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Topics we will discuss in this class

- Finite Difference Methods (First 10 weeks; 5620)
  - Steady state problems
  - initial problems
- Build your own simulation (Last 5 weeks; 5910)

Chapter 1: Finite Difference Approximation

Example: $\frac{dA}{dt} = kA, A(0) = A_0$

$\frac{1}{A}dA = kdt \rightarrow \ln|A| = kt + c_0$

$A(t) = e^{kt+c_0} = c_1 e^{kt} \rightarrow c_1 = A_0$

$\boxed{A(t) = A_0 e^{kt}}$

Example: $\frac{dP}{dt} = \alpha P - \beta P^2, P(0) = P_0$

Example - spring mass system: $\frac{d^2y}{dt^2} = \alpha\frac{dy}{dt} + ky = f(t), y(0) = y_0, \frac{dy}{dt}(0) = v_0$

Example: $y'' + \frac{P}{EI}y = 0, y(0) = 0, y(L) = 0$

This example has the solution: $\boxed{y(x) = c_1\cos(\lambda x) + c_2\sin(\lambda x)}$,
$assuming y(L) = c_2\sin(\lambda L) = 0, c_2 = 0, \sin(\lambda L) = 0, \lambda L = n\pi$.

This means $y_n(x) = c_2\sin(\frac{\pi n}{L}x)$

Example - Heat equation: $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$,
$u(x,0) = f(x), u(o,t) = g_1(t), u(',l) = g_2(l)$

Example - Potential Equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \Delta u = f(x,y)$
This a BVP, 2nd order, partial diff eq.

Example - Conservation Laws: $\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$
$u(x,o) = f(x), x \in \mathbb{R}$
Example - Cahn-Hilliard Equation: we just looked at the wikipedia page lol.

Definitions

- $f'(a) \approx \frac{f(x)-f(a)}{x-a}$. The difference quotient
- Increment Notation: $x - a = h \rightarrow x = a + h \rightarrow f'(a) \approx \frac{f(a+h)-f(a)}{h}$

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