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Internal Assessment Test – III June 2024

Sub:	Mathematics II for ECE Stream							Code:	BMATE201
Date:	24/06/2024	Duration:	90 mins	Max Marks:	50	Sem:	II	Section:	M, N, O, P

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1.	Solve by using modified Euler's method, $y' = \log_{10}(x+y)$, $y(0) = 2$ at $x = 0.2$..	[8]	CO4	L2
2.	Find the interpolation polynomial using Newton's divided difference formula for the following data	[7]	CO3	L3
3.	Using Lagrange's interpolation formulae to find $f(5)$, from the following data	[7]	CO3	L2
4.	Evaluate $\int_0^{0.6} e^{-x^2}\,dx$, by using Simpson's 1/3rd rule by taking 7 ordinates.	[7]	CO3	L2

Data for Q2:

x	0	1	2	5
y	2	3	12	147

Data for Q3:

x	1	3	4	6	9
f(x)	-3	9	30	132	156

for HoD

5.	Given $\sin 45° = 0.7071$, $\sin 50° = 0.7660, \sin 55° = 0.8192, \sin 60° = 0.8660$, find $\sin 52°$ using Newton's forward interpolation formula.	[7]	CO3	L2
6.	Find Taylor's series method the value of y at $x = 0.1$ to 4 place of decimals from $y' = xy^2 - 1$. , $y(0) = 1$	[7]	CO4	L2
7.	Using Runge-Kutta method of order 4, find y at $x = 0.6$, given that $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$, taking $h = 0.2$.	[7]	CO4	L3
8.	Apply Milne's predictor corrector method, find $y(0.4)$ $\frac{dy}{dx} = 2\, e^x y$ x: 0, 0.1, 0.2, 0.3 y: 2.4, 2.473, 3.129, 4.059	[7]	CO4	L2

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

IAT-2

Q.1 Solve by using modified Euler's method $y' = \log_{10}(x+y)$, $y(0) = 2$ at $x = 0.2$.

Sol: Given that, $f(x,y) = \log_{10}(x+y)$

$x_0 = 0$, $y_0 = 2$

To find $y(0.2)$.

Let $h = 0.1$

x	0	0.1	0.2
y	2	?	?

By Euler's formula,

$$y_1^{(0)} = y_0 + (h \cdot f(x_0, y_0))$$

$$= 2 + 0.1[\log_{10}(0+2)]$$

$$\Rightarrow y_1^{(0)} = 2.0301$$

By Modified Euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 2 + \frac{0.1}{2}\left[\log_{10}(0+2) + \log_{10}(0.1+2.0301)\right]$$

$$\Rightarrow y_1^{(1)} = 2.0315$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 2 + \frac{0.1}{1}\left[\log_{10}(0+2) + \log_{10}(0.1 + 2.0315)\right]$$

$$\Rightarrow y_1^{(2)} = 2.0315$$

$$\Rightarrow \boxed{y(0.1) = 2.0315}$$

x	0	0.1	0.2
y	2	2.0315	?

By Euler's method,

$$y_2^{(0)} = y_1 + h\,f(x_1, y_1)$$

$$= 2.0315 + 0.1\, f(0.1, 2.0315)$$

$$\Rightarrow y_2^{(0)} = 2.0644$$

By modified Euler's method,

$$y_2^{(1)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(0)})\right]$$

$$= 2.0315 + \frac{0.1}{2}\left[\log_{10}(0.1 + 2.0315) + \log_{10}(0.2 + 2.0644)\right]$$

$$\Rightarrow y_2^{(1)} = 2.0657$$

$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$

$$= 2.0315 + \frac{0.1}{2}\left[\log_{10}(0.1 + 2.0315) + \log_{10}(0.2 + 2.0657)\right]$$

$$\Rightarrow y_2^{(2)} = 2.0657$$

$$\Rightarrow \boxed{y(0.2) = 2.0657}$$

x	0	0.1	0.2
y	2	2.0315	2.0657

Q.2 Find the interpolation polynomial using Newton's divided difference formula for the following data;

x	0	1	2	5
y	2	3	12	147

Sol^n:- Divided difference table

x	y	1st D.D	2nd D.D	3rd D.D
$x_0 = 0$	2			
		$\frac{3-2}{1-0} = 1$		
$x_1 = 1$	3		$\frac{9-1}{2-0} = 4$	
		$\frac{12-3}{2-1} = 9$		$\frac{9-4}{5-0} = 1$
$x_2 = 2$	12		$\frac{45-9}{5-1} = 9$	
		$\frac{147-12}{5-2} = 45$		
$x_3 = 5$	147			

By Newton's divided difference formula, we have

$$f(x) = y_0 + (x-x_0)\, f(x_0,x_1) + (x-x_0)(x-x_1)\, f(x_0,x_1,x_2)$$
$$+ (x-x_0)(x-x_1)(x-x_2)\, f(x_0,x_1,x_2,x_3)$$

$$= 2 + (x-0)(1) + (x-0)(x-1)\,4 + (x-0)(x-1)(x-2)(1)$$

$$\Rightarrow f(x) = 2 + x + 4x(x-1) + x(x-1)(x-2)$$

$$= 2 + x + 4x^2 - 4x + x(x^2 - 3x + 2)$$

$$= 2 + 4x^2 - 3x + x^3 - 3x^2 + 2x$$

$\boxed{f(x) = x^3 + x^2}$ [redacted] $\boxed{x + 2}$

Q.3 Using Lagrange's interpolation formulae to find $f(5)$, from the following data:

x	1	3	4	6	9
y	-3	9	30	132	156.

Sol: Here, $x_0 = 1$, $x_1 = 3$, $x_2 = 4$, $x_3 = 6$, $x_4 = 9$

$y_0 = -3$, $y_1 = 9$, $y_2 = 30$, $y_3 = 132$, $y_4 = 156$.

By Lagrange's interpolation formula, we have

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_1)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4.$$

$$\Rightarrow f(5) = \frac{(5-3)(5-4)(5-6)(5-9)}{(1-3)(1-4)(1-6)(1-9)}(-3)$$

$$+\frac{(5-1)(5-4)(5-6)(5-9)}{(3-1)(3-4)(3-6)(3-9)}(9)$$

$$+\frac{(5-1)(5-3)(5-6)(5-9)}{(4-1)(4-3)(4-6)(4-9)}(30)$$

$$+\frac{(5-1)(5-3)(5-4)(5-9)}{(6-1)(6-3)(6-4)(6-9)}(132)$$

$$+\frac{(5-1)(5-3)(5-4)(5-6)}{(9-1)(9-3)(9-4)(9-6)}(156)$$

$$= \frac{-1}{10}+(-4)+32+\frac{704}{15}+\left(-26/15\right)$$

$$f(5) = 73.1$$

Q.4 Evaluate $\int_0^{0.6} e^{-x^2}\, dx$, by using simpson's $1/3^{rd}$ rule by taking 7 ordinates.

Sol:

7 ordinates $\Rightarrow$ $(7-1=6)$ subintervals.

i.e., $n = 6$

$$h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1.$$

Here $f(x) = e^{-x^2}$

$f(x) = e^{-x^2}$

x	0	0.1	0.2	0.3	0.4	0.5	0.6
y	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Simpson's $1/3^{rd}$ rule,

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$\int_0^{0.6} e^{-x^2}\,dx = \frac{0.1}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.1}{3}\left[(1+0.6977) + 4(0.99+0.9139+0.7788) + 2(0.9608+0.8521)\right]$$

$$\Rightarrow \int_0^{0.6} e^{-x^2}\,dx = 0.5351.$$

Q.5 Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$

$\sin 60^\circ = 0.8660$, find $\sin 52^\circ$, using Newton's forward interpolation formula.

Sol:- Newton's forward difference table

x	$y = f(x)$	Δy_0	$\Delta^2 y$	$\Delta^3 y$
45°	0.7071			
		0.0589		
50°	0.7660		−0.0057	
		0.0532		−0.0007
55°	0.8192		−0.0064	
		0.0468		
60°	0.8660			

By Newton's forward interpolation formula, we have

$$f(x) = y_0 + r\,\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0$$

$$r = \frac{x - x_0}{h} = \frac{52^\circ - 45^\circ}{5^\circ} = 1.4$$

$$\Rightarrow f(52^\circ) = \sin 52^\circ = 0.7071 + (1.4)(0.0589)$$

$$+ \frac{(1.4)(1.4-1)}{2}(-0.0057)$$

$$+ \frac{(1.4)(1.4-1)(1.4-2)}{6}(-0.0007)$$

$$\Rightarrow \boxed{\sin 52^\circ = 0.788}$$

Q.6 Find Taylor's series method the value of y at $x = 0.1$ to 4 place of decimals from $y' = xy^2 - 1$, with $y(0) = 1$.

Sol^n: Here $x_0 = 0,\ y_0 = 1$

$y' = xy^2 - 1$

$y_0' = x_0 y_0^2 - 1$

$= -1$

$y'' = y^2 + 2xyy'$

$y_0'' = y_0^2 + 2x_0 \cdot y_0 y_0'$

$= 1$

$y''' = 2yy' + 2xyy'' + 2xy'^2 + 2yy'$

$y_0''' = 4y_0 y_0' + 2x_0 y_0 y_0'' + 2x_0 y_0'^2$

$= -4$

By Taylor's series method

$$y(x) = y_0 + (x-x_0)\, y_0' + \frac{(x-x_0)^2}{2!}\, y_0'' + \frac{(x-x_0)^3}{3!}\, y_0'''$$

$$y(0.1) = 1 + (0.1-0)(-1) + \frac{(0.1-0)^2}{2}(4)$$

$$+ \frac{(0.1-0)^3}{6}(-4)$$

$$\Rightarrow y(0.1) = 0.9043$$

Q.7 Using Runge Kutta method of order 4, find y at $x = 0.6$, given that $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$, taking $h = 0.2$.

Sol: Given, $x_0 = 0.4$, $y_0 = 0.41$, $h = 0.2$

$$f(x,y) = \sqrt{x+y}.$$

To find $y(0.6)$.

$$K_1 = hf(x_0, y_0) = 0.2\sqrt{0.4+0.41} = 0.18$$

$$K_2 = hf(x_0 + h/2,\ y_0 + K_1/2) = 0.2$$

$$K_3 = hf(x_0 + h/2,\ y_0 + K_2/2) = 0.2009$$

$$K_4 = hf(x_0 + h,\ y_0 + K_3) = 0.22$$

By Runge Kutta fourth order method, we have

$y_1 = y(0.6) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$\Rightarrow \boxed{y(0.6) = 0.6103}$

8. Apply Milne's predictor corrector method, find $y(0.4)$, $\frac{dy}{dx} = 2e^x y$

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

Sol: Given, $y' = f(x, y) = 2e^x y$.

	x	y	y'
$x_0 =$	0	2.4	4.8
$x_1 =$	0.1	2.473	5.4661
$x_2 =$	0.2	3.129	7.6435
$x_3 =$	0.3	4.059	10.9582
$x_4 =$	0.4		

By Milne's predictor formula,

$$y_4^p = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right)$$

$$= 5.7607$$

$$y_4' = f(x_4, y_4^p) = 2 \times e^{0.4}(5.7607)$$

$$= 17.1879$$

By Milne's corrector formula,

$$y_4^c = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right)$$

$$= 5.4178$$

$$y_4' = f(x_4, y_4^c) = 16.1648$$

$$y_4^c = 5.3837$$

$$y_4' = 16.0630$$

$$y_4^c = 5.3803$$

$$y_4' = 16.0529$$

$$y_4^c = 5.3799.$$

$$\Rightarrow \boxed{y(0.4) = 5.379}$$