**1.** Calculate the de Broglie wavelength associated with a proton moving with a velocity equal to $\frac{1}{20}$ th of the velocity of light.

*Sol.* Velocity of proton $v = \frac{3\times10^8}{20} = 1.5 \times 10^7$ m/s

mass of the proton $m = 1.67 \times 10^{-27}$ kg.

$$\lambda = \frac{h}{mv} = \frac{6.6\times10^{-34}}{1.67\times10^{-27}\times1.5\times10^{7}} = 2.634 \times 10^{-14}\ \text{m}.$$

**2.** Calculate the de Broglie wavelength of neutron of energy 12.8 MeV. Given $h = 6.62 \times 10^{-34}$ J.sec $m = 1.67 \times 10^{-27}$ kg.

*Sol.* $m_o c^2 = \frac{1.507 \times 10^{-10}}{1.6 \times 10^{-19}} = 941.87$ MeV

Since 12.8 MeV is very small compared to rest mass energy hence relativistic consideration may be ignored

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \quad E = Ve \text{ where } V \text{ is voltage in volts}$$

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 12.8 \times 10^{6} \times 1.6 \times 10^{-19}}}$$

$= 8.0 \times 10^{-5}$ Å.

**3.** Show that the de Broglie wavelength for a material particle of rest mass $m_0$ and charge $q$ accelerated from rest through a potential difference of $V$ volts relativistically is given by

$$\lambda = \frac{h}{\sqrt{2m_0qV\left(1 + \frac{qV}{2m_0c^2}\right)}}.$$

*Sol.* Kinetic energy $E_k = Vq$

$Ek \neq \frac{1}{2}mv^2$ because $v$ is relativistic velocity and so, we cannot find momentum directly from $Ek$. Now, we use relativistic formula

$$E^2 = p^2c^2 + {m_0}^2c^4$$

$$E = Ek + m_0c^2 = Vq + m_0c^2$$

$$P^2c^2 = E^2 - {m_0}^2c^4 = (Vq + m_0c^2)^2 - {m_0}^2c^4$$

$$P^2c^2 = V^2q^2 + 2Vq\,m_0c^2$$

$$P^2 = \frac{V^2q^2}{c^2} + 2Vq\,m_0$$

$$= 2m_0\, Vq\left(1+\frac{Vq}{2m_0c^2}\right)$$

$$P = \sqrt{2m_0Vq\left(1+\frac{Vq}{2m_0c^2}\right)}$$

$\therefore$ de Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2m_0Vq\left(1+\dfrac{Vq}{2m_0c^2}\right)}}$.

**4.** **Calculate the wavelength associated with an electron accelerated to a potential difference of 1.25 keV.**

*Sol.* If $E$ is the K.E. of the electron, the de Broglie wavelength of the wave associated with the electron is

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.25\times10^{3}\times1.6\times10^{-19}}}$$

$$= 3.46 \times 10^{-11}\ \text{m}.$$

**5.** What will be the kinetic energy of an electron if its de Broglie wavelength equals the wavelength of the yellow line of sodium (5896 Å). The rest mass of electron is $m_0 = 9.1\times10^{-31}$ kg and $h = 6.63 \times 10^{-34}$ J.sec.

*Sol.* de Broglie wavelength $\lambda = \dfrac{h}{mv}$ or $v = \dfrac{h}{m\lambda}$.

In the absence of relativistic consideration $m = m_0$

$$\lambda = \frac{h}{m_0v}, \text{ Kinetic Energy } K = \frac{1}{2}m_0v^2$$

$$K = \frac{1}{2}m_0\left(\frac{h}{m_0\lambda}\right)^2 = \frac{h^2}{2m_0\lambda^2}.$$

Putting the values of $h$, $m_0$ and $\lambda$

$$K = \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times(5896\times10^{-10})^2}$$

$$= \frac{6.95\times10^{-25}}{1.6\times10^{-19}} = 4.34 \times 10^{-6}\ \text{eV}.$$

**6.** A particle of rest mass $m_0$ has a kinetic energy $K$. Show that its de Broglie wavelength is given by

$$\lambda = \frac{hc}{\sqrt{K(K + 2m_0c^2)}}.$$

Hence, calculate the wavelength of an electron of kinetic energy 1 MeV. What will be the value of $\lambda$ if $K << m_0c^2$?

*Sol.* According to de Broglie's concept of matter wave, the wavelength

$$\lambda = \frac{h}{mv} \qquad m = \frac{m_0}{\left(1 - v^2/c^2\right)^{1/2}}$$

$$\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = \frac{m_0}{m}.$$

**7.** What is de Broglie wavelengths of any electron which has been accelerated from rest through a potential difference of 100V.

*Sol.*
$$\lambda = \frac{12.25}{\sqrt{V}} \text{ Å}$$

$$V = 100 \text{ volts}$$

$$\therefore \qquad \lambda = \frac{12.25}{\sqrt{100}} = 1.225 \text{ Å}.$$

**8.** Can a photon and an electron of the same momentum have the same wavelength? Compare their wavelengths of the two have the same energy.

*Sol.* Using de Broglie concept of matter wave, momentum of the electron may be written as $p_e = \dfrac{h}{\lambda_e}$ and momentum of photon as $p_{ph} = \dfrac{h\nu}{c} = \dfrac{h}{\lambda_p}$.

So, if the electron and photon have same momentum, then we have $\lambda_e = \lambda_p$.
Thus, the photon and electron of the same momentum have the same wavelength.
The de Broglie wavelength of electron is given by

$$\lambda_e = \frac{h}{mv} \quad \text{and} \quad \frac{1}{2}mv^2 = E \quad \text{or} \quad mv = \sqrt{2\,mE}$$

where $E$ is the energy of the electron

So, $$\lambda_e = \frac{h}{\sqrt{2\,mE}}$$

The de Broglie wavelength of photon is given by

$$\lambda_{ph} = \frac{h}{p} \qquad \text{but } E = h\nu = \frac{hc}{\lambda} = pc$$

$$\lambda_{ph} = \frac{hc}{E}$$

$$1 - \frac{m_0^2}{m^2} = \frac{v^2}{c^2}$$

or $$\frac{v^2}{c^2} = \frac{m^2 - m_0^2}{m^2}$$

or $$m^2 v^2 = (m^2 - m_0^2)\, c^2$$

or $$mv = c\,(m^2 - m_0^2)^{1/2}$$

Substituting this value of $mv$ in equation of wavelength we get

$$\lambda = \frac{h}{c\sqrt{m^2 - m_0^2}}$$

or $$\lambda = \frac{hc}{c^2\sqrt{m^2 - m_0^2}}$$

$$\begin{aligned} c^2(m^2 - m_0^2)^{1/2} &= [c^4(m - m_0)\,(m + m_0)]^{1/2} \\ &= [c^2(m - m_0)\,\{(m + m_0)\,c^2\}]^{1/2} \\ &= [(m - m_0)\,c^2\,\{(m - m_0)\,c^2 + 2m_0\,c^2\}]^{1/2} \end{aligned}$$

or $c^2\,(m^2 - m_0^2)^{1/2} = [K(K + 2m_0\,c^2)]^{1/2}$.

Therefore $$\lambda = \frac{hc}{[K(K + 2m_0\,c^2)]^{1/2}}.$$

For an electron $m_0c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$ joule

$$= \frac{81.9 \times 10^{-15}}{1.6 \times 10^{-19}}\ \text{eV} = 0.51 \times 10^6\ \text{eV} = 0.51\ \text{MeV}.$$

For $K = 1$ MeV, $\lambda = \dfrac{hc}{\sqrt{1(1 + (2 \times 0.51)}} = \dfrac{hc}{\sqrt{2.02}}$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{\sqrt{2.02} \times 1.6 \times 10^{-19} \times 10^6}\ \text{m}$$

$$= 8.78 \times 10^{-13}\ \text{m} = 8.78 \times 10^{-3}\ \text{Å}.$$

If $K << m_0c^2$ then $K + 2m_0c^2 = 2m_0c^2$

$$\lambda = \frac{hc}{\sqrt{2m_0Kc^2}} = \frac{h}{\sqrt{2m_0K}}.$$

Now $\frac{\lambda_{ph}}{\lambda_e} = \frac{hc}{E} \cdot \frac{\sqrt{2\,mE}}{h} = c\sqrt{\frac{2\,m}{E}} = \sqrt{\left(\frac{2\,mc^2}{E}\right)}$.

**9.** Find the energy of the neutron in units of electron volt whose de Broglie wavelength is 1 Å.

*Sol.* Given mass of the neutron = $1.674 \times 10^{-27}$ kg
Plancks's constant $h = 6.60 \times 10^{-34}$ Joule/sec

We know that $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2\,mE}}$

or $$E = \frac{h^2}{2\,m\,\lambda^2}$$

where $m = 1.674 \times 10^{-27}$ kg
$\lambda = 1$ Å $= 10^{-10}$ m
$h = 6.60 \times 10^{-34}$ J.s.

$$E = \frac{(6.60 \times 10^{-34})^2}{2 \times 1.674 \times 10^{-27} \times (10^{-10})^2}$$

$= 13.01 \times 10^{-21}$ Joules

$$= \frac{13.01 \times 10^{-21}}{1.6 \times 10^{-19}} = 8.13 \times 10^{-2} \text{ eV}.$$

**10.** What would be the wavelength of quantum of radiant energy emitted, if an electron transmitted into radiation and converted into one quantum.

*Sol.* When the energy of an electron is transmitted into radiation, we use the following relations to get the value of $\lambda$

$$E = mc^2 \quad \text{and} \quad E = h\nu = \frac{hc}{\lambda}$$

So $$\lambda = \frac{h}{mc} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 0.0242 \times 10^{-10} \text{ m}$$

$= 0.0242$ Å.

**11.** Calculate the smallest possible uncertainty in the position of an electron moving with velocity $3 \times 10^7$ m/s.

*Sol.* $(\Delta x)_{min}\,(\Delta p)_{max} = \dfrac{h}{2\pi}$

$$(\Delta p)_{max} = p \text{ (momentum of the electron)}$$

$$= mv = \frac{m_0 v}{\sqrt{(1 - v^2/c^2)}}$$

$$(\Delta x)_{min} = \frac{h\sqrt{1 - v^2/c^2}}{2\pi\, m_0 v}$$

$\dfrac{h}{2\pi} = 1.05 \times 10^{-34}$ joule/sec, $v = 3 \times 10^7$ m/s · $m_0 = 9 \times 10^{-31}$ kg, $c = 3 \times 10^8$ m/s

$$(\Delta x)_{min} = \frac{1.05 \times 10^{-34}\sqrt{\left(1 - \dfrac{(3 \times 10^7)^2}{(3 \times 10^8)^2}\right)}}{9 \times 10^{-31} \times 3 \times 10^7}$$

$$= 3.8 \times 10^{-12} \text{ m} = 0.038 \text{ Å}.$$

**12.** An electron is confined to a box of length $10^{-8}$ meter; calculate the minimum uncertainty in its velocity. Given $m = 9 \times 10^{-31}$ kg; $\hbar = 1.05 \times 10^{-34}$ joule second.

*Sol.*

$$(\Delta x)_{max} = 10^{-8} \text{ meter}$$

$$(\Delta p)_{min} = \frac{\hbar}{(\Delta x)_{max}} = \frac{1.05 \times 10^{-34}}{10^{-8}} \text{ kg-m/s}$$

$$= 1.05 \times 10^{-26} \text{ kg-m/s}$$

$$(\Delta p)_{min} = m\,(\Delta v)_{min} = 1.05 \times 10^{-26}$$

$$(\Delta v)_{min} = \frac{1.05 \times 10^{-26}}{m} = \frac{1.05 \times 10^{-26}}{9 \times 10^{-31}} = 1.17 \times 10^4 \text{ m/s}.$$

**13.** Find the uncertainty in the momentum of a particle when its position is determined within 0.01 cm. Find also the uncertainty in the velocity of an electron and an α-particle respectively when they are located within $5 \times 10^{-8}$ cm.

*Sol.* According to uncertainty principle

$$(\Delta x)(\Delta p) = \frac{h}{2\pi}$$

or

$$\Delta p = \frac{h}{2\pi\,\Delta x}$$

$$\frac{h}{2\pi} = 1.05 \times 10^{-34}\ \text{J/sec}$$

$$\Delta x = 0.01 \times 10^{-2}\ \text{meter}$$

$$\Delta p = \frac{1.05\times10^{-34}}{0.01\times10^{-2}} = 1.05 \times 10^{-30}\ \text{kg m/s}$$

$$\Delta p = m\,\Delta v$$

∴
$$\Delta v = \frac{h}{2\pi m\,\Delta x}.$$

Uncertainty in the velocity of electron

$$m = 9 \times 10^{-31}\ \text{kg}$$

$$\Delta x = 5 \times 10^{-10}\ \text{m}$$

$$\Delta v = \frac{1.05\times10^{-34}}{9\times10^{-31}\times5\times10^{-10}}$$

$$= 2.33 \times 10^{5}\ \text{m/sec.}$$

Uncertainty in the velocity of α-particle, mass of α-particle = 4 × mass of proton

$$= 4 \times 1.67 \times 10^{-27} = 6.68 \times 10^{-27}\ \text{kg}$$

$$\Delta x = 5 \times 10^{-10}\ \text{m}$$

$$\Delta v = \frac{1.05\times10^{-34}}{6.68\times10^{-27}\times5\times10^{-10}}$$

$$= 31.4\ \text{m/sec.}$$

**14.** An electron has a speed $4 \times 10^5$ m/s within the accuracy of 0.01 per cent. Calculate the uncertainty in the position of the electron. Given $h = 6.625 \times 10^{-34}$ J.S. Mass of electron $= 9.11 \times 10^{-31}$ kg.

*Sol.* Uncertainty in velocity $= \dfrac{0.01}{100} \times 4 \times 10^5 = 40$ m/s

$$(\Delta x)(\Delta p) = \frac{h}{2\pi}$$

$$(\Delta x) = \frac{h}{2\pi(\Delta p)} = \frac{1.055 \times 10^{-34}}{m\,\Delta v}$$

$$= \frac{1.055 \times 10^{-34}}{9.11 \times 10^{-31} \times 40} = 2.895 \times 10^{-6} \text{ m.}$$

**15.** An excited atom has an average lifetime of $10^{-8}$ sec. That is during this period it emits a photon and returns to the ground state. What is the minimum uncertainty in the frequency of this photon?

*Sol.*

$$\Delta E\ \Delta t \geq \frac{h}{2\pi}$$

$$E = h\nu$$

or

$$\Delta E = h\,\Delta\nu$$

$$h\,\Delta\nu\ \Delta t \geq \frac{h}{2\pi}$$

or

$$\Delta\nu \geq \frac{1}{2\pi\,\Delta t}$$

$$\Delta t = 10^{-8} \text{ sec}$$

$$\Delta\nu \geq \frac{1}{2 \times 3.14 \times 10^{-8}} = 15.92 \times 10^6 \text{ sec}^{-1}.$$

**16.** If an excited state of hydrogen atom has a lifetime of $2.5 \times 10^{-14}$ sec. What is the minimum error with which the energy of this state can be measured?

*Sol.* The uncertainty in the energy of the photon is equal to the uncertainty in the energy of the excited state of the atom due to energy conservation.

According to uncertainty principle

$$\Delta E \ \Delta t \geq \frac{h}{2\pi}$$

$$\Delta E \geq \frac{h}{2\pi \ \Delta t} = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times (2.5 \times 10^{-14})}$$

$$= 4.22 \times 10^{-21} \text{ J}$$

$$= \frac{4.22 \times 10^{-21}}{1.6 \times 10^{-19}} = 0.026 \text{ eV.}$$

**17.** Using the uncertainty relation $\Delta E \cdot \Delta t = \frac{h}{2\pi}$, calculate the time required for the atomic system to retain rotation energy for a line of wavelength 6000 Å and width $10^{-4}$ Å.

*Sol.* We know that $E = h\nu = \frac{hc}{\lambda}$

$$\Delta E = -\frac{hc}{\lambda^2} \Delta\lambda$$

where $\Delta\lambda$ is the width of the spectral lines. Here $\Delta\lambda = 10^{-4} \times 10^{-10}$ m.

Using uncertainty relation

$\Delta E \cdot \Delta t = \frac{h}{2\pi}$, we have

$$\Delta t = \frac{h}{2\pi \ \Delta E} = \frac{h}{2\pi\left(\frac{hc}{\lambda^2}\right)\Delta\lambda} = \frac{\lambda^2}{2\pi \ c \ \Delta\lambda}$$

Putting the values we get $\Delta t = \dfrac{(6 \times 10^{-7})^2}{2 \times 3.14 \times 3 \times 10^{8} \times 10^{-14}}$

$$= 1.9 \times 10^{-8} \text{ second.}$$

**18.** A nucleon is confined to a nucleus of diameter $5 \times 10^{-4}$ m. Calculate the minimum uncertainty in the momentum of the nucleon. Also calculate the minimum kinetic energy of the nucleon.

*Sol.* We have the relation

$$(\Delta p)_{\min} (\Delta x)_{\max} = \frac{h}{2\pi}$$

$$(\Delta p)_{\min} = \frac{h}{2\pi (\Delta x)_{\max}} = \frac{6.626 \times 10^{-34}}{5 \times 10^{-14} \times 2 \times 3.14}$$

$$= 0.21 \times 10^{-20} \text{ kg. m/sec.}$$

Since $p$ can not be less than $(\Delta p)_{\min}$

$$E_{\min} = \frac{(P_{\min})^2}{2\,\text{m}}$$

Putting the values of $P_{\min} = 0.21 \times 10^{-20}$ kg. m/sec and $m = 1.675 \times 10^{-27}$ kg.

$$E_{\min} = \frac{(0.21 \times 10^{-20})^2}{2 \times 1.675 \times 10^{-27}} = 0.063 \times 10^{-13} \text{ J}$$

$$= \frac{0.063 \times 10^{-13}}{1.6 \times 10^{-19}} = 0.039 \text{ Mev.}$$

**19.** An electron is confined to a box of length $1.1 \times 10^{-8}$ m. Calculate the minimum uncertainty in its velocity. Given $m = 9.1 \times 10^{-31}$ kg and $\frac{h}{2\pi} = 1.05 \times 10^{-34}$ J.s.

*Sol.* We know from uncertainty principle that

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi} \quad \Delta p \simeq \frac{h}{2\pi\,\Delta x}$$

Let $\Delta v$ be the uncertainty in the velocity of a particle of mass $m$, so we have

$$\Delta p = m\,\Delta v \quad \text{or} \quad \Delta v = \frac{\Delta p}{m}$$

$\Delta x = 1.1 \times 10^{-8}$ m, $\frac{h}{2\pi} = 1.05 \times 10^{-34}$ J.s and $m = 9.1 \times 10^{-31}$ kg

$$\Delta v = \frac{h}{2\pi\, m\, \Delta x} = \frac{1.05 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.1 \times 10^{-8}}$$

$$= 1.06 \times 10^4 \text{ m/sec.}$$

**20.** What is the minimum uncertainty in the frequency of a photon whose life time is about $10^{-8}$ sec?

*Sol.* From uncertainty principle, we have

$$\Delta E\,\Delta t \geq \frac{h}{2\pi} \qquad \text{But } E = h\nu \qquad \text{or} \qquad \Delta E = h\Delta\nu$$

$$h\Delta\nu\,\Delta t \geq \frac{h}{2\pi} \qquad \text{or} \qquad \Delta\nu \geq \frac{1}{2\pi\,\Delta t}$$

putting $\Delta t = 10^{-8}$ sec.

$$\Delta\nu = \frac{1}{2 \times 3.14 \times 10^{-8}} = 15.92 \times 10^{6}/\text{sec}.$$

**21.** Show that $\Psi(x) = e^{icx}$ where $e$ is some finite constant is acceptable eigen-functions. Also normalise it over the region $-a \leq x \leq a$.

*Sol.* The wave function $\Psi(x)$ can be written as

$$\Psi(x) = e^{icx} = \cos cx + i \sin cx.$$

Its derivative is given by

$$\frac{d\Psi(x)}{dx} = ic\, e^{icx} = i\,c\,(\cos cx + i \sin cx)$$

$$= -c \sin cx + i\,c \cos cx.$$

The following points may be observed:

(*i*) $\sin cx$ and $\cos cx$ are periodic functions with maximum value 1 and $c$ is finite constant. Thus $\Psi(x)$ and $\frac{d}{dx}\Psi(x)$ are finite for all values of $x$.

(*ii*) The function $\Psi(x)$ and $\frac{d}{dx}\Psi(x)$ are single-valued because cos $cx$ and sin $cx$ are also continuous for all values of $x$.

Hence $\Psi(x)$ is an acceptable form of the eigen-function. To normalise, the wave function we may write $\Psi(x)$ as

$$\Psi(x) = A\,e^{icx}.$$

Now we have to determine the value of $A$ and we may write

$$\int_{-a}^{a} \psi^*(x)\,\psi(x)\,dx = 1$$

$$A^2\int_{-a}^{a} e^{-icx}\,e^{icx}\,dx = 1$$

$$A^2(x)_{-a}^{+a} = 1$$

$$A^2(2a) = 1 \quad \text{or} \quad A = \frac{1}{\sqrt{2a}}.$$

Hence normalised wave function is

$$\Psi(x) = \frac{1}{\sqrt{2a}}\,e^{icx}.$$

**22.** A particle is moving in one-dimensional potential box (of infinite height) of width 25 Å. Calculate the probability of finding the particle within an interval of 5 Å at the centres of the box when it is in its state of least energy.

*Sol.* The wavefunction of the particle can be written as

$$\Psi(x) = \sqrt{\frac{2}{a}}\,\sin\frac{n\pi x}{a}.$$

For the particle in the least energy state $n = 1$ and hence

$$\Psi(x) = \sqrt{\left(\frac{2}{a}\right)}\,\sin\frac{\pi x}{a}.$$

At the centre of the box $x = \frac{a}{2}$, the probability of finding the particle in the interval $\Delta x$ is given as

$$P = \Psi(x)|^2 \, \Delta x$$

$$\Psi(x)|^2 = \left[\sqrt{\frac{2}{a}} \sin \frac{\pi(a/2)}{a}\right]^2$$

$$= \frac{2}{a} \sin^2 \frac{\pi}{2} = \frac{2}{a}$$

$$P = \frac{2}{a} \Delta x = \frac{2 \times 5 \times 10^{-10}}{25 \times 10^{-10}}$$

$$= 0.4 \begin{bmatrix} a = 25 \text{ Å} \\ \Delta x = 5 \text{ Å} \end{bmatrix}.$$

**23.** A particle is in motion along a line between $x = 0$ and $x = a$ with zero potential energy. At points for which $x < 0$ and $x > a$, the potential energy is infinite. The wave function for the particle in the $n$th state is given by

$$\Psi_n = A \sin \frac{n\pi x}{a}.$$

Find the expression for the normalised wave function.

*Sol.* The probability of the particle between $x$ and $x + dx$ for the $n$th state is given as

$$\Psi_n(x)|^2 \, dx = A^2 \sin^2 \frac{n\pi x}{a} \, dx.$$

Since the particle is found in the region $x = 0$ and $x = a$ for all times, we have

$$\int_o^a |\Psi_n|^2 \, dx = 1$$

$$\int_o^a A^2 \sin^2 \frac{n\pi x}{a} \, dx = 1$$

$$A^2 \int_o^a \frac{1}{2}\left(1 - \cos \frac{2\pi n x}{a}\right) dx = 1$$

$$\frac{A^2}{2}\left(x - \frac{a}{2\pi n} \sin \frac{2\pi n x}{a}\right)_o^a = 1$$

$$\frac{A^2}{2}a = 1 \quad \text{or} \quad A = \sqrt{\frac{2}{a}}.$$

Now the normalised wave function is

$$\Psi_n(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}.$$

**24.** Find the energy of an electron moving in one dimension in an infinitely high potential box of width 1 Å, given mass of the electron $9.11 \times 10^{-31}$ kg and $h = 6.63 \times 10^{-34}$ J.s.

*Sol.* Since we know that $E = \dfrac{n^2h^2}{8ma^2}$ $(n = 1, 2, 3 \ldots\ldots)$.

For least energy of the particle $n = 1$.

Now $$\frac{h^2}{8m\,a^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31}(10^{-10})^2} \text{ Joules}$$

$$= 9.1 \times 10^{-19} \text{ Joules}$$

$$= \frac{9.1 \times 10^{-19}}{1.602 \times 10^{-19}} \text{ eV} = 5.68 \text{ eV}.$$

**25.** Compute the lowest energy of a neutron confined to the nucleus where nucleus is considered a box with a size of $10^{-14}$m. ($h = 6.26 \times 10^{-34}$ J.s, $m = 1.6 \times 10^{-24}$g).

*Sol.* Consider nucleus as a cubical box of size $10^{-14}$ m

$\therefore$ $$x = y = z = a = 10^{-14} \text{ m} = l$$

For the neutron to be in the lowest energy state $nx = ny = nz = 1$.

Now . $$E = \frac{\pi^2\hbar^2}{2m}\left[\frac{n_x^2}{l_x^2} + \frac{n_y^2}{l_y^2} + \frac{n_z^2}{l_z^2}\right] = \frac{\pi^2\hbar^2}{2m}\left[\frac{3}{l^2}\right]$$

$$= \frac{3h^2}{8m\,l^2} = \frac{3 \times [6.62 \times 10^{-34}]^2}{8 \times 1.6 \times 10^{-27} \times 10^{-28}}$$

$$= 10.29 \times 10^{-23} \text{ J} = 6.43 \text{ MeV}.$$