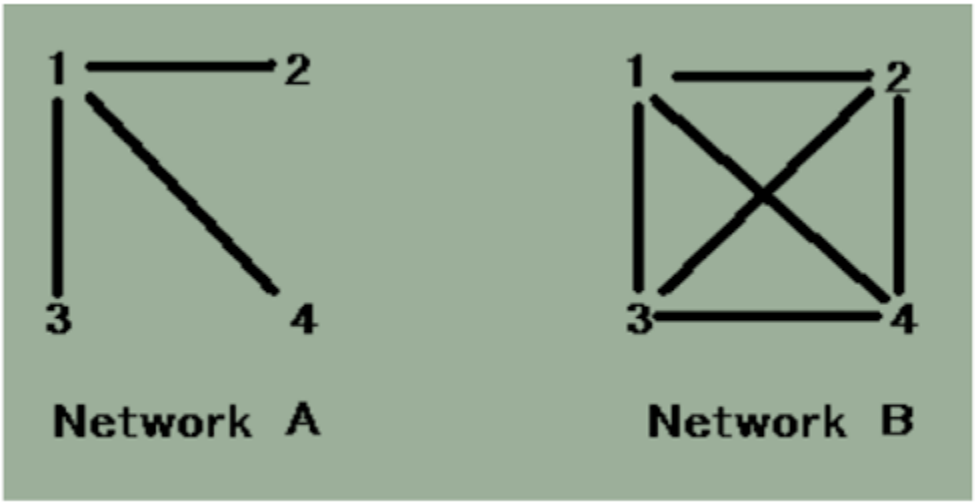
Q1.

(a) For each Network, calculate the three centrality indexes (degree, closeness, and betweenness) for each entire graph (Not for each node of a graph) . Which graph is more centralized ?



Degree Centrality:

The degree centrality(C_D) for a graph G with n nodes is defined as follows.

$C_D(G) = \Sigma(C_D(V^*) - C_D(V_i)) / (n^2 - 3n + 2)$.

Here, $C_D(V^*)$ = maximum degree among all nodes.

The degree of a node in an undirected graph is defined as the number of edges passing through that node.

Network A:

Node	Degree
1	3
2	1
3	1

4	1

Hence, $C_D(V^*) = 3$ and $n^2 - 3n + 2 = 6$.

Hence, $C_D(G) = (3 - 3 + 3 - 1 + 3 - 1 + 3 - 1) / 6 = 1$

Network B:

Node	Degree
1	3
2	3
3	3
4	3

Hence, $C_D(V^*) = 3$ and $n^2 - 3n + 2 = 6$.

Hence, $C_D(G) = (3 - 3 + 3 - 3 + 3 - 3 + 3 - 3) / 6 = 0$.

Since, $0 < 1$, hence network A is more central compared to network B w.r.t. degree centrality.

Closeness Centrality:

The closeness centrality(C_C) for a graph G with n nodes is defined as follows.

$C_C(G) = \Sigma(C_C(V^*) - C_C(V_i)) / ((n^2 - 3n + 2) / (2n - 3))$.

Here, $C_C(V^*)$ = maximum closeness among all nodes.

The closeness of a node in an undirected graph is defined as the inverse of the sum of the distances between this node and all other nodes. It can be normalized by multiplying with (n - 1), where n = number of nodes.

Network A:

The distances of each node to reach every other node are as follows.

	1	2	3	4	$C_C(V_i)$(Normalized)
1	0	1	1	1	3/3
2	1	0	2	2	3/5
3	1	2	0	2	3/5
4	1	2	2	0	3/5

Hence, $C_C(V^*) = 1$ and $(n^2 - 3n + 2) / (2n - 3) = 6/5$.

$C_C(G) = (1 - 1 + 1 - ⅗ + 1 - ⅗ + 1 - ⅗) / (6/5) = 1$

Network B:

The distances of each node to reach every other node are as follows.

	1	2	3	4	$C_C(V_i)$(Normalized)
1	0	1	1	1	3/3
2	1	0	1	1	3/3
3	1	1	0	1	3/3
4	1	1	1	0	3/3

Hence, $C_C(V^*) = 1$ and $(n^2 - 3n + 2) / (2n - 3) = 6/5$.

$C_C(G) = (1 - 1 + 1 - 1 + 1 - 1 + 1 - 1) / (6/5) = 0$.

Since, $0 < 1$, Network A is more centralized than network B w.r.t. closeness of centrality.

Betweenness Centrality:

The Betweenness centrality(C_B) for a graph G with n nodes is defined as follows.

$C_B(G) = \Sigma(C_B(V^*) - C_B(V_i)) / (n - 1)$.

Here, $C_B(V^*)$ = maximum betweenness among all nodes.

The betweenness of a node in an undirected graph is defined as the fraction A / B where B is the sum of the number of shortest paths between any two nodes excluding this node and B is those many shortest paths in which this node is present in that path but not as an end point of that path.

Network A:

The betweenness of each of the nodes is as follows.

Path(src, dest)	1	2	3	4
1, 2	0	0	0	0
1, 3	0	0	0	0
1, 4	0	0	0	0
2, 3	1	0	0	0
2, 4	1	0	0	0
3, 4	1	0	0	0
C_B(Node)	3/3	0	0	0

Hence, $C_B(V^*) = 1$

And $C_B(G) = (1 - 1 + 1 - 0 + 1 - 0 + 1 - 0) / 3 = 1$

Network B:

The betweenness of each of the nodes is as follows.

Path(src, dest)	1	2	3	4
1, 2	0	0	0	0
1, 3	0	0	0	0
1, 4	0	0	0	0
2, 3	0	0	0	0
2, 4	0	0	0	0

3, 4	0	0	0	0
C_B(Node)	0	0	0	0

Hence, $C_B(V^*) = 0$

And $C_B(G) = (0 - 0 + 0 - 0 + 0 - 0 + 0 - 0) / 3 = 0$.

Since 0 < 1, network A is more centralized than network B w.r.t. Betweenness centrality.

(b) Comparing the centralized and non-centralized graph above, discuss advantages and disadvantages of centralized networks.

Advantages and disadvantages of centralized are as follows.

- If we think about a network topology of computers then a centralized network will provide less reliability as compared to a network which is more distributed in nature. This is because, a single point of failure will exist in centralized network where the nodes which are having high measures of centrality may become a single point of failure and can terminate the communication between the entire network.

- Network Traffic can be easily monitored and controlled in a centralized network as all this traffic will pass through few computers where it will be relatively easy to monitor and control the traffic.

Advantages and disadvantages of non-centralized are as follows.

- Network Traffic can’t be easily monitored and controlled in a centralized network as all this traffic will pass through many computers where it will be relatively hard to monitor and control the traffic.

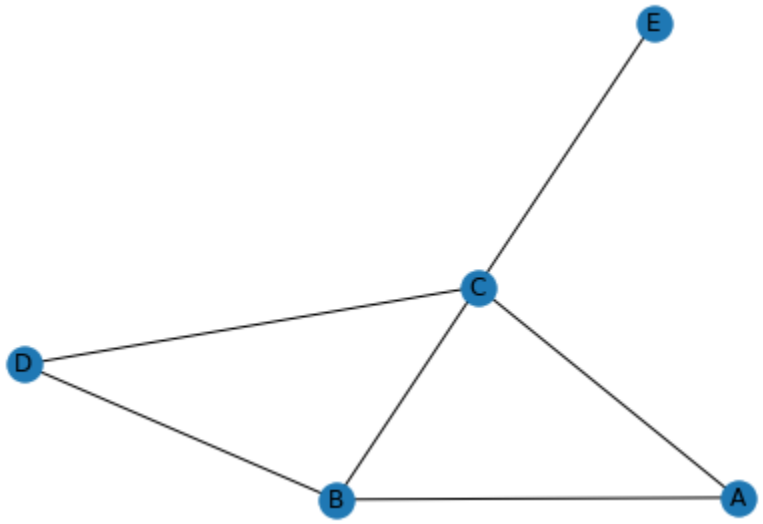
- If we think about a network topology of computers then a non-centralized network will provide high reliability as compared to a network which is more distributed in nature. This is because, a single point of failure will not exist in a non-centralized network.

2. Centrality can be used as an index of power in networks, because the most central node is more likely to occupy the most powerful position. Do you think it is true ? Answer with reasons why you think so.

This will highly depend on the what is the nature of the network and what task the network is trying to accomplish. In some cases, like monitoring the traffic, it will be better to give more responsibility to the computer which has high centrality measure because it can control the traffic well.

In other case, where we want to make the network more resilient to network failures, we won't make any one computer to have more responsibility than others as it can bring the reliability down.

3. Calculate the betweenness centrality of all nodes in the network.



The betweenness of each of the nodes is as follows.

Path(src, dest)	A	B	C	D	E	# of shortest paths
A, B	0	0	0	0	0	1
A, C	0	0	0	0	0	1
A, D	0	1	1	0	0	2
A, E	0	0	1	0	0	1
B, C	0	0	0	0	0	1
B, D	0	0	0	0	0	1
B, E	0	0	1	0	0	1
C, D	0	0	0	0	0	1
C, E	0	0	0	0	0	1

D, E	0	0	1	0	0	1

The betweenness of all nodes is as follows.

Nodes	g(Node)	Normalized g(Node)
A	0	0
B	1/2	0.0833
C	7/2	0.0583
D	0	0
E	0	0