# 1 Results

**Definition 1.** (**Interference**) Given two sets $U, W$, we say $U$ $k-$*interferes* with $W$ if

$$|U \cap W| \geq \frac{|W|}{k} \tag{1}$$

for some $k \in (0, |W|]$

Note that if $|U| = |W|$, then $U$ $k$-interferes with $W$ if and only if $W$ $k$-interferes with $U$. We restrict the upper range of $k$ to $|W|$ for convenience, as beyond that all values of $\frac{|W|}{k}$ will be less than 1. Note that we do not to floor or ceil thw RHS since the LHS will always be an integer.

**Definition 2.** (**(r,T,k)-Subgraph capacity**) Given a vertex set $V = \{v_1, ..., v_n\}$, the $(r, T, k)$-*subgraph capacity* of $V$ is the maximum number of subgraphs that can be generated randomly with expected size $r$ subject to the constraint that for any randomly selected subgraph $U$,

$$\mathbb{E}[|X_U^k|] \leq T \tag{2}$$

where $X_U^k = \{W \in V^r \mid U \text{ } k\text{-interferes with } W\}$ and $V^r$ is the set of all currently generated subgraphs with expected size $r$.

Intuitively, $\mathbb{E}[|X_U^k|]$ represents the expected number of k-interferences caused by randomly picking a subgraph $U$.

**Lemma 3.** *Given a vertex set $V$ with $n$ vertices and two subsets $U, W$ or size $r$, the probability that $U$ $k$-interferes with $W$ is*

$$\sum_{y=\lceil \frac{r}{k} \rceil}^{r} \frac{\binom{r}{y}\binom{n-r}{r-y}}{\binom{n}{r}}$$

*Proof.* if $V = \{v_1, ..., v_n\}$, we can represent a subset $U$ as a vector of length $n$, $u$ defined by

$$u[i] = \begin{cases} 1 & \text{if } v_i \in U \\ 0 & \text{if } v_i \notin U \end{cases}$$

With this representation, $U, W$ intersect that the indices where both vectors $u, w$ have a 1. Let $Y$ be a random variable denoting the number of indices where both $u, w$ have a 1. Then

$$P(Y = y) = \frac{\binom{r}{y}\binom{n-r}{r-y}}{\binom{n}{r}} \tag{3}$$

Then the probability that $U$ $k$-interferes with $W$ is

$$\sum_{y=\lceil \frac{r}{k} \rceil}^{r} \frac{\binom{r}{y}\binom{n-r}{r-y}}{\binom{n}{r}} \tag{4}$$

□

**Theorem 4.** *Given a vertex set $V$ with $n$ vertices, the $(r, T, k)$-subgraph capacity of $V$ is*

$$\frac{T}{\sum_{y=\lceil \frac{r}{k} \rceil}^{r} \frac{\binom{r}{y}\binom{n-r}{r-y}}{\binom{n}{r}}} + 1$$

*Proof.* Let $M$ be the number of subgraphs of size $r$ currently generated. Pick an arbitrary subgraph $u$ of size $r$. From lemma 3., we know that the probability of $U$ $k$-interfering with another subgraph is $F(n, r, k)$. Since there are $M - 1$ such subgraphs, the expected number of $k$-interferences by picking $U$ is

$$\sum_{y=\lceil \frac{r}{k} \rceil}^{r} \frac{\binom{r}{y}\binom{n-r}{r-y}}{\binom{n}{r}}(M-1) \tag{5}$$

From equation (2), we have

$$\sum_{y=\lceil \frac{r}{k} \rceil}^{r} \frac{\binom{r}{y}\binom{n-r}{r-y}}{\binom{n}{r}}(M-1) < T \implies M < \frac{T}{\sum_{y=\lceil \frac{r}{k} \rceil}^{r} \frac{\binom{r}{y}\binom{n-r}{r-y}}{\binom{n}{r}}} + 1 \tag{6}$$

The $(r, T, k)$-subgraph capacity of $V$ is the largest $M$ that satisfies equation (8). $\square$