

We want to find $P(G|R = \text{yes})$, which we can compute using the joint distribution of G and R , evaluated at $R = \text{yes}$, i.e., $P(G, R = \text{yes})$. In particular, we have,

$$P(G|R = \text{yes}) = \frac{P(G, R = \text{yes})}{\sum_{\forall G} P(G, R = \text{yes})}.$$

We compute,

$$\begin{aligned} P(G, R = \text{yes}) &= \sum_{\forall S} \sum_{\forall C} P(G, S, R = \text{yes}, C) \\ &= \sum_{\forall S} \sum_{\forall C} P(G|S, R = \text{yes}) P(S|C) P(R = \text{yes}|C) P(C) \\ &= \sum_{\forall S} P(G|S, R = \text{yes}) \underbrace{\sum_{\forall C} P(S|C) P(R = \text{yes}|C) P(C)}_{F(S)} \\ &= \sum_{\forall S} P(G|S, R = \text{yes}) F(S) \end{aligned}$$

We first compute F . We have,

$$\begin{aligned} F(S) &= \sum_{\forall C} P(S|C) P(R = \text{yes}|C) P(C) \\ &= 0.5(P(S|C = \text{yes}) P(R = \text{yes}|C = \text{yes}) \\ &\quad + P(S|C = \text{no}) P(R = \text{yes}|C = \text{no})) \\ &= 0.5(0.8P(S|C = \text{yes}) + 0.1P(S|C = \text{no})) \\ &= \begin{cases} 0.5 \times (0.8 \times 0.1 + 0.1 \times 0.8) = 0.08, S = \text{on} \\ 0.5 \times (0.8 \times 0.9 + 0.2 \times 0.1) = 0.37, S = \text{off} \end{cases} \end{aligned}$$

Substituting this back into the joint distribution, we have

$$\begin{aligned} P(G, R = \text{yes}) &= 0.08 \times P(G|S = \text{on}, R = \text{yes}) + 0.37 \times P(G|S = \text{off}, R = \text{yes}) \\ &= \begin{cases} 0.08 \times 0.83 + 0.37 \times 0.06 = 0.0886, G = \text{wet} \\ 0.08 \times 0.15 + 0.37 \times 0.9 = 0.345, G = \text{damp} \\ 0.08 \times 0.02 + 0.37 \times 0.04 = 0.0164, G = \text{dry} \end{cases} \end{aligned}$$

$$\therefore P(G = \text{wet}|R = \text{yes}) = \frac{0.0886}{0.0886 + 0.345 + 0.0164} \approx 19.688.$$