Q1.(a) : True False :

i. A matching M in a graph G=(V,E) is a maximum matching if and only if there is no M-augmenting path in G

True

ii. A disconnected simple acyclic graph does not have a spanning tree.

False.

iii. The endpoints of a cut-edge are both cut-vertices.

False. Counterexample: Take any tree of at least two vertices. Every edge is a cut-edge. However, the leaves are not cut-vertices. So any edge with a leaf as an endpoint is a counterexample.

Q1. (b) For each situation, would you find an Euler circuit or a Hamilton Circuit?

i. The department of Public Works must inspect all streets in the city to remove dangerous debris.

Solution : Eulerian circuit (Considering debris as nodes and streets as edges, the problem is traversing every street (edge) once to locate debris ; Eulerian circuit)

ii. Relief food supplies must be delivered to eight emergency shelters located at different sites in a large city.

Solution : Hamiltonian circuit (Emergency shelter = nodes, streets = edges; Considering and The problem is to visit every shelter (node) once ; Hamiltonian circuit)

iii. The Department of Public Works must inspect traffic lights at intersections in the city to determine which are still working.

Solution : Hamiltonian circuit (Intersection = nodes, streets = edges; Considering and The problem is to visit every intersection (node) once ; Hamiltonian circuit)

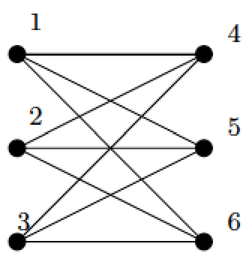
Q2 (a) For each of the graphs K_n, P_n, C_n and W_n, give: the order, the size, the maximum degree and the minimum degree in terms of n.

Solution :

$$\begin{array}{l} K_n = (V,E): \ |V| = n, \ |E| = \binom{n}{2}, \ \delta(K_n) = n-1, \ \Delta(K_n) = n-1 \\ P_n = (V,E): \ |V| = n, \ |E| = n-1, \ \delta(P_n) = 1, \ \Delta(P_n) = 2 \\ C_n = (V,E): \ |V| = n, \ |E| = n, \ \delta(C_n) = 2, \ \Delta(C_n) = 2 \\ W_n = (V,E): \ |V| = n, \ |E| = 2 \cdot n - 2, \ \delta(W_n) = 3, \ \Delta(W_n) = n-1 \end{array}$$

(b). Draw a bipartite graph of order 6. Give its adjacency list and a drawing.

Solution :



1	2	3	4	5	6
4	4	4	1	1	1
5	5	5	2	2	2
6	6	6	3	3	3

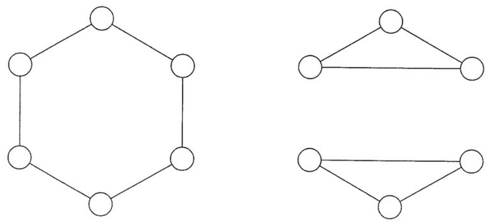
(c). Consider the graphs G1 = (V1; E1) and G2 = (V2; E2). Give the order, the degree of the vertices and the size of G1 × G2 in terms of those of G1 and G2.

Solution :

Order $|V_1||V_2|$, $d_{G_1 \times G_2}(u_1, u_2) = d_{G_1}(u_1) + d_{G_2}(u_2)$ and size $|V_1||E_2| + |V_2||E_1|$.

(d). Prove or give a counterexample: any two graphs with the same degree sequence are isomorphic

Solution : hexagon which is connected and two separated triangles, which is obviously not connected, yet their degree sequences are the same.

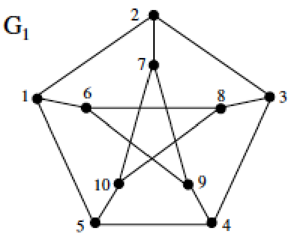


Q3. (a) Graph 1 : Has a

(b) Give the line graph of C_5 and G = ({1, 2, 3, 4, 5}; {1 2 , 2 3 , 2 4 , 2 5 , 3 4; 3 5 , 4 5})

(c) Find the diameter (maximum of all vertex eccentricities) of the Peterson's graph.

G_1



Solution :

$$G_1\colon\ e(v) = 2,\ 1 \le v \le 10;\ r(G) = 2;\ \text{all the vertex are central.}$$

(d) Give a connected graph G = (V, E) and a vertex u є V for the following relation :

D(G) = D(G − u). D is the diameter of G (maximum of all vertex eccentricities)

Solution :

$$G = W_6 \text{ and } u \text{ a vertex of degree 3.}$$

Q4. (a): Let us consider the graphs whose adjacency list is given.

a	b	c	d	e	f	g	h	i	j
d	d	h	a	a	a	b	c	b	b
e	g		b	d	d	i		g	g
f	i		e			j			
	j		f						

i. Draw the graph

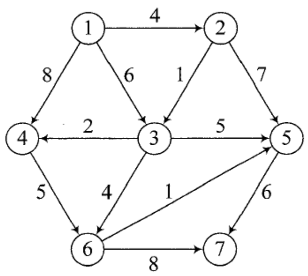
ii. Using the algorithm BFS, find the distance from the vertices a and b to each of the other vertices of the connected component to which they belong

[Note : You are expected to show the BFS traversal as intermediary steps for distance computation]

Solution :

v	a	b	d	e	f	g	i	j
d(a,v)	0	2	1	1	1	3	3	3
d(b,v)	2	0	1	2	2	1	1	1

Q5b. Obtain the shortest distance and shortest path from vertex 1 in the network shown below :



Solution :

Iteration 3:

Step 1. $P = \{1, 2, 3\}$, $L(1) = 0$, $L(2) = 4$, and $L(3) = 5$. $L'(4) = 7$, $L'(5) = 10$, and $L'(6) = 9$. Adjoin vertex 4 to P. The arc (3, 4) is labeled.

Step 2. $P = \{1, 2, 3, 4\}$, and $L(4) = 7$. $L'(5) = \min\{10, L(4) + a(4, 5)\}$, $L'(6) = \min\{9, L(4) + a(4, 6)\}$, and $L'(7) = \min\{\infty, L(4) + a(4, 7)\}$.

Iteration 4:

Step 1. $P = \{1, 2, 3, 4\}$, $L(1) = 0$, $L(2) = 4$, $L(3) = 5$, and $L(4) = 7$. $L'(5) = 10$, and $L'(6) = 9$. Adjoin vertex 6 to P. The arc (3, 6) is labeled.

Step 2. $P = \{1, 2, 3, 4, 6\}$, and $L(6) = 9$. $L'(5) = \min\{10, L(6) + a(6, 5)\}$, and $L'(7) = \min\{\infty, L(6) + a(6, 7)\}$.

Iteration 5:

Step 1. $P = \{1, 2, 3, 4, 6\}$, $L(1) = 0$, $L(2) = 4$, $L(3) = 5$, $L(4) = 7$, and $L(6) = 9$. $L'(5) = 10$, and $L'(7) = 17$. Adjoin vertex 5 to P. The arc (3, 5) is labeled.

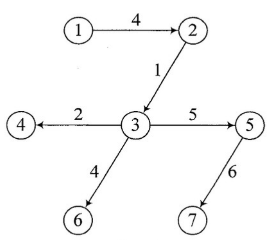
Step 2. $P = \{1, 2, 3, 4, 6, 5\}$, and $L(5) = 10$. $L'(7) = \min\{17, L(5) + a(5, 7)\}$.

Iteration 6:

Step 1. $P = \{1, 2, 3, 4, 6, 5\}$, $L(1) = 0$, $L(2) = 4$, $L(3) = 5$, $L(4) = 7$, $L(6) = 9$, and $L(5) = 10$. $L'(7) = 16$. Adjoin vertex 7 to P. The arc (5, 7) is labeled.

Step 2. $P = \{1, 2, 3, 4, 6, 5, 7\}$, and $L(7) = 16$. At this stage, $P = V$.

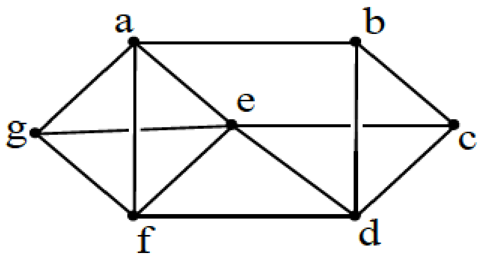
The labeled arcs (1, 2), (2, 3), (3, 4), (3, 5), (3, 6), and (5, 7) constitute a shortest path arborescence rooted at vertex 1, as shown in Fig. 5-2.



Q5.c: Every tree with two or more vertices is 2-chromatic.

Solution: Chose any vertices v in the given tree T. Let T be a rooted tree at vertex v. suppose the first color is assigned to the root v. Paint all the vertices adjacent to v with second color. Next paint the vertices adjacent to this using first color. Continue this process till every vertex in T has been painted. Hence all the vertices at odd distance from v have second color. While v and vertices at even distances from v have first color.

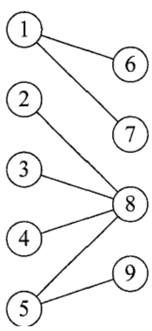
Q6.a: Find the chromatic number of the following graph



Solution: d(b) = d(g) = d(c) = 3 and
d(a) = d(d) = d(f) = 4,d(e)=5.
Using the inequality χ (G) $\leq$ 1+ Δ(G)
χ(G) $\leq$ 5.

Now since G has a triangle sub graph 3 $\leq \chi$(G) $\leq$ 5.
Suppose χ(G) = 5 then G should have 5 vertices with degree at least 4, but there are only 3 vertices in G . χ (G) $\neq$ 5.
Hence 3 $\leq$ χ (G) $\leq$ 4.
Now G is not 3 colourable since a, e, g, f which are connected each other, must be assigned different colours.
Therefore χ (G) = 4.

Q6.b: For the following bipartite graph, provide the maximum matching, minimum vertex cover, maximum independent set and minimum edge cover.



Solution :
i. Maximum matching : {1,6} {2,8} {5,9}
ii. Minimum vertex cover : 1,5,8
iii. Maximum independent set : {2,3,4,6,7,9}
iv. Minimum edge cover : {5,8}