

Problem Set 1

Q1. Draw the diagram for the following graph :

- (a) $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 5\}, \{2, 6\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}\}$.
- (b) $V = \{1, 2, 3, 4, 5\}$ and $E = \{\{1, 2\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}\}$

Q2. Are the graphs mentioned in **Q1.** bipartite ? Explain

Q3. Draw the complement of the graphs mentioned in Q1.

Q4. Is the complement of a bipartite graph bipartite? Comment. You may consider the bipartite test of the graph mentioned in **Q3.** to justify your answer.

Q5. In any group of two or more people, prove that there are always two with the same number of friends.

Q6. [An alternate way of presenting Q5]

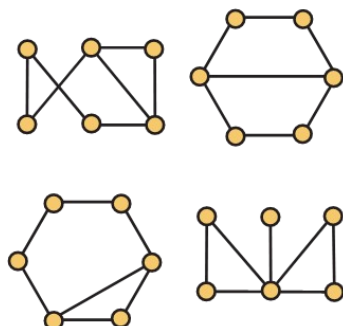
Show that for any graph with $|V| \geq 2$ nodes there exist two vertices with the same degree.

Q7. Prove that the sequence (7, 6, 5, 4, 3, 3, 2) is not graphic.

Q8. For each of the following sequences, either give an example of such a graph, or explain why one does not exist.

- A graph with six vertices whose degree sequence is [5,5,4,3,2,2]
- A graph with six vertices whose degree sequence is [5,5,4,3,3,2]
- A graph with six vertices whose degree sequence is [5,5,5,5,3,3]
- A simple graph with six vertices whose degree sequence is [5,5,5,5,3,3]

Q9. Determine which (if any) pairs of graphs are isomorphic. For pairs that are isomorphic, give an isomorphism between the two graphs. For pairs that are not isomorphic, explain why?



Q10. [Additional Problems]

Exercise 1-7 : The maze problem

Exercise 1-8 : The Decanting problem

from the book **Graph Theory with applications to Engineering and Computer Science** by **Narsingh Deo**