‘Small World’ Networks

(An Introduction)

Presenter : Vishal Asthana (wish@nile.usc.edu)

Small World Networks

Outline

- What are Small World Networks ? Where are they found ?
- Statistical Characteristics(3)
- Watts -Strogatz (WS) Model and its variation
- Examples
- Conclusion and Future work

Small World Networks

What are they ? Where are they found ?

- Most Large Scale ‘Sparse’ Networks are found to be of the small world type e.g. ‘Internet’, ‘Neurons’, ‘Human beings’ (Friendship Networks)
- A.k.a ‘Six Degrees of Separation’ (Strangers -- Sociological Concept)
- Why the name Small World Networks ? (Cliché :People far away know a common friend !)
- ***Mathematically*** : In between ‘Regular Networks’ and ‘Random Networks’

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Statistical Characteristics

Three main attributes used to analyze Small World Graphs :

- Average Vertex Degree (k)
(Avg. of No. of Edges Incident on 'v' over all 'v')

- Average Characteristic Path Length (L)
(Shortest Dist. B/w 2 points Avged over all connected pairs)

- Average Clustering Coefficient (C)
(Prob. Of 2 nodes with a "mutual" friend being connected)

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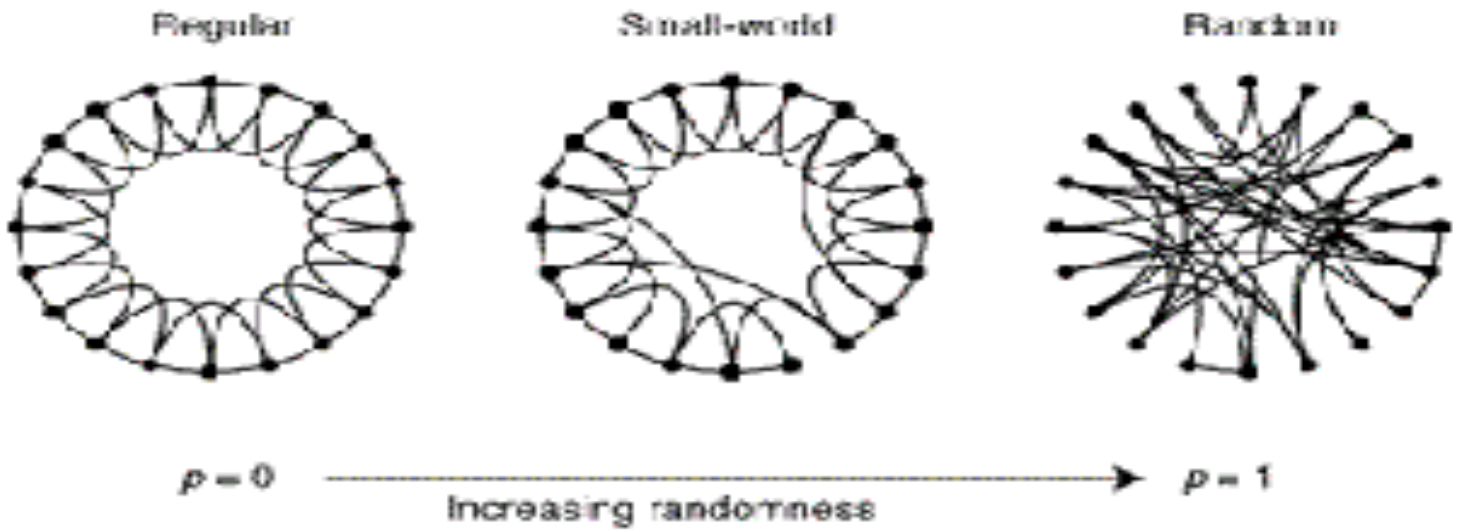
Watts -Strogatz (WS) Model (1998)

- First successful attempt !! (Low ‘L’ and High ‘C’)
- Roots of the Model ? (Friends, neighbors,1-2 far away etc.)



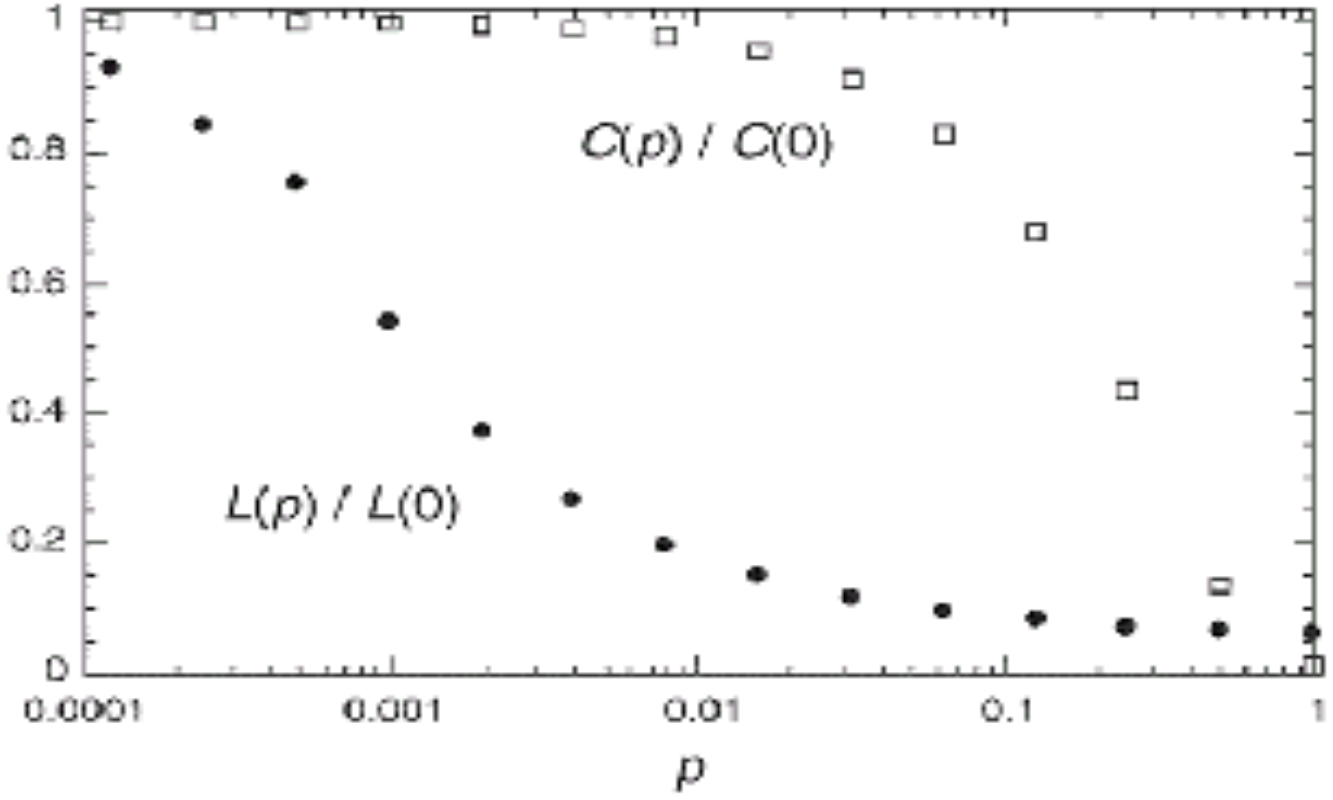
‘k’ (Aver. Vertex Degree) = 4 ,

‘n’(No. of Nodes) = 20 ,

‘p’ (Rewiring probability)

Small World Networks

Watts -Strogatz (WS) Model (1998) (contd..)



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Potential problem with WS Model ?

-Edges allowed to be 'disconnected', therefore chances of 'Isolated Clusters' !!

Solution ?
Variant of WS Model --> Newman and Watts (1999a,1999b)

-Edges 'added' between Randomly chosen pair of sites but 'no edges removed' from the original lattice, therefore easier to analyze !

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Examples !!

- Kevin Bacon Graph (KBG)
- Power Grid (Western US)
- C. elegans Worm
- Infectious Disease Spreading

Studied by Watts-Strogatz

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Examples : **KBG (Kevin Bacon Graph), Grid, Worm**

- Most popular example !!
- Validated using Movie Actors' Database (150,000 films, 300,000 combined actors) (www.us.imdb.com)
- 'Nodes' represent actors who have appeared in one or more films
- 'Edge' is the connection whenever the actors have appeared together in at least 1 feature film
- 90% of actors are part of single 'connected' component KBG* (225K actors in 110K films)
- k = 61 (Sufficiently Sparse). **Grid** -: (4971,2.67) , **Worm** :- (282,14)

Empirical examples of small-world networks				
	L_{actual}	L_{random}	C_{actual}	C_{random}
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
C. elegans	2.65	2.25	0.28	0.05

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Example : **Spread of Infectious Disease**

- Type of Distributed Dynamic System
- Disease spreads from a small set of initiators to a much larger population
- At time (t =0), single infective introduced into a healthy population
- After 1 unit of time, infective is “removed” (dies or becomes immune), but in that interval can infect (with some probability) each of its neighbors

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Example : **Spread of Infectious Disease** (contd..)

• Three distinct regimes of behavior :

- Diseases with Low infectious ness (Infects Little population, then dies)
- Diseases with High infectious ness (Infects Entire population, function of 'L' !!)
- Diseases with Medium infectious ness (Complicated relationship between Structure and Dynamics, not completely characterized)

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Conclusion and Future Work

- Why Small World ?? (Understand a Mix behavior (Regular + Random))
- Great concept, somewhat new , I have just started in this direction .. long way to go !!
- ‘IDS’ System should be based on WS Model ??
- Shamim’s talk (Suggested Application in Multicast for Mobile Ad-Hoc, Freenets !!)