

PD IEC/TR 60865-2:2015**BSI Standards Publication**

Short-circuit currents — Calculation of effects

Part 2: Examples of calculation

National foreword

This Published Document is the UK implementation of IEC/TR 60865-2:2015. It supersedes PD 6575-2:1995 which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee PEL/73, Short circuit currents.

A list of organizations represented on this committee can be obtained on request to its secretary.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

© The British Standards Institution 2015.

Published by BSI Standards Limited 2015

ISBN 978 0 580 84957 2

ICS 17.220.01; 29.020; 29.240.20

Compliance with a British Standard cannot confer immunity from legal obligations.

This Published Document was published under the authority of the Standards Policy and Strategy Committee on 31 May 2015.

Amendments/corrigenda issued since publication

Date	Text affected



IEC TR 60865-2

Edition 2.0 2015-04

TECHNICAL REPORT

**Short-circuit currents – Calculation of effects –
Part 2: Examples of calculation**

Licensed copy: Imperial College London, Version correct as of 02/05/2016

INTERNATIONAL
ELECTROTECHNICAL
COMMISSION

ICS 17.220.01; 29.240.20

ISBN 978-2-8322-2551-6

Warning! Make sure that you obtained this publication from an authorized distributor.

CONTENTS

FOREWORD	5
1 Scope	7
2 Normative references	7
3 Symbols and units	7
4 Example 1 – Mechanical effects on a 10 kV arrangement with single rigid conductors	8
4.1 General	8
4.2 Data	9
4.3 Normal load case: Conductor stress and forces on the supports caused by dead load	9
4.4 Exceptional load case: Effects of short-circuit currents	10
4.4.1 Maximum force on the central main conductor	10
4.4.2 Conductor stress and forces on the supports	11
4.5 Conclusions	13
5 Example 2 – Mechanical effects on a 10 kV arrangement with multiple rigid conductors	14
5.1 General	14
5.2 Data (additional to the data of Example 1)	14
5.3 Normal load case: Conductor stress and forces on the supports caused by dead load	15
5.4 Exceptional load case: Effects of short-circuit currents	15
5.4.1 Maximum forces on the conductors	15
5.4.2 Conductor stress and forces on the supports	16
5.5 Conclusions	20
6 Example 3. – Mechanical effects on a high-voltage arrangement with rigid conductors	20
6.1 General	20
6.2 Data	21
6.3 Normal load case: Conductor stress and forces on the supports caused by dead load	22
6.4 Exceptional load case: Effects of short-circuit currents	23
6.4.1 Maximum force on the central main conductor	23
6.4.2 Conductor stress and forces on the supports	23
6.4.3 Conclusions	29
7 Example 4. – Mechanical effects on a 110 kV arrangement with slack conductors	30
7.1 General	30
7.2 Data	31
7.3 Electromagnetic load and characteristic parameters	32
7.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out	34
7.5 Dynamic conductor sag at midspan	35
7.6 Tensile force $F_{f,d}$ after short-circuit caused by drop	36
7.7 Horizontal span displacement b_h and minimum air clearance a_{min}	36
7.8 Conclusions	36
8 Example 5. – Mechanical effects on strained conductors	37
8.1 General	37
8.2 Common data	37
8.3 Centre-line distance between sub-conductors $a_s = 0,1$ m	38

8.3.1	Electromagnetic load and characteristic parameters	38
8.3.2	Tensile force $F_{t,d}$ during short-circuit caused by swing out	41
8.3.3	Dynamic conductor sag at midspan	41
8.3.4	Tensile force $F_{f,d}$ after short-circuit caused by drop	42
8.3.5	Horizontal span displacement b_h and minimum air clearance a_{min}	43
8.3.6	Pinch force $F_{pi,d}$	43
8.3.7	Conclusions	43
8.4	Centre-line distance between sub-conductors $a_s = 0,4$ m	44
8.4.1	Preliminary remarks	44
8.4.2	Characteristic dimensions and parameters	44
8.4.3	Pinch force $F_{pi,d}$	45
8.4.4	Conclusions	47
9	Example 6 – Mechanical effects on strained conductors with dropper in the middle of the span	47
9.1	General.....	47
9.2	Common data	48
9.3	Plane of the dropper parallel to the main conductors.....	48
9.3.1	General	48
9.3.2	Current flow along the whole length of the main conductor span	49
9.3.3	Current flow along half of the length of the main conductor and along the dropper	57
9.4	Plane of the dropper perpendicular to the main conductors	64
9.4.1	General	64
9.4.2	Current flow along the whole length of the main conductor span	64
9.4.3	Current flow along half of the length of the main conductor and along the dropper.....	69
10	Example 7 – Mechanical effects on vertical main conductors (droppers)	77
10.1	General.....	77
10.2	Data.....	77
10.3	Short-circuit tensile force and maximum horizontal displacement	78
10.4	Pinch force.....	78
10.4.1	Static tensile force regarding droppers	78
10.4.2	Characteristic dimensions and parameters	79
10.4.3	Pinch force $F_{pi,d}$	80
10.5	Conclusions	81
11	Example 8 – Thermal effect on bare conductors	81
11.1	General.....	81
11.2	Data.....	81
11.3	Calculations	82
11.4	Conclusion	82
	Bibliography	83
	Figure 1 – Conductor arrangement	8
	Figure 2 – Position of the sub-conductors and connecting pieces	14
	Figure 3 – Two-span arrangement with tubular conductors.....	21
	Figure 4 – Arrangement with slack conductors	31
	Figure 5 – Arrangement with strained conductors	37
	Figure 6 – Arrangement with strained conductors and droppers in midspan. Plane of the droppers parallel to the main conductors	47

Figure 7 – Possible arrangement of perpendicular droppers in three-phase system and two-line system	64
Figure 8 – Arrangement with strained conductors	77

INTERNATIONAL ELECTROTECHNICAL COMMISSION

**SHORT-CIRCUIT CURRENTS –
CALCULATION OF EFFECTS****Part 2: Examples of calculation****FOREWORD**

- 1) The International Electrotechnical Commission (IEC) is a worldwide organization for standardization comprising all national electrotechnical committees (IEC National Committees). The object of IEC is to promote international co-operation on all questions concerning standardization in the electrical and electronic fields. To this end and in addition to other activities, IEC publishes International Standards, Technical Specifications, Technical Reports, Publicly Available Specifications (PAS) and Guides (hereafter referred to as "IEC Publication(s)"). Their preparation is entrusted to technical committees; any IEC National Committee interested in the subject dealt with may participate in this preparatory work. International, governmental and non-governmental organizations liaising with the IEC also participate in this preparation. IEC collaborates closely with the International Organization for Standardization (ISO) in accordance with conditions determined by agreement between the two organizations.
- 2) The formal decisions or agreements of IEC on technical matters express, as nearly as possible, an international consensus of opinion on the relevant subjects since each technical committee has representation from all interested IEC National Committees.
- 3) IEC Publications have the form of recommendations for international use and are accepted by IEC National Committees in that sense. While all reasonable efforts are made to ensure that the technical content of IEC Publications is accurate, IEC cannot be held responsible for the way in which they are used or for any misinterpretation by any end user.
- 4) In order to promote international uniformity, IEC National Committees undertake to apply IEC Publications transparently to the maximum extent possible in their national and regional publications. Any divergence between any IEC Publication and the corresponding national or regional publication shall be clearly indicated in the latter.
- 5) IEC itself does not provide any attestation of conformity. Independent certification bodies provide conformity assessment services and, in some areas, access to IEC marks of conformity. IEC is not responsible for any services carried out by independent certification bodies.
- 6) All users should ensure that they have the latest edition of this publication.
- 7) No liability shall attach to IEC or its directors, employees, servants or agents including individual experts and members of its technical committees and IEC National Committees for any personal injury, property damage or other damage of any nature whatsoever, whether direct or indirect, or for costs (including legal fees) and expenses arising out of the publication, use of, or reliance upon, this IEC Publication or any other IEC Publications.
- 8) Attention is drawn to the Normative references cited in this publication. Use of the referenced publications is indispensable for the correct application of this publication.
- 9) Attention is drawn to the possibility that some of the elements of this IEC Publication may be the subject of patent rights. IEC shall not be held responsible for identifying any or all such patent rights.

The main task of IEC technical committees is to prepare International Standards. However, a technical committee may propose the publication of a technical report when it has collected data of a different kind from that which is normally published as an International Standard, for example "state of the art".

IEC TR 60865-2, which is a technical report, has been prepared by IEC technical committee 73: Short-circuit currents.

This second edition cancels and replaces the first edition published in 1994. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition.

- a) The determinations for auto reclosure together with rigid conductors have been revised.

- b) The configurations in cases of flexible conductor arrangements have been changed.
- c) The influence of mid-span droppers to the span has been included.
- d) For vertical cable-connection the displacement and the tensile force onto the lower fixing point may be calculated now.
- e) Additional recommendations for foundation loads due to tensile forces have been added.
- f) The subclause for determination of the thermal equivalent short-circuits current has been deleted (is part of IEC 60909-0:2001 now).
- g) The standard IEC 60865-1:2011 has been reorganized and some of the symbols have been changed to follow the conceptual characteristic of international standards.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
73/168/DTR	73/173/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts in the IEC 60865 series, published under the general title *Short-circuit currents – Calculations of effects*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

SHORT-CIRCUIT CURRENTS – CALCULATION OF EFFECTS

Part 2: Examples of calculation

1 Scope

The object of this part of IEC 60865, which is a Technical Report, is to show the application of procedures for the calculation of mechanical and thermal effects due to short circuits as presented in IEC 60865-1. Thus, this technical report is an addition to IEC 60865-1. It does not, however, change the basis for standardized procedures given in that publication.

The following points should particularly be noted:

- a) The examples in this Technical Report illustrate how to make the calculations according to IEC 60865-1 in a simplified and easy-to-follow manner. They are not intended as a check for computer programs.
- b) The numbers in parentheses at the end of the equations refer to the equations in IEC 60865-1:2011.
- c) The system voltages are referred to as nominal voltages.
- d) The results are rounded to three significant digits.
- e) Short-circuit effects appear as exceptional load in addition to the mechanical loads of the normal operation of a switchgear. In the following examples with rigid conductors, a possible static preloading is therefore calculated too. Depending on whether it concerns the load of the normal operation or the load during the short-circuit different safety factors come to use. The height of these factors has been chosen typically and is recommended for the use. However, other safety factors may be necessary depending on the safety concept.

2 Normative references

IEC 60865-1:2011, *Short-Circuit Currents – Calculation of Effects – Part 1: Definitions and calculation methods*

IEC 60909-0:2001, *Short-circuit currents in three-phase AC systems – Part 0: Calculation of currents*

3 Symbols and units

For symbols and units, reference is made to IEC 60865-1:2011.

In addition, the following symbols are used:

$F_{\text{str},k}$	Dead load (characteristic value)	N
$F_{\text{str},d}$	Dead load (design value)	N
$F_{\text{st,r,d}}$	Force on support of rigid conductors (design value) due to dead load	N
h_S, h_I	Height of the substructure, insulator	m
H_s	Horizontal component of the force at the lower fixing point of one sub-conductor of a dropper	N

$J_{st,m}$	Second moment of main conductor area with respect to the direction of the dead load	m^4
I_k	Steady-state short-circuit current (r.m.s) according to IEC 60909-0	A
l_{eff}	Effective length of a span	m
l_f	Form factor of a span	m
l_h	Extend of one head armature and clamp	m
m, n	Factor for heat effect of the d.c. component and a.c. component	1
$M_{S,d}, M_{I,d}$	Bending moment on the bottom on the substructure, insulator (design value)	Nm
V_s	Vertical component of the force at the upper fixing point of one sub-conductor of a dropper	N
$W_{st,m}$	Section modulus of main conductor with respect to the direction of the dead load	m^3
γ_F	Partial safety factor for action	1
γ_M	Partial safety factor for material property	1
$\sigma_{st,m,d}$	Bending stress caused by the dead load (design value)	N/m^2
$\sigma_{st,m,k}$	Bending stress caused by the dead load (characteristic value)	N/m^2

4 Example 1 – Mechanical effects on a 10 kV arrangement with single rigid conductors

4.1 General

The basis for the calculation in this example is a three-phase 10 kV busbar with one conductor per phase. The conductors are continuous beams with equidistant simple supports. The conductor arrangement is shown in Figure 1. According to IEC 61936-1 [1]¹, the calculation is done for the normal load case considering the dead load of the busbar and the exceptional load case considering the combination of effects of short-circuit currents and dead load.

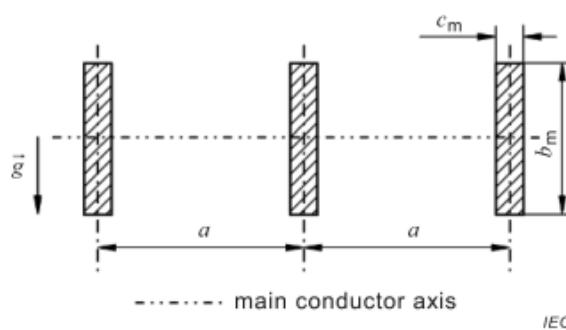


Figure 1 – Conductor arrangement

¹ The numbers in square brackets refer to the Bibliography.

4.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	=	16 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,35
System frequency	f	=	50 Hz
No automatic reclosing			
Number of spans		≥	3
Centre-line distance between supports	l	=	1 m
Centre-line distance between conductors	a	=	0,2 m
Rectangular conductor EN AW-6101B T7			
– Dimensions	b_m	=	60 mm
	c_m	=	10 mm
– Mass per unit length of main conductor	m'_m	=	1,62 kg/m
– Young's modulus	E	=	70 000 N/mm ²
– Stress corresponding to the yield point	f_y	=	120 N/mm ² to 180 N/mm ²
Conventional value of acceleration of gravity	g	=	9,81 m/s ²
Partial safety factors; for example according to EN 1990 [2]			
– Normal load case	γ_F	=	1,35
	γ_M	=	1,1
– Exceptional load case	$\gamma_F \gamma_M$	=	1,0

NOTE Safety factors differ in national standards.

4.3 Normal load case: Conductor stress and forces on the supports caused by dead load

The dead load on the conductor is:

$$F_{\text{str},k} = m'_m l g = 1,62 \frac{\text{kg}}{\text{m}} \cdot 1,00 \text{ m} \cdot 9,81 \frac{\text{m}}{\text{s}^2} = 15,9 \text{ N}$$

$$F_{\text{str},d} = \gamma_F F_{\text{str},k} = 1,35 \cdot 15,9 \text{ N} = 21,5 \text{ N}$$

The conductor bending stress is:

$$\sigma_{\text{st},m,k} = \frac{F_{\text{str},k} l}{8 W_{\text{st},m}} = \frac{15,9 \text{ N} \cdot 1,00 \text{ m}}{8 \cdot 6 \cdot 10^{-6} \text{ m}^3} = 0,33 \cdot 10^6 \text{ N/m}^2 = 0,33 \text{ N/mm}^2$$

$$\sigma_{\text{st},m,d} = \gamma_F \sigma_{\text{st},m,k} = 1,35 \cdot 0,33 \text{ N/mm}^2 = 0,45 \text{ N/mm}^2$$

with

$$J_{st,m} = \frac{c_m b_m^3}{12} = \frac{0,010 \cdot 0,060^3}{12} m^4 = 1,8 \cdot 10^{-7} m^4$$

$$W_{st,m} = \frac{J_{st,m}}{b_m/2} = \frac{1,8 \cdot 10^{-7} m^4}{0,03 m} = 6 \cdot 10^{-6} m^3$$

NOTE The equation for the calculation of $\sigma_{st,m,k}$ gives the maximum value for two spans. The actual value for three or more spans is slightly lower.

The conductors have sufficient strength if

$$\sigma_{st,m,d} \leq \frac{f_y}{\gamma_M}$$

with the lower value of f_y . The partial safety factors for normal load case γ_F , γ_M see 4.2. This gives:

$$\sigma_{st,m,d} = 0,45 \text{ N/mm}^2 \text{ less than } \frac{f_y}{\gamma_M} = \frac{120 \text{ N/mm}^2}{1,1} = 109 \text{ N/mm}^2$$

The forces on the supports are in the direction of the dead load:

- for the outer supports (A) with $\alpha_A = 0,4$, see IEC 60865-1:2011, Table 3:

$$F_{st,r,dA} = \alpha_A F_{str,d} = 0,4 \cdot 21,5 \text{ N} = 8,6 \text{ N}$$

- for the inner supports (B) with $\alpha_B = 1,1$, see IEC 60865-1:2011, Table 3:

$$F_{st,r,dB} = \alpha_B F_{str,d} = 1,1 \cdot 21,5 \text{ N} = 23,7 \text{ N}$$

NOTE In some standards the safety factors for the supports can include the partial safety factor γ_F for action.

4.4 Exceptional load case: Effects of short-circuit currents

4.4.1 Maximum force on the central main conductor

The maximum electromagnetic force on the central main conductor is:

$$F_{m3} = \frac{\mu_0}{2\pi} \frac{\sqrt{3}}{2} i_p^2 \frac{l}{a_m} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \frac{\sqrt{3}}{2} \cdot (30,6 \cdot 10^3 \text{ A})^2 \cdot \frac{1,00 \text{ m}}{0,202 \text{ m}} = 803 \text{ N} \quad (2)$$

where

$$i_p = \kappa \sqrt{2} I_k'' = 1,35 \cdot \sqrt{2} \cdot 16 \text{ kA} = 30,6 \text{ kA} = 30,6 \cdot 10^3 \text{ A}$$

and the effective distance between the main conductors

$$a_m = \frac{a}{k_{12}} = \frac{0,20 \text{ m}}{0,99} = 0,202 \text{ m} \quad (6)$$

with k_{12} according to IEC 60865-1:2011, Figure 1 with $a_{1s} = a$, $b_s = b_m$, $c_s = c_m$, for $b_m/c_m = 60 \text{ mm}/10 \text{ mm} = 6$, and $a/c_m = 200 \text{ mm}/10 \text{ mm} = 20$.

4.4.2 Conductor stress and forces on the supports

4.4.2.1 General

The calculations can be made according to the following 4.4.2.2 or 4.4.2.3.

4.4.2.2 Simplified method

4.4.2.2.1 Conductor bending stress

The maximum bending stress is:

$$\sigma_{m,d} = V_{gm} V_{rm} \beta \frac{F_m 3 l}{8 W_m} = 1,0 \cdot 0,73 \cdot \frac{803 \text{ N} \cdot 1,00 \text{ m}}{8 \cdot 1 \cdot 10^{-6} \text{ m}^3} = 73,3 \cdot 10^6 \text{ N/m}^2 = 73,3 \text{ N/mm}^2 \quad (9)$$

where

$V_{gm} V_{rm} = 1,0$ ($V_{gm} V_{rm}$)_{max} according to IEC 60865-1:2011, Table 2

$\beta = 0,73$ according to IEC 60865-1:2011, Table 3

$$W_m = \frac{J_m}{c_m / 2} = \frac{0,5 \cdot 10^{-8} \text{ m}^4}{0,005 \text{ m}} = 1 \cdot 10^{-6} \text{ m}^3$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{m,d} + \sigma_{st,m,k} \leq q f_y \quad (11)$$

with the lower value of f_y . $\sigma_{st,m,k}$ see 4.3. For rectangular cross-section $q = 1,5$, see IEC 60865-1:2011, Table 4. This gives:

$$\sigma_{m,d} + \sigma_{st,m,k} = 73,3 \text{ N/mm}^2 + 0,33 \text{ N/mm}^2 = 73,6 \text{ N/mm}^2 \quad \text{less than} \quad q f_y = 1,5 \cdot 120 \text{ N/mm}^2 = 180 \text{ N/mm}^2$$

4.4.2.2.2 Forces on the supports

The equivalent static force on the supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_m \quad (15)$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y and $\sigma_{tot,d} = \sigma_{m,d} + \sigma_{st,m,k}$ it is:

$$\frac{\sigma_{tot,d}}{0,8 f_y} = \frac{73,6 \text{ N/mm}^2}{0,8 \cdot 180 \text{ N/mm}^2} = 0,511$$

Therefore, with a three-phase short-circuit we meet range 2 in IEC 60865-1:2011, Table 2,

$$0,370 < \frac{\sigma_{tot,d}}{0,8 f_y} < 1$$

hence

$$V_F V_{rm} = \frac{0,8 f_y}{\sigma_{tot,d}} = \frac{1}{0,511} = 1,96$$

For the outer supports (A) it is with $\alpha_A = 0,4$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 1,96 \cdot 0,4 \cdot 803 \text{ N} = 630 \text{ N}$$

For the inner supports (B) it is with $\alpha_B = 1,1$, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 1,96 \cdot 1,1 \cdot 803 \text{ N} = 1731 \text{ N}$$

4.4.2.3 Detailed method

4.4.2.3.1 Relevant natural frequency f_{cm} and factors V_F , V_{rm} and $V_{\sigma m}$

The relevant natural frequency of the main conductor is:

$$f_{cm} = \frac{\gamma}{l^2} \sqrt{\frac{E J_m}{m'_m}} = \frac{3,56}{(1,00 \text{ m})^2} \cdot \sqrt{\frac{7 \cdot 10^{10} \text{ N/m}^2 \cdot 0,5 \cdot 10^{-8} \text{ m}^4}{1,62 \text{ kg/m}}} = 52,3 \text{ Hz} \quad (16)$$

where

$\gamma = 3,56$ according to IEC 60865-1:2011, Table 3

$J_m = 0,5 \cdot 10^{-8} \text{ m}^4$ see 4.4.2.2.1

The frequency ratio is:

$$\frac{f_{cm}}{f} = \frac{52,3 \text{ Hz}}{50 \text{ Hz}} = 1,05$$

From Figure 4 and 5.7.3 of IEC 60865-1:2011, the following values for the factors V_F , $V_{\sigma m}$ and V_{rm} are obtained:

$$V_F = 1,8$$

$$V_{\sigma m} = 1,0$$

$$V_{rm} = 1,0$$

4.4.2.3.2 Conductor bending stress

The maximum bending stress is:

$$\sigma_{m,d} = V_{\sigma m} V_{rm} \beta \frac{F_{m3} l}{8 W_m} = 1,0 \cdot 1,0 \cdot 0,73 \cdot \frac{803 \text{ N} \cdot 1,00 \text{ m}}{8 \cdot 1 \cdot 10^{-6} \text{ m}^3} = 73,3 \cdot 10^6 \text{ N/m}^2 = 73,3 \text{ N/mm}^2 \quad (9)$$

where

$V_{\sigma m} V_{rm} = 1,0 \cdot 1,0$ according to 4.4.2.3.1

$\beta = 0,73$ according to IEC 60865-1:2011, Table 3

$W_m = 1 \cdot 10^{-6} \text{ m}^3$ see 4.4.2.2.1

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{m,d} + \sigma_{st,m,k} \leq q f_y \quad (11)$$

with the lower value of f_y . $\sigma_{st,m,k}$ see 4.3. For rectangular cross-section $q = 1,5$, see IEC 60865-1:2011, Table 4. This gives:

$$\sigma_{m,d} + \sigma_{st,m,k} = 73,3 \text{ N/mm}^2 + 0,33 \text{ N/mm}^2 = 73,6 \text{ N/mm}^2 \text{ less than } q f_y = 1,5 \cdot 120 \text{ N/mm}^2 = 180 \text{ N/mm}^2$$

4.4.2.3.3 Forces on the supports

The equivalent static force on supports becomes:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3} \quad (15)$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y and $\sigma_{tot,d} = \sigma_{m,d} + \sigma_{st,m,k}$ it is:

$$\frac{\sigma_{tot,d}}{0,8 f_y} = \frac{73,6 \text{ N/mm}^2}{0,8 \cdot 180 \text{ N/mm}^2} = 0,511$$

Therefore, with a three-phase short-circuit we meet range 2 in IEC 60865-1:2011, Table 2,

$$0,370 < \frac{\sigma_{tot,d}}{0,8 f_y} < 1$$

hence

$$V_F V_{rm} = \frac{0,8 f_y}{\sigma_{tot,d}} = \frac{1}{0,511} = 1,96$$

According to 4.4.2.3.1 above, $V_F V_{rm} = 1,8 \cdot 1,0 = 1,8$ which is less than the value 1,96 according to IEC 60865-1:2011, Table 2.

For the outer supports (A) it is with $\alpha_A = 0,4$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 1,8 \cdot 1,0 \cdot 0,4 \cdot 803 \text{ N} = 578 \text{ N}$$

For the inner supports (B) it is with $\alpha_B = 1,1$, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 1,8 \cdot 1,0 \cdot 1,1 \cdot 803 \text{ N} = 1590 \text{ N}$$

4.5 Conclusions

The busbar will withstand the dead load

The calculated bending stress is

$$\sigma_{st,m,d} \quad 1 \text{ N/mm}^2$$

The outer supports have to withstand a vertical force of

$$F_{st,r,dA} \quad 9 \text{ N}$$

The inner supports have to withstand a vertical force of

$$F_{st,r,dB} \quad 24 \text{ N}$$

	Simplified method	Detailed method
The busbar will withstand the short-circuit load		
The calculated bending stress is	$\sigma_{\text{tot,d}}$	74 N/mm ² 74 N/mm ²
The outer supports have to withstand an equivalent static force of	$F_{r,\text{dA}}$	630 N 580 N
The inner supports have to withstand an equivalent static force of	$F_{r,\text{dB}}$	1 740 N 1 590 N

The stresses and forces are rounded.

The forces calculated with the detailed method are less than calculated with the simplified method.

5 Example 2 – Mechanical effects on a 10 kV arrangement with multiple rigid conductors

5.1 General

The basis for the calculation in this example is the same three-phase 10 kV busbar as in Example 1, but now with three sub-conductors per main conductor as shown in Figure 2. The cross-sections of the sub-conductors are 60 mm × 10 mm as the conductors of Example 1. The connecting pieces are spacers. According to IEC 61936-1 [1], the calculation is done for the normal load case considering the dead load of the busbar and the exceptional load case considering the combination of effects of short-circuit currents and dead load.

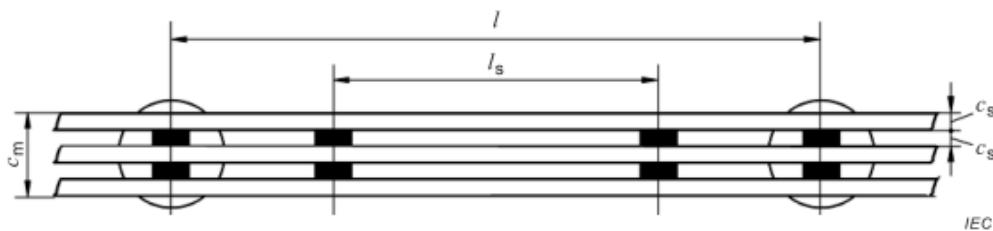


Figure 2 – Position of the sub-conductors and connecting pieces

5.2 Data (additional to the data of Example 1)

Number of sub-conductors	$n = 3$
Dimension of sub-conductor in the direction of the force	$c_s = 10 \text{ mm}$
Number of sets of spacers	$k = 2$
Centre-line distance between connecting pieces	$l_s = 0,5 \text{ m}$
Dimension of spacers of EN AW-6101B T7	60 mm × 60 mm × 10 mm

5.3 Normal load case: Conductor stress and forces on the supports caused by dead load

In 4.3 (Example 1), the following values are calculated for one conductor

Dead load of the conductor	$F_{\text{str},k} = 15,9 \text{ N}$
	$F_{\text{str},d} = 21,5 \text{ N}$
Conductor bending stress	$\sigma_{\text{st},m,k} = 0,33 \text{ N/mm}^2$
	$\sigma_{\text{st},m,d} = 0,45 \text{ N/mm}^2$

In this Example 2, the conductor bending stress is the same as in Example 1, 4.3. According to the number of sub-conductors, the vertical forces on the supports are n times higher

- for the outer supports (A) with $\alpha_A = 0,4$, see IEC 60865-1:2011, Table 3:

$$F_{\text{st},r,dA} = n \alpha_A F_{\text{str},d} = 3 \cdot 0,4 \cdot 21,5 \text{ N} = 25,8 \text{ N}$$

- for the inner supports (B) with $\alpha_B = 1,1$, see IEC 60865-1:2011, Table 3:

$$F_{\text{st},r,dB} = n \alpha_B F_{\text{str},d} = 3 \cdot 1,1 \cdot 21,5 \text{ N} = 71,0 \text{ N}$$

5.4 Exceptional load case: Effects of short-circuit currents

5.4.1 Maximum forces on the conductors

5.4.1.1 Maximum force on the central main conductor

The maximum electromagnetic force on the central main conductor is:

$$F_{m3} = \frac{\mu_0}{2\pi} \frac{\sqrt{3}}{2} i_p^2 \frac{l}{a_m} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \frac{\sqrt{3}}{2} \cdot \left(30,6 \cdot 10^3 \text{ A}\right)^2 \cdot \frac{1,00 \text{ m}}{0,20 \text{ m}} = 811 \text{ N} \quad (2)$$

where

$$i_p = \kappa \sqrt{2} I_k'' = 1,35 \cdot \sqrt{2} \cdot 16 \text{ kA} = 30,6 \text{ kA} = 30,6 \cdot 10^3 \text{ A}$$

and the effective distance between the main conductors

$$a_m = \frac{a}{k_{12}} = \frac{0,2 \text{ m}}{1,00} = 0,20 \text{ m} \quad (6)$$

with k_{12} according to IEC 60865-1:2011, Figure 1 with $a_{1s} = a$, $b_s = b_m$, $c_s = c_m$, for $b_m/c_m = 60 \text{ mm}/50 \text{ mm} = 1,2$ and $a/c_m = 200 \text{ mm}/50 \text{ mm} = 4$. The dimensions b_m and c_m are shown in IEC 60865-1:2011, Figure 2 b).

5.4.1.2 Maximum force on the sub-conductor

The maximum electromagnetic force on the outer sub-conductor between two adjacent connecting pieces is:

$$F_s = \frac{\mu_0}{2\pi} \left(\frac{i_p}{n} \right)^2 \frac{l_s}{a_s} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{30,6 \cdot 10^3 \text{ A}}{3} \right)^2 \cdot \frac{0,5 \text{ m}}{20,2 \cdot 10^{-3} \text{ m}} = 515 \text{ N} \quad (4)$$

where

$$\frac{1}{a_s} = \frac{k_{12}}{a_{12}} + \frac{k_{13}}{a_{13}} = \frac{0,60}{20 \text{ mm}} + \frac{0,78}{40 \text{ mm}} = \frac{1}{20,2 \text{ mm}} \quad (8)$$

with k_{12} and k_{13} from IEC 60865-1:2011, Figure 1:

- $k_{12} = 0,60$ for $a_{12}/c_s = 20 \text{ mm}/10 \text{ mm} = 2$ and $b_s/c_s = b_m/c_s = 60 \text{ mm}/10 \text{ mm} = 6$
 - $k_{13} = 0,78$ for $a_{13}/c_s = 40 \text{ mm}/10 \text{ mm} = 4$ and $b_s/c_s = b_m/c_s = 60 \text{ mm}/10 \text{ mm} = 6$
- or a_s from IEC 60865-1:2011, Table 1.

5.4.2 Conductor stress and forces on the supports

5.4.2.1 General

The calculations can be made according to 5.4.2.2 or 5.4.2.3.

5.4.2.2 Simplified method

5.4.2.2.1 Bending stress caused by the forces between the main conductors

The maximum bending stress caused by the forces between the main conductors is:

$$\sigma_{m,d} = V_{\sigma m} V_{rm} \beta \frac{F_{m3} l}{8 W_m} = 1,0 \cdot 0,73 \cdot \frac{811 \text{ N} \cdot 1,00 \text{ m}}{8 \cdot 3 \cdot 10^{-6} \text{ m}^3} = 24,7 \cdot 10^6 \text{ N/m}^2 = 24,7 \text{ N/mm}^2 \quad (9)$$

where

$$V_{\sigma m} V_{rm} = 1,0 = (V_{\sigma m} V_{rm})_{\max} \quad \text{according to IEC 60865-1, Table 2}$$

$$\beta = 0,73 \quad \text{according to IEC 60865-1, Table 3}$$

$$J_s = \frac{c_s^3 b_s}{12} = \frac{0,010^3 \cdot 0,060}{12} \text{ m}^4 = 0,5 \cdot 10^{-8} \text{ m}^4$$

$$W_s = \frac{J_s}{c_s/2} = \frac{0,5 \cdot 10^{-8} \text{ m}^4}{0,005 \text{ m}} = 1 \cdot 10^{-6} \text{ m}^3$$

$$W_m = n W_s = 3 \cdot 1 \cdot 10^{-6} \text{ m}^3 = 3 \cdot 10^{-6} \text{ m}^3 \quad \text{according to IEC 60865-1, 5.4.2}$$

5.4.2.2.2 Bending stress caused by the forces between the sub-conductors

The maximum bending stress caused by the forces between the sub-conductors is:

$$\sigma_{s,d} = V_{\sigma s} V_{rs} \frac{F_s l_s}{16 W_s} = 1,0 \cdot \frac{515 \text{ N} \cdot 0,5 \text{ m}}{16 \cdot 1 \cdot 10^{-6} \text{ m}^3} = 16,1 \cdot 10^6 \text{ N/m}^2 = 16,1 \text{ N/mm}^2 \quad (10)$$

where

$$V_{\sigma s} V_{rs} = 1,0 = (V_{\sigma s} V_{rs})_{\max} \quad \text{according to IEC 60865-1:2011, Table 2}$$

$$W_s = 1 \cdot 10^{-6} \text{ m}^3 \quad \text{see 5.4.2.2.1}$$

5.4.2.2.3 Total conductor stress

The total conductor stress is with the stresses calculated in 5.4.2.2.1, 5.4.2.2.2 and 5.3:

$$\sigma_{tot,d} = \sigma_{m,d} + \sigma_{s,d} + \sigma_{st,m,k} = 24,7 \text{ N/mm}^2 + 16,1 \text{ N/mm}^2 + 0,33 \text{ N/mm}^2 = 41,1 \text{ N/mm}^2 \quad (12)$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{\text{tot,d}} \leq q f_y \quad (13)$$

with the lower value of f_y . For rectangular cross-sections $q = 1,5$, see IEC 60865-1:2011, Table 4 or 5.4.2. This gives:

$$\sigma_{\text{tot,d}} = 41,1 \text{ N/mm}^2 \text{ less than } q f_y = 1,5 \cdot 120 \text{ N/mm}^2 = 180 \text{ N/mm}^2$$

It is recommended that the stress caused by the forces between sub-conductors holds

$$\sigma_{\text{s,d}} \leq f_y \quad (14)$$

with the lower value of f_y . This gives:

$$\sigma_{\text{s,d}} = 16,1 \text{ N/mm}^2 \text{ less than } f_y = 120 \text{ N/mm}^2$$

5.4.2.2.4 Forces on the supports

The equivalent static force on supports is:

$$F_{\text{r,d}} = V_F V_{\text{rm}} \alpha F_{\text{m3}} \quad (15)$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y it is:

$$\frac{\sigma_{\text{tot,d}}}{0,8 f_y} = \frac{41,1 \text{ N/mm}^2}{0,8 \cdot 180 \text{ N/mm}^2} = 0,285$$

therefore with a three-phase short-circuit we meet range 1 in IEC 60865-1:2011, Table 2,

$$\frac{\sigma_{\text{tot,d}}}{0,8 f_y} < 0,370$$

hence

$$V_F V_{\text{rm}} = 2,7$$

For the outer supports (A) it is with $\alpha_A = 0,4$, see IEC 60865-1:2011, Table 3:

$$F_{\text{r,dA}} = V_F V_{\text{rm}} \alpha_A F_{\text{m3}} = 2,7 \cdot 0,4 \cdot 811 \text{ N} = 876 \text{ N}$$

For the inner supports (B) it is with $\alpha_B = 1,1$, see IEC 60865-1:2011, Table 3:

$$F_{\text{r,DB}} = V_F V_{\text{rm}} \alpha_B F_{\text{m3}} = 2,7 \cdot 1,1 \cdot 811 \text{ N} = 2409 \text{ N}$$

5.4.2.3 Detailed method

5.4.2.3.1 Relevant natural frequency f_{cm} of the main conductors, f_{cs} of the sub-conductors and factors V_F , $V_{\sigma m}$, $V_{\sigma s}$, V_{rm} and V_{rs}

The relevant natural frequency of the main conductors is:

$$f_{cm} = e \frac{\gamma}{l^2} \sqrt{\frac{E J_s}{m'_s}} = 0,97 \cdot \frac{3,56}{(1,00 \text{ m})^2} \cdot \sqrt{\frac{7 \cdot 10^{10} \text{ N/m}^2 \cdot 0,5 \cdot 10^{-8} \text{ m}^4}{1,62 \text{ kg/m}}} = 50,8 \text{ Hz} \quad (17)$$

where

$$e = 0,97 \quad \text{according to IEC 60865-1:2011, Figure 3c), for } k = 2 \left(\frac{l_s}{l} = 0,5 \right) \text{ and the radio}$$

$$\frac{m_z}{nm'_s l} = \frac{1,62 \text{ kg/m} \cdot 0,06 \text{ m} \cdot 2}{3 \cdot 1,62 \text{ kg/m} \cdot 1,00 \text{ m}} = 0,04$$

$$\gamma = 3,56 \quad \text{according to IEC 60865-1:2011, Table 3}$$

$$J_s = 0,5 \cdot 10^{-8} \text{ m}^4 \quad \text{see 5.4.2.2.1}$$

The relevant natural frequency of the sub-conductors is:

$$f_{cs} = \frac{3,56}{l_s^2} \sqrt{\frac{E J_s}{m'_s}} = \frac{3,56}{(0,5 \text{ m})^2} \cdot \sqrt{\frac{7 \cdot 10^{10} \text{ N/m}^2 \cdot 0,5 \cdot 10^{-8} \text{ m}^4}{1,62 \text{ kg/m}}} = 209 \text{ Hz} \quad (18)$$

The frequency ratios are:

$$\frac{f_{cm}}{f} = \frac{50,8 \text{ Hz}}{50 \text{ Hz}} = 1,02$$

$$\frac{f_{cs}}{f} = \frac{209 \text{ Hz}}{50 \text{ Hz}} = 4,18$$

This gives from IEC 60865-1:2011, Figure 4 and 5.7.3, the following values for the factors V_F , $V_{\sigma m}$, $V_{\sigma s}$, V_{rm} and V_{rs} :

$$\begin{aligned} V_F &= 1,8 \\ V_{\sigma m} &= 1,0 & V_{\sigma s} &= 1,0 \\ V_{rm} &= 1,0 & V_{rs} &= 1,0 \end{aligned}$$

5.4.2.3.2 Bending stress caused by the forces between the main conductors

The maximum bending stress caused by the forces between the main conductors is:

$$\sigma_{m,d} = V_{\sigma m} V_{rm} \beta \frac{F_m 3 l}{8 W_m} = 1,0 \cdot 1,0 \cdot 0,73 \cdot \frac{811 \text{ N} \cdot 1,00 \text{ m}}{8 \cdot 3 \cdot 10^{-6} \text{ m}^3} = 24,7 \cdot 10^6 \text{ N/m}^2 = 24,7 \text{ N/mm}^2 \quad (9)$$

where

$$V_{\sigma m} V_{rm} = 1,0 \cdot 1,0 \quad \text{according to 5.4.2.3.1}$$

$$\beta = 0,73 \quad \text{according to IEC 60865-1:2011, Table 3}$$

$$W_m = 3 \cdot 10^{-6} \text{ m}^3 \quad \text{see 5.4.2.2.1}$$

5.4.2.3.3 Bending stress caused by the forces between the sub-conductors

The maximum bending stress caused by the forces between the sub-conductors is:

$$\sigma_{s,d} = V_{os} V_{rs} \frac{F_s l_s}{16 W_s} = 1,0 \cdot 1,0 \cdot \frac{515 \text{ N} \cdot 0,5 \text{ m}}{16 \cdot 1 \cdot 10^{-6} \text{ m}^3} = 16,1 \cdot 10^6 \text{ N/m}^2 = 16,1 \text{ N/mm}^2 \quad (10)$$

where

$$V_{os} V_{rs} = 1,0 \cdot 1,0 \quad \text{according to 5.4.2.3.1}$$

$$W_s = 1 \cdot 10^{-6} \text{ m}^3 \quad \text{see 5.4.2.2.1}$$

5.4.2.3.4 Total bending stress in the busbar

The total conductor stress is with the stresses calculated in 5.4.2.3.2 and 5.4.2.3.3 and 5.3:

$$\sigma_{tot,d} = \sigma_{m,d} + \sigma_{s,d} + \sigma_{st,m,k} = 24,7 \text{ N/mm}^2 + 16,1 \text{ N/mm}^2 + 0,33 \text{ N/mm}^2 = 41,1 \text{ N/mm}^2 \quad (12)$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \leq q f_y \quad (13)$$

with the lower value of f_y . For rectangular cross-sections $q = 1,5$, see IEC 60865-1:2011, Table 4. This gives:

$$\sigma_{tot,d} = 41,1 \text{ N/mm}^2 \text{ less than } q f_y = 1,5 \cdot 120 \text{ N/mm}^2 = 180 \text{ N/mm}^2$$

It is recommended a value

$$\sigma_{s,d} \leq f_y \quad (14)$$

with the lower value of f_y . This gives:

$$\sigma_{s,d} = 16,1 \text{ N/mm}^2 \text{ less than } f_y = 120 \text{ N/mm}^2$$

5.4.2.3.5 Forces on the supports

The equivalent static force on supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3} \quad (15)$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y it is:

$$\frac{\sigma_{tot,d}}{0,8 f_y} = \frac{41,1 \text{ N/mm}^2}{0,8 \cdot 180 \text{ N/mm}^2} = 0,285$$

Therefore with a three-phase short-circuit we meet range 1 in IEC 60865-1:2011, Table 2,

$$\frac{\sigma_{\text{tot,d}}}{0,8 f_y} < 0,370$$

hence

$$V_F V_{\text{rm}} = 2,7$$

According to 5.4.2.3.1 above, $V_F V_{\text{rm}} = 1,8 \cdot 1,0 = 1,8$, which is less than the value 2,7 obtained from IEC 60865-1:2011, Table 2.

For the outer supports (A) it is with $\alpha_A = 0,4$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{\text{rm}} \alpha_A F_{m3} = 1,8 \cdot 1,0 \cdot 0,4 \cdot 811 \text{ N} = 584 \text{ N}$$

For the inner supports (B) it is with $\alpha_B = 1,1$; see IEC 60865-1:2011, Table 3:

$$F_{r,DB} = V_F V_{\text{rm}} \alpha_B F_{m3} = 1,8 \cdot 1,0 \cdot 1,1 \cdot 811 \text{ N} = 1606 \text{ N}$$

5.5 Conclusions

The busbar will withstand the dead load

The calculated bending stress is $\sigma_{st,m,d}$ 1 N/mm²

The outer supports have to withstand a vertical force of $F_{st,r,dA}$ 26 N

The inner supports have to withstand a vertical force of $F_{st,r,DB}$ 71 N

		Simplified method	Detailed method
The busbar will withstand the short-circuit force			
The calculated bending stresses are	$\sigma_{tot,d}$	42 N/mm ²	42 N/mm ²
	$\sigma_{s,d}$	17 N/mm ²	17 N/mm ²
The outer supports have to withstand an equivalent static force of	$F_{r,dA}$	880 N	590 N
The inner supports have to withstand an equivalent static force of	$F_{r,DB}$	2 410 N	1 610 N

The forces calculated with the detailed method are less than calculated with the simplified method.

6 Example 3. – Mechanical effects on a high-voltage arrangement with rigid conductors

6.1 General

The basis for the calculation in this example is a three-phase 380 kV busbar, with one tubular conductor per phase. The conductor arrangement is shown in Figure 3. This example includes calculations without and with automatic reclosing. Without automatic reclosing only one short-circuit current duration exists, with automatic reclosing two short-circuit current durations exist with an interval without current flow.

According to IEC 61936-1 [1], the calculation is done for the normal load case considering the dead load of the busbar and the exceptional load case considering the combination of effects of short-circuit currents and dead load.

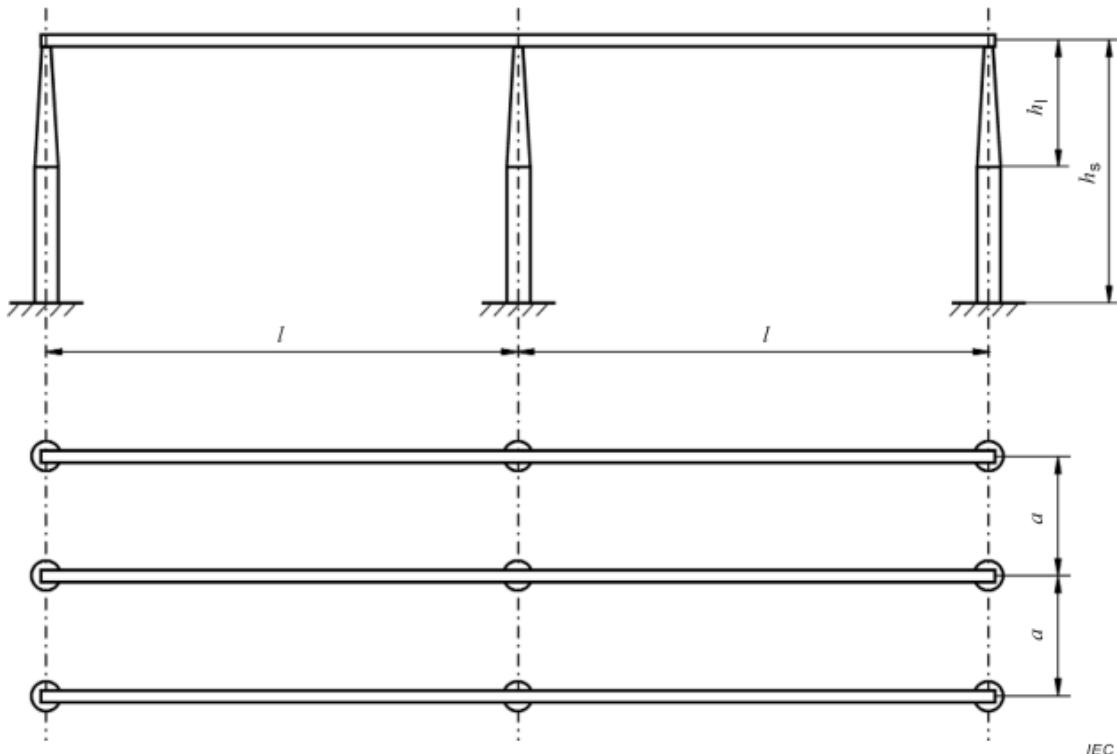


Figure 3 – Two-span arrangement with tubular conductors

6.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	=	50 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,81
System frequency	f	=	50 Hz
Number of spans			2
Centre-line distance between supports	l	=	18 m
Centre-line distance between conductors	a	=	5 m
Height of the insulator with clamp	h_l	=	3,7 m
Height of the support	h_s	=	7,0 m
Tubular conductor 160 mm × 6 mm EN AW-6101B T6			
– Mass per unit length	m_m'	=	7,84 kg/m
– Outer diameter	d	=	160 mm
– Wall thickness	t	=	6 mm
– Young's modulus	E	=	70 000 N/mm ²

– Stress corresponding to the yield point f_y = 160 N/mm² to 240 N/mm²

Conventional value of acceleration of gravity g = 9,81 m/s²

Partial safety factors; for example according to EN 1990 [2]

– Normal load case γ_F = 1,35

γ_M = 1,1

– Exceptional load case $\gamma_F \gamma_M$ = 1,0

NOTE Safety factors differ in national standards.

6.3 Normal load case: Conductor stress and forces on the supports caused by dead load

The dead load on the conductor is

$$F_{\text{str},k} = m'_m / g = 7,84 \frac{\text{kg}}{\text{m}} \cdot 18 \text{ m} \cdot 9,81 \frac{\text{m}}{\text{s}^2} = 1384 \text{ N}$$

$$F_{\text{str},d} = \gamma_F F_{\text{str},k} = 1,35 \cdot 1384 \text{ N} = 1868 \text{ N}$$

The conductor bending stress is

$$\sigma_{\text{st},m,k} = \frac{F_{\text{str}} l}{8 W_m} = \frac{1384 \text{ N} \cdot 18 \text{ m}}{8 \cdot 108 \cdot 10^{-6} \text{ m}^3} = 28,8 \cdot 10^6 \text{ N/m}^2 = 28,8 \text{ N/mm}^2$$

$$\sigma_{\text{st},m,d} = \gamma_F \sigma_{\text{st},m,k} = 1,35 \cdot 28,8 \text{ N/mm}^2 = 38,9 \text{ N/mm}^2$$

with

$$J_m = \frac{\pi}{64} \left(d^4 - (d - 2t)^4 \right) = \frac{\pi}{64} \cdot \left(0,16^4 - (0,16 - 2 \cdot 0,006)^4 \right) \text{m}^4 = 8,62 \cdot 10^{-6} \text{ m}^4$$

$$W_m = \frac{J_m}{d/2} = \frac{8,62 \cdot 10^{-6} \text{ m}^4}{0,16/2 \text{ m}} = 108 \cdot 10^{-6} \text{ m}^3$$

The conductors have sufficient strength if

$$\sigma_{\text{st},m,d} \leq \frac{f_y}{\gamma_M}$$

with the lower value of f_y . The partial safety factors for normal load case γ_F , γ_M see 6.2. This gives:

$$\sigma_{\text{st},m,d} = 38,9 \text{ N/mm}^2 \text{ less than } \frac{f_y}{\gamma_M} = \frac{160 \text{ N/mm}^2}{1,1} = 145 \text{ N/mm}^2$$

The force on the supports are in the direction of the dead load:

– for the outer supports (A) with $\alpha_A = 0,375$, see IEC 60865-1:2011, Table 3:

$$F_{\text{st},r,dA} = \alpha_A F_{\text{str},d} = 0,375 \cdot 1868 \text{ N} = 701 \text{ N} = 0,701 \text{ kN}$$

– for the inner supports (B) with $\alpha_B = 1,25$, see IEC 60865-1:2011, Table 3:

$$F_{st,r,dB} = \alpha_B F_{str,d} = 1,25 \cdot 1868 \text{ N} = 2335 \text{ N} = 2,335 \text{ kN}$$

6.4 Exceptional load case: Effects of short-circuit currents

6.4.1 Maximum force on the central main conductor

The maximum electromagnetic force on the central main conductor becomes:

$$F_{m3} = \frac{\mu_0}{2\pi} \frac{\sqrt{3}}{2} i_p^2 \frac{l}{a_m} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \frac{\sqrt{3}}{2} \cdot (128 \cdot 10^3 \text{ A})^2 \cdot \frac{18 \text{ m}}{5 \text{ m}} = 10200 \text{ N} = 10,2 \text{ kN} \quad (2)$$

where

$$i_p = \kappa \sqrt{2} I_k'' = 1,81 \cdot \sqrt{2} \cdot 50 \text{ kA} = 128 \text{ kA} = 128 \cdot 10^3 \text{ A}$$

and $a_m = a = 5 \text{ m}$ according to IEC 60865-1:2011, 5.3.

6.4.2 Conductor stress and forces on the supports

6.4.2.1 General

The calculations can be made according to the following 6.4.2.2 or 6.4.2.3.

6.4.2.2 Simplified method

6.4.2.2.1 Calculation without three-phase automatic reclosing

6.4.2.2.1.1 Conductor bending stress

The maximum bending stress is:

$$\sigma_{m,d} = V_{om} V_{rm} \beta \frac{F_{m3} l}{8 W_m} = 1,0 \cdot 0,73 \cdot \frac{10,2 \cdot 10^3 \text{ N} \cdot 18 \text{ m}}{8 \cdot 108 \cdot 10^{-6} \text{ m}^3} = 155 \cdot 10^6 \text{ N/m}^2 = 155 \text{ N/mm}^2 \quad (9)$$

where

$$V_{om} V_{rm} = 1,0 = (V_{om} V_{rm})_{\max} \quad \text{according to IEC 60865-1:2011, Table 2}$$

$$\beta = 0,73 \quad \text{according to IEC 60865-1:2011, Table 3}$$

$$W_m = 108 \cdot 10^{-6} \text{ m}^3 \quad \text{see 6.3}$$

For tubular cross-section, the total bending stress becomes:

$$\sigma_{tot,d} = \sqrt{\sigma_{m,d}^2 + \sigma_{st,m,k}^2} = \sqrt{155^2 + 28,8^2} \text{ N/mm}^2 = 158 \text{ N/mm}^2$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \leq q f_y \quad (11)$$

with the lower value of f_y . For tubular cross-section in accordance with IEC 60865-1:2011, Table 4:

$$q = 1,7 \frac{1 - (1 - 2t/d)^3}{1 - (1 - 2t/d)^4} = 1,7 \cdot \frac{1 - (1 - 2 \cdot 0,006 \text{ m}/(0,160 \text{ m}))^3}{1 - (1 - 2 \cdot 0,006 \text{ m}/(0,160 \text{ m}))^4} = 1,32$$

This gives:

$$\sigma_{\text{tot,d}} = 158 \text{ N/mm}^2 \quad \text{less than} \quad q f_y = 1,32 \cdot 160 \text{ N/mm}^2 = 211 \text{ N/mm}^2$$

6.4.2.2.1.2 Forces on the supports and moments on the substructures

The equivalent static force on supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3} \quad (15)$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y it is:

$$\frac{\sigma_{\text{tot,d}}}{0,8 f_y} = \frac{158 \text{ N/mm}^2}{0,8 \cdot 240 \text{ N/mm}^2} = 0,823$$

Therefore we meet range 2 in IEC 60865-1:2011, Table 2,

$$0,370 < \frac{\sigma_{\text{tot,d}}}{0,8 f_y} < 1$$

hence

$$V_F V_{rm} = \frac{0,8 f_y}{\sigma_{\text{tot,d}}} = \frac{1}{0,823} = 1,22$$

For the outer supports (A) it is with $\alpha_A = 0,375$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 1,22 \cdot 0,375 \cdot 10,2 \text{ kN} = 4,67 \text{ kN}$$

For the inner supports (B) it is with $\alpha_B = 1,25$, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 1,22 \cdot 1,25 \cdot 10,2 \text{ kN} = 15,6 \text{ kN}$$

The bending moments on the substructures are:

- on the bottom of the outer insulators

$$M_{IA,d} = F_{r,dA} h_1 = 4,67 \text{ kN} \cdot 3,7 \text{ m} = 17,3 \text{ kNm}$$

- on the bottom of the outer supports

$$M_{SA,d} = F_{r,dA} h_S = 4,67 \text{ kN} \cdot 7,0 \text{ m} = 32,7 \text{ kNm}$$

- on the bottom of the inner insulators

$$M_{IB,d} = F_{r,dB} h_1 = 15,6 \text{ kN} \cdot 3,7 \text{ m} = 57,7 \text{ kNm}$$

- on the bottom of the inner supports

$$M_{SB,d} = F_{r,dB} h_S = 15,6 \text{ kN} \cdot 7,0 \text{ m} = 109 \text{ kNm}$$

6.4.2.2.2 Calculation with three-phase automatic reclosing

6.4.2.2.2.1 General

In networks with three-phase automatic reclosing different mechanical stresses can occur during the first and the second short-circuit current flow duration. In 6.4.2.2.1, the stresses and forces during the first short-circuit current flow duration are calculated.

6.4.2.2.2.2 Conductor bending stress

The maximum bending stress during the second short-circuit current flow duration is:

$$\sigma_{m,d} = V_{om} V_{rm} \beta \frac{F_{m3} l}{8 W_m} = 1,8 \cdot 0,73 \cdot \frac{10,2 \cdot 10^3 \text{ N} \cdot 18 \text{ m}}{8 \cdot 108 \cdot 10^{-6} \text{ m}^3} = 279 \cdot 10^6 \text{ N/m}^2 = 279 \text{ N/mm}^2 \quad (9)$$

where

$$V_{om} V_{rm} = 1,8 = (V_{om} V_{rm})_{\max} \quad \text{according to IEC 60865-1:2011, Table 2}$$

$$\beta = 0,73 \quad \text{according to IEC 60865-1:2011, Table 3}$$

$$W_m = 108 \cdot 10^{-6} \text{ m}^3 \quad \text{see 6.3}$$

The bending stress during the second short-circuit current flow duration is greater than during the first short-circuit current flow duration calculated in 6.4.2.2.1.1.

The total bending stress becomes:

$$\sigma_{tot,d} = \sqrt{\sigma_{m,d}^2 + \sigma_{st,m,k}^2} = \sqrt{279^2 + 28,8^2} \text{ N/mm}^2 = 281 \text{ N/mm}^2$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \leq q f_y \quad (11)$$

with the lower value of f_y and where $q = 1,32$, see 6.4.2.2.1.1. This gives:

$$\sigma_{tot,d} = 281 \text{ N/mm}^2 \quad \text{greater than} \quad q f_y = 1,32 \cdot 160 \text{ N/mm}^2 = 211 \text{ N/mm}^2$$

Considering only the result of the simplified method, the busbar is not assumed to withstand the short-circuit force. Therefore it is necessary to apply the detailed method to verify that the conductors are assumed to withstand the short-circuit force.

6.4.2.2.2.3 Forces on the supports

The following calculation is made only for information, as a result of the conductors not withstanding the short-circuit force according to the simplified method.

The equivalent static force on the supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3} \quad (15)$$

During the first short-circuit current flow it is, see 6.4.2.2.1.2:

$$(V_F V_{rm})_1 = 1,22$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y it is during the second short-circuit current flow:

$$\frac{\sigma_{\text{tot},d}}{0,8 f_y} = \frac{281 \text{ N/mm}^2}{0,8 \cdot 240 \text{ N/mm}^2} = 1,46$$

therefore we meet range 3 in IEC 60865-1:2011, Table 2,

$$1 < \frac{\sigma_{\text{tot},d}}{0,8 f_y}$$

hence during the second short-circuit current flow

$$(V_F V_{rm})_2 = 1,0$$

According to IEC 60865-1:2011, 5.6, the greater of both values is to be inserted in Equation (15):

$$V_F V_{rm} = \max \{(V_F V_{rm})_1; (V_F V_{rm})_2\} = \max \{1,22; 1,00\} = 1,22$$

For the outer supports (A) it is with $\alpha_A = 0,375$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 1,22 \cdot 0,375 \cdot 10,2 \text{ kN} = 4,67 \text{ kN}$$

For the inner supports (B) it is with $\alpha_B = 1,25$, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 1,22 \cdot 1,25 \cdot 10,2 \text{ kN} = 15,6 \text{ kN}$$

6.4.2.3 Detailed method

6.4.2.3.1 Relevant natural frequency f_{cm} and factors V_F , $V_{\sigma m}$ and V_{rm}

The relevant natural frequency of the main conductors is:

$$f_{cm} = \frac{\gamma}{l^2} \sqrt{\frac{E J_m}{m'_m}} = \frac{2,45}{(18 \text{ m})^2} \cdot \sqrt{\frac{7 \cdot 10^{10} \text{ N/m}^2 \cdot 8,62 \cdot 10^{-6} \text{ m}^4}{7,84 \text{ kg/m}}} = 2,10 \text{ Hz} \quad (16)$$

where

$\gamma = 2,45$ according to IEC 60865-1:2011, Table 3

$J_m = 8,62 \cdot 10^{-6} \text{ m}^4$ see 6.3

The frequency ratio is:

$$\frac{f_{cm}}{f} = \frac{2,10 \text{ Hz}}{50 \text{ Hz}} = 0,042$$

For this ratio, the factors V_F , $V_{\sigma m}$ and V_{rm} are according to IEC 60865-1:2011, 5.7.3, Figure 4 and Figure 5:

$$\begin{aligned}V_F &= 0,36 \\V_{\sigma m} &= 0,32 \\V_{rm} &= 1,0 \quad \text{without three-phase automatic reclosing} \\V_{rm} &= 1,8 \quad \text{with three-phase automatic reclosing}\end{aligned}$$

6.4.2.3.2 Calculation without three-phase automatic reclosing

6.4.2.3.2.1 Conductor bending stress

The maximum bending stress is:

$$\sigma_{m,d} = V_{\sigma m} V_{rm} \beta \frac{F_{m3} l}{8W_m} = 0,32 \cdot 1,0 \cdot 0,73 \cdot \frac{10,2 \cdot 10^3 \text{ N} \cdot 18 \text{ m}}{8 \cdot 108 \cdot 10^{-6} \text{ m}^3} = 49,6 \cdot 10^6 \text{ N/m}^2 = 49,6 \text{ N/mm}^2 \quad (9)$$

where

$$\begin{aligned}V_{\sigma m} V_{rm} &= 0,32 \cdot 1,0 = 0,32 && \text{according to 6.4.2.3.1, value which is less than } 1,0 = (V_{\sigma m} V_{rm})_{\max} \\&&& \text{according to IEC 60865-1:2011, Table 2} \\&\beta && \text{according to IEC 60865-1:2011, Table 3} \\&W_m && 108 \cdot 10^{-6} \text{ m}^3 \quad \text{see 6.3}\end{aligned}$$

The total bending stress is:

$$\sigma_{tot,d} = \sqrt{\sigma_{m,d}^2 + \sigma_{st,m,k}^2} = \sqrt{49,6^2 + 28,8^2} \text{ N/mm}^2 = 57,4 \text{ N/mm}^2$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \leq q f_y \quad (11)$$

with the lower value of f_y and where $q = 1,32$, see 6.4.2.2.1.1. This gives:

$$\sigma_{tot,d} = 57,4 \text{ N/mm}^2 \quad \text{less than} \quad q f_y = 1,32 \cdot 160 \text{ N/mm}^2 = 211 \text{ N/mm}^2$$

6.4.2.3.2.2 Forces on the supports and moments on the substructures

The equivalent static force on the supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3} \quad (15)$$

According to 6.4.2.3.1 above, $V_F V_{rm} = 0,36 \cdot 1,0 = 0,36$ which is lower than the value $1,0 = (V_{\sigma m} V_{rm})_{\max}$ according to IEC 60865-1:2011, Table 2.

For the outer supports (A) it is with $\alpha_A = 0,375$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 0,36 \cdot 1,0 \cdot 0,375 \cdot 10,2 \text{ kN} = 1,38 \text{ kN}$$

For the inner supports (B) it is with $\alpha_B = 1,25$, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 0,36 \cdot 1,0 \cdot 1,25 \cdot 10,2 \text{ kN} = 4,59 \text{ kN}$$

The bending moments on the substructures are:

- on the bottom of the outer insulators

$$M_{IA,d} = F_{r,dA} h_I = 1,38 \text{ kN} \cdot 3,7 \text{ m} = 5,11 \text{ kNm}$$

- on the bottom of the outer supports

$$M_{SA,d} = F_{r,dA} h_S = 1,38 \text{ kN} \cdot 7,0 \text{ m} = 9,66 \text{ kNm}$$

- on the bottom of the inner insulators

$$M_{IB,d} = F_{r,dB} h_I = 4,59 \text{ kN} \cdot 3,7 \text{ m} = 17,0 \text{ kNm}$$

- on the bottom of the inner supports

$$M_{SB,d} = F_{r,dB} h_S = 4,59 \text{ kN} \cdot 7,0 \text{ m} = 32,1 \text{ kNm}$$

6.4.2.3.2.3 Calculation with three-phase automatic reclosing

6.4.2.3.2.3.1 Conductor bending stress

The maximum bending stress during the second short-circuit current flow duration is:

$$\sigma_{m,d} = V_{gm} V_{rm} \beta \frac{F_{m3} l}{8 W_m} = 0,58 \cdot 0,73 \cdot \frac{10,2 \cdot 10^3 \text{ N} \cdot 18 \text{ m}}{8 \cdot 108 \cdot 10^{-6} \text{ m}^3} = 90,0 \cdot 10^6 \text{ N/m}^2 = 90,0 \text{ N/mm}^2 \quad (9)$$

where

$V_{gm} V_{rm} = 0,32 \cdot 1,8 = 0,58$ according to 6.4.2.3.1 above, value which is less than $1,8 = (V_{gm} V_{rm})_{max}$

according to IEC 60865-1:2011, Table 2

$\beta = 0,73$ according to IEC 60865-1:2011, Table 3

$W_m = 108 \cdot 10^{-6} \text{ m}^3$ see 6.3

The bending stress during the second short-circuit current flow duration is greater than during the first short-circuit current flow duration, see 6.4.2.3.2.1.

The total bending stress is:

$$\sigma_{tot,d} = \sqrt{\sigma_{m,d}^2 + \sigma_{st,m,k}^2} = \sqrt{90,0^2 + 28,8^2} \text{ N/mm}^2 = 94,5 \text{ N/mm}^2$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \leq q f_y \quad (11)$$

with the lower value of f_y and where $q = 1,32$, see 6.4.2.2.1.1. This gives:

$$\sigma_{tot,d} = 94,5 \text{ N/mm}^2 \text{ less than } q f_y = 1,32 \cdot 160 \text{ N/mm}^2 = 211 \text{ N/mm}^2$$

6.4.2.3.2.3.2 Forces on the supports and moments on the substructures

The equivalent static force on the supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3}$$

(15)

During the first short-circuit current flow it is, see 6.4.2.3.1:

$$(V_F V_{rm})_1 = 0,36 \cdot 1,0 = 0,36$$

During the second current flow it is according to 6.4.2.3.1

$$(V_F V_{rm})_2 = 0,36 \cdot 1,8 = 0,65$$

which is lower than the value 1,0 according to IEC 60865-1:2011, Table 2.

According to IEC 60865-1:2011, 5.6, the greater of both values is to be inserted in Equation (15):

$$V_F V_{rm} = \max \{(V_F V_{rm})_1; (V_F V_{rm})_2\} = \max \{0,36; 0,65\} = 0,65$$

For the outer supports (A) it is with $\alpha_A = 0,375$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 0,65 \cdot 0,375 \cdot 10,2 \text{ kN} = 2,49 \text{ kN}$$

For the inner supports (B) it is with $\alpha_B = 1,25$, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 0,65 \cdot 1,25 \cdot 10,2 \text{ kN} = 8,29 \text{ kN}$$

The bending moments on the substructures are:

- on the bottom of the outer insulators

$$M_{IA,d} = F_{r,dA} h_l = 2,49 \text{ kN} \cdot 3,7 \text{ m} = 9,21 \text{ kNm}$$

- on the bottom of the outer supports

$$M_{SA,d} = F_{r,dA} h_S = 2,49 \text{ kN} \cdot 7,0 \text{ m} = 17,4 \text{ kNm}$$

- on the bottom of the inner insulators

$$M_{IB,d} = F_{r,dB} h_l = 8,29 \text{ kN} \cdot 3,7 \text{ m} = 30,7 \text{ kNm}$$

- on the bottom of the inner supports

$$M_{SB,d} = F_{r,dB} h_S = 8,29 \text{ kN} \cdot 7,0 \text{ m} = 58,0 \text{ kNm}$$

6.4.3 Conclusions

The busbar will withstand the dead load

The calculated bending stress is

$$\sigma_{st,m,d} \quad 39 \text{ N/mm}^2$$

The outer supports have to withstand a vertical force of

$$F_{st,r,dA} \quad 0,701 \text{ kN}$$

The inner supports have to withstand a vertical force of

$$F_{st,r,dB} \quad 2,335 \text{ kN}$$

		Simplified method	Detailed method
a) Without three-phase automatic reclosure			
The busbar will withstand the short-circuit force			
The calculated bending stress is	$\sigma_{\text{tot,d}}$	158 N/mm ²	57 N/mm ²
The outer supports have to withstand an equivalent static force of	$F_{r,\text{dA}}$	4,7 kN	1,38 kN
The inner supports have to withstand an equivalent static force of	$F_{r,\text{dB}}$	15,6 kN	4,59 kN
b) With three-phase automatic reclosure			
The calculated bending stress is	$\sigma_{\text{tot,d}}$	281 N/mm ²	95 N/mm ²
The busbar is assumed to withstand the short-circuit force when using the detailed method but not when using the simplified method			
The outer supports have to withstand an equivalent static force of	$F_{r,\text{dA}}$	4,7 kN	2,49 kN
The inner supports have to withstand an equivalent static force of	$F_{r,\text{dB}}$	15,6 kN	8,29 kN

7 Example 4. – Mechanical effects on a 110 kV arrangement with slack conductors

7.1 General

The basis for the calculations in this example is a three-phase flexible busbar connection with one all-aluminium stranded conductor per phase with varying distances between the conductors. The anchor points at each end of the span are post insulators on steel substructures as shown in Figure 4.

The effective length of the span is the distance between the axis of the supports reduced by

- the extend of the connection plate of the equipment including the clamp, and
- an additional form factor which depends on conductor stiffness and mounting form, for example 0,1 m to 0,3 m.

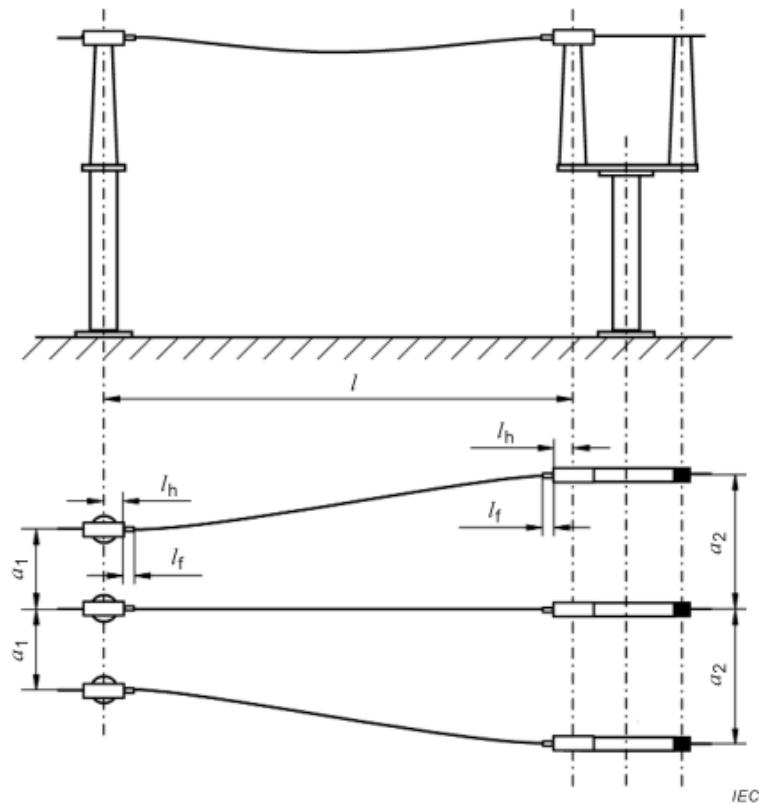


Figure 4 – Arrangement with slack conductors

7.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	=	19 kA
Duration of the first short-circuit current flow	T_{k1}	=	0,3 s
Centre-line distance between supports	l	=	11,5 m
Extend of one head armature and clamp	l_h	=	0,4 m
Form factor	l_f	=	0,15 m
Centre-line distances between conductors	a_1	=	1,6 m
	a_2	=	2,4 m
Resultant spring constant of both span supports	S	=	100 N/mm
All-aluminium stranded conductor EN 243-AL1			
– Number of sub-conductors	n	=	1
– Cross section	A_s	=	243 mm ²
– Mass per unit length	m'_s	=	0,671 kg/m
– Young's modulus	E	=	55 000 N/mm ²
Static tensile force of one flexible main conductor at a temperature of -20°C (local minimum winter temperature)	$F_{st,-20}$	=	350 N
Static tensile force of one flexible main conductor at a temperature of 60°C (maximum operating temperature)	$F_{st,60}$	=	250 N
Conventional value of acceleration of gravity	g	=	9,81 m/s ²

7.3 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0,75 \frac{(I_k'')^2}{a} \frac{l_c}{l_{\text{eff}}} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0,75 \cdot \frac{(19 \cdot 10^3 \text{ A})^2}{2 \text{ m}} \cdot 1,0 = 27,1 \text{ N/m} \quad (19a)$$

with the effective length of the span

$$l_{\text{eff}} = l - 2l_h - 2l_f = 11,5 \text{ m} - 2 \cdot 0,4 \text{ m} - 2 \cdot 0,15 \text{ m} = 10,4 \text{ m}$$

and

$$l_c = l_{\text{eff}}$$

and an equivalent distance

$$a = \frac{a_1 + a_2}{2} = \frac{1,6 \text{ m} + 2,4 \text{ m}}{2} = 2 \text{ m}.$$

The parameter r is:

$$r = \frac{F'}{n m_s' g} = \frac{27,1 \text{ N/m}}{1 \cdot 0,671 \text{ kg/m} \cdot 9,81 \text{ m/s}^2} = 4,12 \quad (20)$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 4,12 = 76,4^\circ \quad (21)$$

The equivalent static conductor sags at midspan are:

$$f_{\text{es},-20} = \frac{n m_s' g l_{\text{eff}}^2}{8 F_{\text{st},-20}} = \frac{1 \cdot 0,671 \text{ kg/m} \cdot 9,81 \text{ m/s}^2 \cdot (10,4 \text{ m})^2}{8 \cdot 350 \text{ N}} = 0,254 \text{ m}$$

$$f_{\text{es},60} = \frac{n m_s' g l_{\text{eff}}^2}{8 F_{\text{st},60}} = \frac{1 \cdot 0,671 \text{ kg/m} \cdot 9,81 \text{ m/s}^2 \cdot (10,4 \text{ m})^2}{8 \cdot 250 \text{ N}} = 0,356 \text{ m} \quad (22)$$

The periods of the conductor oscillation are:

$$T_{-20} = 2\pi \sqrt{0,8 \frac{f_{\text{es},-20}}{g}} = 2\pi \sqrt{0,8 \frac{0,254}{9,81 \text{ m/s}^2}} = 0,904 \text{ s}$$

$$T_{60} = 2\pi \sqrt{0,8 \frac{f_{\text{es},60}}{g}} = 2\pi \sqrt{0,8 \frac{0,356}{9,81 \text{ m/s}^2}} = 1,071 \text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$T_{\text{res},-20} = \frac{T_{-20}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{0,904}{\sqrt[4]{1+4,12^2} \left[1 - \frac{\pi^2}{64} \left(\frac{76,4^\circ}{90^\circ} \right)^2 \right]} = 0,494 \text{ s} \quad (24)$$

$$T_{\text{res},60} = \frac{T_{60}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{1,071}{\sqrt[4]{1+4,12^2} \left[1 - \frac{\pi^2}{64} \left(\frac{76,4^\circ}{90^\circ} \right)^2 \right]} = 0,585 \text{ s}$$

The stiffness norms are:

$$N_{-20} = \frac{1}{S l_{\text{eff}}} + \frac{1}{n E_{\text{eff},-20} A_s} = \frac{1}{10^5 \text{ N/m} \cdot 10,4 \text{ m}} + \frac{1}{1 \cdot 1,82 \cdot 10^{10} \text{ N/m}^2 \cdot 243 \cdot 10^{-6} \text{ m}^2} = 1,188 \cdot 10^{-6} \text{ 1/N} \quad (25)$$

$$N_{60} = \frac{1}{S l_{\text{eff}}} + \frac{1}{n E_{\text{eff},60} A_s} = \frac{1}{10^5 \text{ N/m} \cdot 10,4 \text{ m}} + \frac{1}{1 \cdot 1,78 \cdot 10^{10} \text{ N/m}^2 \cdot 243 \cdot 10^{-6} \text{ m}^2} = 1,193 \cdot 10^{-6} \text{ 1/N}$$

with the actual Young's moduli

$$E_{\text{eff},-20} = E \left[0,3 + 0,7 \sin \left(\frac{F_{\text{st},-20}}{n A_s \sigma_{\text{fin}}} 90^\circ \right) \right] = 55 \cdot 10^9 \frac{\text{N}}{\text{m}^2} \cdot \left[0,3 + 0,7 \sin \left(\frac{1,44 \cdot 10^6 \text{ N/m}^2}{50 \cdot 10^6 \text{ N/m}^2} \cdot 90^\circ \right) \right]$$

$$= 1,82 \cdot 10^{10} \text{ N/m}^2 \quad (26)$$

$$E_{\text{eff},60} = E \left[0,3 + 0,7 \sin \left(\frac{F_{\text{st},60}}{n A_s \sigma_{\text{fin}}} 90^\circ \right) \right] = 55 \cdot 10^9 \frac{\text{N}}{\text{m}^2} \cdot \left[0,3 + 0,7 \sin \left(\frac{1,03 \cdot 10^6 \text{ N/m}^2}{50 \cdot 10^6 \text{ N/m}^2} \cdot 90^\circ \right) \right]$$

$$= 1,78 \cdot 10^{10} \text{ N/m}^2$$

because

$$\frac{F_{\text{st},-20}}{n A_s} = \frac{350 \text{ N}}{1 \cdot 243 \cdot 10^{-6} \text{ m}^2} = 1,44 \cdot 10^6 \text{ N/m}^2 \quad \text{less than} \quad \sigma_{\text{fin}} = 50 \cdot 10^6 \text{ N/m}^2$$

$$\frac{F_{\text{st},60}}{n A_s} = \frac{250 \text{ N}}{1 \cdot 243 \cdot 10^{-6} \text{ m}^2} = 1,03 \cdot 10^6 \text{ N/m}^2 \quad \text{less than} \quad \sigma_{\text{fin}} = 50 \cdot 10^6 \text{ N/m}^2$$

The stress factors are:

$$\zeta_{-20} = \frac{(n g m'_s l_{\text{eff}})^2}{24 F_{\text{st},-20}^3 N_{-20}} = \frac{\left(1 \cdot 9,81 \text{ m/s}^2 \cdot 0,671 \text{ kg/m} \cdot 10,4 \text{ m} \right)^2}{24 (350 \text{ N})^3 \cdot 1,188 \cdot 10^{-6} \text{ 1/N}} = 3,84 \quad (28)$$

$$\zeta_{60} = \frac{(n g m'_s l_{\text{eff}})^2}{24 F_{\text{st},60}^3 N_{60}} = \frac{\left(1 \cdot 9,81 \text{ m/s}^2 \cdot 0,671 \text{ kg/m} \cdot 10,4 \text{ m} \right)^2}{24 (250 \text{ N})^3 \cdot 1,193 \cdot 10^{-6} \text{ 1/N}} = 10,5$$

Because

$$T_{k1} = 0,3 \text{ s} \quad \text{less than} \quad 0,4 T_{-20} = 0,4 \cdot 0,904 \text{ s} = 0,361 \text{ s}$$

$$T_{k1} = 0,3 \text{ s} \quad \text{less than} \quad 0,4 T_{60} = 0,4 \cdot 1,071 \text{ s} = 0,428 \text{ s}$$

in the Equations (29), (32), and (35) it is to be inserted:

$$T_{k1} = 0,3 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{\text{end},-20} = \delta_{\text{end},60} = 2\delta_1 = 2 \cdot 76,4^\circ = 153^\circ \quad (29)$$

because

$$\begin{aligned} \frac{T_{k1}}{T_{\text{res},-20}} &= \frac{0,3 \text{ s}}{0,494 \text{ s}} = 0,607 \quad \text{greater than } 0,5 \\ \frac{T_{k1}}{T_{\text{res},60}} &= \frac{0,3 \text{ s}}{0,585 \text{ s}} = 0,513 \quad \text{greater than } 0,5 \end{aligned}$$

The maximum swing-out angles $\delta_{\text{max},-20}$ and $\delta_{\text{max},60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{\text{end},-20}$ and $\delta_{\text{end},60}$:

For $\delta_{\text{end},-20} = \delta_{\text{end},60} = 153^\circ$ greater than 90° it is:

$$\chi_{-20} = \chi_{60} = 1 - r = 1 - 4,12 = -3,12 \quad (30)$$

and for $\chi_{-20} = \chi_{60} = -3,12$ less than $-0,985$ it is:

$$\delta_{\text{max},-20} = \delta_{\text{max},60} = 180^\circ \quad (31)$$

7.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3 \left(\sqrt{1+r^2} - 1 \right) = 3 \left(\sqrt{1+4,12^2} - 1 \right) = 9,72 \quad (32)$$

because

$$\begin{aligned} T_{k1} = 0,3 \text{ s} &\quad \text{greater than } \frac{T_{\text{res},-20}}{4} = \frac{0,494 \text{ s}}{4} = 0,124 \text{ s} \\ T_{k1} = 0,3 \text{ s} &\quad \text{greater than } \frac{T_{\text{res},60}}{4} = \frac{0,585 \text{ s}}{4} = 0,146 \text{ s} \end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 9,72$ and $\zeta_{-20} = 3,84$:

$$\psi_{-20} = 0,594$$

- for $\varphi_{60} = 9,72$ and $\zeta_{60} = 10,5$:

$$\psi_{60} = 0,745$$

The tensile forces during the short-circuit are:

$$\begin{aligned} F_{t,d,-20} &= F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 350 \text{ N} \cdot (1 + 9,72 \cdot 0,594) = 2371 \text{ N} = 2,37 \text{ kN} \\ F_{t,d,60} &= F_{st,60} (1 + \varphi_{60} \psi_{60}) = 250 \text{ N} \cdot (1 + 9,72 \cdot 0,745) = 2060 \text{ N} = 2,06 \text{ kN} \end{aligned} \quad (33)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{2,37 \text{ kN}; 2,06 \text{ kN}\} = 2,37 \text{ kN}$$

7.5 Dynamic conductor sag at midspan

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of -20°C.

The elastic expansion is:

$$\varepsilon_{ela} = N_{60} (F_{t,d,60} - F_{st,60}) = 1,193 \cdot 10^{-6} \frac{1}{\text{N}} \cdot (2060 \text{ N} - 250 \text{ N}) = 2,16 \cdot 10^{-3} \quad (34)$$

The thermal expansion is:

$$\varepsilon_{th} = c_{th} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{res,60}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{19 \cdot 10^3 \text{ A}}{1 \cdot 243 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{0,585 \text{ s}}{4} = 2,41 \cdot 10^{-4} \quad (35)$$

because

$$T_{k1} = 0,3 \text{ s} \quad \text{greater than} \quad \frac{T_{res,60}}{4} = \frac{0,585 \text{ s}}{4} = 0,146 \text{ s}$$

with

$$c_{th} = 0,27 \cdot 10^{-18} \text{ m}^4 / (\text{A}^2 \text{s}) \quad \text{for all-aluminium conductors}$$

The factor C_D is:

$$C_D = \sqrt{1 + \frac{3}{8} \left(\frac{l_{eff}}{f_{es,60}} \right)^2 (\varepsilon_{ela} + \varepsilon_{th})} = \sqrt{1 + \frac{3}{8} \left(\frac{10,4 \text{ m}}{0,356 \text{ m}} \right)^2 (2,16 \cdot 10^{-3} + 2,41 \cdot 10^{-4})} = 1,33 \quad (36)$$

The factor C_F is:

$$C_F = 1,15 \quad (37)$$

because

$$r = 4,12 \quad \text{greater than} \quad 1,8$$

The dynamic conductor sag at midspan is:

$$f_{ed} = C_F C_D f_{es,60} = 1,15 \cdot 1,33 \cdot 0,356 \text{ m} = 0,55 \text{ m} \quad (38)$$

7.6 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 4,12 \quad \text{greater than} \quad 0,6$$

and

$$\delta_{\max,-20} = \delta_{\max,60} = 180^\circ \quad \text{greater than} \quad 70^\circ$$

the drop force after short-circuit $F_{f,d}$ is significant:

$$\begin{aligned} F_{f,d,-20} &= 1,2 F_{st,-20} \sqrt{1+8 \zeta_{-20} \frac{\delta_{\max,-20}}{180^\circ}} = 1,2 \cdot 350 \text{ N} \cdot \sqrt{1+8 \cdot 3,84 \cdot \frac{180^\circ}{180^\circ}} = 2366 \text{ N} = 2,37 \text{ kN} \\ F_{f,d,60} &= 1,2 F_{st,60} \sqrt{1+8 \zeta_{60} \frac{\delta_{\max,60}}{180^\circ}} = 1,2 \cdot 250 \text{ N} \cdot \sqrt{1+8 \cdot 10,5 \cdot \frac{180^\circ}{180^\circ}} = 2766 \text{ N} = 2,77 \text{ kN} \end{aligned} \quad (43)$$

The drop force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max \{F_{f,-20}; F_{f,d,60}\} = \max \{2,37 \text{ kN}; 2,77 \text{ kN}\} = 2,77 \text{ kN}$$

7.7 Horizontal span displacement b_h and minimum air clearance a_{\min}

The maximum horizontal span displacement is:

$$b_h = f_{ed} = 0,55 \text{ m} \quad (44)$$

because

$$\delta_{\max,60} = 180^\circ \quad \text{greater than} \quad 90^\circ$$

and the minimum air clearance is:

$$a_{\min} = a - 2 b_h = 2 \text{ m} - 2 \cdot 0,55 \text{ m} = 0,90 \text{ m} \quad (48)$$

7.8 Conclusions

According to IEC 60865-1:2011, 6.5.1 and 6.5.3, the supports (post insulators and steel structures) and the foundations have to withstand a bending force

$$\max \{F_{t,d}; F_{f,d}\} = \max \{2,37 \text{ kN}; 2,77 \text{ kN}\} = 2,77 \text{ kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop.

The clamping device for the conductor anchoring shall be specified with a rating based on the force

$$\max \{1,5 F_{t,d} ; 1,0 F_{f,d}\} = \max \{1,5 \cdot 2,37 \text{ kN} ; 1,0 \cdot 2,77 \text{ kN}\} = \max \{3,56 \text{ kN} ; 2,77 \text{ kN}\} = 3,56 \text{ kN}$$

The horizontal displacement is 0,55 m and the minimum air clearance is 0,90 m.

8 Example 5. – Mechanical effects on strained conductors

8.1 General

The basis for the calculations in this example is a three-phase 380 kV arrangement with strained twin-bundle conductors as shown in Figure 5. In the span there are two connections of pantograph-disconnectors, which also operate as spacers, and between the connections one spacer.

The calculation is carried out for two different centre-line distances between sub-conductors showing the effect of the pinch force.

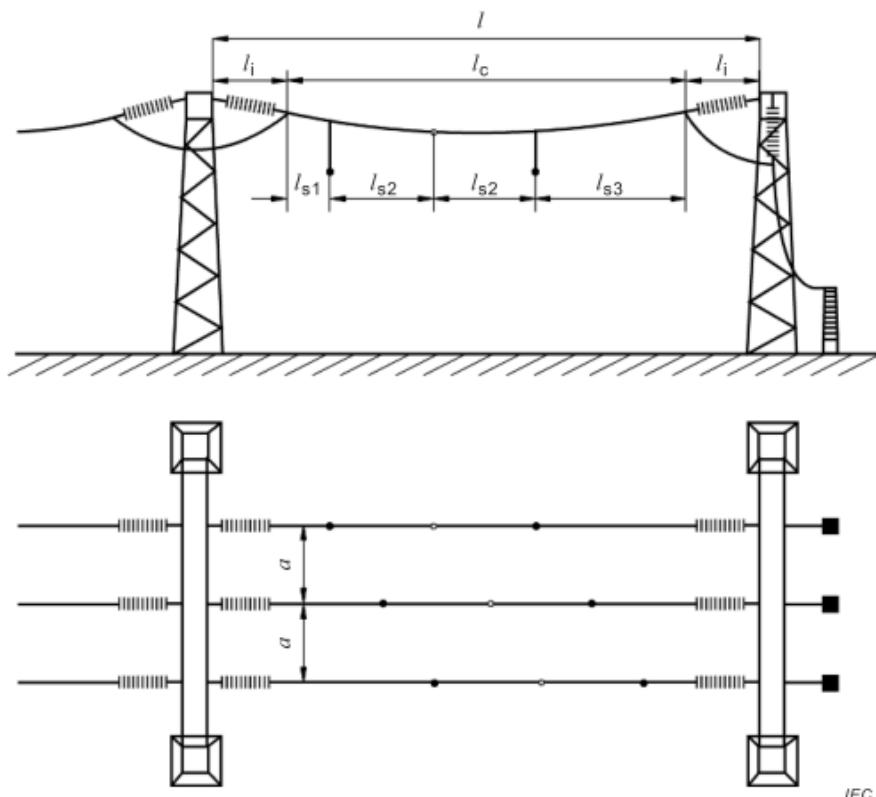


Figure 5 – Arrangement with strained conductors

8.2 Common data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	=	63 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,81
Duration of the short-circuit current flow	T_{k1}	=	0,5 s
System frequency	f	=	50 Hz
Centre-line distance between supports	l	=	48 m
Length of one insulator chain	l_i	=	5,3 m

Cord length $l_c = l - 2l_i$	l_c	=	37,4 m
Centre-line distance between conductors	a	=	5 m
Resultant spring constant of both span supports of one span	S	=	500 N/mm
Twin conductor 2 EN 1046-AL1/45-ST1A			
– Number of sub-conductors	n	=	2
– Outer diameter of one sub-conductor	d	=	43 mm
– Cross-section of one sub-conductor	A_s	=	1 090 mm ²
– Mass per unit length of one sub-conductor	m'_s	=	3,25 kg/m
– Young's modulus	E	=	60 000 N/mm ²
Static tensile force of one flexible main conductor at a temperature of -20°C (local minimum winter temperature)	$F_{st,-20}$	=	17,8 kN
Static tensile force of one flexible main conductor at a temperature of 60°C (maximum operating temperature)	$F_{st,60}$	=	15,4 kN
Additional concentrated masses representing the connections of pantograph disconnectors			
– Number of spacers	n_c	=	3
– Mass of one connection	m_c	=	36 kg
– Mass of one spacer	m_{cs}	=	2 kg
– Distances	l_{s1}	=	4,2 m
	l_{s2}	=	9,5 m
	l_{s3}	=	14,2 m
Conventional value of acceleration of gravity	g	=	9,81 m/s ²

8.3 Centre-line distance between sub-conductors $a_s = 0,1 \text{ m}$

8.3.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0,75 \frac{(I_k')^2}{a} \frac{l_c}{l} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0,75 \cdot \frac{(63 \cdot 10^3 \text{ A})^2}{5 \text{ m}} \cdot \frac{37,4 \text{ m}}{48 \text{ m}} = 92,8 \text{ N/m} \quad (19a)$$

The parameter r is:

$$r = \frac{F'}{n m'_{sc} g} = \frac{92,8 \text{ N/m}}{2 \cdot 4,24 \text{ kg/m} \cdot 9,81 \text{ m/s}^2} = 1,12 \quad (20)$$

where m'_{sc} is the resulting mass per unit length of one sub-conductor including concentrated masses:

$$m'_{sc} = m'_s + \frac{2m_c + m_{cs}}{nl_c} = 3,25 \frac{\text{kg}}{\text{m}} + \frac{2 \cdot 36 \text{ kg} + 2 \text{ kg}}{2 \cdot 37,4 \text{ m}} = 4,24 \text{ kg/m}$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 1,12 = 48,2^\circ \quad (21)$$

The equivalent static conductor sags at midspan are:

$$f_{es,-20} = \frac{n m'_{sc} g l^2}{8 F_{st,-20}} = \frac{2 \cdot 4,24 \text{ kg/m} \cdot 9,81 \text{ m/s}^2 \cdot (48 \text{ m})^2}{8 \cdot 17,8 \cdot 10^3 \text{ N}} = 1,35 \text{ m} \quad (22)$$

$$f_{es,60} = \frac{n m'_{sc} g l^2}{8 F_{st,60}} = \frac{2 \cdot 4,24 \text{ kg/m} \cdot 9,81 \text{ m/s}^2 \cdot (48 \text{ m})^2}{8 \cdot 15,4 \cdot 10^3 \text{ N}} = 1,56 \text{ m}$$

The periods of the conductor oscillation are:

$$T_{-20} = 2\pi \sqrt{0,8 \frac{f_{es,-20}}{g}} = 2\pi \sqrt{0,8 \frac{1,35 \text{ m}}{9,81 \text{ m/s}^2}} = 2,09 \text{ s} \quad (23)$$

$$T_{60} = 2\pi \sqrt{0,8 \frac{f_{es,60}}{g}} = 2\pi \sqrt{0,8 \frac{1,56 \text{ m}}{9,81 \text{ m/s}^2}} = 2,24 \text{ s}$$

The resulting periods of the conductor oscillation are:

$$T_{res,-20} = \frac{T_{-20}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{2,09}{\sqrt[4]{1+1,12^2} \left[1 - \frac{\pi^2}{64} \left(\frac{48,2^\circ}{90^\circ} \right)^2 \right]} = 1,79 \text{ s} \quad (24)$$

$$T_{res,60} = \frac{T_{60}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{2,24}{\sqrt[4]{1+1,12^2} \left[1 - \frac{\pi^2}{64} \left(\frac{48,2^\circ}{90^\circ} \right)^2 \right]} = 1,91 \text{ s}$$

The stiffness norms are:

$$N_{-20} = \frac{1}{Sl} + \frac{1}{n E_{eff,-20} A_s} = \frac{1}{5 \cdot 10^5 \text{ N/m} \cdot 48 \text{ m}} + \frac{1}{2 \cdot 2,87 \cdot 10^{10} \text{ N/m}^2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 5,77 \cdot 10^{-8} \text{ 1/N} \quad (25)$$

$$N_{60} = \frac{1}{Sl} + \frac{1}{n E_{s,60} A_s} = \frac{1}{5 \cdot 10^5 \text{ N/m} \cdot 48 \text{ m}} + \frac{1}{2 \cdot 2,72 \cdot 10^{10} \text{ N/m}^2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 5,85 \cdot 10^{-8} \text{ 1/N}$$

with the actual Young's moduli

$$E_{eff,-20} = E \left[0,3 + 0,7 \sin \left(\frac{F_{st,-20}}{n A_s \sigma_{fin}} 90^\circ \right) \right] = 6 \cdot 10^{10} \frac{\text{N}}{\text{m}^2} \cdot \left[0,3 + 0,7 \sin \left(\frac{8,17 \cdot 10^6 \text{ N/m}^2}{50 \cdot 10^6 \text{ N/m}^2} 90^\circ \right) \right]$$

$$= 2,87 \cdot 10^{10} \text{ N/m}^2$$

$$E_{eff,60} = E \left[0,3 + 0,7 \sin \left(\frac{F_{st,60}}{n A_s \sigma_{fin}} 90^\circ \right) \right] = 6 \cdot 10^{10} \frac{\text{N}}{\text{m}^2} \cdot \left[0,3 + 0,7 \sin \left(\frac{7,06 \cdot 10^6 \text{ N/m}^2}{50 \cdot 10^6 \text{ N/m}^2} 90^\circ \right) \right]$$

$$= 2,72 \cdot 10^{10} \text{ N/m}^2 \quad (26)$$

because

$$\frac{F_{st,-20}}{n A_s} = \frac{17,8 \cdot 10^3 \text{ N}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 8,17 \cdot 10^6 \text{ N/m}^2 \quad \text{less than} \quad \sigma_{fin} = 50 \cdot 10^6 \text{ N/m}^2$$

$$\frac{F_{st,60}}{n A_s} = \frac{15,4 \cdot 10^3 \text{ N}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 7,06 \cdot 10^6 \text{ N/m}^2 \quad \text{less than} \quad \sigma_{fin} = 50 \cdot 10^6 \text{ N/m}^2$$

The stress factors are:

$$\zeta_{-20} = \frac{(ng m'_{sc} l)^2}{24 F_{st,-20}^3 N_{-20}} = \frac{(2 \cdot 9,81 \text{ m/s}^2 \cdot 4,24 \text{ kg/m} \cdot 48 \text{ m})^2}{24 (17,8 \cdot 10^3 \text{ N})^3 \cdot 5,77 \cdot 10^{-8} \text{ 1/N}} = 2,04 \quad (28)$$

$$\zeta_{60} = \frac{(ng m'_{sc} l)^2}{24 F_{st,60}^3 N_{60}} = \frac{(2 \cdot 9,81 \text{ m/s}^2 \cdot 4,24 \text{ kg/m} \cdot 48 \text{ m})^2}{24 (15,4 \cdot 10^3 \text{ N})^3 \cdot 5,85 \cdot 10^{-8} \text{ 1/N}} = 3,11$$

Because

$$T_{k1} = 0,5 \text{ s} \quad \text{less than} \quad 0,4 T_{-20} = 0,4 \cdot 2,09 \text{ s} = 0,836 \text{ s}$$

$$T_{k1} = 0,5 \text{ s} \quad \text{less than} \quad 0,4 T_{60} = 0,4 \cdot 2,24 \text{ s} = 0,896 \text{ s}$$

in the Equations (29), (32), and (35) it is to be inserted:

$$T_{k1} = 0,5 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{end,-20} = \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{res,-20}} \right) \right] = 48,2^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,79 \text{ s}} \right) \right] = 57,0^\circ \quad (29)$$

$$\delta_{end,60} = \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{res,60}} \right) \right] = 48,2^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,91 \text{ s}} \right) \right] = 51,8^\circ$$

because

$$\frac{T_{k1}}{T_{res,-20}} = \frac{0,5 \text{ s}}{1,79 \text{ s}} = 0,279 \quad \text{less than} \quad 0,5$$

$$\frac{T_{k1}}{T_{res,60}} = \frac{0,5 \text{ s}}{1,91 \text{ s}} = 0,262 \quad \text{less than} \quad 0,5$$

The maximum swing-out angles $\delta_{max,-20}$ and $\delta_{max,60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{end,-20}$ and $\delta_{end,60}$:

– for 0 less than $\delta_{end,-20} = 57,0^\circ$ less than 90° is:

$$\chi_{-20} = 1 - r \sin \delta_{end,-20} = 1 - 1,12 \cdot \sin 57,0^\circ = 0,0607 \quad (30)$$

and for $-0,985$ less than $\chi_{-20} = 0,0607$ less than $0,766$:

$$\delta_{max,-20} = 10^\circ + \arccos \chi_{-20} = 10^\circ + \arccos 0,0607 = 96,5^\circ \quad (31)$$

- for 0 less than $\delta_{\text{end},60} = 51,8^\circ$ less than 90° is:

$$\chi_{60} = 1 - r \sin \delta_{\text{end},60} = 1 - 1,12 \cdot \sin 51,8^\circ = 0,120 \quad (30)$$

and for $-0,985$ less than $\chi_{60} = 0,120$ less than $0,766$:

$$\delta_{\max,60} = 10^\circ + \arccos \chi_{60} = 10^\circ + \arccos 0,120 = 93,1^\circ \quad (31)$$

8.3.2 Tensile force $F_{t,d}$ during short-circuit caused by swing out

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3 \left(\sqrt{1+r^2} - 1 \right) = 3 \left(\sqrt{1+1,12^2} - 1 \right) = 1,50 \quad (32)$$

because

$$\begin{aligned} T_{k1} &= 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},-20}}{4} = \frac{1,79 \text{ s}}{4} = 0,448 \text{ s} \\ T_{k1} &= 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},60}}{4} = \frac{1,91 \text{ s}}{4} = 0,478 \text{ s} \end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 1,50$ and $\zeta_{-20} = 2,04$:

$$\psi_{-20} = 0,691$$

- for $\varphi_{60} = 1,50$ and $\zeta_{60} = 3,11$:

$$\psi_{60} = 0,759$$

The tensile forces during the short-circuit forces are:

$$F_{t,d,-20} = F_{st,-20}(1+\varphi_{-20}\psi_{-20}) = 17,8 \text{ kN} \cdot (1+1,50 \cdot 0,691) = 36,3 \text{ kN}$$

$$F_{t,d,60} = F_{st,60}(1+\varphi_{60}\psi_{60}) = 15,4 \text{ kN} \cdot (1+1,50 \cdot 0,759) = 32,9 \text{ kN} \quad (33)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{36,3 \text{ kN}; 32,9 \text{ kN}\} = 36,3 \text{ kN}$$

8.3.3 Dynamic conductor sag at midspan

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than at a conductor temperature of -20°C .

The elastic expansion is:

$$\varepsilon_{\text{ela}} = N_{60} (F_{t,d,60} - F_{st,60}) = 5,85 \cdot 10^{-8} \frac{1}{N} \cdot (32,9 - 15,4) \cdot 10^3 \text{ N} = 1,02 \cdot 10^{-3} \quad (34)$$

The thermal expansion is:

$$\varepsilon_{\text{th}} = c_{\text{th}} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{\text{res},60}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{1,91 \text{ s}}{4} = 1,08 \cdot 10^{-4} \quad (35)$$

because

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},60}}{4} = \frac{1,91 \text{ s}}{4} = 0,478 \text{ s}$$

and for ASCR conductors with $A_{\text{Al}}/A_{\text{St}} = 1046 \text{ mm}/45 \text{ mm} = 23,2$ greater than 6:

$$c_{\text{th}} = 0,27 \cdot 10^{-18} \text{ m}^4/(\text{A}^2 \text{s})$$

The factor C_D is:

$$C_D = \sqrt{1 + \frac{3}{8} \left(\frac{l}{f_{\text{es},60}} \right)^2 (\varepsilon_{\text{ela}} + \varepsilon_{\text{th}})} = \sqrt{1 + \frac{3}{8} \left(\frac{48 \text{ m}}{1,56 \text{ m}} \right)^2 (1,02 \cdot 10^{-3} + 1,08 \cdot 10^{-4})} = 1,18 \quad (36)$$

The factor C_F is:

$$C_F = 0,97 + 0,1r = 0,97 + 0,1 \cdot 1,12 = 1,08 \quad (37)$$

because

$$0,8 \quad \text{less than} \quad r = 1,12 \quad \text{less than} \quad 1,8$$

The dynamic conductor sag at midspan is:

$$f_{\text{ed}} = C_F C_D f_{\text{es},60} = 1,08 \cdot 1,18 \cdot 1,56 \text{ m} = 1,99 \text{ m} \quad (38)$$

8.3.4 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 1,12 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned} \delta_{\text{max},-20} &= 96,5^\circ & \text{greater than} & 70^\circ \\ \delta_{\text{max},60} &= 93,1^\circ & \text{greater than} & 70^\circ \end{aligned}$$

the tensile force after short-circuit $F_{f,d}$ is significant:

$$\begin{aligned} F_{f,d,-20} &= 1,2 \cdot F_{\text{st},-20} \sqrt{1 + 8 \zeta_{-20} \frac{\delta_{\text{max},-20}}{180^\circ}} = 1,2 \cdot 17,8 \text{ kN} \cdot \sqrt{1 + 8 \cdot 2,04 \cdot \frac{96,5^\circ}{180^\circ}} = 66,7 \text{ kN} \\ F_{f,d,60} &= 1,2 \cdot F_{\text{st},60} \sqrt{1 + 8 \zeta_{60} \frac{\delta_{\text{max},60}}{180^\circ}} = 1,2 \cdot 15,4 \text{ kN} \cdot \sqrt{1 + 8 \cdot 3,11 \cdot \frac{93,1^\circ}{180^\circ}} = 68,8 \text{ kN} \end{aligned} \quad (43)$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max \{F_{f,d,-20}; F_{f,d,60}\} = \max \{66,7 \text{ kN}; 68,8 \text{ kN}\} = 68,8 \text{ kN}$$

8.3.5 Horizontal span displacement b_h and minimum air clearance a_{\min}

The maximum horizontal span displacement for strained conductors with $I_c = l - 2l_i$ is:

$$b_h = f_{ed} \sin \delta_1 = 1,99 \text{ m} \cdot \sin 48,2^\circ = 1,48 \text{ m} \quad (45)$$

because

$$\delta_{\max,60} = 93,1^\circ \quad \text{greater than} \quad \delta_1 = 48,2^\circ$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,48 \text{ m} = 2,04 \text{ m} \quad (48)$$

8.3.6 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled:

$$\frac{a_s}{d} = \frac{0,1 \text{ m}}{0,043 \text{ m}} = 2,33 \quad \text{less than} \quad 2,5 \quad (53)$$

and

$$I_s = 9,35 \text{ m} \quad \text{greater than} \quad 70a_s = 70 \cdot 0,1 \text{ m} = 7 \text{ m} \quad (53)$$

with

$$I_s = \frac{I_{s1} + 2I_{s2} + I_{s3}}{4} = \frac{I_c}{4} = \frac{4,2 + 2 \cdot 9,5 + 14,2}{4} \text{ m} = \frac{37,4 \text{ m}}{4} = 9,35 \text{ m}$$

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1,1F_{t,d,-20} = 1,1 \cdot 36,3 \text{ kN} = 39,9 \text{ kN} \\ F_{pi,d,60} &= 1,1F_{t,d,60} = 1,1 \cdot 32,9 \text{ kN} = 36,2 \text{ kN} \end{aligned} \quad (51)$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 8.3.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max \{F_{pi,d,-20}; F_{pi,d,60}\} = \max \{39,9 \text{ kN}; 36,2 \text{ kN}\} = 39,9 \text{ kN}$$

8.3.7 Conclusions

According to IEC 60865-1:2011, 6.5.2, to the structure, the insulators and the connectors the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max\{36,3 \text{ kN}; 68,8 \text{ kN}; 39,9 \text{ kN}\} = 68,8 \text{ kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop.

The maximum horizontal displacement is 1,48 m and the minimum air clearance is 2,04 m.

8.4 Centre-line distance between sub-conductors $a_s = 0,4 \text{ m}$

8.4.1 Preliminary remarks

In this case it is

$$\frac{a_s}{d} = \frac{0,400 \text{ m}}{0,043 \text{ m}} = 9,30$$

and neither Equation (52) nor Equation (53) of IEC 60865-1:2011 is fulfilled. Therefore the pinch force $F_{pi,d}$ is to be calculated with the Equations (54) and following of IEC 60865-1:2011, 6.4. The other results are the same as 8.3.2, 8.3.3, 8.3.4 and 8.3.5, they do not depend on the centre-line distance of the sub-conductors:

Tensile force during short-circuit	$F_{t,d} = 36,3 \text{ kN}$
Tensile force after short-circuit	$F_{f,d} = 68,8 \text{ kN}$
Maximum horizontal displacement	$b_h = 1,48 \text{ m}$
Minimum air clearance	$a_{min} = 2,04 \text{ m}$

8.4.2 Characteristic dimensions and parameters

The short-circuit current force between the sub-conductors is:

$$F_v = (n-1) \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 \frac{l_s}{a_s} v_2 \quad (54)$$

The factor v_1 for calculation of v_2 is:

$$v_1 = f \frac{1}{\sin \frac{180^\circ}{n}} \sqrt{\frac{(a_s - d)m'_s}{\frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 n - 1}} = 50 \frac{1}{\text{s}} \cdot \frac{1}{\sin \frac{180^\circ}{2}} \cdot \sqrt{\frac{(0,400 \text{ m} - 0,043 \text{ m}) \cdot 3,25 \text{ kg/m}}{\frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \left(\frac{63 \cdot 10^3 \text{ A}}{2} \right)^2 \cdot \frac{2-1}{0,400 \text{ m}}}} = 2,42 \quad (55)$$

According to IEC 60865-1:2011, Figure 9, the factor v_2 for $v_1 = 2,42$ and $\kappa = 1,81$ is:

$$v_2 = 2,22$$

According to IEC 60865-1:2011, Figure 10, the factor v_3 for $a_s/d = 9,3$ is:

$$v_3 = 0,250$$

With this the short-circuit current force between the sub-conductors is:

$$F_v = (n-1) \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 \frac{l_s}{a_s} \frac{v_2}{v_3} = (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2} \right)^2 \cdot \frac{9,35 \text{ m}}{0,4 \text{ m}} \cdot \frac{2,22}{0,25} = 41,2 \cdot 10^3 \text{ N} = 41,2 \text{ kN} \quad (54)$$

The strain factors are:

$$\varepsilon_{st,-20} = 1,5 \frac{F_{st,-20} l_s^2 N_{-20}}{(a_s - d)^2} \left(\sin \frac{180^\circ}{n} \right)^2 = 1,5 \cdot \frac{17,8 \cdot 10^3 \text{ N} \cdot (9,35 \text{ m})^2 \cdot 5,77 \cdot 10^{-8} \text{ 1/N}}{(0,400 \text{ m} - 0,043 \text{ m})^2} \cdot \left(\sin \frac{180^\circ}{2} \right)^2 = 1,06 \quad (56)$$

$$\varepsilon_{st,60} = 1,5 \frac{F_{st,60} l_s^2 N_{60}}{(a_s - d)^2} \left(\sin \frac{180^\circ}{n} \right)^2 = 1,5 \cdot \frac{15,4 \cdot 10^3 \text{ N} \cdot (9,35 \text{ m})^2 \cdot 5,85 \cdot 10^{-8} \text{ 1/N}}{(0,400 \text{ m} - 0,043 \text{ m})^2} \cdot \left(\sin \frac{180^\circ}{2} \right)^2 = 0,927$$

$$\varepsilon_{pi,-20} = 0,375 n \frac{F_v l_s^3 N_{-20}}{(a_s - d)^3} \left(\sin \frac{180^\circ}{n} \right)^3 = 0,375 \cdot 2 \cdot \frac{41,2 \cdot 10^3 \text{ N} (9,35 \text{ m})^3 \cdot 5,77 \cdot 10^{-8} \text{ 1/N}}{(0,400 \text{ m} - 0,043 \text{ m})^3} \cdot \left(\sin \frac{180^\circ}{2} \right)^3 = 32,0 \quad (57)$$

$$\varepsilon_{pi,60} = 0,375 n \frac{F_v l_s^3 N_{60}}{(a_s - d)^3} \left(\sin \frac{180^\circ}{n} \right)^3 = 0,375 \cdot 2 \cdot \frac{41,2 \cdot 10^3 \text{ N} (9,35 \text{ m})^3 \cdot 5,85 \cdot 10^{-8} \text{ 1/N}}{(0,400 \text{ m} - 0,043 \text{ m})^3} \cdot \left(\sin \frac{180^\circ}{2} \right)^3 = 32,5$$

The parameters j_{-20} and j_{60} are:

$$j_{-20} = \sqrt{\frac{\varepsilon_{pi,-20}}{1 + \varepsilon_{st,-20}}} = \sqrt{\frac{32,0}{1 + 1,06}} = 3,94$$

$$j_{60} = \sqrt{\frac{\varepsilon_{pi,60}}{1 + \varepsilon_{st,60}}} = \sqrt{\frac{32,5}{1 + 0,927}} = 4,11 \quad (58)$$

8.4.3 Pinch force $F_{pi,d}$

Because

$$j_{-20} = 3,94 \quad \text{greater than} \quad 1$$

$$j_{60} = 4,11 \quad \text{greater than} \quad 1$$

the sub-conductors clash and the tensile forces due to contraction are calculated according to IEC 60865-1:2011, 6.4.2:

$$F_{pi,d,-20} = F_{st,-20} \left(1 + \frac{\nu_{e,-20}}{\varepsilon_{st,-20}} \xi_{-20} \right)$$

$$F_{pi,d,60} = F_{st,60} \left(1 + \frac{\nu_{e,60}}{\varepsilon_{st,60}} \xi_{60} \right) \quad (59)$$

According to IEC 60865-1:2011, Figure 11, and

- with $j_{-20} = 3,94$ and $\varepsilon_{st,-20} = 1,06$ the factor ξ_{-20} is:

$$\xi_{-20} = 2,86$$

- with $j_{60} = 4,11$ and $\varepsilon_{st,60} = 0,927$ the factor ξ_{60} is:

$$\xi_{60} = 2,91$$

The factors $\nu_{e,-20}$ and $\nu_{e,60}$ are:

$$\begin{aligned}\nu_{e,-20} &= \frac{1}{2} + \left[\frac{9}{8} n(n-1) \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 N_{-20} \nu_2 \left(\frac{l_s}{a_s - d} \right)^4 \frac{\left(\sin \frac{180^\circ}{n} \right)^4}{\xi_{-20}^3} \left\{ 1 - \frac{\arctan \sqrt{\nu_4}}{\sqrt{\nu_4}} \right\} - \frac{1}{4} \right]^{1/2} \\ &= \frac{1}{2} + \left[\frac{9}{8} \cdot 2 \cdot (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2} \right)^2 \cdot 5,77 \cdot 10^{-8} \frac{1}{N} \cdot 2,22 \cdot \left(\frac{9,35 \text{ m}}{0,400 \text{ m} - 0,043 \text{ m}} \right)^4 \right. \\ &\quad \left. \cdot \frac{\left(\sin \frac{180^\circ}{2} \right)^4}{2,87^3} \left\{ 1 - \frac{\arctan \sqrt{8,3}}{\sqrt{8,3}} \right\} - \frac{1}{4} \right]^{1/2} \\ &= 1,14\end{aligned}\quad (60)$$

$$\begin{aligned}\nu_{e,60} &= \frac{1}{2} + \left[\frac{9}{8} n(n-1) \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 N_{60} \nu_2 \left(\frac{l_s}{a_s - d} \right)^4 \frac{\left(\sin \frac{180^\circ}{n} \right)^4}{\xi_{60}^3} \left\{ 1 - \frac{\arctan \sqrt{\nu_4}}{\sqrt{\nu_4}} \right\} - \frac{1}{4} \right]^{1/2} \\ &= \frac{1}{2} + \left[\frac{9}{8} \cdot 2 \cdot (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2} \right)^2 \cdot 5,85 \cdot 10^{-8} \frac{1}{N} \cdot 2,22 \cdot \left(\frac{9,35 \text{ m}}{0,400 \text{ m} - 0,043 \text{ m}} \right)^4 \right. \\ &\quad \left. \cdot \frac{\left(\sin \frac{180^\circ}{2} \right)^4}{2,91^3} \left\{ 1 - \frac{\arctan \sqrt{8,3}}{\sqrt{8,3}} \right\} - \frac{1}{4} \right]^{1/2} \\ &= 1,12\end{aligned}\quad (60)$$

with the factor ν_4 :

$$\nu_4 = \frac{a_s - d}{d} = \frac{0,400 \text{ m} - 0,043 \text{ m}}{0,043 \text{ m}} = 8,3 \quad (61)$$

With this, the tensile forces caused by pinch are:

$$\begin{aligned}F_{pi,d,-20} &= F_{st,-20} \left(1 + \frac{\nu_{e,-20}}{\varepsilon_{st,-20}} \xi_{-20} \right) = 17,8 \text{ kN} \cdot \left(1 + \frac{1,14}{1,06} \cdot 2,86 \right) = 72,6 \text{ kN} \\ F_{pi,d,60} &= F_{st,60} \left(1 + \frac{\nu_{e,60}}{\varepsilon_{st,60}} \xi_{60} \right) = 15,4 \text{ kN} \cdot \left(1 + \frac{1,12}{0,927} \cdot 2,91 \right) = 69,5 \text{ kN}\end{aligned}\quad (59)$$

The pinch-force $F_{pi,d}$ is the maximum value of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max \{ F_{pi,d,-20}; F_{pi,d,60} \} = \max \{ 72,6 \text{ kN}; 69,5 \text{ kN} \} = 72,6 \text{ kN}$$

8.4.4 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max \{36,3 \text{ kN}; 68,8 \text{ kN}; 72,6 \text{ kN}\} = 72,6 \text{ kN}$$

given by the pinch force $F_{pi,d}$.

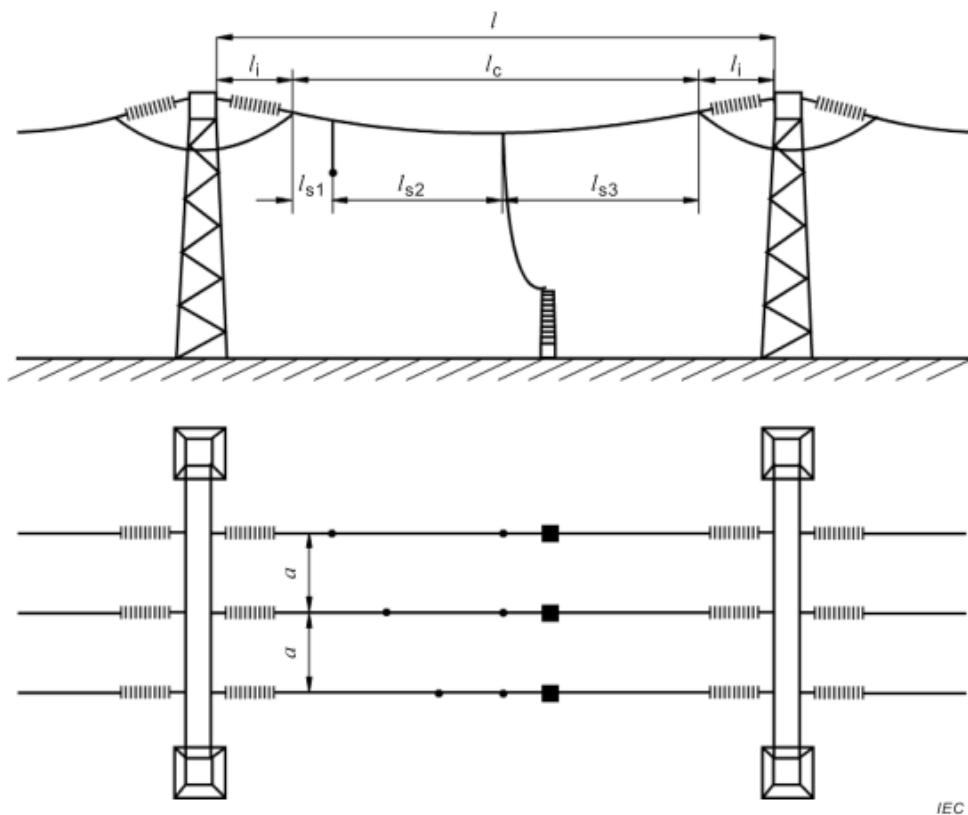
The maximum horizontal displacement is 1,48 m and the minimum air clearance is 2,04 m.

9 Example 6 – Mechanical effects on strained conductors with dropper in the middle of the span

9.1 General

The basis for the calculations in this example is a three-phase 380 kV arrangement with strained twin-bundle conductors as shown in Figure 6. In the span there is one connection of pantograph-disconnectors and one dropper in midspan, both operate as spacers.

The calculation is carried out for the arrangement of droppers with plane parallel to the main conductors and plane perpendicular to the main conductors.



**Figure 6 – Arrangement with strained conductors and droppers in midspan.
Plane of the droppers parallel to the main conductors**

9.2 Common data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	=	63 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,81
Duration of the first short-circuit current flow	T_{k1}	=	0,5 s
System frequency	f	=	50 Hz
Centre-line distance between supports	l	=	48 m
Length of one insulator chain	l_i	=	5,3 m
Cord length $l_c = l - 2l_i$	l_c	=	37,4 m
Centre-line distance between conductors	a	=	5 m
Resultant spring constant of both span supports of one span	S	=	500 N/mm
Twin conductor 2 EN 1046-AL1/45-ST1A			
– Number of sub-conductors	n	=	2
– Centre-line distance between sub-conductors	a_s	=	0,1 m
– Outer diameter of one sub-conductor	d	=	43 mm
– Cross-section of one sub-conductor	A_s	=	1 090 mm ²
– Mass per unit length of one sub-conductor	m'_s	=	3,25 kg/m
– Young's modulus	E	=	60 000 N/mm ²
Static tensile force of one flexible main conductor at a temperature of -20°C (local minimum winter temperature)	$F_{st,-20}$	=	17,4 kN
Static tensile force of one flexible main conductor at a temperature of 60°C (maximum operating temperature)	$F_{st,60}$	=	15,0 kN
Additional concentrated masses representing the connections of pantograph disconnectors			
– Number of spacers	n_c	=	2
– Mass of one connection	m_c	=	36 kg
– Distances	l_{s1}	=	2,5 m
	l_{s2}	=	18,6 m
	l_{s3}	=	16,3 m
Conventional value of acceleration of gravity	g	=	9,81 m/s ²

9.3 Plane of the dropper parallel to the main conductors

9.3.1 General

In addition to 9.2, the following data are given:

Height of the dropper at a main conductor temperature of 60°C (maximum operating temperature)	h	=	7 m
Width of the dropper at a main conductor temperature of 60°C (maximum operating temperature)	w	=	2 m
Cord length of the dropper	l_v	=	7,6 m

9.3.2 Current flow along the whole length of the main conductor span

9.3.2.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0,75 \frac{(I_k')^2}{a} \frac{l_c}{l} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0,75 \cdot \frac{(63 \cdot 10^3 \text{ A})^2}{5 \text{ m}} \cdot \frac{37,4 \text{ m}}{48 \text{ m}} = 92,8 \text{ N/m} \quad (19a)$$

The parameter r is:

$$r = \frac{F'}{nm'_sc g} = \frac{92,8 \text{ N/m}}{2 \cdot 3,73 \text{ kg/m} \cdot 9,81 \text{ m/s}^2} = 1,27 \quad (20)$$

where m'_sc is the resulting mass per unit length of one sub-conductor including concentrated mass:

$$m'_sc = m'_s + \frac{m_c}{nl_c} = 3,25 \frac{\text{kg}}{\text{m}} + \frac{36 \text{ kg}}{2 \cdot 37,4 \text{ m}} = 3,73 \text{ kg/m}$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 1,27 = 51,8^\circ \quad (21)$$

The equivalent static conductor sags at midspan are:

$$f_{es,-20} = \frac{nm'_sc g l^2}{8 F_{st,-20}} = \frac{2 \cdot 3,73 \text{ kg/m} \cdot 9,81 \text{ m/s}^2 (48 \text{ m})^2}{8 \cdot 17,4 \cdot 10^3 \text{ N}} = 1,21 \text{ m}$$

$$f_{es,60} = \frac{nm'_sc g l^2}{8 F_{st,60}} = \frac{2 \cdot 3,73 \text{ kg/m} \cdot 9,81 \text{ m/s}^2 (48 \text{ m})^2}{8 \cdot 15,0 \cdot 10^3 \text{ N}} = 1,41 \text{ m} \quad (22)$$

The periods of the conductor oscillation are:

$$T_{-20} = 2\pi \sqrt{0,8 \frac{f_{es,-20}}{g}} = 2\pi \sqrt{0,8 \cdot \frac{1,21 \text{ m}}{9,81 \text{ m/s}^2}} = 1,97 \text{ s}$$

$$T_{60} = 2\pi \sqrt{0,8 \frac{f_{es,60}}{g}} = 2\pi \sqrt{0,8 \cdot \frac{1,41 \text{ m}}{9,81 \text{ m/s}^2}} = 2,13 \text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$\begin{aligned} T_{\text{res},20} &= \frac{T_{20}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{1,97}{\sqrt[4]{1+1,27^2} \left[1 - \frac{\pi^2}{64} \left(\frac{51,8^\circ}{90^\circ} \right)^2 \right]} = 1,63 \text{ s} \\ T_{\text{res},60} &= \frac{T_{60}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{2,13}{\sqrt[4]{1+1,27^2} \left[1 - \frac{\pi^2}{64} \left(\frac{51,8^\circ}{90^\circ} \right)^2 \right]} = 1,77 \text{ s} \end{aligned} \quad (24)$$

The stiffness norms are:

$$\begin{aligned} N_{20} &= \frac{1}{SI} + \frac{1}{n E_{\text{eff},20} A_s} = \frac{1}{5 \cdot 10^5 \text{ N/m} \cdot 48 \text{ m}} + \frac{1}{2 \cdot 2,84 \cdot 10^{10} \text{ N/m}^2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 5,78 \cdot 10^{-8} \text{ 1/N} \\ N_{60} &= \frac{1}{SI} + \frac{1}{n E_{\text{eff},60} A_s} = \frac{1}{5 \cdot 10^5 \text{ N/m} \cdot 48 \text{ m}} + \frac{1}{2 \cdot 2,70 \cdot 10^{10} \text{ N/m}^2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 5,87 \cdot 10^{-8} \text{ 1/N} \end{aligned} \quad (25)$$

with the actual Young's moduli

$$\begin{aligned} E_{\text{eff},20} &= E \left[0,3 + 0,7 \sin \left(\frac{F_{\text{st},20}}{n A_s \sigma_{\text{fin}}} 90^\circ \right) \right] = 6 \cdot 10^{10} \frac{\text{N}}{\text{m}^2} \cdot \left[0,3 + 0,7 \sin \left(\frac{7,98 \cdot 10^6 \text{ N/m}^2}{5 \cdot 10^7 \text{ N/m}^2} \cdot 90^\circ \right) \right] \\ &= 2,84 \cdot 10^{10} \text{ N/m}^2 \\ E_{\text{eff},60} &= E \left[0,3 + 0,7 \sin \left(\frac{F_{\text{st},60}}{n A_s \sigma_{\text{fin}}} 90^\circ \right) \right] = 6 \cdot 10^{10} \frac{\text{N}}{\text{m}^2} \cdot \left[0,3 + 0,7 \sin \left(\frac{6,88 \cdot 10^6 \text{ N/m}^2}{5 \cdot 10^7 \text{ N/m}^2} \cdot 90^\circ \right) \right] \\ &= 2,70 \cdot 10^{10} \text{ N/m}^2 \end{aligned} \quad (26)$$

because

$$\begin{aligned} \frac{F_{\text{st},20}}{n A_s} &= \frac{17,4 \cdot 10^3 \text{ N}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 7,98 \cdot 10^6 \text{ N/m}^2 && \text{less than} && \sigma_{\text{fin}} = 50 \cdot 10^6 \text{ N/m}^2 \\ \frac{F_{\text{st},60}}{n A_s} &= \frac{15,0 \cdot 10^3 \text{ N}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} = 6,88 \cdot 10^6 \text{ N/m}^2 && \text{less than} && \sigma_{\text{fin}} = 50 \cdot 10^6 \text{ N/m}^2 \end{aligned}$$

The stress factors are:

$$\begin{aligned} \zeta_{20} &= \frac{(n g m'_{\text{sc}} l)^2}{24 F_{\text{st},20}^3 N_{20}} = \frac{(2 \cdot 9,81 \text{ m/s}^2 \cdot 3,73 \text{ kg/m} \cdot 48 \text{ m})^2}{24 (17,4 \cdot 10^3 \text{ N})^3 \cdot 5,78 \cdot 10^{-8} \text{ 1/N}} = 1,69 \\ \zeta_{60} &= \frac{(n g m'_{\text{sc}} l)^2}{24 F_{\text{st},60}^3 N_{60}} = \frac{(2 \cdot 9,81 \text{ m/s}^2 \cdot 3,73 \text{ kg/m} \cdot 48 \text{ m})^2}{24 (15,0 \cdot 10^3 \text{ N})^3 \cdot 5,87 \cdot 10^{-8} \text{ 1/N}} = 2,60 \end{aligned} \quad (28)$$

Because

$$\begin{aligned} T_{k1} &= 0,5 \text{ s} && \text{less than} && 0,4 T_{20} = 0,4 \cdot 1,97 \text{ s} = 0,788 \text{ s} \\ T_{k1} &= 0,5 \text{ s} && \text{less than} && 0,4 T_{60} = 0,4 \cdot 2,13 \text{ s} = 0,852 \text{ s} \end{aligned}$$

in the Equations (29), (32), and (35) it is to be inserted:

$$T_{k1} = 0,5 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\begin{aligned}\delta_{\text{end},-20} &= \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{\text{res},-20}} \right) \right] = 51,8^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,63 \text{ s}} \right) \right] = 69,9^\circ \\ \delta_{\text{end},60} &= \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{\text{res},60}} \right) \right] = 51,8^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,77 \text{ s}} \right) \right] = 62,3^\circ\end{aligned}\quad (29)$$

because

$$\begin{aligned}\frac{T_{k1}}{T_{\text{res},-20}} &= \frac{0,5 \text{ s}}{1,63 \text{ s}} = 0,307 \quad \text{less than} \quad 0,5 \\ \frac{T_{k1}}{T_{\text{res},60}} &= \frac{0,5 \text{ s}}{1,77 \text{ s}} = 0,283 \quad \text{less than} \quad 0,5\end{aligned}$$

The maximum swing-out angles $\delta_{\max,-20}$ and $\delta_{\max,60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{\text{end},-20}$ and $\delta_{\text{end},60}$:

- for 0 less than $\delta_{\text{end},-20} = 69,9^\circ$ less than 90° is:

$$\chi_{-20} = 1 - r \sin \delta_{\text{end},-20} = 1 - 1,27 \cdot \sin 69,9^\circ = -0,193 \quad (30)$$

and for $-0,985$ less than $\chi_{-20} = -0,193$ less than $0,766$:

$$\delta_{\max,-20} = 10^\circ + \arccos \chi_{-20} = 10^\circ + \arccos (-0,193) = 111^\circ \quad (31)$$

- for 0 less than $\delta_{\text{end},60} = 62,3^\circ$ less than 90° is:

$$\chi_{60} = 1 - r \sin \delta_{\text{end},60} = 1 - 1,27 \cdot \sin 62,3^\circ = -0,124 \quad (30)$$

and for $-0,985$ less than $\chi_{60} = -0,124$ less than $0,766$:

$$\delta_{\max,60} = 10^\circ + \arccos \chi_{60} = 10^\circ + \arccos (-0,124) = 107^\circ \quad (31)$$

9.3.2.2 Tensile force $F_{t,d}$ during short-circuit caused by swing out without dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{-20} = \varphi_{60} = 3 \left(\sqrt{1+r^2} - 1 \right) = 3 \left(\sqrt{1+1,27^2} - 1 \right) = 1,85 \quad (32)$$

because

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},-20}}{4} = \frac{1,63 \text{ s}}{4} = 0,408 \text{ s}$$

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},60}}{4} = \frac{1,77 \text{ s}}{4} = 0,443 \text{ s}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 1,85$ and $\zeta_{-20} = 1,69$:

$$\psi_{-20} = 0,641$$

- for $\varphi_{60} = 1,85$ and $\zeta_{60} = 2,60$:

$$\psi_{60} = 0,714$$

The tensile forces during the short-circuit are:

$$F_{t,d,-20} = F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 17,4 \text{ kN} \cdot (1 + 1,85 \cdot 0,641) = 38,0 \text{ kN} \quad (33)$$

$$F_{t,d,60} = F_{st,60} (1 + \varphi_{60} \psi_{60}) = 15,0 \text{ kN} \cdot (1 + 1,85 \cdot 0,714) = 34,8 \text{ kN}$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{38,0 \text{ kN}; 34,8 \text{ kN}\} = 38,0 \text{ kN}$$

9.3.2.3 Dynamic conductor sag at midspan

The elastic expansions are:

$$\varepsilon_{ela,-20} = N_{-20} (F_{t,d,-20} - F_{st,-20}) = 5,78 \cdot 10^{-8} \frac{1}{N} \cdot (38,0 - 17,4) \cdot 10^3 \text{ N} = 1,19 \cdot 10^{-3} \quad (34)$$

$$\varepsilon_{ela,60} = N_{60} (F_{t,d,60} - F_{st,60}) = 5,87 \cdot 10^{-8} \frac{1}{N} \cdot (34,8 - 15,0) \cdot 10^3 \text{ N} = 1,16 \cdot 10^{-3}$$

The thermal expansions are:

$$\varepsilon_{th,-20} = c_{th} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{\text{res},-20}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{1,63 \text{ s}}{4} = 0,919 \cdot 10^{-4} \quad (35)$$

$$\varepsilon_{th,60} = c_{th} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{\text{res},60}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{1,77 \text{ s}}{4} = 0,998 \cdot 10^{-4}$$

because

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},-20}}{4} = \frac{1,63 \text{ s}}{4} = 0,408 \text{ s}$$

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},60}}{4} = \frac{1,77 \text{ s}}{4} = 0,443 \text{ s}$$

and for ASCR conductors with $A_{Al}/A_{St} = 1046 \text{ mm}/45 \text{ mm} = 23,2$ greater than 6

$$c_{\text{th}} = 0,27 \cdot 10^{-18} \text{ m}^4 / (\text{A}^2 \text{s})$$

The factors C_D are:

$$\begin{aligned} C_{D,-20} &= \sqrt{1 + \frac{3}{8} \left(\frac{l}{f_{es,-20}} \right)^2 (\varepsilon_{ela,-20} + \varepsilon_{th,-20})} = \sqrt{1 + \frac{3}{8} \left(\frac{48 \text{ m}}{1,21 \text{ m}} \right)^2 (1,19 \cdot 10^{-3} + 0,919 \cdot 10^{-4})} = 1,33 \\ C_{D,60} &= \sqrt{1 + \frac{3}{8} \left(\frac{l}{f_{es,60}} \right)^2 (\varepsilon_{ela,60} + \varepsilon_{th,60})} = \sqrt{1 + \frac{3}{8} \left(\frac{48 \text{ m}}{1,41 \text{ m}} \right)^2 (1,16 \cdot 10^{-3} + 0,998 \cdot 10^{-4})} = 1,24 \end{aligned} \quad (36)$$

The factor C_F is:

$$C_F = 0,97 + 0,1r = 0,97 + 0,1 \cdot 1,27 = 1,10 \quad (37)$$

because

$$0,8 \quad \text{less than} \quad r = 1,27 \quad \text{less than} \quad 1,8$$

The dynamic conductor sags at midspan are:

$$\begin{aligned} f_{ed,-20} &= C_F C_{D,-20} f_{es,-20} = 1,10 \cdot 1,33 \cdot 1,21 \text{ m} = 1,77 \text{ m} \\ f_{ed,60} &= C_F C_{D,60} f_{es,60} = 1,10 \cdot 1,24 \cdot 1,41 \text{ m} = 1,92 \text{ m} \end{aligned} \quad (38)$$

9.3.2.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\begin{aligned} \sqrt{(h_{-20} + f_{es,-20} + f_{ed,-20})^2 + w^2} &= \sqrt{(7,2 \text{ m} + 1,21 \text{ m} + 1,77 \text{ m})^2 + (2 \text{ m})^2} = 10,4 \text{ m} \quad \text{greater than} \quad l_v = 7,6 \text{ m} \\ \sqrt{(h_{60} + f_{es,60} + f_{ed,60})^2 + w^2} &= \sqrt{(7,0 \text{ m} + 1,41 \text{ m} + 1,92 \text{ m})^2 + (2 \text{ m})^2} = 10,5 \text{ m} \quad \text{greater than} \quad l_v = 7,6 \text{ m} \end{aligned}$$

with the dropper height at -20°C due to the change of sag with the temperature of the main conductor

$$h_{-20} = h_{60} + (f_{es,60} - f_{es,-20}) = 7,0 \text{ m} + (1,41 \text{ m} - 1,21 \text{ m}) = 7,2 \text{ m}$$

and $h_{60} = h = 7,0 \text{ m}$.

The actual swing-out angles are:

$$\begin{aligned} \delta_{-20} &= \arccos \frac{(h_{-20} + f_{es,-20})^2 + f_{ed,-20}^2 - (l_v^2 - w^2)}{2f_{ed,-20}(h_{-20} + f_{es,-20})} \\ &= \arccos \frac{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,77 \text{ m})^2 - ((7,6 \text{ m})^2 - (2,0 \text{ m})^2)}{2 \cdot 1,77 \text{ m} \cdot (7,2 \text{ m} + 1,21 \text{ m})} = 47,5^\circ \end{aligned} \quad (39)$$

$$\begin{aligned}\delta_{60} &= \arccos \frac{(h_{60} + f_{es,60})^2 + f_{ed,60}^2 - (l_v^2 - w^2)}{2f_{ed,60}(h_{60} + f_{es,60})} = \\ &= \arccos \frac{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,92 \text{ m})^2 - ((7,6 \text{ m})^2 - (2,0 \text{ m})^2)}{2 \cdot 1,92 \text{ m} \cdot (7,0 \text{ m} + 1,41 \text{ m})} = 50,2^\circ\end{aligned}\quad (39)$$

The load parameters are:

$$\begin{aligned}\varphi_{-20} &= 3(r \sin \delta_{-20} + \cos \delta_{-20} - 1) = 3(1,27 \sin 47,5^\circ + \cos 47,5^\circ - 1) = 1,84 \\ \varphi_{60} &= 3(r \sin \delta_{60} + \cos \delta_{60} - 1) = 3(1,27 \sin 50,2^\circ + \cos 50,2^\circ - 1) = 1,85\end{aligned}\quad (41)$$

because

$$\begin{aligned}\delta_{-20} &= 47,5^\circ \quad \text{less than} \quad \delta_1 = 51,8^\circ \\ \delta_{60} &= 50,2^\circ \quad \text{less than} \quad \delta_1 = 51,8^\circ\end{aligned}$$

and also

$$\begin{aligned}\delta_{end,-20} &= 69,9^\circ \quad \text{greater than} \quad \delta_{-20} = 47,5^\circ \\ \delta_{end,60} &= 62,3^\circ \quad \text{greater than} \quad \delta_{60} = 50,2^\circ\end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 1,84$ and $\zeta_{-20} = 1,69$:

$$\psi_{-20} = 0,641$$

- for $\varphi_{60} = 1,85$ and $\zeta_{60} = 2,60$:

$$\psi_{60} = 0,714$$

The tensile forces during the short-circuit are:

$$\begin{aligned}F_{t,d,-20} &= F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 17,4 \text{ kN} \cdot (1 + 1,84 \cdot 0,641) = 37,9 \text{ kN} \\ F_{t,d,60} &= F_{st,60} (1 + \varphi_{60} \psi_{60}) = 15,0 \text{ kN} \cdot (1 + 1,85 \cdot 0,714) = 34,8 \text{ kN}\end{aligned}\quad (42)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{37,9 \text{ kN}; 34,8 \text{ kN}\} = 37,9 \text{ kN}$$

9.3.2.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 1,27 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned}\delta_{max,-20} &= 111^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{max,60} &= 107^\circ \quad \text{greater than} \quad 70^\circ\end{aligned}$$

however

$$\begin{array}{lll} \delta_{-20} = 47,5^\circ & \text{less than} & 60^\circ \\ \delta_{60} = 50,2^\circ & \text{less than} & 60^\circ \end{array}$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 1,27 \quad \text{greater than} \quad 0,6$$

and

$$\begin{array}{lll} \delta_{\max,-20} = 111^\circ & \text{greater than} & 70^\circ \\ \delta_{\max,60} = 107^\circ & \text{greater than} & 70^\circ \end{array}$$

The tensile forces after short-circuit are:

$$\begin{aligned} F_{f,d,-20} &= 1,2 \cdot F_{st,-20} \sqrt{1 + 8 \zeta_{-20} \frac{\delta_{\max,-20}}{180^\circ}} = 1,2 \cdot 17,4 \text{ kN} \cdot \sqrt{1 + 8 \cdot 1,69 \cdot \frac{111^\circ}{180^\circ}} = 63,8 \text{ kN} \\ F_{f,d,60} &= 1,2 \cdot F_{st,60} \sqrt{1 + 8 \zeta_{60} \frac{\delta_{\max,60}}{180^\circ}} = 1,2 \cdot 15,0 \text{ kN} \cdot \sqrt{1 + 8 \cdot 2,60 \cdot \frac{107^\circ}{180^\circ}} = 65,8 \text{ kN} \end{aligned} \quad (43)$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max \{F_{f,d,-20}; F_{f,d,60}\} = \max \{63,8 \text{ kN}; 65,8 \text{ kN}\} = 65,8 \text{ kN}$$

9.3.2.6 Horizontal span displacement b_h and minimum air clearance a_{\min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of -20°C .

The maximum horizontal span displacement for stranded conductors with $I_c = I - 2I_i$ is:

$$b_h = f_{ed,60} \sin \delta_{60} = 1,92 \text{ m} \cdot \sin 50,2^\circ = 1,48 \text{ m} \quad (47)$$

because

$$\begin{array}{lll} \delta_{60} = 50,2^\circ & \text{less than} & \delta_{\max,60} = 107^\circ \\ \delta_{60} = 50,2^\circ & \text{less than} & \delta_1 = 51,8^\circ \end{array}$$

The minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,48 \text{ m} = 2,04 \text{ m} \quad (48)$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_h and the minimum air clearance a_{\min} shall be calculated according to IEC 60865-1:2011, 6.2.7.

$$b_h = f_{ed,60} \sin \delta_1 = 1,92 \text{ m} \cdot \sin 51,8^\circ = 1,51 \text{ m} \quad (45)$$

because

$$\delta_{\max,60} = 107^\circ \quad \text{greater than} \quad \delta_1 = 51,8^\circ$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,51 \text{ m} = 1,98 \text{ m} \quad (48)$$

9.3.2.7 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled:

$$\frac{a_s}{d} = \frac{0,1 \text{ m}}{0,043 \text{ m}} = 2,33 \quad \text{less than} \quad 2,5 \quad (53)$$

and

$$l_s = 12,5 \text{ m} \quad \text{greater than} \quad 70a_s = 70 \cdot 0,1 \text{ m} = 7 \text{ m} \quad (53)$$

with

$$l_s = \frac{l_{s1} + l_{s2} + l_{s3}}{3} = \frac{l_c}{3} = \frac{2,5 + 18,6 + 16,3}{3} \text{ m} = \frac{37,4 \text{ m}}{3} = 12,5 \text{ m}$$

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1,1F_{t,d,-20} = 1,1 \cdot 38,0 \text{ kN} = 41,8 \text{ kN} \\ F_{pi,d,60} &= 1,1F_{t,d,60} = 1,1 \cdot 34,8 \text{ kN} = 38,3 \text{ kN} \end{aligned} \quad (51)$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 9.3.2.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max \{F_{pi,d,-20}; F_{pi,d,60}\} = \max \{41,8 \text{ kN}; 38,3 \text{ kN}\} = 41,8 \text{ kN}$$

9.3.2.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max \{37,9 \text{ kN}; 0 \text{ kN}; 41,8 \text{ kN}\} = 41,8 \text{ kN}$$

given by the tensile force $F_{pi,d}$ caused by pinch.

The maximum horizontal displacement is 1,48 m and the minimum air clearance is 2,04 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max\{38,0 \text{ kN}; 65,8 \text{ kN}; 41,8 \text{ kN}\} = 65,8 \text{ kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,51 m and the minimum air clearance is 1,98 m.

9.3.3 Current flow along half of the length of the main conductor and along the dropper

9.3.3.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0,75 \frac{(I_k^*)^2}{a} \frac{l_c/2 + l_v/2}{l} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0,75 \cdot \frac{(63 \cdot 10^3 \text{ A})^2}{5 \text{ m}} \cdot \frac{37,4 \text{ m}/2 + 7,6 \text{ m}/2}{48 \text{ m}} = 55,8 \text{ N/m} \quad (19b)$$

The parameter r is:

$$r = \frac{F'}{nm'_sc g} = \frac{55,8 \text{ N/m}}{2 \cdot 3,73 \text{ kg/m} \cdot 9,81 \text{ m/s}^2} = 0,763 \quad (20)$$

where

$$m'_sc = 3,73 \text{ kg/m} \quad \text{see 9.3.2.1}$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 0,763 = 37,3^\circ \quad (21)$$

The equivalent static conductor sags at midspan are, see 9.3.1.1:

$$f_{es,-20} = 1,21 \text{ m} \quad f_{es,60} = 1,41 \text{ m} \quad (22)$$

The periods of the conductor oscillation are, see 9.3.2.1:

$$T_{-20} = 1,97 \text{ s} \quad T_{60} = 2,13 \text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$T_{res,-20} = \frac{T_{-20}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{1,97}{\sqrt[4]{1+0,763^2} \left[1 - \frac{\pi^2}{64} \left(\frac{37,3^\circ}{90^\circ} \right)^2 \right]} = 1,80 \text{ s} \\ T_{res,60} = \frac{T_{60}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{2,13}{\sqrt[4]{1+0,763^2} \left[1 - \frac{\pi^2}{64} \left(\frac{37,3^\circ}{90^\circ} \right)^2 \right]} = 1,95 \text{ s} \quad (24)$$

The stiffness norms are, see 9.3.2.1:

$$N_{-20} = 5,78 \cdot 10^{-8} \text{ 1/N} \quad N_{60} = 5,87 \cdot 10^{-8} \text{ 1/N} \quad (25)$$

The stress factors are, see 9.3.2.1:

$$\zeta_{-20} = 1,69 \quad \zeta_{60} = 2,60 \quad (28)$$

In the Equations (29), (32), and (35) it is to be inserted, see 9.3.2.1:

$$T_{k1} = 0,5 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\begin{aligned} \delta_{\text{end},-20} &= \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{\text{res},-20}} \right) \right] = 37,3^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,80 \text{ s}} \right) \right] = 43,8^\circ \\ \delta_{\text{end},60} &= \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{\text{res},60}} \right) \right] = 37,3^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,95 \text{ s}} \right) \right] = 38,8^\circ \end{aligned} \quad (29)$$

because

$$\frac{T_{k1}}{T_{\text{res},-20}} = \frac{0,5 \text{ s}}{1,80 \text{ s}} = 0,278 \quad \text{less than} \quad 0,5$$

$$\frac{T_{k1}}{T_{\text{res},60}} = \frac{0,5 \text{ s}}{1,95 \text{ s}} = 0,256 \quad \text{less than} \quad 0,5$$

The maximum swing-out angles $\delta_{\max,-20}$ and $\delta_{\max,60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{\text{end},-20}$ and $\delta_{\text{end},60}$:

– for 0 less than $\delta_{\text{end},-20} = 43,8^\circ$ less than 90° is:

$$\chi_{-20} = 1 - r \sin \delta_{\text{end},-20} = 1 - 0,763 \cdot \sin 43,8^\circ = 0,472 \quad (30)$$

and for $-0,985$ less than $\chi_{-20} = 0,472$ less than $0,766$:

$$\delta_{\max,-20} = 10^\circ + \arccos \chi_{-20} = 10^\circ + \arccos 0,472 = 71,8^\circ \quad (31)$$

– for 0 less than $\delta_{\text{end},60} = 38,8^\circ$ less than 90° is:

$$\chi_{60} = 1 - r \sin \delta_{\text{end},60} = 1 - 0,763 \cdot \sin 38,8^\circ = 0,522 \quad (30)$$

and for $-0,985$ less than $\chi_{60} = 0,522$ less than $0,766$:

$$\delta_{\max,60} = 10^\circ + \arccos \chi_{60} = 10^\circ + \arccos 0,522 = 68,5^\circ \quad (31)$$

9.3.3.2 Tensile force $F_{t,d}$ during short-circuit caused by swing out without dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3\left(\sqrt{1+r^2} - 1\right) = 3\left(\sqrt{1+0,763^2} - 1\right) = 0,774 \quad (32)$$

because

$$\begin{aligned} T_{k1} &= 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{res,-20}}{4} = \frac{1,80 \text{ s}}{4} = 0,450 \text{ s} \\ T_{k1} &= 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{res,60}}{4} = \frac{1,95 \text{ s}}{4} = 0,488 \text{ s} \end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 0,774$ and $\zeta_{-20} = 1,69$:

$$\psi_{-20} = 0,702$$

- for $\varphi_{60} = 0,774$ and $\zeta_{60} = 2,60$:

$$\psi_{60} = 0,774$$

The tensile forces during the short-circuit are:

$$\begin{aligned} F_{t,d,-20} &= F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 17,4 \text{ kN} \cdot (1 + 0,774 \cdot 0,702) = 26,9 \text{ kN} \\ F_{t,d,60} &= F_{st,60} (1 + \varphi_{60} \psi_{60}) = 15,0 \text{ kN} \cdot (1 + 0,774 \cdot 0,774) = 24,0 \text{ kN} \end{aligned} \quad (33)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{26,9 \text{ kN}; 24,0 \text{ kN}\} = 26,9 \text{ kN}$$

9.3.3.3 Dynamic conductor sag at midspan

The elastic expansions are:

$$\begin{aligned} \varepsilon_{ela,-20} &= N_{-20} (F_{t,d,-20} - F_{st,-20}) = 5,78 \cdot 10^{-8} \frac{1}{N} \cdot (26,9 - 17,4) \cdot 10^3 \text{ N} = 0,543 \cdot 10^{-3} \\ \varepsilon_{ela,60} &= N_{60} (F_{t,d,60} - F_{st,60}) = 5,87 \cdot 10^{-8} \frac{1}{N} \cdot (24,0 - 15,0) \cdot 10^3 \text{ N} = 0,528 \cdot 10^{-3} \end{aligned} \quad (34)$$

The thermal expansions are:

$$\begin{aligned} \varepsilon_{th,-20} &= c_{th} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{res,-20}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{1,80 \text{ s}}{4} = 1,02 \cdot 10^{-4} \\ \varepsilon_{th,60} &= c_{th} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{res,60}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{1,95 \text{ s}}{4} = 1,10 \cdot 10^{-4} \end{aligned} \quad (35)$$

because

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},-20}}{4} = \frac{1,80 \text{ s}}{4} = 0,450 \text{ s}$$

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},60}}{4} = \frac{1,95 \text{ s}}{4} = 0,488 \text{ s}$$

and for ASCR conductors with $A_{\text{Al}}/A_{\text{St}} = 1046 \text{ mm}/45 \text{ mm} = 23,2$ greater than 6

$$c_{\text{th}} = 0,27 \cdot 10^{-18} \text{ m}^4/(\text{A}^2\text{s})$$

The factors C_D are:

$$C_{D,-20} = \sqrt{1 + \frac{3}{8} \left(\frac{l}{f_{\text{es},-20}} \right)^2 (\varepsilon_{\text{ela},-20} + \varepsilon_{\text{th},-20})} = \sqrt{1 + \frac{3}{8} \left(\frac{48 \text{ m}}{1,21 \text{ m}} \right)^2 (0,543 \cdot 10^{-3} + 1,02 \cdot 10^{-4})} = 1,18 \quad (36)$$

$$C_{D,60} = \sqrt{1 + \frac{3}{8} \left(\frac{l}{f_{\text{es},60}} \right)^2 (\varepsilon_{\text{ela},60} + \varepsilon_{\text{th},60})} = \sqrt{1 + \frac{3}{8} \left(\frac{48 \text{ m}}{1,41 \text{ m}} \right)^2 (0,528 \cdot 10^{-3} + 1,10 \cdot 10^{-4})} = 1,13$$

The factor C_F is:

$$C_F = 1,05 \quad (37)$$

because

$$r = 0,763 \quad \text{less than} \quad 0,8$$

The dynamic conductor sags at midspan are:

$$f_{\text{ed},-20} = C_F C_{D,-20} f_{\text{es},-20} = 1,05 \cdot 1,18 \cdot 1,21 \text{ m} = 1,50 \text{ m} \quad (38)$$

$$f_{\text{ed},60} = C_F C_{D,60} f_{\text{es},60} = 1,05 \cdot 1,13 \cdot 1,41 \text{ m} = 1,67 \text{ m}$$

9.3.3.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\sqrt{(h_{-20} + f_{\text{es},-20} + f_{\text{ed},-20})^2 + w^2} = \sqrt{(7,2 \text{ m} + 1,21 \text{ m} + 1,50 \text{ m})^2 + (2 \text{ m})^2} = 10,1 \text{ m} \quad \text{greater than} \quad l_v = 7,6 \text{ m}$$

$$\sqrt{(h_{60} + f_{\text{es},60} + f_{\text{ed},60})^2 + w^2} = \sqrt{(7,0 \text{ m} + 1,41 \text{ m} + 1,67 \text{ m})^2 + (2 \text{ m})^2} = 10,3 \text{ m} \quad \text{greater than} \quad l_v = 7,6 \text{ m}$$

with the dropper height at -20°C due to the change of sag with the temperature of the main conductor

$$h_{-20} = h_{60} + (f_{\text{es},60} - f_{\text{es},-20}) = 7,0 \text{ m} + (1,41 \text{ m} - 1,21 \text{ m}) = 7,2 \text{ m}$$

and $h_{60} = h = 7,0 \text{ m}$.

The actual swing-out angles are:

$$\begin{aligned}\delta_{-20} &= \arccos \frac{(h_{-20} + f_{es,-20})^2 + f_{ed,-20}^2 - (l_v^2 - w^2)}{2f_{ed,-20}(h_{-20} + f_{es,-20})} \\ &= \arccos \frac{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,50 \text{ m})^2 - ((7,6 \text{ m})^2 - (2,0 \text{ m})^2)}{2 \cdot 1,50 \text{ m} \cdot (7,2 \text{ m} + 1,21 \text{ m})} = 40,4^\circ\end{aligned}\quad (39)$$

$$\begin{aligned}\delta_{60} &= \arccos \frac{(h_{60} + f_{es,60})^2 + f_{ed,60}^2 - (l_v^2 - w^2)}{2f_{ed,60}(h_{60} + f_{es,60})} \\ &= \arccos \frac{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,67 \text{ m})^2 - ((7,6 \text{ m})^2 - (2,0 \text{ m})^2)}{2 \cdot 1,67 \text{ m} \cdot (7,0 \text{ m} + 1,41 \text{ m})} = 45,3^\circ\end{aligned}\quad (39)$$

The load parameters are:

$$\varphi_{-20} = \varphi_{60} = 3 \left(\sqrt{1+r^2} - 1 \right) = 3 \left(\sqrt{1+0,763^2} - 1 \right) = 0,774 \quad (40)$$

because

$$\begin{array}{lll}\delta_{-20} = 40,4^\circ & \text{greater than} & \delta_1 = 37,3^\circ \\ \delta_{60} = 45,3^\circ & \text{greater than} & \delta_1 = 37,3^\circ\end{array}$$

and also

$$\begin{array}{lll}T_{k1} = 0,5 \text{ s} & \text{greater than} & \frac{T_{res,-20}}{4} = \frac{1,80 \text{ s}}{4} = 0,450 \text{ s} \\ T_{k1} = 0,5 \text{ s} & \text{greater than} & \frac{T_{res,60}}{4} = \frac{1,95 \text{ s}}{4} = 0,488 \text{ s}\end{array}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 0,774$ and $\zeta_{-20} = 1,69$:

$$\psi_{-20} = 0,702$$

- for $\varphi_{60} = 0,774$ and $\zeta_{60} = 2,60$:

$$\psi_{60} = 0,774$$

The tensile forces during the short-circuit are:

$$\begin{aligned}F_{t,d,-20} &= F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 17,4 \text{ kN} \cdot (1 + 0,774 \cdot 0,702) = 26,9 \text{ kN} \\ F_{t,d,60} &= F_{st,60} (1 + \varphi_{60} \psi_{60}) = 15,0 \text{ kN} \cdot (1 + 0,774 \cdot 0,774) = 24,0 \text{ kN}\end{aligned}\quad (42)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{26,9 \text{ kN}; 24,0 \text{ kN}\} = 26,9 \text{ kN}$$

9.3.3.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 0,763 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned}\delta_{\max,-20} &= 71,8^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{\max,60} &= 68,5^\circ \quad \text{less than} \quad 70^\circ\end{aligned}$$

however

$$\begin{aligned}\delta_{-20} &= 40,4^\circ \quad \text{less than} \quad 60^\circ \\ \delta_{60} &= 45,6^\circ \quad \text{less than} \quad 60^\circ\end{aligned}$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 0,763 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned}\delta_{\max,-20} &= 71,8^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{\max,60} &= 68,5^\circ \quad \text{less than} \quad 70^\circ\end{aligned}$$

the drop force becomes:

$$\begin{aligned}F_{f,d,-20} &= 1,2 \cdot F_{st,-20} \sqrt{1+8\zeta_{-20} \frac{\delta_{\max,-20}}{180^\circ}} = 1,2 \cdot 17,4 \text{ kN} \cdot \sqrt{1+8 \cdot 1,69 \cdot \frac{71,8^\circ}{180^\circ}} = 52,8 \text{ kN} \\ F_{f,d,60} &= 0 \text{ kN}\end{aligned}\quad (43)$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max \{F_{f,d,-20}; F_{f,d,60}\} = \max \{52,8 \text{ kN}; 0 \text{ kN}\} = 52,8 \text{ kN}$$

9.3.3.6 Horizontal span displacement b_h and minimum air clearance a_{\min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of -20°C .

The maximum horizontal span displacement for stranded conductors with $I_c = I - 2l_i$ is:

$$b_h = f_{ed,60} \sin \delta_1 = 1,68 \text{ m} \cdot \sin 37,3^\circ = 1,02 \text{ m} \quad (47)$$

because

$$\begin{array}{lll} \delta_{60} = 45,3^\circ & \text{less than} & \delta_{\max,60} = 68,5^\circ \\ \delta_{60} = 45,3^\circ & \text{greater than} & \delta_1 = 37,3^\circ \end{array}$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,02 \text{ m} = 2,96 \text{ m} \quad (48)$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_h and the minimum air clearance a_{\min} shall be calculated according to IEC 60865-1:2011, 6.2.7:

$$b_h = f_{ed,60} \sin \delta_1 = 1,68 \text{ m} \cdot \sin 37,3^\circ = 1,02 \text{ m} \quad (45)$$

because

$$\delta_{\max,60} = 68,5^\circ \quad \text{greater than} \quad \delta_1 = 37,3^\circ$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,02 \text{ m} = 2,96 \text{ m} \quad (48)$$

9.3.3.7 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled, see 9.3.2.7.

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1,1F_{t,d,-20} = 1,1 \cdot 26,9 \text{ kN} = 29,6 \text{ kN} \\ F_{pi,d,60} &= 1,1F_{t,d,60} = 1,1 \cdot 24,0 \text{ kN} = 26,4 \text{ kN} \end{aligned} \quad (51)$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 9.3.3.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max \{F_{pi,d,-20}; F_{pi,d,60}\} = \max \{29,6 \text{ kN}; 26,4 \text{ kN}\} = 29,6 \text{ kN}$$

9.3.3.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max \{26,9 \text{ kN}; 0 \text{ kN}; 29,6 \text{ kN}\} = 29,6 \text{ kN}$$

given by the tensile force $F_{pi,d}$ caused by pinch.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max \{26,9 \text{ kN}; 52,8 \text{ kN}; 29,6 \text{ kN}\} = 52,8 \text{ kN}$$

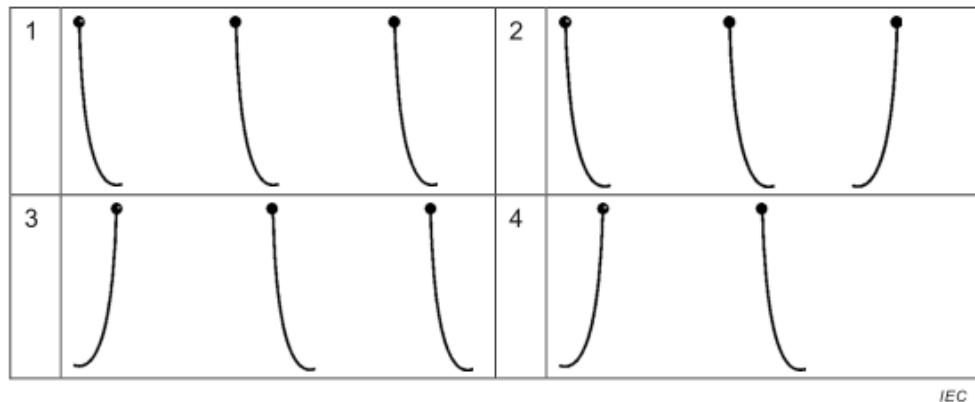
given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

9.4 Plane of the dropper perpendicular to the main conductors

9.4.1 General

Droppers in three-phase systems can be arranged as shown in Figure 7. It is not possible to predict in which pair of main conductors the short-circuit currents flow. In all cases, the calculation should be done according to IEC 60865-1:2011.



In two-line-systems with outwards mounted droppers arranged according to configuration 4 in Figure 1, the maximum horizontal displacement b_h shall be calculated according to IEC 60865-1:2011, 6.2.5. The minimum air-clearance can be calculated by replacing in Equation (39) the plus-sign by a minus sign.

Figure 7 – Possible arrangement of perpendicular droppers in three-phase system and two-line system

In addition to 9.2, the following data are given:

Height of the dropper at a main conductor temperature of 60°C (maximum operating temperature)

$$h = 7 \text{ m}$$

Width of the dropper at a main conductor temperature of 60°C (maximum operating temperature)

$$w = 1,5 \text{ m}$$

Cord length of the dropper

$$l_v = 7,4 \text{ m}$$

9.4.2 Current flow along the whole length of the main conductor span

9.4.2.1 Electromagnetic load and characteristic parameters

The electromagnetic load and the characteristic parameters are calculated in 9.3.2.1, they do not depend on the dropper:

The characteristic electromagnetic load per unit length is:

$$F' = 92,8 \text{ N/m} \quad (19a)$$

The parameter r is:

$$r = 1,27 \quad (20)$$

The direction of the resulting force on the conductor is:

$$\delta_1 = 51,8^\circ \quad (21)$$

The equivalent static conductor sags at midspan are:

$$f_{es,-20} = 1,21 \text{ m} \quad f_{es,60} = 1,41 \text{ m} \quad (22)$$

The periods of the conductor oscillation are:

$$T_{-20} = 1,97 \text{ s} \quad T_{60} = 2,13 \text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$T_{res,-20} = 1,63 \text{ s} \quad T_{res,60} = 1,77 \text{ s} \quad (24)$$

The stiffness norms are:

$$N_{-20} = 5,78 \cdot 10^{-8} \text{ 1/N} \quad N_{60} = 5,87 \cdot 10^{-8} \text{ 1/N} \quad (25)$$

The stress factors are:

$$\zeta_{-20} = 1,69 \quad \zeta_{60} = 2,60 \quad (26)$$

In the Equations (29), (32), and (35) it is to be inserted

$$T_{k1} = 0,5 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{end,-20} = 69,9^\circ \quad \delta_{end,60} = 62,3^\circ \quad (27)$$

The maximum swing-out angles are:

$$\delta_{max,-20} = 111^\circ \quad \delta_{max,60} = 107^\circ \quad (28)$$

9.4.2.2 Tensile force $F_{t,d}$ during short-circuit caused by swing out without dropper in midspan

The tensile force $F_{t,d}$ during short-circuit caused by swing out without dropper in midspan is the same as in 9.3.2.2, it does not depend on the dropper.

The tensile forces during the short-circuit are:

$$F_{t,d,-20} = 38,0 \text{ kN} \quad F_{t,d,60} = 34,8 \text{ kN} \quad (33)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{38,0 \text{ kN}; 34,8 \text{ kN}\} = 38,0 \text{ kN}$$

9.4.2.3 Dynamic conductor sag at midspan

The dynamic conductor sags at midspan are the same as in 9.3.2.3:

$$f_{ed,-20} = 1,77 \text{ m} \quad f_{ed,60} = 1,92 \text{ m} \quad (38)$$

9.4.2.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\begin{aligned} \sqrt{(h_{-20} + f_{es,-20})^2 + w^2} + f_{ed,-20} &= \sqrt{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,5 \text{ m})^2} + 1,77 \text{ m} = 10,3 \text{ m} \quad \text{greater than } l_v = 7,4 \text{ m} \\ \sqrt{(h_{60} + f_{es,60})^2 + w^2} + f_{ed,60} &= \sqrt{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,5 \text{ m})^2} + 1,92 \text{ m} = 10,5 \text{ m} \quad \text{greater than } l_v = 7,4 \text{ m} \end{aligned}$$

with the dropper height at -20°C due to the change of sag with the temperature of the main conductor

$$h_{-20} = h_{60} + (f_{es,60} - f_{es,-20}) = 7,0 \text{ m} + (1,41 \text{ m} - 1,21 \text{ m}) = 7,2 \text{ m}$$

and $h_{60} = h = 7,0 \text{ m}$.

The actual swing-out angles are:

$$\begin{aligned} \delta_{-20} &= \arccos \frac{(h_{-20} + f_{es,-20})^2 + f_{ed,-20}^2 - (l_v^2 - w^2)}{2f_{ed,-20}\sqrt{(h_{-20} + f_{es,-20})^2 + w^2}} + \arccos \frac{h_{-20} + f_{es,-20}}{\sqrt{(h_{-20} + f_{es,-20})^2 + w^2}} \\ &= \arccos \frac{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,77 \text{ m})^2 - ((7,4 \text{ m})^2 - (1,5 \text{ m})^2)}{2 \cdot 1,77 \text{ m} \cdot \sqrt{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,5 \text{ m})^2}} + \arccos \frac{7,2 \text{ m} + 1,21 \text{ m}}{\sqrt{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,5 \text{ m})^2}} \\ &= 55,2^\circ \end{aligned} \quad (39)$$

$$\begin{aligned} \delta_{60} &= \arccos \frac{(h_{60} + f_{es,60})^2 + f_{ed,60}^2 - (l_v^2 - w^2)}{2f_{ed,60}\sqrt{(h_{60} + f_{es,60})^2 + w^2}} + \arccos \frac{h_{60} + f_{es,60}}{\sqrt{(h_{60} + f_{es,60})^2 + w^2}} \\ &= \arccos \frac{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,92 \text{ m})^2 - ((7,4 \text{ m})^2 - (1,5 \text{ m})^2)}{2 \cdot 1,92 \text{ m} \cdot \sqrt{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,5 \text{ m})^2}} + \arccos \frac{7,0 \text{ m} + 1,41 \text{ m}}{\sqrt{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,5 \text{ m})^2}} \\ &= 58,2^\circ \end{aligned}$$

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3\left(\sqrt{1+r^2} - 1\right) = 3\left(\sqrt{1+1,27^2} - 1\right) = 1,85 \quad (32)$$

because

$$\begin{aligned}\delta_{-20} &= 55,2^\circ \quad \text{greater than} \quad \delta_1 = 51,8^\circ \\ \delta_{60} &= 58,2^\circ \quad \text{greater than} \quad \delta_1 = 51,8^\circ\end{aligned}$$

and

$$\begin{aligned}T_{k1} &= 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{res,-20}}{4} = \frac{1,63 \text{ s}}{4} = 0,408 \text{ s} \\ T_{k1} &= 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{res,60}}{4} = \frac{1,77 \text{ s}}{4} = 0,443 \text{ s}\end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 1,85$ and $\zeta_{-20} = 1,69$:

$$\psi_{-20} = 0,641$$

- for $\varphi_{60} = 1,85$ and $\zeta_{60} = 2,60$:

$$\psi_{60} = 0,714$$

The tensile forces during the short-circuit are:

$$\begin{aligned}F_{t,d,-20} &= F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 17,4 \text{ kN} \cdot (1 + 1,85 \cdot 0,641) = 38,0 \text{ kN} \\ F_{t,d,60} &= F_{st,60} (1 + \varphi_{60} \psi_{60}) = 15,0 \text{ kN} \cdot (1 + 1,85 \cdot 0,714) = 34,8 \text{ kN}\end{aligned} \quad (42)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{38,0 \text{ kN}; 34,8 \text{ kN}\} = 38,0 \text{ kN}$$

9.4.2.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 1,27 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned}\delta_{max,-20} &= 111^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{max,60} &= 107^\circ \quad \text{greater than} \quad 70^\circ\end{aligned}$$

however

$$\begin{aligned}\delta_{-20} &= 55,2^\circ \quad \text{less than} \quad 60^\circ \\ \delta_{60} &= 58,2^\circ \quad \text{less than} \quad 60^\circ\end{aligned}$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 1,27 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned}\delta_{\max,-20} &= 111^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{\max,60} &= 107^\circ \quad \text{greater than} \quad 70^\circ\end{aligned}$$

the drop force becomes:

$$\begin{aligned}F_{f,d,-20} &= 1,2 \cdot F_{st,-20} \sqrt{1+8\zeta_{-20} \frac{\delta_{\max,-20}}{180^\circ}} = 1,2 \cdot 17,4 \text{ kN} \cdot \sqrt{1+8 \cdot 1,69 \cdot \frac{111^\circ}{180^\circ}} = 63,8 \text{ kN} \\ F_{f,d,60} &= 1,2 \cdot F_{st,60} \sqrt{1+8\zeta_{60} \frac{\delta_{\max,60}}{180^\circ}} = 1,2 \cdot 15,0 \text{ kN} \cdot \sqrt{1+8 \cdot 2,60 \cdot \frac{107^\circ}{180^\circ}} = 65,8 \text{ kN}\end{aligned}\tag{43}$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max \{F_{f,d,-20}; F_{f,d,60}\} = \max \{63,8 \text{ kN}; 65,8 \text{ kN}\} = 65,8 \text{ kN}$$

9.4.2.6 Horizontal span displacement b_h and minimum air clearance a_{\min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of -20°C .

The maximum horizontal span displacement for stranded conductors with $I_c = l - 2l_i$ is:

$$b_h = f_{ed,60} \sin \delta_1 = 1,92 \text{ m} \cdot \sin 51,8^\circ = 1,51 \text{ m}\tag{47}$$

because

$$\delta_{60} = 58,2^\circ \quad \text{less than} \quad \delta_{\max,60} = 107^\circ$$

and because

$$\delta_{60} = 58,2^\circ \quad \text{greater than} \quad \delta_1 = 51,8^\circ$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,51 \text{ m} = 1,98 \text{ m}\tag{48}$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_h and the minimum air clearance a_{\min} shall be calculated according to IEC 60865-1:2011, 6.2.7:

$$b_h = f_{ed,60} \sin \delta_1 = 1,92 \text{ m} \cdot \sin 51,8^\circ = 1,51 \text{ m}\tag{45}$$

because

$$\delta_1 = 51,8^\circ \quad \text{less than} \quad \delta_{\max,60} = 107^\circ$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,51 \text{ m} = 1,98 \text{ m} \quad (48)$$

9.4.2.7 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled, see 9.3.2.7.

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1,1F_{t,d,-20} = 1,1 \cdot 38,0 \text{ kN} = 41,8 \text{ kN} \\ F_{pi,d,60} &= 1,1F_{t,d,60} = 1,1 \cdot 34,8 \text{ kN} = 38,3 \text{ kN} \end{aligned} \quad (51)$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 9.3.2.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max \{F_{pi,d,-20}; F_{pi,d,60}\} = \max \{41,8 \text{ kN}; 38,3 \text{ kN}\} = 41,8 \text{ kN}$$

9.4.2.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max \{38,0 \text{ kN}; 0 \text{ kN}; 41,8 \text{ kN}\} = 41,8 \text{ kN}$$

given by the tensile force $F_{pi,d}$ caused by pinch.

The maximum horizontal displacement is 1,51 m and the minimum air clearance is 1,98 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max \{38,0 \text{ kN}; 65,8 \text{ kN}; 41,8 \text{ kN}\} = 65,8 \text{ kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,51 m and the minimum air clearance is 1,98 m and has the same value as with dropper.

9.4.3 Current flow along half of the length of the main conductor and along the dropper

9.4.3.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0,75 \frac{(I_k^*)^2}{a} \frac{l_c/2 + l_v/2}{l} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0,75 \cdot \frac{(63 \cdot 10^3 \text{ A})^2}{5 \text{ m}} \cdot \frac{37,4 \text{ m}/2 + 7,4 \text{ m}/2}{48 \text{ m}} = 55,6 \text{ N/m} \quad (19b)$$

The parameter r is:

$$r = \frac{F'}{nm'_sc g} = \frac{55,6 \text{ N/m}}{2 \cdot 3,73 \text{ kg/m} \cdot 9,81 \text{ m/s}^2} = 0,760 \quad (20)$$

where

$$m'_sc = 3,73 \text{ kg/m} \quad \text{see 9.3.2.1}$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 0,760 = 37,2^\circ \quad (21)$$

The equivalent static conductor sags at midspan are, see 9.3.2.1:

$$f_{es,-20} = 1,21 \text{ m} \quad f_{es,60} = 1,41 \text{ m} \quad (22)$$

The periods of the conductor oscillation are, see 9.3.2.1:

$$T_{-20} = 1,97 \text{ s} \quad T_{60} = 2,13 \text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$T_{res,-20} = \frac{T_{-20}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{1,97}{\sqrt[4]{1+0,760^2} \left[1 - \frac{\pi^2}{64} \left(\frac{37,2^\circ}{90^\circ} \right)^2 \right]} = 1,81 \text{ s} \quad (24)$$

$$T_{res,60} = \frac{T_{60}}{\sqrt[4]{1+r^2} \left[1 - \frac{\pi^2}{64} \left(\frac{\delta_1}{90^\circ} \right)^2 \right]} = \frac{2,13}{\sqrt[4]{1+0,760^2} \left[1 - \frac{\pi^2}{64} \left(\frac{37,2^\circ}{90^\circ} \right)^2 \right]} = 1,95 \text{ s}$$

The stiffness norms are, see 9.3.2.1:

$$N_{-20} = 5,78 \cdot 10^{-8} \text{ 1/N} \quad N_{60} = 5,87 \cdot 10^{-8} \text{ 1/N} \quad (25)$$

The stress factors are, see 9.3.2.1:

$$\zeta_{-20} = 1,69 \quad \zeta_{60} = 2,60 \quad (28)$$

In the Equations (29), (32), and (35) it is to be inserted, see 9.3.2.1:

$$T_{k1} = 0,5 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\begin{aligned}\delta_{\text{end},-20} &= \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{\text{res},-20}} \right) \right] = 37,2^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,81 \text{ s}} \right) \right] = 43,3^\circ \\ \delta_{\text{end},60} &= \delta_1 \left[1 - \cos \left(360^\circ \frac{T_{k1}}{T_{\text{res},60}} \right) \right] = 37,2^\circ \cdot \left[1 - \cos \left(360^\circ \cdot \frac{0,5 \text{ s}}{1,95 \text{ s}} \right) \right] = 38,7^\circ\end{aligned}\quad (29)$$

because

$$\begin{aligned}\frac{T_{k1}}{T_{\text{res},-20}} &= \frac{0,5 \text{ s}}{1,81 \text{ s}} = 0,276 \quad \text{less than} \quad 0,5 \\ \frac{T_{k1}}{T_{\text{res},60}} &= \frac{0,5 \text{ s}}{1,95 \text{ s}} = 0,256 \quad \text{less than} \quad 0,5\end{aligned}$$

The maximum swing-out angles $\delta_{\max,-20}$ and $\delta_{\max,60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{\text{end},-20}$ and $\delta_{\text{end},60}$:

- for 0 less than $\delta_{\text{end},-20} = 43,3^\circ$ less than 90° is:

$$\chi_{-20} = 1 - r \sin \delta_{\text{end},-20} = 1 - 0,760 \cdot \sin 43,3^\circ = 0,479 \quad (30)$$

and for $-0,985$ less than $\chi_{-20} = 0,479$ less than $0,766$:

$$\delta_{\max,-20} = 10^\circ + \arccos \chi_{-20} = 10^\circ + \arccos 0,479 = 71,4^\circ \quad (31)$$

- for 0 less than $\delta_{\text{end},60} = 38,7^\circ$ less than 90° is:

$$\chi_{60} = 1 - r \sin \delta_{\text{end},60} = 1 - 0,760 \cdot \sin 38,7^\circ = 0,525 \quad (30)$$

and for $-0,985$ less than $\chi_{60} = 0,525$ less than $0,766$:

$$\delta_{\max,60} = 10^\circ + \arccos \chi_{60} = 10^\circ + \arccos 0,525 = 68,3^\circ \quad (31)$$

9.4.3.2 Tensile force $F_{t,d}$ during short-circuit caused by swing out without dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3 \left(\sqrt{1+r^2} - 1 \right) = 3 \left(\sqrt{1+0,760^2} - 1 \right) = 0,768 \quad (32)$$

because

$$\begin{aligned}T_{k1} = 0,5 \text{ s} &\quad \text{greater than} \quad \frac{T_{\text{res},-20}}{4} = \frac{1,81 \text{ s}}{4} = 0,453 \text{ s} \\ T_{k1} = 0,5 \text{ s} &\quad \text{greater than} \quad \frac{T_{\text{res},60}}{4} = \frac{1,95 \text{ s}}{4} = 0,488 \text{ s}\end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 0,768$ and $\zeta_{-20} = 1,69$:

$$\psi_{-20} = 0,702$$

– for $\varphi_{60} = 0,768$ and $\zeta_{60} = 2,60$:

$$\psi_{60} = 0,775$$

The tensile forces during the short-circuit are:

$$\begin{aligned} F_{t,d,-20} &= F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 17,4 \text{ kN} \cdot (1 + 0,768 \cdot 0,702) = 26,8 \text{ kN} \\ F_{t,d,60} &= F_{st,60} (1 + \varphi_{60} \psi_{60}) = 15,0 \text{ kN} \cdot (1 + 0,768 \cdot 0,775) = 23,9 \text{ kN} \end{aligned} \quad (33)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{26,8 \text{ kN}; 23,9 \text{ kN}\} = 26,8 \text{ kN}$$

9.4.3.3 Dynamic conductor sag at midspan

The elastic expansions are:

$$\begin{aligned} \varepsilon_{ela,-20} &= N_{-20} (F_{t,d,-20} - F_{st,-20}) = 5,78 \cdot 10^{-8} \frac{1}{N} \cdot (26,8 - 17,4) 10^3 \text{ N} = 0,543 \cdot 10^{-3} \\ \varepsilon_{ela,60} &= N_{60} (F_{t,d,60} - F_{st,60}) = 5,87 \cdot 10^{-8} \frac{1}{N} \cdot (23,9 - 15,0) 10^3 \text{ N} = 0,522 \cdot 10^{-3} \end{aligned} \quad (34)$$

The thermal expansions are:

$$\begin{aligned} \varepsilon_{th,-20} &= c_{th} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{res,-20}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{1,81 \text{ s}}{4} = 1,02 \cdot 10^{-4} \\ \varepsilon_{th,60} &= c_{th} \left(\frac{I_k''}{n A_s} \right)^2 \frac{T_{res,60}}{4} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{\text{A}^2 \text{s}} \cdot \left(\frac{63 \cdot 10^3 \text{ A}}{2 \cdot 1090 \cdot 10^{-6} \text{ m}^2} \right)^2 \cdot \frac{1,95 \text{ s}}{4} = 1,10 \cdot 10^{-4} \end{aligned} \quad (35)$$

because

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{res,-20}}{4} = \frac{1,81 \text{ s}}{4} = 0,453 \text{ s}$$

$$T_{k1} = 0,5 \text{ s} \quad \text{greater than} \quad \frac{T_{res,60}}{4} = \frac{1,95 \text{ s}}{4} = 0,488 \text{ s}$$

and for ASCR conductors with $A_{Al}/A_{St} = 1046 \text{ mm}^2/45 \text{ mm}^2 = 23,2$ greater than 6

$$c_{th} = 0,27 \cdot 10^{-18} \frac{\text{m}^4}{(\text{A}^2 \text{s})}$$

The factors C_D are:

$$\begin{aligned} C_{D,-20} &= \sqrt{1 + \frac{3}{8} \left(\frac{l}{f_{es,-20}} \right)^2 (\varepsilon_{ela,-20} + \varepsilon_{th,-20})} = \sqrt{1 + \frac{3}{8} \left(\frac{48 \text{ m}}{1,21 \text{ m}} \right)^2 (0,543 \cdot 10^{-3} + 1,02 \cdot 10^{-4})} = 1,18 \\ C_{D,60} &= \sqrt{1 + \frac{3}{8} \left(\frac{l}{f_{es,60}} \right)^2 (\varepsilon_{ela,60} + \varepsilon_{th,60})} = \sqrt{1 + \frac{3}{8} \left(\frac{48 \text{ m}}{1,41 \text{ m}} \right)^2 (0,522 \cdot 10^{-3} + 1,10 \cdot 10^{-4})} = 1,13 \end{aligned} \quad (36)$$

The factor C_F is:

$$C_F = 1,05 \quad (37)$$

because

$$r = 0,760 \quad \text{less than} \quad 0,8$$

The dynamic conductor sags at midspan are:

$$\begin{aligned} f_{ed,-20} &= C_F C_{D,-20} f_{es,-20} = 1,05 \cdot 1,18 \cdot 1,21 \text{ m} = 1,50 \text{ m} \\ f_{ed,60} &= C_F C_{D,60} f_{es,60} = 1,05 \cdot 1,13 \cdot 1,41 \text{ m} = 1,67 \text{ m} \end{aligned} \quad (38)$$

9.4.3.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\begin{aligned} \sqrt{(h_{-20} + f_{es,-20})^2 + w^2} + f_{ed,-20} &= \sqrt{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,5 \text{ m})^2} + 1,77 \text{ m} = 10,3 \text{ m} \quad \text{greater than} \quad l_v = 7,4 \text{ m} \\ \sqrt{(h_{60} + f_{es,60})^2 + w^2} + f_{ed,60} &= \sqrt{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,5 \text{ m})^2} + 1,92 \text{ m} = 10,5 \text{ m} \quad \text{greater than} \quad l_v = 7,4 \text{ m} \end{aligned}$$

with the dropper height at -20°C due to the change of sag with the temperature of the main conductor

$$h_{-20} = h_{60} + (f_{es,60} - f_{es,-20}) = 7,0 \text{ m} + (1,41 \text{ m} - 1,21 \text{ m}) = 7,2 \text{ m}$$

and $h_{60} = h = 7,0 \text{ m}$.

The actual swing-out angles are:

$$\begin{aligned}\delta_{-20} &= \arccos \frac{(h_{-20} + f_{es,-20})^2 + f_{ed,-20}^2 - (l_v^2 - w^2)}{2f_{ed,-20}\sqrt{(h_{-20} + f_{es,-20})^2 + w^2}} + \arccos \frac{h_{-20} + f_{es,-20}}{\sqrt{(h_{-20} + f_{es,-20})^2 + w^2}} \\ &= \arccos \frac{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,50 \text{ m})^2 - ((7,4 \text{ m})^2 - (1,5 \text{ m})^2)}{2 \cdot 1,50 \text{ m} \cdot \sqrt{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,5 \text{ m})^2}} + \arccos \frac{7,2 \text{ m} + 1,21 \text{ m}}{\sqrt{(7,2 \text{ m} + 1,21 \text{ m})^2 + (1,5 \text{ m})^2}} \\ &= 47,1^\circ\end{aligned}\quad (39)$$

$$\begin{aligned}\delta_{60} &= \arccos \frac{(h_{60} + f_{es,60})^2 + f_{ed,60}^2 - (l_v^2 - w^2)}{2f_{ed,60}\sqrt{(h_{60} + f_{es,60})^2 + w^2}} + \arccos \frac{h_{60} + f_{es,60}}{\sqrt{(h_{60} + f_{es,60})^2 + w^2}} = \\ &= \arccos \frac{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,67 \text{ m})^2 - ((7,4 \text{ m})^2 - (1,5 \text{ m})^2)}{2 \cdot 1,67 \text{ m} \cdot \sqrt{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,5 \text{ m})^2}} + \arccos \frac{7,0 \text{ m} + 1,41 \text{ m}}{\sqrt{(7,0 \text{ m} + 1,41 \text{ m})^2 + (1,5 \text{ m})^2}} \\ &= 52,7^\circ\end{aligned}$$

The load parameters are:

$$\varphi_{-20} = \varphi_{60} = 3 \left(\sqrt{1+r^2} - 1 \right) = 3 \left(\sqrt{1+0,760^2} - 1 \right) = 0,768 \quad (40)$$

because

$$\begin{aligned}\delta_{-20} &= 47,1^\circ \quad \text{greater than} \quad \delta_1 = 37,2^\circ \\ \delta_{60} &= 52,7^\circ \quad \text{greater than} \quad \delta_1 = 37,2^\circ\end{aligned}$$

and also

$$\begin{aligned}T_{k1} = 0,5 \text{ s} &\quad \text{greater than} \quad \frac{T_{res,-20}}{4} = \frac{1,81 \text{ s}}{4} = 0,453 \text{ s} \\ T_{k1} = 0,5 \text{ s} &\quad \text{greater than} \quad \frac{T_{res,60}}{4} = \frac{1,95 \text{ s}}{4} = 0,488 \text{ s}\end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

- for $\varphi_{-20} = 0,768$ and $\zeta_{-20} = 1,69$:

$$\psi_{-20} = 0,702$$

- for $\varphi_{60} = 0,768$ and $\zeta_{60} = 2,60$:

$$\psi_{60} = 0,775$$

The tensile forces during the short-circuit are:

$$\begin{aligned}F_{t,d,-20} &= F_{st,-20} (1 + \varphi_{-20} \psi_{-20}) = 17,4 \text{ kN} \cdot (1 + 0,768 \cdot 0,702) = 26,8 \text{ kN} \\ F_{t,d,60} &= F_{st,60} (1 + \varphi_{60} \psi_{60}) = 15,0 \text{ kN} \cdot (1 + 0,768 \cdot 0,775) = 23,9 \text{ kN}\end{aligned}\quad (42)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max \{F_{t,d,-20}; F_{t,d,60}\} = \max \{26,8 \text{ kN}; 23,9 \text{ kN}\} = 26,8 \text{ kN}$$

9.4.3.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 0,760 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned}\delta_{\max,-20} &= 71,4^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{\max,60} &= 68,3^\circ \quad \text{less than} \quad 70^\circ\end{aligned}$$

however

$$\begin{aligned}\delta_{-20} &= 47,1^\circ \quad \text{less than} \quad 60^\circ \\ \delta_{60} &= 52,7^\circ \quad \text{less than} \quad 60^\circ\end{aligned}$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 0,760 \quad \text{greater than} \quad 0,6$$

and

$$\begin{aligned}\delta_{\max,-20} &= 71,4^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{\max,60} &= 68,3^\circ \quad \text{less than} \quad 70^\circ\end{aligned}$$

the drop forces become:

$$\begin{aligned}F_{f,d,-20} &= 1,2 \cdot F_{st,-20} \sqrt{1 + 8 \zeta_{-20} \frac{\delta_{\max,-20}}{180^\circ}} = 1,2 \cdot 17,4 \text{ kN} \cdot \sqrt{1 + 8 \cdot 1,69 \cdot \frac{71,4^\circ}{180^\circ}} = 52,7 \text{ kN} \\ F_{f,d,60} &= 0 \text{ kN}\end{aligned}\tag{43}$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max \{F_{f,d,-20}; F_{f,d,60}\} = \max \{52,7 \text{ kN}; 0 \text{ kN}\} = 52,7 \text{ kN}$$

9.4.3.6 Horizontal span displacement b_h and minimum air clearance a_{\min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of -20°C .

The maximum horizontal span displacement for strained conductors with $I_c = l - 2l_i$ is:

$$b_h = f_{ed,60} \sin \delta_1 = 1,67 \text{ m} \cdot \sin 37,2^\circ = 1,02 \text{ m}\tag{47}$$

because

$$\begin{aligned}\delta_{60} &= 57,2^\circ \quad \text{less than} \quad \delta_{\max,60} = 68,3^\circ \\ \delta_{60} &= 52,7^\circ \quad \text{greater than} \quad \delta_1 = 37,2^\circ\end{aligned}$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,02 \text{ m} = 2,96 \text{ m} \quad (48)$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_h and the minimum air clearance a_{\min} shall be calculated according to IEC 60865-1:2011, 6.2.7:

$$b_h = f_{ed,60} \sin \delta_1 = 1,67 \text{ m} \cdot \sin 37,2^\circ = 1,02 \text{ m} \quad (45)$$

because

$$\delta_{\max,60} = 68,3^\circ \quad \text{greater than} \quad \delta_1 = 37,2^\circ$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1,02 \text{ m} = 2,96 \text{ m} \quad (48)$$

9.4.3.7 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short circuit because Equation (53) is fulfilled, see 9.3.2.6.

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1,1F_{t,d,-20} = 1,1 \cdot 26,8 \text{ kN} = 29,5 \text{ kN} \\ F_{pi,d,60} &= 1,1F_{t,d,60} = 1,1 \cdot 23,9 \text{ kN} = 26,3 \text{ kN} \end{aligned} \quad (51)$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 9.4.3.4.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max \{F_{pi,d,-20}; F_{pi,d,60}\} = \max \{29,5 \text{ kN}; 26,3 \text{ kN}\} = 29,5 \text{ kN}$$

9.4.3.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max \{26,8 \text{ kN}; 0 \text{ kN}; 29,5 \text{ kN}\} = 29,5 \text{ kN}$$

given by the tensile force $F_{pi,d}$ caused by pinch.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\{F_{t,d}; F_{f,d}; F_{p,d}\} = \max\{26,8 \text{ kN}; 52,8 \text{ kN}; 29,5 \text{ kN}\} = 52,8 \text{ kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

10 Example 7 – Mechanical effects on vertical main conductors (droppers)

10.1 General

The basis for the calculation in this example is a three-phase 380-kV-arrangement with droppers as shown in Figure 8. The droppers are fixed at the lower fixing point with horizontal connectors. At the upper end a V-shaped insulator-chain is given to prevent a swing-out of the fixing-point in the direction of the short-circuit force.

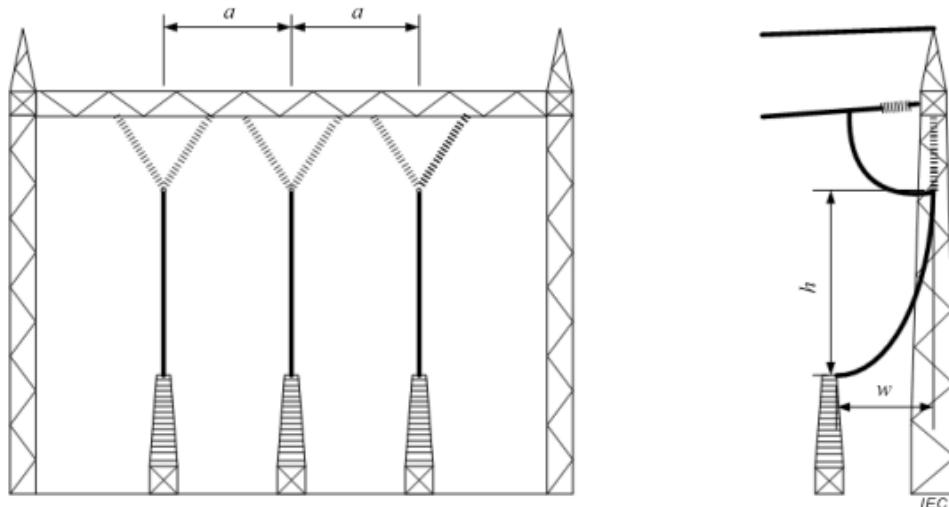


Figure 8 – Arrangement with strained conductors

10.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	= 40 kA
Factor for the calculation of the peak short-circuit current	κ	= 1,81
System frequency	f	= 50 Hz
Height of dropper	h	= 12,3 m
Width of dropper	w	= 5 m
Centre-line distance between supports $l = \sqrt{h^2 + w^2} = \sqrt{(12,3 \text{ m})^2 + (5 \text{ m})^2}$	l	= 13,28 m
Cord length of the dropper	l_v	= 14 m
Centre-line distance between conductors	a	= 6 m
Centre-line distance between connecting pieces	l_s	= 13,28 m
Centre-line distance between sub-conductors	a_s	= 0,1 m
Resultant spring constant of both span supports of one span	S	= 100 N/mm

Twin conductor 2 EN 550-AL1/71-ST1A

– Number of sub-conductors	$n = 2$
– Sub-conductor diameter	$d = 32,2 \text{ mm}$
– Sub-conductor cross-section	$A_s = 611,2 \text{ mm}^2$
– Sub-conductor mass per unit length	$m'_s = 1,95 \text{ kg/m}$
– Young's modulus	$E = 62\,000 \text{ N/mm}^2$

Conventional value of acceleration of gravity $g = 9,81 \text{ m/s}^2$

10.3 Short-circuit tensile force and maximum horizontal displacement

The short-circuit tensile force is:

$$F_{t,d} = \frac{5}{3} l_v \frac{\mu_0}{2\pi} \frac{(I_k'')^2}{a} \frac{l_v}{w} = \frac{5}{3} \cdot 14 \text{ m} \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \frac{(40 \cdot 10^3 \text{ A})^2}{6 \text{ m}} \cdot \frac{14 \text{ m}}{5 \text{ m}} = 3484 \text{ N} = 3,48 \text{ kN} \quad (49)$$

for cable length

$$1,4w = 1,4 \cdot 5 \text{ m} = 7 \text{ m} \quad \text{less than} \quad l_v = 14 \text{ m} \quad \text{less than} \quad 3,3w = 3,3 \cdot 5 \text{ m} = 16,5 \text{ m}$$

The maximum horizontal displacement is:

$$\begin{aligned} b_h &= \left[0,6 \sqrt{\frac{l_v}{l}} - 1 + 0,44 \left(\frac{l_v}{l} - 1 \right) - 0,32 \ln \frac{l_v}{l} \right] \frac{l^2}{l_v} \\ &= \left[0,6 \sqrt{\frac{14 \text{ m}}{13,28 \text{ m}} - 1} + 0,44 \left(\frac{14 \text{ m}}{13,28 \text{ m}} - 1 \right) - 0,32 \ln \frac{14 \text{ m}}{13,28 \text{ m}} \right] \cdot \frac{(13,28 \text{ m})^2}{14 \text{ m}} \\ &= 1,85 \text{ m} \end{aligned} \quad (50)$$

for cable length

$$l_v = 14 \text{ m} \quad \text{less than} \quad 2l = 2 \cdot 13,28 \text{ m} = 26,6 \text{ m}$$

The minimum air clearance is:

$$a_{\min} = a - 2b_h = 6 \text{ m} - 2 \cdot 1,85 \text{ m} = 2,3 \text{ m} \quad (48)$$

10.4 Pinch force

10.4.1 Static tensile force regarding droppers

The horizontal component of the force caused by one sub-conductor at the lower fixing point may be calculated with the conductor sagging curve as a parabola, for example as in [3]:

$$H_s = \sqrt{\frac{1}{24} \frac{(m'_s g)^2 w^2}{\sqrt{l_v^2 - h^2} - 1}} = \sqrt{\frac{1}{24} \frac{(1,95 \text{ kg/m} \cdot 9,81 \text{ m/s}^2)^2 \cdot (5 \text{ m})^2}{\sqrt{(14 \text{ m})^2 - (12,3 \text{ m})^2} - 1}} = 33,6 \text{ N} = 34 \text{ N}$$

The vertical component of the force caused by one sub-conductor at the upper fixing point is:

$$V_s = m'_s l_v g = 1,95 \frac{\text{kg}}{\text{m}} \cdot 14 \text{ m} \cdot 9,81 \frac{\text{m}}{\text{s}^2} = 268 \text{ N}$$

The average tensile force in the dropper is:

$$F_{st} = n \frac{(H_s + V_s)}{2} = 2 \cdot \frac{(268 \text{ N} + 34 \text{ N})}{2} = 302 \text{ N}$$

10.4.2 Characteristic dimensions and parameters

Because

$$\frac{a_s}{d} = \frac{0,100 \text{ m}}{0,0322 \text{ m}} = 3,11 \quad \text{and} \quad \frac{l_s}{a_s} = \frac{13,28 \text{ m}}{0,100 \text{ m}} = 133$$

neither Equation (52) nor Equation (53) of IEC 60865-1:2011 is fulfilled, the pinch force $F_{pi,d}$ is to be calculated with the Equations (54) and following of IEC 60865-1:2011, 6.4.

The short-circuit current force between the sub-conductors is:

$$F_v = (n-1) \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 \frac{l_s}{a_s} \nu_2 \quad (54)$$

The factor ν_1 for calculation of ν_2 is:

$$\nu_1 = f \frac{1}{\sin \frac{180^\circ}{n}} \sqrt{\frac{(a_s - d) m'_s}{\frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 \frac{n-1}{a_s}}} = 50 \frac{1}{\text{s}} \cdot \frac{1}{\sin \frac{180^\circ}{2}} \cdot \sqrt{\frac{(0,100 \text{ m} - 0,0322 \text{ m}) \cdot 1,95 \text{ kg/m}}{\frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{40 \cdot 10^3 \text{ A}}{2} \right)^2 \cdot \frac{2-1}{0,100 \text{ m}}}} = 0,643 \quad (55)$$

According to IEC 60865-1:2011, Figure 9, the factor ν_2 for $\nu_1 = 0,643$ and $\kappa = 1,81$ is:

$$\nu_2 = 2,11$$

According to IEC 60865-1:2011, Figure 10, the factor ν_3 for $a_s/d = 3,11$ is:

$$\nu_3 = 0,483$$

With this the short-circuit current force between the sub-conductors is:

$$F_v = (n-1) \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 \frac{l_s}{a_s} \frac{\nu_2}{\nu_3} = (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{40 \cdot 10^3 \text{ A}}{2} \right)^2 \cdot \frac{13,28 \text{ m}}{0,1 \text{ m}} \cdot \frac{2,11}{0,483} = 46411 \cdot 10^3 \text{ N} = 46,4 \text{ kN} \quad (54)$$

The stiffness norm is:

$$N = \frac{1}{S l} + \frac{1}{n E_{eff} A_s} = \frac{1}{1 \cdot 10^5 \text{ N/m} \cdot 13,28 \text{ m}} + \frac{1}{2 \cdot 1,89 \cdot 10^{10} \text{ N/m}^2 \cdot 611,2 \cdot 10^{-6} \text{ m}^2} = 79,6 \cdot 10^{-8} \text{ 1/N} \quad (25)$$

with the actual Young's modulus

$$E_{\text{eff}} = E \left[0,3 + 0,7 \sin \left(\frac{F_{\text{st}}}{n A_s \sigma_{\text{fin}}} \cdot 90^\circ \right) \right] = 6,2 \cdot 10^{10} \frac{\text{N}}{\text{m}^2} \cdot \left[0,3 + 0,7 \sin \left(\frac{0,247 \cdot 10^6 \text{ N/m}^2}{50 \cdot 10^6 \text{ N/m}^2} \cdot 90^\circ \right) \right] \quad (26)$$

$$= 1,89 \cdot 10^{10} \text{ N/m}^2$$

because

$$\frac{F_{\text{st}}}{n A_s} = \frac{302 \text{ N}}{2 \cdot 611,2 \cdot 10^{-6} \text{ m}^2} = 0,247 \cdot 10^6 \text{ N/m}^2 \quad \text{less than} \quad \sigma_{\text{fin}} = 50 \cdot 10^6 \text{ N/m}^2$$

The strain factors are:

$$\varepsilon_{\text{st}} = 1,5 \frac{F_{\text{st}} l_s^2 N}{(a_s - d)^2} \left(\sin \frac{180^\circ}{n} \right)^2 = 1,5 \cdot \frac{302 \text{ N} \cdot (13,28 \text{ m})^2 \cdot 79,6 \cdot 10^{-8} \text{ 1/N}}{(0,100 \text{ m} - 0,0322 \text{ m})^2} \cdot \left(\sin \frac{180^\circ}{2} \right)^2 = 13,8 \quad (56)$$

$$\varepsilon_{\text{pi}} = 0,375 n \frac{F_v l_s^3 N}{(a_s - d)^3} \left(\sin \frac{180^\circ}{n} \right)^3 = 0,375 \cdot 2 \cdot \frac{46,4 \cdot 10^3 \text{ N} \cdot (13,28 \text{ m})^3 \cdot 79,6 \cdot 10^{-8} \text{ 1/N}}{(0,100 \text{ m} - 0,0322 \text{ m})^3} \cdot \left(\sin \frac{180^\circ}{2} \right)^3 = 2,08 \cdot 10^5 \quad (57)$$

The parameter j is:

$$j = \sqrt{\frac{\varepsilon_{\text{pi}}}{1 + \varepsilon_{\text{st}}}} = \sqrt{\frac{2,08 \cdot 10^5}{1 + 13,8}} = 119 \quad (58)$$

10.4.3 Pinch force $F_{\text{pi,d}}$

Because

$$j = 119 \quad \text{greater than} \quad 1$$

the sub-conductors clash and the tensile forces due to contraction are calculated according to IEC 60865-1:2011, 6.4.2:

$$F_{\text{pi,d}} = F_{\text{st}} \left(1 + \frac{\nu_e}{\varepsilon_{\text{st}}} \xi \right) \quad (59)$$

According to IEC 60865-1:2011, Figure 11, and with $j = 119$ and $\varepsilon_{\text{st}} = 13,8$ the factor ξ is:

$$\xi = 55,0$$

The factor ν_e is:

$$\begin{aligned}
 v_e &= \frac{1}{2} + \left[\frac{9}{8} n(n-1) \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n} \right)^2 N v_2 \left(\frac{l_s}{a_s - d} \right)^4 \frac{\left(\sin \frac{180^\circ}{n} \right)^4}{\xi^3} \left\{ 1 - \frac{\arctan \sqrt{v_4}}{\sqrt{v_4}} \right\} - \frac{1}{4} \right]^{1/2} \\
 &= \frac{1}{2} + \left[\frac{9}{8} \cdot 2 \cdot (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{Vs}{Am} \cdot \left(\frac{40 \cdot 10^3 A}{2} \right)^2 \cdot 79,6 \cdot 10^{-8} \frac{1}{N} \cdot 2,11 \cdot \left(\frac{13,28 m}{0,100 m - 0,0322 m} \right)^4 \right. \\
 &\quad \left. \cdot \frac{\left(\sin \frac{180^\circ}{2} \right)^4}{55,0^3} \cdot \left\{ 1 - \frac{\arctan \sqrt{2,11}}{\sqrt{2,11}} \right\} - \frac{1}{4} \right]^{1/2} \\
 &= 1,30
 \end{aligned} \tag{60}$$

with the factor v_4 :

$$v_4 = \frac{a_s - d}{d} = \frac{0,100 m - 0,0322 m}{0,0322 m} = 2,11 \tag{61}$$

With this, the tensile force is:

$$F_{pi,d} = F_{st} \left(1 + \frac{v_e}{v_{st}} \right) = 302 N \cdot \left(1 + \frac{1,30}{13,8} \cdot 55,0 \right) = 1878 N = 1,88 kN \tag{59}$$

10.5 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max \{ F_{t,d}; F_{pi,d} \} = \max \{ 3,48 kN; 1,88 kN \} = 3,48 kN$$

given by the short-circuit tensile force $F_{t,d}$.

The maximum horizontal displacement is 1,85 m and the minimum air clearance is 2,3 m.

11 Example 8 – Thermal effect on bare conductors

11.1 General

The basis for the calculation is a three-phase 10 kV busbar with one conductor per phase.

11.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	$I_k'' = 24,0 \text{ kA}$
Steady-state short-circuit current (r.m.s.)	$I_k = 19,2 \text{ kA}$
Factor for the calculation of the peak short-circuit current	$\kappa = 1,8$
Duration of short-circuit current	$T_k = 0,8 \text{ s}$
System frequency	$f = 50 \text{ Hz}$

Rectangular conductor of EN AW-6101B with cross-section	$A = 600 \text{ mm}^2$
Conductor temperature at the beginning of short-circuit	$\vartheta_b = 65^\circ\text{C}$
Conductor temperature at the end of short-circuit	$\vartheta_e = 170^\circ\text{C}$

11.3 Calculations

For $\vartheta_b = 65^\circ\text{C}$ and $\vartheta_e = 170^\circ\text{C}$, the rated short-time withstand current density is found from IEC 60865-1:2011, Figure 13b):

$$S_{\text{thr}} = 80,7 \text{ A/mm}^2$$

The thermal equivalent short-circuit current is according to Equation (103) of IEC 60909-0:2001:

$$I_{\text{th}} = I_k'' \sqrt{m+n} = 24,0 \text{ kA} \cdot \sqrt{0,056+0,86} = 23,0 \text{ kA}$$

m and n are found from IEC 60909-0:2001, Figures 21 and 22, for

$$f T_k = 50 \text{ 1/s} \cdot 0,8 \text{ s} = 40 ; \quad \kappa = 1,8 ; \quad I_k''/I_k = 24,0 \text{ kA}/19,2 \text{ kA} = 1,25$$

For the conductor cross-section $A = 600 \text{ mm}^2$ the thermal equivalent short-circuit current density is:

$$S_{\text{th}} = \frac{I_{\text{th}}}{A} = \frac{23,0 \cdot 10^3 \text{ A}}{600 \text{ mm}^2} = 38,3 \text{ A/mm}^2$$

The busbar has sufficient thermal strength if:

$$S_{\text{th}} = 38,3 \text{ A/mm}^2 \quad \text{less than} \quad S_{\text{thr}} \sqrt{\frac{T_{kr}}{T_k}} = 80,7 \frac{\text{A}}{\text{mm}^2} \cdot \sqrt{\frac{1\text{s}}{0,8\text{s}}} = 90,2 \text{ A/mm}^2 \quad (65)$$

11.4 Conclusion

The busbar has sufficient thermal strength.

Bibliography

- [1] IEC 61936-1, *Power installations exceeding 1 kV a.c. – Part 1: Common rules*
 - [2] EN 1990, *Eurocode: Basis of structural design*
 - [3] Kiessling, F., Nefzger, P., Nolasco, J.F., Kaintzyk, U.: Overhead Power Lines. Planning, Design, Construction. Berlin: Springer, 2003
-

NO COPYING WITHOUT BSI PERMISSION EXCEPT AS PERMITTED BY COPYRIGHT LAW

British Standards Institution (BSI)

BSI is the national body responsible for preparing British Standards and other standards-related publications, information and services.

BSI is incorporated by Royal Charter. British Standards and other standardization products are published by BSI Standards Limited.

Licensed copy: Imperial College London, Version correct as of 02/05/2016

About us

We bring together business, industry, government, consumers, innovators and others to shape their combined experience and expertise into standards-based solutions.

The knowledge embodied in our standards has been carefully assembled in a dependable format and refined through our open consultation process. Organizations of all sizes and across all sectors choose standards to help them achieve their goals.

Information on standards

We can provide you with the knowledge that your organization needs to succeed. Find out more about British Standards by visiting our website at bsigroup.com/standards or contacting our Customer Services team or Knowledge Centre.

Buying standards

You can buy and download PDF versions of BSI publications, including British and adopted European and international standards, through our website at bsigroup.com/shop, where hard copies can also be purchased.

If you need international and foreign standards from other Standards Development Organizations, hard copies can be ordered from our Customer Services team.

Subscriptions

Our range of subscription services are designed to make using standards easier for you. For further information on our subscription products go to bsigroup.com/subscriptions.

With **British Standards Online (BSOL)** you'll have instant access to over 55,000 British and adopted European and international standards from your desktop. It's available 24/7 and is refreshed daily so you'll always be up to date.

You can keep in touch with standards developments and receive substantial discounts on the purchase price of standards, both in single copy and subscription format, by becoming a **BSI Subscribing Member**.

PLUS is an updating service exclusive to BSI Subscribing Members. You will automatically receive the latest hard copy of your standards when they're revised or replaced.

To find out more about becoming a BSI Subscribing Member and the benefits of membership, please visit bsigroup.com/shop.

With a **Multi-User Network Licence (MUNL)** you are able to host standards publications on your intranet. Licences can cover as few or as many users as you wish. With updates supplied as soon as they're available, you can be sure your documentation is current. For further information, email bsmusales@bsigroup.com.

BSI Group Headquarters

389 Chiswick High Road London W4 4AL UK

Revisions

Our British Standards and other publications are updated by amendment or revision. We continually improve the quality of our products and services to benefit your business. If you find an inaccuracy or ambiguity within a British Standard or other BSI publication please inform the Knowledge Centre.

Copyright

All the data, software and documentation set out in all British Standards and other BSI publications are the property of and copyrighted by BSI, or some person or entity that owns copyright in the information used (such as the international standardization bodies) and has formally licensed such information to BSI for commercial publication and use. Except as permitted under the Copyright, Designs and Patents Act 1988 no extract may be reproduced, stored in a retrieval system or transmitted in any form or by any means – electronic, photocopying, recording or otherwise – without prior written permission from BSI. Details and advice can be obtained from the Copyright & Licensing Department.

Useful Contacts:

Customer Services

Tel: +44 845 086 9001

Email (orders): orders@bsigroup.com

Email (enquiries): cservices@bsigroup.com

Subscriptions

Tel: +44 845 086 9001

Email: subscriptions@bsigroup.com

Knowledge Centre

Tel: +44 20 8996 7004

Email: knowledgecentre@bsigroup.com

Copyright & Licensing

Tel: +44 20 8996 7070

Email: copyright@bsigroup.com



...making excellence a habit.TM