Convex Optimization

(January – June 2022)

Assignment 1

(Un-Graded)

1. What is dual norm. Derive the dual norm of the $L_1, L_2\ and\ L_\infty$ norm
2. *Gram-Schmidt*: Let {(1,-1,1,1), (1,0,1,0), (0,1,0,1) be a linearly independent set in R^4. Find an orthonormal set $\{v_1, v_2, v_3\}$ st $L(\ (1,-1,1,1), (1,0,1,0), (0,1,0,1)) \ = \ L(v_1, v_2, v_3\ )$
3. Prove that every real, symmetric matrix X has the decomposition $X = Q\Lambda Q^T$. Q is an orthogonal matrix and Λ is a diagonal matrix with eigenvalues as elements?
4. Can an orthogonal matrix have an entry $U_{ij}\ >\ 1$? Why?
5. When is a diagonal matrix orthogonal?
6. When is an upper triangular matrix orthogonal?
7. Is the inverse of an orthogonal matrix orthogonal?
8. What are eigenvalues and eigenvectors of a diagonal matrix?
9. Can you find vectors that attain each of the equality cases in $\lambda_{min} \leq \frac{x^T A x}{\|x\|^2} \leq \lambda_{max}$

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Assignment 2

(Un-Graded)

1. If $A = U\Sigma V^T$ is the SVD of a square invertible A, what is the SVD of A^{-1}?
2. If A is positive semidefinite, is its eigendecomposition $A = U\Lambda U^T$ also an SVD?
3. If A is not positive semidefinite, how can we modify signs in $A = U\Lambda U^T$ to obtain an SVD?
4. Show that for a square $A = USU^T$one has $AA^T = US^2U^T$ and $A^TA = \mathrm{V}^{\mathrm{T}}S^2\mathrm{V}$, and that these are eigendecompositions.