1. (3 points) Find the product $(5 + 3i)(2 - i)$.

$$= 10 - 5i + 6i - 3i^2$$
$$= 10 + i - 3(-1)$$
$$= 10 + i + 3$$
$$= 13 + i$$

2. (5 points) Find the quotient and write down the answer in Standard form $\frac{2+3i}{1-i}$.

$$\frac{2+3i}{1-i} \times \frac{1+i}{1+i}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$= \frac{2 + 2i + 3i + 3i^2}{1^2 - (i)^2}$$

$$= \frac{2 + 5i + 3(-1)}{1 - (-1)}$$

$$= \frac{-1 + 5i}{2}$$

3. (20 points) Solve the following equation by the indicated techniques

(a) $x^2 = 36$ by square root

$$x = \pm \sqrt{36}$$
$$\boxed{x = \pm 6}$$

(b) $3x^2 + 5x + 2 = 0$ factoring

$3 \times 2 = 6$

$$3x^2 + 3x + 2x + 2 = 0$$
$$3x(x+1) + 2(x+1) = 0$$
$$(x+1)(3x+2) = 0$$
$$x+1 = 0 \quad \text{or} \quad 3x+2 = 0$$

$3 \quad 2$

$x = -1 \quad \text{or} \quad 3x = -2$

$x = -2/2$

$\therefore x = -1 \text{ or } -2/3$

(c) $x^2 - 6x = 13$ by completing square

$$x^2 - 6x + 3^2 = 13 + 3^2$$

$$(x-3)^2 = 22$$

$$(x-3) = \pm \sqrt{22}$$

$$x = 3 \pm \sqrt{22}$$

(d) $x^2 - 6x + 10 = 0$ by quadratic formula

comparing this with $ax^2 + bx + c = 0$

$$a = 1 \qquad b = -6 \qquad c = 10$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$

$$= \frac{6 \pm \sqrt{36 - 40}}{2}$$

$$= \frac{6 \pm \sqrt{-4}}{2}$$

$$= \frac{6 \pm 2i}{2} = \frac{2(3 \pm i)}{2}$$

$$= 3 \pm i \qquad \#$$

4. (20 points) Word problems

(a) (5 points) A wool suit, discounted by 30% for a clearance sale, has a price tag of $399. What was the suit's original price?

Let the price of suit before discount $= x$

then discount $= 30\% \text{ of } x = \dfrac{80x}{100}$

Actual price $=$ original price $-$ discount

$$399 = x - \dfrac{30x}{100} = x - 0.3x$$

$$399 = 0.7x$$

$$x = \dfrac{399}{0.7} = 570$$

$\therefore \boxed{x = 570}$

(b) (7 points) A bank loaned out $12,000$, part of it at the rate 8% per year and the rest at the rate of 18% per year. If the interest received totaled $1000, how much was loaned at 8% ?

Total money $= 12,000$

Let Bank $A = x$

then Bank $B = 12000 - x$ (rest after x)

A

$8\% = 0.08$

x

Interest $= 0.08x$

B

$18\% = 0.18$

$12000 - x$

$0.18(12000 - x)$

$= 2160 - 0.18x$

But total earned

$$0.08x + 2160 - 0.18x = 1000$$

$$2160 - 0.1x = 1000$$

$$-0.1x = -1160$$

$x = \dfrac{-1160}{-0.1} = 11600$

$\therefore$ Bank $A = 11600$

Bank $B = 400$

(c) (8 points) A coffee manufacturer wants to market a new blend of coffee that sells for \$3.90 per pound by mixing two coffee that sells for \$2.75 and \$5 per pound, respectively. What amounts of each coffee should be blended to obtain the desire mixture of 100 pound?

Bran A   Brand B   mixed

\$2.75   \$5   \$3.90

X   100-X   100

2.75x   5(100-x)   390

$2.75x + 500 - 5x = 390$

$500 - 2.25x = 390$

$-2.25x = -110$

$x = \dfrac{-110}{-2.25}$

$x = 49$

∴ Blend $A = x = 49$

Blend $B = 100 - 49 = 51$

5. (23 points) Word problems

   (a) (7 points)

$$\sqrt{3x+1} - \sqrt{x-1} = 2$$

$$\sqrt{3x+1} = 2 + \sqrt{x+1}$$

squaring on both sides

$$(\sqrt{3x+1})^2 = (2 + \sqrt{x+1})^2$$

$$3x+1 = 2 + 4\sqrt{x+1} + (\sqrt{x+1})^2$$

$$3x+1 = 2 + 4\sqrt{x+1} + x+1$$

$$3x+1 = 3 + 4\sqrt{x+1} + x$$
$$-x$$

$$2x+1 = 3 + 4\sqrt{x+1}$$

$$2x-2 = 4\sqrt{x+1}$$

dividing by 2

$$x-1 = 2\sqrt{x+1}$$

squaring on both sides

$$(x-1)^2 = 4(x+1)$$

$$x^2 - 2x + 1 = 4x + 4$$

$$x^2 - 6x + 1 - 4 = 0$$

$$x^2 - 6x = 0$$

   (b) (5 points)

$$2(s+1)^2 - 5(s+1) = 3$$

Let $s+1 = t$ $\Rightarrow$ $2t^2 - 5t - 3 = 0$

$$2t^2 - 6t + t - 3 = 0$$

$$2t(t-3) + 1(t-3) = 0$$

$$(t-3)(2t+1) = 0$$

$$t - 3 = 0 \quad \sim \quad 2t+1 = 0$$

$$t = 3 \quad \sim \quad 2t = -1$$

$$s+1 = 3 \quad or \quad t = -\frac{1}{2}$$

$$\therefore \quad s = 2 \quad or \quad s+1 = -\frac{1}{2}$$

$$s = -\frac{1}{2} - 1 = -\frac{3}{2}$$

$$\begin{array}{c} -6 \\ \frown \\ \boxed{-6 \quad 1} \\ 6 \quad -1 \end{array}$$

   (c) (5 points)

$$\left|\frac{3x-2}{2x-3}\right| = 2$$

$$\frac{3x-2}{2x-3} = 2 \qquad\qquad \frac{3x-2}{2x-3} = -2$$

$$3x - 2 = 2(2x - 3)$$
$$3x - 2 = 4x - 6$$
$$3x - 4x = -6 + 2$$
$$-x = -4$$
$$x = 4$$

$$3x - 2 = -2(2x - 3)$$
$$3x - 2 = -4x + 6$$
$$3x + 4x = 6 + 2$$
$$7x = 8$$
$$x = \frac{8}{7}$$

(d) (6 points)

$$\frac{4}{x-2} = \frac{-3}{x+5} + \frac{7}{(x+5)(x-2)} \qquad LCD = (x-2)(x+5)$$

$$4(x+5) = -3(x-2) + 7$$

$$4x + 20 = -3x + 6 + 7$$

$$4x = -3x + 13 - 20$$

$$4x + 3x = -7$$

$$7x = -7$$

$$\boxed{x = -1}$$

6. (12 points) Solve the following inequality

(a) (4 points)

$$8 - 4(2-x) \leq -2x$$

$$8 - 8 + 4x \leq -2x$$

$$0 + 4x \leq -2x$$

$$4x + 2x \leq 0$$

$$6x \leq 0$$

$$x \leq 0$$

$$(-\infty, 0]$$

(b) (4 points)

$$|3t - 2| \leq 4$$
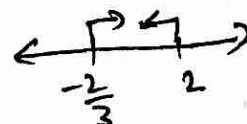
$$3t - 2 \leq 4 \qquad -(3t-2) \leq 4$$

$$3t \leq 4 + 2 \qquad -3t + 2 \leq 4$$

$$t \leq \frac{6}{3} \qquad -3t \leq 2$$

$$t \leq 2 \qquad t \geq -\frac{2}{3}$$

(c) (4 points)

$$|2 - 3x| > 1$$

$$2 - 3x > 1 \qquad -(2-3x) > 1$$

$$-3x > 1-2 \qquad -2+3x > 1$$

$$-3x > -1 \qquad 3x > 3$$

$$x < \frac{1}{3} \qquad x > 1$$

$$\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$$

7. (17 points) (a) (7 points) Determine whether or not the three are the vertices of a right angle triangle?

$$A(-2,5), B(12,3), C(10,-11)$$

Here, $AB = \sqrt{(12+2)^2 + (3-5)^2} = \sqrt{14^2 + (2)^2} = \sqrt{196+4} = \sqrt{200}$

$BC = \sqrt{(10-12)^2 + (-11-3)^2} = \sqrt{(-2)^2 + (-14)^2} = \sqrt{4+196} = \sqrt{200}$

$AC = \sqrt{(10+2)^2 + (-11-5)^2} = \sqrt{12^2 + (-16)^2} = \sqrt{144+256} = \sqrt{400}$

$$AB^2 = 200$$
$$BC^2 = 200$$
$$AC^2 = 400$$

$$\therefore \quad AB^2 + BC^2 = AC^2$$

Then by pythagorean $\triangle ABC$ is a right angle triangle.

(b) (5 points) Determine the domain intercepts and symmetry fro the following equation

$$y = \frac{3x}{x^2 + 9}$$ domain $D = \mathbb{R}$

x-intercept ($y = 0$)

$$\frac{3x}{x^2+9} = 0 \Rightarrow 3x = 0 \Rightarrow x = 0$$

y-intercept ($x = 0$) $y = \frac{3 \cdot 0}{0^2 + 9} = 0$

Symmetry

About y-axis: y is not even
function so it is not symmetry about y-axis

it is ~~not~~ symmetry About origin
~~because i~~

To check symmetry about x-axis (put $y \to -y$)

$-y = \frac{3x}{x^2+9}$ it is

different so it is not symmetry about x-axis #

$-y = \frac{-3x}{(-x)^2+9} \Rightarrow y = \frac{3x}{x^2+9}$

(c) (5 points) Find at least 5 ordered pairs that are solutions, and graph the equation

$$y = -x^2 + 9$$

x	$y = -x^2 + 9$	point (x, y)
0	$y = -0^2 + 9 = 9$	$(0, 9)$
1	$y = -1^2 + 9 = 8$	$(1, 8)$
2	$y = -2^2 + 9 = 5$	$(2, 9)$
-1	$y = -(-1)^2 + 9 = 8$	$(-1, 8)$
-2	$y = -(-2)^2 + 9 = 5$	$(-2, 5)$

x intercept

$0 = -x^2 + 9$

$x^2 = 9$

$x = \pm 3$

y-intercept

$y = 0 + 9$

$= 9$
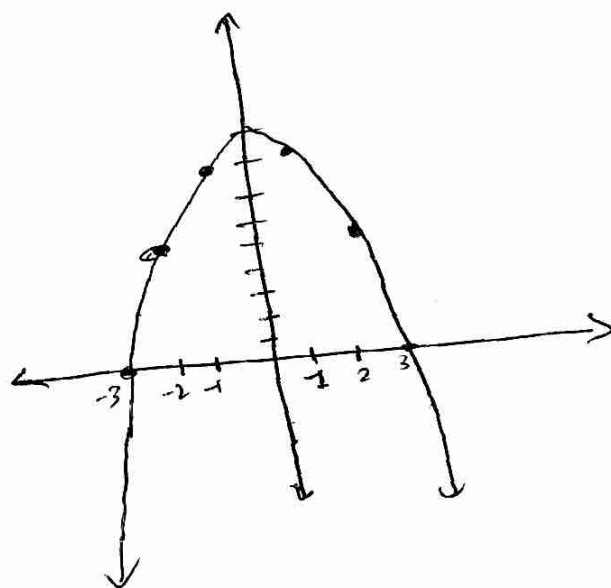
Bonus

8. (5 points) Solve the inequality

(Hint: use sign test)

$$(2x - 3)(x + 2)(x - 3) \geq 0$$

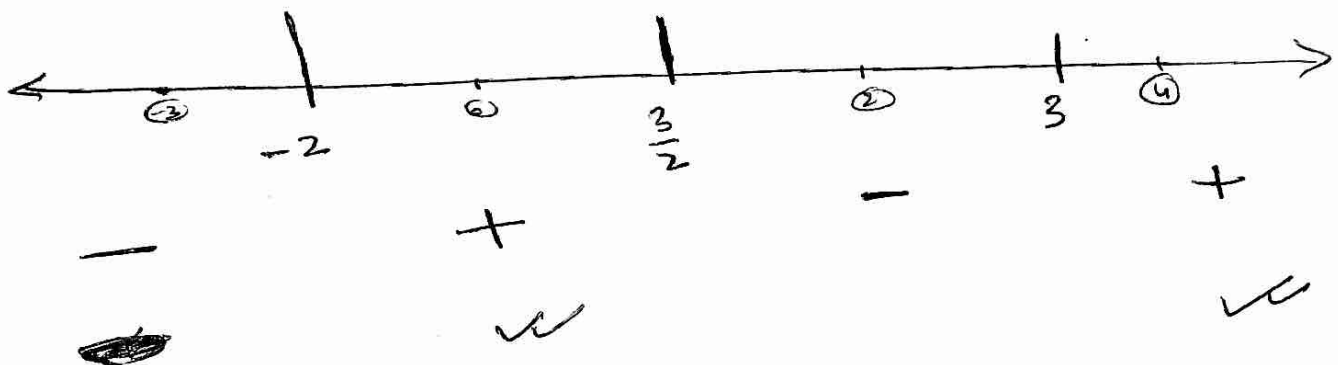$$2x - 3 = 0 \qquad x + 2 = 0 \qquad x - 3 = 0$$
$$2x = 3 \qquad x = -2 \qquad x = 3$$
$$x = \frac{3}{2}$$

$$[-2, \tfrac{3}{2}] \cup [3, \infty)$$