1. (15 points) Find the value of unknown a such that the function f is continuous at the given points

$$f(x) = \begin{cases} 3x^2 & for \quad x \geq 1 \\ ax - 4 & for \quad x < 1 \end{cases} \text{ at x=1}$$

2. (20 points) Evaluate the following limits

(a) $\lim\limits_{x \to 0} \frac{sinx}{5x} =$

(b) $\lim\limits_{t \to 0} \frac{sin2t}{3t} =$

(c) $\lim\limits_{x \to 2} \sqrt[3]{12x + 3} =$

(d) $\lim\limits_{x \to -1} f(x)$ if it exists for the given function

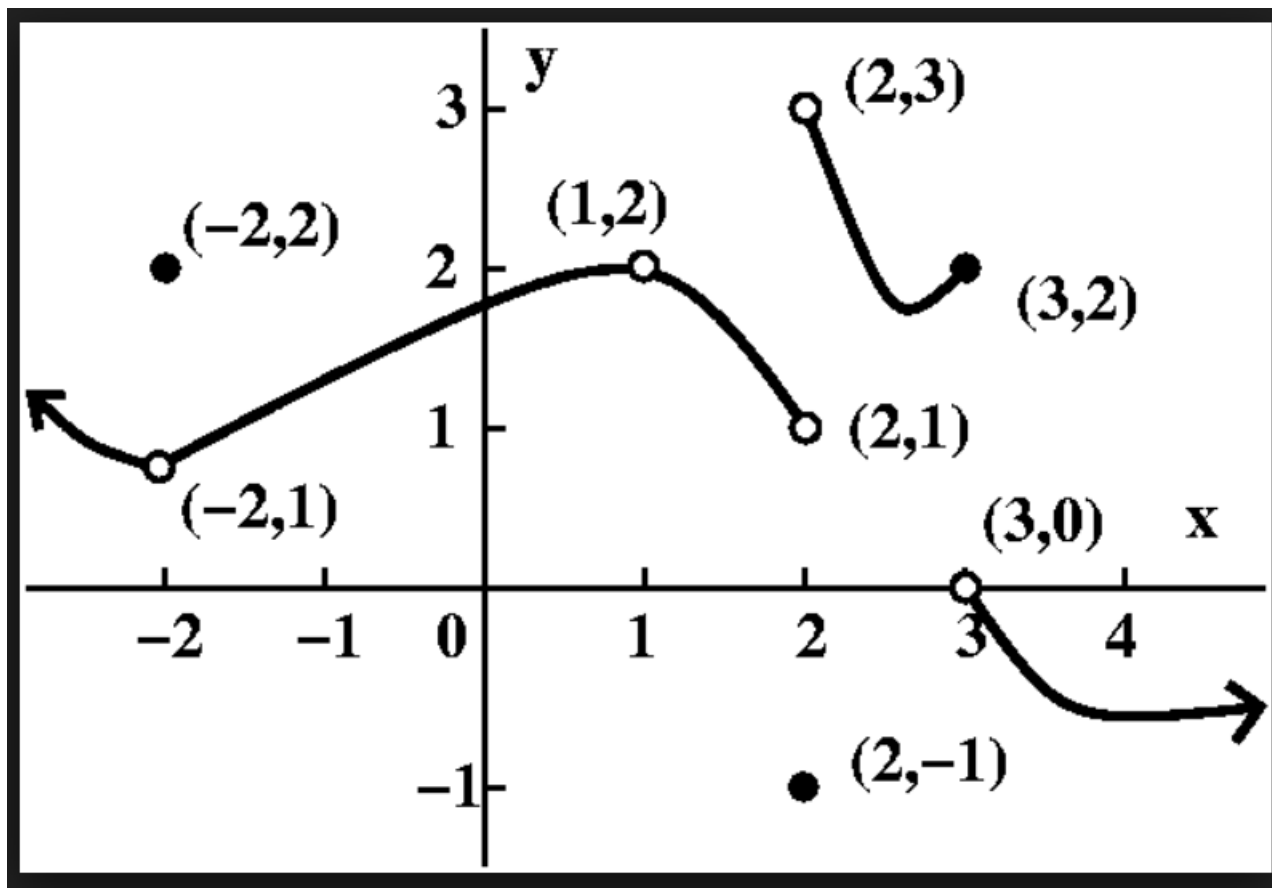$$f(x) = \begin{cases} 3x^2 & for \quad x \geq -1 \\ 2x + 4 & for \quad x < -1 \end{cases}$$

(e) $\lim\limits_{x \to 5} \frac{x-5}{x^2-25}$

(f) $\lim\limits_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$

3. (15 points) (a) (5 points) Write down the definition of the derivative of the function $f(x)$

(b) (10 points) Find the derivative of $f(x) = x^2 - 5$ by using the definition of derivative.

4. (20 points) For the function f(x) given in the graph, evaluate the following (if they exit)

(a) $\lim\limits_{x \to -2} f(x) =$

(b) $\lim\limits_{x \to 1} f(x) =$

(c) $\lim\limits_{x \to 2} f(x) =$

(d) $\lim\limits_{x \to -1} f(x) =$

(e) $\lim\limits_{x \to 3} f(x) =$

(f) Is $f(x)$ is continuous at x=-2 if not specify the types of discontinuity.

(g) Is $f(x)$ is continuous at x=0 if not specify the types of discontinuity.

(h) Is $f(x)$ is continuous at x=1 if not specify the types of discontinuity.

(i) Is $f(x)$ is continuous at x=2 if not specify the types of discontinuity.

(j) Is $f(x)$ is continuous at x=3 if not specify the types of discontinuity.

5. (10 points) Find the point x at which $f(x)$ is not continuous, then explain which types of discontinuity it has?

   (a) (4 points)

$$f(x) = \frac{4}{x - 6}$$

(b) (6 points)

$$f(x) = \begin{cases} x^2 & for \quad x < 1 \\ x + 1 & for \quad x \geq 1 \end{cases}$$

6. (10 points) Is piecewise function f(x) is continuous or not at x=0? Give reason why it is continuous or why not?

$$f(x) = \begin{cases} 5x^2 + 3x + 1 & for \quad x < 0 \\ x + 1 & for \quad x \geq 0 \end{cases}$$

7. (10 points) Explain why the function $f(x) = x^3 + 5x - 3$ has at least one zero in the given interval [0,1].

BonusBonusBonus

8. (5 Bonus points) Evaluate the limit $\lim\limits_{x \to 0} \frac{sin2x}{sin3x} =$