

CFD Code Development Frameworks

Python, C, OpenFOAM, Fenics

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Table of Content

1 Governing Equations

2 Solution algorithm

3 Grid

4 Problem Statement

Governing Equations

Incompressible Navier-Stokes Equations (INSE) in Eulerian form:

$$\partial_t \rho + \rho \nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} \quad (2)$$

Eqn. 1: Density can be omitted, but let's keep it for now.

Eqn. 2:

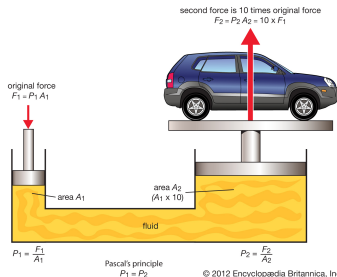
- LHS is acceleration which is intrinsically Lagrangian and when is translated to Eulerian coordinates it becomes non-linear
- RHS describes influence of pressure gradient, viscosity and body forces.

Governing Equations

Complexities

Difficulties for numerical solution:

- Nonlocality of pressure gradient (Solution: None)
- Nonlinearity of acceleration term (Solution: e.g. Picard's Method)
- Pressure and velocity are coupled and there is no separate equation for pressure (Solution: e.g. SIMPLE, PISO, ACM)



Governing Equations

Artificial Compressibility Method

ACM first developed by A.J. Chorin in 1967 assumes a small compressibility for the fluid and isothermal condition for the flow:

$$\rho = \rho(p) \therefore \partial_t \rho = \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t}$$

where c is artificial sound speed. So INSE becomes:

$$\begin{aligned}\partial_t P + c^2 \nabla \cdot \mathbf{u} &= 0 \\ \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{g}\end{aligned}$$

where $P = \frac{p}{\rho}$.

Governing Equations

Remarks on ACM

- Limited scope of applicability: incompressible steady state problems.
- Small times step: t is pseudo-time step and depends on c . Therefore, if an explicit discretization method is used time step should have a small value.
- Efficient parallelization since there is no elliptic PDE

Although applicability of the method is very limited, it provides a simple yet informative framework for pedagogical purposes

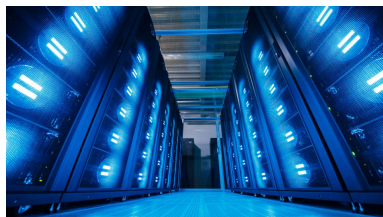


Table of Content

1 Governing Equations

2 Solution algorithm

3 Grid

4 Problem Statement

Solution algorithm

- 1 Initialization of the domain and the variables (u , v , p)
- 2 While convergence is reached do
 - 1 Solve momentum equation in x -direction to get u
 - 2 Solve momentum equation in y -direction to get v
 - 3 Update the boundary conditions for u and v
 - 4 Solve continuity equation to get P
 - 5 Check convergence criteria

Solution algorithm

Discretization

Spatial: central difference, second order

$$\partial_x \phi = \frac{\phi_{i+1} - \phi_{i-1}}{\Delta x}$$

Temporal: backward difference, first order

$$\partial_t \phi = \frac{\phi^{n+1} - \phi^n}{\Delta t}$$

Note: for brevity we use the superscript n to denote the variable in the current time step and no superscript for the previous time step.

Solution algorithm

Momentum Equation in x-direction

$$\partial_t u + \partial_x(uu) + \partial_y(uv) = -\partial_x P + \nu (\partial_{xx} u + \partial_{yy} u)$$

$$\begin{aligned} & \frac{u_{i,j}^n - u_{i,j}}{\Delta t} + \frac{\left(\frac{u_{i+1,j} + u_{i,j}}{2}\right)^2 - \left(\frac{u_{i,j} + u_{i-1,j}}{2}\right)^2}{\Delta x} \\ & + \frac{\left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) \left(\frac{v_{i+1,j} + v_{i,j}}{2}\right) - \left(\frac{u_{i,j} + u_{i,j-1}}{2}\right) \left(\frac{v_{i+1,j-1} + v_{i,j-1}}{2}\right)}{\Delta y} \\ & = \\ & - \frac{P_{i+1,j} - P_{i,j}}{\Delta x} \\ & + \nu \left(\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} \right) \end{aligned}$$

Solution algorithm

Momentum Equation in y -direction

$$\begin{aligned}
 \partial_t v + \partial_x(uv) + \partial_y(vv) &= -\partial_y P + \nu (\partial_{xx} v + \partial_{yy} v) \\
 \frac{v_{i,j}^n - v_{i,j}}{\Delta t} + \frac{\left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) \left(\frac{v_{i+1,j} + v_{i,j}}{2}\right) - \left(\frac{u_{i-1,j+1} + u_{i-1,j}}{2}\right) \left(\frac{v_{i,j} + v_{i-1,j}}{2}\right)}{\Delta x} \\
 + \frac{\left(\frac{v_{i,j+1} + v_{i,j}}{2}\right)^2 - \left(\frac{v_{i,j} + v_{i,j-1}}{2}\right)^2}{\Delta y} \\
 &= \\
 - \frac{P_{i,j+1} - P_{i,j}}{\Delta y} \\
 + \nu \left(\frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{\Delta x^2} + \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{\Delta y^2} \right)
 \end{aligned}$$

Solution algorithm

Continuity Equation

$$\partial_t P + c^2 (\partial_x u + \partial_y v) = 0$$

$$\frac{P_{i,j}^n - P_{i,j}}{\Delta t} + c^2 \left(\frac{u_{i,j}^n - u_{i-1,j}^n}{\Delta x} + \frac{v_{i,j}^n - v_{i,j-1}^n}{\Delta y} \right) = 0$$

Solution algorithm

Boundary Conditions

The two common boundary conditions can be formulated as follows:

- Dirichlet: $\phi_g = 2\phi_b - \phi_i$
- Neumann: $\phi_g = \phi_i - \left(\Delta n \frac{\partial \phi}{\partial n} \right)_b$

Indices: g , b and i denote a ghost, boundary and inner node, respectively

Variables: ϕ is a quantity of interest (u , v or P) and n represents a direction normal to the boundary cell face

Solution algorithm

Convergence Criteria

$$1 \quad E_u = \sqrt{(\Delta t \Delta x \Delta y) \sum_{i,j} (u_{i,j}^{n+1} - u_{i,j}^n)^2}$$

$$2 \quad E_v = \sqrt{(\Delta t \Delta x \Delta y) \sum_{i,j} (v_{i,j}^{n+1} - v_{i,j}^n)^2}$$

$$3 \quad E_p = \sqrt{\frac{\Delta t \Delta x \Delta y}{c^2} \sum_{i,j} (P_{i,j}^{n+1} - P_{i,j}^n)^2}$$

$$4 \quad E_{\nabla} = (\Delta t \Delta x \Delta y) \nabla \cdot \mathbf{u}$$

$$5 \quad E_{tot} = \max\{E_u, E_v, E_p, E_{\nabla}\} < \varepsilon$$

where ε is the desired tolerance.

Table of Content

1 Governing Equations

2 Solution algorithm

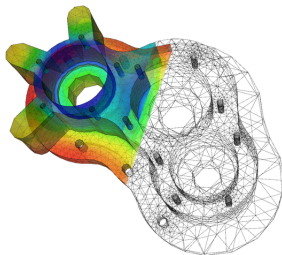
3 Grid

4 Problem Statement

Grid

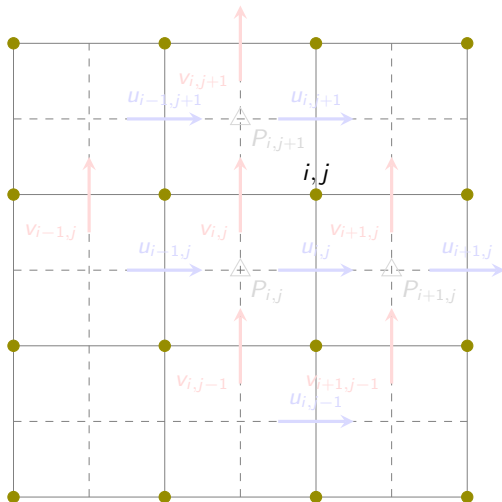
There are two types of grids from variable placement point of view:

- Collocated: all the variables at the node centers; easier to implement and extend while issues such as checkerboard problem arise
- Staggered: scalar variables at the center of a node and vector variables at the face center; more accurate though more difficult to implement



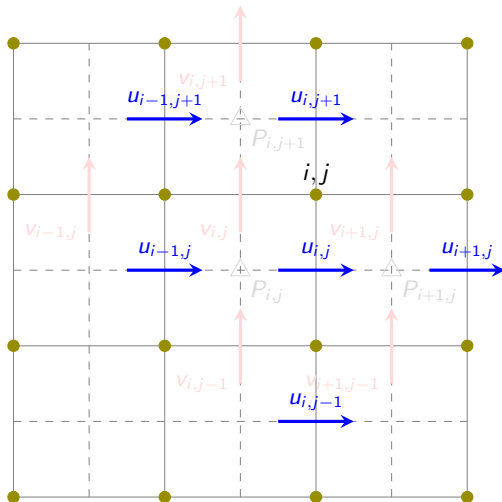
Grid

Staggered Grid



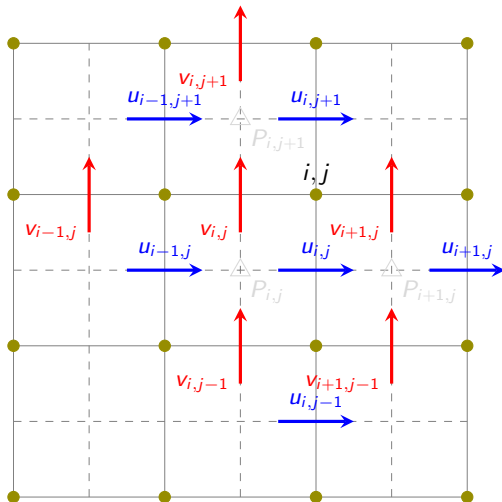
Grid

Staggered Grid



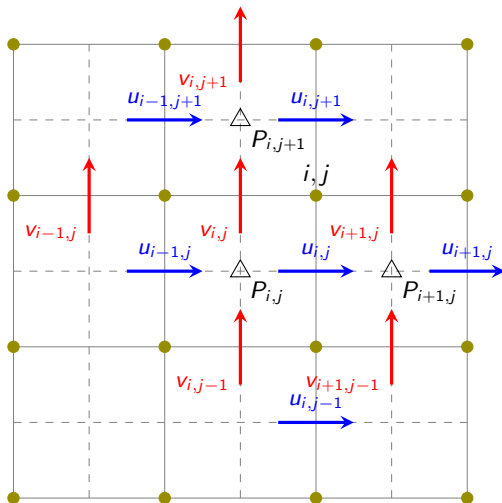
Grid

Staggered Grid



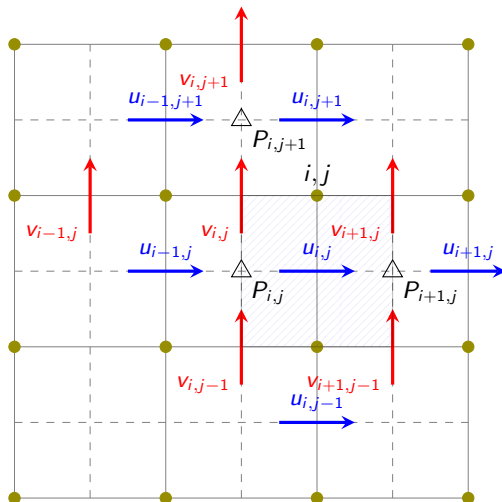
Grid

Staggered Grid



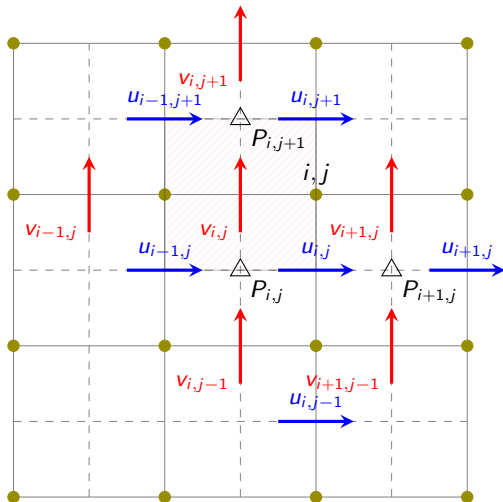
Grid

Staggered Grid



Grid

Staggered Grid



Grid

Staggered Grid

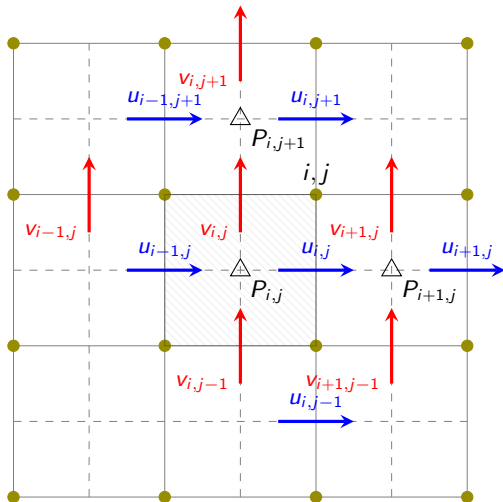


Table of Content

1 Governing Equations

2 Solution algorithm

3 Grid

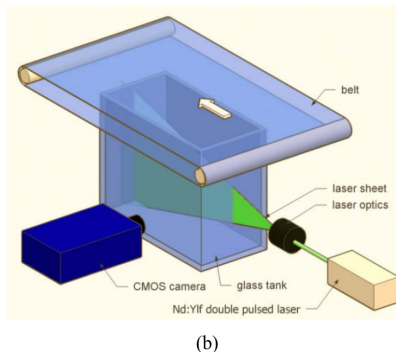
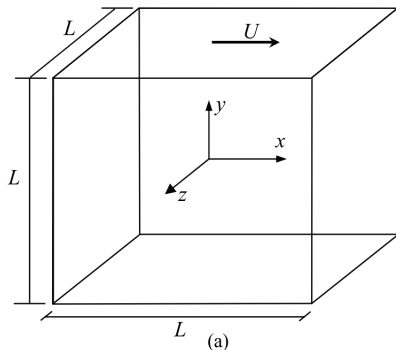
4 Problem Statement

Problem Statement

Lid-Driven Cavity

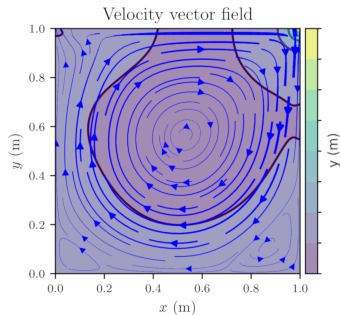
Characteristics: steady-state, incompressible

BC: walls everywhere (top wall is moving)

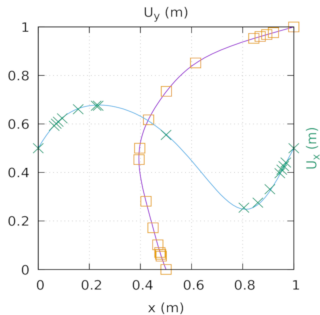


Problem Statement

Outputs



Velocity fluctuations at the middle for $Re=100$



Numerical, U_x —
Ghia, U_x □
Numerical, U_y —
Ghia, U_y ×

Problem Statement

Workshop

A numerical solution to this benchmark problem will be presented based on:

- (Finite Difference) Coding from scratch: Python, C
- (Finite Volume) Modifying one of the solver in OpenFOAM
- (Finite Element) Using libraries provided by FEniCS



Let's Get Started!

```
$ mkdir UHWS
```

```
$ cd UHWS
```

```
$ git clone
```

```
https://github.com/taataam/UHOFWorkshop.git
```

```
$ cd UHOFWorkshop/workshop3
```

