

Probability & Random Variable Assignment-1

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Question:

It is known that 10 % of certain articles manufactured are defective.What is the probability that in a random sample space of 12 such articles,9 are defective?

Solution:

The repeated selections of articles in a random sample space are bernauli trials.Let X denote the number of times of selective defective article in random sample space of 12 articles.

Here,

$$p = 10\% = \frac{10}{100} = \frac{1}{10} \quad (1)$$

$$q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10} \quad (2)$$

Clearly, X ha s a Binomial distribution with n=12,

$$p = \frac{1}{10} \text{ and } q = \frac{9}{10} \quad (3)$$

The Binomial distribution of X is given by,

$$P(X = r) = \binom{n}{r} p^r q^{n-r} \quad (4)$$

To calculate the probability of getting exactly 9 defective articles in a sample of 12 is:

$$P(X = 9) = \binom{12}{9} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \quad (5)$$

$$= \frac{12!}{9!3!} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \quad (6)$$

$$= \frac{12 \times 11 \times 10 \times 9!}{9! \times 3 \times 2 \times 1} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \quad (7)$$

$$= 22 \frac{9^3}{10^{11}} \quad (8)$$

(9)

Hence, Probability of getting 9 defective articles is $22 \left(\frac{9^3}{10^{11}}\right)$