Exam - 3
Review

1) Check whether x-c is the facter of the polynomial by using the facter Theorem. Also, compute the remainder theorem.

ⓐ $f(x) = x^3 - 5x^2 + 3x + 1$ by $x-1$

Sol^n:

$x - 1 = 0$
$x = 1$

Then by Remainder theorem

$f(1) = 1^3 - 5\cdot 1^2 + 3\cdot 1 + 1 = 5 - 5 = 0$

Remaind $= 0$

By facter theorem, being Remainder $R = 0$
$x-1$ is the facter.

ⓑ $f(x) = -3x^4 + x^3 - 5x^2 + 2x + 4$ by $x-1$

Sol^n:

$x - 1 = 0$
$x = 1$

$f(1) = -3\cdot 1^4 + 1^3 - 5\cdot 1^2 + 2\cdot 1 + 4$
$= -8 + 7 = -1$

Remaind $= -1$

By facter theorem $R \neq 0 \Rightarrow x-1$ is not a Facter.

② ②

Factor f(x) into linear factors

(a) $f(x) = 6x^3 + 13x^2 - 14x + 3$ $k = -3$

Sol^n

By synthetic division, $x + 3 = 0$

$$\begin{array}{r|rrrr} -3 & 6 & 13 & -14 & 3 \\ & \downarrow & -18 & 15 & -3 \\ \hline & 6 & -5 & 1 & \boxed{0} \end{array}$$

$$f(x) = (x+3)(6x^2 - 5x + 1)$$
$$= (x+3)\left(x - \frac{3}{6}\right)\left(x - \frac{2}{6}\right)$$
$$= (x+3)\left(x - \frac{1}{2}\right)\left(x - \frac{1}{3}\right)$$

$$\begin{array}{c} 6 \\ \wedge \\ -3 \quad -2 \end{array}$$

(b) $f(x) = x^4 + 2x^3 - 7x^2 - 20x - 12$ $k = -2$ multiplicity 2

$(x+2)^2$

By synthetic division,

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & -7 & -20 & -12 \\ & \downarrow & -2 & 0 & 14 & +12 \\ \hline & 1 & 0 & -7 & -6 & \boxed{0} \end{array}$$

$$f(x) = (x+2)^2 (x^3 - 7x - 6)$$

For $x^3 - 7x - 6$ $x = -1$ is a zero

$x + 1$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & \downarrow & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$f(x) = (x+2)^2 (x+1)(x^2-x-6)$

-6 ; $-3 \quad 2$

$= (x+2)^2 (x+1)(x-3)(x+2)$

$= (x+2)^3 (x+1)(x-3)$ #

③ Find all zeros and their multiplicity.

ⓐ $f(x) = (x+1)^2 (x-1)^3 (x^2-10)$

sol^n: $(x+1)^2 = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1$ with multiplicity 2

$(x-1)^3 = 0 \Rightarrow x-1 = 0 \Rightarrow x = 1$ with multiplicity 3

$x^2 - 10 = 0 \Rightarrow x^2 = 10 \Rightarrow x = \pm\sqrt{10}$

$\therefore x = \sqrt{10}$ multiplicity 1

$x = -\sqrt{10}$ multiplicity 1

ⓑ $f(x) = 3x(x-2)(x+3)(x^2-1)$

sol^n: $3x = 0 \Rightarrow x = 0$ multiplicity 1

$x-2 = 0 \Rightarrow x = 2$ multiplicity 1

$x+3 = 0 \Rightarrow x = -3$ multiplicity 1

$x^2 - 1 = 0 \Rightarrow x = \pm 1$

$x = 1$ multiplicity 1

$x = -1$ multiplicity 1

④ find the polynomial with given zeros:

ⓐ Zeros of -4 ~~and~~ multiplicity 2
zeros of 0 multiplicity 2
$f(-1) = -6$

Sol^n: one factor $(x+4)^2$

2nd factor $x-0 = x^2$
and $f(-1) = -6 \Rightarrow (x+1)$ divides $f(x)$.

$$f(x) = x^2(x+4)^2(x+1) + \frac{-6}{x+1}$$

$$= x^2(x+4)^2(x+1) - \frac{6}{x+1}$$

ⓑ zeros of 2 and 4 multiplicity 2 and $f(1) = -18$

Sol^n: one factor $(x-2)^2$

second factor $(x-4)^2$

$f(1) = -18$ means $x-1$ divides $f(x)$

$$\therefore f(x) = (x-1)(x-2)^2(x-4)^2 + \frac{-18}{x-1}$$

$$= (x-1)(x-2)^2(x-4)^2 - \frac{18}{x-1}$$

5

⑤ List all possible rational zeros and by using Descarte's Rule of sign, How many positive and negative solutions does f(x) has?

ⓐ $f(x) = x^3 - x^2 - 10x - 8$

sol^n: c = factors of $-8 = \pm 1, \pm 2, \pm 4, \pm 8$

a = Factors of $1 = \pm 1$

possible sol^n!

$\therefore \frac{c}{a} = \pm 1, \pm 2, \pm 4, \pm 8$

But By Descarte's Rule of sign.

$f(x) = \underbrace{x^3 -}_{1} x^2 - 10x - 8$

Just 1 sign change ⇒ Just 1 positive solution

But for Negative solution.

$f(-x) = (-x)^3 - (-x)^2 - 10(-x) - 8$

$= -x^3 \underbrace{- x^2 +}_{1} \underbrace{10x - 8}_{2}$

2 sign change

⇒ 2 or 0 Negative solutions.

(b) $f(x) = 6x^3 + 17x^2 - 31x - 12$

solutions:

$c =$ factors of $-12 = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$a =$ factors of $6 = \pm 1, \pm 2, \pm 3, \pm 6$

Thus, all possible solutions:

$\frac{c}{a} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{3}{2},$
$\pm \frac{4}{3}, \pm \frac{4}{6} = \frac{2}{3}, \pm \frac{4}{12} = \pm \frac{1}{4},$

By Descartes Rule of sign.

$f(x) = 6x^3 \underbrace{+ 17x^2 -}_{1} 31x - 12$

1 change

Just 1 positive solutions.

Again for negative solution,

$f(-x) = 6(-x)^3 + 17(-x)^2 - 31(-x) - 12$

$= \underbrace{-6x^3 +}_{1} 17x^2 \underbrace{+ 31x -}_{2} 12$

2 changes

$\Rightarrow$ 2 or 0 negative solutions.

6 Find the End behavior of the followings.

(a) $f(x) = 10x^6 - x^5 + 8x - 2$

Sol^n: It has similar ending behavior as of $10x^6$ → Both ends up

(b) $f(x) = 3 + 2x - 4x^3 - 7x^8$

Sol^n: It has similar ending behavior as of $-7x^8$ → Both ends down

(c) $f(x) = -4x^7 - x^5 + x^3 - 1$

Solution: It has similar ending behavior as of $-4x^7$

Left up and right down

(d) $f(x) = 5x^5 + 2x^3 - 3x + 14$

Solution: It has similar Ending behavior as of $5x^5$

Left down and right up

7 Sketch the graph by using zeros and their multiplicity.

(a) $f(x) = x^2 (x-5)(x+3)(x-1)$

Soln: This polynomial has same ending behavior as of x^5 left down, right up.

$\downarrow \quad \nearrow$

zeros of the polynomial:

$x^2 = 0 \Rightarrow x = 0$ multiplicity 2

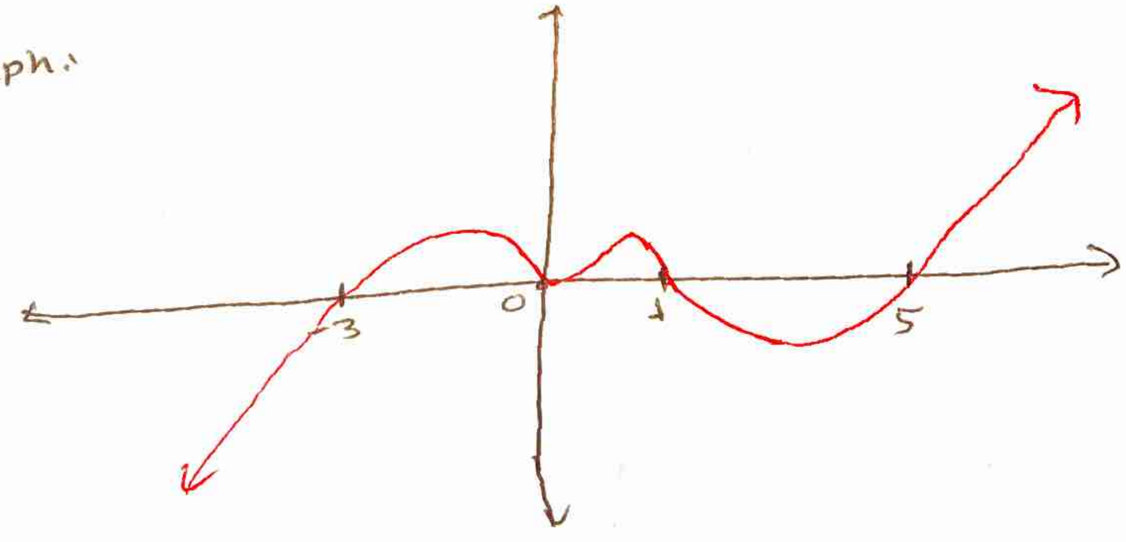
$x-5=0 \Rightarrow x=5$ multiplicity 1

$x+3=0 \Rightarrow x=-3$ multiplicity 1

$x-1=0 \Rightarrow x=1$ multiplicity 1

So the graph bouncing back at 0 and crossing x-axis at 5, -3, and 1

Graph:



6 Sketch the graph

$(3x-1)(x+2)^2(x+1)$

Sol^n: It has the same ending behavior as of

$3x \cdot x^2 \cdot x = 3x^4$ Both ending up

↖ ↗

zeros of the polynomial

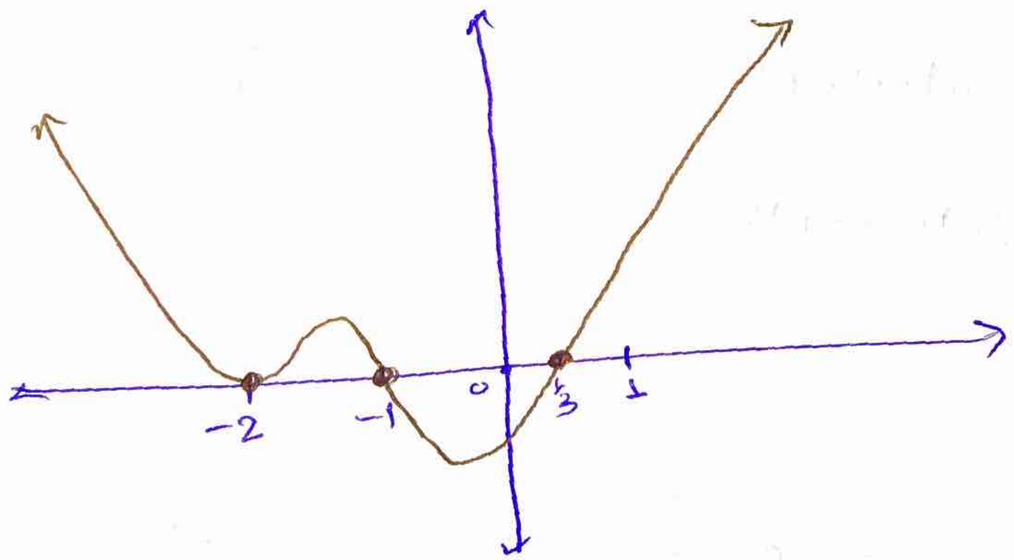
$3x-1=0 \quad 3x=1 \Rightarrow x=\frac{1}{3}$ multiplicity 1

$(x+2)^2=0 \Rightarrow x+2=0 \Rightarrow x=-2$ multiplicity 2

$x+1=0 \Rightarrow x=-1$ multiplicity 1

$\therefore$ graph crosses x-axis at $x=\frac{1}{3}$ and $x=-1$

but bounces back at $x=-2$

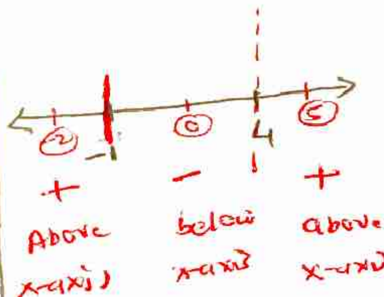


8. Graph the following rational function by determining domain, vertical Asymptotes, Horizontal Asymptotes, x-intercepts and y-intercepts and Also oblique Asymptotes.

(a) $f(x) = \dfrac{x+1}{x-4}$

Domain $= \{x : x \neq 4\}$

sign test



Vertical Asymptotes: $x-4=0$

$x=4$

Horizontal Asymptotes: $y = \frac{1}{1} = 1$

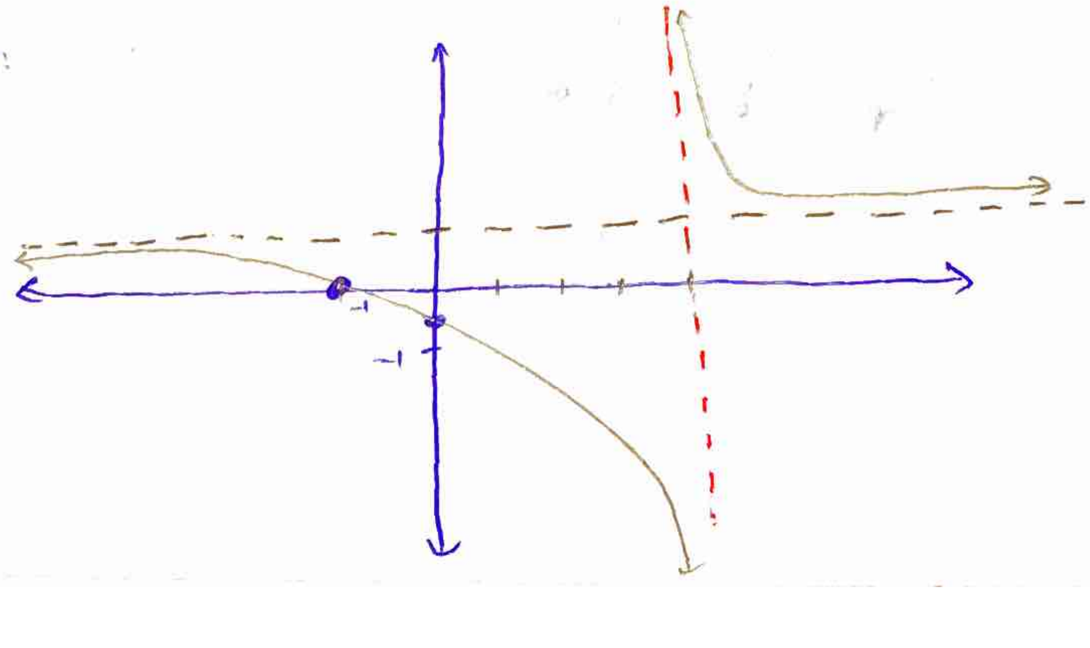
No oblique Asymptotes.

y-intercept: $y = \frac{1}{-4} = -\frac{1}{4}$

x-intercepts: $x+1=0$

$x=-1$

Graph:



b) very imp

$$f(x) = \frac{3x}{x^2-x-2}$$

Solution: By factorization $x^2-x-2 = (x-2)(x+1)$

$$\therefore f(x) = \frac{3x}{(x-2)(x+1)}$$

Domain: $D = \{x : x \neq 2, -1\}$

V.A: $(x-2)(x+1) = 0$

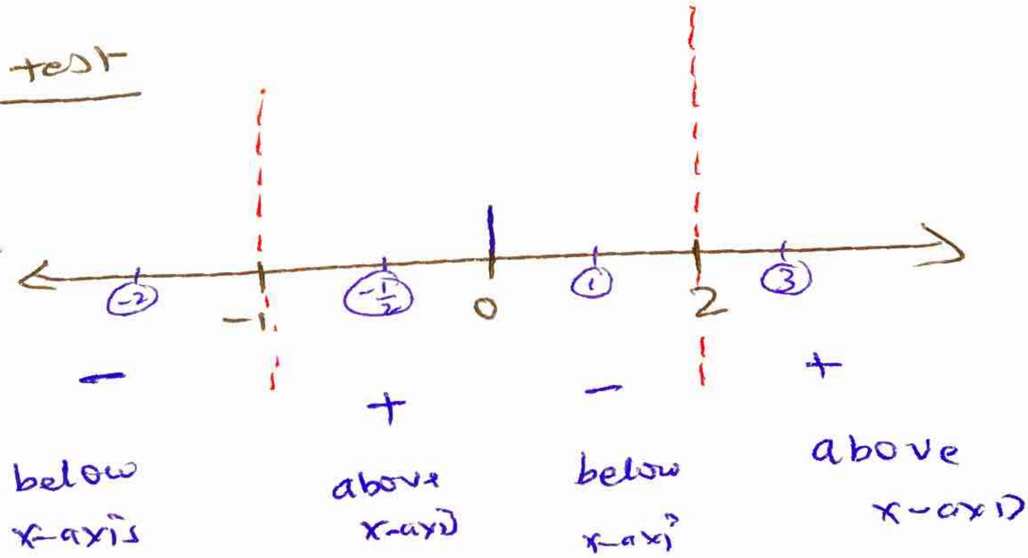
$x = 2, \ x = -1$

H.A: since the bottom power bigger $y = 0$ is H.A.

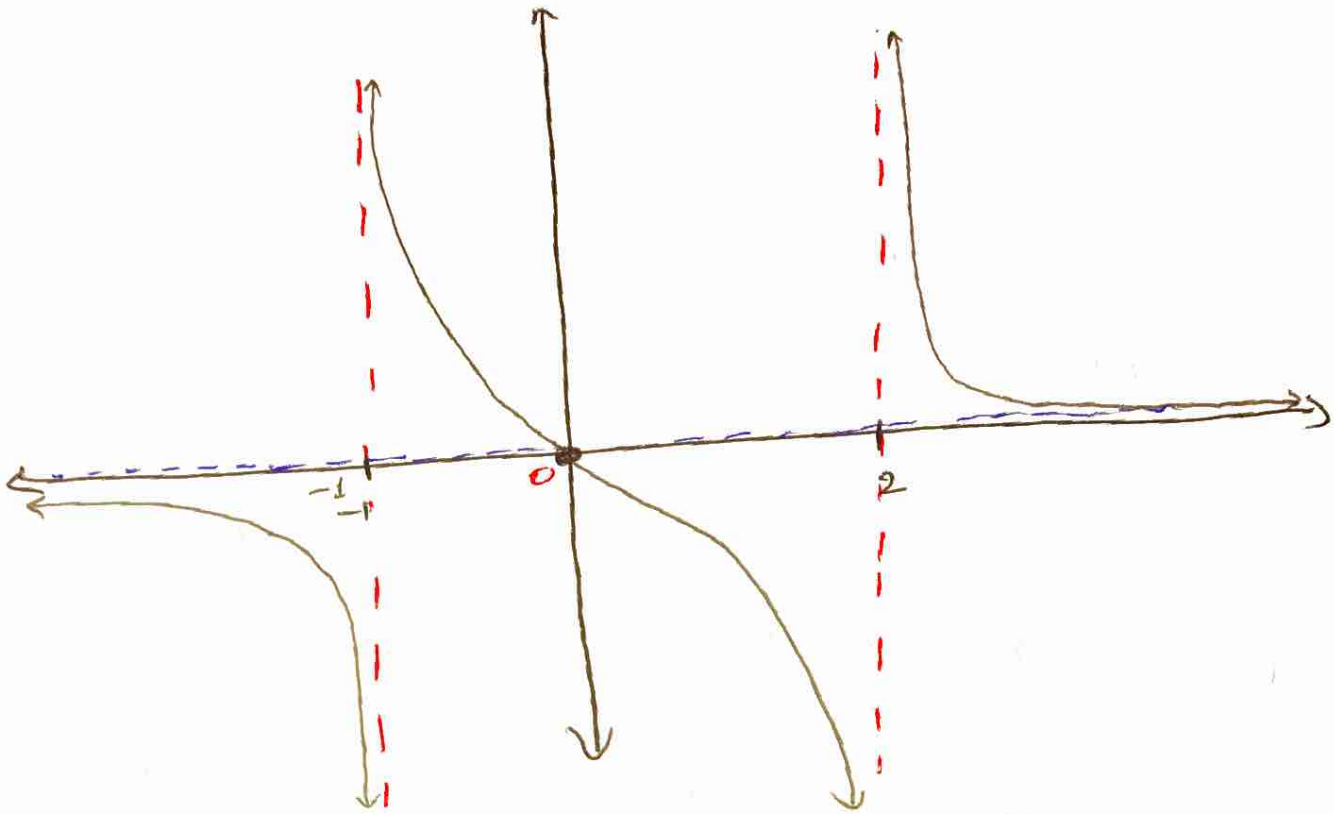
y-intercepts: $y = \frac{0}{2} = 0 \quad \therefore y = 0$

x-intercepts: $3x = 0 \Rightarrow x = 0$

Sign test



graph is ~~back~~ on the back page



-1
-1
0
2

Very imp

c) $f(x) = \dfrac{x^2+1}{x+3}$

sol^n: Domain $D = \{x : x \neq -3\}$

V.A: $x+3=0 \Rightarrow x=-3$

H.A: oblique only (because top is bigger)

To find oblique Asymptotes:

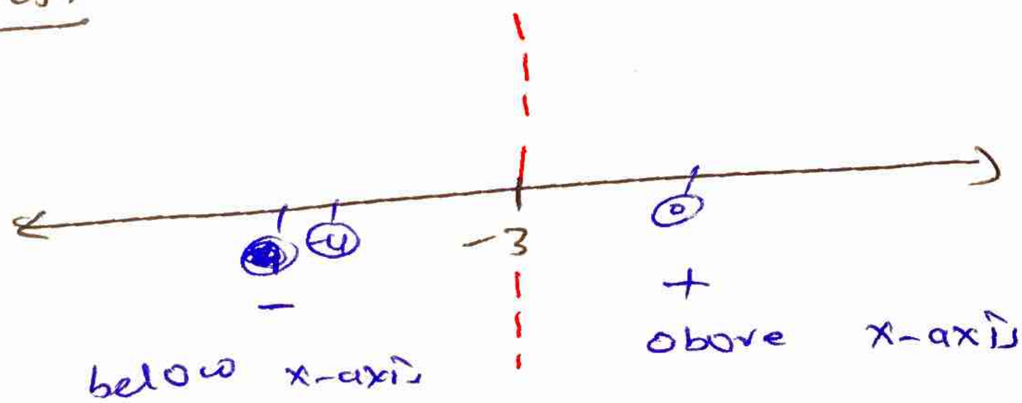
$$\begin{array}{r|l} & x-3 \\ \hline x+3 & x^2+1 \\ & \underline{-\,x^2 \;-\; 3x} \\ & -3x+1 \\ & \underline{+\,3x \;+\; 9} \\ & 10 \end{array}$$

$\therefore$ oblique Asymptotes $y = x-3$

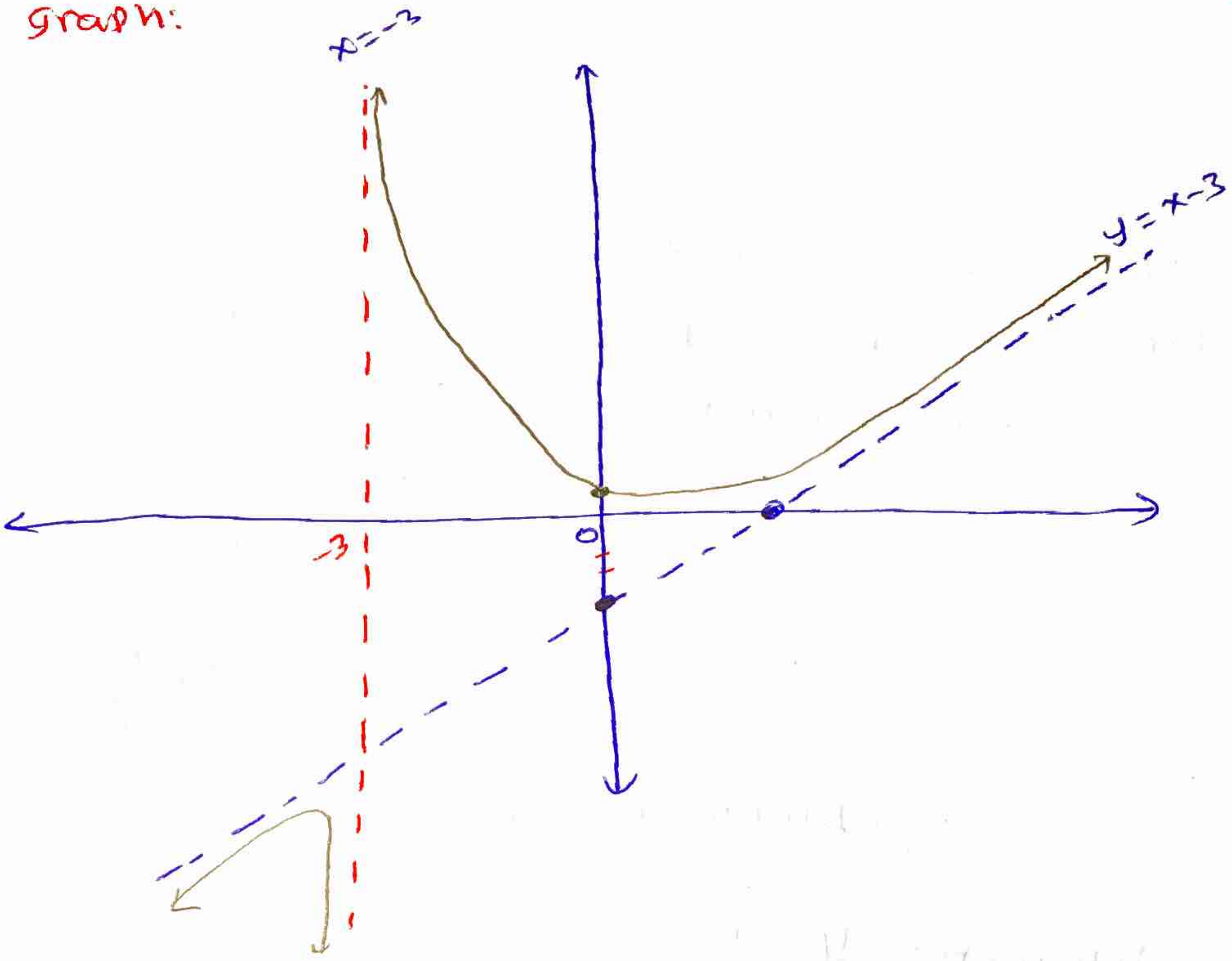
y-intercepts: $y = \frac{1}{3}$

x-intercepts: $x^2+1=0 \Rightarrow x^2=-1$ there NO x-intercepts.

sign test

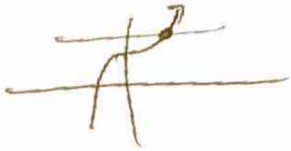


Graph:



Q Check whether following functions are one-one, if they are one-one then find f^{-1} and its domain (domain of f^{-1})

(a) $f(x) = x^3 + 1$



Yes it is one-one

Now for the inverse.

$y = x^3 + 1$

Interchanging x and y

$x = y^3 + 1$

$x - 1 = y^3$

$\sqrt[3]{x-1} = y$

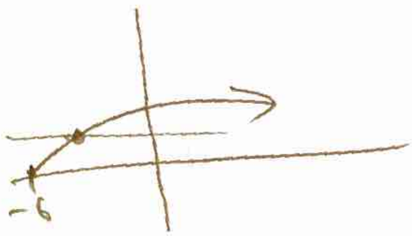
$\therefore y = \sqrt[3]{x-1}$

$f^{-1}(x) = \sqrt[3]{x-1}$

domain $= \mathbb{R}$.

(b) $f(x) = \sqrt{x+6}$ $\quad x \geq -6$

Yes it is one-one



Now for the inverse,

$$y = \sqrt{x+6}$$

Interchanging x and y

$$x = \sqrt{y+6}$$

squaring.

$$x^2 = y+6$$

$$x^2 - 6 = y$$

$$\therefore f^{-1}(x) = x^2 - 6$$

Domain is real number

(c) for the following one-one function

$$f(x) = \frac{x+2}{x-1}, \quad x \neq 1$$

$$y = \frac{x+2}{x-1}$$

Interchanging x and y

$$x = \frac{y+2}{y-1}$$

$$(y-1) \cdot x = \frac{y+2}{\cancel{(y-1)}} \cdot \cancel{(y-1)}$$

$$xy - x = y+2$$

combining like terms:

$$xy - y = x + 2$$

$$\cancel{y(x+1)} \quad y(x-1) = x+2$$

$$y = \frac{x+2}{x-1}$$

$$\therefore f^{-1}(x) = \frac{x+2}{x-1}$$

$$\text{Domain} = \{x : x \neq 1\}$$

10 Solve the following.

ⓐ $4^{x-2} = 2^{3x+3}$

sol^n:

$$(2^2)^{x-2} = 2^{3x+3}$$

$$2^{2x-4} = 2^{3x+3}$$

$$\Rightarrow 2x - 4 = 3x + 3$$

$$\Rightarrow 2x - 3x = 3 + 4$$

$$\Rightarrow -x = 7$$

$$\Rightarrow x = -7$$

ⓑ $32^{2x} = 16^{x-1}$

sol^n: $(2^5)^{2x} = (2^4)^{x-1}$

$$2^{10x} = 2^{4x-1}$$

$$10x = 4x - 1$$

$$10x - 4x = 1$$

$$6x = 1$$

$$x = 1/6$$

(11) Find the following

(a) Find the future value and interest earned if \$ 8000.0 deposited for 9 years at 3% compounded

(i) quarterly

(ii) continuously:

Soln:

$P = 8000$

$T = 9$

$R = 3\% = 0.03$

Future values.

(i) quaterly, compounding

$n = 4$

$$\therefore A = P\left(1+\frac{R}{n}\right)^{nT}$$

$$= 8000\left(1+\frac{0.03}{4}\right)^{4\times 9}$$

$$= 8000(1+0.0075)^{36}$$

$$= 8000(1.0075)^{36}$$

$$= 8000 \times 1.308$$

$$= 10,469.16$$

Interest $= A - P = 10,469.16 - 8,000$

$= 2469.16$

(ii) compounded continuously

$$A = Pe^{RT}$$

$$A = 8000\,e^{0.03\times 9}$$

$$= 8000\,e^{0.27}$$

$$= 8000 \times 1.3099$$

$$= 10,4\cancel{79}9.72$$

Interest $= A - P$

$= 10,479.72 - 8000$

$= 2479.72$

(5) Find the present value if the total amount is to get \$25,000 in 5 years compounded semi-annually at the rate of 4%.

Soln:

$A = 25000$

$R = 4\% = 0.04$

$T = 5$

$n = 2$

$$A = P\left(1 + \frac{R}{n}\right)^{nT}$$

$$25000 = P\left(1 + \frac{0.04}{2}\right)^{2 \times 5}$$

$$25000 = P\,(1 + 0.02)^{10}$$

$$25000 = P\,(1.02)^{10}$$

$$25000 = P \times 1.21899$$

$$\therefore P = \frac{25000}{1.21899}$$

$$= 20,508.78$$

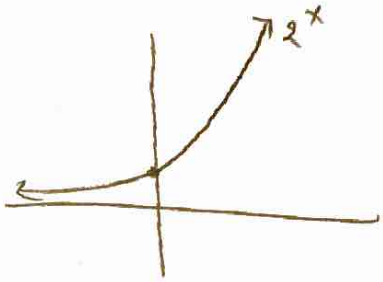
present value $= 20,508.78$.

12 Sketch the graph.

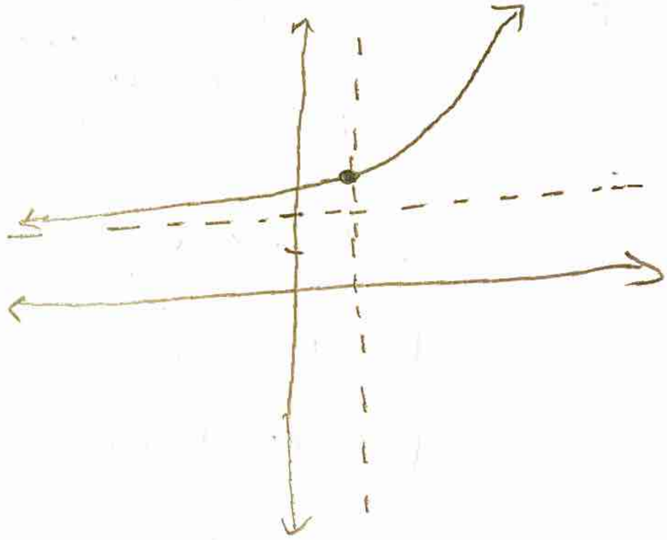
a $f(x) = 2^{x-1} + 2$

sol^n:

parent function



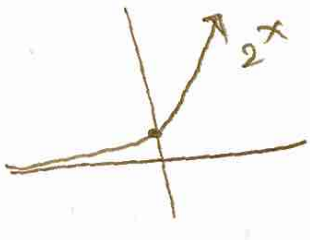
shifting 1 right
2 up



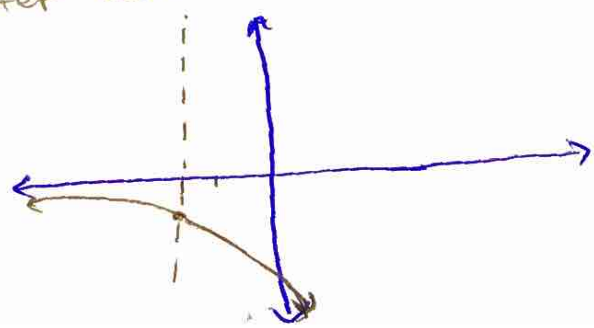
b $f(x) = -2^{x+2}$

sol^n.

parent function

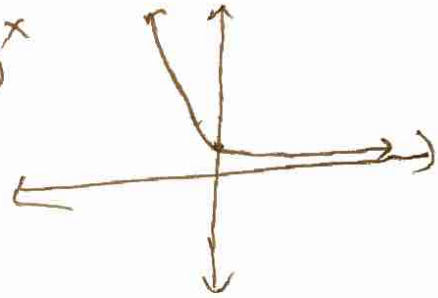


- Flipping down
- shifting 2 step left

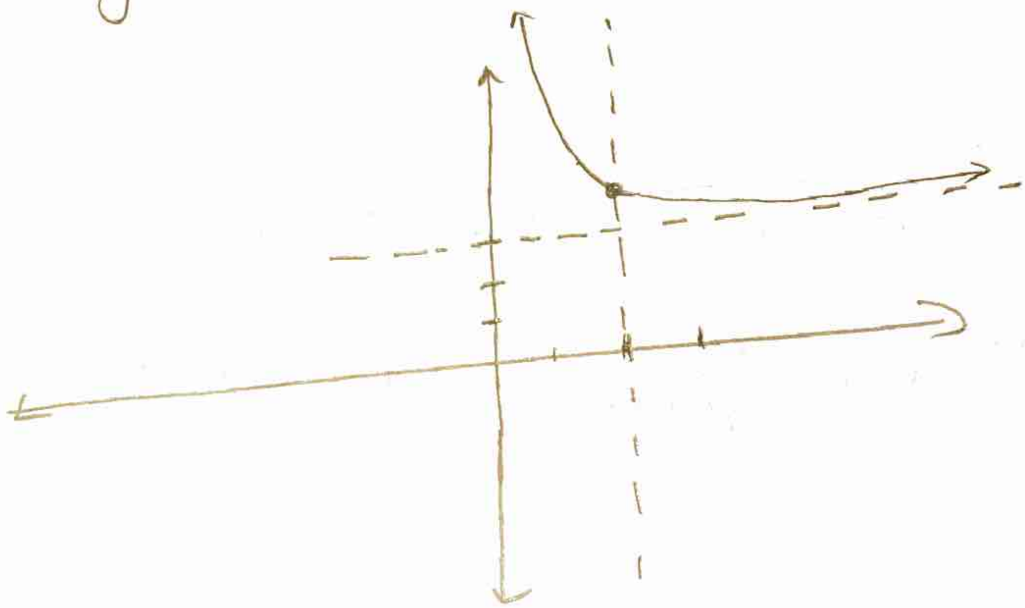


(c) $f(x) = \left(\frac{1}{3}\right)^{x-2} + 3$

Soln: Parent function $f(x) = \left(\frac{1}{3}\right)^{x}$



Shifting 2 right
shifting 3 up



(13) Solve the following:

(a) $x = \log_5 \frac{1}{625}$

Soln: $x = \log_5 \frac{1}{5^4}$

$x = \log_5 5^{-4}$

$x = -4$

(b) $\log_x 25 = -2$

Soln: $x^{-2} = 25$

$x^{-2} = 5^2$

$\therefore x^{-2} = \left(\frac{1}{5}\right)^{-2}$

$x = 1/5$

(c) $\log_{\frac{1}{2}}(x+3) = 4$

Sol^n: $4^{\frac{1}{2}} = x+3$

$2^{2 \cdot \frac{1}{2}} = x+3$

$2 = x+3$

$x = -3+2$

$x = -1$

(d) $\log_{(x+3)} 6 = 1$

Sol^n $(x+3)^1 = 6$

$x = 3$

(e) $\log_2(x+3) - \log_2(x+4)$

Solution: $= \log_2 \frac{x+3}{x+4}$

$\left(\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)\right)$

(f) $-\frac{2}{3}\log_5 5m^2 + \frac{1}{2}\log_5 25m^2$

Sol^n: ~~$-\frac{2}{3}\log_5$~~

$-\frac{2}{3}(\log_5 5 + \log_5 m^2) + \frac{1}{2}(\log_5 25 + \log_5 m^2)$

$= -\frac{2}{3} \cdot 1 - \frac{2}{3}\log_5 m^2 + \frac{1}{2}(\log_5 5^2 + 2\log_5 m)$

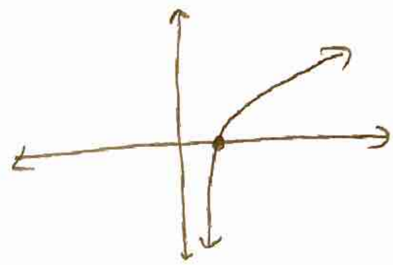
$= -\frac{2}{3} - \frac{4}{3}\log_5 m + \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2\log_5 m$

$= -2/3 - \frac{4}{3}\log_5 m + 1 + \log_5 m$

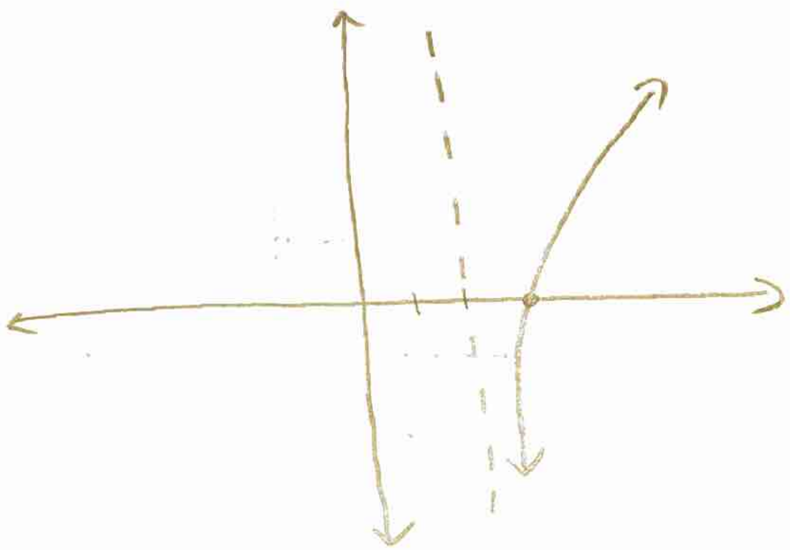
14 Graph the following:

@ $f(x) = \log_6 (x-2)$

sol: parent function $\log_6 x$

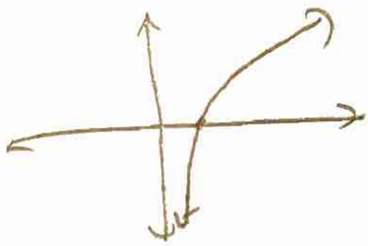


shifting 2 Right

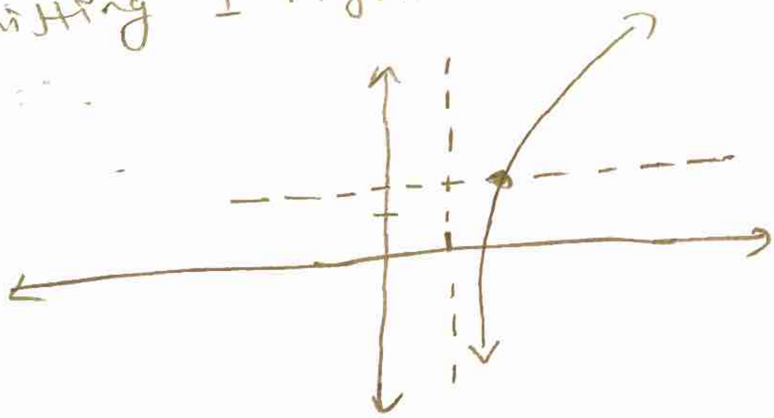


ⓑ $f(x) = \log_3 (x-1) + 2$

sol: parent function $\log_3 x$

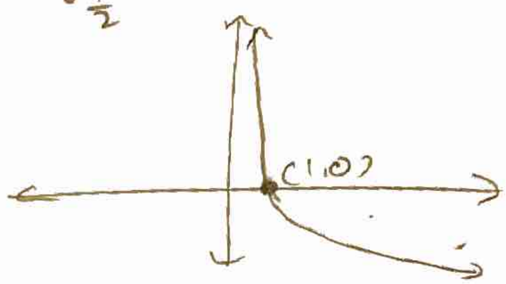


shifting 1 right and 2 up

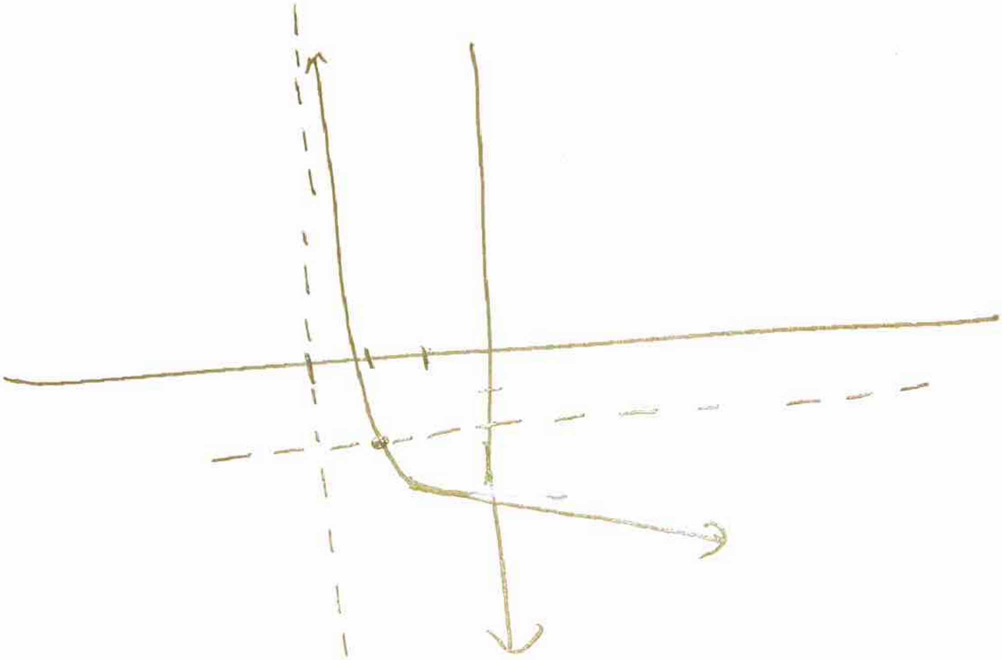


(c) $f(x) = \log_{\frac{1}{2}}(x+3) - 2$

Sol^n: parent function $f(x) = \log_{\frac{1}{2}} x$



Shifting 3 left
Shifting 2 down



15 solve,

(a) $\log x + \log x^2 = 3$

Sol^n: $\log x \cdot x^2 = 3$

$\log x^3 = 3$

$x^3 = 10^3$

$x = 10$

(b) $\ln x + \ln x^2 = 3$

sol^n: $\ln x \cdot x^2 = 3$

$\ln x^3 = 3$

$x^3 = e^3$

$x = e$ #

(c) $\log_7 (x^3 + 65) = 0$

sol^n: $x^3 + 65 = 7^0$

$x^3 + 65 = 1$

$x^3 = -64$

$x^3 = (-4)^3$

$\therefore x = -4$

(d) $\log_2 (2x-3) + \log_2 (x+1) = 1$

sol^n: $\log_2 (2x-3)(x+1) = 1$

$\therefore (2x-3)(x+1) = 2^1$

$2x^2 + 2x - 3x - 3 = 2$

$2x^2 - x - 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-5)}}{2 \cdot 2}$

$= \frac{1 \pm \sqrt{1 + 40}}{4}$

$= \frac{1 \pm \sqrt{41}}{4}$

© For $f(x) = \sqrt{x+4}$ $g(x) = \frac{1}{x+5}$

(i) $fog(x)$

(ii) $gof(x)$

(iii) Their domain

Sol: (i) $f \circ g(x) = f(g(x))$

$$= f\left(\frac{1}{x+5}\right)$$

$$= \sqrt{\frac{1}{x+5} + 4} = \sqrt{\frac{4x+21}{x+5}}$$

Domain $= \{x : x \geq -\frac{21}{4}, x \neq -5\}$

(ii) $gof(x) = g(f(x))$

$$= g(\sqrt{x+4})$$

$$= \frac{1}{\sqrt{x+4} + 5}$$

Domain $= \{x : x \neq -4\}$

2) $f(x) = \sqrt{x}$ $\quad g(x) = x+3$

find (i) $f \circ g\,(x)$

(ii) $g \circ f\,(x)$

(iii) Their Domain

(i) $f \circ g(x) = f(g(x))$

$= f(x+3)$

$= \sqrt{x+3}$

Domain $= \{x : x \geq -3\}$

(ii) $g \circ f(x) = g(f(x))$

$= g(\sqrt{x})$

$= \sqrt{x} + 3$

Domain $= \{x : x \geq 0\}$