# practice-Copy1

March 8, 2020

[52]:
```
from IPython.display import Image
from sklearn.externals.six import StringIO
from sklearn.tree import export_graphviz
import pydotplus
import imblearn
import lightgbm
import hyperopt
```

[30]:
```
import pandas as pd
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
%matplotlib inline
import plusmodules as pm
import warnings
warnings.filterwarnings('ignore')
```

[17]:
```
df=pd.read_csv('US_Heart_Patients.csv')
```

[29]:
```
df=df.sample(frac=1, random_state=3)
```

[19]:
```
df.head()
```

[19]:

| | male | age | education | currentSmoker | cigsPerDay | BPMeds | \ |
|---|---|---|---|---|---|---|---|
| 3546 | 0 | 54 | 1.0 | 0 | 0.0 | 0.0 | |
| 1127 | 0 | 42 | 3.0 | 1 | 10.0 | 0.0 | |
| 3088 | 0 | 58 | 1.0 | 0 | 0.0 | 1.0 | |
| 437 | 1 | 45 | 1.0 | 1 | 30.0 | 0.0 | |
| 3188 | 1 | 63 | 1.0 | 0 | 0.0 | 0.0 | |

| | prevalentStroke | prevalentHyp | diabetes | totChol | sysBP | diaBP | BMI | \ |
|---|---|---|---|---|---|---|---|---|
| 3546 | 0 | 0 | 0 | 241.0 | 106.0 | 77.0 | 27.64 | |
| 1127 | 0 | 0 | 0 | 253.0 | 109.0 | 74.0 | 24.38 | |
| 3088 | 1 | 1 | 0 | 274.0 | 159.0 | 90.0 | 28.40 | |
| 437 | 0 | 0 | 0 | 240.0 | 141.0 | 89.0 | 25.01 | |

```
3188               0             1         0    190.0  148.0   90.0  27.13

      heartRate  glucose  TenYearCHD
3546       78.0     74.0           0
1127       88.0     60.0           0
3088       72.0     81.0           0
437        95.0     76.0           0
3188       72.0     86.0           0
```

[20]:
```
df
```

[20]:
```
      male  age  education  currentSmoker  cigsPerDay  BPMeds  \
3546     0   54        1.0              0         0.0     0.0
1127     0   42        3.0              1        10.0     0.0
3088     0   58        1.0              0         0.0     1.0
437      1   45        1.0              1        30.0     0.0
3188     1   63        1.0              0         0.0     0.0
…    …   …         …              …         …      …
789      1   63        1.0              0         0.0     0.0
968      0   54        1.0              0         0.0     0.0
1667     0   56        1.0              1         3.0     0.0
3321     0   58        2.0              0         0.0     0.0
1688     0   40        4.0              1        15.0     0.0

      prevalentStroke  prevalentHyp  diabetes  totChol  sysBP  diaBP    BMI  \
3546                0             0         0    241.0  106.0   77.0  27.64
1127                0             0         0    253.0  109.0   74.0  24.38
3088                1             1         0    274.0  159.0   90.0  28.40
437                 0             0         0    240.0  141.0   89.0  25.01
3188                0             1         0    190.0  148.0   90.0  27.13
…                 …             …       …       …    …     …     …
789                 0             1         0    260.0  159.5   91.0  27.01
968                 0             1         0    266.0  137.0   88.0  29.76
1667                0             1         0    285.0  145.0  100.0  30.14
3321                0             1         1    265.0  143.5   85.0  21.68
1688                0             0         0    155.0  121.0   86.0  23.16

      heartRate  glucose  TenYearCHD
3546       78.0     74.0           0
1127       88.0     60.0           0
3088       72.0     81.0           0
437        95.0     76.0           0
3188       72.0     86.0           0
…          …      …         …
789        68.0     66.0           0
968        80.0     80.0           0
1667       80.0     86.0           0
```

```
3321       91.0    107.0           0
1688       70.0     59.0           0

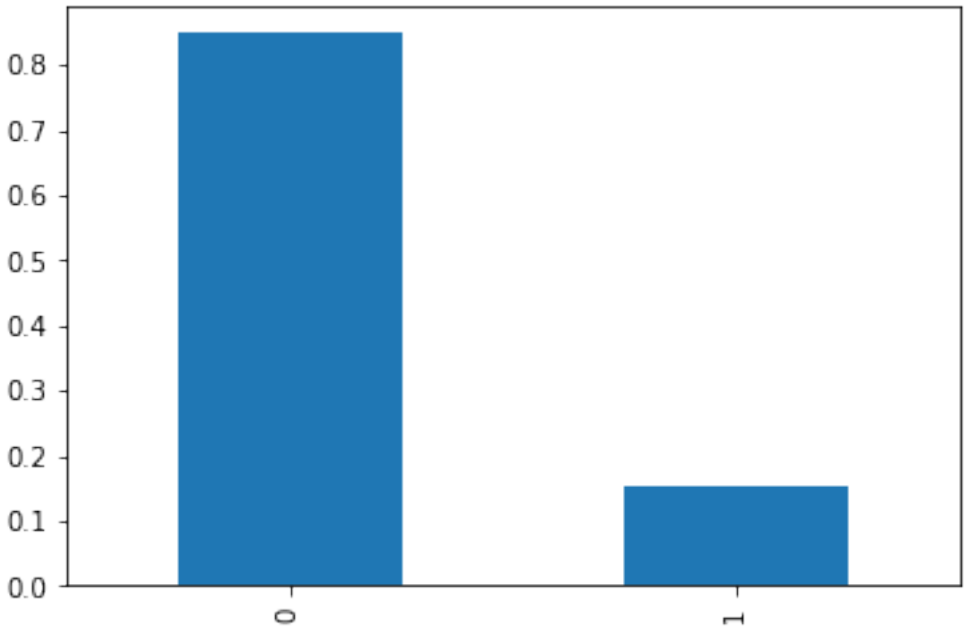
[4240 rows x 16 columns]
```

[21]:
```
df['TenYearCHD'].value_counts()
```

[21]:
```
0    3596
1     644
Name: TenYearCHD, dtype: int64
```

[22]:
```
df['TenYearCHD'].value_counts(normalize=True).plot.bar()
```

[22]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x2b0bcaca390>
```




[23]:
```
mv=df.isnull().sum()
mv[mv>0]
```

[23]:
```
education     105
cigsPerDay     29
BPMeds         53
totChol        50
BMI            19
heartRate       1
glucose       388
dtype: int64
```

[24]:
```
df=df.fillna(method='ffill')
df.head()
# filling all the null values using forward filing method in order not to␣
 ↪change the distribution
```

[24]:

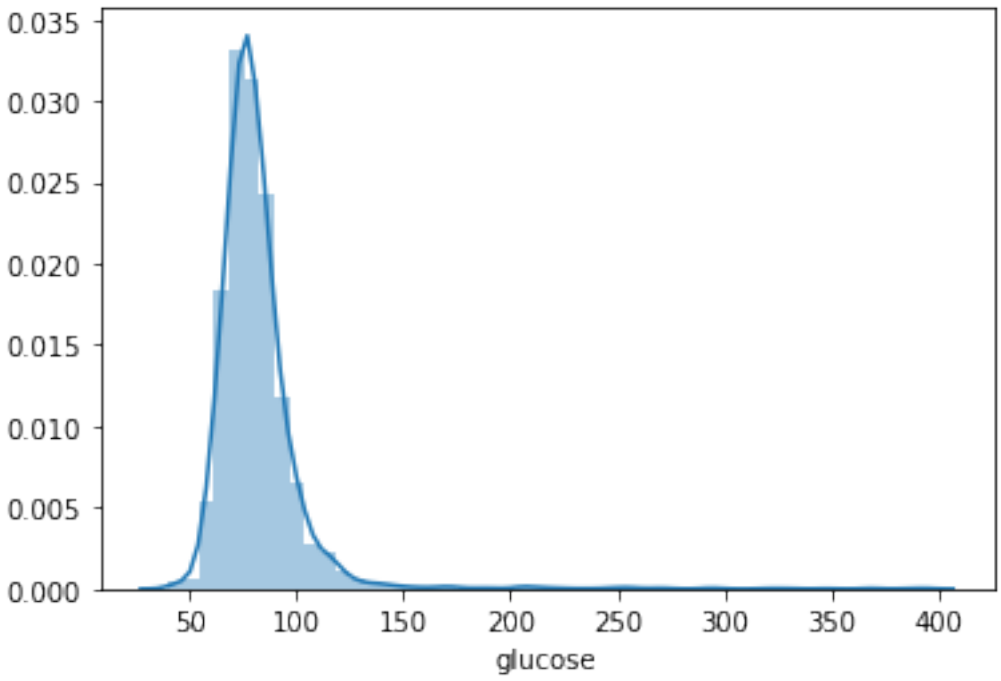
| | male | age | education | currentSmoker | cigsPerDay | BPMeds | \ |
|---|---|---|---|---|---|---|---|
| 3546 | 0 | 54 | 1.0 | 0 | 0.0 | 0.0 | |
| 1127 | 0 | 42 | 3.0 | 1 | 10.0 | 0.0 | |
| 3088 | 0 | 58 | 1.0 | 0 | 0.0 | 1.0 | |
| 437 | 1 | 45 | 1.0 | 1 | 30.0 | 0.0 | |
| 3188 | 1 | 63 | 1.0 | 0 | 0.0 | 0.0 | |

| | prevalentStroke | prevalentHyp | diabetes | totChol | sysBP | diaBP | BMI | \ |
|---|---|---|---|---|---|---|---|---|
| 3546 | 0 | 0 | 0 | 241.0 | 106.0 | 77.0 | 27.64 | |
| 1127 | 0 | 0 | 0 | 253.0 | 109.0 | 74.0 | 24.38 | |
| 3088 | 1 | 1 | 0 | 274.0 | 159.0 | 90.0 | 28.40 | |
| 437 | 0 | 0 | 0 | 240.0 | 141.0 | 89.0 | 25.01 | |
| 3188 | 0 | 1 | 0 | 190.0 | 148.0 | 90.0 | 27.13 | |

| | heartRate | glucose | TenYearCHD |
|---|---|---|---|
| 3546 | 78.0 | 74.0 | 0 |
| 1127 | 88.0 | 60.0 | 0 |
| 3088 | 72.0 | 81.0 | 0 |
| 437 | 95.0 | 76.0 | 0 |
| 3188 | 72.0 | 86.0 | 0 |

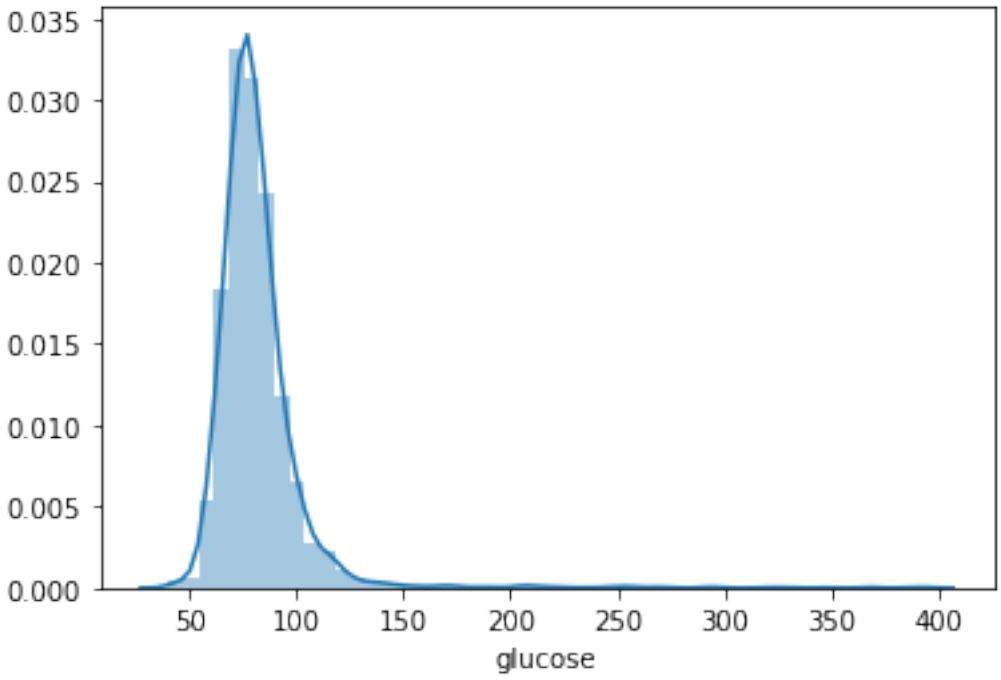
[13]:
```
sns.distplot(df['glucose'].dropna())
# this plot is for column glucose without null values
```

[13]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x2274bb30630>
```




[14]:
```
sns.distplot(df['glucose'])
#this plot is after filling the null values with the median value for column␣
 ↪glucose
```

[14]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x227502cd7f0>
```



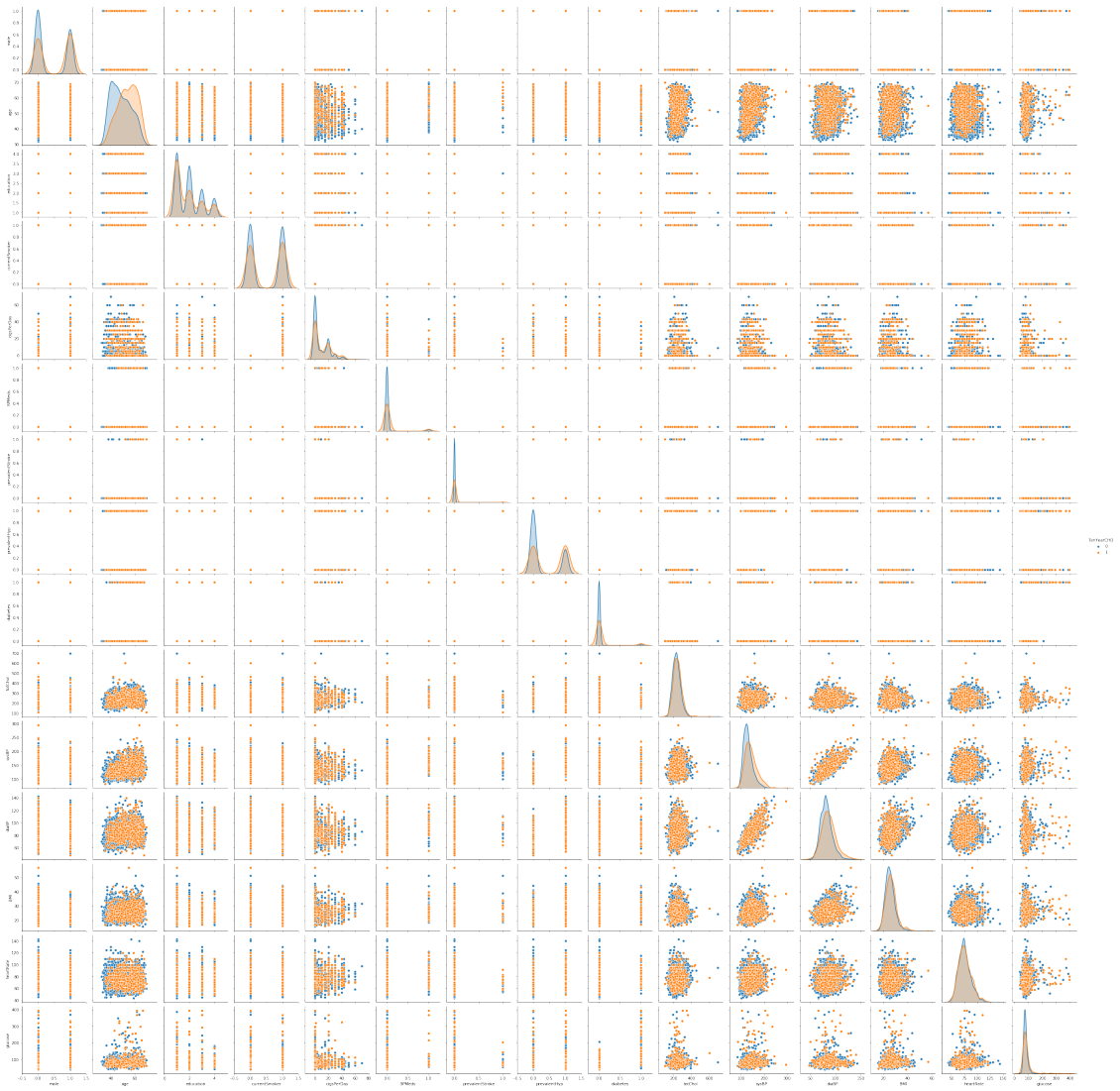
[15]:
```
df['glucose'].describe()
```

[15]:
```
count    4240.000000
mean       81.780896
std        23.257604
min        40.000000
25%        71.000000
50%        78.000000
75%        87.000000
max       394.000000
Name: glucose, dtype: float64
```

if we see the values that we have change in the distribution in order not to change the distribution we just change the format of filling the null values to forward and backward filling

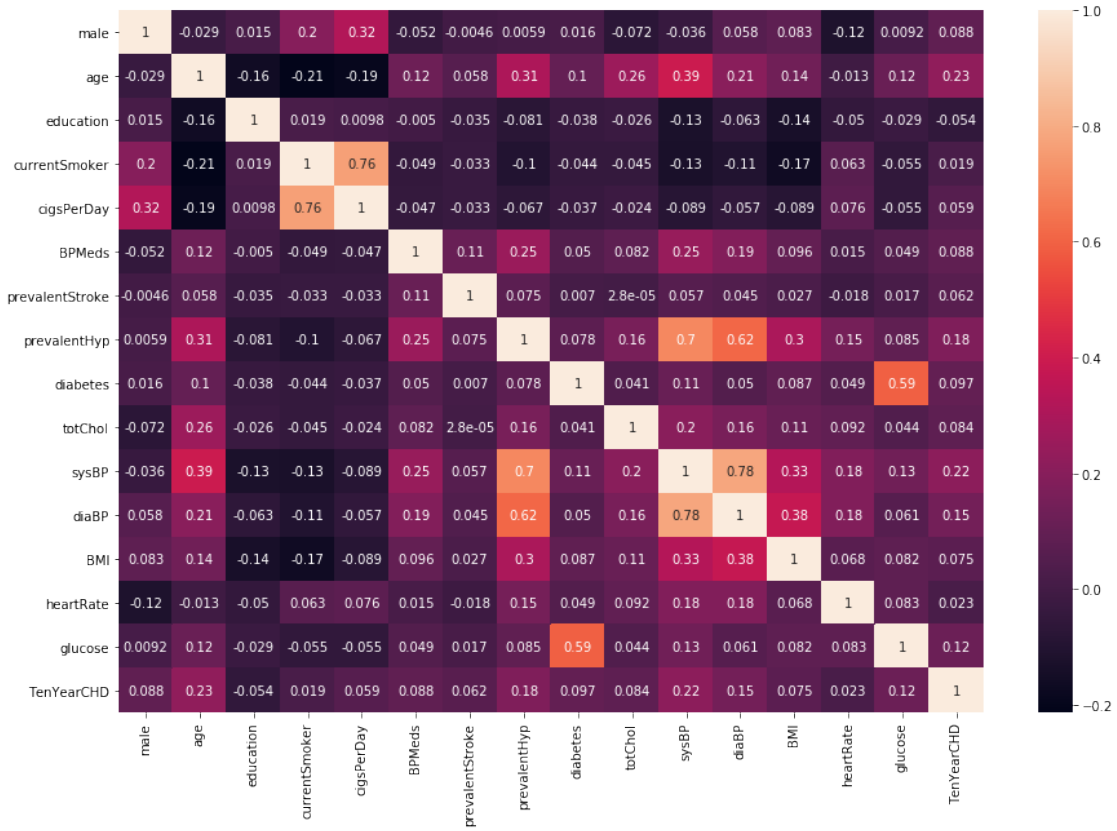
[16]:
```
sns.pairplot(df, hue='TenYearCHD')
```

[16]:
```
<seaborn.axisgrid.PairGrid at 0x2274bc53a20>
```

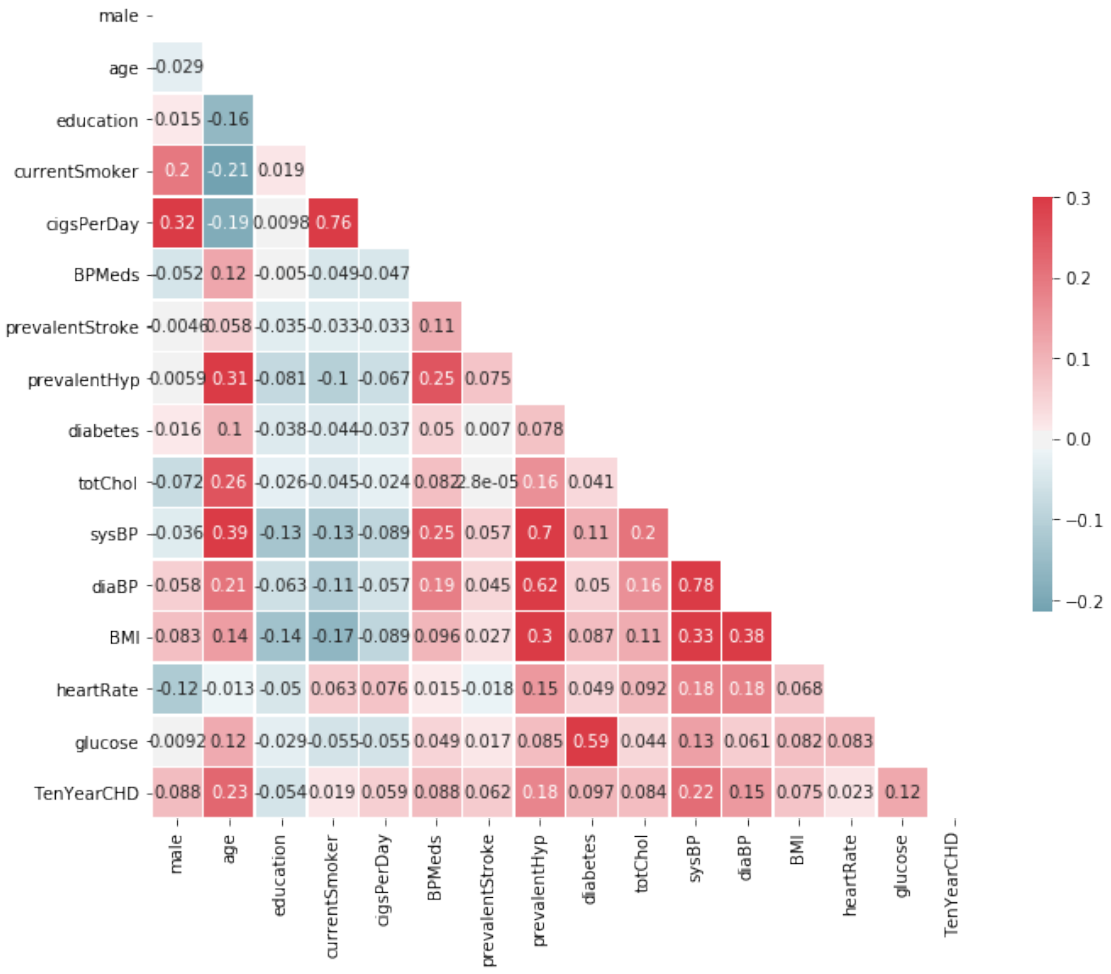


[17]:
```
plt.figure(figsize=(15,10))
sns.heatmap(df.corr(), annot=True)
```

[17]: <matplotlib.axes._subplots.AxesSubplot at 0x227573f2320>

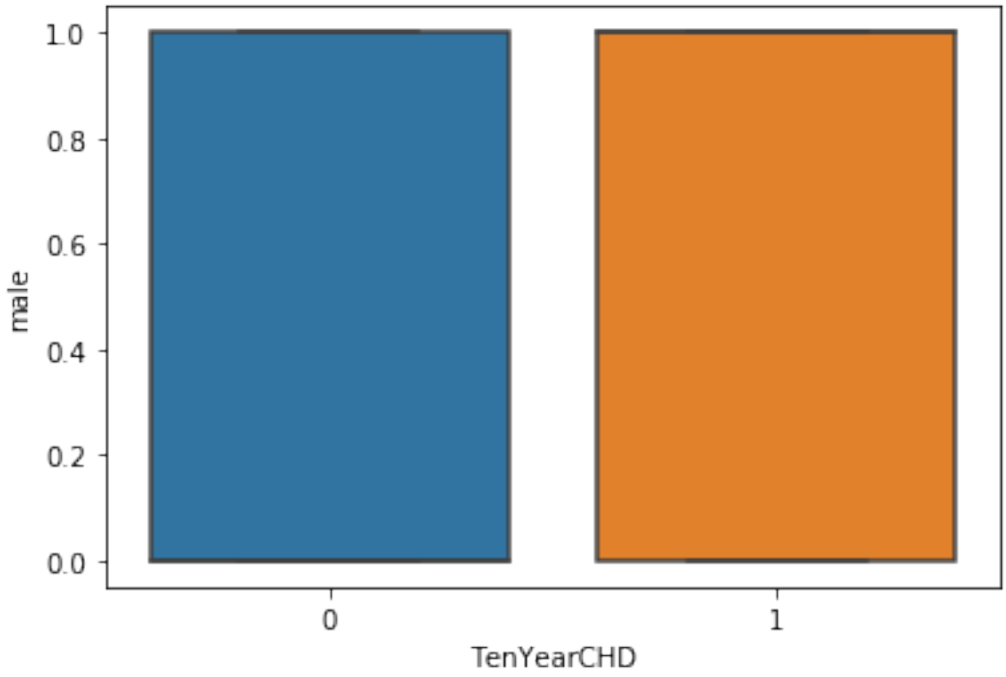


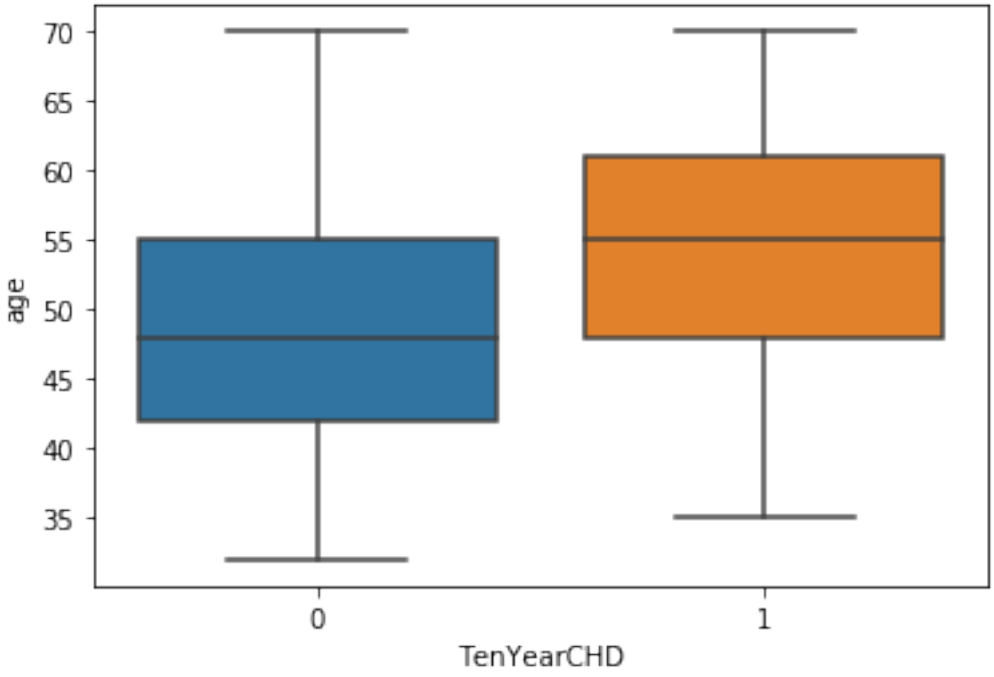

[18]:
```
import plusmodules as pm
pm.corr_matrix(df)
```

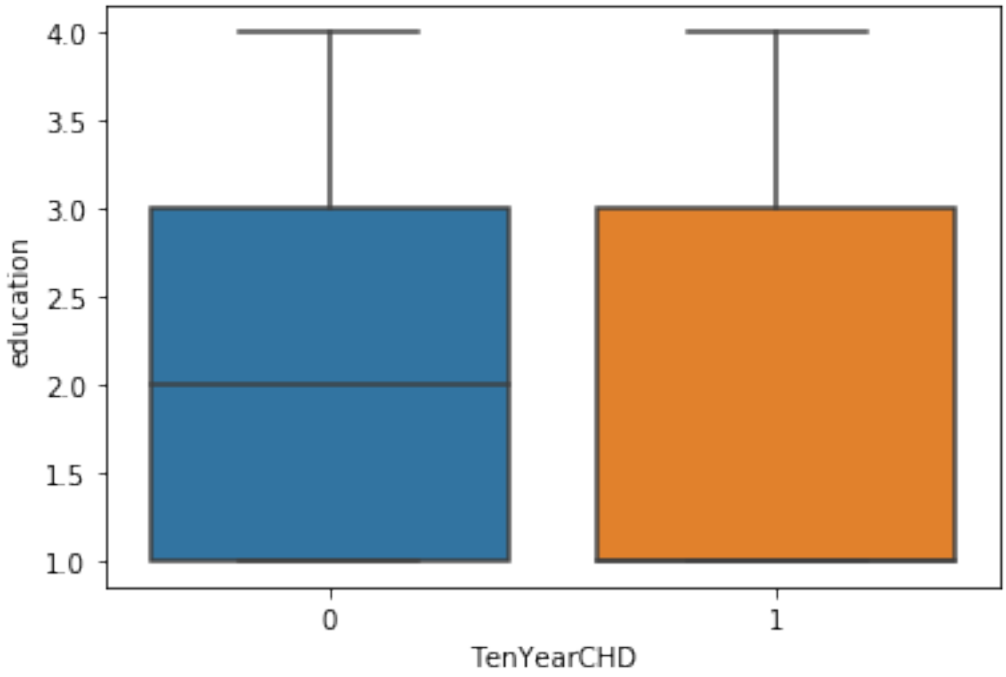


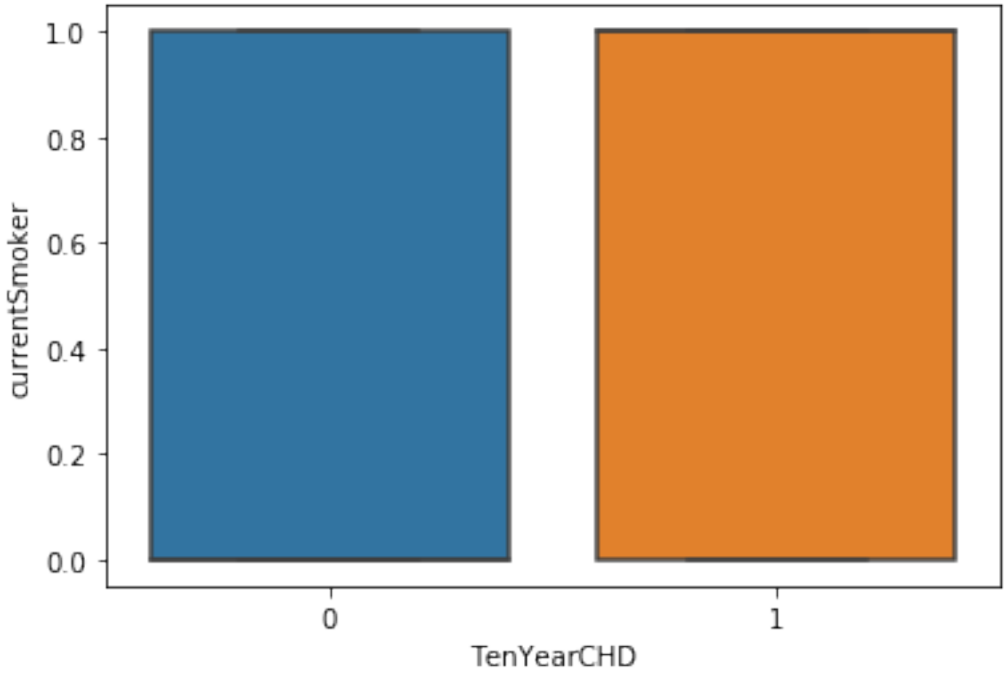

[19]:
```
cols=list(df.columns)

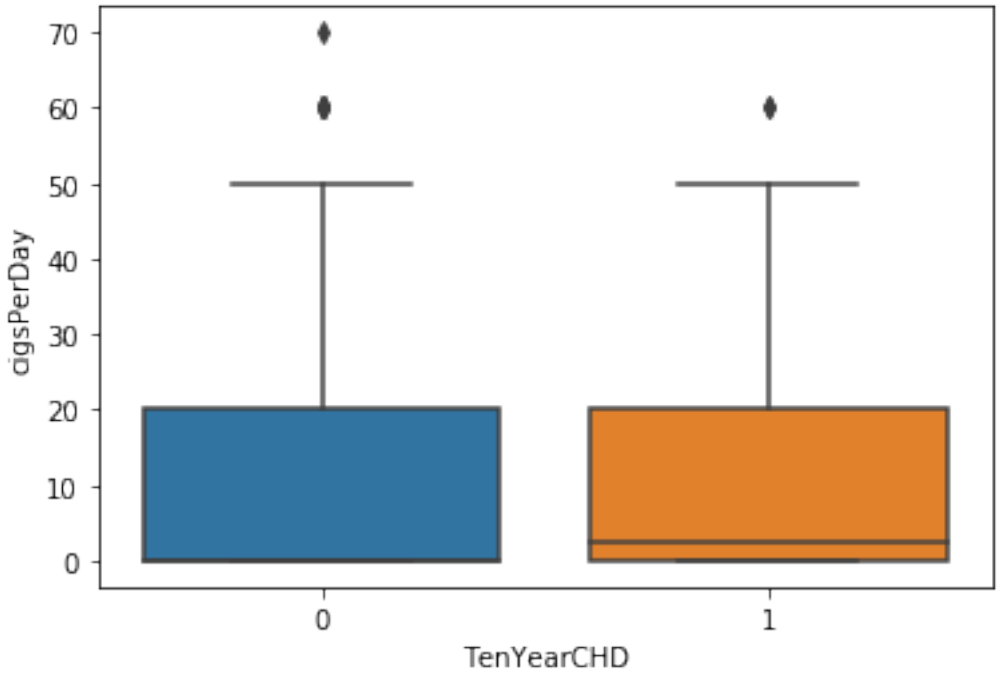
for col in cols:
    sns.boxplot(y=df[col],x=df['TenYearCHD'])
    plt.show()
```

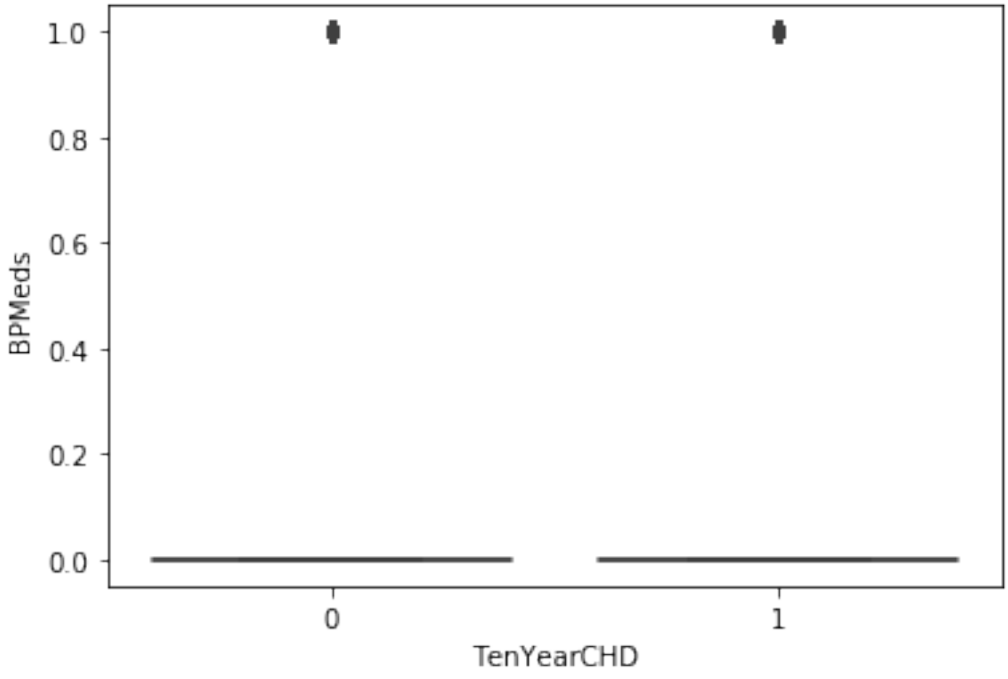

1.0
0.8
0.6
0.4
0.2
0.0
male
0
1
TenYearCHD

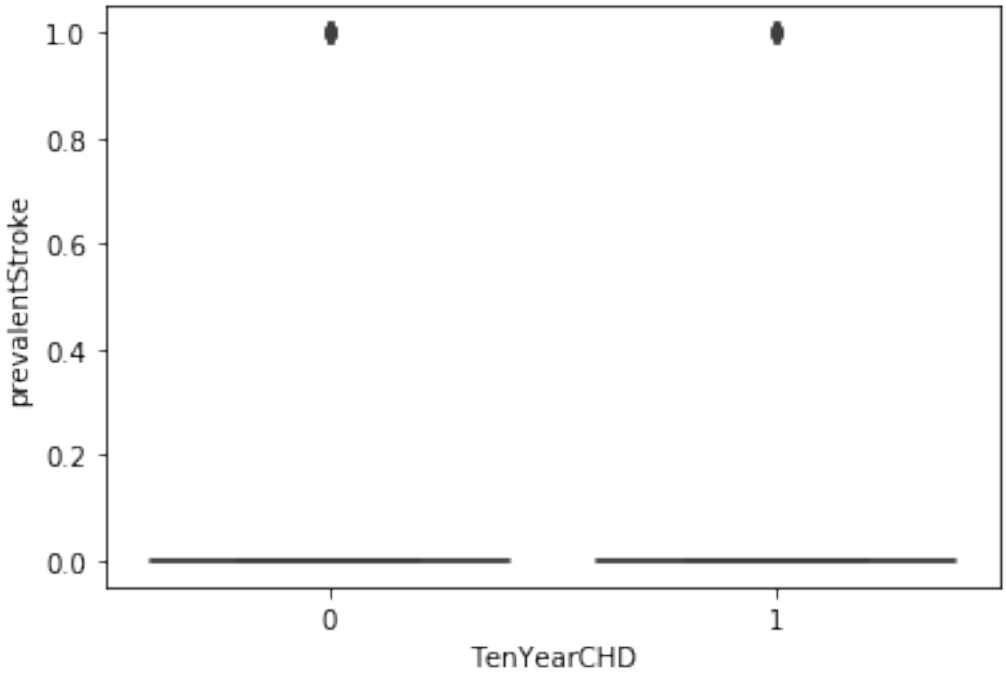

70
65
60
55
50
45
40
35
age
0
1
TenYearCHD

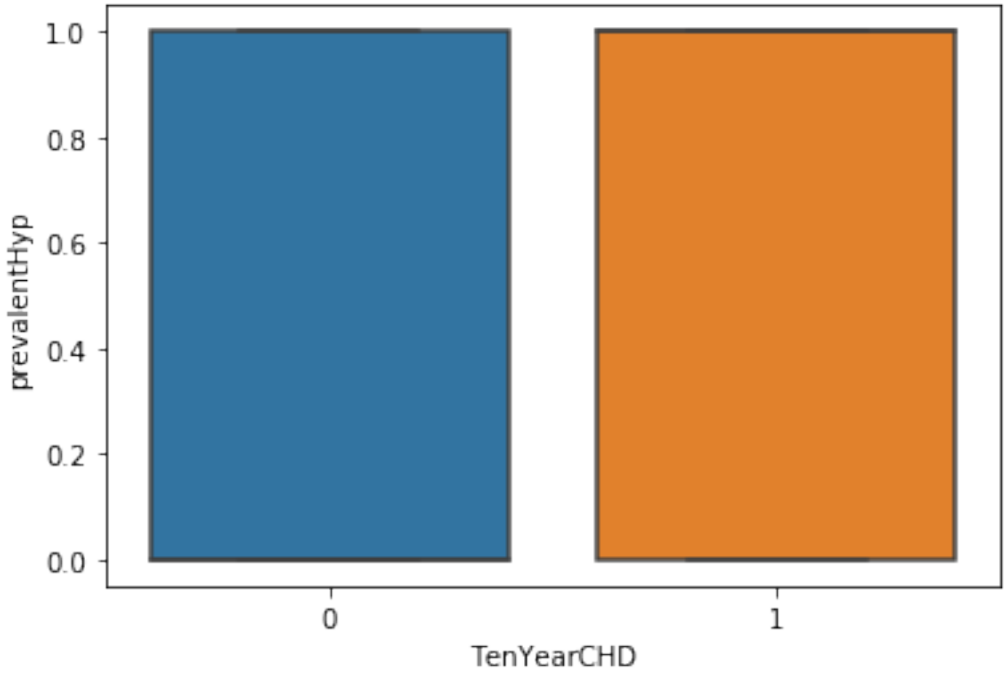

4.0
3.5
3.0
2.5
2.0
1.5
1.0
education
0
1
TenYearCHD

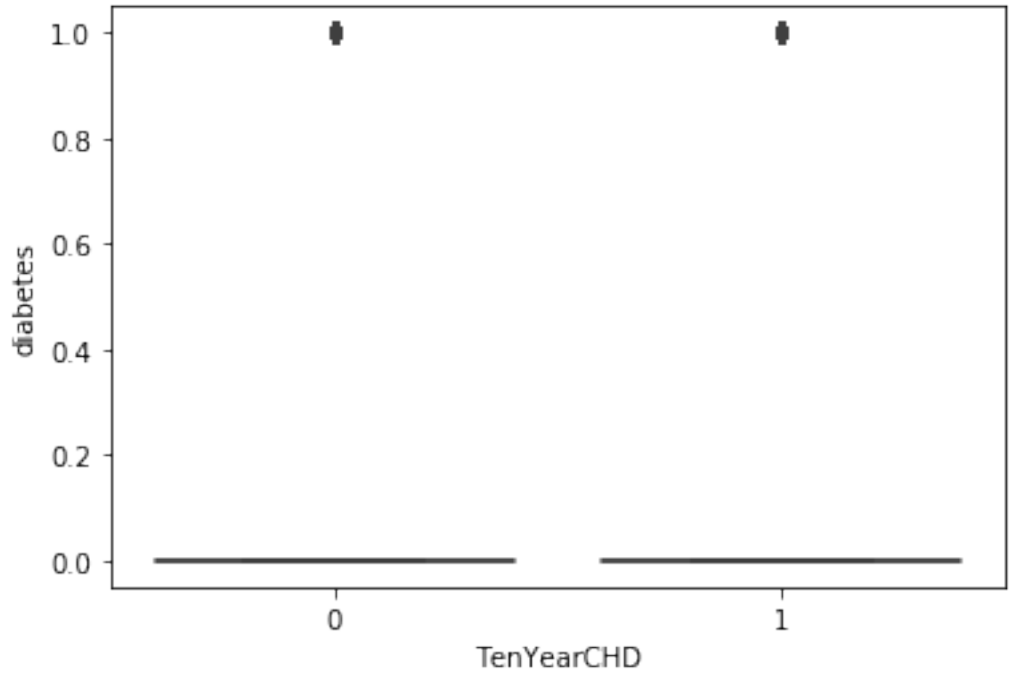

1.0
0.8
0.6
0.4
0.2
0.0
currentSmoker
0
1
TenYearCHD

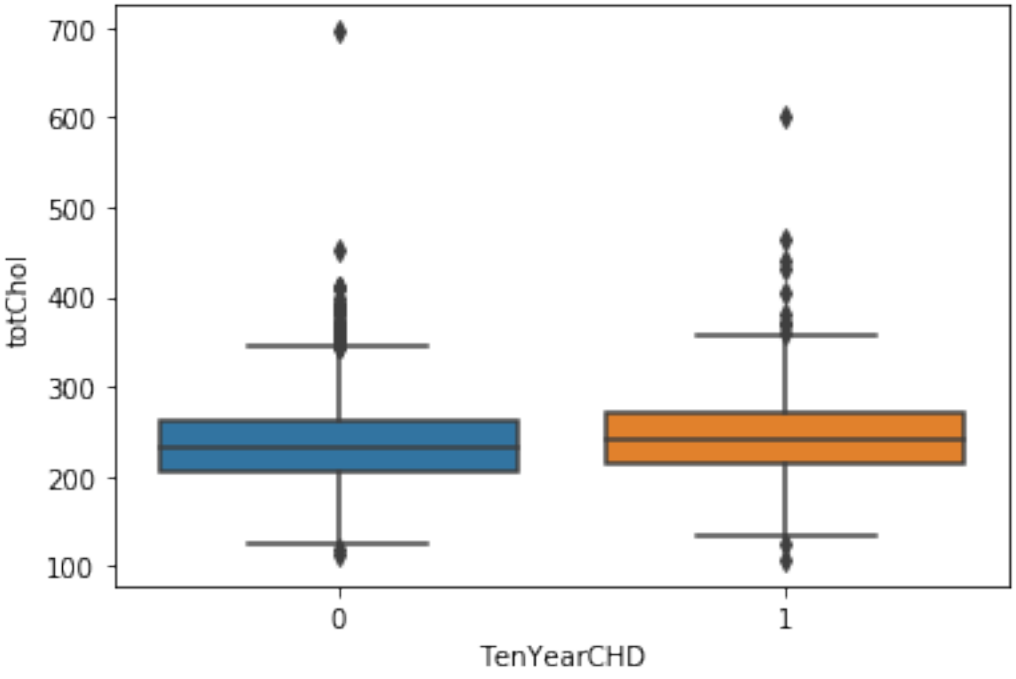

70
60
50
40
30
20
10
0
cigsPerDay
0
1
TenYearCHD

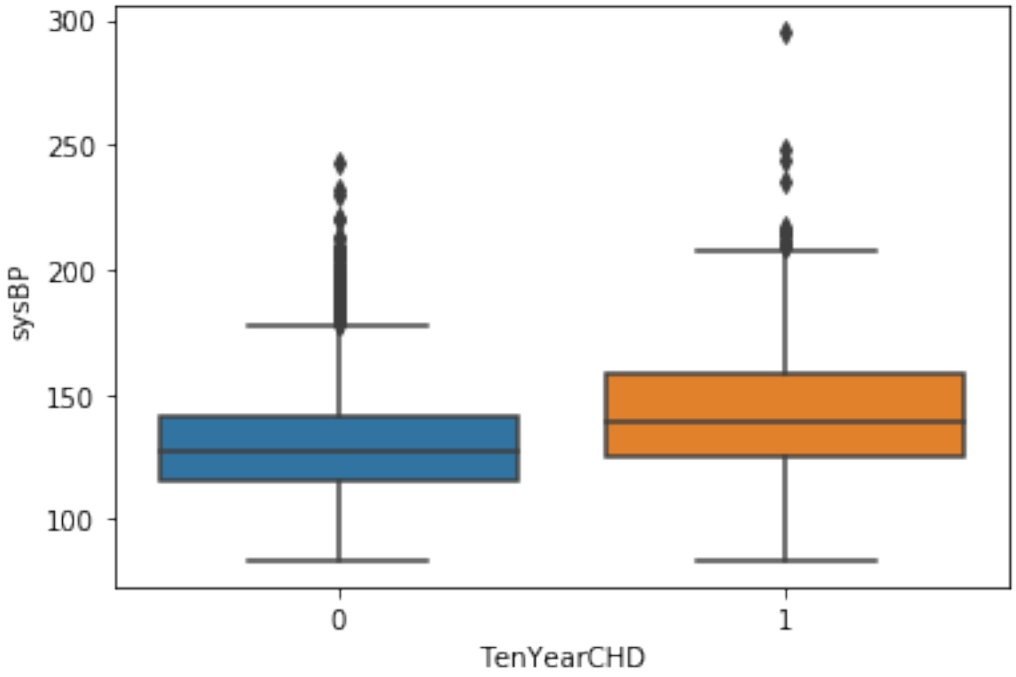

1.0
0.8
0.6
0.4
0.2
0.0
BPMeds
0
1
TenYearCHD

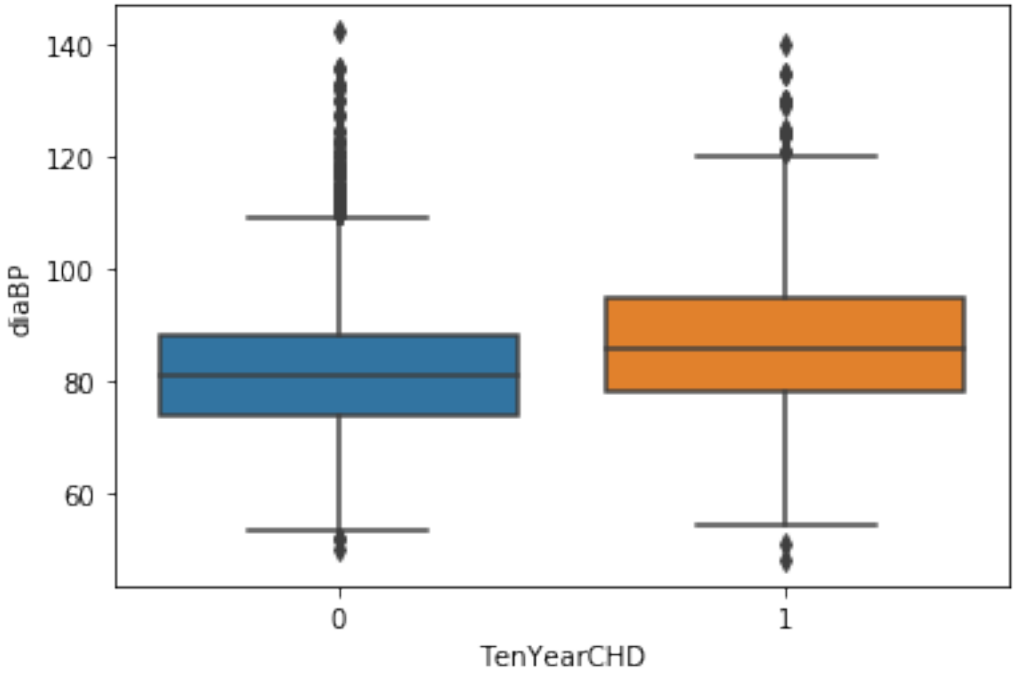

1.0
0.8
0.6
0.4
0.2
0.0
prevalentStroke
0
1
TenYearCHD

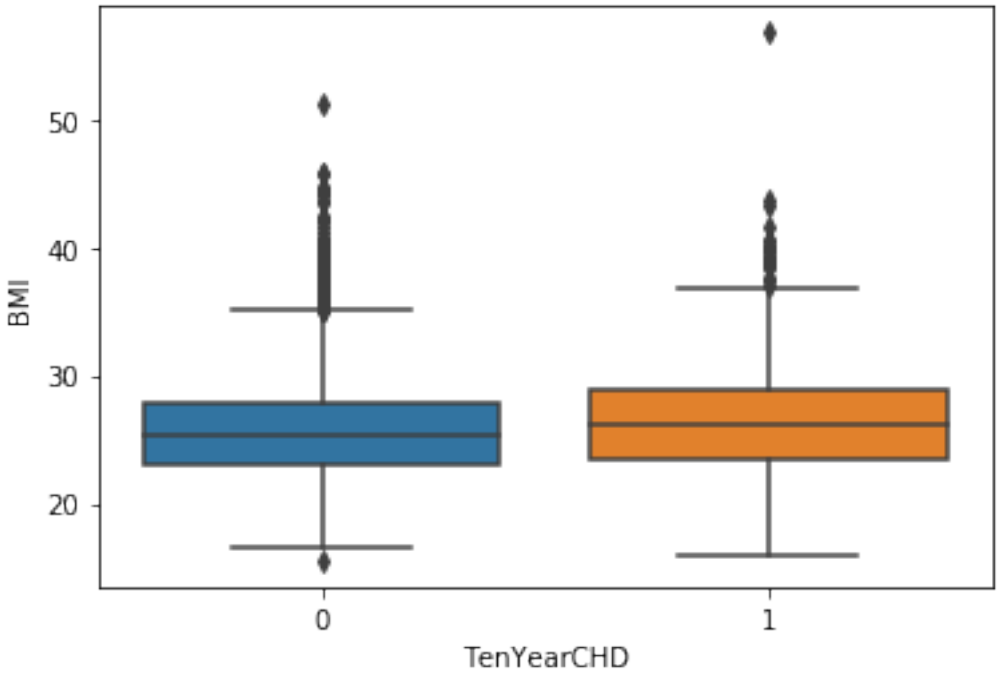

1.0
0.8
0.6
0.4
0.2
0.0
prevalentHyp
0
1
TenYearCHD

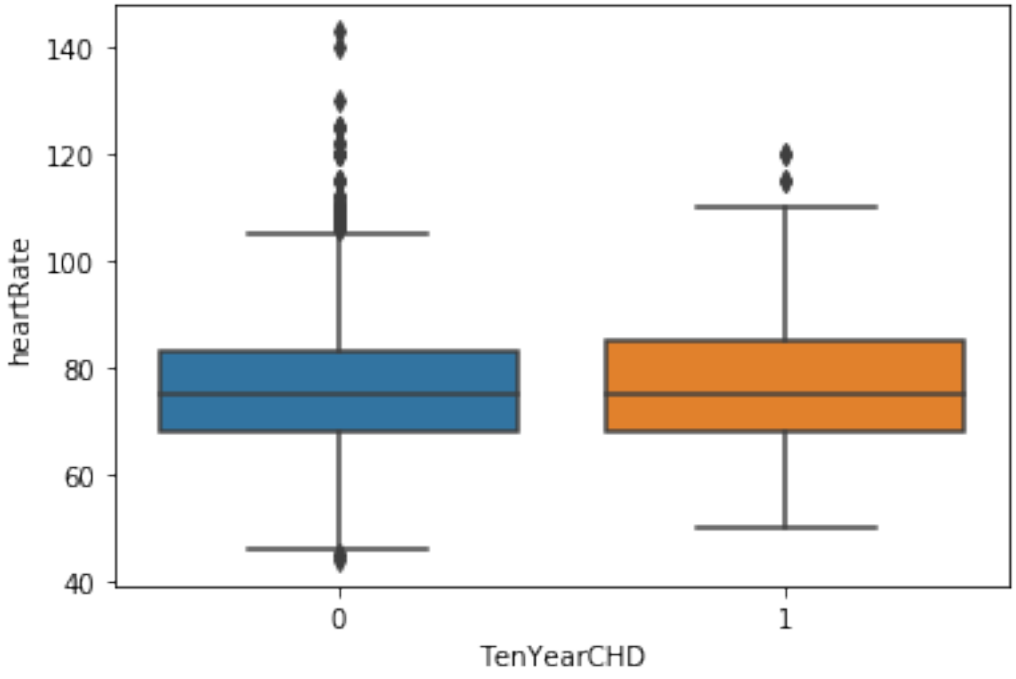

1.0
0.8
0.6
0.4
0.2
0.0
diabetes
0
1
TenYearCHD

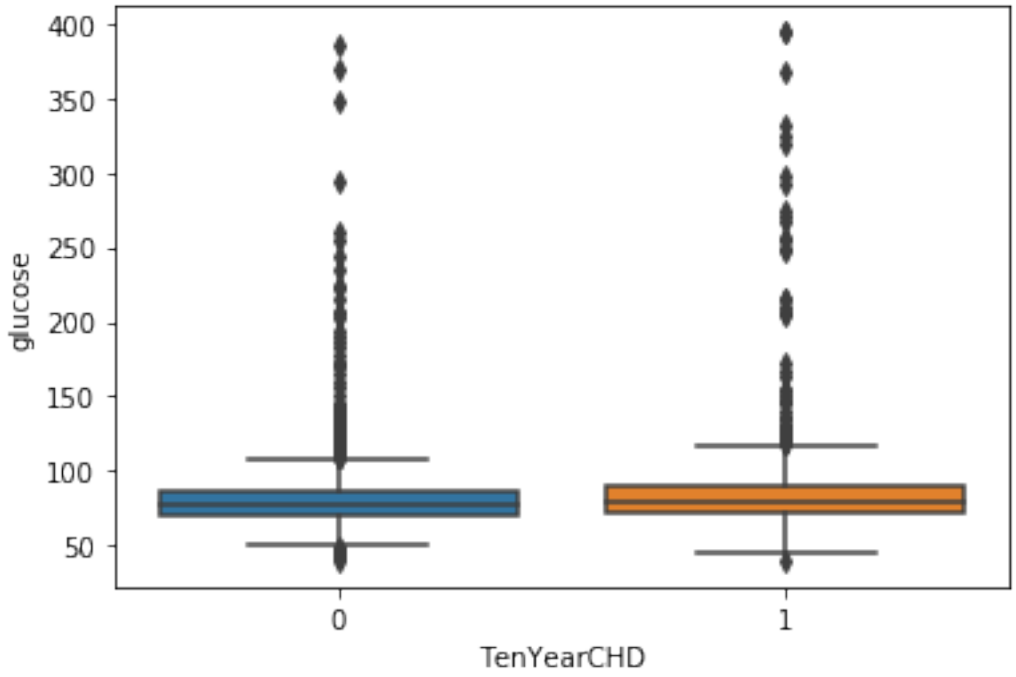

700
600
500
400
300
200
100
totChol
0
1
TenYearCHD

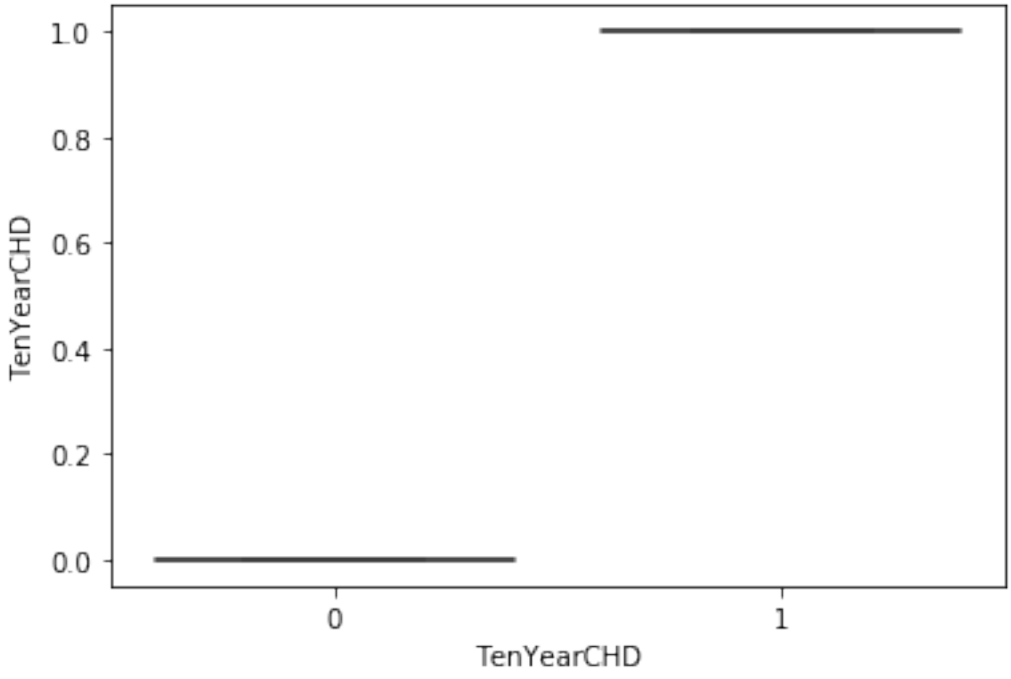

300
250
200
150
100
sysBP
0
1
TenYearCHD


140
120
100
80
60
diaBP
0
1
TenYearCHD


50
40
30
20
BMI
0
1
TenYearCHD


140
120
100
80
60
40
heartRate
0
1
TenYearCHD

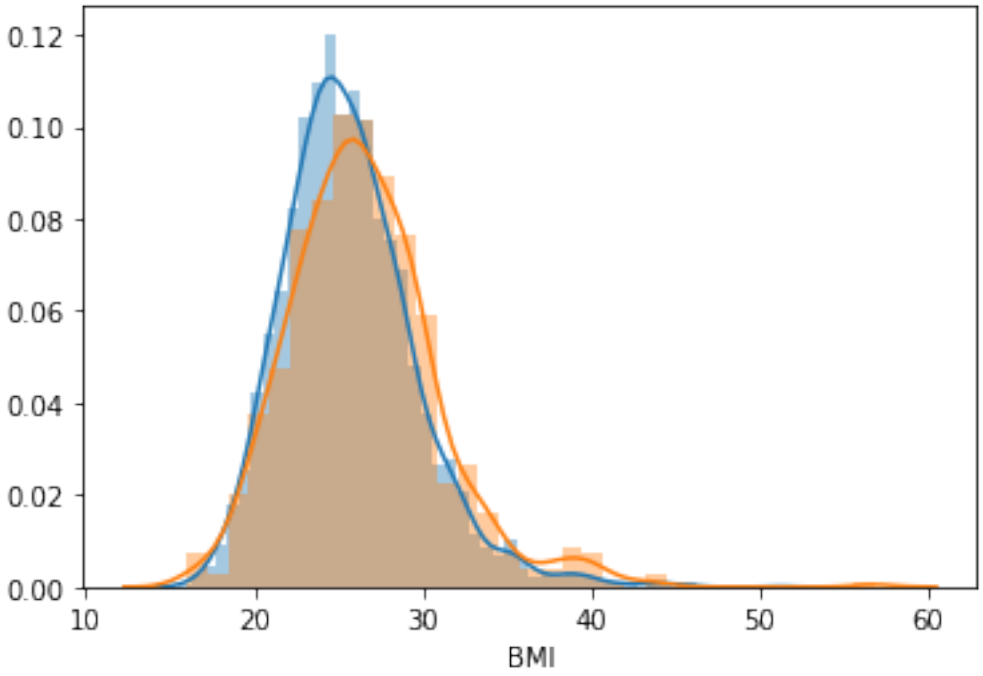



[20]:
```
df0= df[df['TenYearCHD']==0]
df1= df[df['TenYearCHD']==1]
```

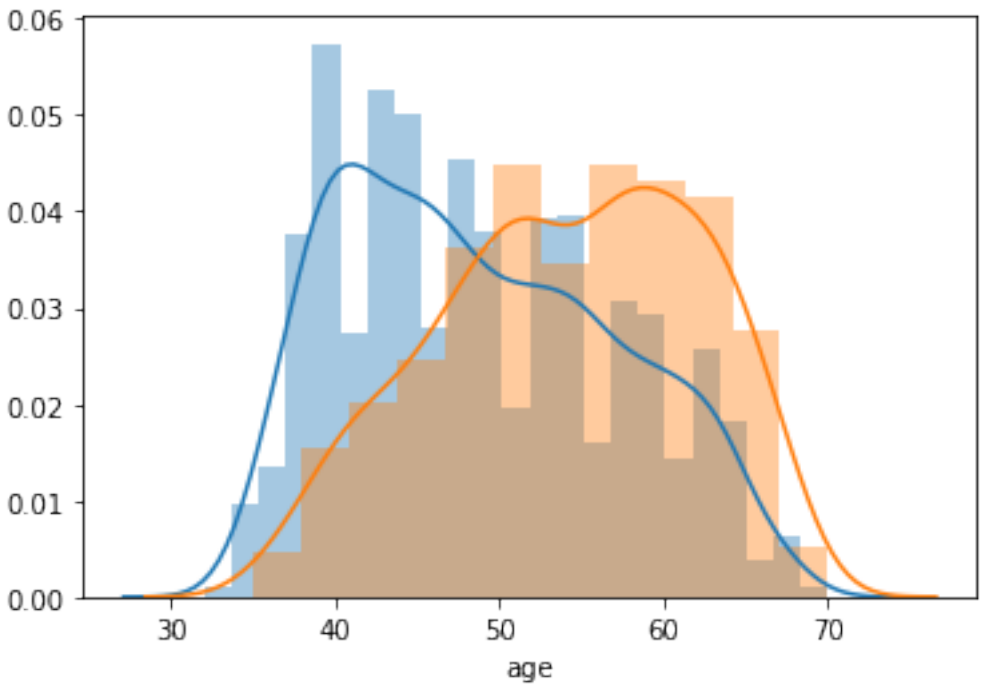
```
sns.distplot(df0['BMI'])
sns.distplot(df1['BMI'])
```

[20]: <matplotlib.axes._subplots.AxesSubplot at 0x2275bb69518>




[21]:
```
df0= df[df['TenYearCHD']==0]
df1= df[df['TenYearCHD']==1]
sns.distplot(df0['age'])
sns.distplot(df1['age'])
```

[21]: <matplotlib.axes._subplots.AxesSubplot at 0x2275bcbce10>




[22]:
```
x=df.drop('TenYearCHD', axis=1)
y=df['TenYearCHD']
con=sm.add_constant(x)
```

[23]:
```
from statsmodels.stats.outliers_influence import variance_inflation_factor as␣
↪vif

pd.DataFrame([vif(con.values , i) for i in range(con.shape[1])], index=con.
↪columns, columns=['VIF'])
```

[23]:

| | VIF |
|---|---|
| const | 197.703826 |
| male | 1.197415 |
| age | 1.368623 |
| education | 1.052772 |
| currentSmoker | 2.490359 |
| cigsPerDay | 2.618425 |
| BPMeds | 1.097285 |
| prevalentStroke | 1.018798 |
| prevalentHyp | 2.050602 |
| diabetes | 1.551588 |
| totChol | 1.104573 |
| sysBP | 3.733003 |
| diaBP | 2.961913 |

```
BMI                1.234955
heartRate          1.095442
glucose            1.569448
```

[ ]:

BUILDING LOGISTIC REGRESSION MODEL

[24]:
```
model=sm.Logit(y,x).fit()
model.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.397264
         Iterations 6
```

[24]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                           Logit Regression Results
==============================================================================
Dep. Variable:             TenYearCHD   No. Observations:                 4240
Model:                          Logit   Df Residuals:                     4225
Method:                           MLE   Df Model:                           14
Date:                Tue, 03 Mar 2020   Pseudo R-squ.:                 0.06739
Time:                        10:40:25   Log-Likelihood:                -1684.4
converged:                       True   LL-Null:                       -1806.1
Covariance Type:            nonrobust   LLR p-value:                 6.601e-44
==============================================================================
===
                     coef    std err          z      P>|z|      [0.025
0.975]
------------------------------------------------------------------------------
---
male                0.3752      0.097      3.863      0.000       0.185
0.566
age                 0.0282      0.005      5.177      0.000       0.018
0.039
education          -0.1652      0.044     -3.732      0.000      -0.252
-0.078
currentSmoker      -0.2588      0.140     -1.842      0.065      -0.534
0.017
cigsPerDay          0.0232      0.006      4.114      0.000       0.012
0.034
BPMeds              0.4356      0.214      2.035      0.042       0.016
0.855
prevalentStroke     0.8754      0.436      2.006      0.045       0.020
1.731
prevalentHyp        0.9055      0.116      7.839      0.000       0.679
1.132
```

```
diabetes            0.8494      0.275       3.092       0.002       0.311
1.388
totChol            -0.0011      0.001      -1.112       0.266      -0.003
0.001
sysBP               0.0114      0.004       3.233       0.001       0.004
0.018
diaBP              -0.0245      0.006      -4.305       0.000      -0.036
-0.013
BMI                -0.0507      0.011      -4.447       0.000      -0.073
-0.028
heartRate          -0.0200      0.004      -5.460       0.000      -0.027
-0.013
glucose             0.0015      0.002       0.746       0.456      -0.002
0.005
================================================================================
===
"""
```

coeff values gives the limit of the curve which is for age we get it as 0.0619 and the area for age lies in between +0.0619 and -0.0619 and the pvalue is 0.000 so based on the pvalue, if we get higher p value then the feature is not significant otherwise it is significant.

based on the p vlaue and the significance we remove the features which have the highest pvalue

[25]:
```
# we do backward elimination process to remove the insignificant features which␣
 ↪are not required for the model
```

[26]:
```
p=model.pvalues
p
```

[26]:
```
male               1.120983e-04
age                2.255895e-07
education          1.903272e-04
currentSmoker      6.542441e-02
cigsPerDay         3.885736e-05
BPMeds             4.185155e-02
prevalentStroke    4.481306e-02
prevalentHyp       4.523480e-15
diabetes           1.987960e-03
totChol            2.661457e-01
sysBP              1.224859e-03
diaBP              1.666984e-05
BMI                8.698494e-06
heartRate          4.759197e-08
glucose            4.557022e-01
dtype: float64
```

[27]:
```
cols=list(con.columns)

while len(cols)>1:
    x1=con[cols]
    model=sm.Logit(y, x1).fit()
    p=model.pvalues
    max_p=max(p)
    feature_maxp=p.idxmax()

    if max_p>0.05:
        print('\n')
        print('column removed: ', feature_maxp, 'prob :', max_p)
        cols.remove(feature_maxp)
    else:
        break
cols
```

```
Optimization terminated successfully.
         Current function value: 0.378369
         Iterations 7


column removed:  currentSmoker prob : 0.9248067145320612
Optimization terminated successfully.
         Current function value: 0.378370
         Iterations 7


column removed:  BMI prob : 0.818347435155813
Optimization terminated successfully.
         Current function value: 0.378376
         Iterations 7


column removed:  heartRate prob : 0.6936547572101218
Optimization terminated successfully.
         Current function value: 0.378395
         Iterations 7


column removed:  education prob : 0.7021584464005117
Optimization terminated successfully.
         Current function value: 0.378412
         Iterations 7


column removed:  diaBP prob : 0.6177168524266301
```

```
Optimization terminated successfully.
         Current function value: 0.378441
         Iterations 7


column removed:  diabetes prob : 0.4300198717944336
Optimization terminated successfully.
         Current function value: 0.378513
         Iterations 7


column removed:  BPMeds prob : 0.20712144906607255
Optimization terminated successfully.
         Current function value: 0.378697
         Iterations 7


column removed:  prevalentHyp prob : 0.06386738963096576
Optimization terminated successfully.
         Current function value: 0.379099
         Iterations 7
```

[27]:
```
['const',
 'male',
 'age',
 'cigsPerDay',
 'prevalentStroke',
 'totChol',
 'sysBP',
 'glucose']
```

[28]:
```
model=sm.Logit(y, con[cols]).fit()
model.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.379099
         Iterations 7
```

[28]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                           Logit Regression Results
==============================================================================
Dep. Variable:             TenYearCHD   No. Observations:                 4240
Model:                          Logit   Df Residuals:                     4232
Method:                           MLE   Df Model:                            7
Date:                Tue, 03 Mar 2020   Pseudo R-squ.:                 0.1100
Time:                        10:40:25   Log-Likelihood:                -1607.4
converged:                       True   LL-Null:                       -1806.1
```

```
Covariance Type:            nonrobust   LLR p-value:                 8.410e-82
================================================================================
===
                       coef    std err          z      P>|z|      [0.025
0.975]
--------------------------------------------------------------------------------
---
const               -8.8484      0.438    -20.220      0.000      -9.706
-7.991
male                 0.5096      0.098      5.184      0.000       0.317
0.702
age                  0.0635      0.006     10.632      0.000       0.052
0.075
cigsPerDay           0.0212      0.004      5.510      0.000       0.014
0.029
prevalentStroke      1.0675      0.437      2.443      0.015       0.211
1.924
totChol              0.0021      0.001      2.081      0.037       0.000
0.004
sysBP                0.0167      0.002      8.299      0.000       0.013
0.021
glucose              0.0074      0.002      4.579      0.000       0.004
0.011
================================================================================
===
"""
```

AGE

1. positive sign of age indicates that probability of CHD increases with age
2. log(odds) of CHD increased by 0.0646 when age increased by 1 year
3. as the age increases by 1 years then odds(CHD) increases by 6 percent (1.066-1)

[29]:
```
exp_b=pd.DataFrame({'coef': model.params, 'exp_coef':np.exp(model.params)})
exp_b
```

[29]:

| | coef | exp_coef |
|---|---|---|
| const | -8.848368 | 0.000144 |
| male | 0.509623 | 1.664664 |
| age | 0.063459 | 1.065516 |
| cigsPerDay | 0.021153 | 1.021378 |
| prevalentStroke | 1.067451 | 2.907957 |
| totChol | 0.002113 | 1.002115 |
| sysBP | 0.016687 | 1.016827 |
| glucose | 0.007373 | 1.007400 |

#MALE

1. positive value of male says that there is more chances for CHD in male when compared to

female
2. log(odds) of CHD for male is 0.4897 higher when compared to female
3. odds(CHD) of male is 63% (exp_coef-1) greater than female

[30]:
```
x_prob=con[cols]
x_prob['y']=y.values
```

[31]:
```
x_prob.head()
```

[31]:
```
      const  male  age  cigsPerDay  prevalentStroke  totChol  sysBP  glucose  \
3546    1.0     0   54         0.0                0    241.0  106.0     74.0
1127    1.0     0   42        10.0                0    253.0  109.0     60.0
3088    1.0     0   58         0.0                1    274.0  159.0     81.0
437     1.0     1   45        30.0                0    240.0  141.0     76.0
3188    1.0     1   63         0.0                0    190.0  148.0     86.0

      y
3546  0
1127  0
3088  0
437   0
3188  0
```

[32]:
```
x_prob.to_csv('x_prob.csv')
```

[33]:
```
prob=model.predict(con[cols])
```

[34]:
```
type(prob)
```

[34]:
```
pandas.core.series.Series
```

[35]:
```
prob
```

[35]:
```
3546    0.069264
1127    0.040086
3088    0.432654
437     0.193365
3188    0.302438
          …
789     0.344482
968     0.120925
1667    0.171440
3321    0.193964
1688    0.038748
Length: 4240, dtype: float64
```

[36]:
```
prob=pd.DataFrame(prob, columns=['prob'])
prob.head()
```

[36]:
```
          prob
3546  0.069264
1127  0.040086
3088  0.432654
437   0.193365
3188  0.302438
```

[44]:
```
prob['y_est']=prob['prob'].apply(lambda x: 1 if x>0.5 else 0)
```

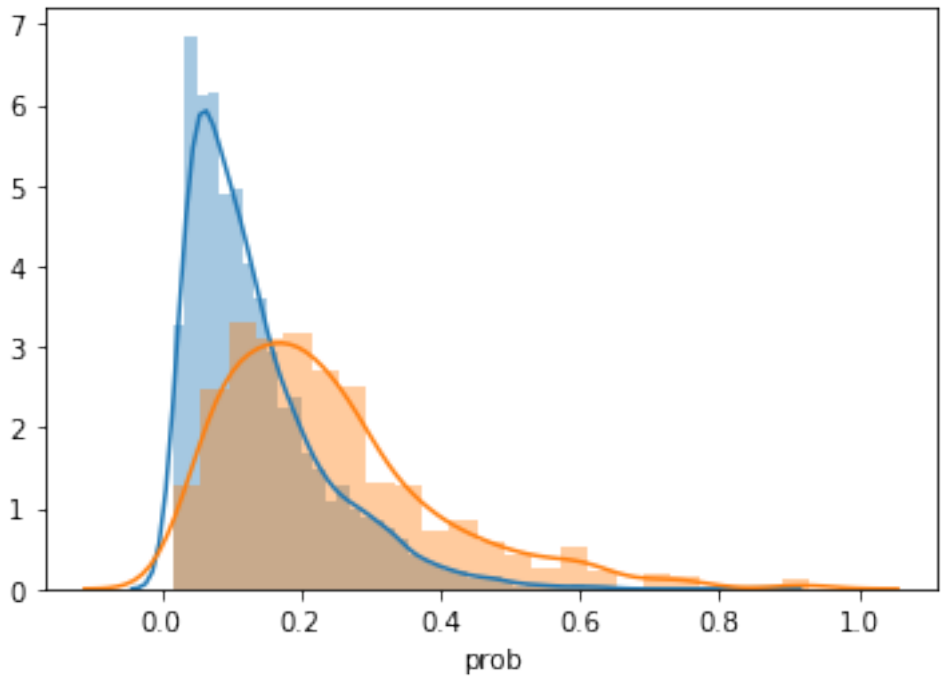
[45]:
```
from sklearn.metrics import confusion_matrix
confusion_matrix(y, prob['y_est'])
```

[45]:
```
array([[3573,   23],
       [ 595,   49]], dtype=int64)
```

[46]:
```
prob['y']=y.values
```

[47]:
```
df0=prob[prob['y']==0]
df1=prob[prob['y']==1]
sns.distplot(df0['prob'])
sns.distplot(df1['prob'])
```

[47]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x2275c89d710>
```



[48]:
```
#Classification report
from sklearn.metrics import classification_report, accuracy_score,␣
↪confusion_matrix
```

[51]:
```
confusion_matrix(y, prob['y_est'])
```

[51]:
```
array([[3573,   23],
       [ 595,   49]], dtype=int64)
```

[50]:
```
accuracy_score(y,prob['y_est'])
```

[50]:
```
0.8542452830188679
```

# 1 ROC analysis

[52]:
```
from sklearn.metrics import roc_auc_score, roc_curve
```

[55]:
```
print('AUC value for the model is :', roc_auc_score(y, prob['prob']))
```

```
AUC value for the model is : 0.7304199282847056
```

[68]:
```
fpr, tpr, thresholds = roc_curve(y, prob['prob'])
```

[69]:
```
fpr
```

[69]:
```
array([0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 2.78086763e-04,
       2.78086763e-04, 5.56173526e-04, 5.56173526e-04, 8.34260289e-04,
       8.34260289e-04, 1.39043382e-03, 1.39043382e-03, 2.22469410e-03,
       2.22469410e-03, 2.50278087e-03, 2.50278087e-03, 2.78086763e-03,
       2.78086763e-03, 3.33704116e-03, 3.33704116e-03, 3.61512792e-03,
       3.61512792e-03, 3.89321468e-03, 3.89321468e-03, 4.44938821e-03,
       4.44938821e-03, 4.72747497e-03, 4.72747497e-03, 5.00556174e-03,
       5.00556174e-03, 5.28364850e-03, 5.28364850e-03, 6.67408231e-03,
       6.67408231e-03, 7.23025584e-03, 7.23025584e-03, 8.89877642e-03,
       8.89877642e-03, 9.73303671e-03, 9.73303671e-03, 1.02892102e-02,
       1.02892102e-02, 1.16796440e-02, 1.16796440e-02, 1.25139043e-02,
       1.25139043e-02, 1.27919911e-02, 1.27919911e-02, 1.36262514e-02,
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       1.47385984e-02, 1.50166852e-02, 1.50166852e-02, 1.52947720e-02,
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       1.80756396e-02, 1.83537264e-02, 1.83537264e-02, 1.89098999e-02,
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0.10042395, 0.10037017, 0.09990133, 0.0997378 , 0.09957168,
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0.09377728, 0.09363914, 0.09361491, 0.09259582, 0.09245396,
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0.08362683, 0.08350795, 0.08298388, 0.08291183, 0.08230025,
0.08226383, 0.08100847, 0.08090965, 0.08072979, 0.08071016,
0.08045552, 0.08041438, 0.08026502, 0.08019655, 0.0799844 ,
0.07998133, 0.07995473, 0.07995433, 0.0797089 , 0.0797061 ,
0.07945378, 0.07943751, 0.07888998, 0.07885991, 0.07770182,
0.07769342, 0.07759494, 0.0774411 , 0.07519614, 0.07489458,
0.07418652, 0.07417896, 0.07354856, 0.07354527, 0.07337354,
0.07336796, 0.07322231, 0.07319695, 0.07307742, 0.07299536,
0.07283037, 0.07280322, 0.07217041, 0.07216143, 0.07187547,
0.07186352, 0.0714693 , 0.07142604, 0.07119708, 0.07113929,
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0.06684286, 0.06524281, 0.06522665, 0.06402206, 0.06398712,
0.06334123, 0.06327201, 0.06159922, 0.06158461, 0.06125926,
0.06115098, 0.05975845, 0.05975037, 0.05964408, 0.05964223,
0.05948584, 0.05948231, 0.05911416, 0.05908679, 0.05868924,

```
       0.05862304, 0.0577153 , 0.05759739, 0.05747826, 0.05742848,
       0.05668755, 0.05666189, 0.05593665, 0.05590074, 0.05521105,
       0.05508452, 0.0549536 , 0.05493949, 0.05430349, 0.05429474,
       0.05316018, 0.05315129, 0.05163659, 0.0515894 , 0.05148726,
       0.05146957, 0.05137055, 0.05126952, 0.05095405, 0.05095118,
       0.05071836, 0.05067791, 0.05034513, 0.05031417, 0.04949195,
       0.04949127, 0.04869796, 0.04866308, 0.0480071 , 0.04785995,
       0.04661886, 0.04656894, 0.04505122, 0.04501014, 0.04500188,
       0.04500046, 0.04498397, 0.04495405, 0.04376022, 0.04373156,
       0.0434092 , 0.04340905, 0.04322769, 0.04320108, 0.04175277,
       0.04172554, 0.04089151, 0.04080126, 0.04071272, 0.04069495,
       0.04036055, 0.04023161, 0.03407794, 0.03401422, 0.03132938,
       0.03122749, 0.03051799, 0.03049861, 0.02486645, 0.024665  ,
       0.01832377, 0.01813727, 0.01746985, 0.01744419, 0.01615037])
```

[72]:
```
roc=pd.DataFrame({'fpr': fpr,'tpr': tpr,'thresholds': thresholds, })
```

[73]:
```
roc
```

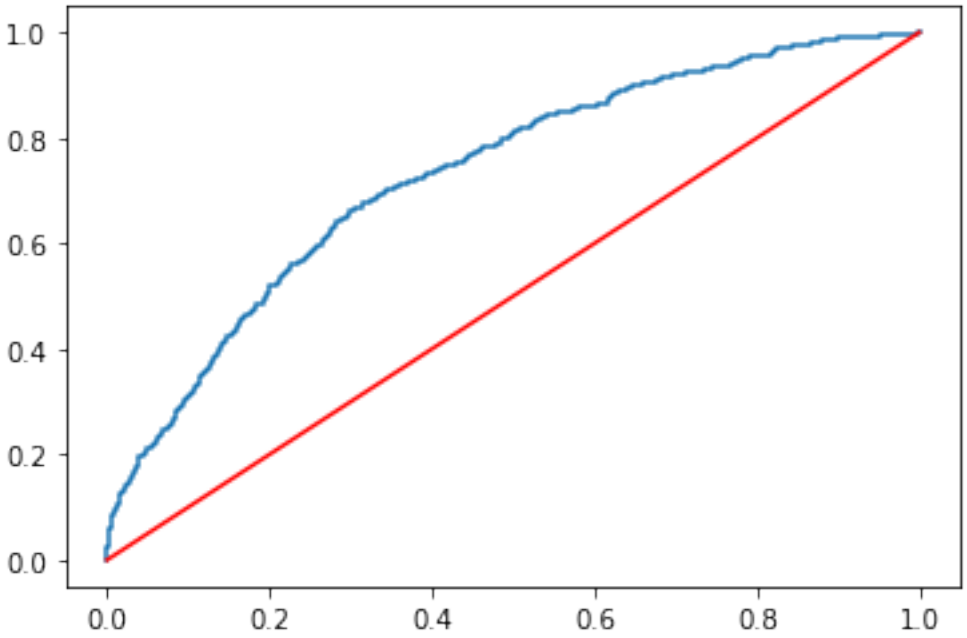
[73]:

| | fpr | tpr | thresholds |
|---|---|---|---|
| 0 | 0.000000 | 0.000000 | 0.928660 |
| 1 | 0.000000 | 0.001553 | 0.928660 |
| 2 | 0.000000 | 0.004658 | 0.913129 |
| 3 | 0.000278 | 0.004658 | 0.858071 |
| 4 | 0.000278 | 0.021739 | 0.644514 |
| .. | … | … | … |
| 975 | 0.998888 | 0.996894 | 0.018324 |
| 976 | 0.998888 | 0.998447 | 0.018137 |
| 977 | 0.999166 | 0.998447 | 0.017470 |
| 978 | 0.999166 | 1.000000 | 0.017444 |
| 979 | 1.000000 | 1.000000 | 0.016150 |

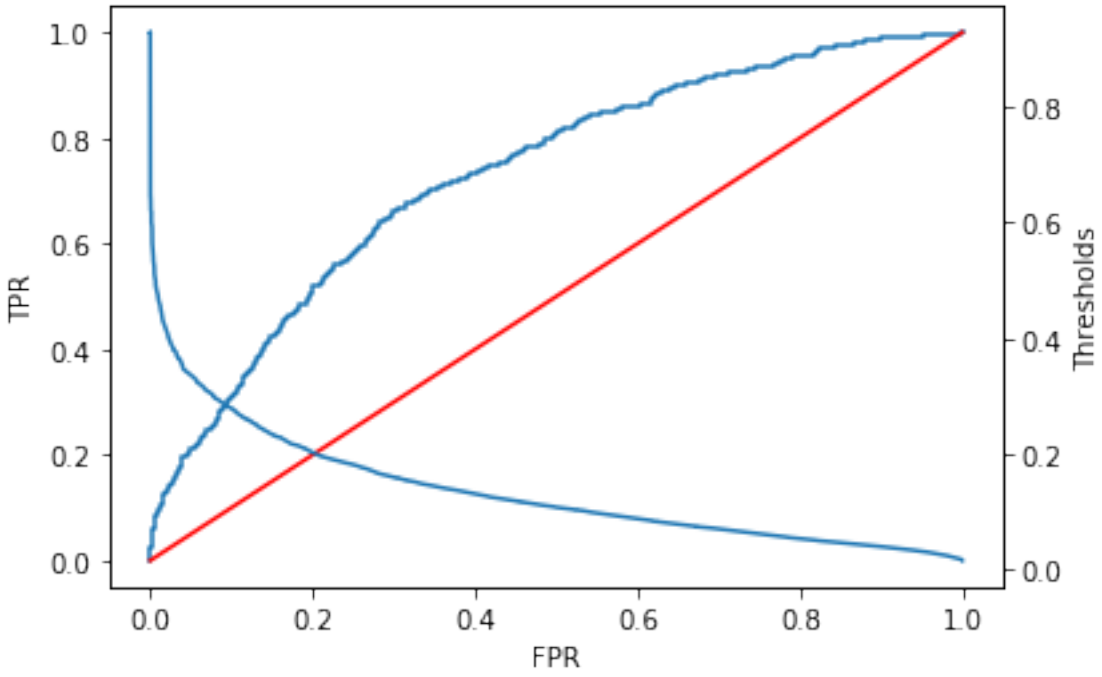
[980 rows x 3 columns]

[90]:
```
plt.plot(fpr, tpr)
plt.plot(fpr, fpr, 'red')
```

[90]:
```
[<matplotlib.lines.Line2D at 0x2275adefda0>]
```




[86]:
```
fig, ax= plt.subplots()
ax.plot(fpr, tpr)
ax.plot(fpr, fpr, 'red')
ax.set_xlabel('FPR')
ax.set_ylabel('TPR')
ax1=ax.twinx()
ax1.set_ylabel('Thresholds')
ax1.plot(fpr, thresholds)
plt.show()
```




[92]:
```
roc[(roc['tpr']>=0.80) & (roc['tpr']<=0.81)]
```

[92]:

| | fpr | tpr | thresholds |
|---|---|---|---|
| 738 | 0.494160 | 0.801242 | 0.109596 |
| 739 | 0.496663 | 0.801242 | 0.109258 |
| 740 | 0.496663 | 0.802795 | 0.109103 |
| 741 | 0.497497 | 0.802795 | 0.109003 |
| 742 | 0.497497 | 0.804348 | 0.108997 |
| 743 | 0.498331 | 0.804348 | 0.108638 |
| 744 | 0.498331 | 0.805901 | 0.108583 |
| 745 | 0.498610 | 0.805901 | 0.108400 |
| 746 | 0.498610 | 0.809006 | 0.108310 |
| 747 | 0.499166 | 0.809006 | 0.108284 |

if the model is good then we might have got the value of fpr as very low with higher tpr, higher tpr and lower fpr says the threshold value

[ ]:
```

```