

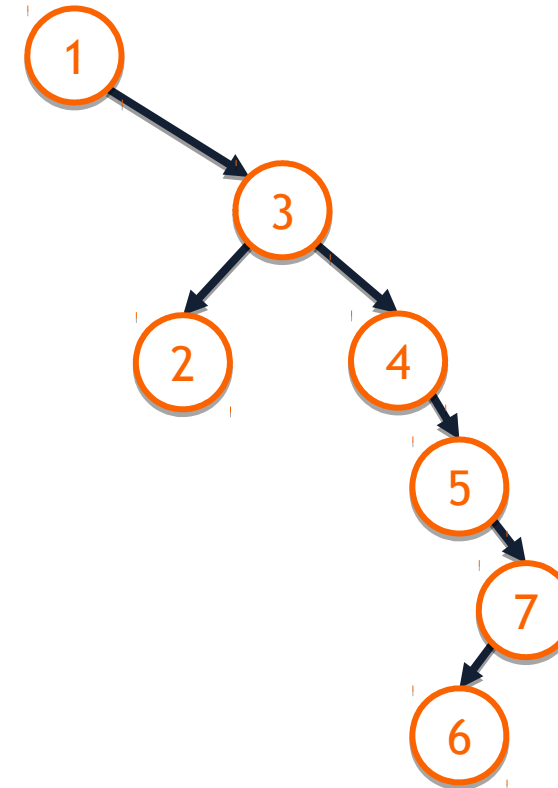
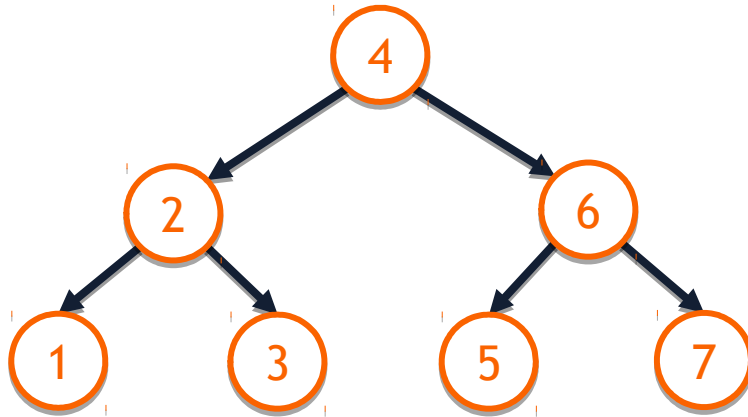
Balanced BST

Prof. Wade Fagen-Ulmschneider

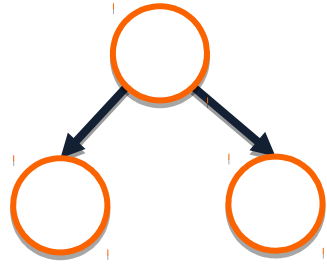
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TO BRIGHT CHILDREN
OF THE FUTURE

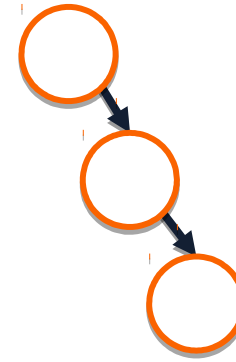
Balanced BSTs are height-balanced trees that ensures nearly half of the data is located in each subtree:



BST Sub-structures



Mountain



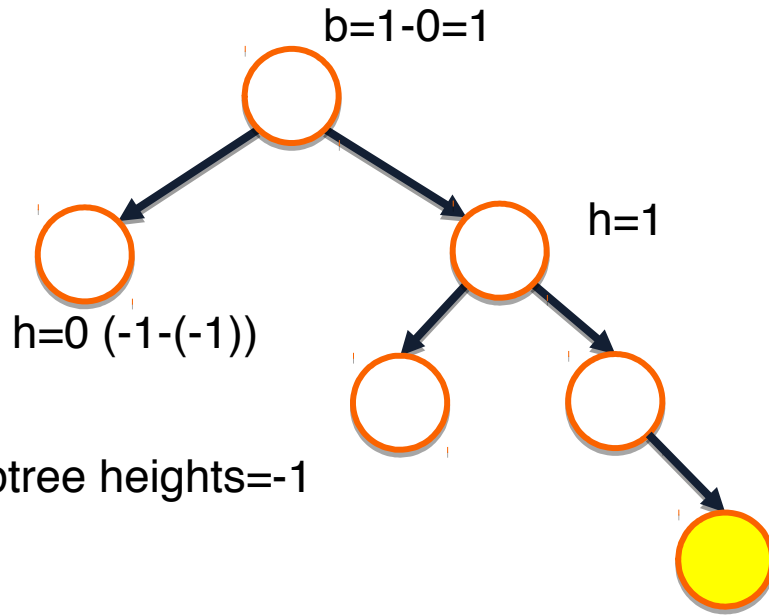
Stick

Example: BST Insert

Consider a new node inserted into an initially balanced BST:

balance factor:
height of left child tree -
height of right child tree

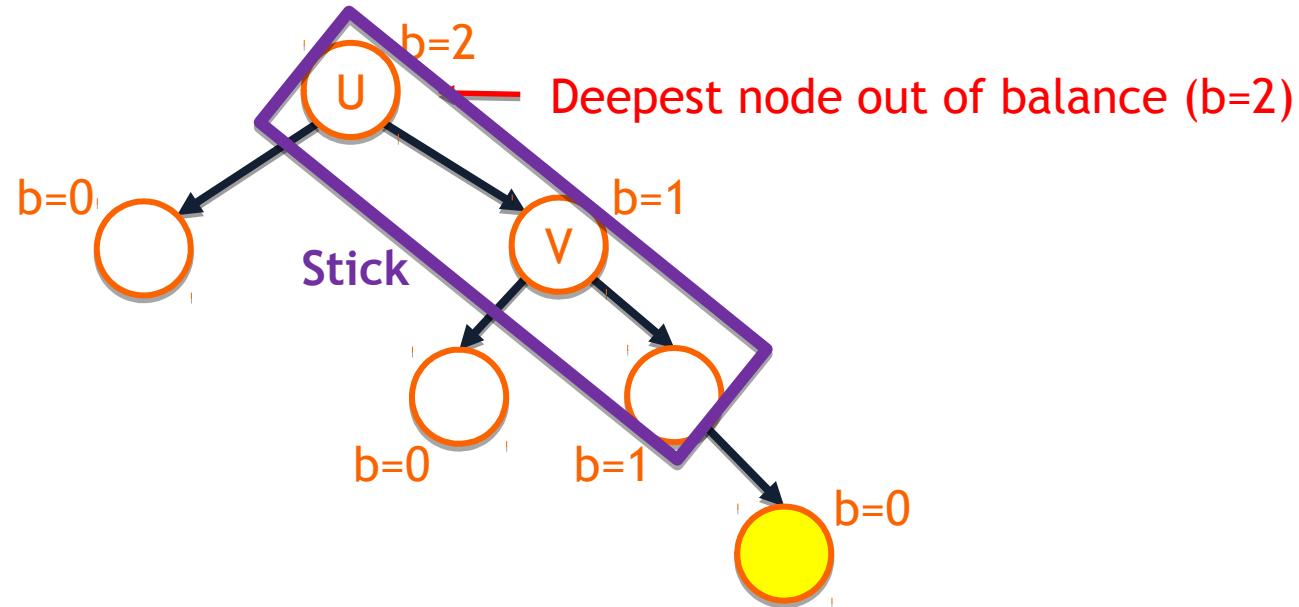
for leaf nodes, left and right subtree heights=-1



original tree (before the insertion)
has a balance factor of 1

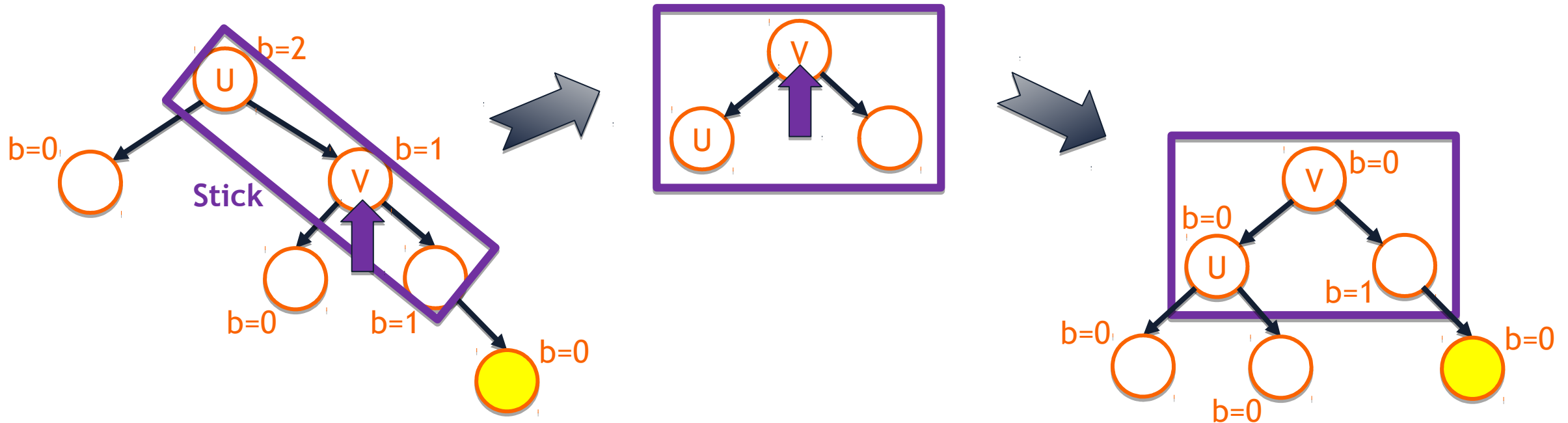
BST Insert

We identify the deepest node in the tree that is out of balance:



BST Rotation

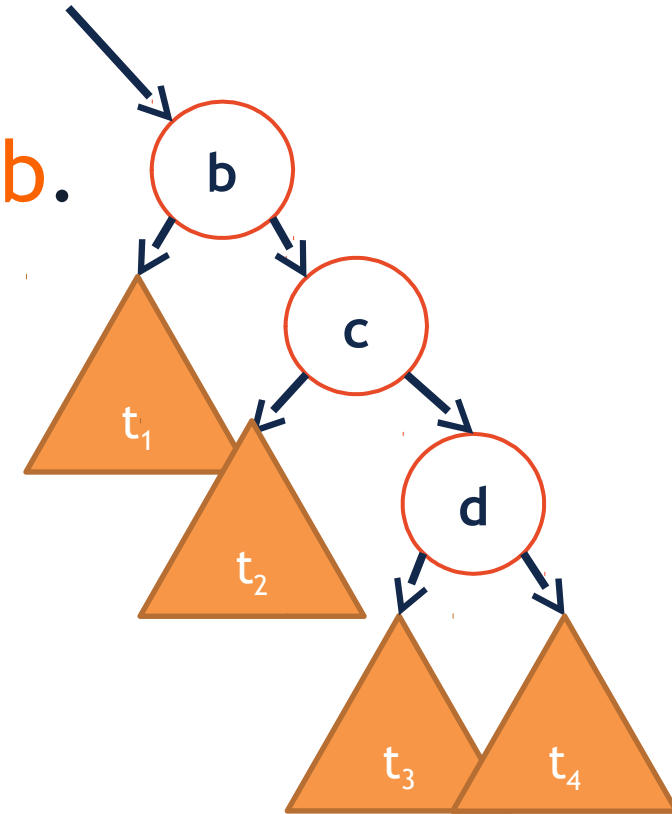
To turn a stick into a mountain: Break the stick in half, raise it up as a mountain, and re-attach the children:



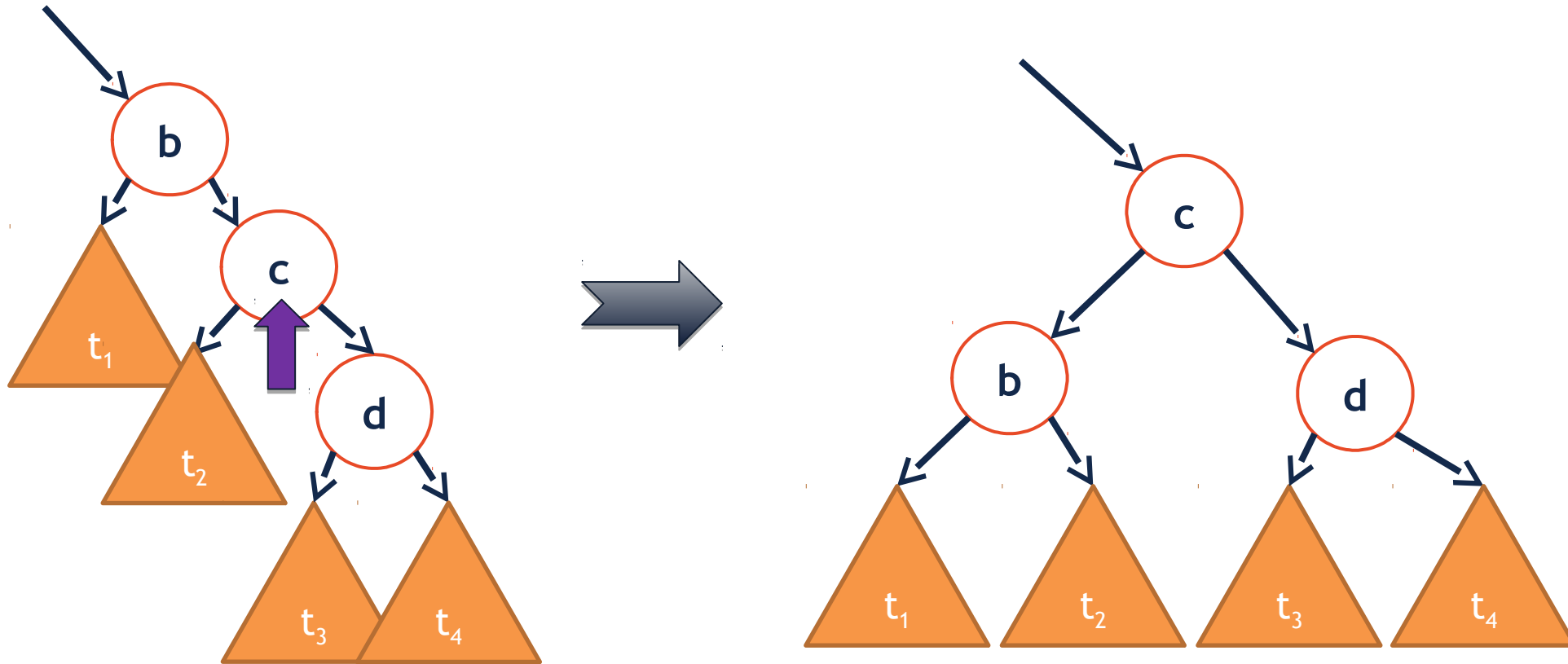
Generic Left Rotation

Consider an arbitrary tree such that:

- The deepest point of imbalance is at node **b**.
- The balance factor of **b** is 2.
- The balance factor of **c** is 1.
- An insert in t_3 or t_4 caused the imbalance.

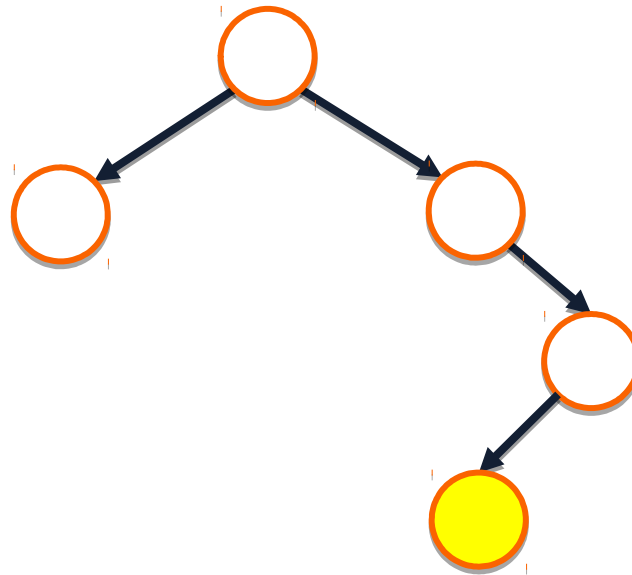


Generic Left Rotation



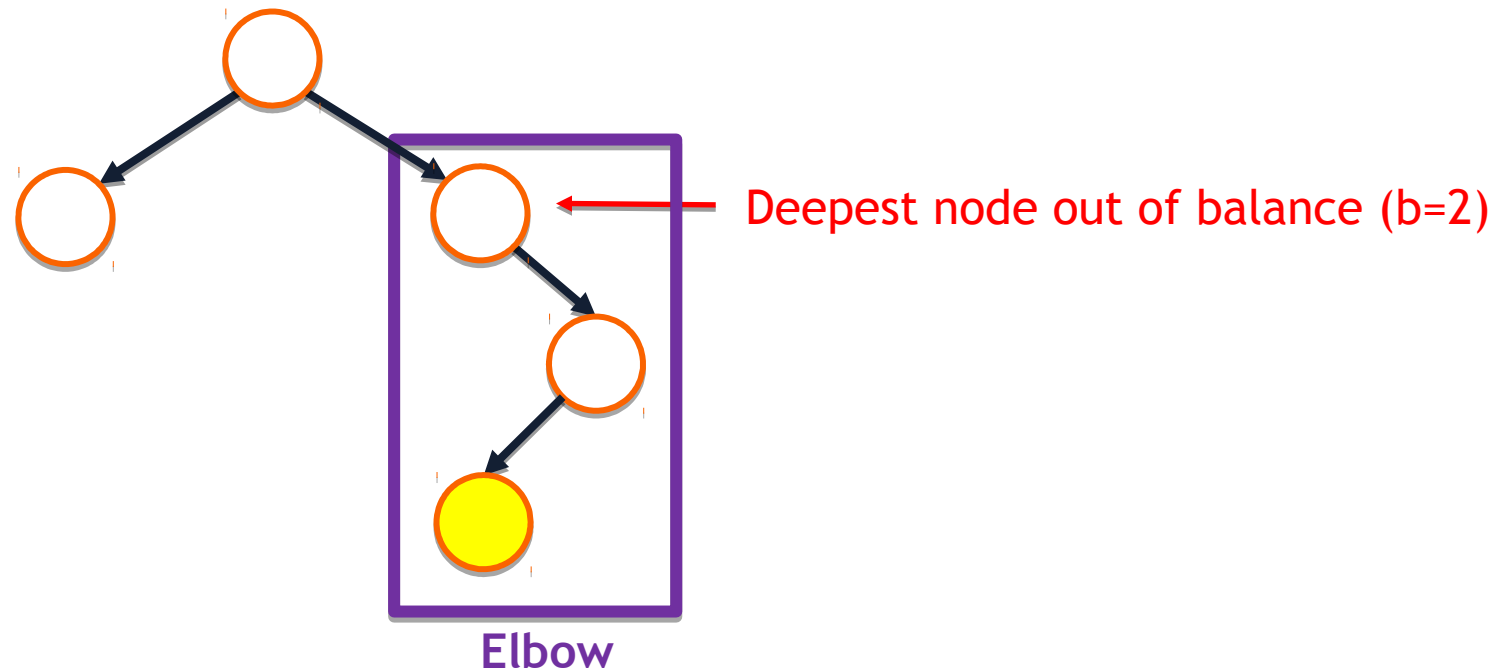
BST Insert, Example #2

Consider a new node inserted into an initially balanced BST:



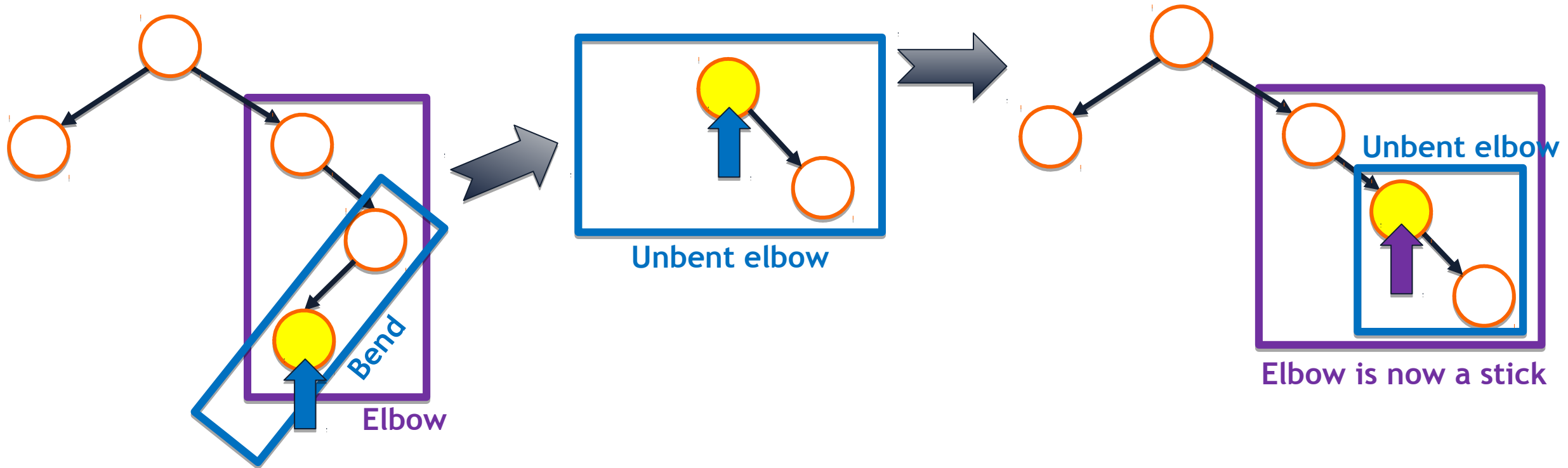
BST Insert

We identify the deepest node in the tree that is out of balance:



BST Rotation

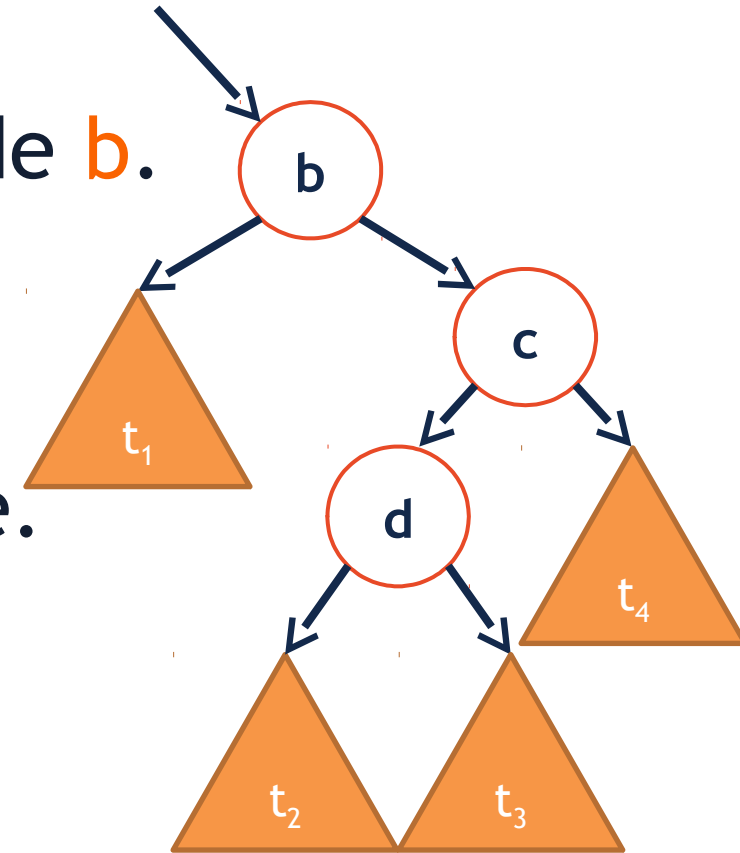
We “unbend” the elbow with a rotation about the bend and then we have a stick:



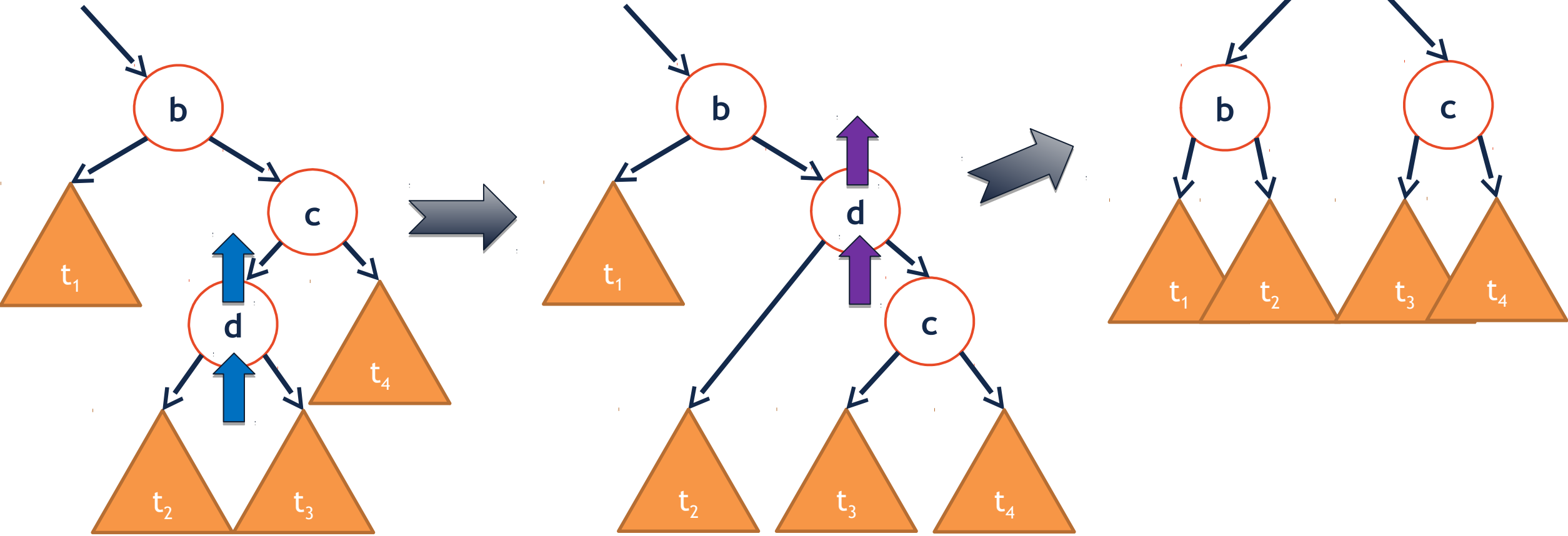
Generic Right-Left Rotation

Consider an arbitrary tree such that:

- The deepest point of imbalance is at node **b**.
- The balance factor of **b** is 2.
- The balance factor of **c** is -1.
- An insert in t_2 or t_3 caused the imbalance.



Generic Right-Left Rotation



Rotation Summary

	Balance factor (b) of the lowest point of imbalance	Balance factor (b) of the node in the direction of imbalance
Left Rotation	2	1
Right Rotation	-2	-1
Right-Left Rotation	2	-1
Left-Right Rotation	-2	1

BST Rotations

- BST rotations restore the balance property to a tree after an insert causes a BST to be out of balance.
- Four possible rotations: L, R, LR, RL
 - Rotations are local operations.
 - Rotations do not impact the broader tree.
 - Rotations run in $O(1)$ time.
- These trees are called “AVL Trees”

