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Where we are going with the class

- Complete Assn. 7 today
- Assn 8 - Using Jacobi to compute iteration solns to $Ax = b$
- Assn 9 - Speed code up using OpenMP

Iterative methods to computing solutions: Jacobi iteration & more
let: $A = L + D + U$
$Ax = b \to (L + D + U)x = b \to Dx = b - (L + U)x$
$x = D^{-1}(b - (L + U)x)$

- L: A lower triangular matrix with zeros on the diagonal
- D: A Matrix with non-zero values only in the diagonal
- U: An upper triangular matrix with zeros on the diagonal

$x^{k+1} = D^{-1}(b - (L + U)x^{k})$
$x^{k+1} \to Ax^{*} = b$
$x^{k+1} \to x^{*}$
Note: A is diagonally dominant

Implementation for Jacobi
$Ax + b \to x^{k+1} = D^{-1}(b - (L + U)x^{k}) \to x^{*}$
input: $A \in \mathbb{R}^{n \times m}, b \in \mathbb{R}^{n}, x^{0} \in \mathbb{R}^{n}$, tol maxIter
initialize: error = 10 * tol; iter = 0;

loop:
while (error > tol && iter < maxIter){
for(int i = 0; i ¡ n; i++){
double sum = b_i;
for(int j = 0; j ¡ n; j++){
sum += $a_{i,j} * x_0$;
for(int j = i + 1; j ¡ n; j++){
sum += $a_{i,j} * x_0$}
$x_i = sum/a_{i,i};$}
double error = 0.0;
for(int i = 0; i ¡ n; i++){
double value = $x1_i - x0_i$;
$error + = val + val_i$}
error = sqrt(error);
iter++;}

November 29, 2023

Guass-Seidel Iteration
$A = (L + D + U) \to Ax = b$
$(D + U)x = b - Lx$

$x = (D+U)^{-1}(b - Lx)$
$x^{(k+1)} = (D+U)^{-1}(b - Lx^k)$
$(D+U)^{-1}x$ (Use back substitution to obtain $(D+U)^{-1}$)
Residual formula: $x^{k+1} = (D+U)^{-1}(b - (L+D+U-D-U)x^k)$
$\to (D+U)^{-1}(b - Ax^k + (D+U)x^k) \to x^k + (D+U)^{-1}r^k$
$r^k = b - Ax^k$

Example - Leslie Matrix

$$Ł = \begin{bmatrix} f_0 & f_1 & ... & f_n \\ s_0 & 0 & ... & 0 \\ 0 & s_1 & ... & 0 \\ ... & & & \\ 0 & ... & s_{n-1} & 0 \end{bmatrix}$$

$LN_0 \to N_1 \to N_0 = L^{-1}N$
We can decompose this matrix into three separate matrices L, U, D. We cannot apply iterative methods to finding solutions for this decomposition (or the original matrix) because there are too many zero values, which have an inverse of ∞.

Time complexity of iterative solvers

- GE + BS $\approx O(n^3) + O(n^2) \to O(n^3)$
- LU + FS + BS $\approx O(n^3) + O(n^2) + O(n^2)$
- Jacobi $\approx O(n^2)/$ iteration
- Guass-Seidel $\approx O(n^2)/$ iteration

December 1, 2023

Guass-seidel elimination cont.

$x^{k+1} = x^k + "(D+U)^{-1}"r^k$
$x^0, x^1, x^2, ... \to Ax^k = b$

LU-factorization
Let $A \in \mathbb{R}^{nxm}$
note: A is diagonally dominant

some table koebbe wrote on the board

LU-factorization	jacobi	guass-seidel
num. flops and error	num. flops and error	**ditto**

We need to compare and constrast the results of these metrics to determine which approach is best given the input.

Overview of upcoming homework

- hw08: due at the end of finals week
- Using OpenMP to parallelize 2 linear solving algorithms (will use code from hw08). Due at end of finals week