Fast and Robust Spectrum Sensing via Kolmogorov-Smirnov Test

Guowei Zhang, Xiaodong Wang, *Fellow, IEEE*, Ying-Chang Liang, and Ju Liu

***Abstract*—A new approach to spectrum sensing in cognitive radio systems based on the Kolmogorov-Smirnov (K-S) test is proposed. The K-S test is a non-parametric method to measure the goodness of fit. The basic procedure involves computing the empirical cumulative distribution function (ECDF) of some decision statistic obtained from the received signal, and comparing it with the ECDF of the channel noise samples. A sequential version of the K-S-based spectrum sensing technique is also proposed. Extensive simulation results demonstrate that compared with the existing spectrum detection methods, such as the energy detector and the eigenvalue-based detector, the proposed K-S detectors offer superior detection performance and faster detection, and is more robust to channel uncertainty and non-Gaussian noise.**

***Index Terms*—Cognitive radio, spectrum sensing, Kolmogorov-Smirnov test, sequential detection, non-Gaussian noise.**

I. INTRODUCTION

TO cope with the recent reality of stringent shortage in frequency spectrum due to the proliferation of wireless services, cognitive radio has been considered as an attractive technique to improve spectrum utilization for future wireless systems. In cognitive radio networks, one important function of the secondary users is to detect the presence of primary users utilizing the channel, and to access the channel in such a way that it causes no performance degradation to the primary users. Designing fast and accurate spectrum sensing algorithms is a challenging task especially at the low signal-to-noise ratio (SNR) region [1], [2].

The energy detector is one of the most commonly employed spectrum sensing schemes, since it does not require any prior knowledge about the structure of the primary user's signal. However, the energy detector does require to know the noise variance to properly perform the detection. In practice, the noise variance may not be perfectly known and the uncertainty in the noise variance can significantly degrade the performance of the energy detector. Other spectrum sensing methods take advantage of the statistical properties of the signal to be detected, including the cyclostationarity models [3], [4], the matched-filtering-based methods [5]–[7], and the filter-bank techniques [8], [9]. Recently the eigenvalue-based blind spectrum sensing has been proposed in [10]–[12] that makes use of the random matrix theory. The test statistic is the ratio between the largest and the smallest eigenvalues of the covariance matrix of the received signal. This detector requires a relatively large number of signal samples to reach satisfactory performance, resulting in long sensing time. Moreover, its performance degrades substantially in the presence of non-Gaussian noise.

To achieve fast spectrum sensing, sequential algorithms have been proposed in [13], [14] based on the sequential probability ratio test (SPRT) and the cumulative sum (CUSUM) test, respectively. Both approaches require that the probability density functions (pdf) f_0 and f_1, corresponding to the cases of noise only and signal-plus-noise, respectively, be known or estimated in advance. Moreover, they are restricted to detect the primary user with a specific SNR. On the other hand, typically the SNR of the primary user is unknown to the secondary user; and moreover, in the presence of non-Gaussian noise whose pdf is not exactly known, these sequential detectors become less effective.

In this paper, we propose a new approach to fast and robust spectrum sensing based on the Kolmogorov-Smirnov (K-S) test. The K-S detector does not require the prior knowledge of the signal and it needs only a short sequence of noise samples. Note that this is essentially the same requirement as the energy detector, since the energy detector needs to know the channel noise variance, which is estimated from the channel noise samples obtained prior to the start of the communication session. The proposed detector significantly outperforms the existing detectors in the presence of non-Gaussian noise. In Gaussian noise, it is less sensitive to the noise uncertainty than the energy detector; and it requires much less signal samples (i.e., faster sensing) compared with the eigenvalue-based detector. We also propose a sequential K-S detector and show its advantage over the existing sequential spectrum sensing methods.

The remainder of this paper is organized as follows. In Section II, we formulate the general spectrum sensing problem in a dispersive MIMO channel. In Section III, we propose the K-S-based spectrum sensing technique and its sequential variant. In Section IV, we provide extensive simulation results to compare the proposed K-S detector with some existing spectrum sensing algorithms under various conditions. Finally, Section V concludes the paper.

II. THE SPECTRUM SENSING PROBLEM

We consider the problem of detecting the presence of one or several primary users at a given channel based on the signal observed by the cognitive radio's receiver. Specifically, assuming a general multiple-input multiple-output (MIMO) frequency-selective fading channel between the primary and

Paper approved by M. R. Beuhrer, the Editor for Cognitive Radio and UWB of the IEEE Communications Society. Manuscript received October 21, 2009; revised March 3, 2010 and May 24, 2010.

G. Zhang was supported in part by a scholarship from the China Scholarship Council.

G. Zhang and J. Liu are with the School of Information Science and Engineering, Shandong University, Jinan, 250100, P.R. China.

X. Wang is with the Electrical Engineering Department, Columbia University, New York, NY 10027 (e-mail: wangx@ee.columbia.edu).

Y.-C. Liang is with the Institute for Infocomm Research, A*STAR, Singapore.

Digital Object Identifier 10.1109/TCOMM.2010.11.090209

secondary users, the sampled signal $\boldsymbol{y}(t)$ received by a secondary user, defined as $\boldsymbol{y}[n] \triangleq \boldsymbol{y}(nT_s)$ with $1/T_s$ being the sampling rate, is given by

$$\boldsymbol{y}[n] = \sum_{k=1}^{K} \sum_{\ell=0}^{L-1} \boldsymbol{H}_k[n, \ell] \boldsymbol{s}_k[n-\ell] + \boldsymbol{v}[n], \tag{1}$$

when there are K primary users present and transmitting over the sensed channel. In (1), L is the multipath channel delay spread in terms of the number of symbol intervals; $\boldsymbol{s}_k[n] \in \mathbb{C}^{N_t}$ is the n-th transmitted symbol vector by the k-th primary user with N_t being the number of transmit antennas; $\boldsymbol{y}[n] \in \mathbb{C}^{N_r}$ is the n-th received signal vector by the secondary user with N_r being the number of receive antennas; $\boldsymbol{H}_k[n, \ell] \in \mathbb{C}^{N_r \times N_t}$ is the time-variant MIMO channel tap matrix of the k-th user; $\boldsymbol{v}[n] \in \mathbb{C}^{N_r}$ is the noise vector.

On the other hand, if no primary user is transmitting over the sensed channel, the observations consist of noise only, i.e.,

$$\boldsymbol{y}[n] = \boldsymbol{v}[n]. \tag{2}$$

Denote the received signals samples as $\boldsymbol{Y} \triangleq \{\boldsymbol{y}[n], n = 1, \ldots, M\}$. Then the spectrum sensing problem becomes the following hypothesis test problem

$$\begin{array}{rll} & \mathcal{H}_0: & \text{No primary user is present:} \\ & & \boldsymbol{Y} \text{ follows model (2),} \\ \text{and} & \mathcal{H}_1: & \text{One or more primary users are present:} \\ & & \boldsymbol{Y} \text{ follows model (1).} \end{array} \tag{3}$$

Note there are several special cases of the general signal model in (1) that are of interest, as follows.

- Slow-fading frequency-flat MIMO channels: $L = 1$ and $\boldsymbol{H}_k[n, 0] \equiv \boldsymbol{H}_k, \forall n$.
- Slow-fading frequency-selective MIMO channels: $\boldsymbol{H}_k[n, \ell] \equiv \boldsymbol{H}_k[\ell], \forall n$.
- MIMO-OFDM channels: $L = 1$ and $\boldsymbol{H}_k[n, 0]$ are obtained by the DFT of the time-domain channel coefficients. (Here n becomes the subcarrier index.)

III. Spectrum Sensing via K-S Test

The Kolmogorov-Smirnov (K-S) test is an non-parametric test of goodness of fit for the continuous cumulative distribution of the data samples [15], [16]. It can be used to approve the null hypothesis that two data populations are drawn from the same distribution to a certain required level of significance. On the other hand, failing to approve the null hypothesis shows that they are from different distributions.

A. Two-sample K-S Test

In the two-sample K-S test, we observe a sequence of i.i.d. real-valued data samples $z_1, z_2, \ldots, z_N$ with the underlying cumulative distribution function (cdf) $F_1(z)$. We are also given another i.i.d. sequence $\xi_1, \xi_2, \ldots, \xi_{N_0}$ drawn from a hypothesized distribution $F_0(\xi)$. The null hypothesis to be tested is

$$H_0: \quad F_1 = F_0. \tag{4}$$

The K-S test first forms the empirical cdf from the observed data samples

$$\hat{F}_1(z) \triangleq \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}(z_n \leq z), \tag{5}$$

where $\mathbb{I}(\cdot)$ is the indicator function, which is equal to one if the input is true, and equal to zero otherwise. It also forms the empirical cdf $\hat{F}_0$

$$\hat{F}_0(\xi) \triangleq \frac{1}{N_0} \sum_{n=1}^{N_0} \mathbb{I}(\xi_n \leq \xi). \tag{6}$$

The largest absolute difference between the two cdfs is used as the goodness-of-fit statistic, given by [15], [16]

$$D \triangleq \sup_{z \in \mathbb{R}} |F_1(z) - F_0(z)|; \tag{7}$$

and in practice, it is calculated by

$$\hat{D} = \max_i |\hat{F}_1(z_i) - \hat{F}_0(z_i)|, \tag{8}$$

for a set of uniformly spaced sample points $\{z_i\}$. The details of the numeric procedure can be found in [17]. The significance level $\hat{\alpha}$ of the observed value $\hat{D}$ is given by

$$\hat{\alpha} \triangleq P(D > \hat{D}) = \Psi\Big(\big[\sqrt{\tilde{N}} + 0.12 + \frac{0.11}{\sqrt{\tilde{N}}}\big]\hat{D}\Big), \tag{9}$$

$$\text{with} \quad \Psi(x) \triangleq 2 \sum_{m=1}^{\infty} (-1)^{m-1} e^{-2m^2x^2}, \tag{10}$$

where $\tilde{N}$ is the equivalent sample size, given by

$$\tilde{N} = \frac{NN_0}{N + N_0}. \tag{11}$$

Note that $\Psi(\cdot)$ is a monotonically decreasing function with $\Psi(0) = 1$ and $\Psi(\infty) = 0$.

The hypothesis H_0 is rejected at a significance level α if $\hat{\alpha} = P(D > \hat{D}) < \alpha$. The significance level α is an input of the K-S test to specify the false alarm probability under the null hypothesis, i.e.,

$$\alpha \triangleq P(D \geq \tau \mid H_0), \tag{12}$$

where τ is a threshold or the critical value, that can be obtained given a level of significance α by inverting (9). Note that the relationship of critical value τ and the significance level α depends on the equivalent sample size $\tilde{N}$. Hence given α, H_0 is accepted, i.e., $F_1 = F_0$, if $\hat{D} \leq \tau$; and otherwise H_0 is rejected, i.e., $F_1 \neq F_0$.

B. Two-dimensional K-S Test

Since the signals in (1) are complex-valued, the corresponding distributions are two-dimensional (2D). Consider a sequence of 2D real-valued data samples $(u_1, v_1), \ldots, (u_N, v_N)$. In the 2D K-S test, the cdfs for all four quadrants (I, II, III, and IV) of the 2D plane are examined, i.e.,

$$F^{I}(u, v) \triangleq P(U < u, V < v), \quad F^{II}(u, v) \triangleq P(U > u, V < v), \tag{13}$$

$$F^{III}(u, v) \triangleq P(U > u, V > v), \quad F^{IV}(u, v) \triangleq P(U < u, V > v). \tag{14}$$

In [18], it is suggested to calculate the four empirical cdfs using all possible combinations of the 2D data samples. For example, the first quadrant empirical cdf is given by

$$\hat{F}_1^{\mathrm{I}}(u,v) = \frac{1}{N^2} \sum_{(i,j)\in\{1,\ldots,N\}\times\{1,\ldots,N\}} \mathbb{I}(u_i < u)\mathbb{I}(v_j < v). \tag{15}$$

The empirical cdfs for the other quadrants can be computed similarly. On the other hand, in [19], it is proposed to use the 2D samples directly rather than all possible combinations for forming the empirical cdfs. Thus the first quadrant empirical cdf becomes

$$\hat{F}_1^{\mathrm{I}}(u,v) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}(u_n < u)\mathbb{I}(v_n < v). \tag{16}$$

The two methods are compared in [19], where it is shown that when the data components from the two dimensions are uncorrelated their performance is similar.

The statistic of the 2D K-S test is the largest absolute difference between the the empirical cdfs among all four quadrants under H_0 and H_1, i.e.,

$$\hat{D} = \max_{q\in\{\mathrm{I,II,III,IV}\}} \max_{1\le n\le N} |\hat{F}_1^q(u_n,v_n) - \hat{F}_0^q(u_n,v_n)|. \tag{17}$$

As in the 1D test, for a given significance level or critical value, using $\hat{D}$ in (17) we can then test to approve or disapprove the hypothesis H_0.

C. K-S-based Spectrum Sensing

The proposed K-S-based spectrum sensing algorithm makes use of a sequence of noise-only samples to form the null hypothesis distribution F_0. It performs the two-sample K-S test to accept or reject the null hypothesis test based on the received signal samples. Since the received signals in (1) and (2) are complex-valued, we will consider the following ways to form the decision statistics $\{z_n\}$.

- Magnitude-based 1D K-S detector: from M received signal vectors $\{\boldsymbol{y}[n], n = 1,\ldots,M\}$, we obtain the following MN_r decision statistics:

  $$z_{(n-1)N_r+j} = |y_j[n]|,\ j = 1,\ldots,N_r;\ n = 1,\ldots,M.$$

- Quadrature-based 1D K-S detector: from M received signal vectors $\{\boldsymbol{y}[n], n = 1,\ldots,M\}$, we obtain the following $2MN_r$ decision statistics

  $$z_{2[(n-1)N_r+j]} = \Re\{y_j[n]\},\ z_{2[(n-1)N_r+j]+1} = \Im\{y_j[n]\},$$
  $$j = 1,\ldots,N_r;\ n = 1,\ldots,M.$$

- Quadrature-based 2D K-S detector: from M received signal vectors $\{\boldsymbol{y}[n], n = 1,\ldots,M\}$, we obtain the following MN_r decision statistic pairs

  $$z_{(n-1)N_r+j} = (\Re\{y_j[n]\}, \Im\{y_j[n]\}),\ j = 1,\ldots,N_r;$$
  $$n = 1,\ldots,M.$$

The K-S-based spectrum sensing then involves the following steps.

- The cognitive user obtains the noise statistics by collecting M_0 noise-only sample vectors $\{\boldsymbol{v}[n], n = 1,\ldots,M_0\}$ and the corresponding amplitude or quadrature statistics $\{\xi_n\}$. It computes the empirical 1D or 2D noise cdf $\hat{F}_0$.
- Spectrum sensing: The cognitive user collects M received signal vectors $\{\boldsymbol{y}[n], n = 1,\ldots,M\}$ and form the corresponding amplitude or quadrature statistics $\{z_n\}$. It computes the empirical 1D or 2D cdf $\hat{F}_1$. It then computes the maximum difference $\hat{D}$ in (8), and the threshold τ based on the given false alarm rate α using (9) and (12). If $\hat{D} > \tau$, then declare the primary users' presence; otherwise no primary user is present.

D. Sequential K-S Spectrum Sensing

Given the sequential nature of the problem, in which the secondary user acquires observations in a sequential manner, it is also of interest to approach the spectrum sensing problem in a sequential manner. That is, instead of using a fixed number of samples for each decision, some realizations of the observation sequences may allow us to make decision after only a few samples, whereas for other realizations we may need to continue sampling to make a decision.

While the algorithm described in the previous subsection is designed for a fixed number of samples, the significance values of the K-S statistic (9) allows us to make sequential decisions on the null hypothesis. With each new sample, the empirical cdf $\hat{F}_1$ can be updated and the K-S statistic reevaluated.

A sequential K-S test can be formed by concatenating P K-S tests, starting with q samples and adding q samples at each subsequent stage up to P stages, where P is the truncation point of the test. We fix the desired false alarm probability of the sequential test to $P_{FA} = \alpha$. Because the sequential test is composed of P tests, we need to calculate the false alarm probability of each stage in order to meet the overall P_{FA}. Let β be the false alarm probability of each stage. Then the resulting P_{FA} of the P-truncated sequential K-S test can be approximated as

$$\begin{aligned} P_{FA} &\cong \beta + (1-\beta)\beta + (1-\beta)^2\beta + \ldots + (1-\beta)^{P-1}\\ \beta &= 1 - (1-\beta)^P = \alpha. \end{aligned} \tag{18}$$

Note that (18) is exact when the signal vectors are independent. Here since the adjacent signal vectors share overlapping segments, (18) becomes an approximation. Hence each individual K-S test should use the threshold

$$\beta = 1 - \sqrt[P]{1-\alpha}. \tag{19}$$

Algorithm 1 describes the P-truncated K-S spectrum sensing algorithm in detail. Note that the empirical cdf update of Step 5 can be easily done sequentially, as it only needs to include the new samples.

IV. Simulation Results

In this section, we provide simulation results to illustrate the performance of the proposed K-S spectrum sensing algorithm, and compare it with that of the energy detector as well as the eigenvalue-based detector, under various scenarios. In the sequel, the desired false alarm level is set to $\alpha = 0.01$.

Algorithm 1 P-truncated sequential K-S spectrum sensing with $P_{FA} = \alpha$

1: Using the noise-only samples, compute the empirical distribution $\hat{F}_0$.
2: $\beta = 1 - \sqrt[P]{1-\alpha}$; $p = 0$.
3: Take q new received signal vectors $\{\boldsymbol{y}[n], n = pq + 1, \ldots, (p+1)q\}$ and form the corresponding decision statistics.
4: Compute the threshold τ_s based on the given false alarm rate β by inverting (9) with $\tilde{N} \leftarrow (p+1)q$.
5: Update the empirical cdf $\hat{F}_1$ with the new decision statistic samples $\{z_n\}$.
6: Compute the maximum difference $\hat{D}$ in (8).
7: **if** $\hat{D} > \tau_s$ **then**
8: reject H_0. The primary use is present.
9: **else if** $p = P$ **then**
10: do not reject H_0. The primary is not present.
11: **else**
12: $p \leftarrow p + 1$. Go to 3.
13: **end if**

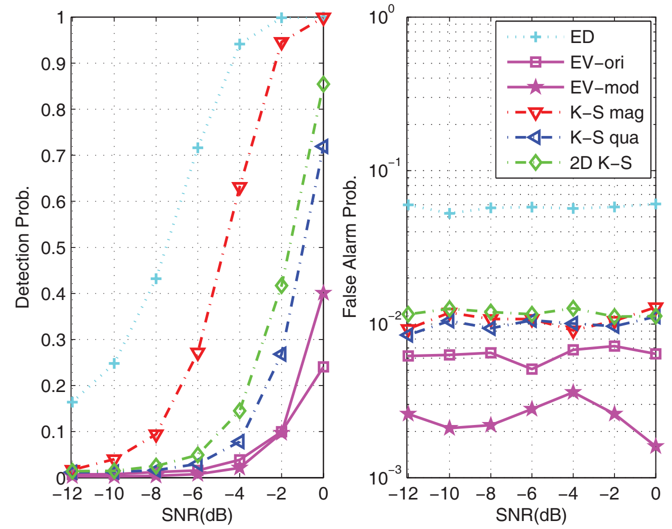

Fig. 1. Performance of different detectors in multipath fading MIMO (SCME) channels with Gaussian noise. $L = 6, N_t = 2, N_r = 4, K = 2, M_0 = 100, M = 50$.

Simulation Setup and Implementations of Existing Methods: We consider a multipath fading 2×4 MIMO channel with two primary users transmitting QPSK signals, i.e., $K = 2, N_t = 2, N_r = 4$. The 3GPP SCM and its extension SCME are employed in the simulations [20], [21] where we use the default settings or urban-micro scenario in MATLAB implementation of SCME, i.e., the channel consists of $L = 6$ paths. The number of noise-only training samples $M_0 = 100$, and the number of received signal samples $M = 50$. The implementation of the K-S detectors follows the procedure described in Section III-C. For the energy detector, the test statistic is given by $T_{ED} = \sum_{n=1}^{M}\sum_{j=1}^{N_r}|y_j[n]|^2$, which follows a χ^2 distribution with $2N_rM$ degrees of freedom under H_0. Based on the M_0 noise-only training samples we obtain an estimate of the noise variance $\widehat{\sigma^2}$. The threshold for the energy detector is then computed according to $\gamma_{ED} = \widehat{\sigma^2} \cdot \chi^{-2}_{2N_rM}(1-\alpha)$.

The original eigenvalue-based detector does not require the noise variance or the noise-only training samples. It makes use of the asymptotic property of the ratio of the largest eigenvalue to the smallest one of the sample covariance matrix. Specifically, it first computes the sample covariance matrix of the stacked received signal $\hat{\boldsymbol{C}} = \frac{1}{M-m+1}\sum_{n=1}^{M-m+1}\tilde{\boldsymbol{y}}[n]\tilde{\boldsymbol{y}}[n]^H$, where $\tilde{\boldsymbol{y}}[n] \triangleq [\boldsymbol{y}[n]^T, \boldsymbol{y}[n+1]^T, \ldots, \boldsymbol{y}[n+m-1]^T]^T$ with m being the smoothing factor ($m = 3$ is used in the simulations), and finds its eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{mN_r}$. Then it compares the ratio $\frac{\lambda_1}{\lambda_{mN_r}}$ to a threshold γ_{EV}, and declares the presence of primary users if $\frac{\lambda_1}{\lambda_{mN_r}} > \gamma_{EV}$, and otherwise no primary user is present. The threshold is given by $\gamma_{EV} = (\frac{\sqrt{U}+\sqrt{D}}{\sqrt{U}-\sqrt{D}})^2 . (1 + \frac{(\sqrt{U}+\sqrt{D})^{-2/3}}{(UD)^{1/6}} . F^{-1}_{TW2}(1-\alpha))$ where $F^{-1}_{TW_2}(\cdot)$ is the inverse Tracy-Widom cumulative distribution function (cdf) of order 2 [11], $U = M - m + 1$ is the sample size, and $D = mN_r$ is the signal dimension.

In order to make a fair comparison with the proposed K-S detectors, we modify the eigenvalue-based detector so that it also makes use of the noise-only samples. In particular, denote $\boldsymbol{v}[1], \ldots, \boldsymbol{v}[M_0]$ as the noise-only samples and assume that $M_0/M = \eta$ is an integer. We first form the $(\eta+1)N_r$-dimensional vectors $\bar{\boldsymbol{y}}[n] \triangleq [\boldsymbol{y}[n]^T, \boldsymbol{v}[n]^T, \ldots, \boldsymbol{v}[(\eta-1)M + n]^T]^T, n = 1, \ldots, M$ and compute the sample covariance matrix $\bar{\boldsymbol{C}}$ as well as its eigenvalues. Then we compare the ratio $\frac{\lambda_1}{\lambda_{(\eta+1)N_r}}$ to a threshold, where the threshold is calculated with the sample size $U = M$ and signal dimension $D = (\eta+1)N_r$.

Performance under Gaussian noise: We first consider spectrum sensing in multipath fading channels by various methods and the results in terms of detection probabilities and false alarm probabilities are shown in Fig. 1. It is seen that the false alarm probability of the energy detector (ED) is significantly higher than the target value $\alpha = 0.01$. This is because the energy detector is sensitive to the uncertainty of noise variance. On the other hand, both the original eigenvalue-based detector (EV-ori) and the modified version that incorporates noise-only samples (EV-mod), can keep the false alarm rate below the target value. And the modified version does improves upon the original version. The magnitude-based K-S detector (K-S mag), the quadrature-based detector (K-S qua), as well as the two-dimensional K-S detector can all meet the target false alarm rate, and provide superior detection performance to the eigenvalue-based detectors. In particular, the magnitude-based K-S detector exhibits the best performance. Note that although the energy detector has an even higher detection probability, it is jeopardized by the very high false alarm rate.

Next we illustrate the spectrum sensing performance in MIMO-OFDM systems. The total number of subcarriers is 512 which is divided into 8 independent subbands. Each sub-band can be occupied by the primary users and sensed by the cognitive radio. Hence the effective sample size $M = 512/8 = 64$. Noise-only samples are collected on any one subband, i.e., $M_0 = 64$. Note that in this case, no signal stacking is needed for the eigenvalue-based methods, i.e., $m = 1$. The detection and false alarm rates of various detectors are averaged over the subbands and shown in Fig. 2. Again it is seen that the energy detector is virtually unusable due to its high false alarm rate. The K-S detectors and the eigenvalue-based detectors can all

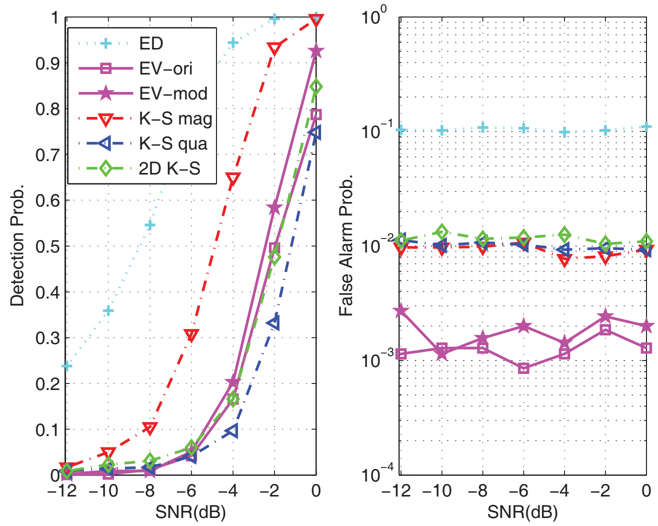

Fig. 2. Performance of different detectors in MIMO-OFDM (SCME) channels with Gaussian noise. $L = 6, N_t = 2, N_r = 4, K = 2$, 512 subcarriers.

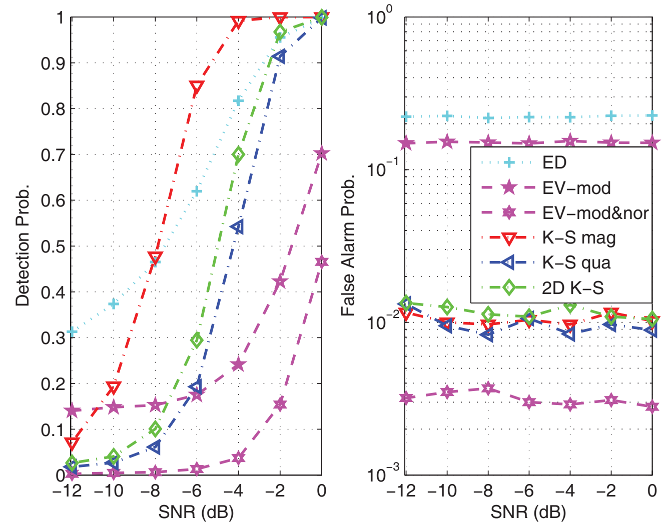

Fig. 4. Performance of different detectors in multipath fading MIMO (SCME) channels with Middleton class-A noise. $L = 6, N_t = 2, N_r = 4, K = 2, A = 0.2, \Gamma = 0.5$.

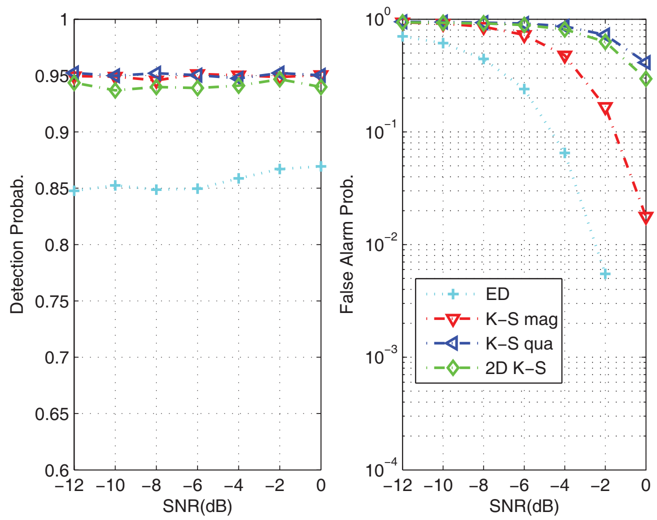

Fig. 3. Performance of different detectors designed under constant detection probability, in multipath fading MIMO (SCME) channels with Gaussian noise. $L = 6, N_t = 2, N_r = 4, K = 2, M_0 = 25, M = 50$. Target $P_d = 0.95$.

meet the target false alarm rate, while the magnitude-based K-S detector still offers the best detection performance.

For spectrum sensing in cognitive radio systems, it is crucial to keep the detection probability high to avoid disrupting the primary user's communications. Therefore a more desirable design is to keep the detection probability fixed and to minimize the false alarm probability. The difficulty, however, is that the secondary user may not have access to the primary user's signals and therefore cannot compute the corresponding detector with fixed detection probability. Nevertheless, here we consider a hypothetical scenario that the primary can access a small segment of the primary user's signal before spectrum sensing starts. Then similarly as before, we can compute both the energy detector and the K-S detector. The performance of both detectors under such a scenario is shown in Fig. 3, where the target detection probability is set as $P_d = 0.95$. Again we can see that the K-S detectors can meet the target P_d whereas the energy detector's detection performance is significantly lower than the target value.

Performance under non-Gaussian noise: We now evaluate the performance of the spectrum sensing algorithms in multipath fading MIMO channels with non-Gaussian or impulsive noise. The noise distribution follows the Middleton class-A model [22] with parameters $A = 0.2$ and $\Gamma = 0.5$. The other system parameters are the same as those for Fig. 1. Note that in non-Gaussian noise the eigenvalue-based detectors have an extremely high false alarm probability, and therefore cannot be used. To circumvent this problem we make another modification by normalizing the received signal first before computing the sample covariance matrix, i.e., $\boldsymbol{y}[n] \leftarrow \boldsymbol{y}[n]/\|\boldsymbol{y}[n]\|, n = 1, \dots, M$. We find that such a normalized eigenvalue-based detector incurs a slight performance degradation in Gaussian noise compared with the original one, but it offers substantial robustness in non-Gaussian noise. The performance of various detectors is illustrated in Fig. 4. It is evident that the energy detector and the unnormalized eigenvalue-based detector (EV-mod) are virtually not functional due to the very high false alarm rate. The normalized eigenvalue-based detector (EV-mod&nor) significantly improves upon the original one. On the other hand, the proposed K-S detectors now exhibits great advantage over the other detectors, in that they not only meet the target false alarm rate closely, but also provide significant improvement in detection performance. Moreover, the magnitude-based K-S detector offers the best performance in non-Gaussian noise. In Fig. 5, we show the performance of the K-S detector and the energy detector with 95% confidence interval in both Gaussian and Middleton class-A noise.

Performance of Sequential K-S Spectrum Sensing: We now illustrate the performance of the sequential K-S spectrum sensing algorithm outlined in Section III-D in multipath fading MIMO channels. The simulation setup is the same as before. The number of samples taken at each step is $q = 10$. We will compare the sequential K-S algorithm with both the fixed-size K-S algorithm and the sequential probability ratio test (SPRT)

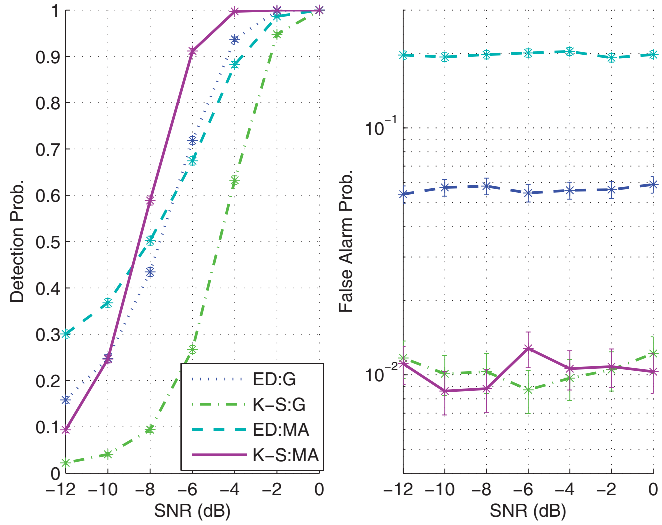

Fig. 5. Confidence intervals of the performance of different detectors in Gaussian and Middleton Class-A noise. $M = 50, M_0 = 100$.

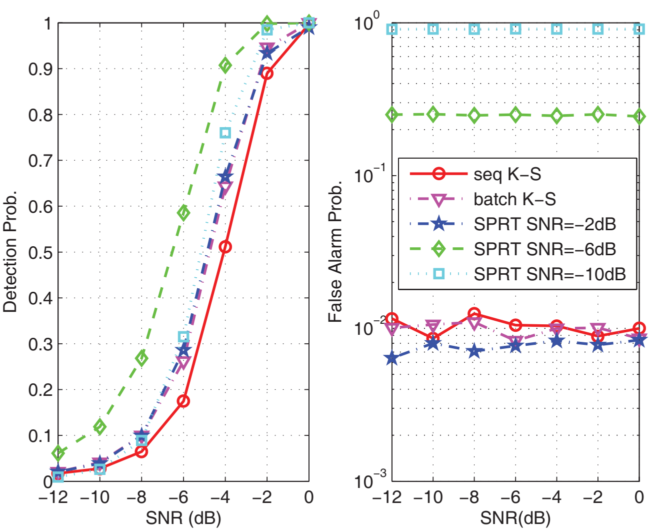

Fig. 6. Performance of different sequential detectors in multi-path fading MIMO (SCME) channels with Gaussian noise. $L = 6, N_t = 2, N_r = 4, K = 2, M_0 = 100, M = 50, q = 10$.

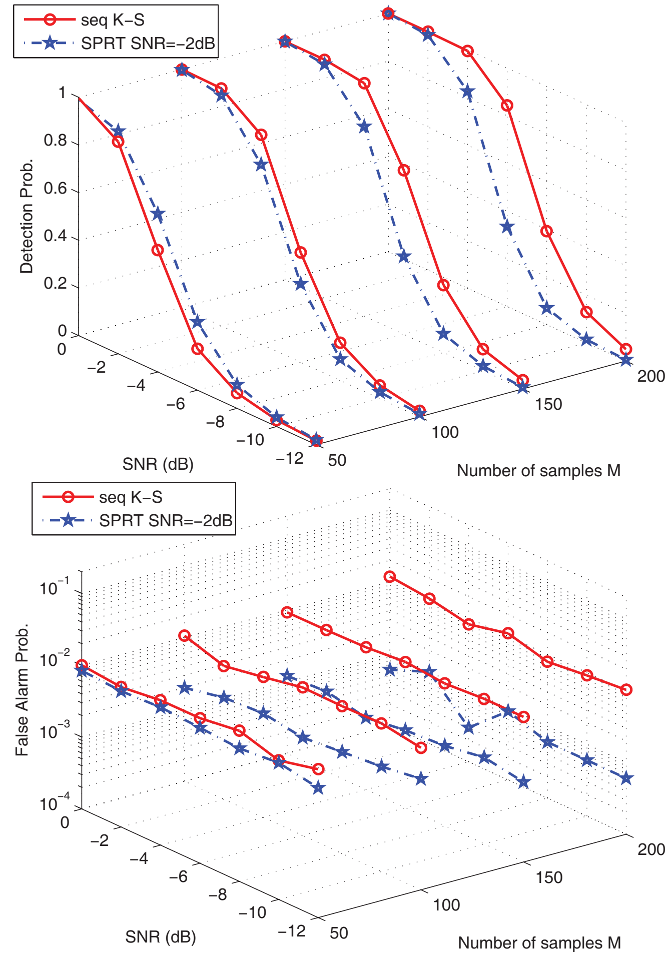

Fig. 7. Performance of sequential K-S and SPRT in multi-path fading MIMO (SCME) channels with Gaussian noise under different sample sizes M. $L = 6, N_t = 2, N_r = 4, K = 2, q = 10$.

in terms of both the detection performance and the number of signal samples required. Note that the SPRT requires to know the pdfs under both H_0 and H_1. In our implementations of SPRT, we assume zero-mean Gaussian distributions under both H_0 and H_1. The variance σ_0^2 under H_0 is obtained from the sample estimate based on the M_0 noise-only samples. On the other hand, under H_1, we used a fixed and possibly mismatched variance σ_1^2. In other words, we assume a fixed SNR$= 10\log[(\sigma_1^2 - \sigma_0^2)/\sigma_0^2]$.

The performance of the sequential K-S detector and that of the SPRT under different assumed SNR values, is illustrated in Fig. 6. Note that for SPRT, the false alarm probability illustrated here corresponds to $P(H_1 \mid H_0) + P(\text{no decision} \mid H_0)$. The performance of the batch (i.e., fixed-size) K-S test is also shown in the same figure. It is seen that the false alarm performance of the SPRT varies significantly under different assumed SNR values. In particular, for assumed SNR$= -6$ dB or -10dB, the false alarm rate is well above the target value $\alpha = 0.01$. On the other hand, for assumed SNR$= -2$dB, the false alarm rate is below 0.01; and the corresponding detection performance is actually better than both the sequential K-S and the batch K-S. However, for this assumed value of SNR$= -2$dB, if we increase the numbers of signal and noise samples (M, M_0) from (50, 100) to, e.g., (100, 200), then the performance of the SPRT will degrade and be outperformed by both the sequential K-S and batch K-S, as shown in Fig. 7.

Next we consider the performance of the sequential K-S detector and that of the SPRT in Middleton class-A noise with $A = 0.2$ and $\Gamma = 0.5$. The number of samples are $M = 100, M_0 = 200$ and $q = 10$. The results are shown in Fig. 8. It is seen that with non-Gaussian noise, the performance of the SPRT under the Gaussian approximation with fixed SNR is substantially inferior to that of the sequential K-S detector. Fig. 9 illustrates the number of samples needed to make a decision under H_1 for various sequential detectors in Middleton class-A noise. The sequential K-S detector is seen to require a much smaller number of samples to reach a decision compared with other methods, i.e., it achieves fast spectrum sensing in non-Gaussian noise.

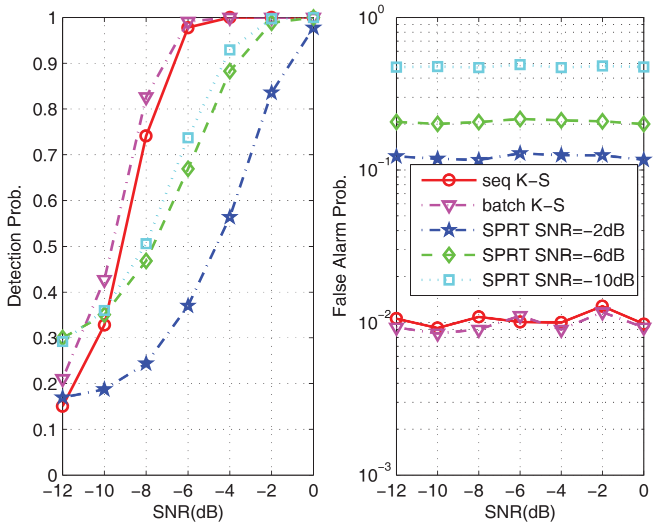

Fig. 8. Performance of different detectors in multi-path fading MIMO (SCME) channels with Middleton class-A noise. $L = 6, N_t = 2, N_r = 4, K = 2, M_0 = 200, M = 100, q = 10, A = 0.2, \Gamma = 0.5$.

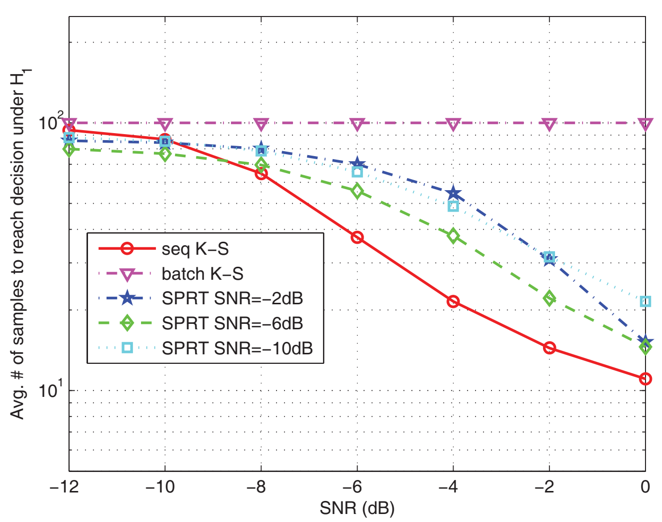

Fig. 9. Average number of samples needed to reach a decision for different detectors in multi-path fading MIMO (SCME) channels with Middleton class-A noise under H_1. $L = 6, N_t = 2, N_r = 4, K = 2$.

V. Conclusions

We have proposed a new nonparametric method for spectrum sensing in cognitive radio systems based on the Kolmogorov-Smirnov (K-S) test. The basic procedure involves computing the empirical cumulative distribution function (ECDF) of the received signal magnitude, and comparing it with the ECDF of the magnitude of the channel noise samples. A sequential version of the K-S-based spectrum sensing technique is also proposed. We have provided extensive simulation results that demonstrate that compared with the existing spectrum detection methods, such as the energy detector and the eigenvalue-based detector, the proposed K-S detectors offer superior detection performance and faster detection, and is more robust to channel uncertainty and non-Gaussian noise. Finally we note that the K-S test has also been recently applied to modulation classification in digital communications [23].

References

[1] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys Tutorials*, vol. 11, no. 1, pp. 116–130, 2009.
[2] A. Ghasemi and E. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Commun. Mag.*, vol. 46, no. 4, pp. 32–39, Apr. 2008.
[3] P. Sutton, K. Nolan, and L. Doyle, "Cyclostationary signatures in practical cognitive radio applications," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 13–24, Jan. 2008.
[4] S. Haykin, D. J. Thomson, and J. H. Reed, "Spectrum sensing for cognitive radio," *Proc. IEEE*, vol. 97, no. 5, pp. 849–877, May 2009.
[5] A. Sahai and D. Cabric, "Spectrum sensing: fundamental limits and practical challenges," *IEEE DySPAN 2005*, Nov 2005.
[6] H.-S. Chen, W. Gao, and D. Daut, "Signature based spectrum sensing algorithms for IEEE 802.22 WRAN," in *Proc. IEEE ICC 2007*, pp. 6487–6492, June 2007.
[7] J. Ma, G. Y. Li, and B. H. Juang, "Signal processing in cognitive radio," *Proc. IEEE*, vol. 97, no. 5, pp. 805–823, May 2009.
[8] M. Hernandez, M. Rojas, and P. Stoica, "New spectral estimation based on filterbank for spectrum sensing," in *Proc. IEEE ICASSP 2008*, pp. 3509–3512, Apr. 2008.
[9] B. Farhang-Boroujeny, "Filter bank spectrum sensing for cognitive radios," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 1801–1811, May 2008.
[10] L. Cardoso, M. Debbah, P. Bianchi, and J. Najim, "Cooperative spectrum sensing using random matrix theory," in *Proc. 3rd International Symposium on Wireless Pervasive Computing (ISWPC) 2008*, May 2008, pp. 334–338.
[11] Y. Zeng and Y.-C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *IEEE Trans. Commun.*, vol. 57, no. 6, pp. 1784–1793, June 2009.
[12] F. Penna, R. Garello, and M. A. Spirito, "Cooperative spectrum sensing based on the limiting eigenvalue ratio distribution in wishart matrices," *IEEE Commun. Lett.*, vol. 13, no. 7, pp. 507–509, July 2009.
[13] S. Chaudhari, V. Koivunen, and H. V. Poor, "Distributed autocorrelation-based sequential detection of OFDM signals in cognitive radios," in *Proc. Ann. 3rd. Conf. on Cogonitive Radio Oriented Wireless Networks and Communications*, Singapore, May 2008, pp. 1–6.
[14] L. Lai, Y. Fan, and H. V. Poor, "Qickest detection in cognitive radio: a sequential change detectionframework," in *Proc. IEEE Global Communications Conf.*, New Orleans, LA, Nov. pp. 1–5.
[15] W. Conover, *Practical Nonparametric Statistics*. John Wiley and Sons, 1980.
[16] W. Press *et al.*, *Numerical Recipes in C*. Cambridge University Press, 1992.
[17] MIT Lecture Notes, http://ocw.mit.edu/NR/rdonlyres/Mathematics/18-443Fall-2006/BD432998-421E-4FE3-8B59-FA3859D4F445/0/lecture14.pdf.
[18] J. Peacock, "Two-dimensional goodness-of-fit testing in astronomy," *Monthly Notices Royal Astronomy Society*, vol. 202, pp. 615–627, 1983.
[19] G. Fasano and A. Franceschini, "A multidimensional of the Kolmogorov-Smirnov test," *Monthly Notices Royal Astronomy Society*, vol. 225, pp. 155–170, 1987.
[20] MATLAB implementation of the 3GPP spatial channel model (3GPP TR 25.996), http://www.tkk.fi/Units/Radio/scm/, Jan. 2005.
[21] Spatial channel model for multiple input multiple output (MIMO) simulations, 3GPP TR 25.996 V8.0.0, http://www.3gpp.org, Dec. 2008.
[22] D. Middleton, "Non-Gaussian noise models in signal processing for telecommunications: new methods and results for class A and class B noisemodels," *IEEE Trans. Inf. Theory*, vol. 45, no. 4, pp. 1129–1149, May 1999.
[23] F. Wang and X. Wang, "Fast and robust modulation classification via Kolmogorov-Smirnov test," *IEEE Trans. Commun.*, vol. 58, no. 8, pp. 2324-2332, Aug. 2010.