

Bayesian Statistics - Notes

Introduction

Bayesian Statistics is an approach to statistical inference where probability expresses a degree of belief in an event. This belief is updated as new evidence is introduced. It is based on Bayes' Theorem, which describes how to update probabilities when given new data.

Bayes' Theorem

Bayes' Theorem provides a mathematical way to update our beliefs based on evidence. It is given by the formula:

$$P(A | B) = [P(B | A) * P(A)] / P(B)$$

Where:

- $P(A | B)$ is the posterior probability: probability of event A occurring given that B is true.
- $P(B | A)$ is the likelihood: probability of observing B given that A is true.
- $P(A)$ is the prior probability: our initial belief before observing B.
- $P(B)$ is the marginal probability: the total probability of B occurring.

Components of Bayesian Statistics

1. Prior Probability ($P(A)$): This represents our initial belief about an event before any new evidence is introduced.
2. Likelihood ($P(B | A)$): This measures how likely it is to observe the given data assuming the hypothesis is true.
3. Posterior Probability ($P(A | B)$): This is the updated probability of the hypothesis after incorporating new evidence.
4. Marginal Probability ($P(B)$): This is the probability of the observed evidence, considering all possible hypotheses.

Applications of Bayesian Statistics

- Machine Learning & AI: Used in Bayesian Networks, Naïve Bayes classifiers, and probabilistic models.
- Medical Diagnosis: Helps in updating probabilities of diseases given new test results.
- Finance: Used in risk assessment and financial forecasting.
- Spam Filtering: Determines whether an email is spam based on prior probabilities and observed features.
- A/B Testing: Helps in updating beliefs about which variation performs better.

Conclusion

Bayesian Statistics provides a flexible and powerful framework for statistical inference. It is widely used in many domains where probability needs to be updated with new evidence. Understanding Bayes' Theorem and its applications can greatly enhance decision-making in uncertain environments.