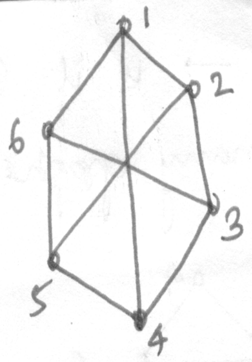
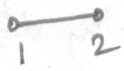
Subgraph

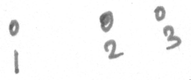


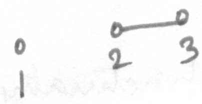
H is a subgraph of G ($H \subseteq G$)

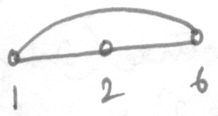
if $V(H) \subseteq V(G)$

$E(H) \subseteq E(G)$

H_1 

H_2 

H_3 

× H_4 

1-2

1-6

edge b/w 2-6.

When considering subgraph of graph G, the original graph must not be altered by identifying two distinct vertices or by adding new edges or vertices.

- every graph is its own subgraph
- a subgraph of a subgraph of G is a ~~subg~~ subgraph of G.
- a single edge in G together with its end vertices is also a subgraph of G

* Two or more ~~graphs~~ subgraphs g_1 and g_2 of a graph G are said to be edge disjoint if g_1 and g_2 do not have any edges in common (may have vertices in common)

Subgraphs which do not have even vertices in common are called vertex disjoint

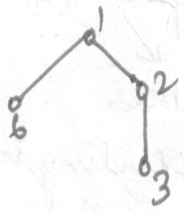
Induced Subgraphs

→ is there a one to correspondence b/w graph given & graph considered

H is an induced subgraph of G

if $V(H) \subseteq V(G)$ &

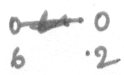
$\{u, v\} \in E(H)$ iff $\{u, v\} \in E(G)$

H_1 

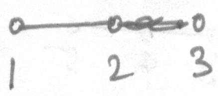
Subgraph

Induced ×

there is an edge b/w 6 & 3 in G but not present in H

 Sub

ind



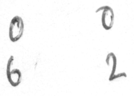
Subgraph

Induced ×

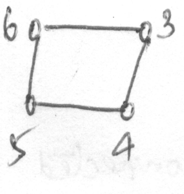
not 2-3 edge in H but in G

if an edge is present in H it must be present in G & if an edge is present in G it must be present in H.

for the vertices given all edges in G must be in H as well

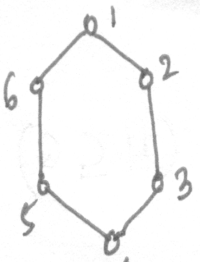
 Sub ✓

Ind ✓

 Sub ✓

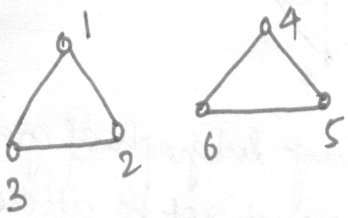
ind ✓

(2, 2, 2, 2, 2, 2) → is it Graphic
↳ how many graphs?



G_1

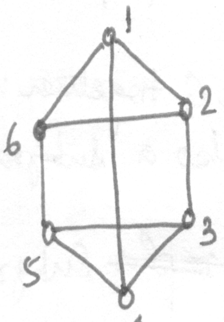
I can reach from 1 to 4



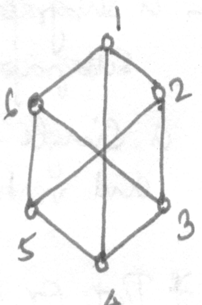
G_2

I cannot reach from 1 to 4 ⟹ G_1 & G_2 are structurally different

(3, 3, 3, 3, 3, 3)



G_3



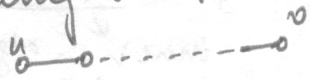
G_4

G_3 & G_4 are structurally different
⇓
there is a substructure (subgraph) ~~triangle~~ or cycle of length 3) present in G_3 but not present in G_4

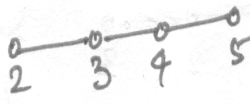
Connectedness

A Graph G is connected if ∀ u, v there exists a path (u, v) in G

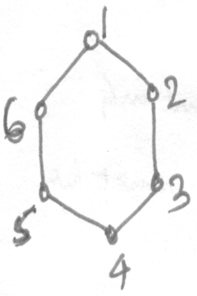
Sequence of vertices starting at u & ending in v



Path (2, 5)

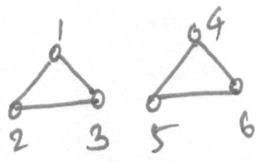


Ex:

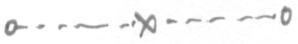


if it true for all u, v ∈ 6 ⟹ graph is connected

Ex: G_2



is disconnected



∄ p(2, 6) ∄ P(1, 5) ⟹ G_2 is disconnected

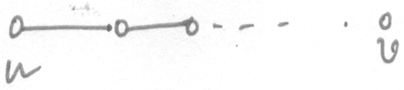
NOTE:
If G is disconnected G is a collection of connected components

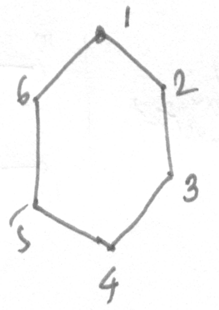
maximal connected sub-graphs

For G_2 which is disconnected, ... 4, 5, 6 are connected

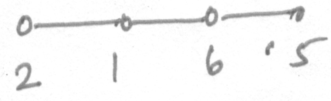
Connected Components

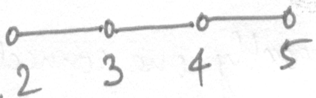
A Graph G is Connected if ∀ u, v there exists a path (u, v) in G



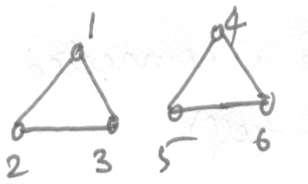


G_1

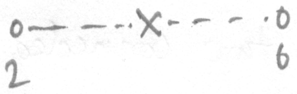
Path(2,5) 



G_1: Connected



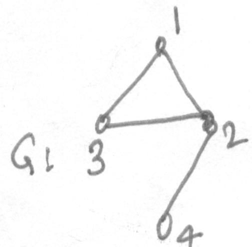
G_2

G_2 

∄ P(2,6) ∄ 1,5

G_2 → disconnected

Note: If G is disconnected then G is a collection of Connected Components which ~~are~~ Maximally Connected ~~Subcomponents~~ Subgraphs

G 

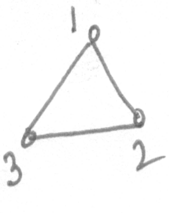
not max sub ind  → Subgraph of G

⊑ G

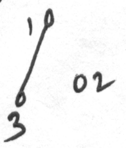
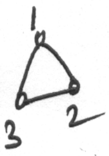
→ Is it connected? Yes.

Is it maximally Connected? → can you add any other vertex and still make the graph connected

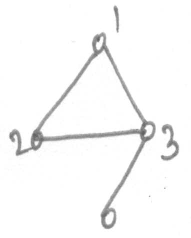
I should not be able to add any more vertex to the Graph & still maintain connectedness ⇓ say we add a vertex 2 -

Sub ind not Maximal —  ⊑ G ⇒ Subgraph of G ⇒ it is a Supergraph that contains graph 

but this supergraph is not connected since you donot have a path to 4 ↳ it is not Maximal

 ⇒ induced subgraph is  Subgraph which is connected.

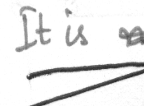
Maximal → there is no Supergraph containing it which is connected

 ⊑ G ⇒ Subgraph ⇒ it is a supergraph that is connected (same as G)

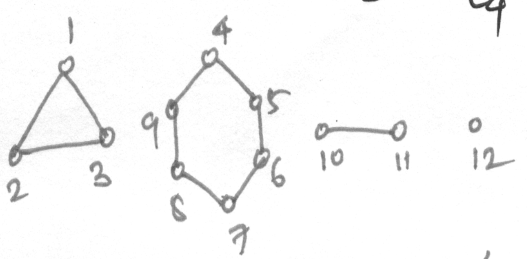
∴ It is ~~not~~ maximal

~~it should be a supergraph which should not be connected~~

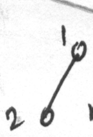
it has no supergraph containing it that is connected

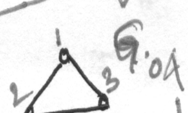


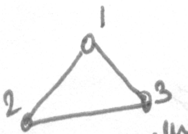
G C_2 C_3 C_4



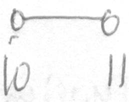
G

 not max

 sub ind × Max connected ∄ P(2,4)

 maximally connected

G is a dis-connected graph with 4 connected components i.e. 4 ~~max~~ maximally connected sub-graphs of G.

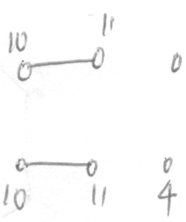


10 11 Sub ✓ ind ✓ max. connected subgraph ? → is there are supergraph containing 10, 11 that is connected? No.

⇒ it is a maximally connected subgraph

or a connected component

max connected

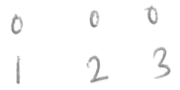


$\nexists P(11,12)$

$\nexists P(10,4)$

Obs.1: → If G is connected then G has exactly one connected component (G itself)

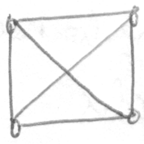
Obs 2: → If G is disconnected then G has atleast two connected components



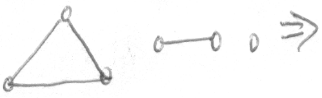
(3 connected components)

$\underbrace{\quad}_{G}$

#



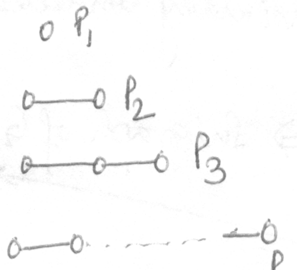
⇒ Connected, Max. no. of edges



⇒ Disconnected, each component has max. no. of edges

Special Graphs

① Path Graph (P_n)

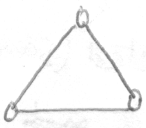


$|V(P_n)| = n$

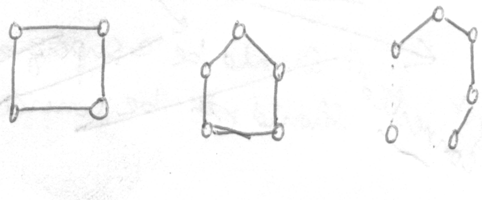
$|E(P_n)| = n-1$

② Cycle Graphs

Closed circuit



C_3



$|V(C_n)| = n$

$|E(C_n)| = n$

③ K-Regular Graph

$\forall v,\ d_G(v) = k$

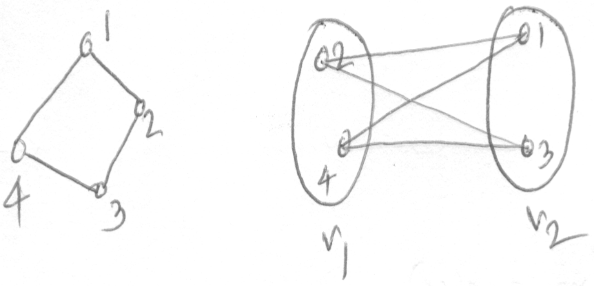
regular 2 regular regular

(4) Bipartite Graphs

G is bipartite if

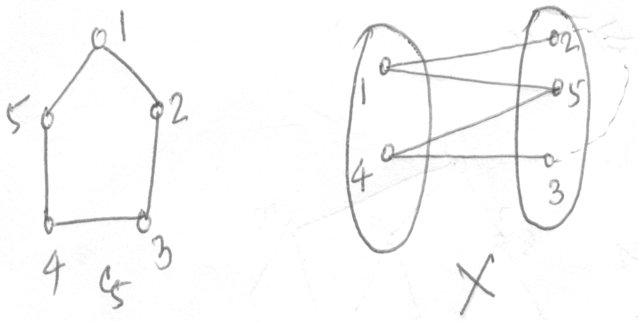
$V(G) = V_1 \cup V_2$ s.t. ~~$V_1 \cap V_2 \neq \emptyset$~~ $V_1 \cap V_2 = \emptyset$

$\forall \{u, v\} \in E(G)$ $u \in V_1$ and $v \in V_2$

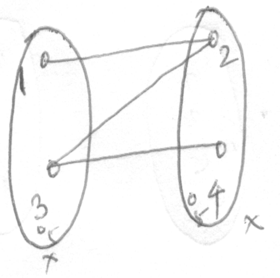


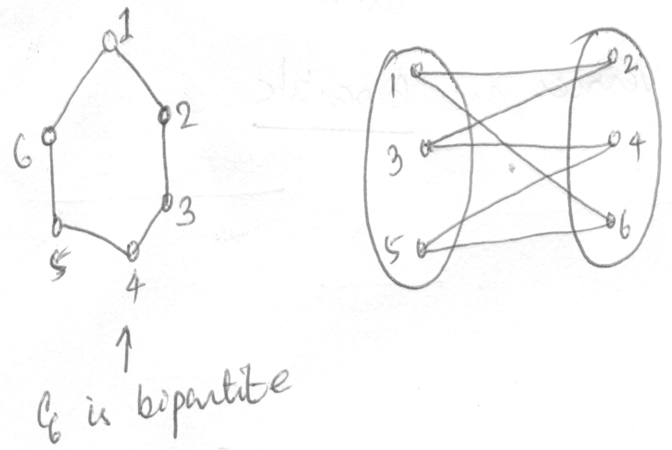
Given a graph G if we can have a decomposition st ~~one~~ of vertices into two sets such that for every edge one vertex is in one set and the other vertex in the other set, then G is called **Bipartite graph**

Ex:



Non-bipartite





C_6 is bipartite

"All even cycles are Bipartite" $\Rightarrow$ C_{2n} is bipartite

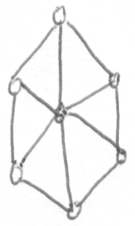
C_{2n+1} is not-bipartite

Theorem: G is bipartite if G has no odd cycles

$\Rightarrow$ G is bipartite if G is odd cycle free

$\Rightarrow$ G is " " " does not have C_{2n+1} as a sub-graph

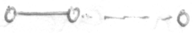
Wheel Gr.
↓
non-bipartite



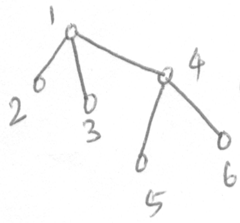
$W_7 = C_6 + v$
↑
v is universal to C_n

P_n 
↳ Bipartite (no cycles)

Trees → connected acyclic graphs (no cycles)

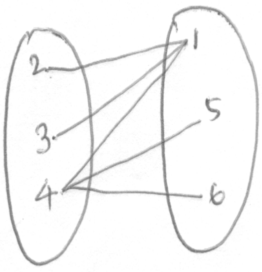
P_n 
↳ Trees

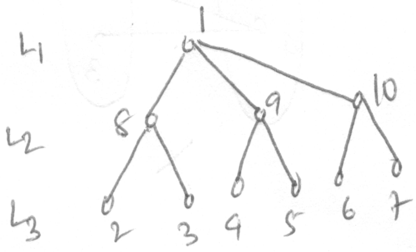
C_n ! Not trees

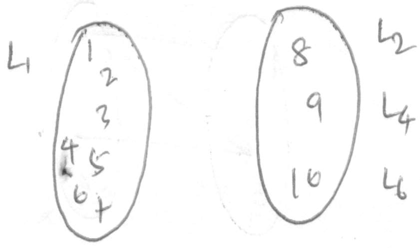


connected & acyclic ⇒ Tree

T_1:







vertices in odd level

vertices in even levels

⇒ All vertices are Bipartite