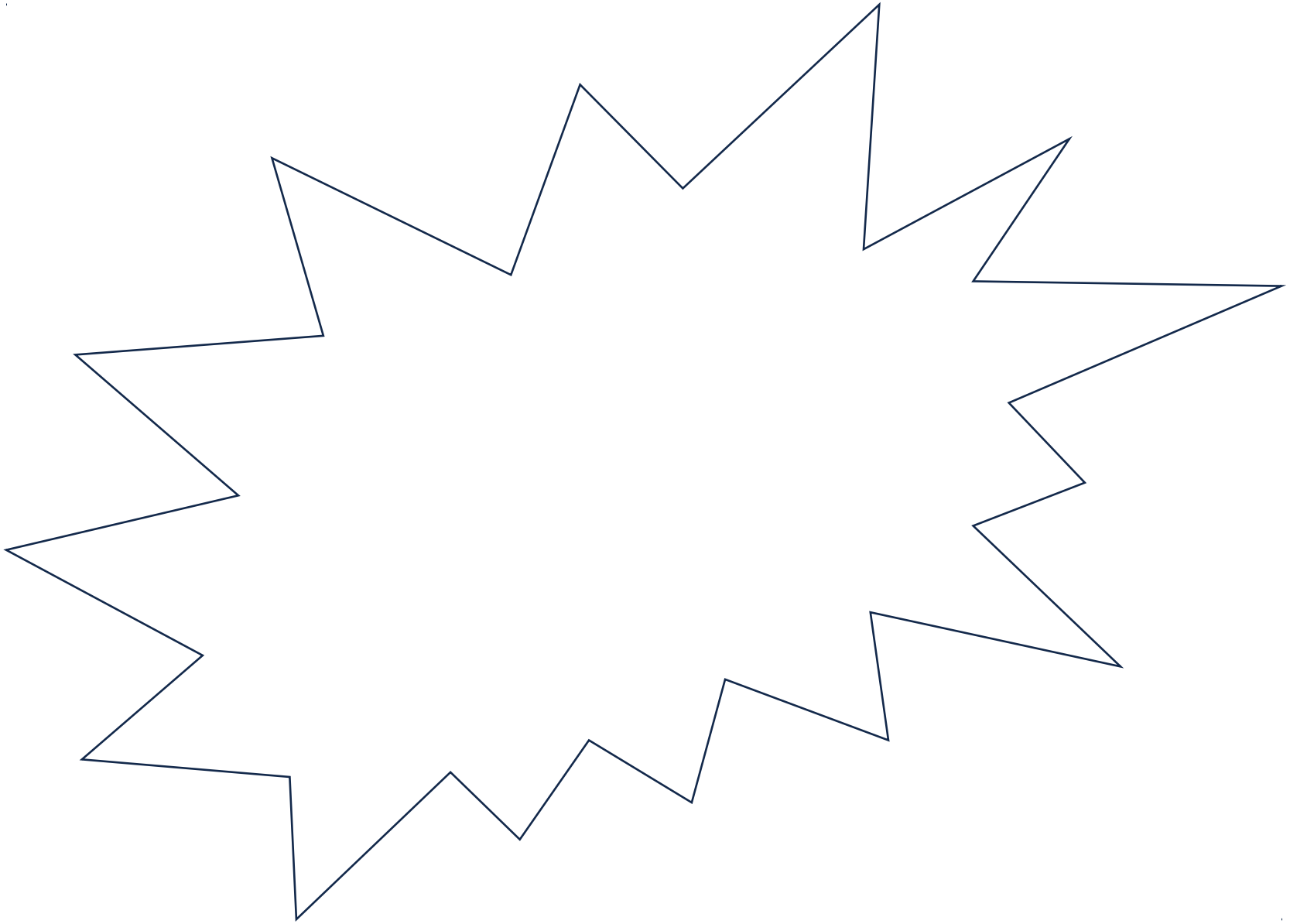
CS 400

Heap - Introduction

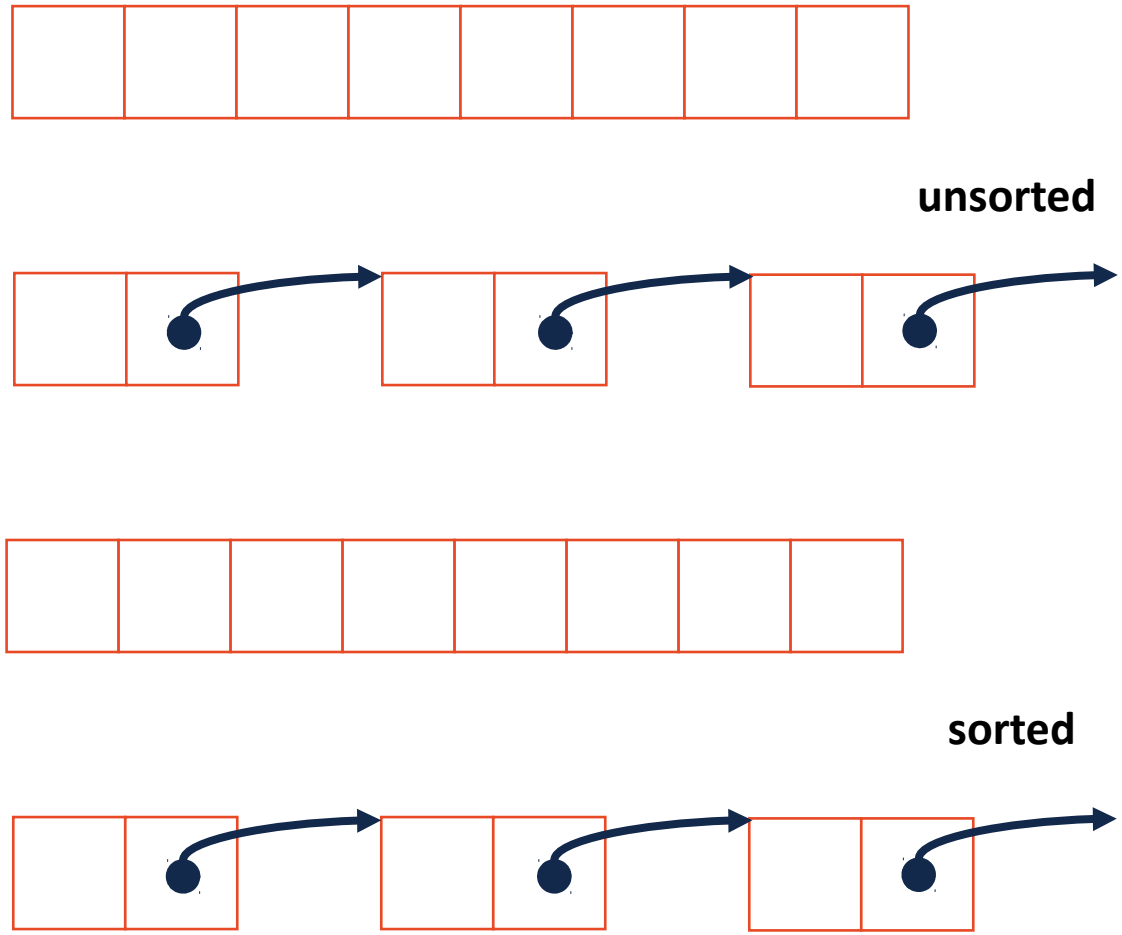
ID: 10-01

Priority Queue

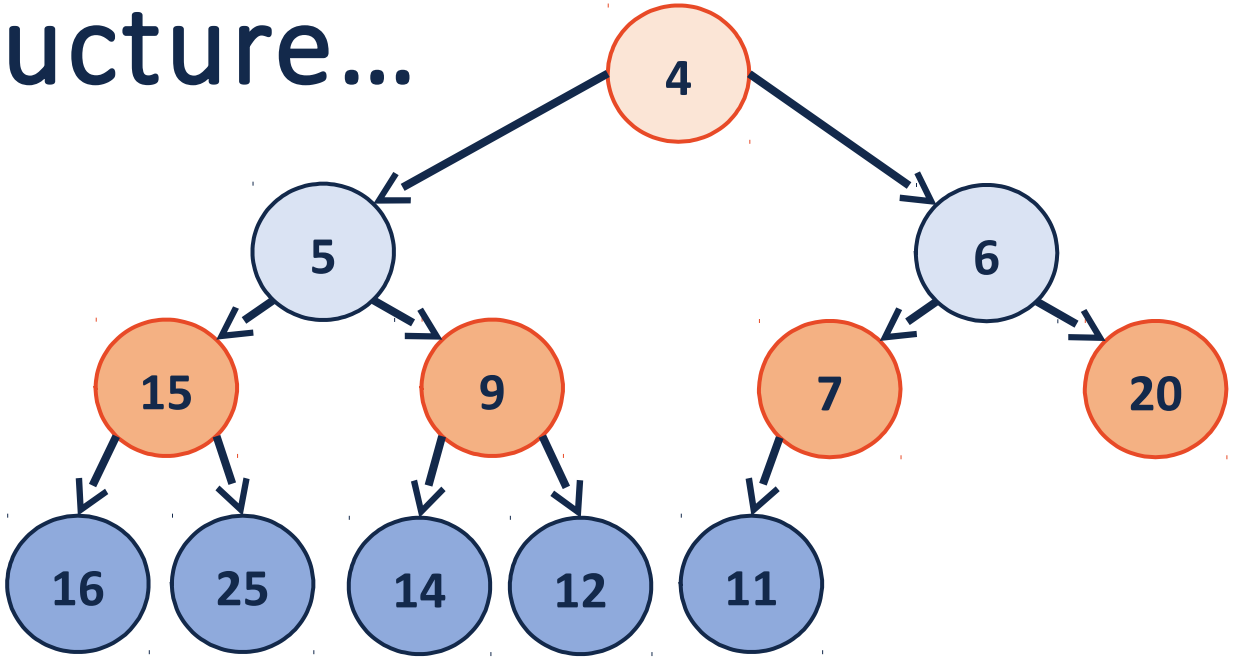


Priority Queue Implementation

insert	removeMin
O(1)*	O(n)
O(1)	O(n)
O(n)	O(1)
O(n)	O(1)



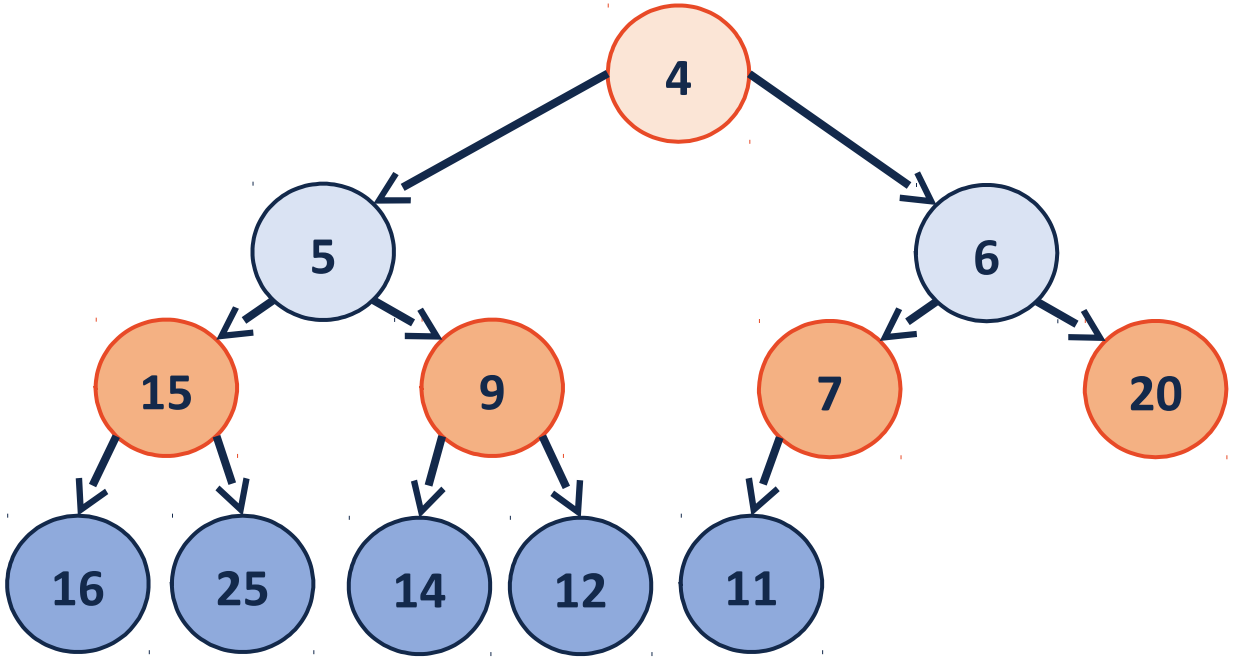
Another possibly structure…



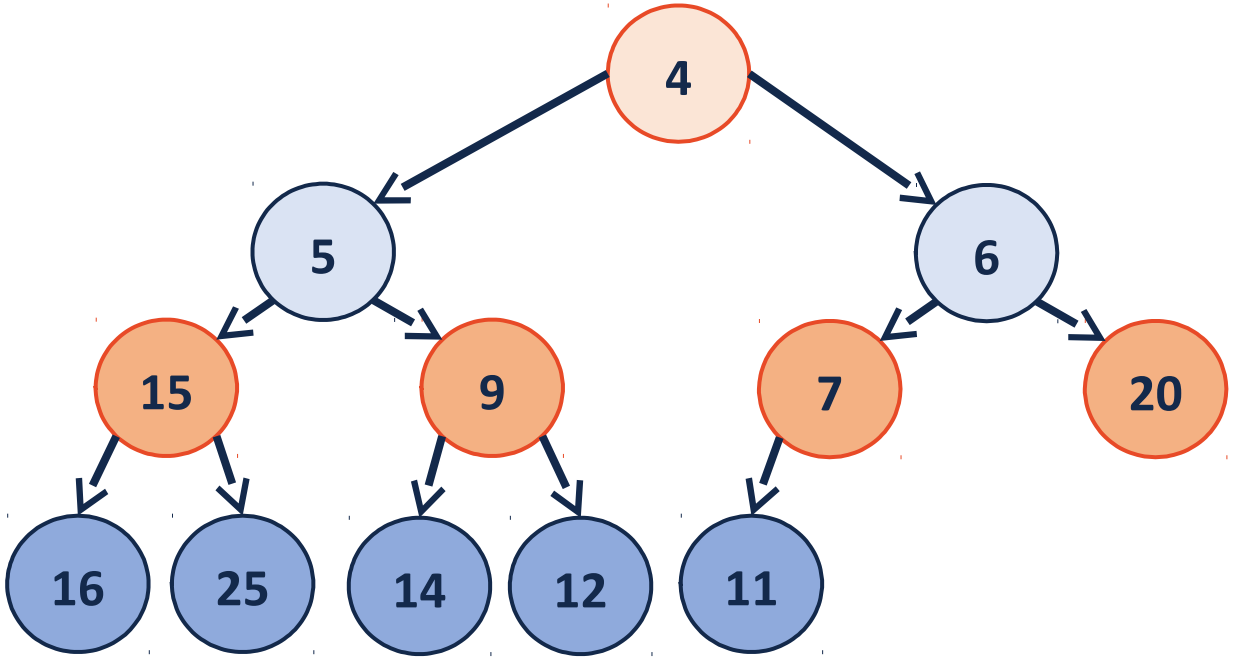
(min)Heap

A complete binary tree T is a min-heap if:

- **T = {}** or
- **T = {r, T_L, T_R}**, where **r** is less than the roots of {T_L, T_R} and {T_L, T_R} are min-heaps.



(min)Heap



4	5	6	15	9	7	20	16	25	14	12	11			

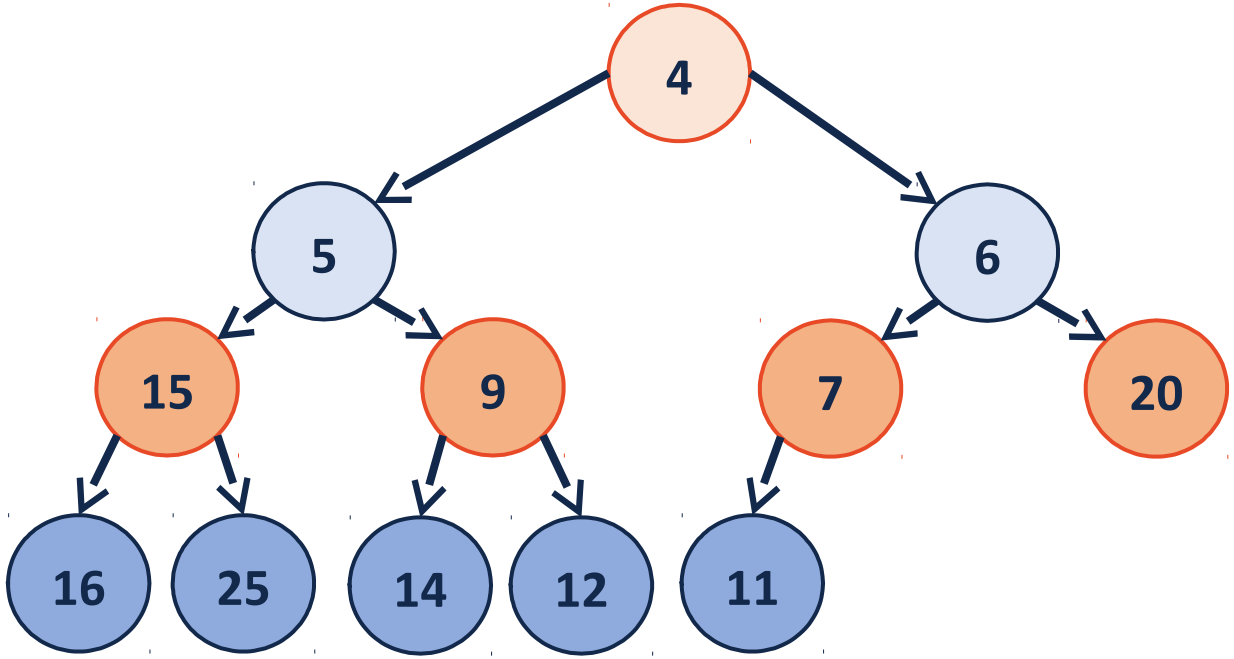
CS 400

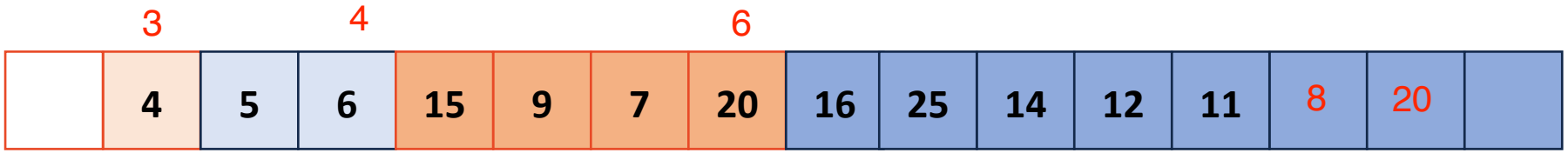
Heap – Insert and removeMin

ID: 10-02

insert

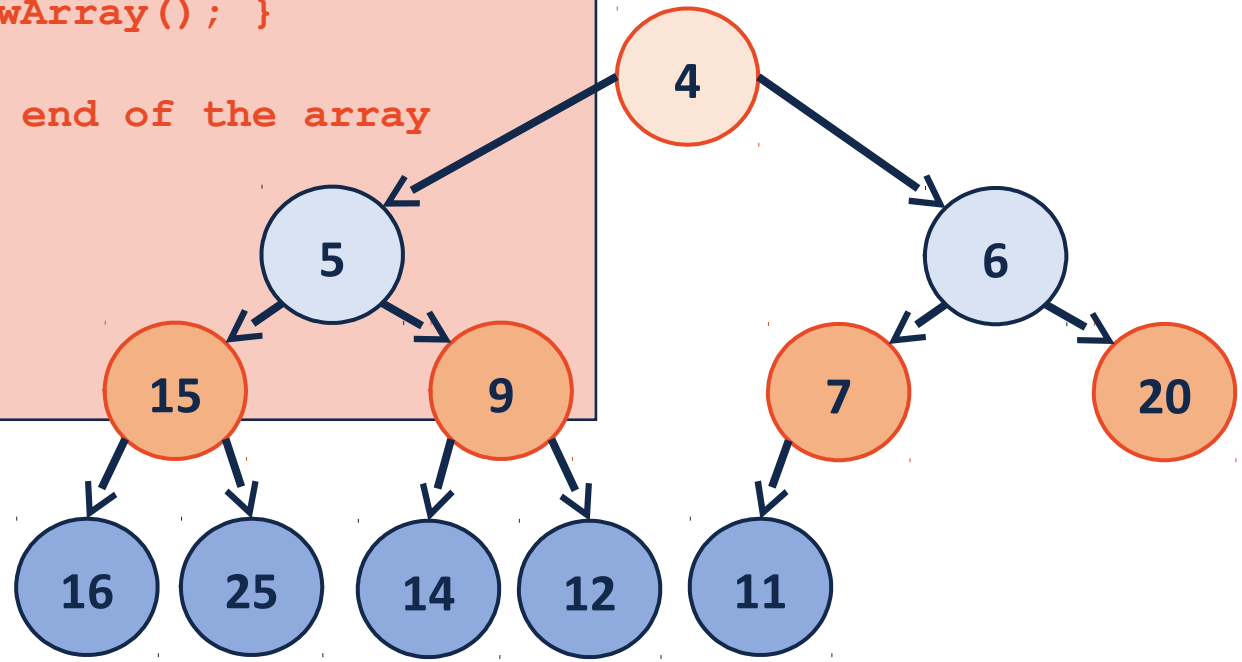
Insert(8)
Insert(3)

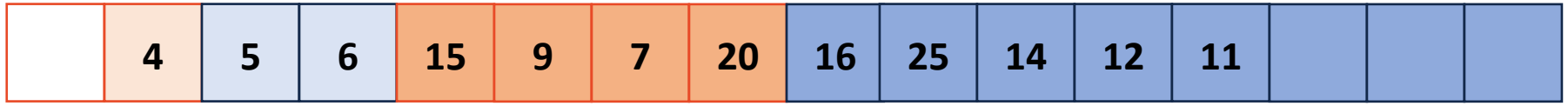




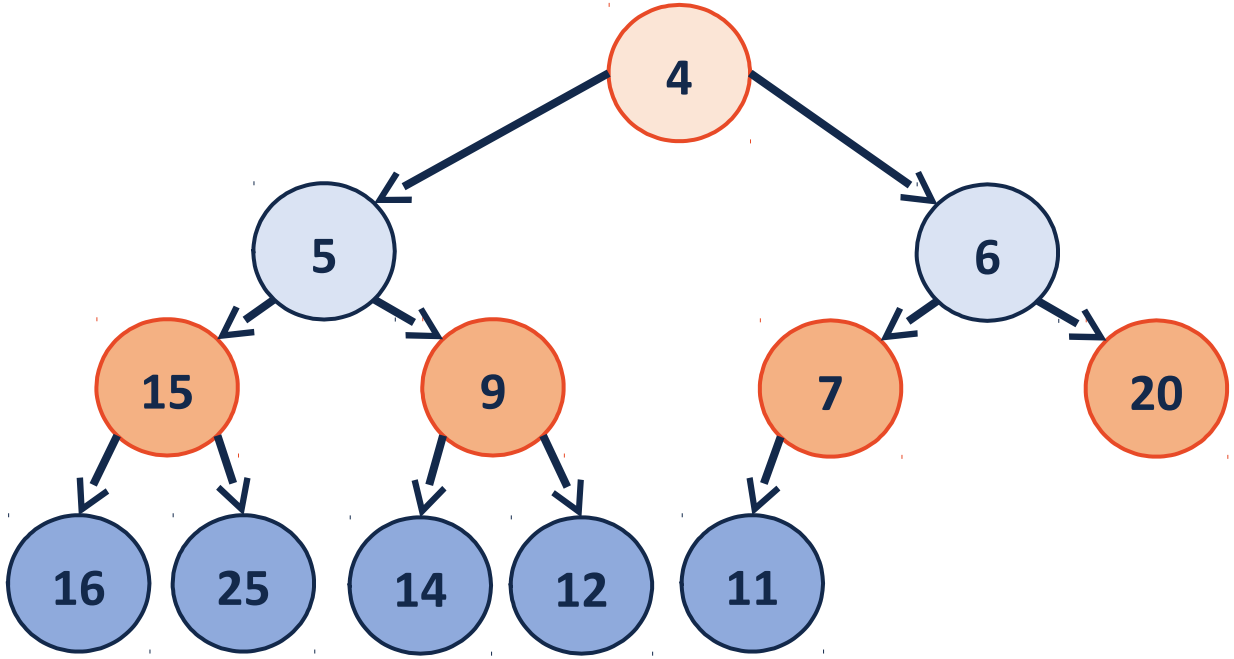
insert

```
 1  template <class T>
 2  void Heap<T>::_insert(const T & key) {
 3    // Check to ensure there's space to insert an element
 4    // ...if not, grow the array
 5    if ( size_ == capacity_ ) { _growArray(); }
 6
 7    // Insert the new element at the end of the array
 8    item_[++size] = key;
 9
10    // Restore the heap property
11    _heapifyUp(size);
12  }
```





growArray

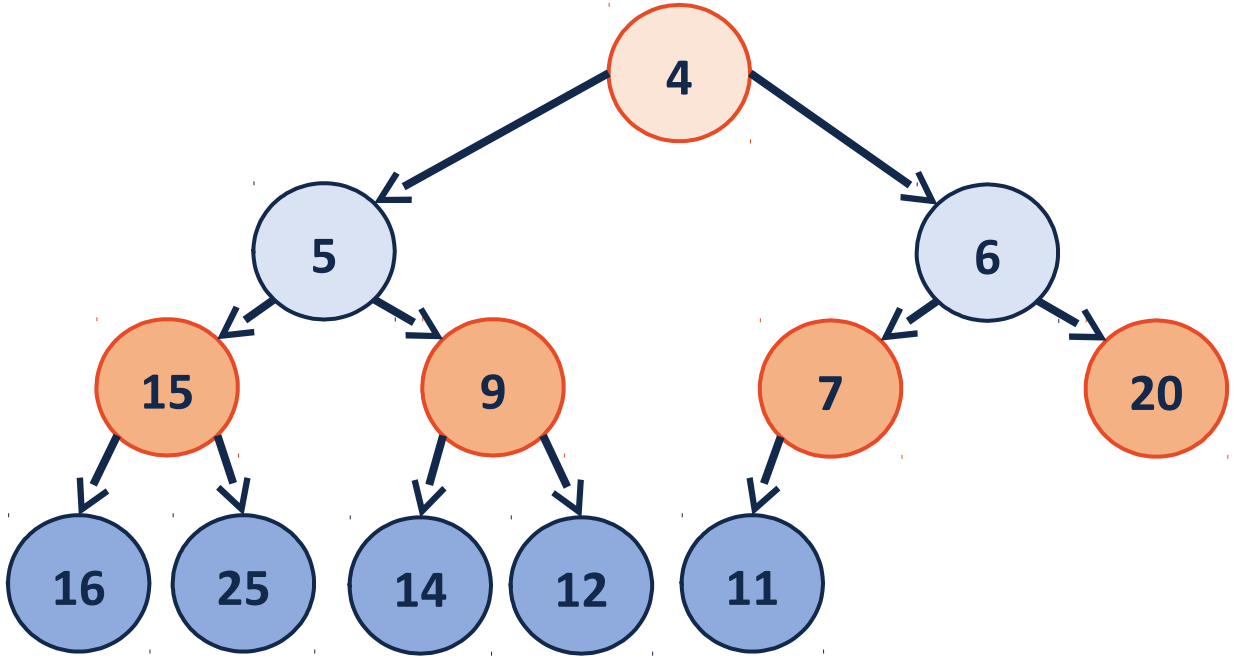


insert- heapifyUp

```
1  template <class T>
2  void Heap<T>::_insert(const T & key) {
3    // Check to ensure there's space to insert an element
4    // ...if not, grow the array
5    if ( size_ == capacity_ ) { _growArray(); }
6
7    // Insert the new element at the end of the array
8    item_[++size] = key;
9
10   // Restore the heap property
11   _heapifyUp(size);
12 }
```

```
1  template <class T>
2  void Heap<T>::_heapifyUp( int index ) {
3    if ( index > 1 ) {
4      if ( item_[index] < item_[ parent(index) ] ) {
5        std::swap( item_[index], item_[ parent(index) ] );
6        _heapifyUp( parent(index) );
7      }
8    }
9  }
```

removeMin

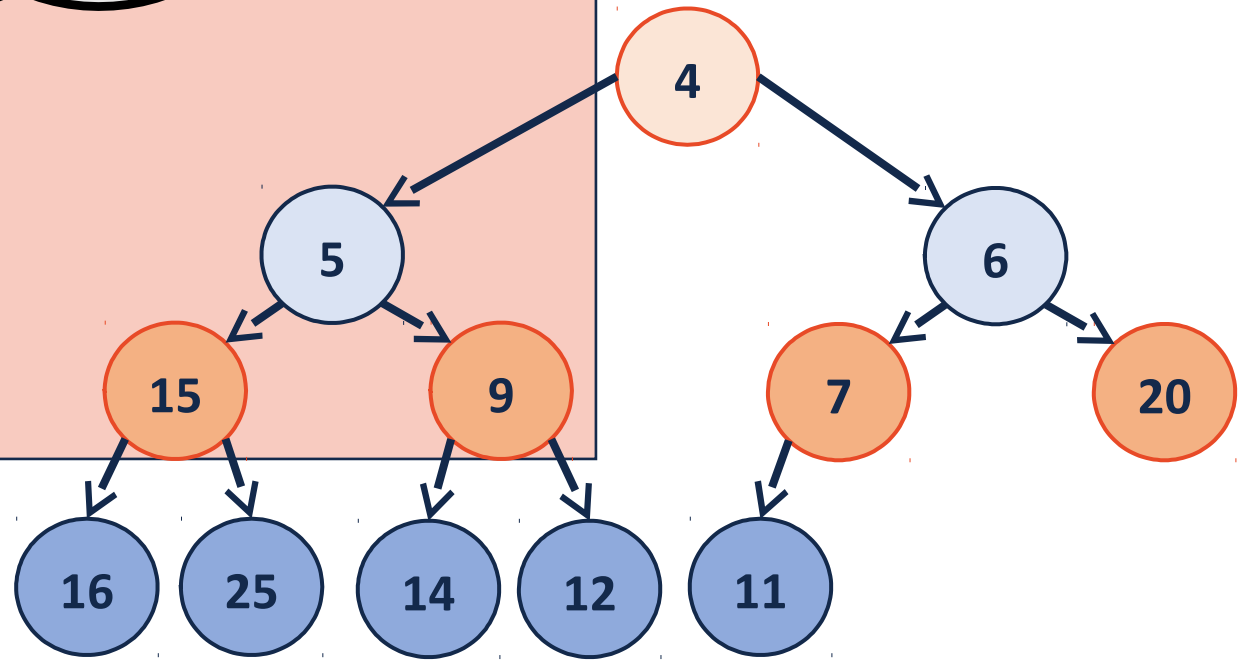


	4	5	6	15	9	7	20	16	25	14	12	11		

removeMin

```
1  template <class T>
2  void Heap<T>::_removeMin() {
3    // Swap with the last value
4    T minValue = item_[1];
5    item_[1] = item_[size_];
6    size--;
7
8    // Restore the heap property
9    heapifyDown();
10
11   // Return the minimum value
12   return minValue;
13 }
```

swap with last element



removeMin - heapifyDown

```
 1  template <class T>
 2  void Heap<T>::_removeMin() {
 3    // Swap with the last value
 4    T minValue = item_[1];
 5    item_[1] = item_[size_];
 6    size--;
 7
 8    // Restore the heap property
 9    _heapifyDown(1);
10
11    // Return the minimum value
12    return minValue;
13  }
```

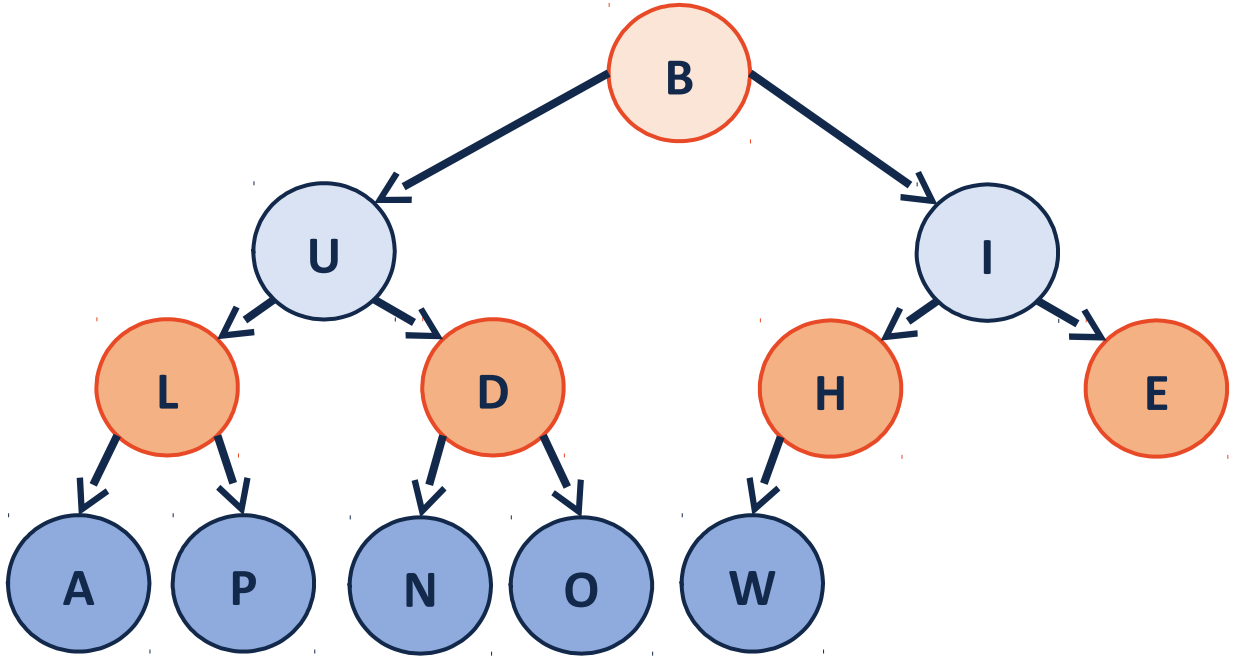
```
 1  template <class T>
 2  void Heap<T>::_heapifyDown(int index) {
 3    if ( !_isLeaf(index) ) {
 4      T minChildIndex = _minChild(index);
 5      if ( item_[index] ___ item_[minChildIndex] ) {
 6         std::swap( item_[index], item_[minChildIndex] );
 7         _heapifyDown( ____minChildIndex____ );
 8      }
 9    }
10  }
```

CS 400

Heap – buildHeap

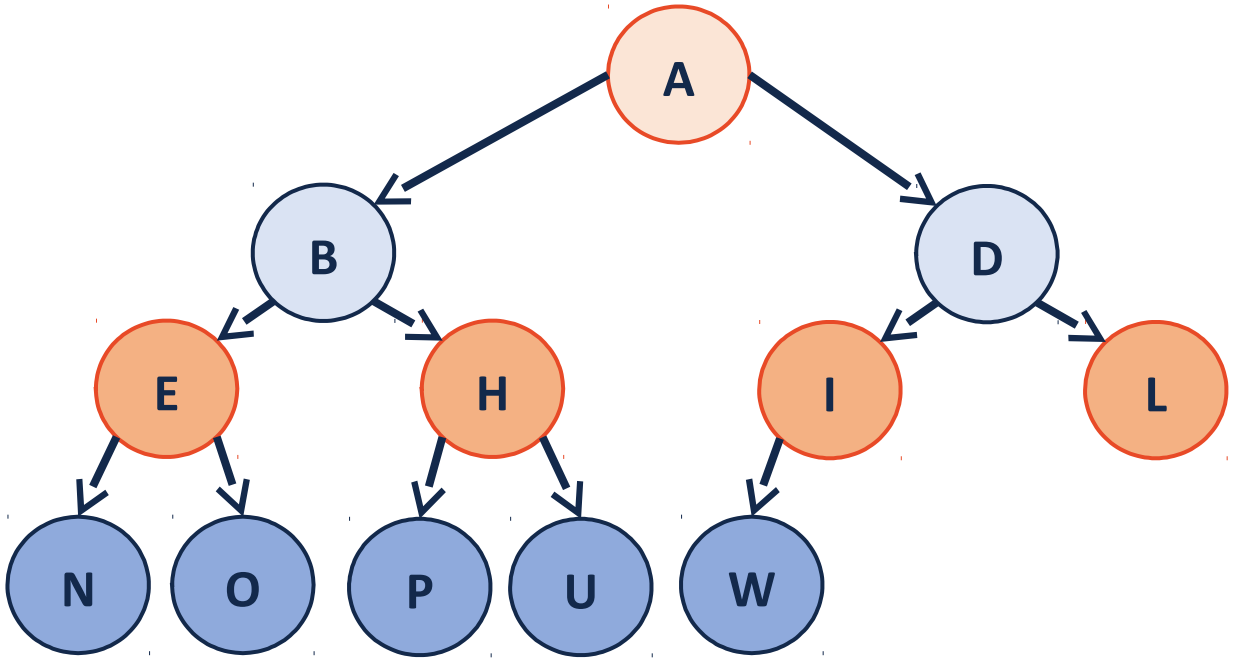
ID: 10-03

buildHeap



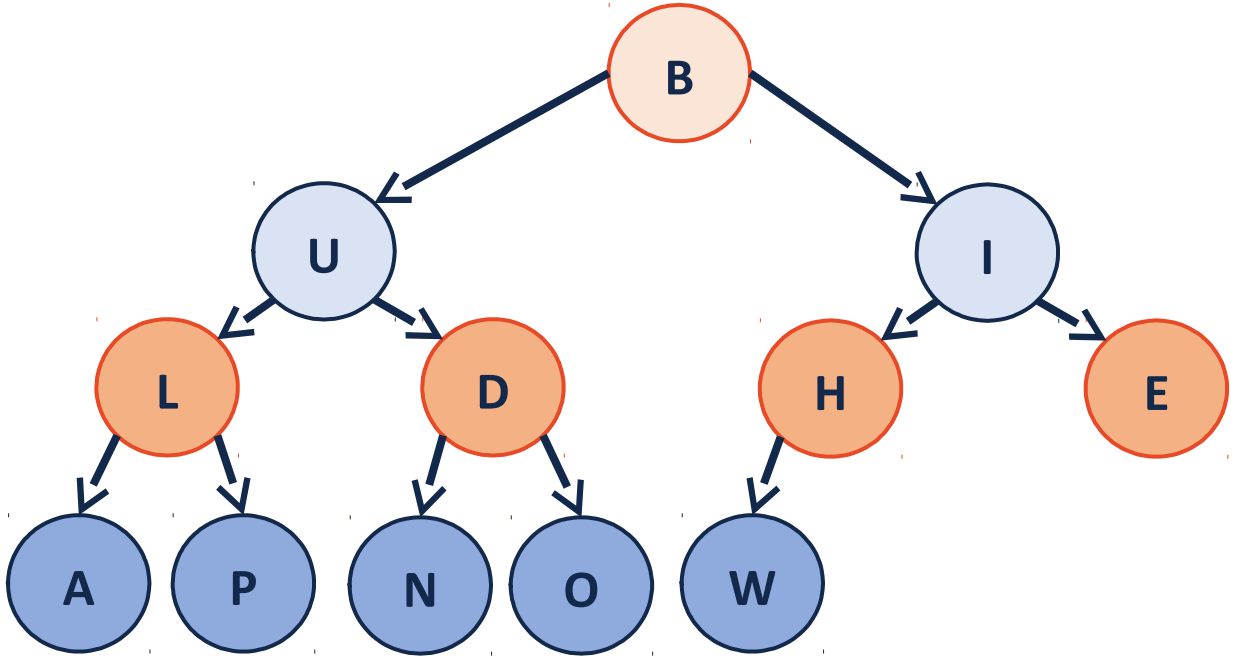
buildHeap – sorted array

	B	U	I	L	D	H	E	A	P	N	O	W			

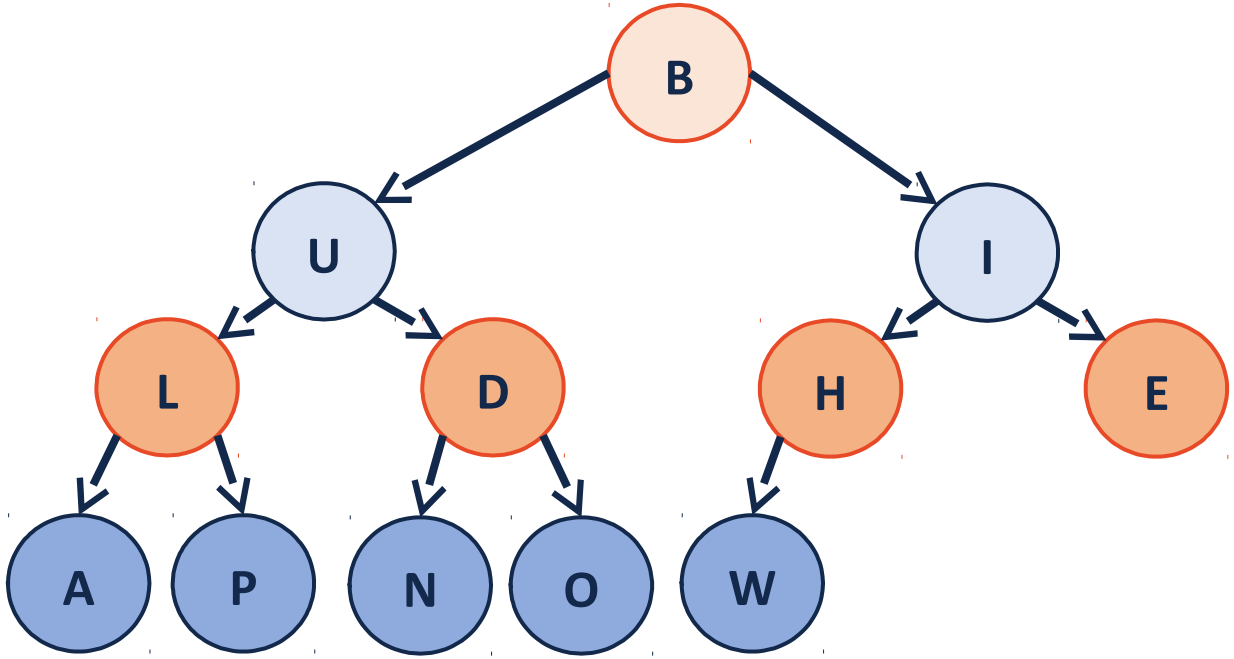


	A	B	D	E	H	I	L	N	O	P	U	W			

buildHeap- heapifyUp

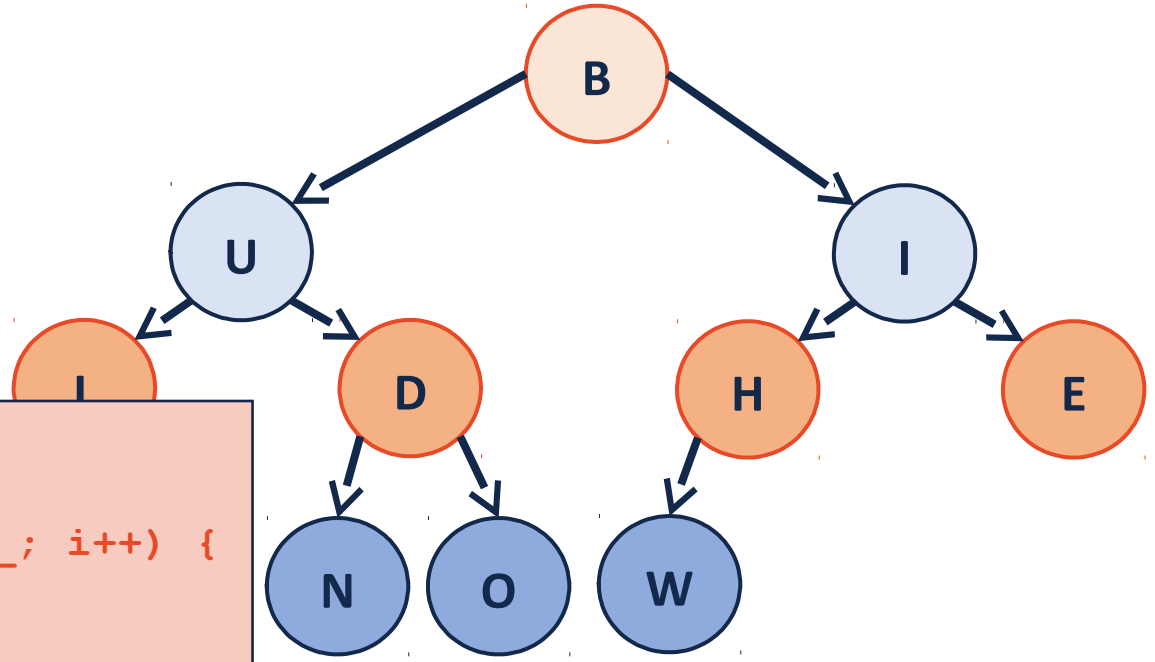


buildHeap- heapifyDown



buildHeap

1. Sort the array – it's a heap!

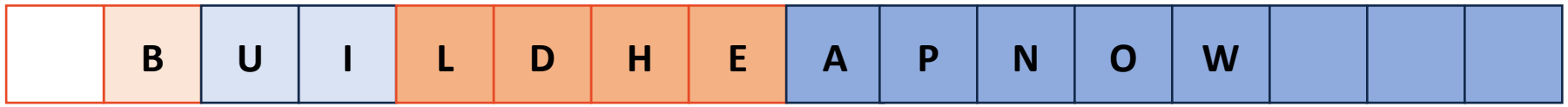


2.

```
1  template <class T>
2  void Heap<T>::buildHeap() {
3    for (unsigned i = 2; i <= size_; i++) {
4      heapifyUp(i);
5    }
6  }
```

3.

```
1  template <class T>
2  void Heap<T>::buildHeap() {
3    for (unsigned i = parent(size); i > 0; i--) {
4      heapifyDown(i);
5    }
6  }
```

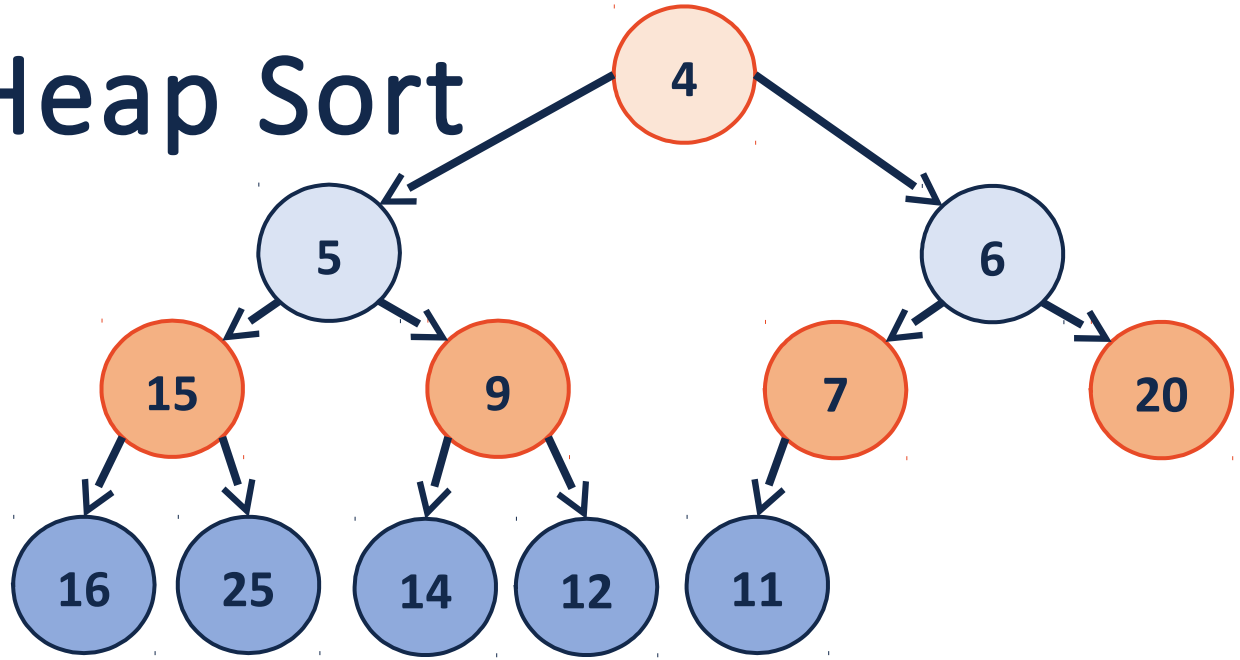


CS 400

Heap – Runtime Analysis

ID: 10-04

Heap Sort



1. Build Heap O(n)
2. n x Remove Min
3. Swap elements to order (ascending or descending)

Running Time?

Why do we care about another sort?