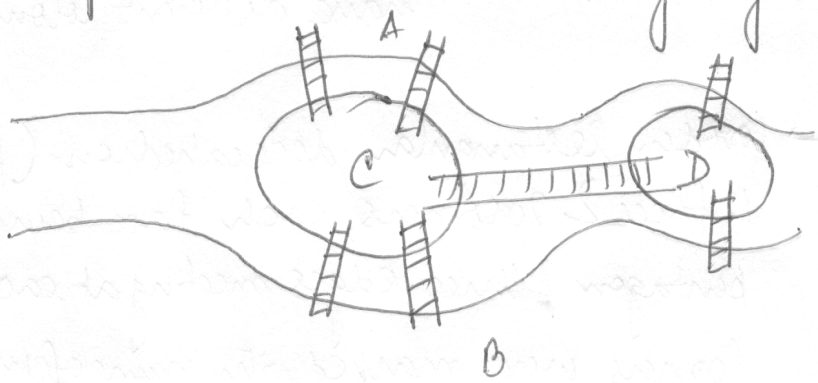
tory of Graph Theory

= 1736 : Euler's Paper which solved the Königsberg bridge problem



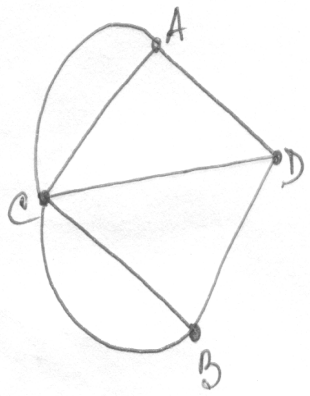
Two Islands C&D, formed by Pregel River in Königsberg (Kaliningrad, West Soviet Russia) were connected to each other and to the two banks A&B using 7 bridges

Problem: Start from any land mass A, B, C or D, walk over each bridge exactly once and return to the starting point
(without swimming across the river of course)

Solution: It was open problem till in 1736 Euler proposed a solution to the problem using graphs.

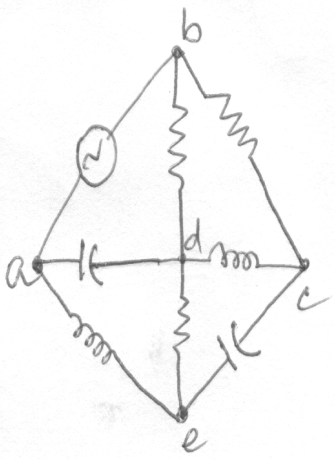
⇓

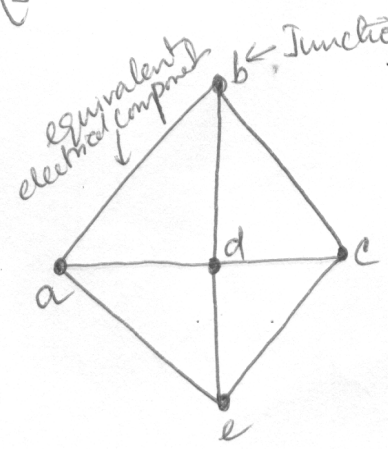
equivalent to drawing figures without lifting a pen



* The solution for this problem does not exist

- 1847, G.R. Kirchoff developed theory of trees for their applications in electrical networks





1840: Four Colour conjecture ⇒ Four colours are sufficient for
(A.F. Mobius) Colouring a map such that the
Countries with common boundaries
have different colours

1859 : Puzzle → wooden rectangular dodecahedron (polyhedron with
(Hamilton) 12 faces & 20 corners, each face being a regular
pentagon & three edges meeting at each corner
Corners were marked with name of important cities

Objective → Find a route along the edges of the dodecahedr
passing through each of the 20 cities exactly once

≡ Hamiltonian circuit

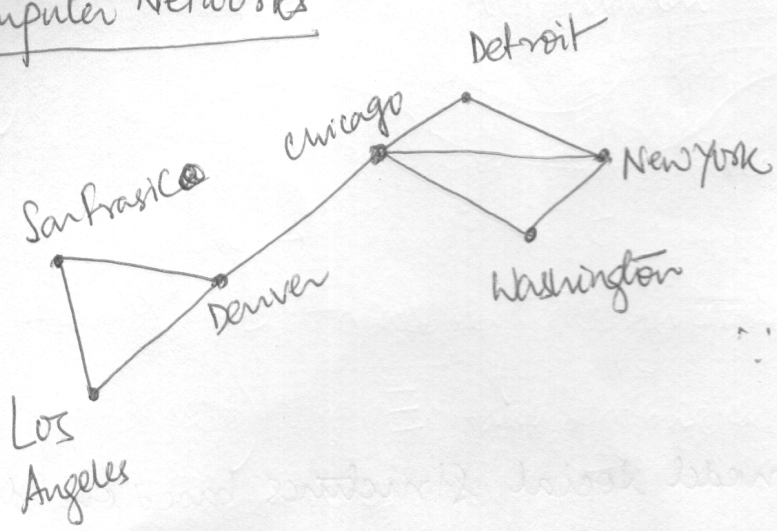
↳ no necessary & sufficient condition till date

* A lot of work has been done (publications)

~~A340 : Four colour conjecture~~

Applications of Graph Models

(1) Computer Networks –

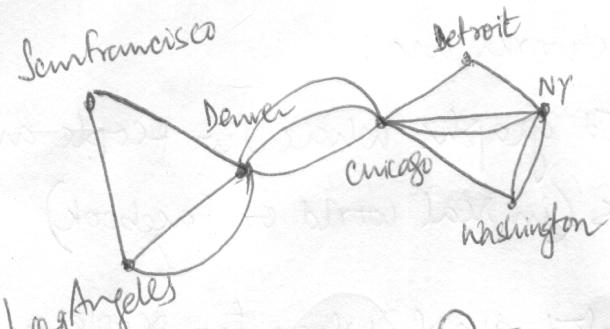


Data centres – Nodes

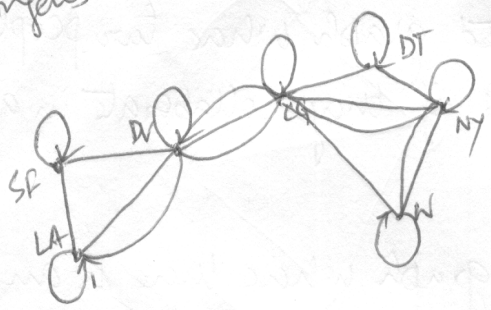
Links ——— Communication Links

Model → Simple Graph

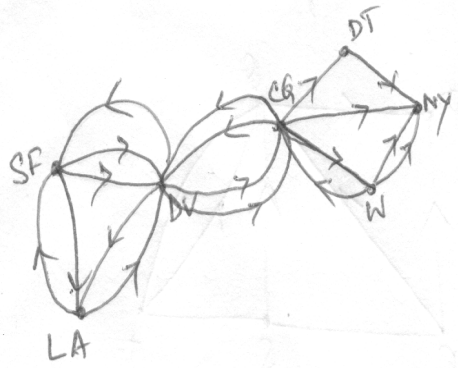
∵ Here we care about only ~~two vertices~~ whether the two datacentres are ~~mutually~~ directly linked (not how many links are there) and all communication links work in both directions



* To model a computer N/w where we care about no of links b/w data centres, we use a multigraph



* To model a computer n/w with diagnostic links, we use pseudograph as loops are needed



* To model a n/w with multiple one-way links, we use a directed multi-graph
  – we use directed graph without multiple edges if we care whether there is atleast one link from a data centre to another data centre

Other application of Graphs

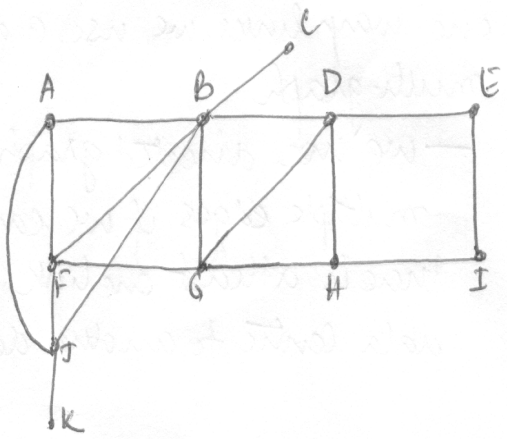
- Social Networks
- Communication Networks
- information Networks
- Software Design
- Transportation N/W
- Biological networks

(1) Social Networks

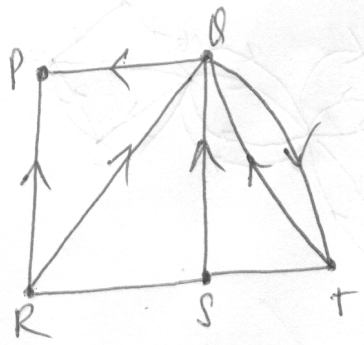
- Graphs can be used to model social structures based on different kinds of relationship between people or groups.

- In a social network, Vertices represent individuals or Organizations and Edges represent Relationship between them

- useful graph models of social networks are:
  - friendship graph – undirected graphs where two people are connected if they are friends (in real world or Facebook)
  - collaboration graph — undirected graph where two people are connected if they collaborate in a specific way
  - influence graph — directed graph where there is an edge from one person to another if the first person can influence the second perso



Friendship Graph



Influence Graph

Collaboration Graph

Academic Collaboration Graph → models collaboration of researchers who have jointly written a paper in a particular subject

- We represent researchers in a particular academic discipline using vertices
- we connect the vertices representing two researchers in this discipline if they are co-authors of a paper

Film Collaboration Graph → Collaboration of actors in films
(Actors - Vertices; ~~Edges~~ Edge - appearance in same movie)

III Information Network

Graphs can be used to model different types of networks that link different types of information

(i) Web Graph → Web Pages ~~represented~~ represented by Vertices & links are represented by edges

A web graph models the web at a particular time & is used by Search engine

(ii) Citation Network → Research Papers in a particular discipline are represented by Vertices

When a paper cites another paper as a reference, there is an edge from the vertex representing this paper to the ~~paper~~ vertex representing the second paper

[To be used in the Lab/Group Project]

IV. Transportation Graphs

Airline Network

- modelled using directed multigraphs
- each flight represented by a directed edge from vertex representing departure airport to vertex representing destination airport

Road Network

- Vertices represent intersections and edges represent Roads
- undirected edges represent two way roads
- directed edges represent one way road

I. Software Design Applications

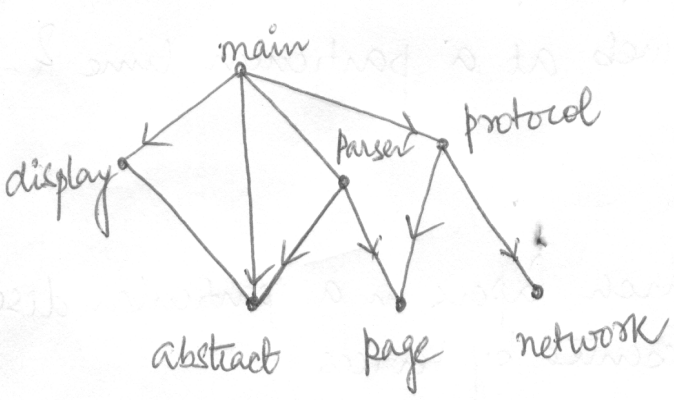
Graph models are used extensively in S/W design. We will introduce two such models:

(i) model representing dependency between modules of a S/W app.
(ii) " " restrictions in execution of statements in computer programs

- When a top down approach is used ~~in a~~ to design software, the system is divided into modules, each performing a specific task
- We use module dependency graph to represent dependency between modules. These dependencies need to be understood before coding can be done

- In a module dependency graph,
  Vertices represent Software modules
  There is an edge from one module to another if the second module depends on first

Example: The dependencies between seven modules in design of a web browser

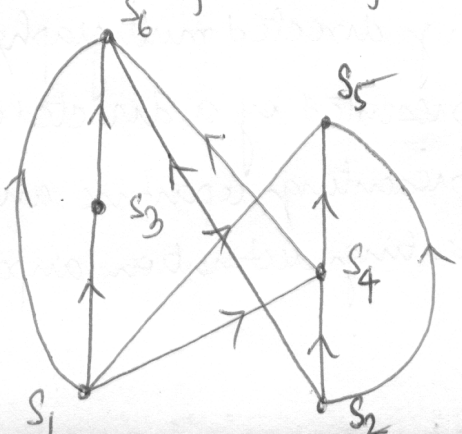


Precendence Graph

We can use a directed graph called precedence graph to represent which statements must have already been executed before we execute each statement

- Vertices represent statements in a program
- there is a directed edge from a vertex to a second vertex if the second vertex cannot be executed before the first

S_1 a := 0
S_2 b := 1
S_3 c := a+1
S_4 d := b+a
S_5 e := d+1
S_6 e := c+d

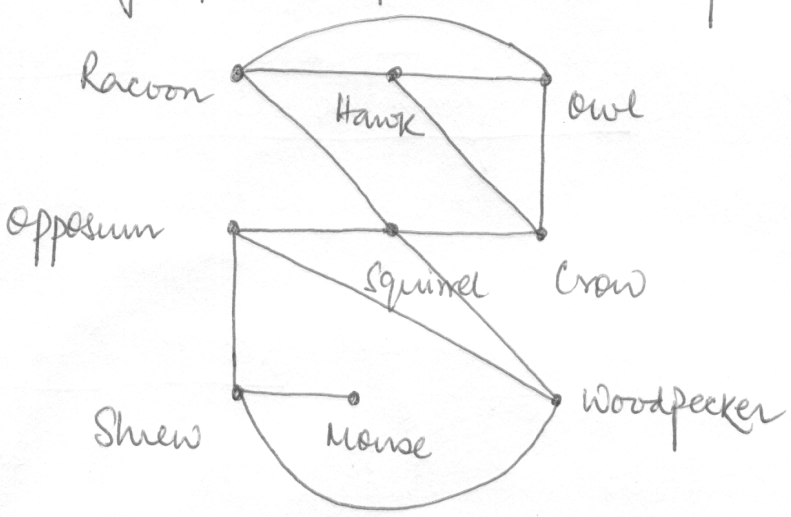


Biological Applications

Graph models are used extensively in many areas of biological science
We describe two such models — one to ecology and the other molecul
biolog

Niche overlap graphs → model competition between species in an ecosys

- Vertices represent species and edges connects two vertices wh
they represent species who compete for food resources.



Similar Resource allo
Graph can be construct
for processes running i
Computer System who Co
for shared resources

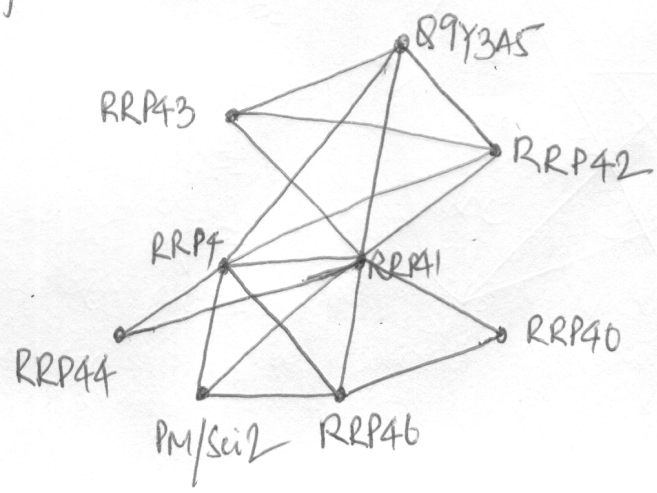
Protein-Interaction Network → Model interaction of proteins in a cell

Vertices – represent proteins

Edges – connecting two vertices — ~~Test~~ Proteins represented by the two
Connected vertices interact

Protein interaction graph can be huge and can contain 100,000 vertices, each representing a different protein, and more than 1,000,000 edges, each representing an interaction between proteins

Protein interaction graphs are often split into smaller graphs, called **modules** which involve interaction between proteins involv in a particular function



Protein interaction graph proteins that degrade RNA i a human cell