Optimization Assignment - 2

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***Problem Statement* -let x and y be two real variables such that $x \geq 0$ and $xy = 1$ Find the minimum value of $x + y$**

Solution

Theoretical approximation

Let x and y variables of the given function

Given

$$xy = 1 \tag{1}$$

can be written in quadratic form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

The given constraints can be expressed as conics with parameters

$$\mathbf{V} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -1 \tag{3}$$

$$\tag{4}$$

And

$$x \geq 0 \tag{5}$$

The minimum value of the function is given by,
Generally,

$$\begin{aligned} \left(x^2 - 1\right) &\leq 0 \quad \forall\, x \in \mathbf{R} \\ \implies -\left(x-1\right)^2 &\leq 1 \\ \implies x^2 &\leq 1 \end{aligned}$$

By solving this ,we get

$$x = \pm 1 \tag{6}$$

$$\boxed{x = 1} \tag{7}$$

$$since, itsgivenx \geq 0 \tag{8}$$

$$\tag{9}$$

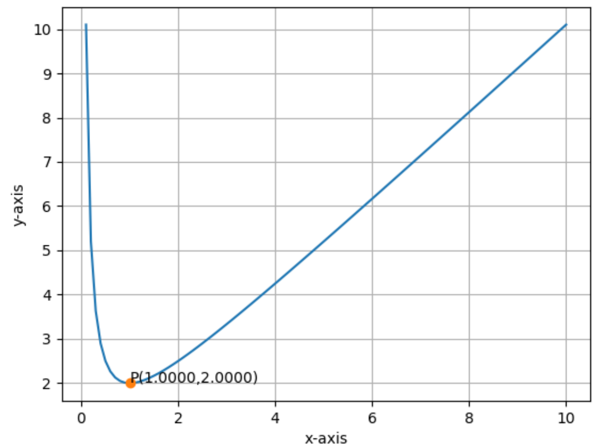

Figure 1: Graph of $f(x)$ vs x

Gradient descent

Let x be the variable of the given function
The minimum value of given function is expressed as

$$f = \min_{x,y} x + y \tag{10}$$

$$f = \min_{\mathbf{x}} \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} \tag{11}$$

$$\mathbf{x}^\top \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} 0 & 0 \end{pmatrix}\mathbf{x} = 1 \tag{12}$$

$$x \geq 0 \tag{13}$$

Using gradient ascent method we can find its minima

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{14}$$

$$\implies x_{n+1} = x_n + \alpha\left(\frac{1 - x^2}{x^2}\right) \tag{15}$$

Taking $x_0 = 1, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Minima} = 2} \tag{16}$$

$$\boxed{\text{Minima Point} = 1} \tag{17}$$