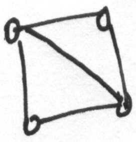
Planar Graphs

A graph with a plane drawing
↳ drawing of edges s.t. no two edges cross each other


← Plane drawing ⇒ Planar graph

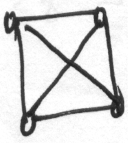


Fig2

← Edges cross eachother
← Not a plane drawing
(Does not conclude G is non-planar)

Fig 3 is an alternate plane drawing of Fig2.

alternate drawing of

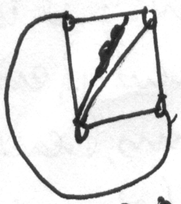
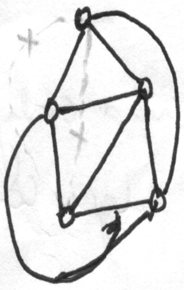


Fig3

← Plane drawing
K_4: Planar graph



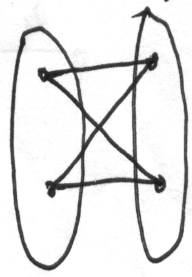
K_5: ~~No planar~~ No ~~place~~ plane drawing (Non planar)

$K_5 - e$ (from ~~K5~~ K_5 Remove any edge)
↳ as a plane drawing ⇒ $K_5 - e$ is planar

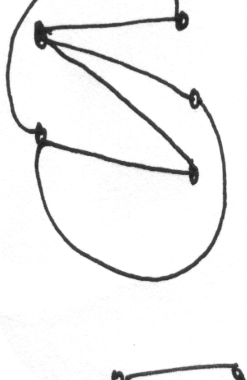
$K_{3,3}$ (Complete Bipartite graph with 3 vertices on each side)
↳ Non-planar while $K_{3,3} - e$ is planar

K_n → Complete graph

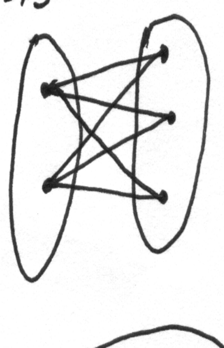
$K_{2,2}$ → complete graph



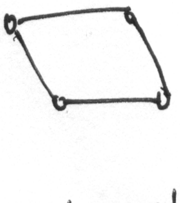
$K_{2,3}$ Can have alternate planar drawing → Planar



$K_{2,3}$

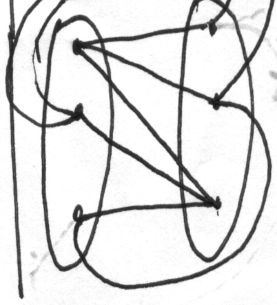


$K_{2,2}$



$\equiv C_4 \Rightarrow$ Planar
→ alternate drawing of $K_{2,2}$

$K_{3,3}$



$K_{n \geq 5}$ → Non-planar

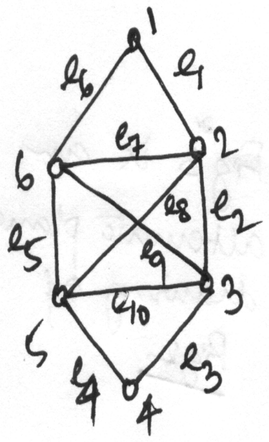
∵ $K_5 \subseteq K_6$ & K_5 is non planar ∴ K_6 is non-planar

If $H \subseteq G$ and H is non-planar then G is non-planar

* ∵ $K_{3,3}$ is non-planar ⇒ $K_{3,4}$ & $K_{4,4}$ is non-planar

Eulerian Graphs

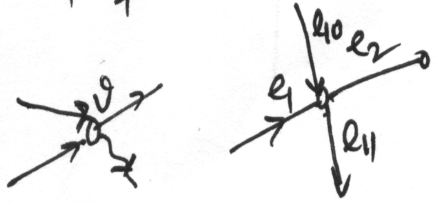
Eulerian Paths ⇒ A Spanning path containing all edges s.t. each edge is visited exactly once (vertices may be repeated)



~~1 e_1 2 e_2 3 e_3 4 e_4 5 e_5 6 . e_7 2 e_8 5~~

1 e_1 2 e_2 3 e_3 4 e_4 5 e_{10} 3 e_9 6 e_7 2 e_8 5 e_5 6 . e_6 1

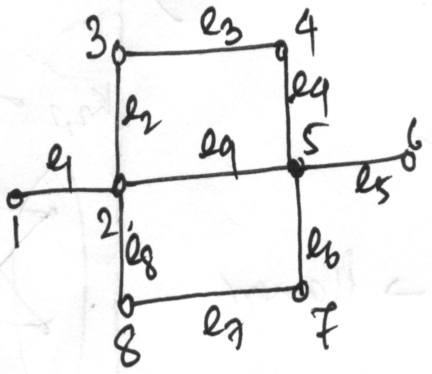
Eulerian circuit



* If in a eulerian circuit a vertex v is visited more than once, then each entry & exit has to be via different edges. Thus, each entry ~~exit~~ exit contributes two edges (edges of degree 2)

Theorem:

A Graph is Eulerian (has eulerian circuit) if and only if degree of every vertex is Even (∵ each entry & exit edges contributes to degree 2 for the vertex)



1 e_1 2 e_2 3 e_3 4 5 e_9 2 e_8 8 e_7 7 e_6 5 e_5 6

Eulerian Path ✓

Eulerian Cycle ✗