

Probability Random Variable

Assignment-1

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QUESTION:

It is known that 10 % of certain articles manufactured are defective. What is the probability that in a random sample space of 12 such articles, 9 are defective?

SOLUTION:

The repeated selections of articles in a random sample space are bernauli trails. Let X denote the number of times of selective defective article in random sample space of 12 articles.

Here,

$$p = 10\% = \frac{10}{100} = \frac{1}{10}$$

$$q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

Clearly, X has a Binomial distribution with $n=12$,

$$p = \frac{1}{10} \text{ and } q = \frac{9}{10}$$

The Binomial distribution of X is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

To calculate the probability of getting exactly 9 defective articles in a sample of 12 is:

$$\begin{aligned} P(X = 9) &= {}^{12}C_9 \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \\ &= \frac{12!}{9!3!} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \\ &= \frac{12 \times 11 \times 10 \times 9!}{9! \times 3 \times 2 \times 1} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \\ &= 22 \frac{9^3}{10^{11}} \end{aligned}$$

Hence, Probability of getting 9 defective articles is $22 \left(\frac{9^3}{10^{11}}\right)$