
November 27, 2023

Where we are going with the class

- Complete Assn. 7 today
- Assn 8 - Using Jacobi to compute iteration solns to $Ax = b$
- Assn 9 - Speed code up using OpenMP

Iterative methods to computing solutions: Jacobi iteration & more

let: $A = L + D + U$

$Ax = b \rightarrow (L + D + U)x = b \rightarrow Dx = b - (L + U)x$

$x = D^{-1}(b - (L + U)x)$

- L: A lower triangular matrix with zeros on the diagonal
- D: A Matrix with non-zero values only in the diagonal
- U: An upper triangular matrix with zeros on the diagonal

$x^{k+1} = D^{-1}(b - (L + U)x^k)$

$x^{k+1} \rightarrow Ax^* = b$

$x^{k+1} \rightarrow x^*$

Note: A is diagonally dominant

Implementation for Jacobi

$Ax + b \rightarrow x^{k+1} = D^{-1}(b - (L + U)x^k) \rightarrow x^*$

input: $A \in \mathbb{R}^{n \times m}$, $b \in \mathbb{R}^n$, $x^0 \in \mathbb{R}^n$, tol maxIter

initialize: error = 10 * tol; iter = 0;

loop:

```
while (error > tol && iter < maxIter){
for(int i = 0; i < n; i++){
double sum = b[i];
for(int j = 0; j < n; j++){
sum += a[i,j] * x[j];
for(int j = i + 1; j < n; j++){
sum += a[i,j] * x[j];
}
x[i] = sum/a[i,i];
double error = 0.0;
for(int i = 0; i < n; i++){
double value = x[i] - x0[i];
error += val * val;
}
error = sqrt(error);
iter++;
}
```

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Guass-Seidel Iteration

$A = (L + D + U) \rightarrow Ax = b$

$(D + U)x = b - Lx$

$$\begin{aligned}
x &= (D + U)^{-1}(b - Lx) \\
x^{(k+1)} &= (D + U)^{-1}(b - Lx^k) \\
(D + U)^{-1}x & \text{ (Use back substitution to obtain } (D + U)^{-1}) \\
\text{Residual formula: } x^{k+1} &= (D + U)^{-1}(b - (L + D + U - D - U)x^k) \\
&\rightarrow (D + U)^{-1}(b - Ax^k + (D + U)x^k) \rightarrow x^k + (D + U)^{-1}r^k \\
r^k &= b - Ax^k
\end{aligned}$$

Example - Leslie Matrix

$$L = \begin{bmatrix} f_0 & f_1 & \dots & f_n \\ s_0 & 0 & \dots & 0 \\ 0 & s_1 & \dots & 0 \\ \dots & & & \\ 0 & \dots & s_{n-1} & 0 \end{bmatrix}$$

$$LN_0 \rightarrow N_1 \rightarrow N_0 = L^{-1}N$$

We can decompose this matrix into three separate matrices L, U, D . We cannot apply iterative methods to finding solutions for this decomposition (or the original matrix) because there are too many zero values, which have an inverse of ∞ .

Time complexity of iterative solvers

- GE + BS $\approx O(n^3) + O(n^2) \rightarrow O(n^3)$
- LU + FS + BS $\approx O(n^3) + O(n^2) + O(n^2)$
- Jacobi $\approx O(n^2)$ / iteration
- Gauss-Seidel $\approx O(n^2)$ / iteration

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Gauss-seidel elimination cont.

$$x^{k+1} = x^k + (D + U)^{-1}r^k$$
