A report on

Scale-Free Network

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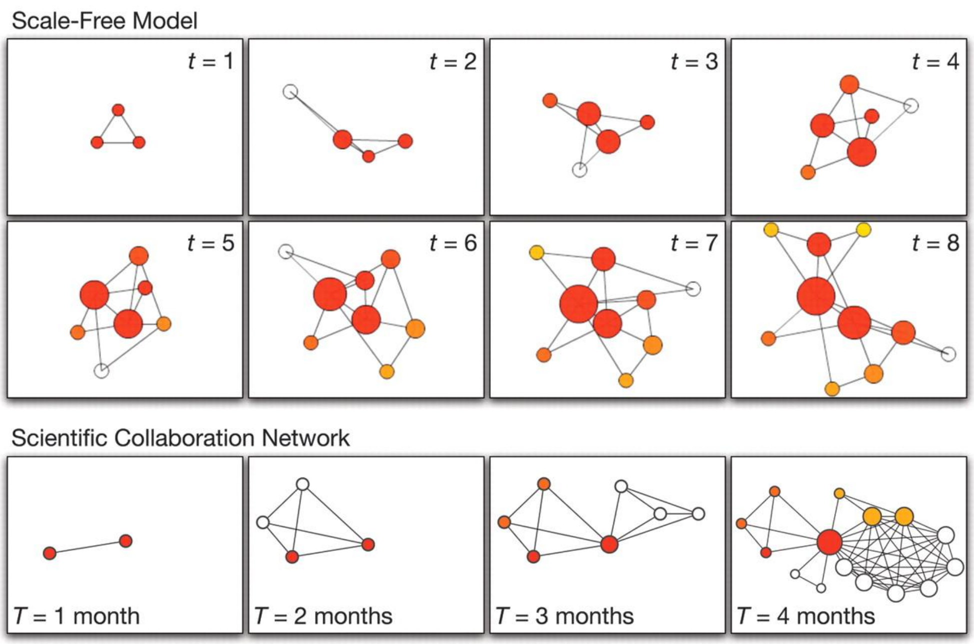


Fig: Some Scale Free Networks for different parameters of t

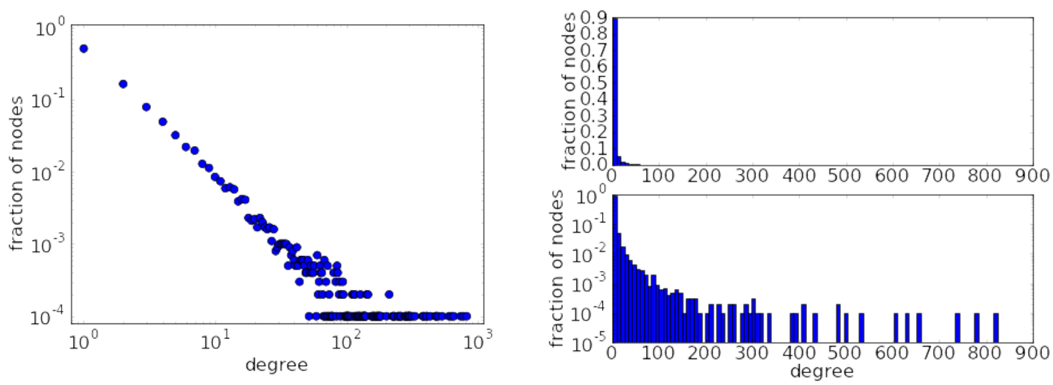
Definition

A scale-free network is a type of network where the degree distribution of vertices of the network roughly follows a power law pattern. Formally speaking, if *P(k)* denotes the fraction of nodes in the network that have k degree then $P(k) \sim k^{-y}$.

Here, *y* is a parameter whose value typically lies between [2, 3] but can sometimes lie outside of the range as well.

Such networks are called scale-free because they have essentially the same functional form at all levels of scales.

Example



In the above figure(right one), the degree distribution of a network with 10000 nodes and power-law exponent y = 2 is represented. About 75% of the nodes have a degree of 3 or less and the average degree is about 7. In the left figure, we can see that there are some nodes with degree > 100. So, although most of the nodes have very small degrees, few of the nodes have a degree close to 500.

History of Scale-Free Network

One of the first records of studying such a network dates back to 1976 when Derek de Solla Price studied the citation network of authors of scientific papers. He wanted to measure the "scientific productivity" of various authors.

He found out that the number of links to papers—i.e., the number of citations they receive—had a heavy-tailed distribution following a power law, and thus that the citation network is scale-free. He did not however use the term "scale-free network", which was not coined until some decades later.

In the year 1999, Albert-László Barabási and his colleagues at the University of Notre Dame, mapped a portion of the topology of the world wide web. They found that some nodes, which they termed as “hubs”, had more connections to other nodes than many other nodes in the network. The number of connections roughly follow a power law and thus this network was scale-free.

Other researchers such as Barabási and Réka Albert proposed a generative mechanism to explain the appearance of power-law distributions. They named this mechanism "preferential attachment" which is essentially the same process that was described by Price in his original paper but he called it as “cumulative advantage”.

Properties of Scale-Free Network

There are 2 important properties of a scale-free network which are growth and preferential attachment.

1. Growth:

Growth means how a network grows or develops over time. This includes how newly joined nodes are attached to the already existing ones and how they choose the criteria to connect to other nodes.

2. Preferential Attachment:

This is a typical tendency of the newly joined nodes to connect to those existing ones that already have a high degree. Hence the probability that the nodes that already have a high degree will have even higher degree increases. This is usually referred to as “rich gets richer and poor gets poorer“ concept.

Examples of scale-free network

Some real life examples of scale-free networks are as follows.

1. Social networks like collaboration networks including collaboration of movie actors in films and co-authorship by mathematicians of papers.
2. Computer network like internet and World Wide Web(WWW)
3. Dependencies of Softwares
4. Network of internet payment banks
5. Protein-protein interaction
6. Semantic Network
7. Airline Network

Generative Models

Scale-free networks don't happen by accident. Erds and Rényi (1960) investigated a network growth model in which two nodes are picked evenly at random and a link is inserted between them at each phase. The features of these random graphs differ from those of scale-free networks, necessitating the development of a model for this growth process. Barabási and Albert's rich get richer generative model, in which each new Web page creates links to existing Web pages with a probability distribution that is not uniform, but proportional to the current in-degree of Web pages, is the most widely known generative model for a subset of scale-free networks.

Applications

The relative commonness of vertices with a degree that substantially surpasses the average is the most noteworthy feature of a scale-free network. The highest-degree nodes are sometimes referred to as "hubs," and they are assumed to have specific functions in their networks, however this varies widely depending on the domain.

1. Immunization

The subject of how to effectively immunise scale free networks that mimic reality networks like the Internet and social networks has been intensively researched. One such technique is to immunise the highest degree nodes, i.e., targeted (deliberate) attacks, because the likelihood of attack is relatively high in this situation and fewer nodes need to be immunised. However, in many real-world situations, the global structure is unavailable, and the highest-degree nodes are unknown.

2. Epidemiology

Until now, it was assumed that eliminating sexually transmitted illnesses necessitated reaching or immunising a substantial part of the population; most encounters would be safe, and the disease would be eradicated. However, unless health offensives target the very connected individuals of scale-free networks—individuals who have sex lives that are quantitatively different from those of their peers—health offensives will fail. No matter how many of their more restrained neighbours are immunised, these people will spread the disease.

References

1. https://en.wikipedia.org/wiki/Scale-free_network

2. https://www.pnas.org/doi/10.1073/pnas.202301299

3.https://arxiv.org/abs/1811.02071?utm_source=feedburner&utm_medium=feed&utm_campaign=Feed%253A+arxiv%252FQSXk+%2528ExcitingAds%2521+cs+updates+on+arXiv.org%2529