Convex Optimization

(January – June 2022)

Assignment 4

(Graded)

Is the intersection of two convex sets convex? Is the union of two convex sets convex?
Prove that the following sets are convex.

(a) **Polyhedra:** sets of the form $K = \{x \in \mathbb{R}_n : \langle a_i, x\rangle \leq b_i$ for $i = 1,2,\ldots, m\}$, where $a_i \in \mathbb{R}_n$ and $b_i \in \mathbb{R}$ for $i = 1,2,\ldots,m$.

(b) **Ellipsoids:** sets of the form $K = \{x \in \mathbb{R}^n : x^\top Ax \leq 1\}$ where $A \in \mathbb{R}^{n\times n}$ is a PD matrix.

(c) **Unit balls in ℓ_p-norms for $p \geq 1$:** $B_p(a, 1) \coloneqq \{x \in \mathbb{R}^n : \|x - a\|_p \leq 1\}$, where a $\in \mathbb{R}^n$ is a vector.