

Review Final

① solve

$$\frac{5}{3x-3} = \frac{2}{x+5}$$

soln:

$$5(x+5) = 2(3x-3)$$

$$5x+25 = 6x-6$$

$$5x-6x = -6-25$$

$$-x = -31$$

$$x = 31$$

② solve

$$\frac{1}{x} = \frac{3}{4}$$

soln:

$$3x = 4$$

$$x = 4/3$$

③ solve

the inequality

$$|2x-4| \leq 2$$

soln:

$$-2 \leq 2x-4 \leq 2$$

$$-2+4 \leq 2x \leq 2+4 \quad \text{Adding 4}$$

$$2 \leq 2x \leq 6 \quad \text{dividing by 2}$$

$$1 \leq x \leq 3$$

$$[1, 3]$$

④ solve

the inequality

$$|x-3| < 2$$

⑤

soln:

$$-2 < x-3 < 2$$

$$-2+3 < x < 2+3$$

$$1 < x < 5 \quad (1, 5)$$

(5)

$$\text{Solve } x^2 + 6x + 9 = 0$$

$$\text{Soln: } (x+3)(x+3) = 0$$

$$x = -3$$

(6)

$$\text{Solve } x^2 + 2x + 1 = 0$$

$$\text{Soln: } (x+1)^2 = 0$$

$$x+1 = 0$$

$$x = -1$$

(7)

Find the eqn of circle with center

a)  $(h, k) = (2, -3)$  and radius  $r=3$

$$\text{Soln: } (x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-(-3))^2 = 3^2$$

$$(x-2)^2 + (y+3)^2 = 9$$

b)  $(h, k) = (1, 0)$  and radius  $r=1$

$$\text{Soln: } (x-1)^2 + (y-0)^2 = 1^2$$

$$(x-1)^2 + y^2 = 1$$

c) check whether following lines  
are parallel or perpendicular or  
neither

$$\text{a) } y = 3x + 9$$

$$\text{b) } y = -\frac{1}{3}x + 4$$

Sol<sup>n</sup>: slope  $m_1 = 3$

$$m_2 = -\frac{1}{3}$$

$m_1 \neq m_2$  Not parallel

$$m_1 \cdot m_2 = 3(-\frac{1}{3}) = -1$$

$\Rightarrow$  perpendicular

⑤  $y = 2x - 3$

~~⑥~~  $y = 2x + 4$

$$m_1 = 2$$

$$m_2 = 2$$

$$m_1 = m_2$$

they are parallel.

⑦ find the x-intercept, y-intercept,  
check the symmetry also plot the  
graph. for  $y = x^2 - 3$

Sol<sup>n</sup>:

- x-intercept ( $y=0$ )  $\Rightarrow 0 = x^2 - 3$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

- y-intercept ( $x=0$ )  $\Rightarrow y = 0 - 3 = -3$

symmetry:

- $x$ -axis ( $y \rightarrow -y$ )  $\Rightarrow -y = x^2 - 3$

Not same  $\Rightarrow$  Not symmetric about  $x$ -axis

- $y$ -axis ( $y \rightarrow -y$ )  $\Rightarrow y = (-x)^2 - 3$

$$y = x^2 - 3$$

same  $\Rightarrow$  symmetric about  $y$ -axis

- origin ( $x \rightarrow -x$ ,  $y \rightarrow -y$ )

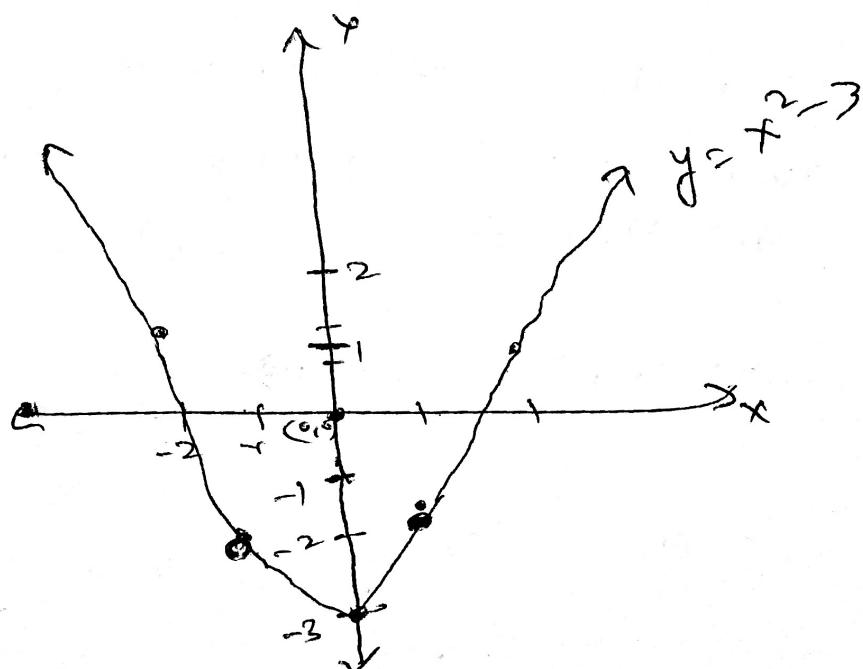
$$-y = (-x)^2 - 3$$

$$-y = x^2 - 3$$

Not same  $\Rightarrow$  NO symmetry about ~~origin~~ origin

graph:

$x$	$y$	point
-1	-2	(-1, -2)
-2	1	(-2, 1)
0	-3	(0, -3)
1	-2	(1, -2)
2	1	(2, 1)



10

$$\text{for } f(x) = 2x+4 \quad g(x) = 3x-3$$

find i)  $f(1)$  ii)  $g(0)$  iii)  $g(1)$

soln:

$$\begin{aligned} f(1) &= 2 \cdot 1 + 4 \\ &= 2+4 \\ &= 6 \end{aligned}$$

$$\begin{aligned} g(0) &= 3 \cdot 0 - 3 \\ &= -3 \end{aligned}$$

$$\begin{aligned} g(1) &= 3 \cdot 1 - 3 \\ &= 3-3 \\ &= 0 \end{aligned}$$

11 If  $f(x) = \frac{3}{5+x}$  is one-one? if so why?

Soln

$$f(a) = f(b)$$

$$\frac{3}{5+a} = \frac{3}{5+b}$$

$$3(5+b) = 3(5+a)$$

$$5+b = 5+a$$

$b = a \Rightarrow f$  is one-one.

12 And the inverse of  $f(x) = 3x+2$

$$\text{soln: } y = 3x+2$$

interchanging  $x$  and  $y$

$$x = 3y+2$$

$$x-2 = 3y$$

$$\frac{x-2}{3} = y$$

$$\therefore f'(x) = \frac{x-2}{3} \quad \#$$

13 Solve for  $x$

(a)  $2^x = 16$       (b)  $3^x = 27$       (c)  $3^x = 9$

$$2^x = 2^4$$

$$3^x = 3^3$$

$$3^x = 3^2$$

$$\therefore x = 4$$

$$\therefore x = 3$$

$$\therefore x = 2$$

(d)  $4^{3x+3} = \frac{1}{4}$

$$4^{3x+3} = 4^{-1}$$

(e)  $\log_2(x) = 3$

$$2^3 = x$$

$$\Rightarrow 8 = x$$

$$\therefore x = 8$$

$$3x + 3 = -1$$

$$3x = -1 - 3$$

$$3x = -4$$

$$x = -\frac{4}{3}$$

(f)  $\log_6(36) = 4x + 3$

$$6^{4x+3} = 36$$

$$6^{4x+3} = 6^2$$

$$\therefore 4x + 3 = 2$$

$$4x = 2 - 3 = -1$$

$$x = -\frac{1}{4}$$

14) find  $fog(x)$  for  $f(x) = \frac{3}{2+x}$   $g(x) = 3x$

$$fog(x) = f(g(x))$$

$$= f(3x)$$

$$= \frac{3}{2+3x} \#$$

15) find  $fog(x)$  for  $f(x) = 2x+3$  &  
 $g(x) = x^2$

$$fog(x) = f(g(x))$$

$$= f(x^2)$$

$$= 2x^2 + 3 \#$$

16) find the exact value of

(a)  $\log_2(8) = 3$

(b)  $\log_2(8) = \log_2(2^3) = 3$

(c)  $\log_3(27)$

Q11) Let  $\log_{\frac{1}{3}}(27) = x$

$$\left(\frac{1}{3}\right)^x = 27 = 3^3$$

$$\left(\frac{1}{3}\right)^x = \frac{1}{3^{-3}} = \left(\frac{1}{3}\right)^{-3}$$

$$\therefore x = -3$$

(a)  $\log_3(27) = \log_3(3^3) = 3$

(e)  $\log_3(81) = \log_3(3^4) = 4$

12) Use the properties of logarithms to express as sum and difference.

$$\begin{aligned} \text{(a)} \quad \log_2(4x^2) &= \log_2(4) + \log_2(x^2) \\ &= \log_2(2^2) + 2 \log_2(x) \\ &= 2 + 2 \log_2(x) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \ln(4xe^x) &= \ln(4) + \ln(x) + \ln(e^x) \\ &= \ln(4) + \ln(x) + x \end{aligned}$$

③  ~~$\log_3(9x^2)$~~   $\log_3(9x^2)$

$$= \log_3(9) + \log_3(x^2)$$

$$= \log_3(3^2) + 2\log_3(x)$$

$$= 2 + 2\log_3(x).$$

18 Sketch the graph  $f(x) = x^2$   
and ~~or~~ write the function for the  
following

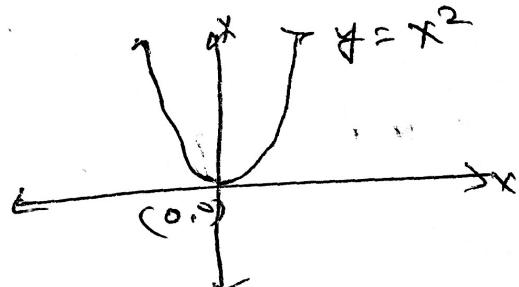
(a) 3 unit shifting down

(b) 2 unit shifting up

(c) 5 unit shifting right

(d) 4 unit shifting left

Soln: graph



(a)  $f(x) = x^2 - 3$

(b)  $f(x) = x^2 + 2$

(c)  $f(x) = (x - 5)^2$

(d)  $f(x) = (x + 4)^2$