Project part 2:

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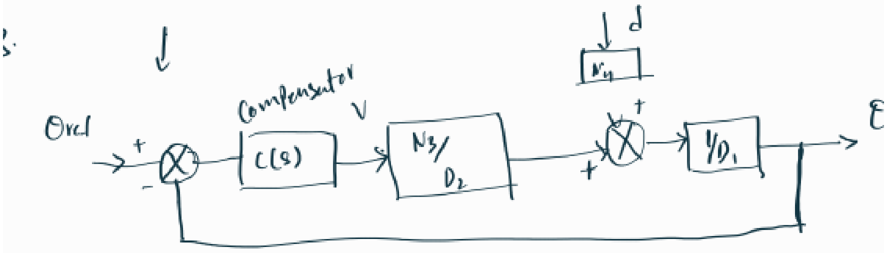
1. Instead of using a PID controller, propose a customized high-order compensator with gain K in order to stabilize θ(t) of the Segway vehicle (this compensator will have at least two stable poles). Show, using Root Locus and Matlab simulations, that the designed compensator stabilizes the system. You do NOT need to use angle deficiency to design the compensator but you can follow a qualitative approach and place the compensator poles/zeros to move the root locus to the stable left-half plane. Pole/zero cancellation is NOT allowed.



$$D_1 = s^2 - 0.0014s - 12.077 \qquad D_2 = (38.5s + 10.35)$$

$$N_3 = -(1.08s + 0.0094) \qquad N_4 = 12.422$$

Let $C(s) = K$

$\theta_{ref} = 0$ $\quad \theta_{CL}(s) = \dfrac{N_4}{\cancel{D_1}} \times \dfrac{\cancel{D_1} D_2}{D_1 D_2 + N_3 \cdot C(s)}$

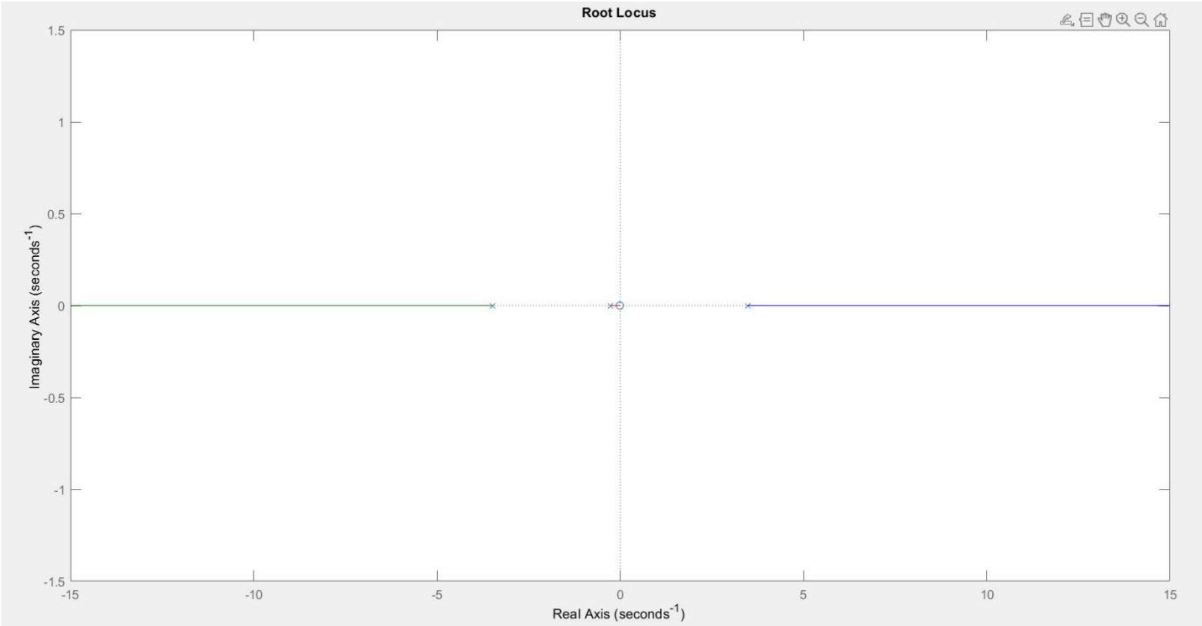
$$\theta_{CL}(s) = \frac{N_4 D_2}{D_1 D_2 + N_3 \cdot K}$$

R Locus

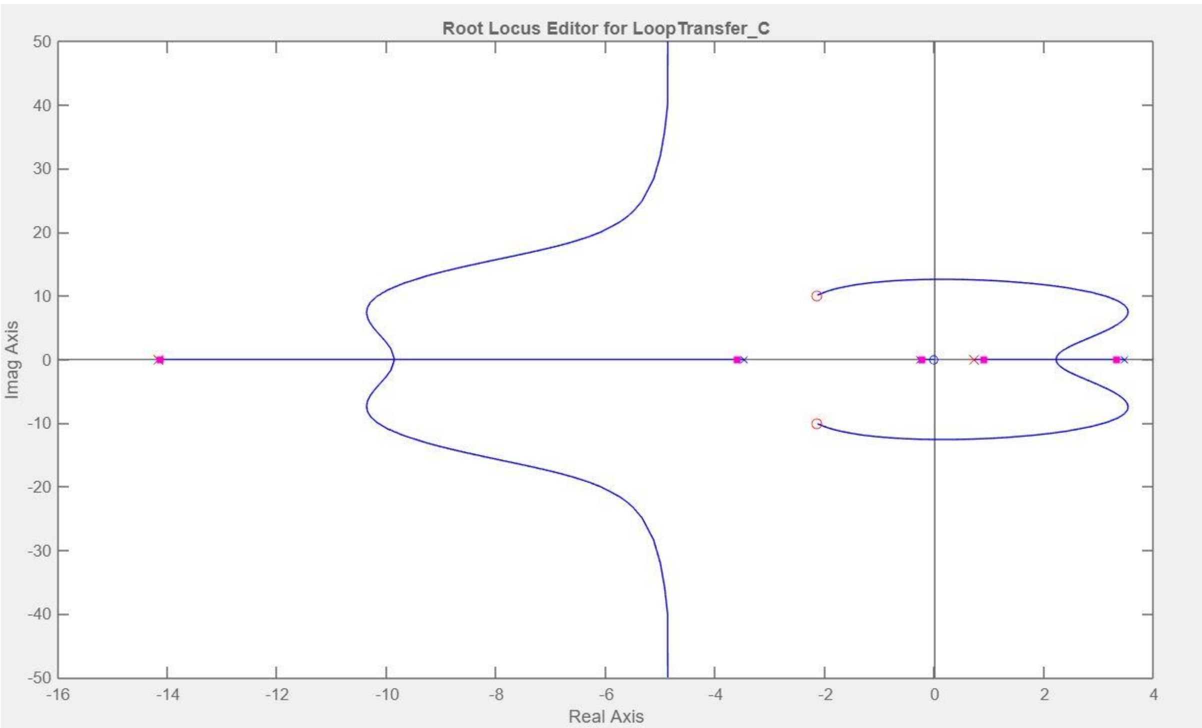
$$K\left(\frac{N_3}{D_1 D_2}\right) = -1$$

$$\frac{N_3}{D_1 D_2} = \frac{-(1.08s + 0.0094)}{(s^2 - 0.0014s - 12.077)(38.5s + 10.35)}$$

Root Locus of N3/D1*D2(Uncompensated system).



Root Locus after adding high-order compensator



→ From the Root locus, we can observe that the unstable pole is going towards $+\infty$, as k value increases

→ I've thought of adding zero's close to imaginary axis on left side to attract the unstable poles.

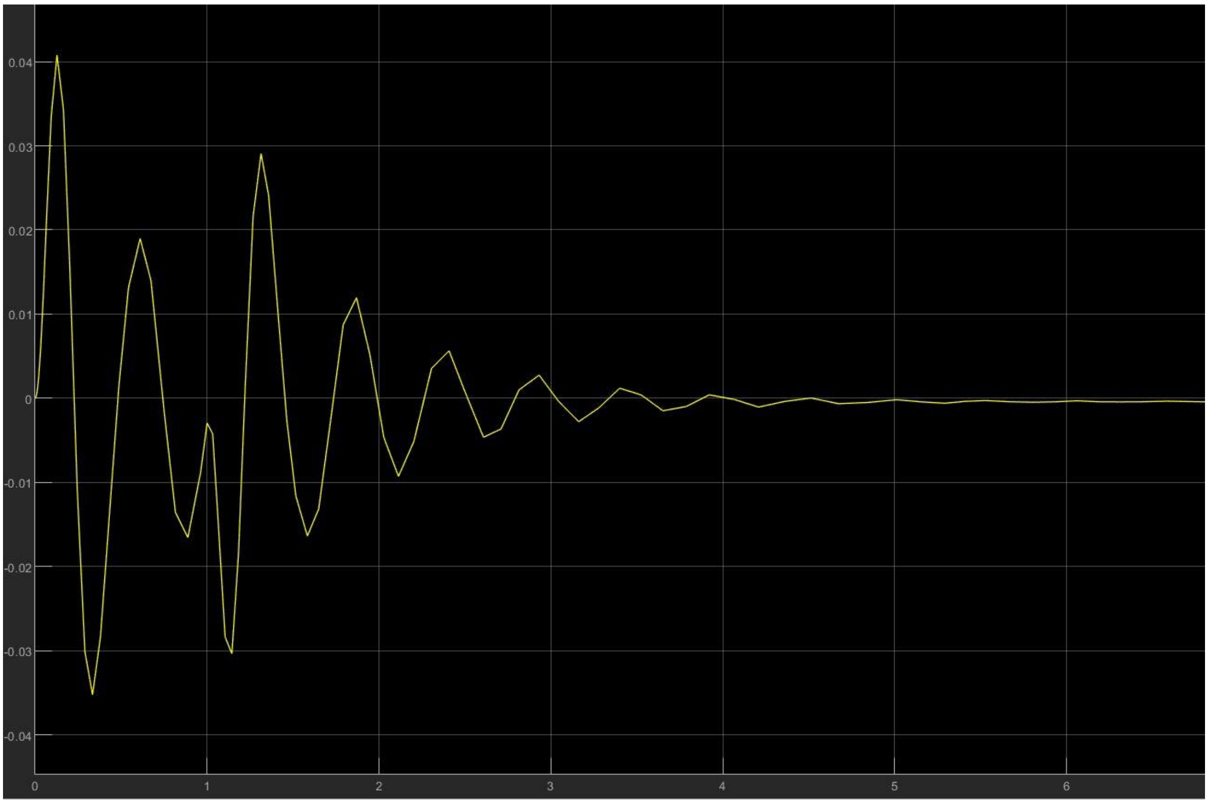
→ To create a break out point on positive real axis, I've added a pole on the right side before the unstable pole

→ Another pole is added to extreme left

Finally, for some values of k, all the poles are stable. According I've choosen the gain.

$$C(s) = \frac{2.7653 \times e^{05} \left(1 + 0.045s + (0.097s)^2\right)}{(1 + 0.0715)(1 - 1.45)}$$

Impulse disturbance response of theta w.r.t time



2. Write the governing dynamics in state space form z˙(t) = Az(t)+Bu(t)+T, where z(t) = (θ, ˙θ, x˙) > is the state vector, A and B are state and control matrices, respectively, u is the controller (which contains the voltage supplied from the motor) and T is the vector containing the disturbance torque mgd/J. Assuming no disturbance and no controller input, analyze the system's stability

2.

$$\dot{z}(t) = A z(t) + B u(t) + T$$

$$z(t) = (\theta, \dot{\theta}, \dot{x})^T$$

$$Y = C z(t) + D \cdot u(t)$$

$$\dot{z}(t) = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \ddot{x} \end{bmatrix} \quad \text{let } C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Measurement matrix

$$\ddot{\theta}(t) = \frac{MgL + mgl}{J}\theta(t) + \frac{\beta}{J}\dot{x}(t) - \frac{a}{J}V + \frac{mg}{J}d - \frac{C_1}{J}\dot{\theta}(t)$$

$$\ddot{x}(t) = \frac{a}{R(M+m)}V - \frac{\beta/R + C_2}{M+m}\dot{x}(t)$$

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \ddot{x} \end{bmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \frac{MgL + mgl}{J} & -\frac{C_1}{J} & \frac{\beta}{J} \\ 0 & 0 & \frac{-(\beta/R + C_2)}{M+m} \end{pmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{x} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ -\frac{\alpha}{J} \\ \frac{\alpha}{R(M+m)} \end{bmatrix} V + \begin{bmatrix} 0 \\ \frac{mg}{J} \\ 0 \end{bmatrix} d$$

U(t) = V

$$\dot{z}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 12.077 & -0.0014 & 0.1407 \\ 0 & 0 & -0.2606 \end{bmatrix} z(t) + \begin{bmatrix} 0 \\ -0.0281 \\ 0.0519 \end{bmatrix} V$$

$$+ \begin{bmatrix} 0 \\ 12.4221 \\ 0 \end{bmatrix} d$$

Poles of the System without disturbance Can be Calculated by finding Eigen values of A

Using Matlab eig(A) = 3.4745, -3.4759
-0.2606

The System is Unstable due to the Presence of Unstable Pole 3.4745

3. Using Matlab, show whether or not the system is controllable and/or observable.

MATLAB Code to find controllability

```
A = [ 0 1 0; 12.077 -0.0014 0.1407; 0 0 -0.2606];
eig(A);

B = [0; -0.0281; 0.0519];

C = [1 0 0; 0 1 0; 0 0 1];
D = [ 0; 0; 0];
T = [0; 12.4221; 0];

pc = ctrb(A,B);
rank(pc)
```

```
ans =

     3
```

```
O = obsv(A,C);
rank(O);
```

```
ans =

     3
```

Since rank of Controllability matrix & observability matrix is 3, the system is controllable & observable.

4. Disregarding the disturbance, use Matlab to transform the state-space representation into control canonical form and modal canonical form.

Matlab Code to transform the state-space representation into control canonical form and modal canonical form.

```
sys = ss(A,B,C, D);
modal_canon_form = canon(sys,'modal')
control_canon_form = transpose(canon(sys,'companion'))
```

Result:

```
modal_canon_form =

  A =
             x1        x2        x3
   x1     3.474         0         0
   x2         0    -3.476         0
   x3         0         0   -0.2606

  B =
             u1
   x1  -0.07972
   x2  -0.09262
   x3    0.2078

  C =
               x1          x2          x3
   y1     0.04718    -0.04718   -0.002926
   y2      0.1639       0.164   0.0007624
   y3           0           0      0.2497

  D =
       u1
   y1   0
   y2   0
   y3   0

Continuous-time state-space model.
```

```
control_canon_form =

  A =
             x1        x2        x3
   x1         0         1         0
   x2         0         0         1
   x3     3.147     12.08    -0.262

  B =
               u1          u2          u3
   x1           0     -0.0281      0.0519
   x2     -0.0281    0.007342    -0.01353
   x3    0.007342     -0.3413    0.003525

  C =
        x1  x2  x3
   y1    1   0   0

  D =
        u1  u2  u3
   y1    0   0   0

Continuous-time state-space model.
```

5. We next aim to stabilize the system by designing a controller that calculates the voltage V based on full-state feedback (all states are measurable). Disregarding the disturbance, use the ACKER command in Matlab to design a control law K such that V = −Kz(t) places the closed loop poles at some desired positions determined by the control designer (student picks the poles).

P contains desired poles.

Matlab code

```
p = [-15 -3+3i -3-3i];

k = acker(A,B,p)
```

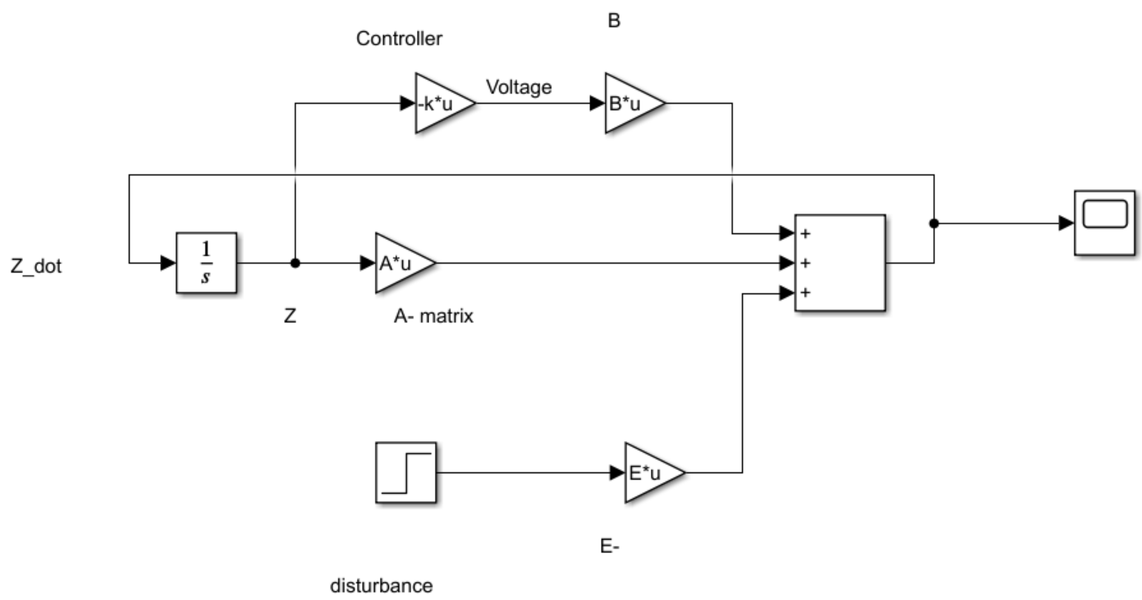
Result:

```
k =

   1.0e+03 *

   -4.2732   -1.5426   -0.4356
```

6. With the control law K from the previous step, and considering the rider's disturbance (impulse and/or step), simulate the states. Discuss the results in connection with the results obtained in Part 1 of the project where only θ was available for the controller.



Z-Vector plot

The graph below was obtained for disturbance input (Impulse),

yellow – theta

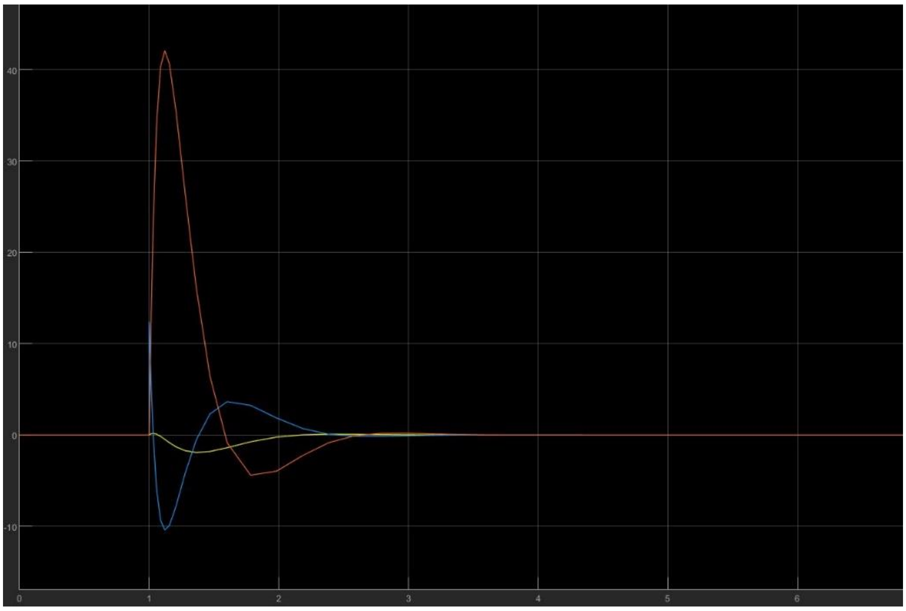
blue - theta_dot

red line - X_dot

Since, we have freedom to pick desired poles, Poles are chosen a bit far from Imaginary axis, which reduced the settling time of theta, X_dot compared to part 1 for both step input & impulse input.

Since we have have used PID & compensators in part 1, we have access to only one state variable for control, where as in state space, we have access to all the relevant state variables, which give more control to design transient characteristics such as overshoot, peak time & rise time.

The graph below was obtained for Step input



Bonus: Assuming $\dot{x}$ is not measurable, design a state observer that predicts speed as $\hat{\dot{x}}$. Use this prediction along with the available measurements to control the Segway, with the same control law K designed above. Comment on the behavior of Segway using the estimation $\hat{\dot{x}}$ instead of measuring/utilizing the actual $\dot{x}$ without the need for an observer.

Given X_dot is not observable. so I've chosen the measurement matrix M as [1 1 0].

Observer poles are chosen at 10 times compared to system poles, as sensor response should be at least ten time faster.

Acker command is used to calculate the respective gains based on pole values.

Matlab code -

```
obsv_p = [-150; -30+3i; -30-30i]
M = [1 1 0];
Y = acker(A', M', obsv_p)
Y = Y'
```

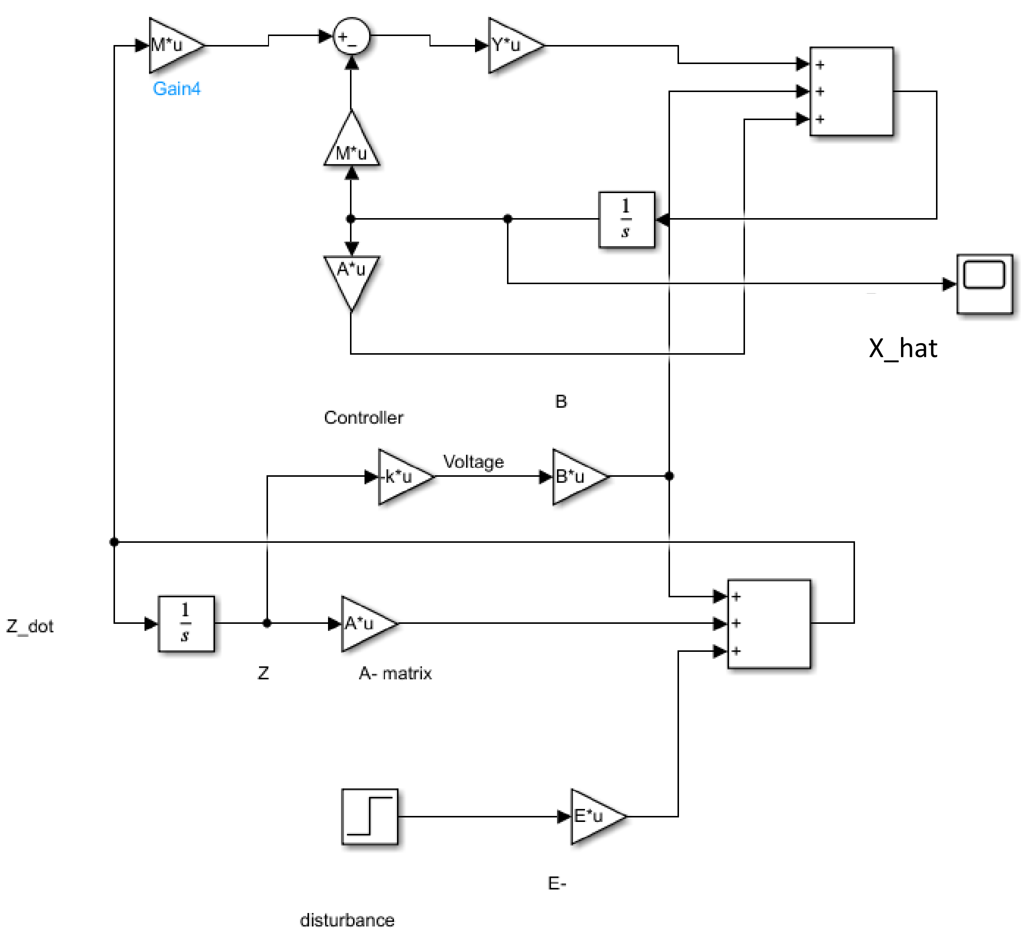
Result -

```
Y =

   1.0e+06 *

   -0.0169
    0.0171
    1.4025
```

Simulink Modelled with Observer



yellow – theta

blue - theta_dot

red line - X_dot

We can observe from the above plot that state variable theta & theta_dot doesn't vary as expected, but the unmeasurable state variable x_dot magnitude increased rapidly. This may be due to reduction in settling time of observer.