Exam Review 2

Chapter - 2 and 3

① Find the Equation of the line containing the points $(-3,4)$ and $(2,5)$

Sol^n: we have $(x_1, y_1) = (-3,4)$

$(x_2, y_2) = (2,5)$

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-4}{2+3} = \frac{1}{5}$$

Equation of the line,

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{5}(x+3)$$

$$y - 4 = \frac{x}{5} + \frac{3}{5}$$

$$y = \frac{x}{5} + \frac{3}{5} + 4$$

$$y = \frac{x}{5} + \frac{23}{5} \quad \#$$

② Find the Equations of the line through $(-1,2)$ and

ⓐ Parallel to $y = -3x$

b) Perpendicular to $y = 2x - 3$

13
12

sol^n: $(x_1, y_1) = (-1, 2)$

(a) parallel to $y = -3x$

the line is parallel so $m = -3$

$\therefore$ ~~eqn~~ Equation of the line,

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3(x+1)$$
$$y - 2 = -3x - 3$$
$$y = -3x - 3 + 2$$
$$y = -3x - 1 \quad \#$$

(b) perpendicular to $y = 2x - 3$

$\therefore$ slope $m = -\frac{1}{2}$

Equation of the line,

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{1}{2}(x+1)$$
$$y - 2 = -\frac{1}{2}x - \frac{1}{2}$$
$$y = -\frac{1}{2}x - \frac{1}{2} + 2$$
$$y = -\frac{1}{2}x + \frac{3}{2} \quad \#$$

③ Find the slope, y-intercept and graph the Line.

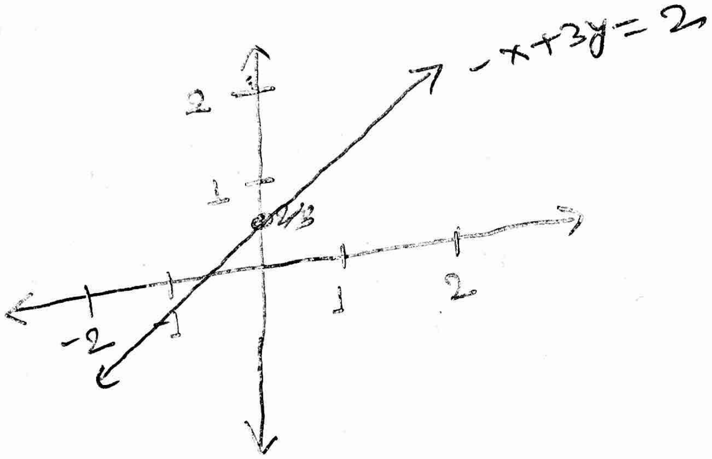
ⓐ $-x+3y=2$

Sol^n: $3y = x+2$

$y = \frac{x}{3} + \frac{2}{3}$

slope $= \frac{1}{3}$, ← positive slope (line going up)

y-intercept $= \frac{2}{3}$



④ Determine whether lines are parallel or perpendicular.

ⓐ $y = 4x+5$

$y = -\frac{1}{4}x + 2$

Sol^n:

$m_1 = 4$ $\quad$ $m_1 \neq m_2 \Rightarrow$ they are not parallel

$m_2 = -\frac{1}{4}$ $\quad$ $m_1 \cdot m_2 = 4(-\frac{1}{4}) = -1 \Rightarrow$ They are perpendicular

(b) $y = -2x + 3$

$y = -\frac{1}{2}x + 2$

Sol^n: $m_1 = -2$

$m_2 = -\frac{1}{2}$

$m_1 \neq m_2 \Rightarrow$ They are not parallel

$m_1 \cdot m_2 = -2(-\frac{1}{2}) = 1 \Rightarrow$ They are not perpendicular

(5) For the following functions

$f(x) = 4x+3 \qquad g(x) = \frac{1}{x+3}$

find (a) $f(-1)$, $g(2)$, $g(7)$.

(b) $\frac{f(x+h) - f(x)}{h}$, $\frac{g(x+h) - g(x)}{h}$

Sol^n:

(a) $f(-1) = 4(-1) + 3 = -4+3 = -1$

$g(2) = \frac{1}{2+3} = \frac{1}{5}$

$g(7) = \frac{1}{7+3} = \frac{1}{10}$

(b) $f(x+h) = 4(x+h) + 3$

$= 4x + 4h + 3$

$\therefore \frac{f(x+h) - f(x)}{h} = \frac{4x+4h+3-(4x+3)}{h}$

$$= \frac{\cancel{4x} + 4h + \cancel{3} - \cancel{4x} - \cancel{3}}{h}$$

$$= \frac{4h}{h}$$

$$= 4 \quad \#$$

And

$$\frac{g(x+h) - g(x)}{h} = ?$$

$$g(x+h) = \frac{1}{x+h+3}$$

Then $\dfrac{g(x+h) - g(x)}{h} = \dfrac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h}$

$$= \frac{1}{h} \left\{ \frac{x+3-(x+h+3)}{(x+h+3)(x+3)} \right\} \quad LCD = (x+3)(x+3+h)$$

$$= \frac{1}{h} \left\{ \frac{\cancel{x} + \cancel{3} - \cancel{x} - h - \cancel{3}}{(x+3)(x+h+3)} \right\}$$

$$= \frac{1}{\cancel{h}} \left\{ \frac{-\cancel{h}}{(x+3)(x+h+3)} \right\}$$

$$= \frac{-1}{(x+3)(x+h+3)} \quad \#$$

6 For $f(x)=\frac{2x}{x-2}$

(a) Is the point $(\frac{1}{2},-\frac{2}{3})$ on the graph of f?

Solⁿ: since $x=\frac{1}{2}$ then

$$y=f(x)=f(\tfrac{1}{2})=\frac{2(\frac{1}{2})}{\frac{1}{2}-2}=\frac{1}{\frac{1-4}{2}}=\frac{1}{\frac{-3}{2}}$$

$$=1\times(-\tfrac{2}{3})$$
$$=-\frac{2}{3}$$

∴ Yes $(\frac{1}{2},-\frac{2}{3})$ is on the graph of f

(b) If $x=4$ what is $f(x)$? and find the point on the graph of f?

Solⁿ: $x=4 \Rightarrow f(4)=\frac{2(4)}{4-2}=\frac{8}{2}=4$

∴ point $=(x,y)=(4,4)$

(c) If $f(x)=1$ then what is x? and find the point on the graph

Solⁿ: $f(x)=1$

$$\frac{2x}{x-2}=1$$

$2x = (x-2)$

$2x = x-2$

$2x - x = -2$

$x = -2$ $\quad\because y = f(x) = 1$

$\therefore$ point $(x, y) = (-2, 1)$

(D) Domain of f ?

Sol^n: Domain of $f = \{x : x \neq 2\}$

(E) Find the x-intercepts and y-intercepts if there is any

Sol^n.

x - intercepts $(y=0)$

$\therefore 0 = \frac{2x}{x-2}$

$\Rightarrow 2x = 0$

$\Rightarrow x = 0$ $\quad \Rightarrow (0,0)$ is the x-intercept

y- intercept $(x=0)$

$y = \frac{2 \cdot 0}{0-2} = \frac{0}{-2} = 0$

$\therefore (0,0)$ is the y-intercepts

7. Determine whether following functions are Even or Odd functions?

ⓐ $f(x) = 3x^3 + 5$

soln: $f(x) = 3x^3 + 5x^0$ This is Neither

Because power are mixed

ⓑ $f(x) = 2x^4 - x^2 + 4$

soln: $f(x) = 2x^4 - x^2 + 4x^0$

All powers are Even so

f(x) is Even.

ⓒ $f(x) = \frac{x^3}{x^5 - x}$

soln: All powers are odd so this is a odd function.

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8. Find the Average rate of change of the function $f(x) = x^3 - 2x + 1$

ⓐ From -1 to 1

ⓑ From 2 to 5

Soln:

ⓐ $f(x) = x^3 - 2x + 1$ from $\underset{a}{\underset{\downarrow}{-1}}$ to $\underset{b}{\underset{\downarrow}{1}}$

$f(a) = f(-1) = (-1)^3 - 2(-1) + 1$
$= -1 + 2 + 1$
$= 2$

$f(b) = f(1) = 1^3 - 2 \cdot 1 + 1$
$= 1 - 2 + 1$
$= 2 - 2 = 0$

Average rate of change $= \dfrac{f(b) - f(a)}{b - a}$

$= \dfrac{0 - 2}{1 - (-1)} = \dfrac{-2}{2} = -1$ #

ⓑ from $\underset{a}{\underset{\downarrow}{2}}$ to $\underset{b}{\underset{\downarrow}{5}}$

$f(a) = f(2) = 2^3 - 2 \cdot 2 + 1 = 8 - 4 + 1 = 5$
$f(b) = f(5) = 5^3 - 2 \cdot 5 + 1 = 125 - 10 + 1 = 116$

Average rate of change $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{116 - 5}{5 - 2}$

$= \dfrac{111}{3}$ #

9 Graph the following piecewise function and find the indicated values:

ⓐ $f(x) = \begin{cases} -3x & \text{if } x < -1 \\ 0 & \text{if } x = -1 \\ 2x^2+1 & \text{if } x > -1 \end{cases}$

$f(0)$ $f(-2)$, $f(3)$, $f(-1)$

$f(0) = 2x^2+1 = 2 \cdot 0^2 + 1 = 0 + 1 = 1$

$f(-2) = -3x = -3(-2) = 6$

$f(3) = 2x^2+1 = 2 \cdot 3^2 + 1 = 18+1 = 19$

$f(-1) = 0$

graph: at $x = -1$ $y = f(x) = -3x$

x	y	(x,y)
-1	$-3(-1) = 3$	$(-1, 3)$

at $x = -1$ $f(x) = 0$ $(x,y) = (-1, 0)$

At $x = -1$

$y = f(x) = 2x^2 + 1$

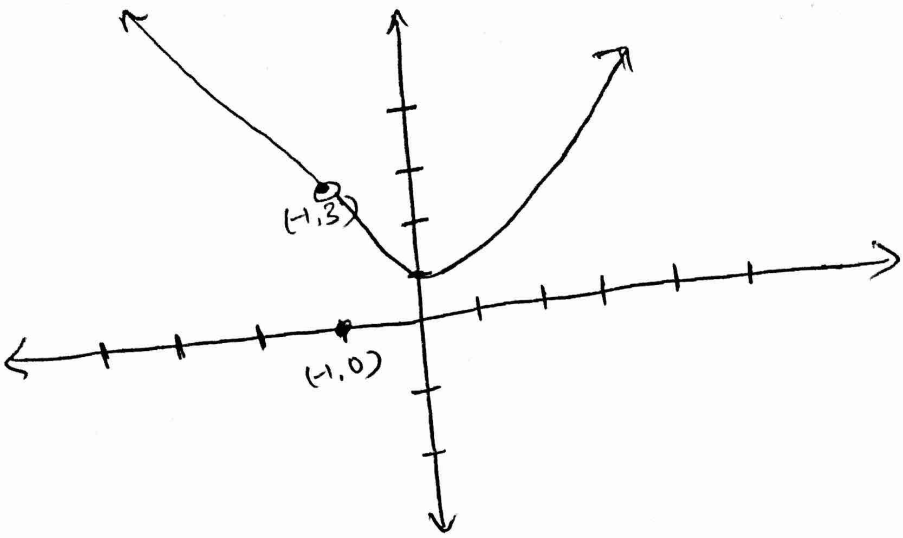
↳ y-intercept $= 1$

x	y	
-1	$2(-1)^2+1 = 3$	$(-1, 3)$

Left side of $x = -1$ Straight line with slope $= -3$

At $x = -1$ $y = 0$

Right side of $x = -1$ parabola (+)

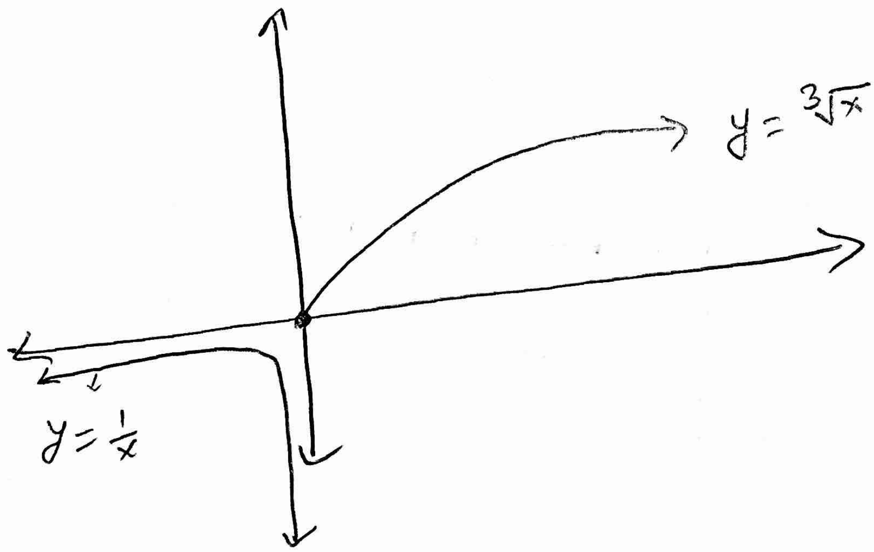


① graph $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$

sol^n:- For first part, left side of $x=0$

$y = \frac{1}{x}$

right side of $x = 0$ $y = \sqrt[3]{x}$



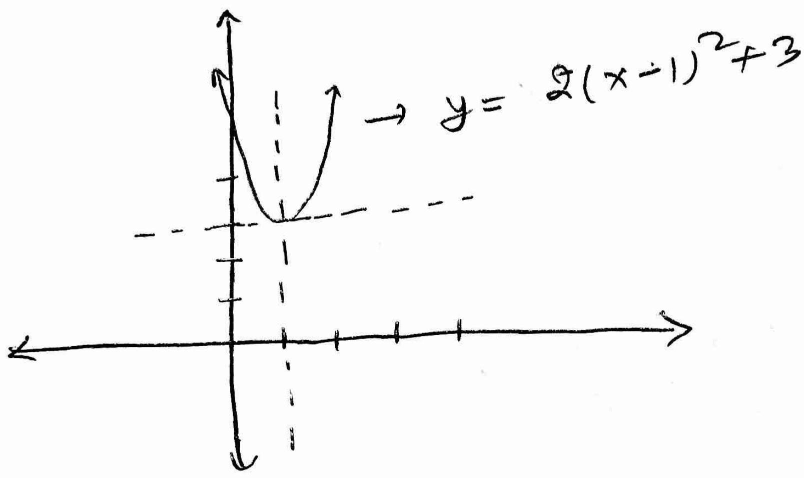
10. Sketch the following graphs:

(a) $f(x) = 2(x-1)^2 + 3$ ↑3 up

↙ vertically stretch

→ 1 right

Parent fun = x^2

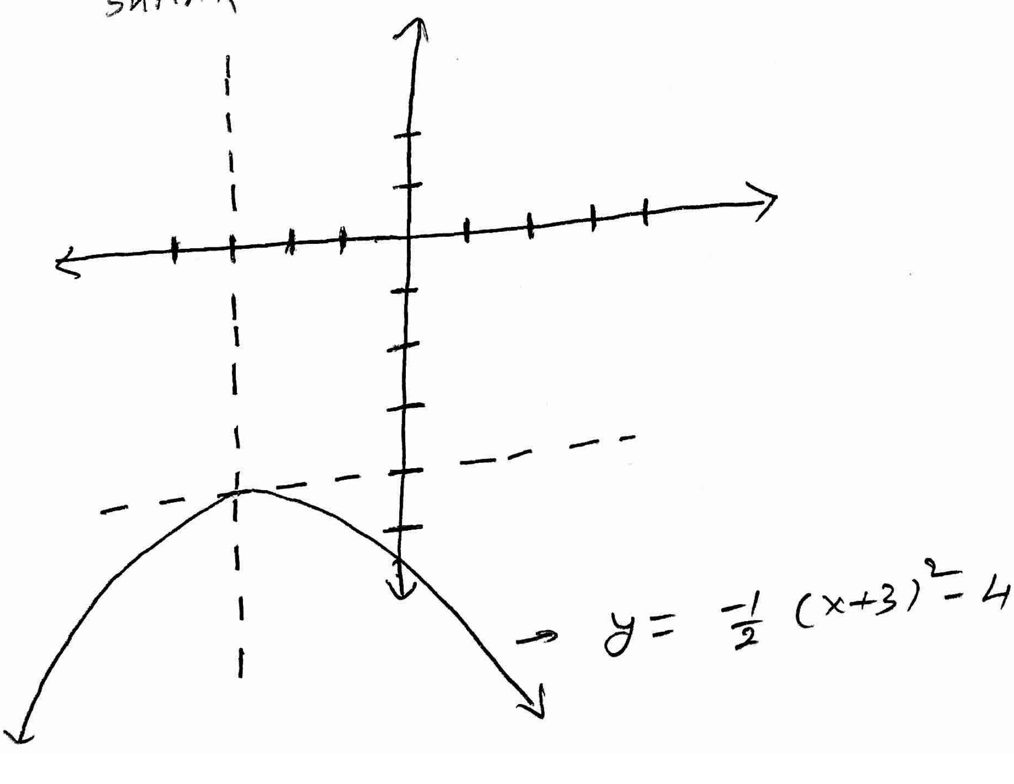


(b) $f(x) = -\frac{1}{2}(x+3)^2 - 4$ ↓4 down

↙ Vertically shrink

← 3 left

parent = $-x^2$

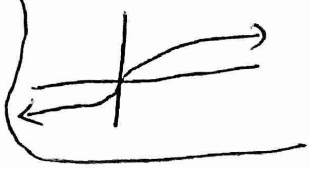


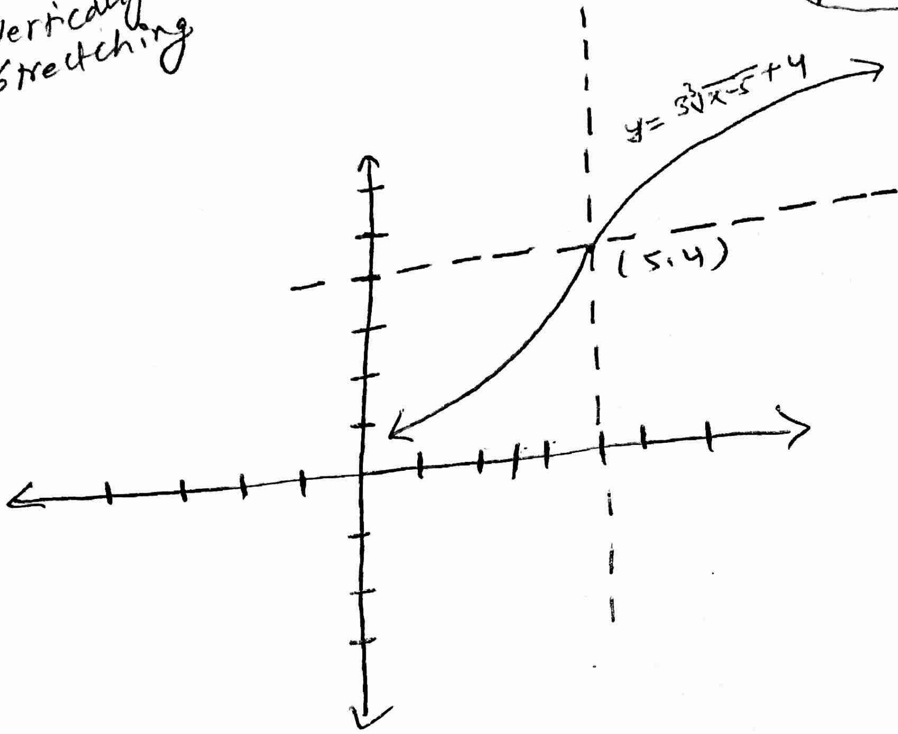
(c) $f(x) = 3\sqrt[3]{x-5} + 4$ ↑ 4 up

↙ vertically stretching

→ 5 right

parent $y = \sqrt[3]{x}$





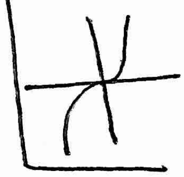
(d) $f(x) = -\frac{1}{4}(x+2)^3 - 3$ ↓ 3 down

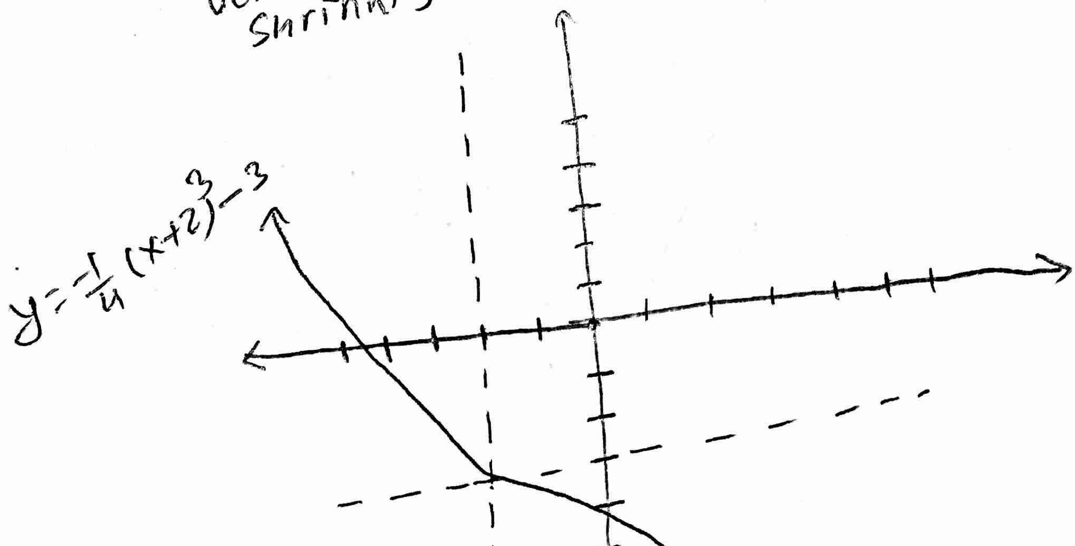
↙ flipping

↓ vertically shrinking

← 2 left

parent fun^n $y = x^3$

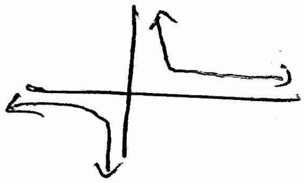


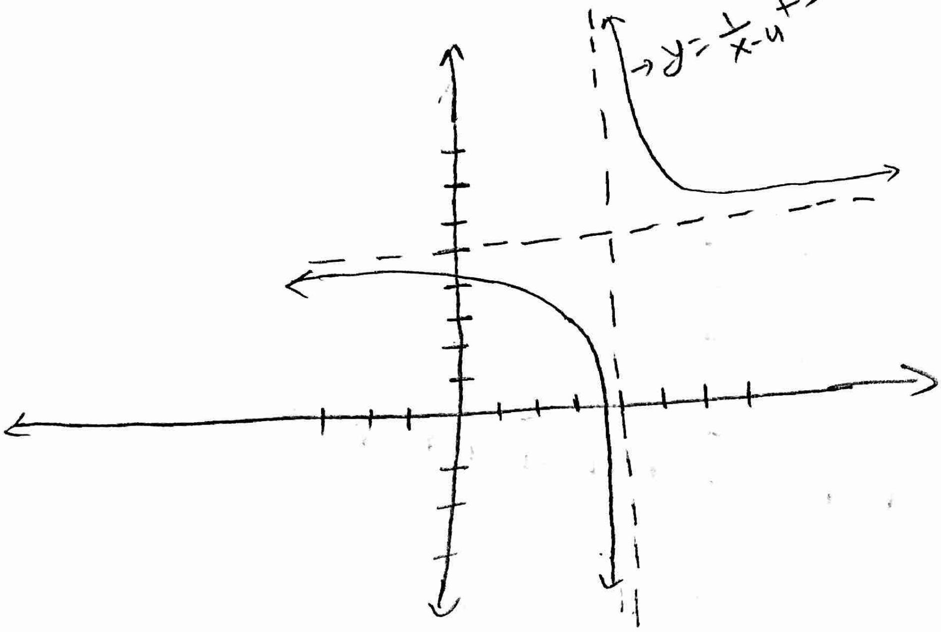


(E) $f(x) = \dfrac{1}{x-4} + 5$ ↑ 5 up

$\xrightarrow{}$ 4 right

parent $f(x) = \dfrac{1}{x}$



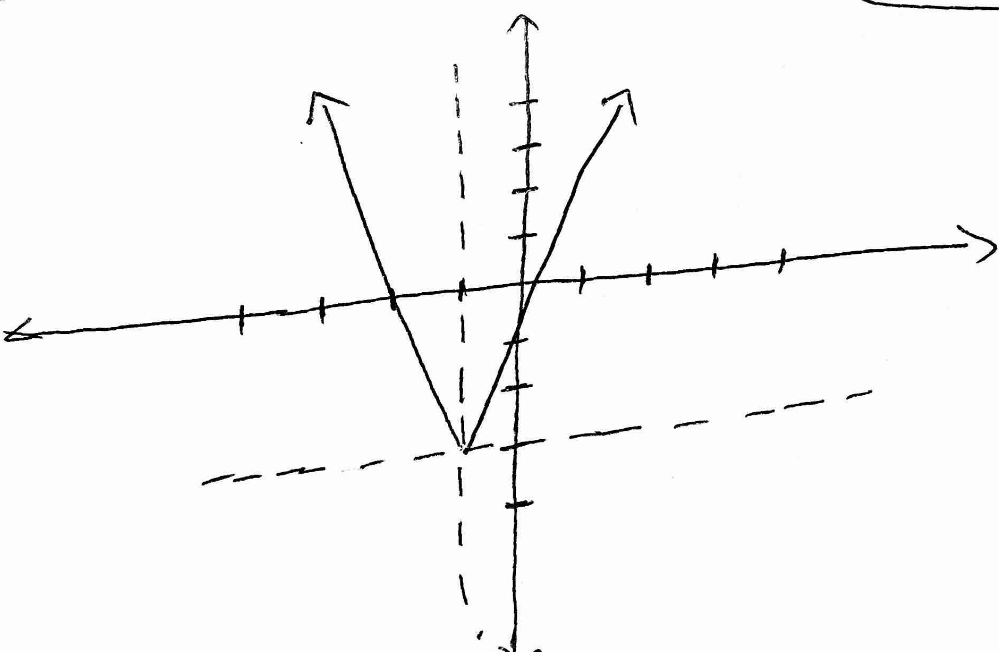


(F) $g(x) = 3\,|x+1| - 3$ ↓ 3 down

vertically stretching

$\longleftarrow$ 1 left

parent $g(x) = |x|$



CHAPTER 5

1) Determine whether followings are polynomials or not? If yes Find the degree, Find the leading term (Term with highest degree) and coefficient of leading term

ⓐ $f(x) = 4x + 2x^3$

Sol^n: Yes it is polynomial because all powers are positive numbers

$\Rightarrow$ highest power = 3

$\therefore$ Degree = 3

Thus leading term $= 2x^3$ (Term with degree 3)

coefficient of leading Term = 2

ⓑ $f(x) = \dfrac{x+1}{x-4}$

Sol^n: It is not polynomial.

ⓒ $f(x) = \sqrt{x}(\sqrt{x} - 1)$

$= \sqrt{x} \cdot \sqrt{x} - \sqrt{x}$

$= (\sqrt{x})^2 - \sqrt{x}$

$= x - \sqrt{x}$

$= x - x^{1/2}$ Not a polynomial.

12. Using the transformation Techniques graph the following & find domain and Range.

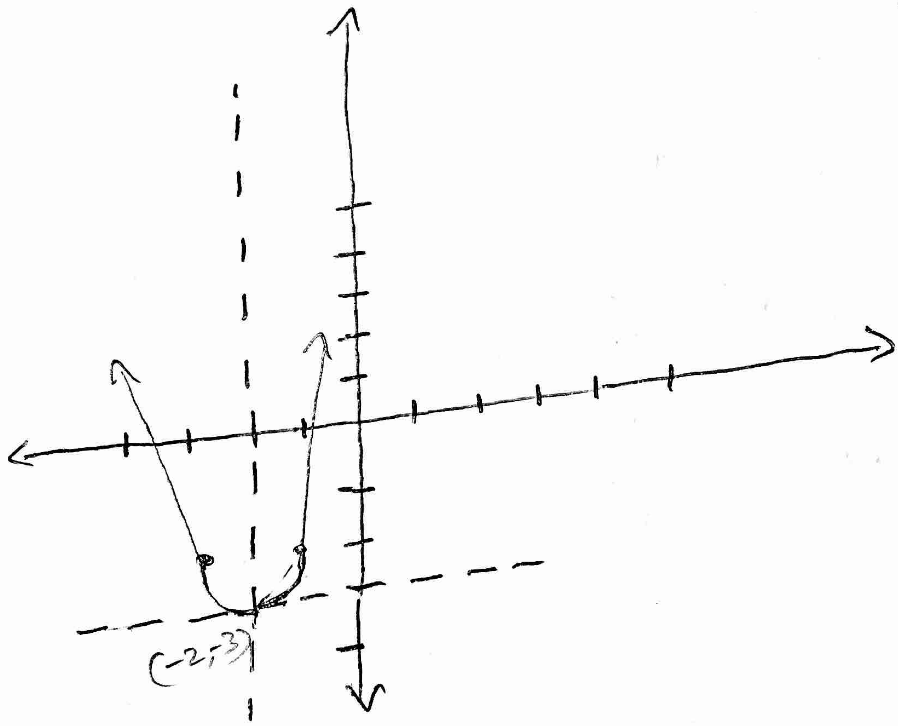
ⓐ $f(x) = (x+2)^4 - 3$ ⓑ $f(x) = \frac{1}{2}(x-5)^5 - 2$

ⓒ $f(x) = -(x-2)^5 + 4$ ⓓ $f(x) = -2(x+1)^4 + 1$

Soln: ⓐ $f(x) = (x+2)^4 - 3$ ↓ 3 down

← 2 Left

Coefficient = +1 ⇒ Both facing up



Domain = $\mathbb{R}$

Range = $y \geq -3$

parent $f(x) = x^4$

points = (0,0)
(1,1)
(-1,1)

Shifting $x = -2$
$y = -3$

point = $(0-2, 0-3)$
$= (-2,-3)$

$(1-2, 1-3)$
$= (-1,-2)$

$(-1-2, 1-3)$
$(-3,-2)$

b) $f(x) = \frac{1}{2}(x-5)^5 - 2$ ↓ 2 down

↓ Vertically shrink

5 right

points are shifting $x = 5$, $y = -2$

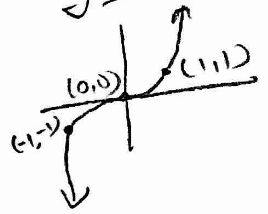
$(0,0) = (0+5, 0-2) = (5,-2)$

$(1,1) = (1+5, 1-2) = (6,-1)$

$(-1,-1) = (-1+5, -1-2) = (4,-3)$

Parent

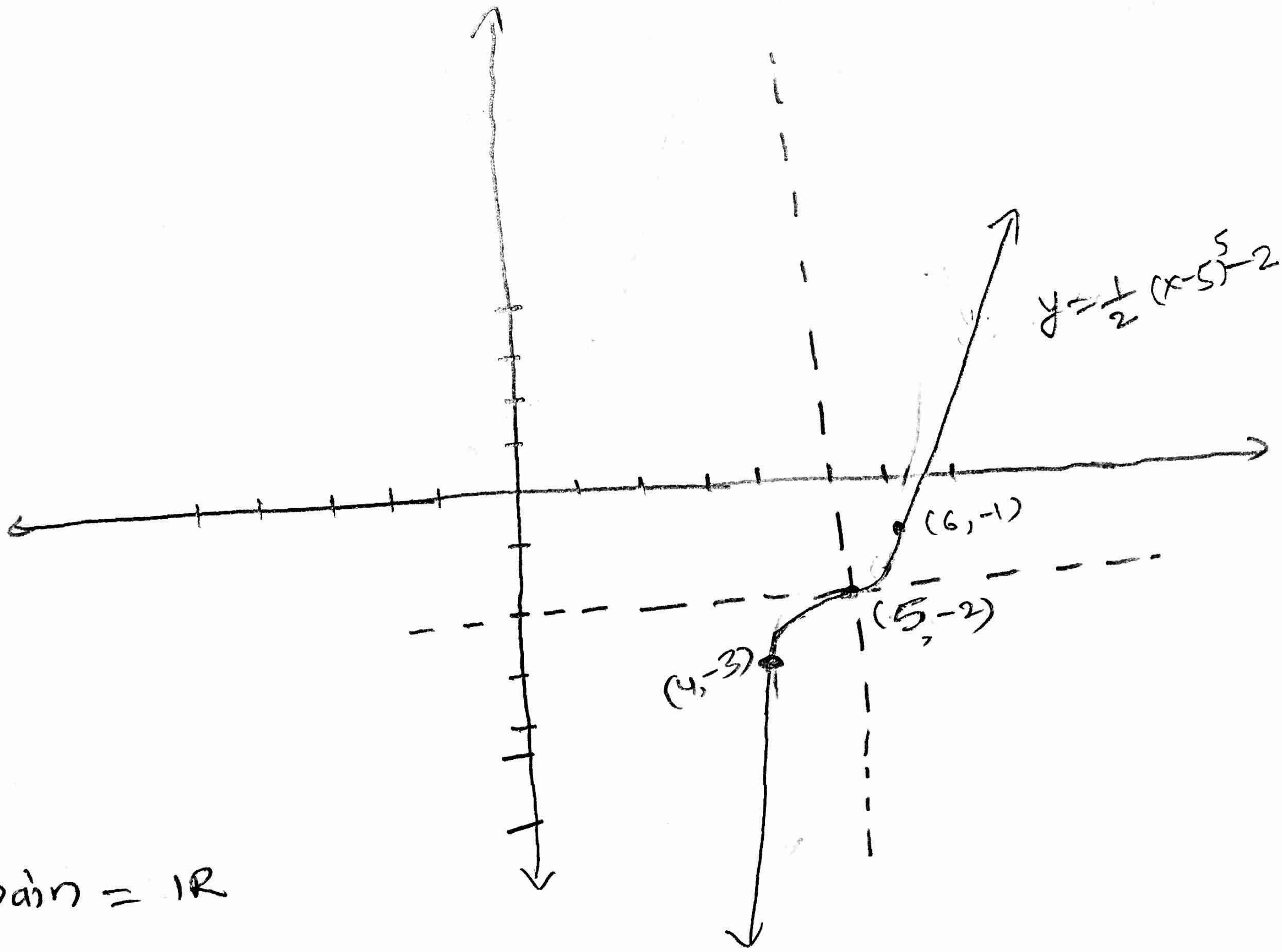
$y = x^5$



Left down
$x \to -\infty, y \to -\infty$

Right up
$x \to +\infty, y \to +\infty$

points =



Domain = $\mathbb{R}$

Range = $y = \mathbb{R}$

(c) $f(x) = -(x-2)^5 + 4$ ↑ 4 up

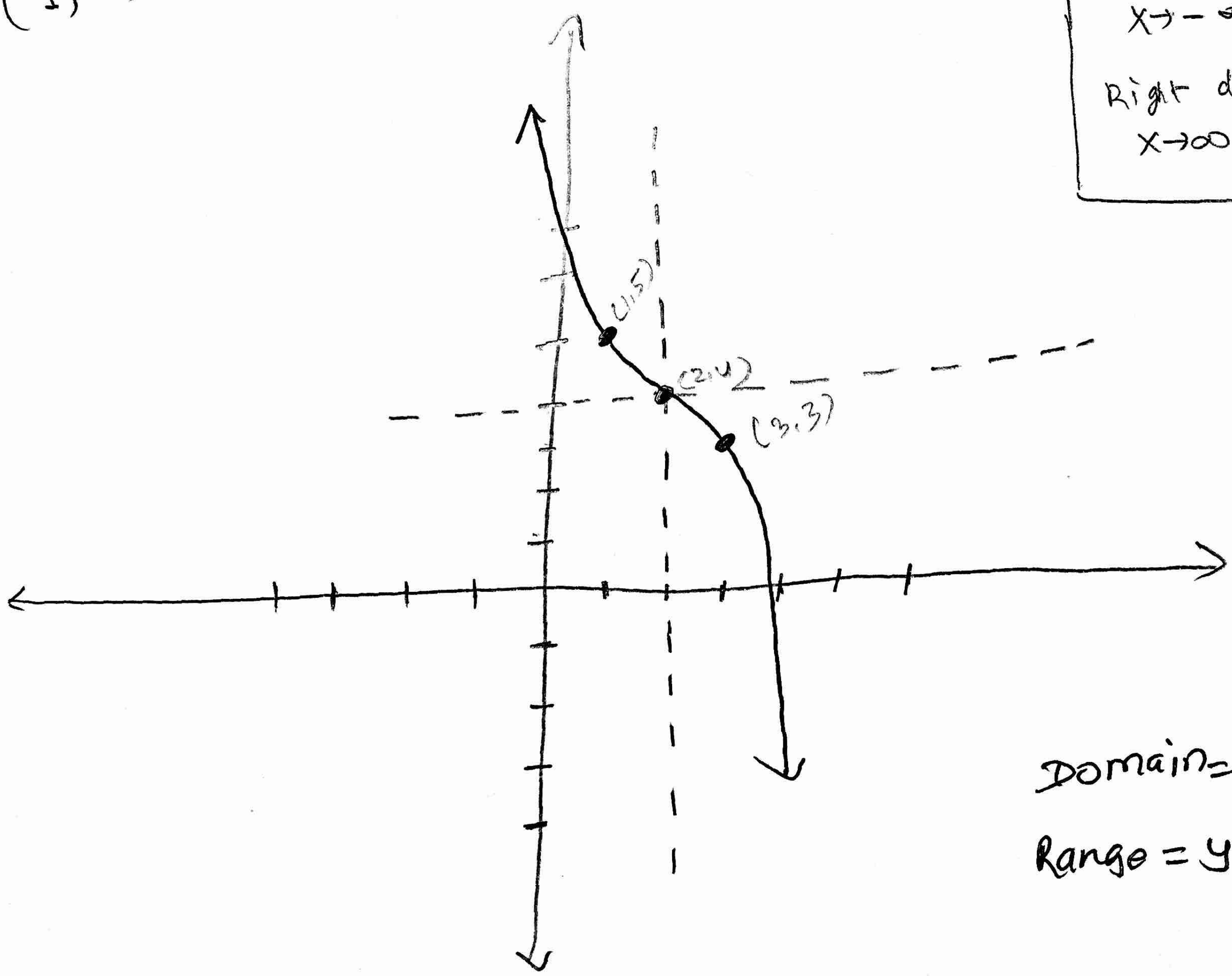
flipping 2 right

points are shifted $x = 2$
$y = 4$

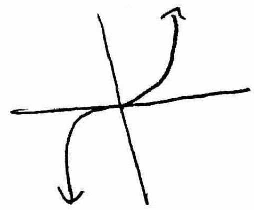
$(0,0) = (0+2, 0+4) = (2,4)$

$(-1,1) = (-1+2, 1+4) = (1,5)$

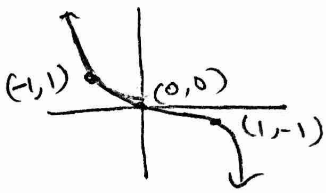
$(1,-1) = (1+2, -1+4) = (3,3)$



parent $f(x) = x^5$



flipping



points = (0,0)
(-1, 1)
(1,-1)

Left up
$x \to -\infty,\ y \to +\infty$

Right down
$x \to \infty,\ y \to -\infty$

Domain = $\mathbb{R}$

Range = $y = \mathbb{R}$

(d)

$f(x) = -2(x+1)^4 + 1$

v. stretching

flipping down

1 left

↑ 1 up

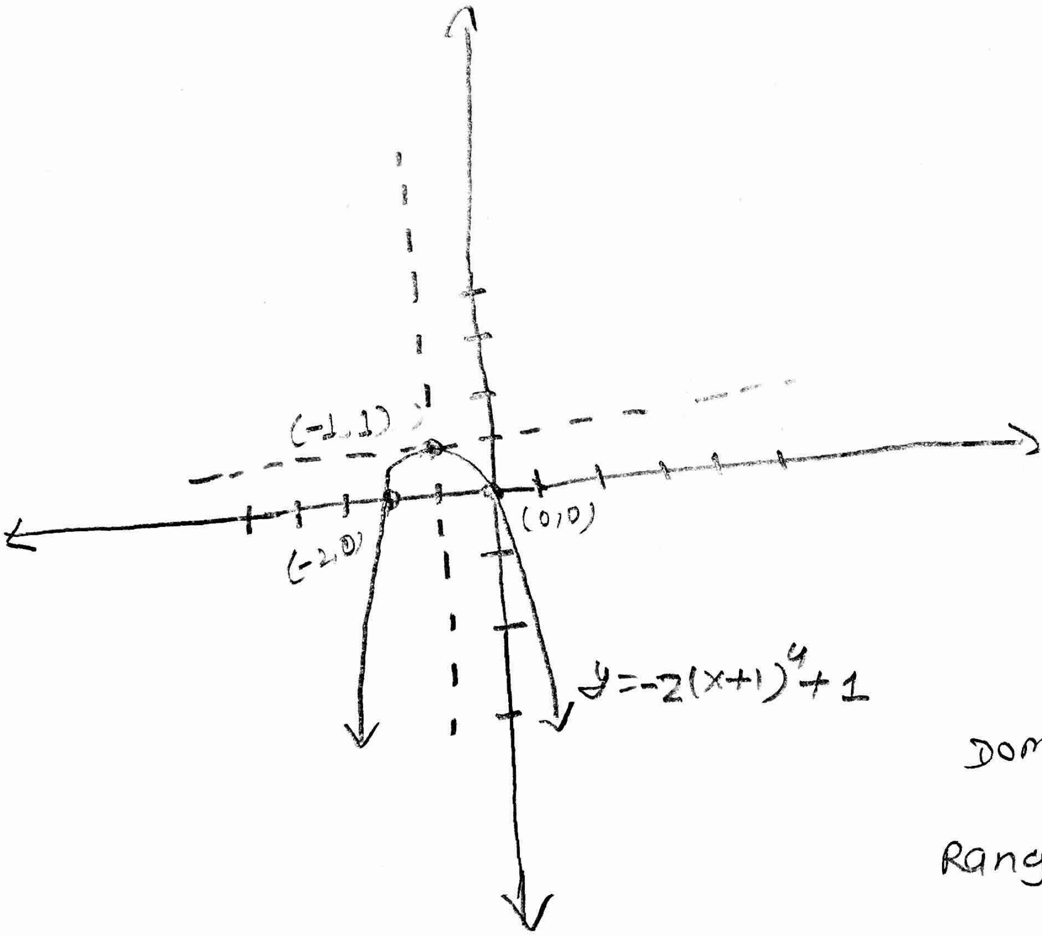
Both facing down

Shifting the points $x = -1$, $y = 1$

$(0,0) = (0-1, 0+1) = (-1,1)$

$(-1,-1) = (-1-1, -1+1) = (-2,0)$

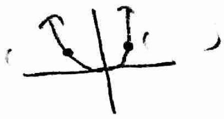
$(1,-1) = (1-1, -1+1) = (0,0)$



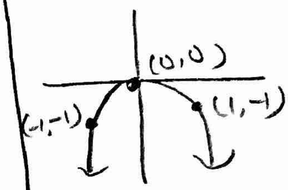
Domain = $\mathbb{R}$

Range = $y \le 1$

parent $f(x) = x^4$



flipping



points = (0,0)
(-1,-1)
(1,-1)

Facing down

13 Find the polynomial with given real zeros and degree chose any coefficient.

ⓐ zeros $-2, 2, 3$ and degree $= 3$

$\therefore$ Factors $(x+2), (x-2), (x-3)$

$\therefore$ $f(x) = (x+2)(x-2)(x-3)$

which is degree 3

ⓑ zeros -2 with multiplicity 2

zeros 4 with multiplicity 1

and degree $= 3$

factors $(x+2)^2, (x-4)^1$

$f(x) = (x+2)^2(x-4)$

which is degree 3.

14 For the following functions:

ⓐ $f(x) = 4(x^2+1)(x-2)^3$

ⓑ $f(x) = (x-5)^3(x+4)^2$ Find

(i) real zeros and their multiplicity

(ii) Determine where graph cross or touches (bounces back) at x-axis at the x-~~intercep~~ intercepts?

(iii) Find number of maximum turning point

(iv) Determine End behavior

(v) Graph it

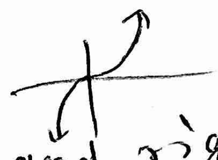
ⓐ $f(x) = 4(x^2+1)(x-2)^3$

coefficient $= +4$

Degree $= x^2 \cdot x^3 = x^5$

x^5 with positive coefficient

Should behave like x^5



left down and right up

(i) real zeros

$x^2+1 = 0$

None

$(x-2)^3 = 0$

$x-2 = 0$

$x = 2$ multiplicity 3

(ii) we have only one x-intercepts

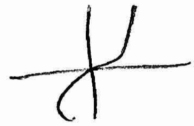
$x = 2$ which has power 3

$\Rightarrow$ it crosses

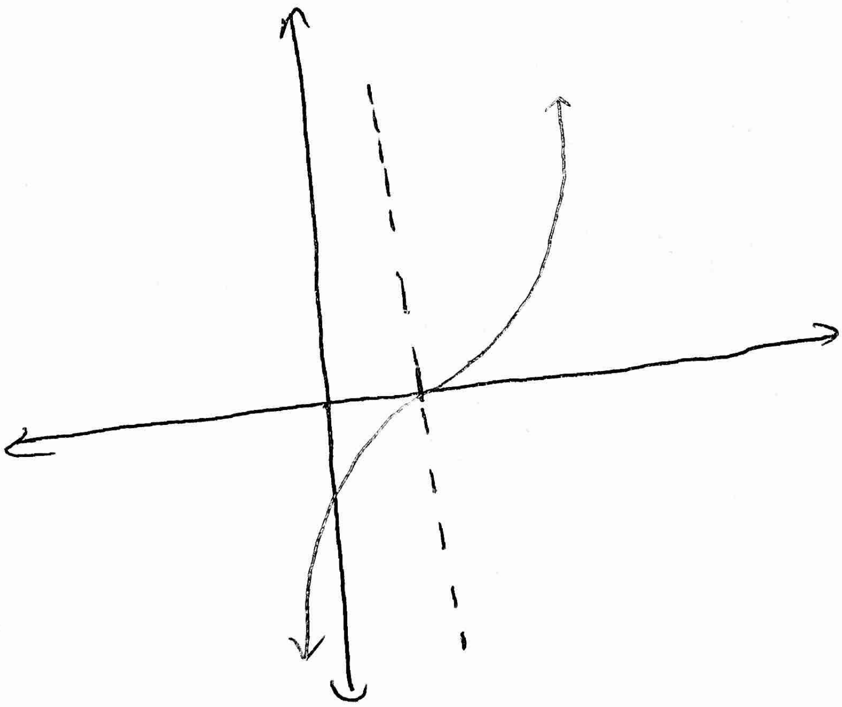
iii) maximum turning point = upto $5-1=4$

iv) End Behavior same like of $4x^5$

Left down
Right up.



(v)

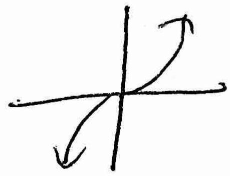


(b) $f(x) = (x-5)^3 (x+4)^2$

coefficient = +1

Degree $x^3 \cdot x^2 = x^5$

x^5 with positive coefficient should have same behavior as like of x^5



left down
right up

(i) real zeros

$(x-5)^3 = 0$
$x-5=0$
$x=5$

multiplicity = 3

$(x+4)^2 = 0$
$x+4=0$
$x=-4$

multiplicity = 2

(ii) $x=5$ has multiplicity 3 so graph crosses the x-axis

$x=-4$ has multiplicity 2 so it touches (bounces back) the x-axis

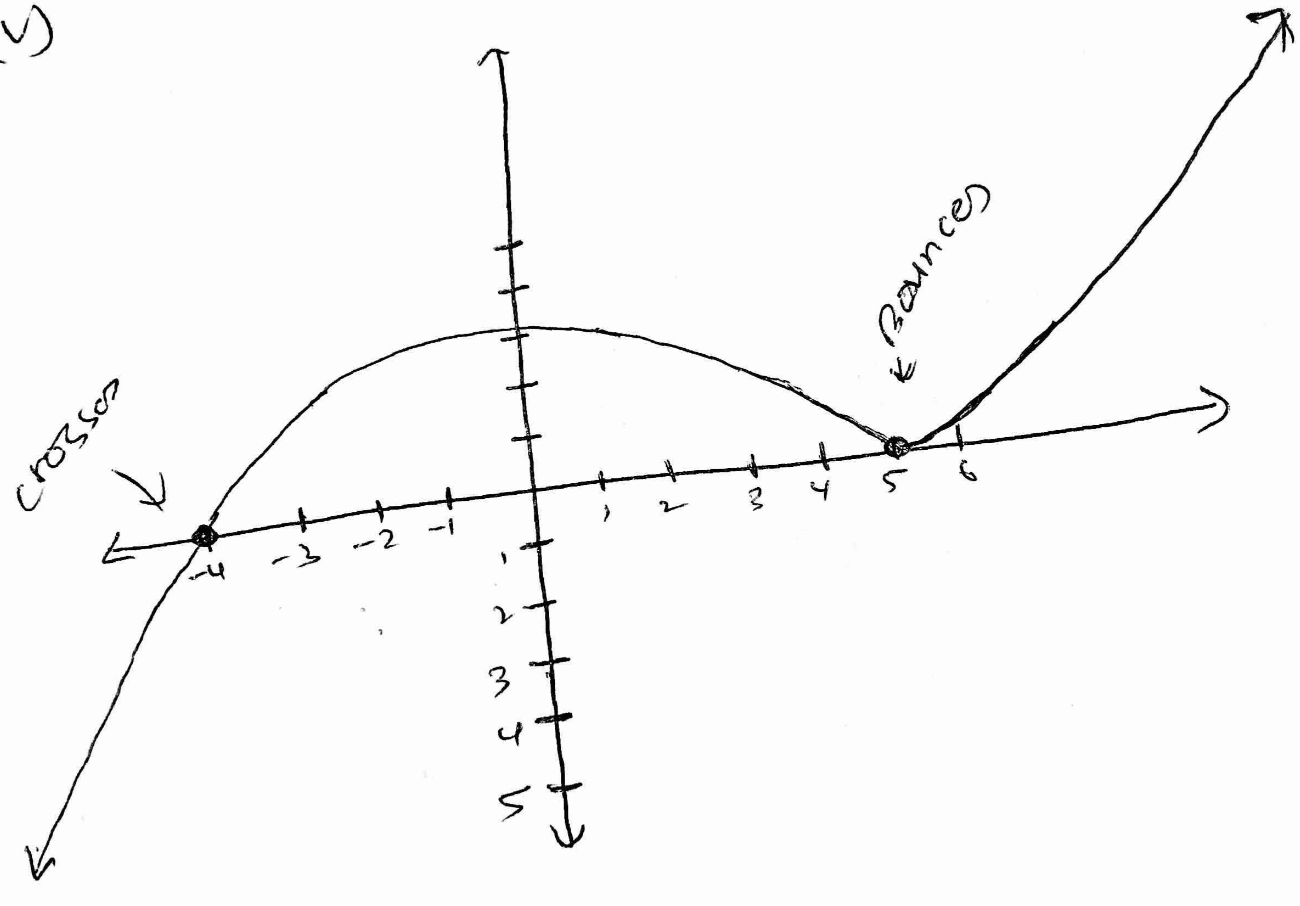
(iii) maximum turning points $= 5-1 = 4$

(iv) End behavior

$= $ same like of x^5 

left down
right up

(V)



15 For the following Rational functions Find vertical, horizontal and oblique Asymptotes.

ⓐ $f(x) = \dfrac{6}{(x+3)(4-x)}$ ⓑ $g(x) = \dfrac{3(x^2-x-6)}{4(x^2-9)}$

Sol^n:

ⓐ $f(x) = \dfrac{6}{(x+3)(4-x)}$

Vertical Asymptotes: $(x+3)(4-x) = 0$

$x+3=0$ or $4-x=0$

$x=-3$ $\quad$ $4=x$

V.A $\begin{cases} x=-3 \\ x=4 \end{cases}$

Horizontal Asymptotes:

Since down power = 2

upper power = 0

H.A $\quad y = 0$

No oblique Asymptotes.

(b) $g(x) = \dfrac{3(x^2-x-6)}{4(x^2-9)} = \dfrac{3x^2-3x-6}{4x^2-36}$

vertical Asymptotes

$4x^2-36=0$

$4x^2=36$

$x^2=\dfrac{36}{4}=9$

$x=\pm 3$

$\therefore$ V.A $\begin{cases} x=3 \\ x=-3 \end{cases}$

Horizontal Asymptotes

upper power $= 2$

Lower power $= 2$

Same power

H.A $y = \dfrac{\text{coefficient of } x^2}{\text{Coefficient of } x^2} = \dfrac{3}{4}$

No oblique Asymptotes.