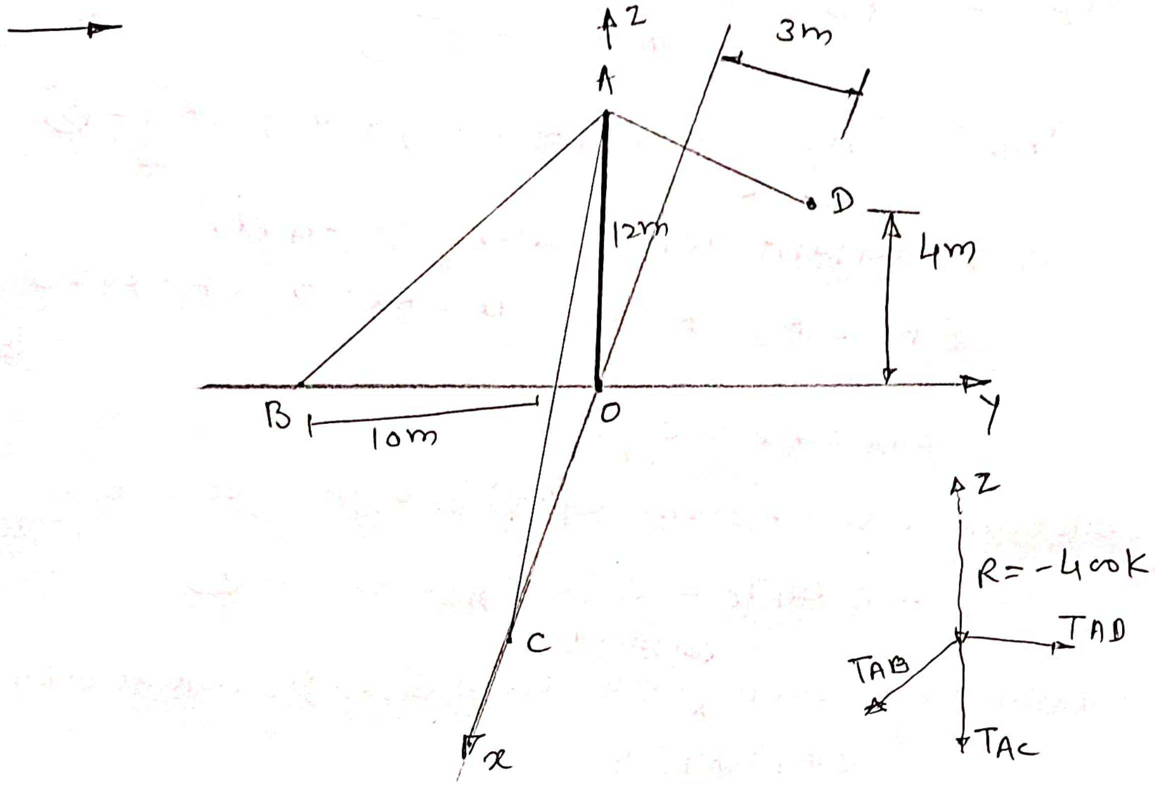
IV) Direction Rx, Ry & Rz

$$\theta x = \cos^{-1}\left(\frac{Rx}{R}\right) = \cos^{-1}\left(\frac{15.81}{25.74}\right) = 52.10^\circ$$

$$\theta y = \cos^{-1}(Ry/R) = \cos^{-1}\left(\frac{17.46}{25.74}\right) = 47.28^\circ$$

$$\theta z = \cos^{-1}(Rz/R) = \cos^{-1}\left(\frac{10.38}{25.74}\right) = 66.21^\circ$$

* Three cables are connected at A as shown (11)
If resultant of three tension at A is
$R = (-400)k$ (Newton). Find the magnitude of each cable tension.



* coordinates

A (0, 0, 12) B (0, −10, 0) C (10, 0, 0) D (−4, 3, 0)

* Force vector ($\overline{T_{AB}}$)

A (0, 0, 12) $x_1 y_1 z_1$
B (0, −10, 0) $x_2 y_2 z_2$

$$\overline{T_{AB}} = T_{AB} \cdot \overline{e_{AB}}$$

$$= T_{AB} \left[\frac{-10j - 12k}{\sqrt{10^2 + 12^2}} \right]$$

$$= T_{AB} \left[-0.64j - 0.77k \right] \quad - (1)$$

* Force Vector ($\overline{T_{AC}}$)

A (0, 0, 12) $x_1 y_1 z_1$
C (10, 0, 0) $x_2 y_2 z_2$

$$\overline{T_{AC}} = \overline{T_{AC}} \cdot \overline{e_{AC}}$$

$$= T_{AC} \left[\frac{-12k + 6i}{\sqrt{12^2 + 6^2}} \right]$$

$$\overline{T_{AC}} = T_{AC} \left[0.44i - 0.894j \right] \quad - (2)$$

* Force vecter ($\overline{T_{AD}}$)

A (0, 0, 12) (x_1, y_1, z_1)
D (-4, 3, 0) $x_2 y_2 z_2$

$$\overline{T_{AD}} = T_{AD}\, \overline{e_{AD}}$$

$$\overline{T_{AD}} = T_{AD}\left[\frac{-4i+3j-12K}{\sqrt{4^2+3^2+12^2}}\right]$$

$$\overline{T_{AD}} = T_{AD}\left[-0.307i + 0.23j - 0.923K\right] \quad -(3)$$

if resultant acting along z-axis

$$\Sigma F_z = R_z = R \quad \& \quad \Sigma F_x = R_x = 0, \; \Sigma F_y = R_y = 0$$

$$\Sigma F_x = R_x = 0$$

collecting i vecter coefficient from eqn (1), (2) & (3) & equate with 0 (zero)

$$0 + 0.44 T_C - 0.307 T_{AD} = 0 \quad -(A)$$

collecting j vecter coefficient from from (1), (2) & (3) & equate with zero.

$$\Sigma F_y = R_y = 0$$

$$-0.64\, T_{AB} + 0.23\, T_{AD} = 0 \quad -(B)$$

$$\Sigma F_z = R_z = -400\, K$$

collecting K vecter coefficient term from (1), (2) (3) & equate ~~with zero~~ with -400 N.

$$-0.77\, T_{AB} - 0.894\, T_{AC} - 0.923\, T_{AD} = -400 \quad -(C)$$

solving eqn (A), (B) & (C)

we get $T_{AC} = 151.46$ N.

$T_{AD} = 220.54$ N.

$T_{AB} = 79.26$ N.