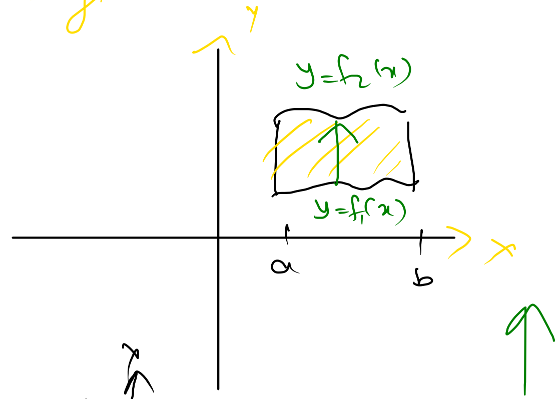
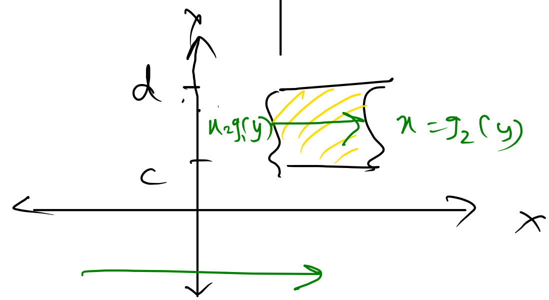
Evaluation over the region.

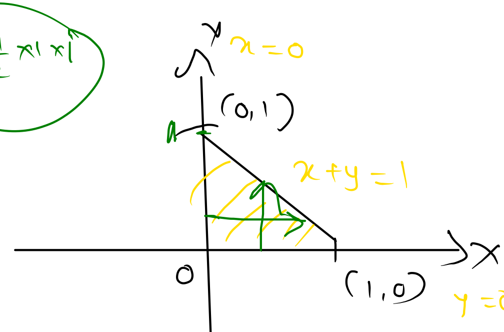
$$\int_{x=a}^{b}\int_{y=f_1(x)}^{f_2(x)} f(x,y)\; dy\, dx$$



$$\int_{y=c}^{d}\int_{x=g_1(y)}^{g_2(y)} f(x,y)\; dx\, dy$$



$\frac{1}{2}\times 1\times 1$



$$\int_{x=0}^{1}\int_{y=0}^{1-x} dy\, dx$$

$$\int_{y=0}^{1}\int_{x=0}^{1-y} dx\, dy$$

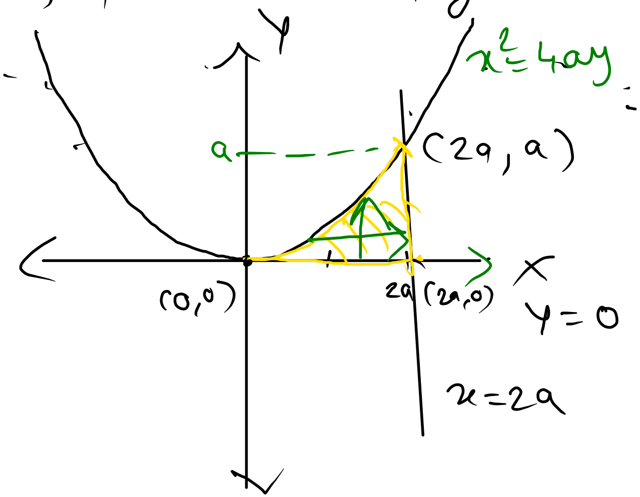
$\cancel{(x)_0^{1-y}}$ $\quad \int (1-y)\, dy \quad y - \frac{y^2}{2}\Big|_0^1$

$1 - \frac{1}{2} = \frac{1}{2}$

$$\frac{(y-y_1)}{(x-x_1)} = \frac{(x_2-x_1)}{(y_2-y_1)}$$

$\iint xy \, dx \, dy$ over the region bounded by

x-axis, ordinate at $x = 2a$, parabola $x^2 = 4ay$



$x^2 = 4ay$

vertex $(0,0)$

$x = \pm\sqrt{4ay}$

$x = 2a$

$x^2 = 4ay$

$4a^2 = 4ay$

$$\int_{x=0}^{2a}\int_{y=0}^{x^2/4a} xy \, dy \, dx \qquad\Bigg|\qquad \int_{y=0}^{a}\int_{x=\sqrt{4ay}}^{2a} xy \, dx \, dy$$

$=$

$\underline{\underline{a^4/3}}$

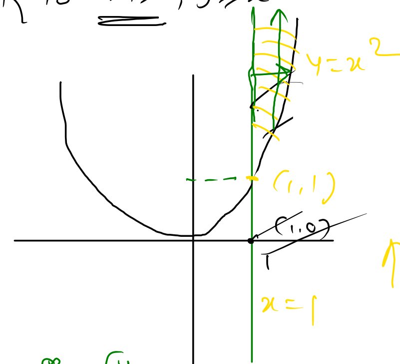
$$\iint_R \frac{1}{x^4+y^2}\,dx\,dy \quad \text{where } R \text{ is } x \ge 1,\ y \ge x^2$$

$x = 1$

$y = x^2$

$x = \pm\sqrt{y}$

$x = 1,\ y = 1$



$$\int_{x=1}^{\infty}\int_{y=x^2}^{\infty} \frac{1}{x^4+y^2}\,dy\,dx$$

$y = x^2$

$x = \pm\sqrt{y}$

$$= \int_1^{\infty}\int_{x^2}^{\infty} \frac{1}{y^2+(x^2)^2}\,dy\,dx$$

$$\int_{y=1}^{\infty}\int_{x=1}^{\sqrt{y}} \frac{1}{x^4+y^2}\,dx\,dy$$

$$= \int_1^{\infty}\left(\frac{1}{x^2}\tan^{-1}\frac{y}{x^2}\right)_{x^2}^{\infty} dx$$

$$= \int_1^{\infty}\left[\frac{1}{x^2}\tan^{-1}\infty - \frac{1}{x^2}\tan^{-1}(1)\right] dx$$

$$= \int_1^{\infty}\left[\frac{1}{x^2}\frac{\pi}{2} - \frac{1}{x^2}\frac{\pi}{4}\right] dx$$

$$= \int_1^{\infty}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)\frac{1}{x^2}\,dx = \frac{\pi}{4}\int_1^{\infty} x^{-2}\,dx$$

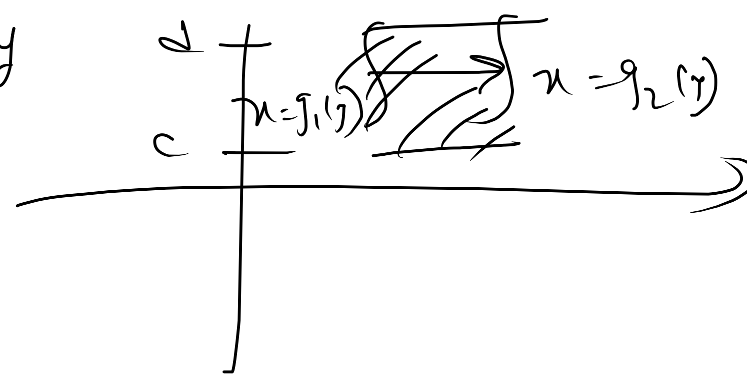
$$= \frac{\pi}{4}\left(\frac{x^{-1}}{-1}\right)_1^{\infty}$$

$$= \frac{\pi}{4}\left(-\frac{1}{x}\right)_1^{\infty}$$

$$= \frac{\pi}{4}\left[0+1\right] = \pi/4$$

$$\int_{x=a}^{b}\int_{y=f_1(x)}^{f_2(x)} f(x,y)\,dy\,dx$$

$$\int_{y=c}^{d}\int_{x=g_1(y)}^{g_2(y)} f(x,y)\,dx\,dy$$



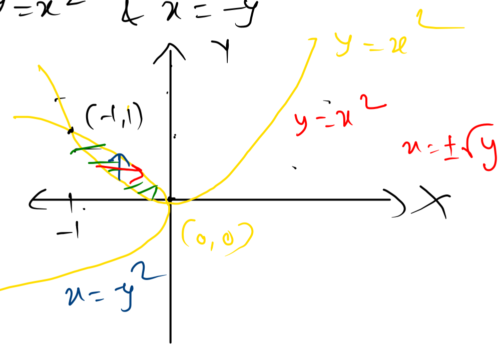
$\iint xy\,dx\,dy$ over area bdd by parabolas $y=x^2$ & $x=-y^2$

$y=x^2$ | $x=-y^2$

vertex $(0,0)$ | $(0,0)$

$x=\pm\sqrt{y}$ | $y^2=-x$

$y=\pm\sqrt{-x}$



$y=x^2$ $\quad$ $x=-y^2$

$y=y^4$

$y(y^3-1)=0$

$y=0$ $\quad$ $y^3=1$

$y=1$

$$\int_{x=-1}^{0}\int_{y=x^2}^{y=\sqrt{-x}} xy\;dy\,dx$$

$$\int_{y=0}^{1}\int_{x=-\sqrt{y}}^{x=-y^2} xy\;dx\,dy$$

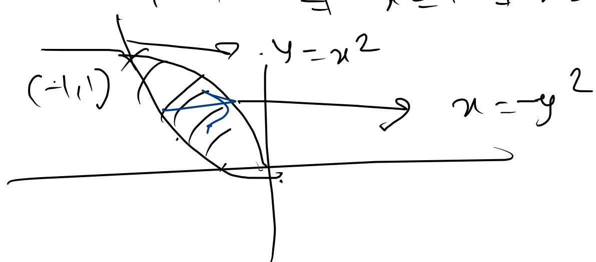
$= -\frac{1}{12}$

for $y=x^2$

if $y=0 \Rightarrow x=0$

$y=1 \Rightarrow x^2=1 \Rightarrow x=-1$

$x=\pm\sqrt{y}$



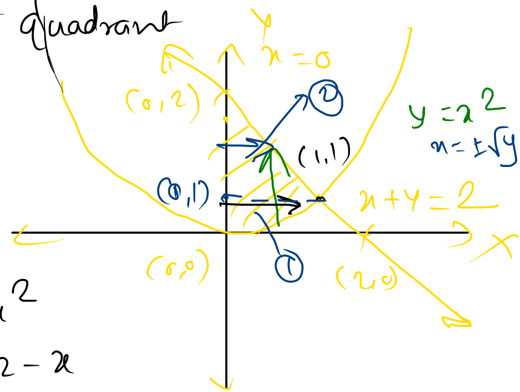
$\iint y\,dx\,dy$ over the area bdd by $x=0$, $y=x^2$, $x+y=2$ in 1'st quadrant

$y=x^2$

$x=\pm\sqrt{y}$

$x+y=2$

x	0	2
y	2	0



$y=x^2$

$y=2-x$

$\Rightarrow x^2=2-x \Rightarrow x^2+x-2=0$

$x=-2,\ x=1$

for $x=1$ as $y=2-x$

$y=1$

$$\int_{x=0}^{1}\int_{y=x^2}^{2-x} y\,dy\,dx$$

$= \frac{16}{15}$

$$\int_{y=0}^{1}\int_{x=0}^{\sqrt{y}} y\,dx\,dy + \int_{y=1}^{2}\int_{x=0}^{2-y} y\,dx\,dy$$

$$\iint_R \frac{y}{(a-x)\sqrt{ax-y^2}}\,dx\,dy$$

R is region bdd by $y^2=ax$ & $y=x$

$y^2=ax$

vertex $(0,0)$

$y=\pm\sqrt{ax}$

$y^2=ax,\ y=x$

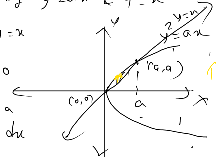
$x^2=ax$

$x(x-a)=0$

$x=0,\ x=a$

as $y=x$

$\therefore y=0,\ y=a$



$$\int_{x=0}^{a}\int_{y=x}^{\sqrt{ax}} \frac{y}{(a-x)\sqrt{ax-y^2}}\,dy\,dx$$

$$=\int_0^a \frac{1}{(a-x)}\left(\int_x^{\sqrt{ax}} \frac{y}{\sqrt{ax-y^2}}\,dy\right)dx$$

$ax-y^2=t$

$0-2y\,dy=dt \Rightarrow y\,dy=\frac{-dt}{2}$

as $y: x\to\sqrt{ax}$

$t: ax-x^2\to 0$

$$=\int_0^a \frac{1}{(a-x)}\int_{ax-x^2}^{0} \frac{-dt}{2(t)^{1/2}}\,dx$$

$$=\frac{1}{2}\int_0^a \frac{1}{(a-x)}\int_0^{ax-x^2} t^{-1/2}\,dt\,dx$$

$$=\frac{1}{2}\int_0^a \frac{1}{(a-x)}\left[\frac{(t)^{1/2}}{1/2}\right]_0^{ax-x^2}dx$$

$$=\frac{1}{2}\int_0^a \frac{1}{a-x}\,2\left((ax-x^2)^{1/2}\right)dx$$

$$=\cancel{\frac{1}{2}\times 2}\int_0^a \frac{1}{a-x}\,x^{1/2}(a-x)^{1/2}\,dx$$

$$=\int_0^a x^{1/2}(a-x)^{-1/2}\,dx$$

$$=\int_0^a x^{1/2}\,a^{-1/2}\left(1-\frac{x}{a}\right)^{-1/2}dx$$

$\frac{x}{a}=t \Rightarrow x=at$

$dx=a\,dt$

$x: 0\to a \quad t: 0\to 1$

$$=a^{-1/2}\int_0^1 (at)^{1/2}(1-t)^{-1/2}\,a\,dt$$

$$=a^{-1/2}a^{1/2}a\int_0^1 t^{1/2}(1-t)^{-1/2}\,dt$$

$$=a\,\beta\left(\tfrac{1}{2}+1,\ -\tfrac{1}{2}+1\right)=a\,\beta\left(\tfrac{3}{2},\tfrac{1}{2}\right)$$

$$=a\,\frac{\Gamma\frac{3}{2}\,\Gamma\frac{1}{2}}{\Gamma 2}$$

$$=a\,\frac{\frac{1}{2}\Gamma\frac{1}{2}\,\sqrt{\pi}}{1!}$$

$\beta(m,n)=\frac{\Gamma m\,\Gamma n}{\Gamma m+n}$

$\Gamma n=(n-1)!$

$$=\frac{a}{2}\pi$$

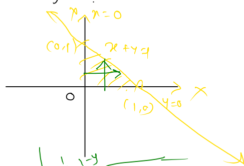
$\Gamma n=(n-1)\Gamma n-1$

$\iint \sqrt{xy(1-x-y)}\;dx\,dy$ over the area

bdd by $x=0$, $y=0$ & $x+y=1$.

$x+y=1$

x	0	1
y	1	0



$$\int_{x=0}^{1}\int_{y=0}^{1-x} \sqrt{xy(1-x-y)}\;dy\,dx$$

$$\int_{y=0}^{1}\int_{x=0}^{1-y} \sqrt{xy(1-x-y)}\;dx\,dy$$

$$= \int_0^1 \sqrt{x}\int_0^{1-x} y^{1/2}(1-x-y)^{1/2}\,dy\,dx \quad (1)$$

Consider: $\int^{1-x} y^{1/2}(1-x-y)^{1/2}\,dy$

put $1-x=a$

$$= \int^{a} y^{1/2}(a-y)^{1/2}\,dy$$

$$= \int^{a} y^{1/2}\,a^{1/2}\left(1-y/a\right)^{1/2}\,dy$$

$y/a=t \Rightarrow y=at$

$dy = a\,dt$

$y: 0\to a \quad t: 0\to 1$

$$= a^{1/2}\int^1 (at)^{1/2}(1-t)^{1/2}\,a\,dt$$

$$= a^{1/2}\,a^{1/2}\,a\int^1 t^{1/2}(1-t)^{1/2}\,dt$$

$$= a^2\,\beta\left(\tfrac{3}{2},\tfrac{3}{2}\right) = a^2\,\frac{\Gamma\frac{3}{2}\,\Gamma\frac{3}{2}}{\Gamma\frac{6}{2}} = \frac{a^2\left(\frac{1}{2}\Gamma\frac{1}{2}\right)^2}{\Gamma 3}$$

$$= \frac{a^2}{4}\,\frac{(\sqrt{\pi})^2}{2!} = \pi\frac{a^2}{8}$$

$\int_0^1 x^m(1-x)^n\,dx$

$= \beta(m+1,n+1)$

$\Gamma n = (n-1)\Gamma n-1$

from (1)

$$\int_0^1 \sqrt{x}\int_0^{1-x} y^{1/2}(1-x-y)^{1/2}\,dy\,dx$$

$$= \int_0^1 \sqrt{x}\;\pi\frac{(a^2)}{8}\,dx$$

$$= \frac{\pi}{8}\int_0^1 \sqrt{x}(1-x)^2\,dx$$

$$= \frac{\pi}{8}\int_0^1 x^{1/2}(1-x)^2\,dx$$

$$= \frac{\pi}{8}\,\beta\left(\tfrac{3}{2},3\right)$$

$$= \frac{\pi}{8}\,\frac{\Gamma\frac{3}{2}\,\Gamma 3}{\Gamma\frac{3}{2}+3}$$

$$= \frac{\pi}{8}\,\frac{\Gamma\frac{3}{2}\;2!}{\Gamma\frac{9}{2}}$$

$$= \frac{\pi}{\cancel{8}_4}\,\frac{\cancel{\Gamma\frac{3}{2}}\,\cancel{(2)}^1}{\frac{7}{2}\,\frac{5}{2}\,\frac{3}{2}\,\cancel{\Gamma\frac{3}{2}}}$$

$$= \frac{\pi\times\cancel{8}\,2}{\cancel{4}_1\times 7\times 5\times 3} = \frac{2\pi}{105}$$

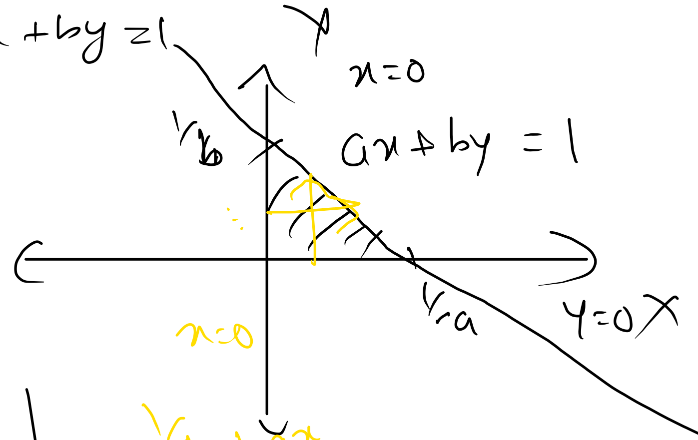
HW:

$=\frac{1}{ab}$

$R = \frac{1}{2} \times \frac{1}{a} \times \frac{1}{b} = \frac{1}{2ab}$

P.T. $\iint_R e^{ax+by}\,dx\,dy = 2R$ where R is area of Δ whose boundaries are

$x=0,\ y=0$ & $ax+by=1$



$ax+by=1$

x	0	$1/a$
y	$1/b$	0

$$\int_{y=0}^{1/b}\int_{x=0}^{\frac{1-by}{a}} e^{ax}\cdot e^{by}\,dx\,dy$$

$$\int_{x=0}^{1/a}\int_{y=0}^{\frac{1-ax}{b}} e^{ax}\, e^{by}\,dy\,dx$$

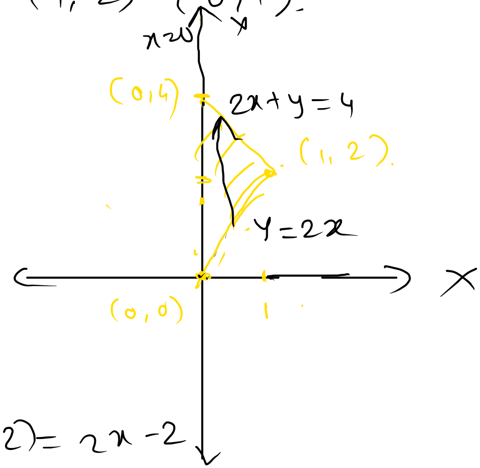
$$\iint_R x(x-y)\,dx\,dy$$ where R is Δ with vertices $(0,0)$ $(1,2)$ $(0,4)$.

$$\int_{x=0}^{1}\int_{y=2x}^{4-2x}(x^2-xy)\,dy\,dx$$

$(0,0)$ $(1,2)$

$$\frac{(y-y_1)}{(x-x_1)} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y}{x} = \frac{2}{1} \Rightarrow y = 2x$$



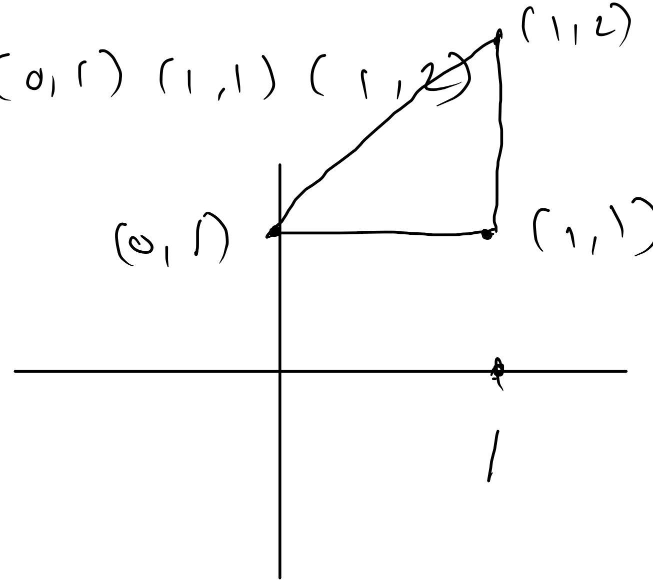
$(1,2)$ $(0,4)$

$$\frac{y-2}{x-1} = \frac{2}{-1} \Rightarrow (y-2) = 2x-2$$

$$\Rightarrow -y+2 = 2x-2 \Rightarrow 2x+y=4$$

HW

$\iint (x^2+y^2)\,dx\,dy$ over area of Δ whose vertices are $(0,1)$ $(1,1)$ $(1,2)$



HW

$$\iint_R \sqrt{xy - y^2}\, dx\, dy \quad \text{where } R \text{ is } \Delta$$

vertices are $(0,0)$ $(10,1)$ & $(1,1)$

$(0,0)$ $(10,1)$

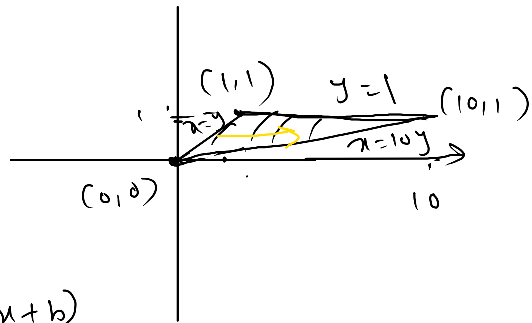
$$\frac{y-0}{x-0} = \frac{1}{10}$$

$$10y = x$$

$$\iint \sqrt{xy - y^2}\, dx\, dy$$

$$\int_{y=0}^{1} \sqrt{y} \int_{x=y}^{10y} (x-y)^{1/2}\, dx\, dy$$

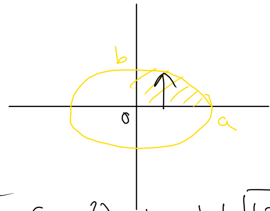
$$\int (ax+b)$$



$$\int \sqrt{y} \left(\frac{(x-y)^{3/2}}{3/2} \right)_{y}^{10y} dy$$

$\iint x^{m-1} y^{n-1}\,dx\,dy$ over +ve quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\int_{x=0}^{a}\int_{y=0}^{b\sqrt{1-\frac{x^2}{a^2}}} x^{m-1}y^{n-1}\,dy\,dx$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$y^2=b^2\left(1-\frac{x^2}{a^2}\right) \qquad y=\pm\, b\sqrt{1-\frac{x^2}{a^2}}$$

$$=\int_0^a x^{m-1}\int_0^{b\sqrt{1-\frac{x^2}{a^2}}} y^{n-1}\,dy\,dx$$

$$=\int_0^a x^{m-1}\left(\frac{y^n}{n}\right)_0^{b\sqrt{1-\frac{x^2}{a^2}}}dx$$

$$=\int_0^a x^{m-1}\,\frac{1}{n}\,b^n\left(\sqrt{1-\frac{x^2}{a^2}}\right)^n dx.$$

$$=\frac{b^n}{n}\int_0^a x^{m-1}\left(1-\frac{x^2}{a^2}\right)^{n/2}dx$$

$=$

$$\frac{x^2}{a^2}=t$$

$$x^2=a^2t$$

$$x=a\,t^{1/2}$$

$$dx=a\,\frac{1}{2}\,t^{-1/2}\,dt$$

$x: 0\to a \qquad t\,|\,0\to 1$

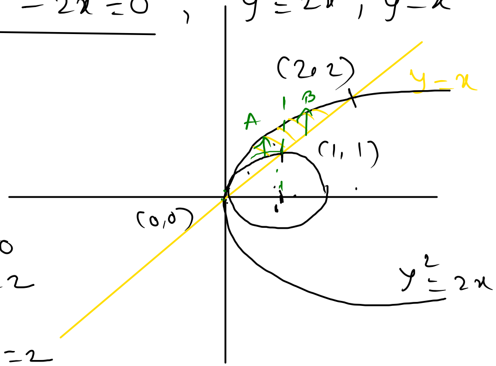
$$=\frac{b^n}{n}\int_0^1 (a\,t^{1/2})^{m-1}(1-t)^{n/2}\,a\,\frac{1}{2}\,t^{-1/2}\,dt$$

$$=\frac{b^n}{n}\,a^{m-1}\,\frac{a}{2}\int_0^1 t^{\frac{m}{2}-\frac{1}{2}-\frac{1}{2}}(1-t)^{n/2}\,dt$$

$$=\frac{b^n a^m}{n\,2}\int_0^1 t^{\frac{m}{2}-1}(1-t)^{n/2}\,dt$$

$$=\frac{b^n a^n}{2n}\,\beta\left(\frac{m}{2},\frac{n}{2}+1\right)$$

$\iint_R xy\,dx\,dy$ over the region R given by $x^2+y^2-2x=0$, $y^2=2x$, $y=x$



$y^2=2x \Rightarrow y=\pm\sqrt{2x}$

$y^2=2x \quad y=x$

$x^2=2x \Rightarrow x(x-2)=0$

$\Rightarrow x=0 \quad x=2$

as $y=x$

$y=0, \; y=2$

$(0,0) \quad (2,2)$

$$\underline{\underline{x^2-2x}}+y^2=0$$

$\Rightarrow x^2-2x+1+y^2=1 \Rightarrow (x-1)^2+y^2=1$

center $=(1,0)$

& rad ①

$\frac{M^2}{4r} = \frac{4}{4}=1$

$x^2+y^2-2x=0$

$$\int_{x=0}^{1}\int_{y=\sqrt{2x-x^2}}^{\sqrt{2x}} xy\,dy\,dx + \int_{x=1}^{2}\int_{y=x}^{\sqrt{2x}} xy\,dy\,dx$$

$=\frac{7}{12}$

$x^2+y^2-2x=0$

$y^2=2x-x^2$

$y=\pm\sqrt{2x-x^2}$

$y^2=2x$

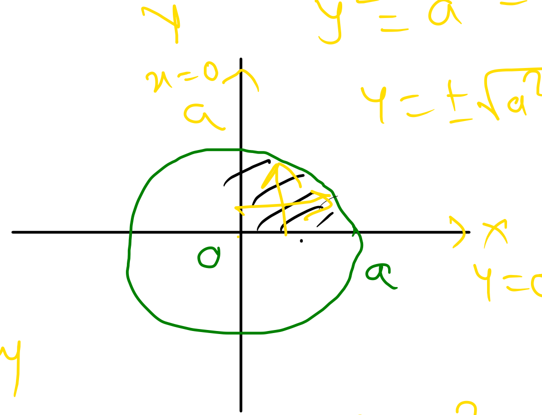
$y=\pm\sqrt{2x}$

$$\iint (x^2-y^2)x\,dx\,dy$$ over the +ve quadrant of circle $x^2+y^2=a^2$

$$y^2=a^2-x^2$$

$$y=\pm\sqrt{a^2-x^2}$$

$$\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} (x^2-y^2)\,x\,dy\,dx$$



$$\int_{y=0}^{a}\int_{x=0}^{\sqrt{a^2-y^2}} (x^2-y^2)\,x\,dx\,dy$$

$$=\frac{a^5}{15}$$

$$x^2=a^2-y^2$$

$$x=\pm\sqrt{a^2-y^2}$$

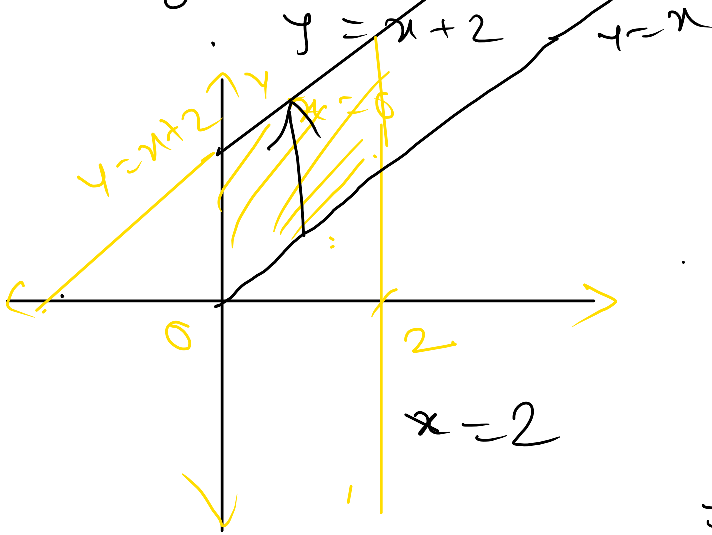
evaluate $\iint_R (x+y)\,dx\,dy$ where R is the region bdd by $x=0$, $x=2$, $y=x$, $y=x+2$

$y = x+2$

$x=0 \quad y=2$

$y=0 \quad x=-2$

$(0,2) \quad (-2,0)$



$$\int_{x=0}^{2}\int_{y=x}^{x+2} (x+y)\,dy\,dx$$

$= 12$

$$\iint_R x^2\, dA$$ where R is region in 1^{st} quad. bdd by hyperbola $xy = 16$ & line $x = y$, $y = 0$ & $x = 8$.

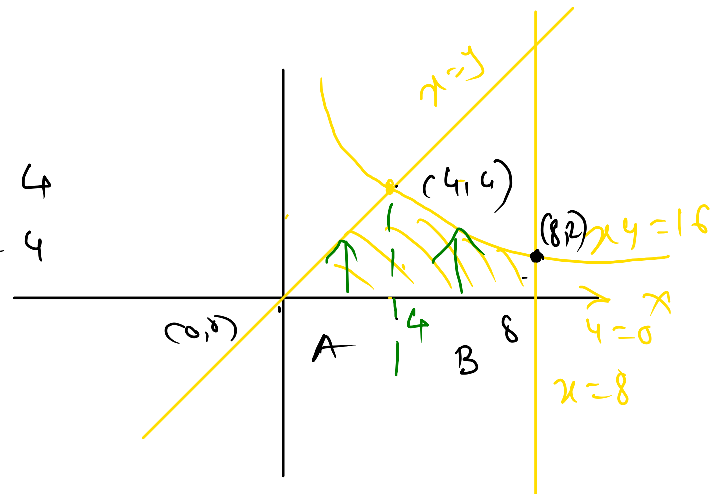
$xy = 16$

$x = y$

$y^2 = 16 \Rightarrow y = \pm 4$

as $x = y \Rightarrow x = \pm 4$

$(4, 4)$ ~~$(-4, -4)$~~



for $x = 8$ $xy = 16 \Rightarrow y = 2$

$$\int_{x=0}^{4}\int_{y=0}^{y=x} x^2\, dy\, dx + \int_{x=4}^{8}\int_{y=0}^{y=16/x} x^2\, dy\, dx$$

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