

# Diffie Hellman Key Exchange

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- First published public key algorithm.
- Enables two users to securely exchange a secret key that can be used for subsequent encryption of messages.

- Suppose Alice & Bob wish to exchange a key.
- Both of them agree upon a prime number,  $p$  & an integer,  $g$ , which is a primitive root of  $p$ .
- $g$  is a generator of order  $p-1$  in the group  $\langle Z_p^*, x \rangle$
- The group and the generator need not be confidential.



- Alice selects a large random integer,  $x$  such that  $0 \leq x \leq p-1$ , computes,  $R_1 = g^x \bmod p$  & sends  $R_1$  to Bob.
- Bob selects another large random integer,  $y$  such that  $0 \leq y \leq p-1$ , computes,  $R_2 = g^y \bmod p$  & sends  $R_2$  to Alice.
- Both Alice and Bob keeps the  $x$  and  $y$  values private.
- The values  $R_1$  and  $R_2$  are publicly available to the other side.

- Alice computes the key,  $K = R_2^x \bmod p$ .
- Bob computes the key,  $K = R_1^y \bmod p$ .
- $K = (g^x \bmod p)^y \bmod p = (g^y \bmod p)^x \bmod p = g^{xy} \bmod p$
- Both of these calculations produce identical results.
- Thus both of them have successfully exchanged a secret key value.

- The security of Diffie Hellman lies on the discrete logarithmic problem.
- Let  $p$  be a very large prime number,  $g$  is a primitive root in group  $G = \langle \mathbb{Z}_p^*, \times \rangle$  and  $x$  is an integer, then it's easy to compute  $R_1 = g^x \bmod p$
- Given  $R_1, g, p$  it's computationally infeasible to calculate  $x = \log_g R_1 \bmod p$ .



# D-H Key Exchange Example

- Suppose A & B agree to use  $p = 23$  &  $g = 7$ .
- A & B select secret keys,  $x = 3$  &  $y = 6$ .
- Each of them computes its public key as,  
$$R_1 = 7^3 \bmod 23 = 21.$$
$$R_2 = 7^6 \bmod 23 = 4.$$
- After exchanging the public keys, the secret key is calculated as,  
By Alice,  $K = 4^3 \bmod 23 = 18.$   
By Bob,  $K = 21^6 \bmod 23 = 18.$

# Discrete Logarithmic Attack

- To make the scheme safe from discrete logarithmic attack, the following are recommended.
- The prime  $p$  must be very large.
- Bob and Alice should destroy the values of  $x$  and  $y$  after they have calculated the secret key.



# Man-in-the-Middle Attack

- Suppose Alice & Bob wish to exchange keys & Darth is an attacker. The attack proceeds as follows.
  - Alice chooses  $x$ , calculates  $R_1 = g^x \bmod p$  and sends  $R_1$  to Bob.
  - Darth intercepts  $R_1$ .
  - Darth chooses  $z$ , calculates  $R_2 = g^z \bmod p$  and sends  $R_2$  to both Alice and Bob.
  - Bob chooses  $y$ , calculates,  $R_3 = g^y \bmod p$  and sends  $R_3$  to Alice.
  - $R_3$  is intercepted by Darth and never reaches Alice.

- Alice and Darth calculates,  $K_1 = g^{xz} \bmod p$  and it's the shared secret key between Alice and Darth.
- Bob and Darth calculates,  $K_2 = g^{zy} \bmod p$  and it's the shared secret key between them.
- Alice and Bob never knows about it.

- Alice & Bob think that they share a secret key.
- Actually, Alice & Darth share secret key  $K_1$  & Alice & Bob share secret key  $K_2$ .
- All future communication is compromised .



- This problem occurs because this algorithm does not authenticate the participants.
- This can be overcome with the use of digital signatures.