

## Convex Optimization

C2 Review 2021

Time: 2hrs

1. Consider the function  $f(x) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ . Use Newton's method to minimize  $f(x)$ . Take  $\mathbf{x}^1 = [9, 8]$ .
  - a. How many iterations does it take to reach the minimum?
  - b. Is the minimum also a global minimum?

2. Derive the optimal step factor for a line search on a quadratic function

$$\min_x f(\mathbf{x}) = \frac{1}{2} \mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{b}^T \mathbf{x} + c$$

3. Discuss KKT conditions in detail. Determine the KKT conditions for the following

$$\min_x f(\mathbf{x}) = \frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{b}^T \mathbf{x} + c$$

subject to  $\mathbf{A} \mathbf{x} = 0$

It is given that  $\mathbf{Q} \succcurlyeq 0$

4. Discuss Complementary Slackness. For the following

$$\text{Maximize } 2x_1 + 16x_2 + 2x_3$$

subject to

$$2x_1 + x_2 - x_3 \leq 3$$

$$-3x_1 + x_2 + 2x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Check whether each of the following is an optimal solution, using complementary slackness

- a.  $x_1 = 6, x_2 = 0, x_3 = 12$
- b.  $x_1 = 0, x_2 = 2, x_3 = 5$
- c.  $x_1 = 0, x_2 = 0, x_3 = 6$