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Vandermonde matrix

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ (x_1 - x) & (x_2 - x) & (x_3 - x) & \dots & (x_n - x) \\ (x_1 - x)^2 & (x_2 - x)^2 & (x_3 - x)^2 & \dots & (x_n - x)^2 \\ \dots & \dots & \dots & \dots & \dots \\ (x_1 - x)^n & (x_2 - x)^n & (x_3 - x)^n & \dots & (x_n - x)^n \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \dots \\ 0 \end{bmatrix}$$

- We can move the 1 in the matrix in the vector on the RHS to compute different derivatives.
- $n > k + 1$ is required to get a good approximation of the derivative

Example: $D_0 u(x) = \dfrac{u(x+h) - u(x-h)}{2h}$

Let $k = 1$, then we rewrite our approximation as

$$C_1 u(x+h) + C_0(x) + C_{-1} a(x-h)$$

Now that we have rephrased our approximation, we can see $n = 3$ (where n is the number of coefficients)

Chapter 2: Building methods for solving DEs

Example: Heat equation. Consider the flow of heat in a 1D rod of material that conducts heat. What happens as time progresses with the rod in regards to its temperature. Let the length of the rod be $L = 1$.
The DE used to heat in relation to time is

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(k(x)\frac{\partial u}{\partial x} = f(x))$$

If k is constant, then

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + f(x)$$

Steady state

$$\frac{\partial u}{\partial t} = 0$$

$$\Rightarrow k\frac{\partial^2 u}{\partial x^2} - f(x) = 0$$

$$\frac{d^2 u}{dx^2} = f(x) \Rightarrow u'' = f$$

$$\Rightarrow u(0) = \alpha, u(1) = \beta$$

Structured Stability

$$\begin{cases} y'' + \frac{P}{EI} y = 0 \\ y(0) = 0 \\ y(c) = 0 \end{cases}$$

this problem has ∞ solutions, so it is a bad idea to use numerical methods on this DE. We will want to use finite difference methods to get around this.

Finite Methods

$$u'' = f(x) \to u' = \int f(x)dx + C_1 = g(x) + C_1$$

$$u = \int g(x)dx + C_1 x + C_2$$

$$u'' = \frac{u(x+h) - 2a(x) + u(x+h)}{h^2} + E$$

Need some points for our linear approx: take equally spaced points on $[0,1]$

$$h = \frac{1-0}{m} \Rightarrow h = 1/m$$
