

C3 Review Test, 2022Paper Title: Graph Theory

Max Marks: 60

Duration: 2 hours

Answer All questions. The marks for each question has been provided alongside.

Q1. (a) State True or False (with Justification): [5+5=10]i. A matching M in a graph $G=(V,E)$ is a maximum matching if and only if there is no M -augmenting path in G .

ii. A disconnected simple acyclic graph does not have a spanning tree.

iii. The endpoints of a cut-edge are both cut-vertices.

Q1. (b) For each situation, would you find an Euler circuit or a Hamilton Circuit?

i. The department of Public Works must inspect all streets in the city to remove dangerous debris.

ii. Relief food supplies must be delivered to eight emergency shelters located at different sites in a large city.

iii. The Department of Public Works must inspect traffic lights at intersections in the city to determine which are still working.

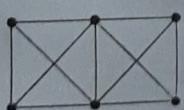
Q2. (a) For each of the graphs : K_n , P_n , C_n and W_n . [6+3+3+3=15]Give the order, the size, the maximum degree and the minimum degree in terms of n .**Q2. (b) Draw a bipartite graph of order 6. Give its adjacency list and a drawing.****Q2. (c) Consider the graphs $G_1 = (V_1; E_1)$ and $G_2 = (V_2; E_2)$. Give the order, the degree of the vertices and the size of $G_1 \times G_2$ in terms of those of G_1 and G_2 .****Q2. (d) Prove or give a counterexample: any two graphs with the same degree sequence are isomorphic****Q3. Consider the following graphs : [5]**

(a) Do they have an Eulerian circuit? (b) Do they have an Hamiltonian Cycle?

(c) Exhibit a matching if they have?

Q4. (a) Is this graph a planar graph? Justify.

[3+2+2+3=10]

(b) Give the line graph of C_5 and $G = (\{1, 2, 3, 4, 5\}; \{12, 23, 24, 25, 34, 35, 45\})$

(c) Find the diameter (maximum of all vertex eccentricities) of the Peterson's graph.

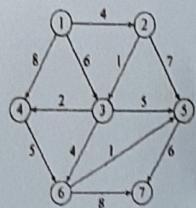
(d) Give a connected graph $G = (V, E)$ and a vertex $u \in V$ for the following relation : $D(G) = D(G - u)$. D is the diameter of G (maximum of all vertex eccentricities)**Q5. (a): Let us consider the graph whose adjacency list is given.**

[5+5+5=15]

a	b	c	d	e	f	g	h	i	j
d	d	h	a	a	a	b	c	b	b
e	g		b	d	d	i		g	g
f	i		e			j			
j		f							

- i. Draw the graph
ii. Using the algorithm BFS, find the distance from the vertices a and b to each of the other vertices of the connected component to which they belong
[Note : You are expected to show the BFS traversal as intermediary steps for distance computation]

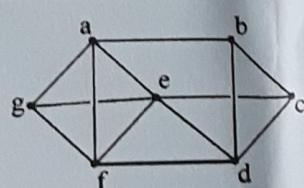
Q5.(b) Using the Dijkstra's algorithm, obtain the shortest distance and shortest path from vertex 1 in the network shown below :



Q5. (c) Prove that : Every tree with two or more vertices is 2-chromatic.

Q6. (a) Find the chromatic number of the following graph

|2+3=5|



Q6.(b) For the following bipartite graph, provide the maximum matching, minimum vertex cover, maximum independent set and minimum edge cover

