Q. Write a short note on scale-free networks and small-world networks. Give some real world examples; related problems and their solutions.

**Scale-free Network:**

A scale-free network is one in which the degree distribution, at least asymptotically, follows a power law. That is, for large values of k, the fraction P(k) of nodes in the network with k connections to other nodes increases as P(k) ~ k-S

Although statistical analysis has disputed several of these assertions and significantly questioned others, numerous networks have been found to be scale-free. Furthermore, some have claimed that understanding whether a network is scale-free according to statistically rigorous definitions is more important than knowing whether a degree distribution is fat-tailed. The fitness model and preferential attachment have both been presented as strategies for explaining conjectured power law degree distributions in real networks. Alternative models such as superlinear preferential attachment and second neighbor preferential attachment may appear to yield scale-free networks in the short term, but as networks grow larger, the degree distribution deviates from a power law.

The relative commonness of vertices with a degree that substantially surpasses the average is the most notable feature of a scale-free network. The highest-degree nodes are sometimes referred to as "hubs," and they are assumed to have specific functions in their networks, however this varies widely depending on the domain.

The clustering coefficient distribution, which decreases as the node degree grows, is another significant feature of scale-free networks. A power law also governs this allocation. This means that the low-degree nodes are part of incredibly dense sub-graphs that are linked together by hubs. Consider a social network where the nodes are people and the linkages represent people's acquaintance ties.

The more detailed properties of scale-free networks now differ depending on the generative method utilised to construct them. Preferential attachment networks, for example, often position the high-degree vertices in the centre of the network, connecting them to form a core, with increasingly lower-degree nodes forming the regions between the core and the periphery. The removal of even a substantial fraction of vertices at random has minimal effect on the network's overall connectedness, implying that such topologies could be effective for security, but focused attacks swiftly destroy the network's connectivity.

Many real-world networks are (roughly) scale-free, necessitating the use of scale-free models to describe them. Price's plan calls for two ingredients in order to develop a scale-free model:

1. Adding or deleting nodes is the first step. Typically, we focus on expanding the network by adding nodes.

2. Preferential attachment: The probability that fresh nodes will be attached to the "old" node (displaystyle Pi Pi).

Some models (see Dangalchev[34] and Fitness model below) can also work statically, with the number of nodes remaining constant. It should also be noted that the fact that "preferential attachment" models produce scale-free networks does not imply that this is the process driving the evolution of real-world scale-free networks, as multiple mechanisms may be at work in real-world scale-free networks.

**Examples**

Several attempts have been made to generate scale-free network properties. Some instances are as follows:

**1. The Barabási–Albert model**

An undirected variation of the Price Model, the Barabási–Albert model, features a linear preferential attachment.

**2. Two-level network model**

Dangalchev constructs a 2-L model by taking into account the relevance of each of a target node's neighbors in preference attachment. In the 2-L model, a node's attractiveness is determined not just by the number of nodes linked to it, but also by the number of linkages in each of these nodes.

**Small-world Network**

A small-world network is a mathematical graph in which the majority of nodes are not neighbours, but the neighbours of any given node are likely to be neighbours, and most nodes may be accessed by a minimal number of hops or steps from any other node. A small-world network is defined as one in which the typical distance L (number of steps necessary) between two randomly chosen nodes grows according to the logarithm of the number of nodes N in the network.

The small-world effect can be seen in many empirical graphs, such as social networks, wikis like Wikipedia, gene networks, and even the Internet's underlying architecture. Many network-on-chip architectures in modern computer hardware are based on this concept.

Cliques and near-cliques, or sub-networks with connections between practically any two nodes within them, are common in small-world networks. This is because a high clustering coefficient is a defining trait. Second, most pairs of nodes will have at least one short path connecting them. The defining characteristic of the mean-shortest path length being tiny explains this. Small-world networks are commonly connected with a number of additional characteristics. Hubs – network nodes with a large number of connections – are typically over-represented (known as high degree nodes). These hubs act as common links, bridging the short distances between adjacent edges.

By analogy, because many flights are routed through hub cities, the small-world network of airline flights has a short mean-path length (i.e., between any two locations, you are likely to take three or fewer flights). This feature is frequently investigated by looking at the percentage of nodes in the network that have a specific number of connections (the degree distribution of the network). Networks with more hubs than expected would have a higher proportion of high-degree nodes, resulting in a degree distribution that is enriched at high-degree values. A fat-tailed distribution is the term for this. As long as the two definitional requirements are met, graphs with highly varied topologies can be classified as small-world networks.

**Examples of small-world networks**

Many real-world phenomena, such as websites with navigation menus, food webs, electric power grids, metabolite processing networks, brain neuron networks, voter networks, telephone call graphs, and airport networks, exhibit small-world traits. Small-world networks have also been discovered in cultural networks[10] and word co-occurrence networks].

Small world traits such as power-law obeying degree distributions exist in networks of linked proteins.

Similarly, transcriptional networks have small world network qualities when the nodes are genes and they are coupled if one gene has an up or down-regulatory genetic influence on the other.

**Construction of small-world networks**

The Watts–Strogatz process is the basic mechanism for creating small-world networks.

Small-world networks can also be introduced with time-delay, which will produce fractals as well as chaos under the correct conditions, or shift to chaos in dynamics networks.

Degree–diameter graphs are built so that each vertex in the network has a finite number of neighbors and the distance between any two vertex (the network's diameter) is kept to a minimum. Small-world networks are constructed as part of the search for order graphs near to the Moore bound.

Barmpoutis et al. describe another technique to build a small world network from scratch.

Another method for building a small world network from the ground up is described by Barmpoutis et al who establish a network with a very short average distance and a very big average clustering. A quick algorithm with constant complexity is provided, as well as measurements of the graphs' robustness. Depending on the application, one can start with a single "ultra small-world" network and then rewire certain edges, or employ numerous small "ultra small-world" networks as subgraphs to a larger graph.