Math 2413 Calculus-I — Summer 2018 — Exam II

G-Number-

Name (Print): ______________

1. (10 points) Find the Equation of Tangent and Normal line for the graph of the function $f(x) = x^4 - 2x^2 + 3$ at $x = 2$. Also find the point at which the Tangent of this graph is horizontal.

$f' = 4x^3 - 4x$

$x = 2$

$y = 2^4 - 2 \cdot 2^2 + 3$

$= 16 - 8 + 3$

$= 11$

$m = f'(2) = 32 - 8 = 24$

$(2, 11)$

tangent $y - 11 = 24(x-2)$

Normal $y - 11 = -\frac{1}{24}(x-2)$

$4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$\boxed{x = 0, \quad x = \pm 1}$

2. (12 points) Find the derivative of the following

(a) $f(x) = sin(2x)cos(2x)$

$f' = \cos 2x \, 2 \cos(2x) - 2\sin(2x)\sin 2x$

$= 2\cos^2 2x - 2\sin^2(2x) = 2\cos(4x)$

(b) $g(x) = (3x-4)(x^3+5)$

$g'(x) = (x^3+5)(3) + (3x-4)3x^2$

$= 3x^3 + 15 + 9x^3 - 12x^2 = 12x^3 - 12x^2 + 15$

(c) $h(x) = \frac{3x^2-1}{2x+5}$

$$\frac{(2x+5)(6x) - (3x^2-1)2}{(2x+5)^2} = \frac{12x^2 + 30x - 6x^2 + 2}{(2x+5)^2}$$

$$= \frac{6x^2 + 30x + 2}{(2x+5)^2}$$

(d) Find the derivative of the function $f(x)$ at $x = 3$ if it exists

$$f(x) = \begin{cases} x^2 & for \quad x \geq 3 \\ 2x+3 & for \quad x < 3 \end{cases}$$

at $x = 3$

LHD $= 2$

RHD $= 2x = 6$

Derivative DNE

(e) $\phi(x) = \sqrt{4-3x^2}$

$$\phi'(x) = \frac{1}{2}(4-3x^2)^{-1/2}(-6x) = \frac{-3x}{\sqrt{4-3x^2}}$$

(f) $\psi(x) = \sqrt{\frac{2x}{x+1}}$

$$\psi'(x) = \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-1/2} \cdot \left(\frac{(x+1)2 - 2x}{(x+1)^2}\right) = \frac{2}{(x+1)^2} \frac{(x+1)^{1/2}}{(2x)^{1/2}}$$

$$= \frac{1}{(x+1)^{3/2}\sqrt{2x}}$$

3. (15 points) (a) (5 points) Find the derivative of the function $x^3y^3 - y = x$

$$y^3 3x^2 + x^3 3y^2 \cdot y' - y' = 1$$

$$y'(3x^3y^2 - 1) = 1 - 3x^2y^3$$

$$y' = \frac{1-3x^2y^3}{3x^3y^2-1}$$

(b) (10 points) Find the Equation of tangent line to the graph of $x^3 + y^3 = 6xy - 1$ at $(2, 3)$.

$$3x^2 + 3y^2 \cdot y' = 6xy' + 6y$$

$$(3y^2 - 6x)\, y' = 6y - 3x^2$$

$$y' = \frac{6y-3x^2}{3y^2-6x}$$

at $(2,3)$ $\quad y' = \frac{18-12}{27-12} = \frac{6}{15} = \frac{2}{5}$

eqn of tangent

$$y - 3 = \frac{2}{5}(x-2)$$

4. (10 points) Water is pumped into a cylindrical tank at the rate of 240 cubic inch per second. While the height of the tank is 3 times the radius then at what rate the height is changing when the height is 5 inch. (Note: volume of cylinder is V=$\pi r^2 h$)

$\frac{dV}{dt} = 240$ $\quad h = 3r \Rightarrow r = \frac{h}{3}$

$V = \pi r^2 h = \pi \frac{h^2}{9} \cdot h = \frac{\pi}{9} h^3$

$\frac{dv}{dt} = 3\frac{\pi}{9} h^2 \cdot \frac{dh}{dt}$

$240 = \frac{\pi}{3} \cdot 5^2 \frac{dh}{dt}$ $\quad \frac{dh}{dt} = \frac{240 \times 3}{25\pi} = \frac{720}{25\pi}$

5. (13 points) A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. Find the rate of change pf the radius when $r = 30$cm. Explain why the rate of change of the radius of the sphere is not constant even though te rate of change of volume is constant.

$\frac{dv}{dt} = 800$

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4\pi}{\cancel{3}} \cancel{3} r^2 \cdot \frac{dr}{dt}$

$800 = 4\pi \, 900 \, \frac{dr}{dt}$

$\frac{\cancel{800}^{2}}{\cancel{3600}_{9}\pi} = \frac{dr}{dt}$ $\quad \frac{dr}{dt} = \frac{2}{9\pi}$

If $V = \alpha r \Rightarrow r = \frac{V}{\alpha}$

6. (10 points) Is the Rolles' Theorem can be applied to $f(x) = (x-2)^2(x-3)$ on the interval $[2,3]$ or Not? Give reason. If Rolle's Theorem can be applied, find all numbers c in the open interval (2,3)such that $f'(c) = 0$.

Yes. f is contin & diff. everywhere

$f(x) = (x^2-4x+4)(x-3)$
$= x^3 - 3x^2 - 4x^2 + 12x + 4x - 12$
$= x^3 - 7x^2 + 16x - 12$

48
8.6

$f' = 3x^2 - 14x + 16 = 0$
$3x^2 - 6x - 8x + 16 = 0$
$3x(x-2) - 8(x-2) = 0$
$(x-2)(3x-8)$
$x = \cancel{2}, \quad x = 8/3$

$x = 8/3$.

7. (10 points) Find the Absolute extrema of the function on the closed interval [-2,1]

$$g(x) = \frac{6x^2}{x-2}$$

$D = \{x : x \neq 2\}$

$$g'(x) = \frac{(x-2)\,12x - 6x^2}{(x-2)^2} = \frac{12x^2 - 24x - 6x^2}{(x-2)^2}$$

$$= \frac{6x^2 - 24x}{(x-2)^2} = \cancel{\frac{6x(x-2)(x+2)}{(x-2)^2}} = \cancel{\frac{6x}{(x-2)}}$$

$6x^2 - 24x = 0$
$6x(x-24) = 0$
$x = 0, \quad \cancel{x = 24}$

$g' = 0 \quad 6x = 0 \Rightarrow x = 0$

$g(0) = 0 \rightarrow$ Abs max

$g(1) = \frac{6}{-1} = -6$

$g(-2) = \frac{24}{-4} = -6$

Abs min

8. (10 points) Find the point on the interval (0, 6) at which the tangent line is parallel to the secant line of the graph $f(x) = x^2 - 2x + 2$.

$f' = 2x - 2$ $\quad f(c) = 2c - 2$

f cont^n & diff every by MVT

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c - 2 = \frac{(6^2 - 12 + 2) - 2}{6 - 0}$$

$$2c - 2 = \frac{24}{6} = 4$$

$$2c = 6 \qquad c = 3.$$

9. (10 points) Find all the critical points of the function $f(x) = 2sin(x) - cos(2x)$ [Note: Sin(2x)= 2sin(x)cos(x)].

$$f' = 2\cos x + 2\sin(2x) = 0$$

$$\cancel{2}\cos x = -\cancel{2}\sin 2x$$

$$1 = \frac{\sin 2x}{\cos x} = -\frac{2\cos x \sin x}{\cos x}$$

$$-\frac{1}{2} = \sin x$$

$$x = 7\pi/6,\ 11\pi/6.$$

$$\frac{7\pi}{6} + 2n\pi,\ \frac{11\pi}{6} + 2n\pi$$

$\frac{\pi}{6} + \pi/2$

$2\pi + \frac{7\pi}{6} = \frac{19\pi}{6}$

BonusBonusBonus

10. (5 Bonus points) Evaluate the derivative $y = sin(xy)$

$$y' = \cos(xy) \cdot (xy' + y)$$

$$y' - \cos xy\ xy' = y\cos(xy)$$

$$y'(1 - x\cos xy) = y\cos(xy)$$

$$y' = \frac{y\cos(xy)}{1 - x\cos xy}$$

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Choose Any 10 questions but Questions 5 and 7 are mandatory

1. (15 points) Find the value of unknown a such that the function f is continuous at the given points

$$f(x) = \begin{cases} 3x^2 & for \quad x \geq 1 \\ ax - 4 & for \quad x < 1 \end{cases} \text{ at x=1}$$

at x=1

LHL $= \lim_{x \to 1^-} ax - 4 = a - 4$

RHL $= \lim_{x \to 1^+} 3x^2 = 3$

$a - 4 = 3$

$a = 7$

2. (20 points) Evaluate the following limits

(a) $\lim_{x\to 0} \frac{\sin x}{5x} =$ $\frac{1}{5}$

(b) $\lim_{t\to 0} \frac{\sin 2t}{3t} =$ $\frac{2}{3}$

(c) $\lim_{x\to 2} \sqrt[3]{12x+3} =$ $\sqrt[3]{24+3} = \sqrt[3]{27} = 3$

(d) $\lim_{x\to -1} f(x)$ **if it exists for the given function**

$$f(x) = \begin{cases} 3x^2 & for \quad x \geq -1 \\ 2x+4 & for \quad x < -1 \end{cases}$$

RHL $= 3(-1)^2 = 3$.
RHL $= 2(-1)+4$
$= 2$

DNE

(e) $\lim_{x\to 5} \frac{x-5}{x^2-25}$

$= \frac{\cancel{(x-5)}}{\cancel{(x-5)}(x+5)} = \frac{1}{10}$

(f) $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$

$= \lim \frac{\sqrt{x+5}-3}{x-4} \times \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$= \frac{(x+5)-9}{(x-4)(\sqrt{x+5}+3)} = \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x+5}+3)}$

$= \frac{1}{3+3} = \frac{1}{6}$

3. (15 points) (a) (5 points) Write down the definition of the derivative of the function $f(x)$

$$f' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) (10 points) Find the derivative of $f(x) = x^2 - 5$ **by using the definition of derivative.**

$$f(x+h) = (x+h)^2 - 5$$

$$f' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

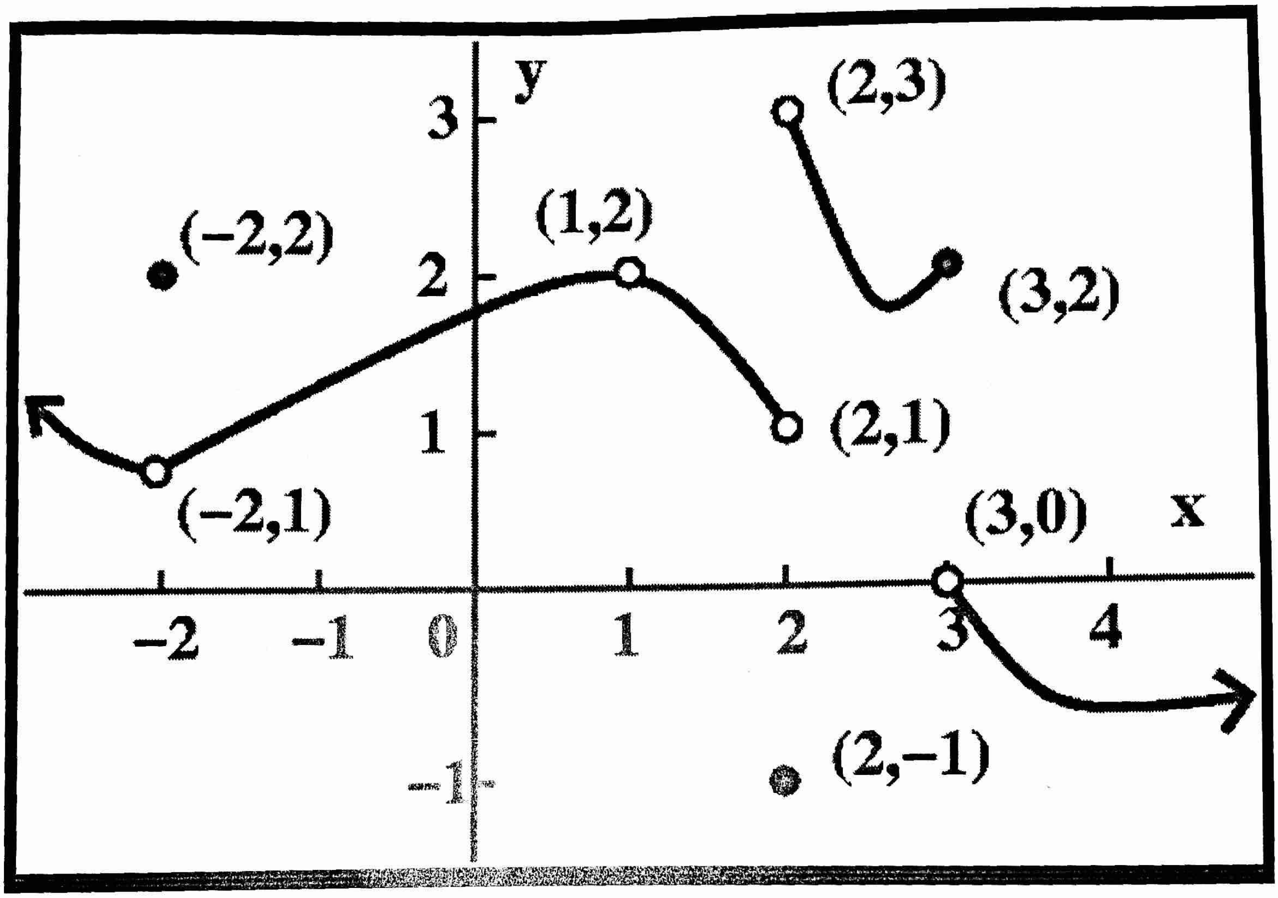
$$= \lim_{h \to 0} \frac{(x+h)^2 \cancel{-5} - x^2 \cancel{+5}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x$$

4. (20 points) **For the function f(x) given in the graph, evaluate the following (if they exit)**



(a) $\lim_{x\to -2} f(x) =$ 1

(b) $\lim_{x\to 1} f(x) =$ 2

(c) $\lim_{x\to 2} f(x) =$ DNE

(d) $\lim_{x\to -1} f(x) =$ 1.5

(e) $\lim_{x\to 3} f(x) =$ DNE

(f) Is $f(x)$ is continuous at x=-2 if not specify the types of discontinuity.

DIS cont Removable

(g) Is $f(x)$ is continuous at x=0 if not specify the types of discontinuity.

continu

(h) Is $f(x)$ is continuous at x=1 if not specify the types of discontinuity.

DiScont Remov

(i) Is $f(x)$ is continuous at x=2 if not specify the types of discontinuity.

DiScon Jump

(j) Is $f(x)$ is continuous at x=3 if not specify the types of discontinuity.

DiScont ~~Re~~ Jump

5. (10 points) Find the point x at which $f(x)$ is not continuous, then explain which types of discontinuity it has?

(a) (4 points)

$$f(x) = \frac{4}{x-6}$$

Infinite at x = 6

(b) (6 points)

$$f(x) = \begin{cases} x^2 & for \quad x < 1 \\ x+1 & for \quad x \geq 1 \end{cases}$$

at $x=1$

LHL= 1

RHL= 2

Jump

6. **(10 points) Is piecewise function f(x) is continuous or not at x=0? Give reason why it is continuous or why not?**

$$f(x) = \begin{cases} 5x^2 + 3x + 1 & for \quad x < 0 \\ x + 1 & for \quad x \geq 0 \end{cases}$$

at x=0

LHL = 1

RHL = 1 f(0) = 1

contin at x=0

7. **(10 points) Explain why the function $f(x) = x^3 + 5x - 3$ has at least one zero in the given interval [0,1].**

f(0) = -3

f(1) = 3 -3 < 0 < 3

By IVT

BonusBonusBonus

8. **(5 Bonus points) Evaluate the limit** $\lim_{x \to 0} \frac{sin2x}{sin3x} =$ $= \frac{\sin 2x}{x} \cdot \frac{x}{\sin 3x}$

$= \frac{2}{1} \cdot \frac{1}{3} = 2/3$

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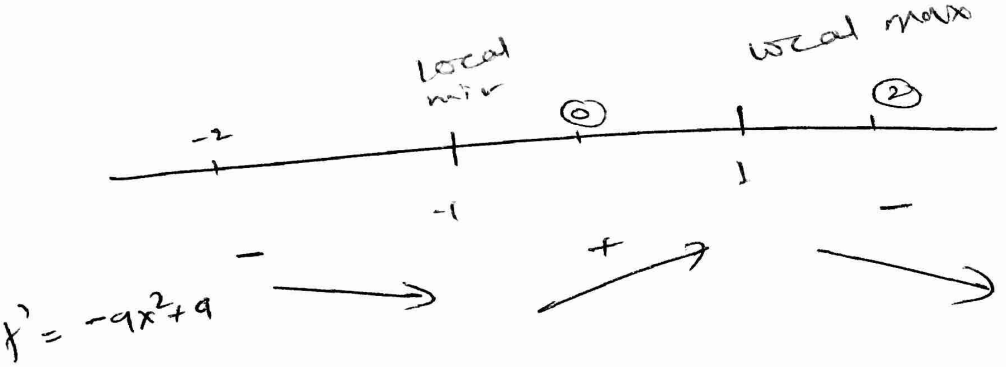
1. (10 points) For the given function $f(x) = -3x^3 + 9x - 12$, find the critical points, classify the critical points (local maximum or local minimum) also find the relative extreme values.

$f'(x) = -9x^2 + 9 = 0$

$x^2 = 1$ $\quad x = \pm 1$



$x = -1$ Local min

$x = 1$ Loccal max

$f(-1) = -3(-1)^3 + 9(-1) - 12 = -3(-1) - 9 - 12$

$= 3 - 9 - 12$

$= -18$ ← Local min

$f(1) = -3 \cdot 1^3 + 9 \cdot 1 - 12$

$= -3 - 12 + 9$

$= 0$ ← local max

2. (20 points) Find the Domain, Vertical asymptotes, Horizontal asymptotes, intercepts of the graph. Find all the critical points, interval of increasing and decreasing, locate the point at which $f(x)$ has local maximum and local minimum. Find all the point of inflection and also determine the interval on which the graph of the function is concave up and concave down for the graph Also Sketch the graph.

$$f(x) = \frac{x+8}{x-7}$$

$\boxed{D = \{x : x \neq 7\}}$

x-int $0 = x+8 \Rightarrow \boxed{x = -8}$

V.A $x - 7 = 0$ $\boxed{\Rightarrow x = 7}$

y-int $\boxed{y = -\frac{8}{7}}$

$\boxed{\text{H.A} \quad y = 1}$

~~x-int~~

$f'(x) = \frac{(x-7) - (x+8)}{(x-7)^2} = \frac{-15}{(x-7)^2}$

$f' \to \infty$ at $x = 7 \notin D$

$f' \neq 0$

$\boxed{\text{No c.p.}}$

0 7 8

$f' = \frac{-15}{(x-7)^2}$ $\searrow$ 7 $\searrow$

$\boxed{\text{always decreasing}}$

$\boxed{\text{No Local max or min}}$

$f'' = \frac{30}{(x-7)^3} = 0$

$\boxed{\text{No POI}}$

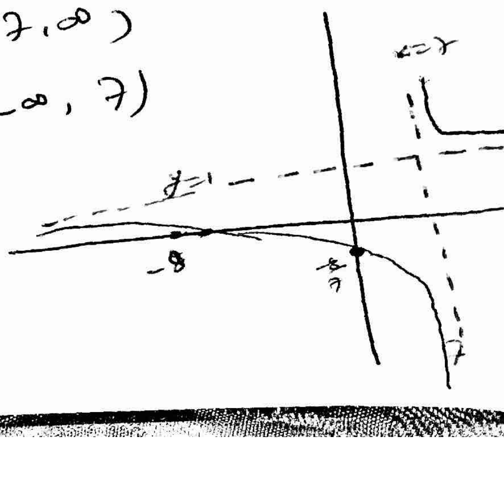
f is $\cup$ on $(7, \infty)$

f is $\cap$ on $(-\infty, 7)$

0 7 8

$f'' = \frac{30}{(x-7)^3}$ $-$ 7 $+$ $\cup$

$\cap$



3. (5 points) Evaluate the infinite limit

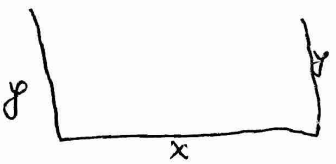
$$\lim_{x\to\infty} \frac{5x^3+1}{10x^3-3x^2+7}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^3}\left(5+\frac{1}{x^3}\right)}{\cancel{x^3}\left(10-\frac{3}{x}+\frac{7}{x^3}\right)}$$

$$= \lim_{x\to\infty} \frac{5+\frac{1}{x^3}}{10-\frac{3}{x}+\frac{7}{x^3}}$$

$$= \frac{5}{10} = \frac{1}{2}$$

4. (10 points) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain $405,000m^2$ in order to provide enough grass for the herd. No fencing is needed along the river. What dimension will require the least amount of fencing?



$A = xy$

$405,000 = xy$

$y = \frac{405,000}{x}$

$P = x + 2y$

$= x + \frac{810,000}{x}$

$P' = 1 - \frac{810,000}{x^2} = 0$

$x^2 = 810,000$

$x = 900$

$\therefore y = \frac{405,000}{900}$

$= 450$

$P'' = \frac{1620000}{x^3} > 0$

minim

5. (10 points) A rectangular solid with square base has a surface area of $625cm^2$. Find the dimension that will result in a solid with maximum volume.

$625 = A = 2x^2 + 4xh$

$\frac{625 - 2x^2}{4x} = h$

$V = x^2h = x^2\left(\frac{625-2x^2}{4x}\right) = \frac{625x}{4} - \frac{x^3}{2}$

$v' = \frac{625}{4} - \frac{3}{2}x^2 = 0 \Rightarrow \frac{3}{2}x^2 = \frac{625}{4} \Rightarrow x^2 = 104.16$

$x = 10.20$

$$\therefore h = \frac{625 - 2(10.20)^2}{4(14.43)} = \frac{625 - 208.33}{57.74} = \frac{416.67}{57.74} = 7.22$$

$$V'' = -\frac{6}{4}x < 0$$

$\Rightarrow$ maximum.

6. (5 points) Calculate the three Iteration of Newton method to approximate a zero of the function using the given initial guess. $x_1 = 2$.

$$f(x) = x^2 - 5$$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x) = 2x$

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	$x_n = 2$	-1	4	$-\frac{1}{4}$	$2 + \frac{1}{4} = \frac{9}{4}$
2	$x_2 = 2.25$	0.0625	~~0.0625~~ 4.5	0.0138	2.236
3	$x_3 = 2.236$	0	4.47	0	2.236

x = 2.236 is the zero.

7. (10 points) Approxite the value of $\sqrt{99.4}$ by using the differential.

$$\boxed{g(x) = \frac{6x^2}{x-2}}$$

$f(x) = \sqrt{x}$

$f' = \frac{1}{2\sqrt{x}}$

$99.4 = x + h$
$100 = x$
$-0.6 = h$

$f(x+h) = f(x) + hf'(x)$

$f(99.4) = f(100) - 0.6 \times \frac{1}{2\sqrt{100}}$

$\sqrt{99.4} = 10 - \frac{0.6 \times 1}{20}$

$= 10 - 0.03$

$= 9.97$

8. (10 points) Find the following integration

(a) $\int(9x^3 - 2x^2 - 6)dx$ $= \frac{9}{4}x^4 - \frac{2}{3}x^3 - 6x + C$

(b) $\int(\sqrt{x} + \frac{1}{2\sqrt{x}})dx$ $= \int x^{1/2} + \frac{1}{2}x^{-1/2}\, dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2}\frac{x^{-1/2}}{1/2} + C$

$= \frac{2}{3}x^{3/2} + x^{1/2} + C$

(c) $\int \frac{x^4-3x^2+5}{x^4}dx$ $= \int 1 - \frac{3}{x^2} + \frac{5}{x^4}\, dx = \int 1 - 3x^{-2} + 5x^{-4} dx$

$= x + 3x^{-1} - \frac{5}{3}x^{-3} + C$

$= x + \frac{3}{x} - \frac{5}{3x^3} + C$

9. (10 points) Find the solution of the differential equation $f'(x) = 10x - 12x^3, f(3) = 2$

$f(x) = \frac{10x^2}{2} - \frac{12}{4}x^4 + C$

$2 = f(3) = 5\times 9 - 3 \cdot 81 + C$ $\therefore f(x) = 5x^2 - 3x^4 + 200$

$2 - 45 + 243 = C \Rightarrow 200 = C$

10. (10 points) Find the interval of concavity (concave upward and concave downward) for the function on $[-2\pi, 2\pi]$

$$f(x) = sinx + cosx$$

$f' = \cos x - \sin x = 0 \Rightarrow \tan x = 1 \Rightarrow x = \pi/4, \ 5\pi/4$

$f'' = -\sin x - \cos x = 0$

$\tan x = -1$

$x = 3\pi/4, \ \frac{7\pi}{4}$

$f'' = -\sin x - \cos x$

$\pi/2$, $3\pi/4$, π, $\frac{7\pi}{4}$, 2π

$-$ ∩ $+$ ∪ $-$ ∩

f ∩ $= (-, 3\pi) \cup (7\pi/4, 2\pi)$

f ∪ $= (3\pi/4, 7\pi/4)$

11. (5 Bonus points) Find the increasing and decreasing interval for the function.

$$f(x) = x - 2sinx$$

$f' = 1 - 2\cos x = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{6}, \ \frac{11\pi}{6}$

$2\pi - \frac{\pi}{6}$
$=$

$(\frac{\pi}{4})$, $\pi/6$, π, $\frac{11\pi}{16}$, 2π

$f' = 1 - 2\cos x$

$-$ $+$ $-$

f is increasing on $(\pi/6, 11\pi/6)$

Everywhere else decreasing

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Summer 2018 Exam III

Name (Print): ____________

$-2(x^2-4x)-2 \quad = -2(x-2)^2+8-2$

$= -2(x-2)^2+6$

1. (10 points) For the given function $f(x) = -2x^2 + 4x - 2$, find the critical points, classify the critical points (local maximum or local minimum) also find the relative extreme values.

$f' = -4x + 4 = 0$

$x = 1$ c.p

local max

$0 \quad 1 \quad 2$

$+ \quad -$

$f' = -4x+4$

$-4(x-1)$

$x=1$ local max

$f(1) = -2 + 4 - 2$

$= 0$ local. max

2. (20 points) Find the Domain, Vertical asymptotes, Horizontal asymptotes, intercepts of the graph. Find all the critical points, interval of increasing and decreasing, locate fhe point at which $f(x)$ has local maximum and local minimum. Find all the point of inflection and also determine the interval on which the graph of the function is concave up and concave down for the graph Also Sketch the graph.

$$f(x) = \frac{x^2 - 1}{2x - 1}$$

$\boxed{\text{Domain} = \{x : x \neq \tfrac{1}{2}\}}$

V.A $\Rightarrow$ $2x - 1 = 0 \Rightarrow \boxed{x = 1/2}$

H.A $\Rightarrow$ None

$2x-1 \,)\, x^2 - 1 \,(\, \tfrac{1}{2}x + \tfrac{1}{4}$

$x^2 - \tfrac{1}{2}x$

$\tfrac{1}{2}x - 1$

$\tfrac{1}{2}x - \tfrac{1}{4}$

$\boxed{-\tfrac{3}{4}}$

$\boxed{S.A = \frac{x}{2} + \frac{1}{4}}$

$\boxed{x\text{-int}: \quad x = \pm 1}$

$\boxed{y\text{-int} = +1}$

$$f' = \frac{(2x-1)\cdot 2x - 2(x^2-1)}{(2x-1)^2}$$

$$= \frac{4x^2 - 2x - 2x^2 + 2}{(2x-1)^2} = \frac{2x^2 - 2x + 2}{(2x-1)^2} = 0$$

$$x^2 - x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+1 \pm \sqrt{1 - 4\cdot 1 \cdot 1}}{2\cdot 1}$$

None.

0 1/2 1

$+ \quad + $

$f' = \frac{2x^2 - 2x + 2}{(2x-1)^2}$

$$f'' = \frac{(2x-1)^2(4x-2) - (2x^2 - 2x + 2)(4(2x-1))}{(2x-1)^4}$$

$$= \frac{\cancel{(2x-1)}\left((2x-1)(4x-2) - 4(2x^2 - 2x + 2)\right)}{(2x-1)^{\cancel{4}\,3}} \quad \cancel{-\frac{(4x^2 - 8x + 4)(4x-2) - (2x^2 - 2x + 2)(8x-4)}{(2x-1)^4}}$$

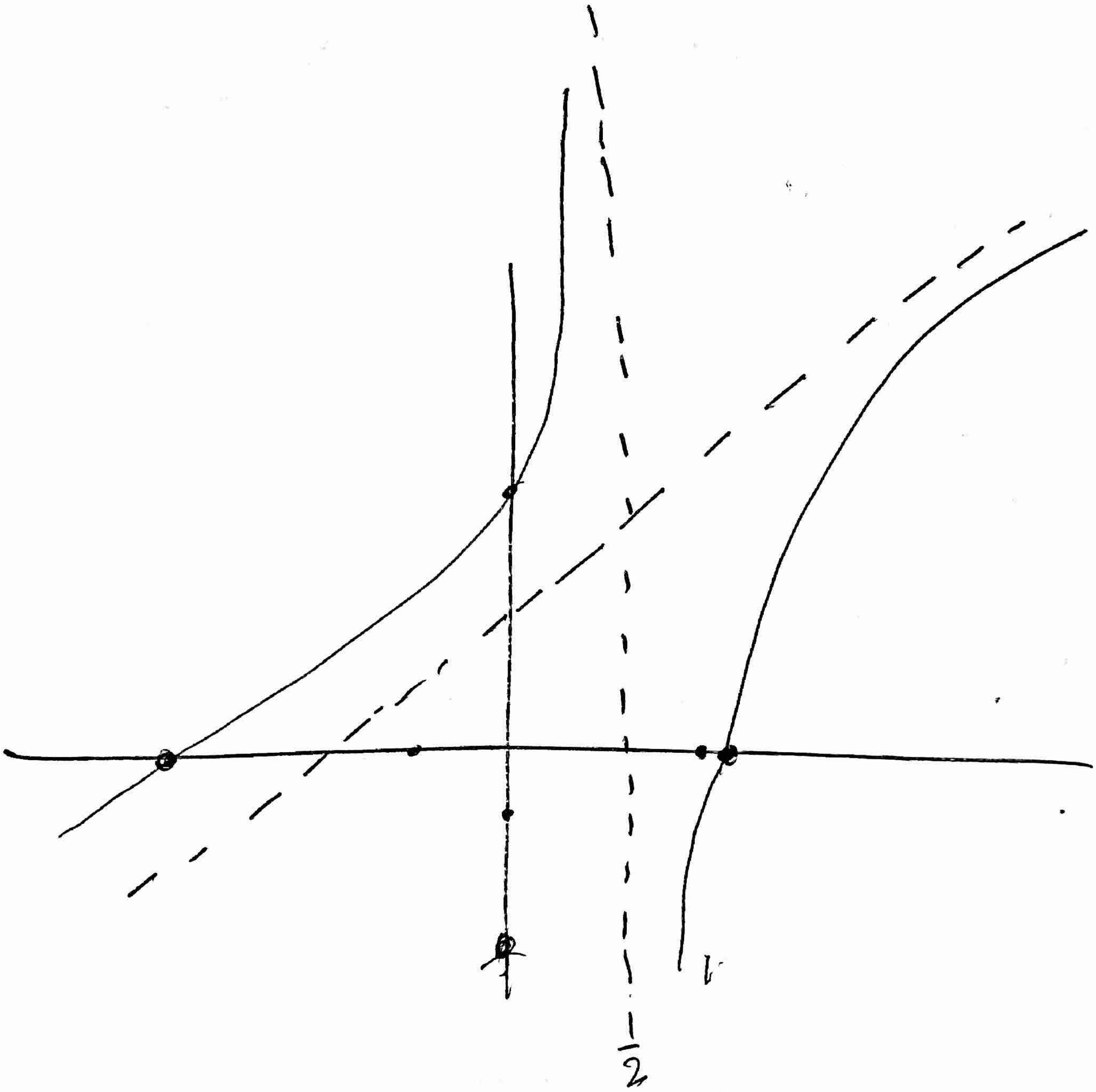
$$= \frac{\cancel{8x^2} - \cancel{4x} - \cancel{4x} + \cancel{2} - \cancel{8x^2} + 8x - 8}{(2x-1)^3} = \cancel{(4x^2 - 4x + 1)}$$

$$= \frac{-6}{(2x-1)^3}$$

$f'' =$ 0 1/2 1

$+ \quad -$

$\cup \quad \cap$



1
2

3. (5 points) Evaluate the infinite limit

$$\lim_{x\to\infty} \frac{5x^3+1}{10x^3-3x^2+7}$$

$$= \frac{5}{10} = \frac{1}{2}$$

4. (10 points) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain $405,000m^2$ in order to provide enough grass for the herd. No fencing is needed along the river. What dimension will require the least amount of fencing?

$A = 405,000$

$xy = 405,000$

$y = \frac{405,000}{x}$

$P = x + 2y$

$= x + \frac{2 \times 405000}{x}$

$= x + 810000x^{-1}$

$P' = 1 - 810,000x^{-2} = 0$

$1 = \frac{810000}{x^2}$

$x^2 = 810000$

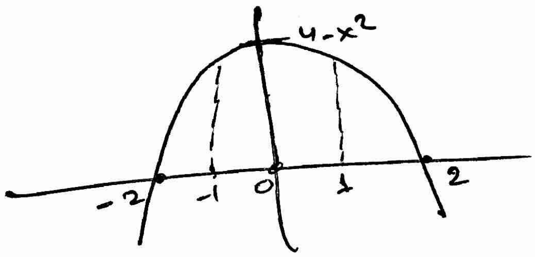
$x = 900$

$\therefore y = \frac{405\cancel{000}}{\cancel{900}} = 450$

$P'' = +\frac{1620000}{x^3} = \frac{1620000}{(900)^3} > 0$ ∪ ← minimum

5. (15 points) For the function $f(x) = 4 - x^2$ on the interval [-2,2] with 4 equal intervals. find the following
   (a) Lower sums $L_p(f)$
   (b) Upper sums $U_p(f)$
   (c) Sums at the left end points
   (d) Sums at the mid-point

$\Delta x_i = \text{Length} = \frac{b-a}{n} = \frac{2+2}{4} = 1$



(a) $L_p(f) = \sum \Delta x_i f(x_i)$
$= 1\cdot f(-2) + 1\cdot f(-1) + 1\cdot f(1) + 1\cdot f(2)$
$= 0 + 1\cdot 3 + 1\cdot 3 + 0 = 6$

(b) $U_p(f) = \sum \Delta x_i\, f(x_i)$
$= 1\cdot f(-1) + 1\cdot f(0) + 1\cdot f(0) + 1\cdot f(1)$
$= 1\cdot 3 + 1\cdot 4 + 1\cdot 4 + 1\cdot 3$
$= 14$

(c) Sums at left end points
$= 1\cdot f(-2) + 1\cdot f(-1) + 1\cdot f(0) + 1\cdot f(1)$
$= 0 + 1\cdot 3 + 1\cdot 4 + 1\cdot 3$
$= 10$

(d) Sums at mid point
$= 1\cdot f(-1.5) + 1\cdot f(-0.5) + 1\cdot f(0.5) + 1\cdot f(1.5)$
$= 1.75 + 3.75 + 3.75 + 1.75$
$= 11$

6. (5 points) Calculate the three Iteration of Newton method to approximate a zero of the function using the given initial guess. $x_1 = 2$.

$$f(x) = x^2 - 5$$

$f' = 2x$

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	2	-1	4	$-\frac{1}{4}$	2.25
2	2.25	0.063	4.5	0.014	2.236
3	2.236	0	4.47	0	2.236

$\boxed{x = 2.236}$

7. (8 points) Approxite the value of $\sqrt{65}$ by using the differential.

$f(x) = \sqrt{x}$ $f'(x) = \frac{1}{2\sqrt{x}}$

65	$x+h$
64	x
1	h

$f(x+h) = f(x) + h f'(x)$

$f(65) = f(64) + 1\, f'(64)$

$\sqrt{65} = 8 + 1 \cdot \frac{1}{2\sqrt{64}} = 8 + \frac{1}{16}$

$\boxed{= 8.063}$

8. (10 points) Find the following integration

(a) $\int_1^3 (9x^3 - 2x^2 - 6)dx$

$\left(\frac{9x^4}{4} - \frac{2x^3}{3} - 6x\right)_1^3$

$= \left(\frac{9}{4} \cdot 3^{4} - \frac{2}{3} 3^3 - 18\right) - \left(\frac{9}{4} - \frac{2}{3} - 6\right)$

$= \left(\frac{729}{4} - \frac{54}{3} - 18\right) - \left(\frac{27 - 8 - 72}{12}\right)$

$= \frac{729\times3 - 54\times4 - 18\times12}{12}$

$= \frac{2187 - 216 - 216}{12} + \frac{52}{12}$

$= \frac{1807}{12}$

$= 150.58$

(b) $\int_0^2 (\sqrt{x} + 2x + 4)dx$

$\left(\frac{2}{3} x^{3/2} + x^2 + 4x\right)_0^2$

$= \left(\frac{2}{3} 2^{3/2} + 4 + 8\right) = (1.06 + 4 + 8)$

$= 13.06$

(c) $\int 2sinx - 4cosx + 2xdx$

$= -2 \cos x - 4 \sin x + 2 + c$

9. (7 points) Find the solution of the differential equation $f'(x) = 10x - 12x^3, f(3) = 2$

$f(x) = \frac{10x^2}{2} - \frac{12x^4}{4} + c$

$2 = f(3) = 5 \times 3^2 - 3 \cdot 3^4 + c$

$2 - 45 + 243 = c$ $\quad c = 200$

$\therefore f(x) = \frac{10}{2}x^2 - \frac{12}{4}x^4 + 200$

$= 5x^2 - 3x^4 + 200$

10. (10 points) Find the interval of concavity (concave upward and concave downward) for the function $[0, 2\pi]$

$$f(x) = sinx + cosx$$

$f' = \cos x - \sin x = 0$

$\tan x = 1$ $x = \frac{\pi}{4}, \frac{5\pi}{4}$

$f'' = -\sin x - \cos x = 0$

$\tan x = -1$

$x = \frac{3\pi}{4}, 7\pi/4$

$\frac{\pi}{2}$ $\frac{3\pi}{4}$ π $\frac{7\pi}{4}$ 2π

$-$ $\cap$ $+$ $\cup$ 4 $\cap$

$f'' = -\sin x - \cos x$

11. (5 Bonus points) Evaluate the indefinite integral

$$\int \frac{x}{\sqrt{1-x^2}}dx$$

$u = 1 - x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$= \int \frac{1}{\sqrt{u}} du$

$= \int u^{-1/2} \frac{du}{-2}$

$= -\frac{1}{2} \frac{u^{-1/2+1}}{-\frac{1}{2}+1} + c$

$= -\frac{1}{2} \frac{u^{1/2}}{\frac{1}{2}} + c$

$= -\frac{2}{2} u^{1/2} + c$

$= -\cancel{2} \sqrt{1-x^2} + c$