Convex Optimization
Assignment -6
(Ungraded)

1. Consider the function

$$f(x, y) = x^2 + y^2 + \beta xy + x + 2y$$

For what values of β, does this function have a unique global minimum?

2. Suppose the one dimensional function $f(x_k + \alpha d_k)$ is unimodal and differentiable. Let α^* be the minimum of the function. If any $\alpha > \alpha^*$ is selected, show that $\nabla f(x_{k+1})^T d_k > 0$.

3. Consider the following problem.

$$f(x, y) = \exp\left(-\frac{1}{3}x^3 + x - y^2\right)$$

Suppose you want to do it using pure Newton's method. Is $x_0 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ good starting point?

4. Find the rectangle of given perimeter that has greatest area by using Lagrange multiplier theorem. Verify it using second order conditions.

5. Consider

minimize

$$f(x, y) = -x$$

subject to

$$y - (1 - x)^3 \leq 0$$

$$-y \leq 0$$

a. Find the optimal solution by solving it graphically.
b. Do Lagrange multipliers exist? How could you say without actually solving the problem?