CHANDANA R. GALGALI

Somaiya TRUST

Batch: PE-I Roll No. 16010422234
Experiment / assignment / tutorial No. 04
Grade: AA / AB / BB / BC / CC / CD / DD

Signature of the Faculty In-Charge with date

Q.1 solve: $\frac{dx}{dy} = xy + x^2 y^3$

Sol^n: Arranging the given equation in the form of Bernoulli's eq^n

$$\frac{dx}{dy} - yx = y^3 x^2$$

Multiplying throughout by $\frac{1}{x^2}$

$$\frac{1}{x^2}\frac{dx}{dy} - \frac{1}{\cancel{x^2}} \cdot y \cdot \cancel{x} = \frac{1}{\cancel{x^2}} \cdot y^3 \cdot \cancel{x^2}$$

$$\frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x} y = y^3 \rightarrow ①$$

Put $-\frac{1}{x} = v$

$$+\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$$

Substituting it in eq^n ① we get,

$$+\frac{dv}{dy} + vy = y^3$$

~~Multiplying throughout by -1,~~

$$\cancel{\frac{dv}{dy} + yv = -y^3}$$

The above equation is a linear differential equation in v

Its solution would be,

$$ve^{\int P dy} = \int [\int e^{\int P dy} Q] dy + c \; ; \quad P = y$$
$$Q = +y^3$$

$$e^{\int P dy} = e^{\int y\, dy} = e^{y^2/2}$$

$$\therefore \; ve^{y^2/2} = +\int \left[e^{y^2/2} y^3 \right] dy + c \rightarrow ②$$

Let $\frac{y^2}{2} = t$

$$y^2 = 2t$$

$$\cancel{2}y = \cancel{2}\frac{dt}{dy}$$

$$y\, dy = dt$$

Rewriting eq^n ②,

$$ve^{y^2/2} = +\int \left[e^{y^2/2} \cdot y \cdot y^2\right] dy + c$$

substituting the values,

$$ve^{t} = +\int \left[e^{t} \cdot 2t \cdot dt\right] + c$$

$$ve^{t} = +2\left[\int e^{t} t\right] dt + c$$

$$ve^{t} = +2\left[e^{t} t - e^{t}(1)\right] + c$$

$$ve^{t} = +2e^{t} t \mp 2e^{t} + c$$

Resubstituting the values of v and t,

$$\frac{-e^{y^2/2}}{x} = -2e^{y^2/2} + e^{y^2/2} y^2 + c$$

$$\frac{-e^{y^2/2}}{x} = e^{y^2/2}\left[-2 + y^2\right] + c$$

Q.2 solve: $\frac{dy}{dx} + \frac{2}{x} y = \frac{y^3}{x^3}$

sol^n: Arranging the given equation in the form of Bernoulli's eq^n,

$$\frac{dy}{dx} + \frac{2}{x} y = \frac{1}{x^3} y^3$$

Dividing throughout by y^3 we get,

$$\frac{1}{y^3} \frac{dy}{dx} + \frac{1}{y^{\cancel{3}2}} \frac{2}{x} \cancel{y} = \frac{1}{x^3} \cdot \cancel{y^3} \cdot \frac{1}{\cancel{y^3}}$$

$$\frac{1}{y^3} \frac{dy}{dx} + \frac{1}{y^2} \cdot \frac{2}{x} = \frac{1}{x^3} \rightarrow ①$$

put $\frac{1}{y^2} = v$

$$\frac{-2}{y^3} \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{1}{y^3} \frac{dy}{dx} = -\frac{1}{2} \frac{dv}{dx}$$

substituting the above values in ①,

$$-\frac{1}{2} \frac{dv}{dx} + v \cdot \frac{2}{x} = \frac{1}{x^3}$$

CHANDANA R. GALGALI

Multiplying throughout by -2,

$$\frac{dv}{dx} + v\left(-\frac{4}{x}\right) = \frac{-2}{x^3}$$

The above eq^n is a linear DE in v;
with $P = -\frac{4}{x}$ and $Q = \frac{-2}{x^3}$

Its solution would be:

$$ve^{\int P\,dx} = \int\left[\left(e^{\int P\,dx}\right)\cdot Q\right]dx + c$$

$$e^{\int P\,dx} = e^{\int -\frac{4}{x}dx} = e^{-4\int \frac{1}{x}dx} = e^{-4\log x} = e^{\log_e x^{-4}}$$

$$\therefore e^{\int P\,dx} = x^{-4} = \frac{1}{x^4}$$

$$\therefore v\cdot\frac{1}{x^4} = \int\left[\frac{1}{x^4}\cdot\left(-\frac{2}{x^3}\right)\right]dx + c$$

$$v\cdot\frac{1}{x^4} = -2\int\left[\frac{1}{x^7}\right]dx + c$$

$$v\cdot\frac{1}{x^4} = -2\left[\frac{x^{-7+1}}{-7+1}\right] + c$$

$$v\cdot\frac{1}{x^4} = +\cancel{2}\left[\frac{x^{-6}}{+\cancel{6}\,3}\right] + c$$

$$\frac{1}{y^2x^4} = \frac{1}{3x^6} + c$$

... [Resubstituting the value of v]

Q.3 Evaluate: $\lim_{x\to 1} \frac{x - x^x}{1 + \log x - x}$

Sol^n: $\lim_{x\to 1} \frac{x - x^x}{1 + \log x - x} \left[\frac{0}{0}\right] = I$

$$I = \lim_{x\to 1} \frac{x - e^{x\log x}}{1 + \log x - x} \left[\frac{0}{0}\right]$$

Applying L'Hospital's rule;

$$I = \lim_{x\to 1} \frac{1 - e^{x\log x}\left[\cancel{x}\cdot\frac{1}{\cancel{x}} + 1\cdot\log x\right]}{0 + \frac{1}{x} - 1}$$

$$I = \lim_{x\to 1} \frac{1 - e^{x\log x}\left[1 + \log x\right]}{\frac{1}{x} - 1} \left[\frac{0}{0}\right]$$

Applying L'Hospital's rule,

$$I = \lim_{x \to 1} \frac{0 - \left[e^{x\log x}\left(0 + \frac{1}{x}\right) + e^{x\log x}\left(\cancel{x} \cdot \frac{1}{\cancel{x}} + 1 \cdot \log x\right)(1+\log x)\right]}{-\frac{1}{x^2} - 0}$$

$$I = \lim_{x \to 1} \frac{-\frac{e^{x\log x}}{x} - e^{x\log x}(1+\log x)^2}{-\frac{1}{x^2}}$$

$$I = 2$$

$$\therefore \lim_{x \to 1} \frac{x - x^x}{1 + \log x - x} = 2$$

Q.4 Prove that : $\lim_{x \to a} \frac{\log(x-a)}{\log(a^x - a^a)} = 1$

$$LHS = \lim_{x \to a} \frac{\log(x-a)}{\log(a^x - a^a)} \quad \left[\frac{\infty}{\infty}\right]$$

Applying L'Hospital's rule,

$$LHS = \lim_{x \to a} \frac{\frac{1}{(x-a)}}{\frac{a^x \log a}{a^x - a^a}}$$

$$= \lim_{x \to a} \frac{a^x - a^a}{(x-a)\, a^x \log a} \quad \left[\frac{0}{0}\right]$$

Applying L'Hospital's rule,

$$LHS = \lim_{x \to a} \left[\frac{a^x \log a - 0}{\log a \left[a^x \log a (x-a) + a^x \right]} \right]$$

$$= \lim_{x \to a} \frac{a^x \cancel{\log a}}{\cancel{\log a} \left[a^x \log a (x-a) + a^x \right]}$$

$$= \frac{a^a}{a^a}$$

$$= 1$$

LHS = RHS

Hence, proved.