Optimization Assignment

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Problem Statement

let x and y be two real variables such that $x \geq 0$ **and**
$xy = 1$ **Find the minimum value of** $x + y$

Given

The function is
$$x + y \tag{1}$$

using cvxpy we can the formulate the given function as

$$f = \mathbf{cx} \tag{2}$$
$$c = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{3}$$

Given constraints are,
$$xy = 1 \tag{4}$$

can be written in quadratic form as

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{5}$$

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + f = 0 \tag{6}$$

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + f = 0 \tag{7}$$

Using SDR(Semi Definite Relaxation), it can be rewritten as

$$Tr(\mathbf{V}\mathbf{X}) + f = 0 \tag{8}$$
$$\tag{9}$$

Such that

$$Tr(\mathbf{V}\mathbf{X}) = -f \tag{10}$$
$$\tag{11}$$

Here , X is a 2×2 matrix of variables where

$$\mathbf{X} = \mathbf{x}^\top \mathbf{x} \tag{12}$$
$$\tag{13}$$

$$X \geq 0 \tag{14}$$

Where

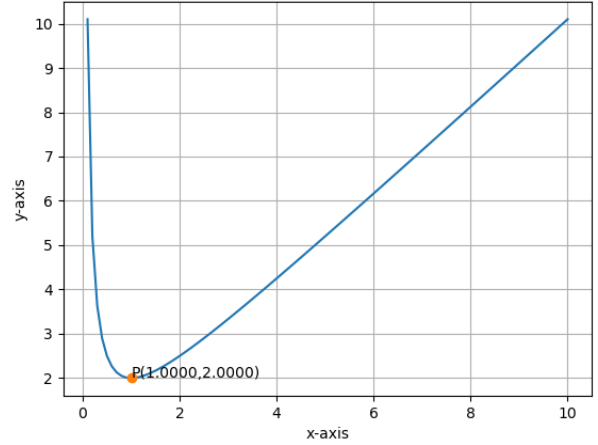$$\mathbf{V} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{15}$$
$$f = -1 \tag{16}$$

Figure 1: Graph of $f(x)$ vs x