Term weighting and Vector space retrieval

Vector Space Model

- It represents documents and queries as vectors of features representing terms
- features are assigned some numerical value that is usually some function of frequency of terms
- Ranking algorithm compute similarity between document and query vectors to yield a retrieval score to each document.

Documents as vectors

- Each doc *d* is viewed as a vector of *tf*×*idf* values, one component for each term
- So we have a vector space
  - terms are axes
  - docs live in this space

Vector Space Model

- Given a finite set of n documents:

  $D = \{d_1, d_2, ..., d_j, ..., d_n\}$

  and a finite set of m terms:

  $T = \{t_1, t_2, ..., t_i, ..., t_m\}$

- Each document will be represented by a column vector of weights as follows:

  $(w_{1j}, w_{2j}, w_{3j}, \ldots \quad w_{ij}, \quad \ldots w_{mj})^t$

  w_{ij} is the weight of term t_i in document d_j.

Vector Space Model

The document collection as a whole will be represented by an m x n term–document matrix as:

$$\begin{pmatrix} W_{11} & W_{12} & \ldots. & W_{1j} & \ldots & W_{1n} \\ W_{21} & W_{22} & \ldots. & W_{2j} & \ldots & W_{2n} \\ W_{i1} & W_{i2} & \ldots. & W_{ij} & \ldots & W_{in} \\ W_{m1} & W_{m2} & \ldots. & W_{mj} & \ldots & W_{mn} \end{pmatrix}$$

Example: Vector Space Model

D1 = Information retrieval is concerned with the organization, storage, retrieval and evaluation of information relevant to user's query.

D2 = A user having an information need formulates a request in the form of query written in natural language.

D3 = The retrieval system responds by retrieving document that seems relevant to the query.

Example: Vector Space Model

Let the weights be assigned based on the frequency of the term within the document.

The term – document matrix is:

$$\begin{pmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Vector Space Model

- Raw term frequency approach gives too much importance to the absolute values of various coordinates of each document

Consider two document vectors

$(2, 2, 1)^t$

$(4, 4, 2)^t$

The documents look similar except the differences in magnitude of term weights.

Normalizing term weights

- To reduce the importance of the length of document vectors we normalize document vectors
- Normalization changes all the vectors to a standard length.

We can convert document vectors to unit length by dividing each dimension by the overall length of the vector.

Normalizing the term-document matrix:

$$\begin{pmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

We get

$$\begin{pmatrix} 0.67 & 0.71 & 0 \\ 0.67 & 0 & 0.71 \\ 0.33 & 0.71 & 0.71 \end{pmatrix}$$

Elements of each column are divided by the length of the column vector ($\sqrt{\sum_i w_{ij}^2}$)

Term weighting

Postulates

1. The more a document contains a given word the more that document is about a concept represented by that word.
2. The less a term occurs in particular document in a collection, the more discriminating that term is.

Term weighting

- The first factor simply means that terms that occur more frequently represent its meaning more strongly than those occurring less frequently
- The second factor considers term distribution across the document collection.

Term weighting

→ a measure that favors terms appearing in fewer documents is required

The fraction n/n_i, exactly gives this measure

where,

n is the total number of the document in the collection

& n_i is the number of the document in which term i occurs

Term weighting

- As the number of documents in any collection is usually large, log of this measure is usually taken, resulting in the following form of inverse document frequency (idf) term weight:

$$idfi = \log\left(\frac{n}{ni}\right)$$

Tf-idf weighting scheme

$$w_{ij} = tf_{ij} \times \log\left(\frac{n}{n_i}\right)$$

tf - document specific statistic

idf - is global statistic and attempts to include distribution of term across document collection.

Tf-idf weighting scheme

- The term frequency (tf) component is document specific statistic that measures the importance of term within the document
- The inverse document frequency (idf) is global statistic and attempts to include distribution of term across document collection.

Tf-idf weighting scheme

Example: Computing tf-idf weight(total docs=100)

term	frequency (tf)	Document frequency (n_j)	idf ($\log(n/n_j)$)	Weight (tf x idf)
Tornado	4	15	0.824	3.296
Swirl	1	20	0.699	0.699
Wind	1	40	0.398	0.389

Normalizing tf and idf factors

- by dividing the term frequency by the frequency of the most frequent term in the document
- idf can be normalized by dividing it by the logarithm of the collection size (n).

$$w_{ij} = \frac{tf_{ij}}{\max(tf_{ij})} \times \log\left(\frac{n}{n_i}\right) / \log(n)$$

Term weighting schemes

- A third factor that may affect weighting function is the document length
- the weighting schemes can thus be characterized by the following three factors:

  1. Within-document frequency or term frequency
  2. Collection frequency or inverse document frequency
  3. Document length

- term weighting scheme can be represented by a triple ABC

  A - tf component

  B - idf component

& C - length normalization component.

- Different combinations of options can be used to represent document and query vectors.
- The retrieval model themselves can be represented by a pair of triples like nnn.nnn (doc = “nnn”, query = “nnn”)

Options for the three weighting factors

- **Term frequency within document (A)**

n Raw term frequency tf = tfij

b tf = 0 or 1 (binary weight)

a $tf = 0.5 + 0.5\left(\frac{tf_{ij}}{\max tf \text{ in } D_j}\right)$ Augmented term frequency

l tf = ln(tfij) + 1.0 Logarithmic term frequency

- **Inverse Document frequency (B)**

n wt = tf no conversion

t Multiply tf with idf

Options for the three weighting factors

- **Document length (C)**

n w_{ij} = wt (no conversion)

c w_{ij} is obtained by dividing each wt by sqrt(sum of(wts squared))

Indexing Algorithm

Step 1. Tokenization: This extracts individual terms (words) in the document, converts all the words in the lower case and removes punctuation marks. The output of the first stage is a representation of the document as a stream of terms.

Step 2. Stop word elimination: Removes words that appear more frequently in the document collection.

Step 3. Stemming: reduce remaining terms to their linguistic root, to get index terms.

Step 4. Term weighting: Assigns weights to term according to their importance in the document, in the collection or some combination of both.

Example: Document Representation

Document 1: Vector space model

Document 2: Probbailistic retrieval model

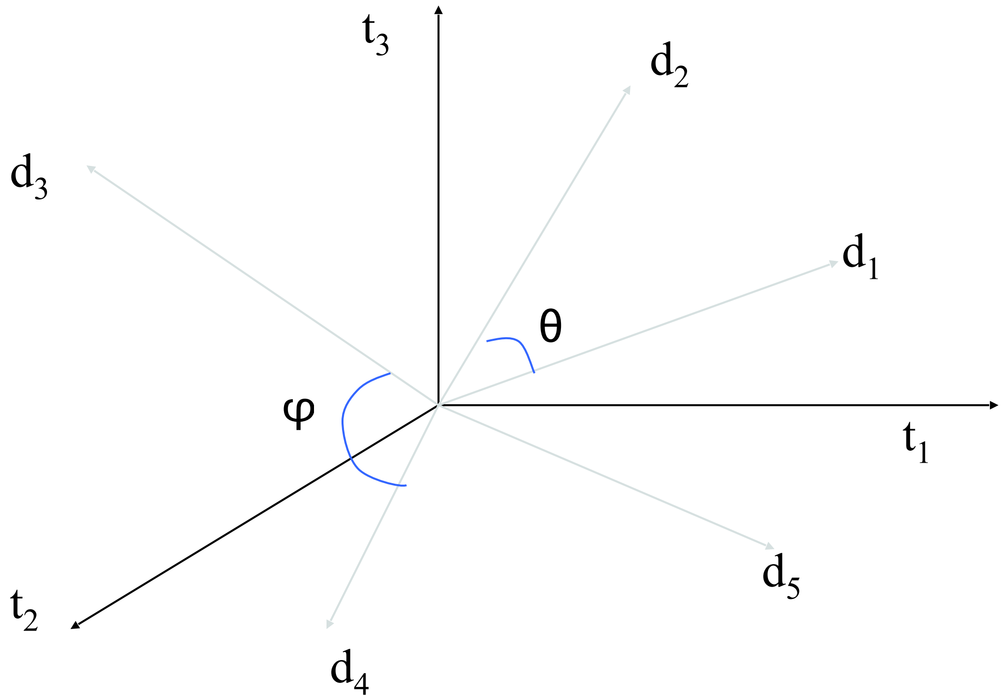
Document 3: Intelligent techniques in information retrieval

Stemmed terms	Document 1	Document 2	Document 3
inform	0	0	1
intellig	0	0	1
model	1	1	0
probabilist	0	1	0
retriev	0	1	1
space	1	0	0
technique	0	0	1
vector	1	0	0

Why turn docs into vectors?

- First application: Query-by-example
  - Given a doc *d*, find others “like” it.
- Now that *d* is a vector, find vectors (docs) “near” it.

Intuition



Postulate: Documents that are "close together" in the vector space talk about the same things.

Desiderata for proximity

- If d_1 is near d_2, then d_2 is near d_1.
- If d_1 near d_2, and d_2 near d_3, then d_1 is not far from d_3.
- No doc is closer to d than d itself.

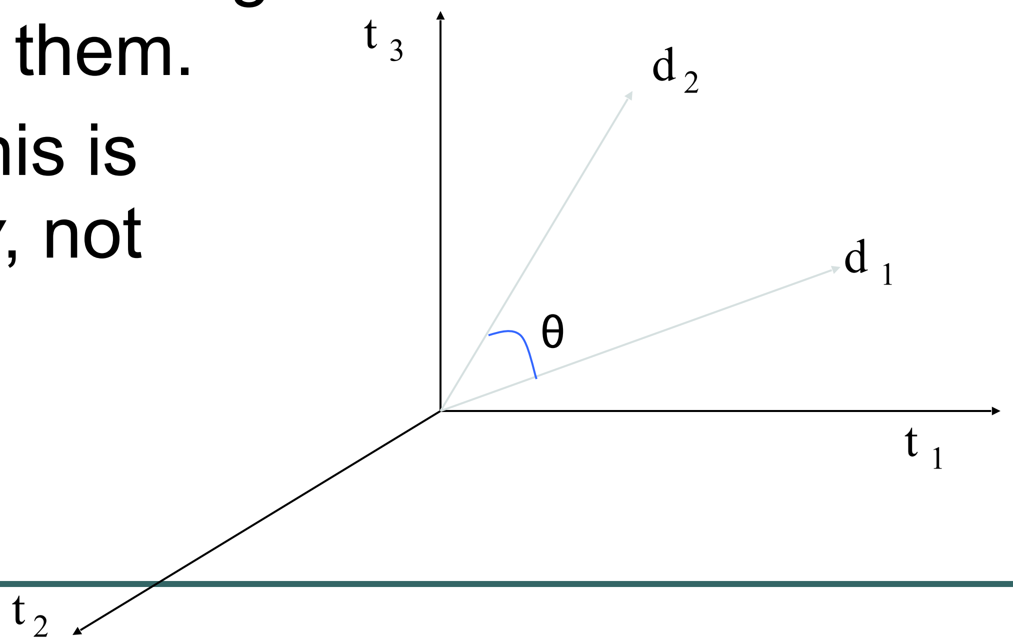
First cut

- Idea: Distance between d_1 and d_2 is the length of the vector $|d_1 - d_2|$.
  - Euclidean distance
- Why is this not a great idea?
  - Short documents would be more similar to each other by virtue of length, not topic
- However, we can implicitly normalize by looking at *angles* instead

Cosine similarity

- Distance between vectors d_1 and d_2 *captured* by the cosine of the angle x between them.
- Note – this is *similarity*, not distance

$$sim(d_j, d_k) = \frac{\sum_{i=1}^{n} w_{i,j} w_{i,k}}{\sqrt{\sum_{i=1}^{n} w_{i,j}^2} \sqrt{\sum_{i=1}^{n} w_{i,k}^2}}$$



Cosine similarity

- A vector can be *normalized* (given a length of 1) by dividing each of its components by its length

$$\|\mathbf{x}\|_2 = \sqrt{\sum_i x_i^2}$$

- This maps vectors onto the unit sphere:
- Then,
- Longer documents don't get more weight

Cosine similarity

- Cosine of angle between two vectors
- The denominator involves the lengths of the vectors.

Normalization

Normalized vectors

- For normalized vectors, the cosine is simply the dot product:

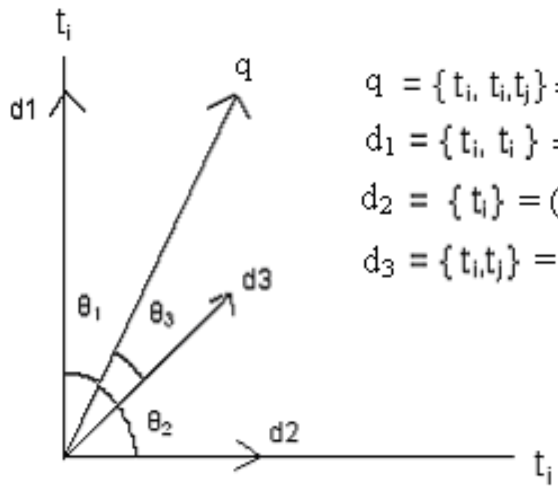
Queries in the vector space model

Central idea: the query as a vector:

- We regard the query as short document
- We return the documents ranked by the closeness of their vectors to the query, also represented as a vector.
- Note that d_q is very sparse!

$$sim(d_j, d_q) = \frac{\sum_{i=1}^{n} w_{i,j} w_{i,q}}{\sqrt{\sum_{i=1}^{n} w_{i,j}^2} \sqrt{\sum_{i=1}^{n} w_{i,q}^2}}$$

Similarity Measures



$q = \{ t_i, t_i, t_j \} = (2,1)$

$d_1 = \{ t_i, t_i \} = (2,0)$

$d_2 = \{ t_i \} = (1,0)$

$d_3 = \{ t_i, t_j \} = (1,1)$

- cosine similarity

$$sim(d_j, q_k) = \frac{(d_j, q_k)}{\|d_j\| \|q_k\|} = \frac{\sum_{i=1}^{m} w_{ij} \times w_{ik}}{\sqrt{\sum_{i=1}^{m} w_{ik}^2} \times \sqrt{\sum_{i=1}^{m} w_{ij}^2}}$$

Let

$D = (0.67 \;\; 0.67 \;\; 0.33)^t$

and $Q = (0.71 \;\; 0.71 \;\; 0)^t$

then the cosine similarity between D and Q will be :

$$\mathrm{Sim}(D,Q) = \frac{0.67 \times 0.71 + 0.67 \times 0.71 + 0.33 \times 0}{\sqrt{(0.67^2 + 0.67^2 + 0.33^2)} \times \sqrt{(0.71^2 + 0.71^2 + 0^2)}}$$

Summary: What's the point of using vector spaces?

- A well-formed algebraic space for retrieval
- Key: A user's query can be viewed as a (very) short document.
- Query becomes a vector in the same space as the docs.
- Can measure each doc's proximity to it.
- Natural measure of scores/ranking – no longer Boolean.
    - Queries are expressed as bags of words

Interaction: vectors and phrases

- Scoring phrases doesn't fit naturally into the vector space world:
  - ***"tangerine trees" "marmalade skies"***
  - Positional indexes don't calculate or store tf.idf information for ***"tangerine trees"***
- Biword indexes (extend the idea to vector space)
  - For these, we can pre-compute tf.idf.
- We can use a positional index to boost or ensure phrase occurrence

Vectors and wild cards

- How about the query ***tan* marm****?
  - Can we view this as a bag of words?
  - Thought: expand each wild-card into the matching set of dictionary terms.
- Danger – unlike the Boolean case, we now have *tf*s and *idf*s to deal with.

Vector spaces and other operators

- Vector space queries are apt for no-syntax, bag-of-words queries (i.e. free text)
  - Clean metaphor for similar-document queries
- Not a good combination with Boolean, wild-card, positional query operators

Query language vs. scoring

- May allow user a certain query language, say
  - Free text basic queries
  - Phrase, wildcard etc. in Advanced Queries.
- For scoring (oblivious to user) may use all of the above, e.g. for a free text query
  - Highest-ranked hits have query as a phrase
  - Next, docs that have all query terms near each other
  - Then, docs that have some query terms, or all of them spread out, with tf x idf weights for scoring

Exercises

- How would you augment the inverted index built in lectures 1–3 to support cosine ranking computations?
- Walk through the steps of serving a query.
- *The math of the vector space model is quite straightforward, but being able to do cosine ranking efficiently at runtime is nontrivial*

Efficient cosine ranking

- Find the k docs in the corpus "nearest" to the query $\Rightarrow$ k largest query-doc cosines.
- Efficient ranking:
  - Computing a single cosine efficiently.
  - Choosing the k largest cosine values efficiently.
    - Can we do this without computing all n cosines?
      - n = number of documents in collection

Limiting the accumulators: Frequency/impact ordered postings

- Idea: we only want to have accumulators for documents for which $wf_{t,d}$ is high enough
- We sort postings lists by this quantity
- We retrieve terms by idf, and then retrieve only one block of the postings list for each term
- We continue to process more blocks of postings until we have enough accumulators
  - Can continue one that ended with highest $wf_{t,d}$
  - The number of accumulators is bounded
- Anh et al. 2001

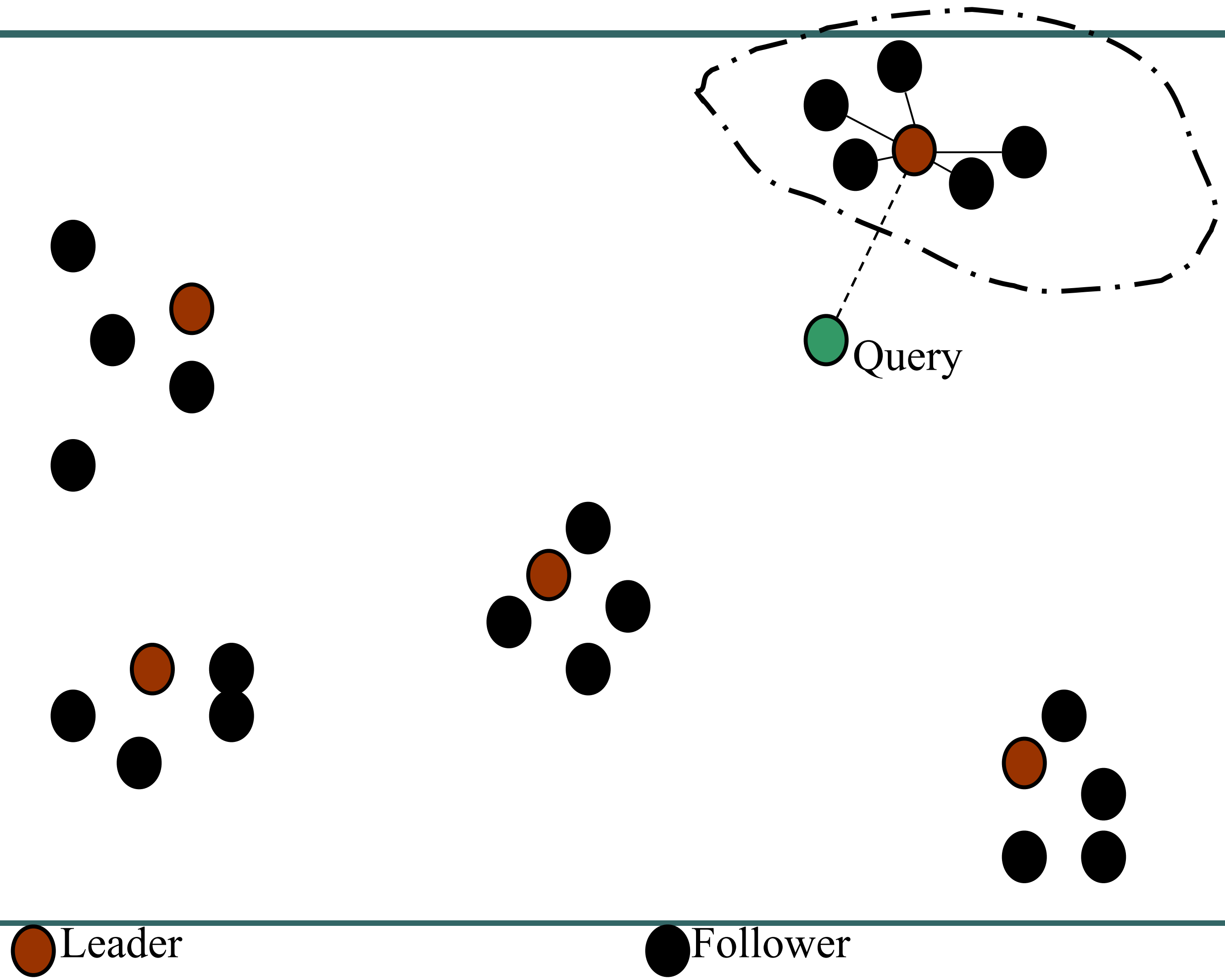
Cluster pruning: preprocessing

- Pick √n *docs* at random: call these *leaders*
- For each other doc, pre-compute nearest leader
  - Docs attached to a leader: its *followers;*
  - Likely: each leader has ~ √*n* followers.

Cluster pruning: query processing

- Process a query as follows:
  - Given query *Q*, find its nearest *leader L.*
  - Seek *k* nearest docs from among *L*'s followers.

Visualization



Why use random sampling

- Fast
- Leaders reflect data distribution

General variants

- Have each follower attached to a=3 (say) nearest leaders.
- From query, find b=4 (say) nearest leaders and their followers.
- Can recur on leader/follower construction.

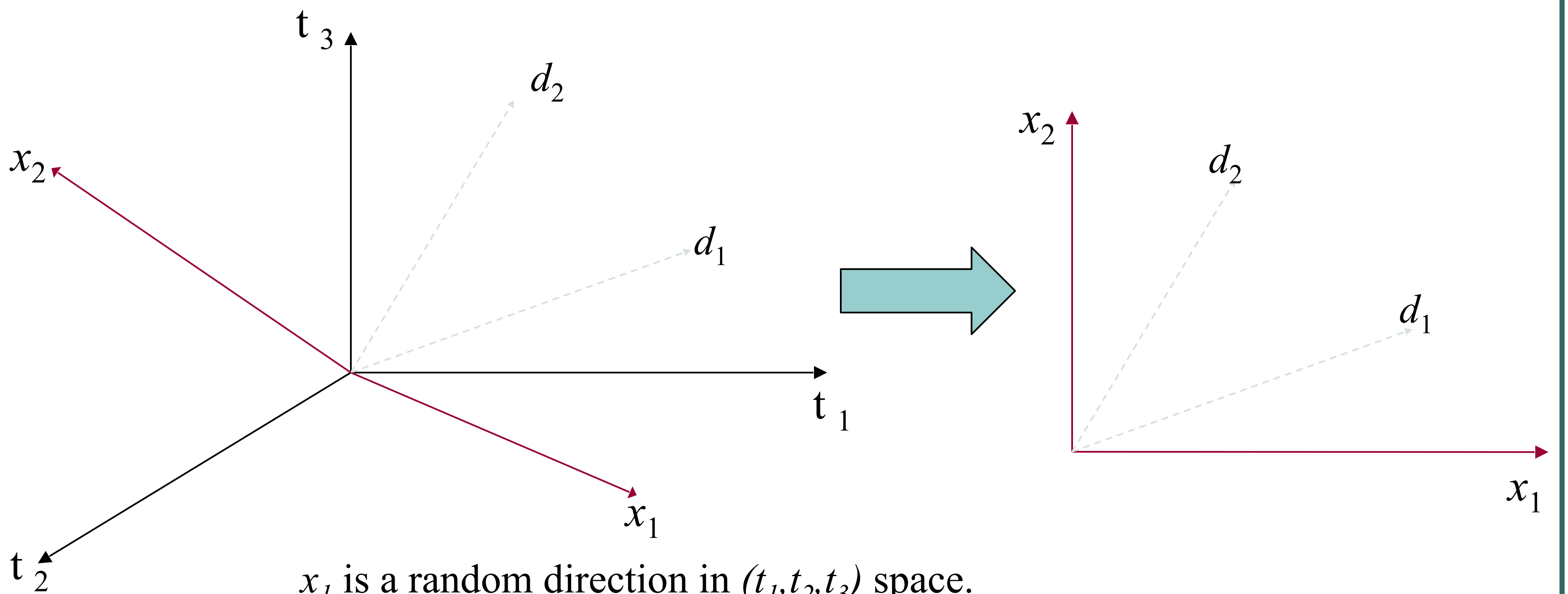
Dimensionality reduction

- What if we could take our vectors and "pack" them into fewer dimensions (say 50,000→100) while preserving distances?
- (Well, almost.)
  - <u>Speeds up cosine computations.</u>
- Two methods:
  - Random projection.
  - "Latent semantic indexing".

Random projection onto k<<m axes

- Choose a random direction x_1 in the vector space.
- For $i = 2$ to k,
  - Choose a random direction x_i that is orthogonal to $x_1, x_2, \ldots x_{i-1}$.
- Project each document vector into the subspace spanned by $\{x_1, x_2, \ldots, x_k\}$.

E.g., from 3 to 2 dimensions



x_1 is a random direction in (t_1,t_2,t_3) space.

x_2 is chosen randomly but orthogonal to x_1.

Dot product of x_1 and x_2 is zero.

Guarantee

- With high probability, relative distances are (approximately) preserved by projection.
- Pointer to precise theorem in Resources.

Computing the random projection

- Projecting n vectors from m dimensions down to k dimensions:
  - Start with $m \times n$ matrix of terms $\times$ docs, A.
  - Find random $k \times m$ orthogonal projection matrix R.
  - Compute matrix product $W = R \times A$.
- j^{th} column of W is the vector corresponding to doc j, but now in $k << m$ dimensions.

Cost of computation

- This takes a total of *kmn* multiplications.
- Expensive

Why?

Latent semantic indexing (LSI)

- Another technique for dimension reduction
- Random projection was data-*independent*
- LSI on the other hand is data-*dependent*
  - Eliminate redundant axes
  - Pull together “related” axes – hopefully
    - ***car*** and ***automobile***