

CFD Code Development Frameworks

Python, C, OpenFOAM, Fenics

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Governing Equations

Incompressible Navier-Stokes Equations (INSE) in Eulerian form:

$$\partial_t \rho + \rho \nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} \quad (2)$$

Eqn. 1: Density can be omitted, but let's keep it for now.

Eqn. 2:

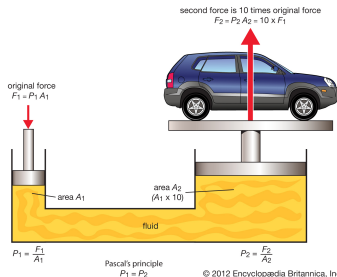
- LHS is acceleration which is intrinsically Lagrangian and when is translated to Eulerian coordinates it becomes non-linear
- RHS describes influence of pressure gradient, viscosity and body forces.

Governing Equations

Complexities

Difficulties for numerical solution:

- Nonlocality of pressure gradient (Solution: None)
- Nonlinearity of acceleration term (Solution: e.g. Picard's Method)
- Pressure and velocity are coupled and there is no separate equation for pressure (Solution: e.g. SIMPLE, PISO, ACM)



Governing Equations

Artificial Compressibility Method

ACM first developed by A.J. Chorin in 1967 assumes a small compressibility for the fluid and isothermal condition for the flow:

$$\rho = \rho(p) \therefore \partial_t \rho = \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t}$$

where c is artificial sound speed. So INSE becomes:

$$\begin{aligned}\partial_t P + c^2 \nabla \cdot \mathbf{u} &= 0 \\ \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{g}\end{aligned}$$

where $P = \frac{p}{\rho}$.

Governing Equations

Remarks on ACM

- Limited scope of applicability: incompressible steady state problems.
- Small times step: t is pseudo-time step and depends on c . Therefore, if an explicit discretization method is used time step should have a small value.
- Efficient parallelization since there is no elliptic PDE

Although applicability of the method is very limited, it provides a simple yet informative framework for pedagogical purposes

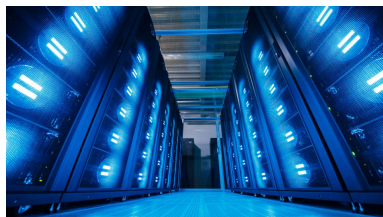


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Solution algorithm

- 1 Initialization of the domain and the variables (u , v , p)
- 2 While convergence is reached do
 - 1 Solve momentum equation in x -direction to get u
 - 2 Solve momentum equation in y -direction to get v
 - 3 Update the boundary conditions for u and v
 - 4 Solve continuity equation to get P
 - 5 Check convergence criteria

Solution algorithm

Discretization

Spatial: central difference, second order

$$\partial_x \phi = \frac{\phi_{i+1} - \phi_{i-1}}{\Delta x}$$

Temporal: backward difference, first order

$$\partial_t \phi = \frac{\phi^{n+1} - \phi^n}{\Delta t}$$

Note: for brevity we use the superscript n to denote the variable in the current time step and no superscript for the previous time step.

Solution algorithm

Momentum Equation in x-direction

$$\partial_t u + \partial_x(uu) + \partial_y(uv) = -\partial_x P + \nu (\partial_{xx} u + \partial_{yy} u)$$

$$\begin{aligned} & \frac{u_{i,j}^n - u_{i,j}}{\Delta t} + \frac{\left(\frac{u_{i+1,j} + u_{i,j}}{2}\right)^2 - \left(\frac{u_{i,j} + u_{i-1,j}}{2}\right)^2}{\Delta x} \\ & + \frac{\left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) \left(\frac{v_{i+1,j} + v_{i,j}}{2}\right) - \left(\frac{u_{i,j} + u_{i,j-1}}{2}\right) \left(\frac{v_{i+1,j-1} + v_{i,j-1}}{2}\right)}{\Delta y} \\ & = \\ & - \frac{P_{i+1,j} - P_{i,j}}{\Delta x} \\ & + \nu \left(\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} \right) \end{aligned}$$

Solution algorithm

Momentum Equation in y -direction

$$\begin{aligned}
 \partial_t v + \partial_x(uv) + \partial_y(vv) &= -\partial_y P + \nu (\partial_{xx} v + \partial_{yy} v) \\
 \frac{v_{i,j}^n - v_{i,j}}{\Delta t} + \frac{\left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) \left(\frac{v_{i+1,j} + v_{i,j}}{2}\right) - \left(\frac{u_{i-1,j+1} + u_{i-1,j}}{2}\right) \left(\frac{v_{i,j} + v_{i-1,j}}{2}\right)}{\Delta x} \\
 + \frac{\left(\frac{v_{i,j+1} + v_{i,j}}{2}\right)^2 - \left(\frac{v_{i,j} + v_{i,j-1}}{2}\right)^2}{\Delta y} \\
 &= \\
 - \frac{P_{i,j+1} - P_{i,j}}{\Delta y} \\
 + \nu \left(\frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{\Delta x^2} + \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{\Delta y^2} \right)
 \end{aligned}$$

Solution algorithm

Continuity Equation

$$\partial_t P + c^2 (\partial_x u + \partial_y v) = 0$$

$$\frac{P_{i,j}^n - P_{i,j}}{\Delta t} + c^2 \left(\frac{u_{i,j}^n - u_{i-1,j}^n}{\Delta x} + \frac{v_{i,j}^n - v_{i,j-1}^n}{\Delta y} \right) = 0$$

Solution algorithm

Boundary Conditions

The two common boundary conditions can be formulated as follows:

- Dirichlet: $\phi_g = 2\phi_b - \phi_i$
- Neumann: $\phi_g = \phi_i - \left(\Delta n \frac{\partial \phi}{\partial n} \right)_b$

Indices: g , b and i denote a ghost, boundary and inner node, respectively

Variables: ϕ is a quantity of interest (u , v or P) and n represents a direction normal to the boundary cell face

Solution algorithm

Convergence Criteria

$$1 \quad E_u = \sqrt{(\Delta t \Delta x \Delta y) \sum_{i,j} (u_{i,j}^{n+1} - u_{i,j}^n)^2}$$

$$2 \quad E_v = \sqrt{(\Delta t \Delta x \Delta y) \sum_{i,j} (v_{i,j}^{n+1} - v_{i,j}^n)^2}$$

$$3 \quad E_p = \sqrt{\frac{\Delta t \Delta x \Delta y}{c^2} \sum_{i,j} (P_{i,j}^{n+1} - P_{i,j}^n)^2}$$

$$4 \quad E_{\nabla} = (\Delta t \Delta x \Delta y) \nabla \cdot \mathbf{u}$$

$$5 \quad E_{tot} = \max\{E_u, E_v, E_p, E_{\nabla}\} < \varepsilon$$

where ε is the desired tolerance.

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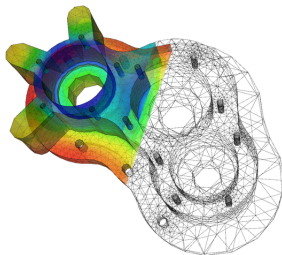
3 Grid

4 Problem Statement

Grid

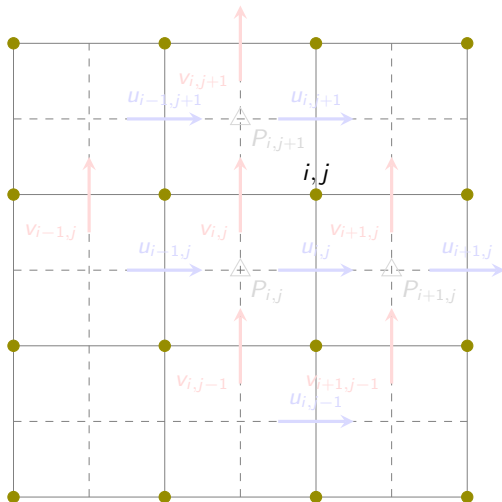
There are two types of grids from variable placement point of view:

- Collocated: all the variables at the node centers; easier to implement and extend while issues such as checkerboard problem arise
- Staggered: scalar variables at the center of a node and vector variables at the face center; more accurate though more difficult to implement



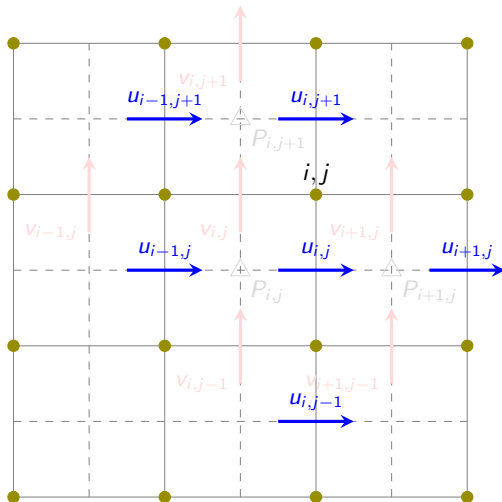
Grid

Staggered Grid



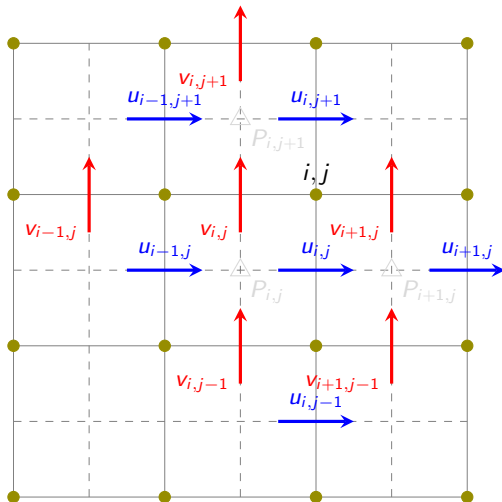
Grid

Staggered Grid



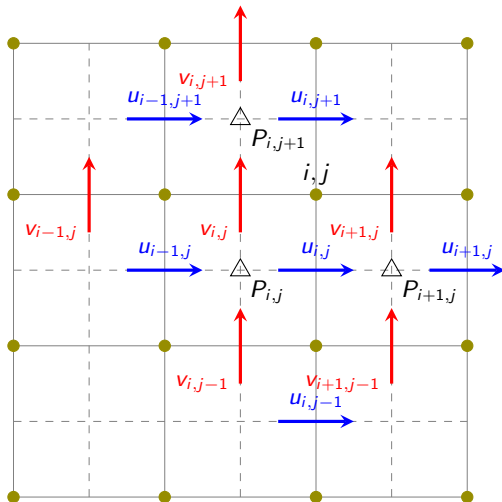
Grid

Staggered Grid



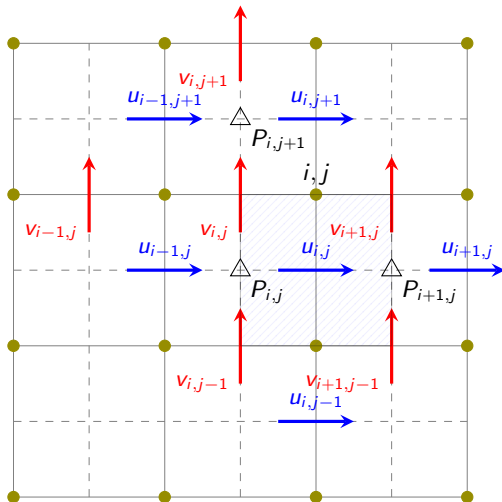
Grid

Staggered Grid



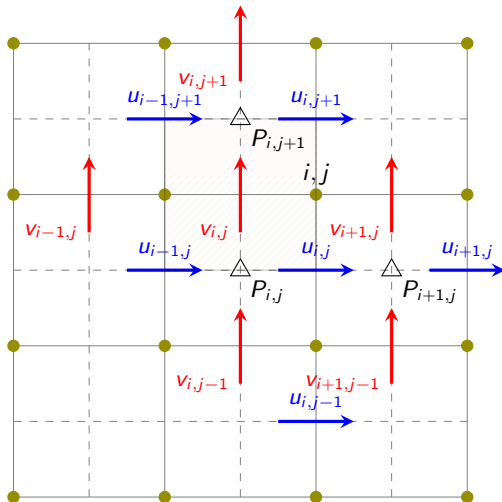
Grid

Staggered Grid



Grid

Staggered Grid



Grid

Staggered Grid

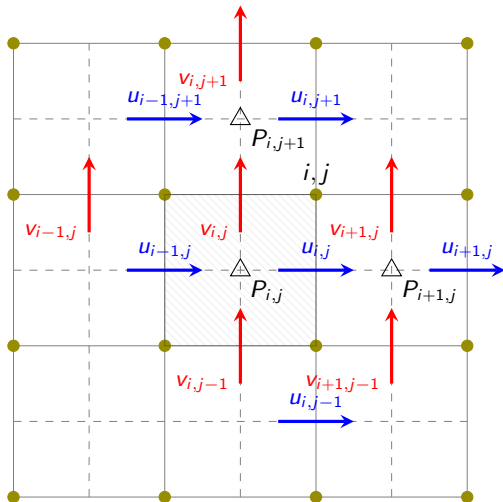


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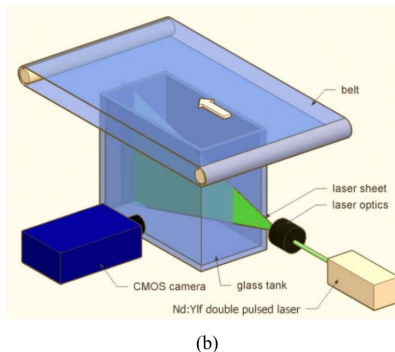
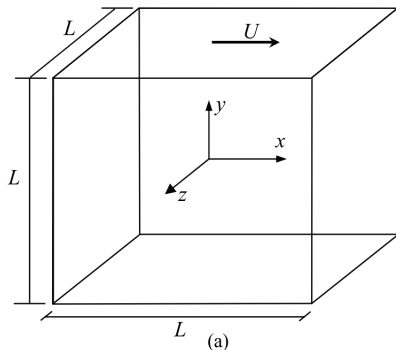
4 Problem Statement

Problem Statement

Lid-Driven Cavity

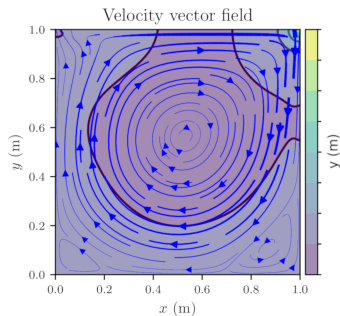
Characteristics: steady-state, incompressible

BC: walls everywhere (top wall is moving)

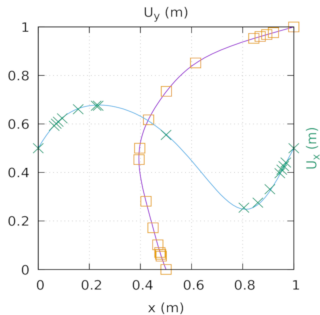


Problem Statement

Outputs



Velocity fluctuations at the middle for $Re=100$



Numerical, U_x —
Ghia, U_x □
Numerical, U_y —
Ghia, U_y ×

Problem Statement

Workshop

A numerical solution to this benchmark problem will be presented based on:

- (Finite Difference) Coding from scratch: Python, C
- (Finite Volume) Modifying one of the solvers in OpenFOAM
- (Finite Element) Using libraries provided by FEniCS



Let's Get Started!

```
$ mkdir UHWS
```

```
$ cd UHWS
```

```
$ git clone
```

```
https://github.com/taataam/UHOFWorkshop.git
```

```
$ cd UHOFWorkshop/workshop3
```

