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Where we are going with the class

- Complete Assn. 7 today
- Assn 8 - Using Jacobi to compute iteration solns to $Ax = b$
- Assn 9 - Speed code up using OpenMP

Iterative methods to computing solutions: Jacobi iteration & more
let: $A = L + D + U$
$Ax = b \to (L + D + U)x = b \to Dx = b - (L + U)x$
$x = D^{-1}(b - (L + U)x)$

- L: A lower triangular matrix with zeros on the diagonal
- D: A Matrix with non-zero values only in the diagonal
- U: An upper triangular matrix with zeros on the diagonal

$x^{k+1} = D^{-1}(b - (L + U)x^k)$
$x^{k+1} \to Ax^* = b$
$x^{k+1} \to x^*$
Note: A is diagonally dominant

Implementation for Jacobi
$Ax + b \to x^{k+1} = D^{-1}(b - (L + U)x^k) \to x^*$
input: $A \in \mathbb{R}^{nxm}, b \in \mathbb{R}^n, x^0 \in \mathbb{R}^n$, tol maxIter
initialize: error = 10 * tol; iter = 0;

loop:
while (error > tol && iter < maxIter){
for(int i = 0; i ¡ n; i++){
double sum = b_i;
for(int j = 0; j ¡ n; j++){
sum += $a_{i,j} * x_0$;
for(int j = i + 1; j ¡ n; j++){
sum += $a_{i,j} * x_0$}
$x_i = sum/a_{i,i};$}
double error = 0.0;
for(int i = 0; i ¡ n; i++){
double value = $x1_i - x0_i$;
$error+ = val + val_i$}
error = sqrt(error);
iter++;}

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Guass-Seidel Iteration
$A = (L + D + U) \to Ax = b$
$(D + U)x = b - Lx$

$x = (D + U)^{-1}(b - Lx)$
$x^{(k+1)} = (D + U)^{-1}(b - Lx^k)$
$(D + U)^{-1}x$ (Use back substitution to obtain $(D + U)^{-1}$)
Residual formula: $x^{k+1} = (D + U)^{-1}(b - (L + D + U - D - U)x^k)$
$\rightarrow (D + U)^{-1}(b - Ax^k + (D + U)x^k) \rightarrow x^k + (D + U)^{-1}r^k$
$r^k = b - Ax^k$

Example - Leslie Matrix

$$Ł = \begin{bmatrix} f_0 & f_1 & ... & f_n \\ s_0 & 0 & ... & 0 \\ 0 & s_1 & ... & 0 \\ ... & & & \\ 0 & ... & s_{n-1} & 0 \end{bmatrix}$$

$LN_0 \rightarrow N_1 \rightarrow N_0 = L^{-1}N$
We can decompose this matrix into three separate matrices L, U, D. We cannot apply iterative methods to finding solutions for this decomposition (or the original matrix) because there are too many zero values, which have an inverse of ∞.

Time complexity of iterative solvers

- GE + BS $\approx O(n^3) + O(n^2) \rightarrow O(n^3)$
- LU + FS + BS $\approx O(n^3) + O(n^2) + O(n^2)$
- Jacobi $\approx O(n^2)$/ iteration
- Guass-Seidel $\approx O(n^2)$/ iteration
