

## Recursion - 1

What is Recursion?

(bookish lang.) → when a function calls itself (directly or indirectly).  
Desi lang. → when solution of a bigger problem is dependent upon similar type of smaller problem.

When to apply recursion → Jd bigger problem ki solution depend krta hai small krke similar problem pr,

eg →  $2^5$  ← Bigger problem  
          ↓  
           $2 \times 2^4$  ← choti problem

⇒

$$2^5 = 2 \times 2^4$$

$$f(n) = 2^n$$

$$f(n-1) = 2^{n-1}$$

$$f(n) = 2 \times 2^{n-1}$$

$$f(n) = 2 \times f(n-1)$$

① Factorial →

Bigger Problem =  $5!$

↓  
 $5 \times 4!$

← smaller problem (similar type)

$$\text{B.P } (5!) = 5 \times (4!) \text{ C.P}$$

② Counting (Reverse) →

$$n = 5$$

o/b → 5, 4, 3, 2, 1

$f(n)$  → print counting from  $n$  to 1

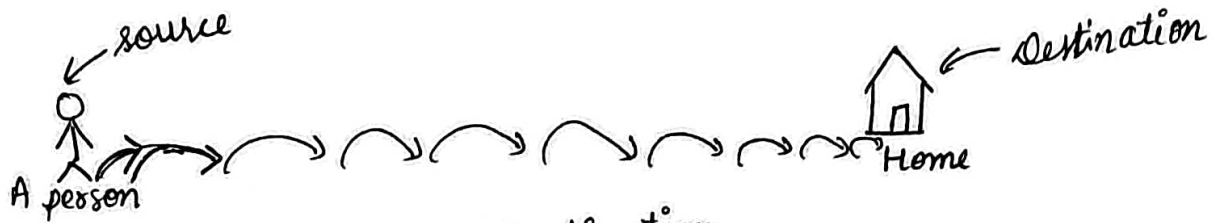
$f(5)$  → print counting from 5 to 1

$f(5)$  → print 5 +  $f(4)$

void solve()

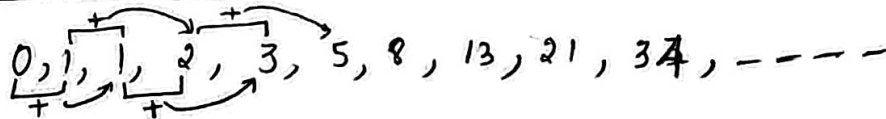
    solve();

Problem Statement  $\rightarrow$  if bigger problem of similar type depending upon similar prob  $\Rightarrow$  Use recursion



We have to reach to destination.  
We will start by taking one step and repeat this step.  
And we will stop whenever we reach destination.

eg  $\rightarrow$  Fibonacci Series  $\rightarrow$



$$0+1=1, 1+1=2, 1+2=3, 2+3=5, 3+5=8, 5+8=13, 8+13=21$$

$$n^{\text{th}} \text{ term} = (n-1)^{\text{th}} \text{ term} + (n-2)^{\text{th}} \text{ term}$$

bigger problem

$$f(n) = f(n-1) + f(n-2)$$

similar type of smaller problems.

Ques

$$i/p \Rightarrow n$$

$$o/p \Rightarrow 2^n$$

$$\text{eg} \rightarrow n=3$$

$$o/p \rightarrow 2^3 = 8$$

$$n=6$$

$$o/p \rightarrow 2^6 = 64$$

Bigger problem  $\rightarrow 2$  to the power of  $n \rightarrow 2^n$

$$2^n = \underbrace{2 \times 2 \times 2 \times \dots \times 2}_{n \text{ times}}$$

def problem

$$2^n = 2 \times 2^{n-1}$$

$\leftarrow$  def problem  $n-1$  times

$$f(n) = 2 \times f(n-1)$$

$$\text{eg} \Rightarrow n=3$$

$$o/p = 6$$

$$n=5 \rightarrow o/p \Rightarrow 120$$

$\Rightarrow$  factorial  $\rightarrow$

$$i/p \rightarrow n$$

$$o/p \rightarrow n!$$

Bigger problem is to find  $n!$

$$n! = \underbrace{n \times (n-1) \times (n-2) \times (n-3) \times \dots \times 3 \times 2 \times 1}_{n!}$$

$$n! = n \times (n-1)!$$

$n! = n \times (n-1)!$   
 Bigger prob.  $\swarrow$   $\searrow$  smaller prob.

$$f(n) = n \times f(n-1)$$

$\Rightarrow$  Counting (Reverse)  $\rightarrow$  i/p = n  
 o/p  $\Rightarrow$  reverse counting from n to 1.

$f(n) \rightarrow$  reverse counting from n to 1

$$f(n) \Rightarrow \text{print } n + f(n-1)$$

$f(5) \Rightarrow$  print 5 then  $f(4)$

① Codes  $\rightarrow$

factorial  $\rightarrow$

```
int factorial(int n){
```

```
    int ans = n * factorial(n-1);
```

```
    return ans;
```

```
}
```

we will end up  
 getting segmentation  
fault.

calls goes for  $n = 5, 4, 3, 2, 1,$   
 $0, -1, -2, -3, \dots$

① Component of Recursive Code  $\rightarrow$  3 components  
 tells the function when to stop?  
 (Rukna kab hai?)

Mandatory  $\left\{ \begin{array}{l} \rightarrow \text{Base condition} \\ \rightarrow \text{Recursive Relation} \leftarrow \text{Relation} \end{array} \right.$

Optional  $\rightarrow$  Processing

new

```
int factorial(int n){
```

```
    // base case.
```

```
    if(n == 1)
```

```
        return 1;
```

```
    return n * factorial(n-1);
```

```
    int chotiProblemKaAns = factorial(n-1);
```

```
    int badiProblemKaAns = n * chotiProblemKaAns;
```

```
}
```

Recursion call will be made  
 only till 1 and 1 will  
 be returned.

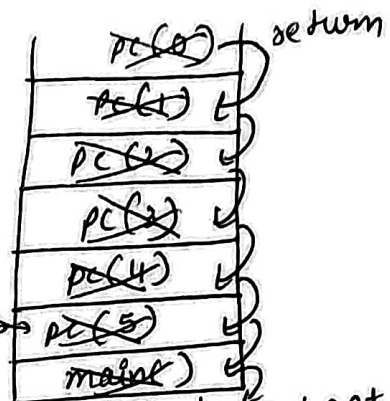
```

void printCounting (int n) {
    if (n == 0) ← base case
        return;
    // Processing
    cout << n;
    // Recursive Relation
    printCounting (n-1);
}

```

we have to always use return keyword in base case.

whenever we call a function, it's entry is stored in the stack.



Stack → Last In First Out

let  $n = 5$   
 $5 == 0 \rightarrow F$   
 $cout << 5$   
 func call for  $n-1=4$

$pc = printCounting$

o/p  $\Rightarrow 5 \ 4 \ 3 \ 2 \ 1$

$n = 4$   
 $4 == 0 \rightarrow F$   
 $cout << 4$   
 func call for  $n-1=3$

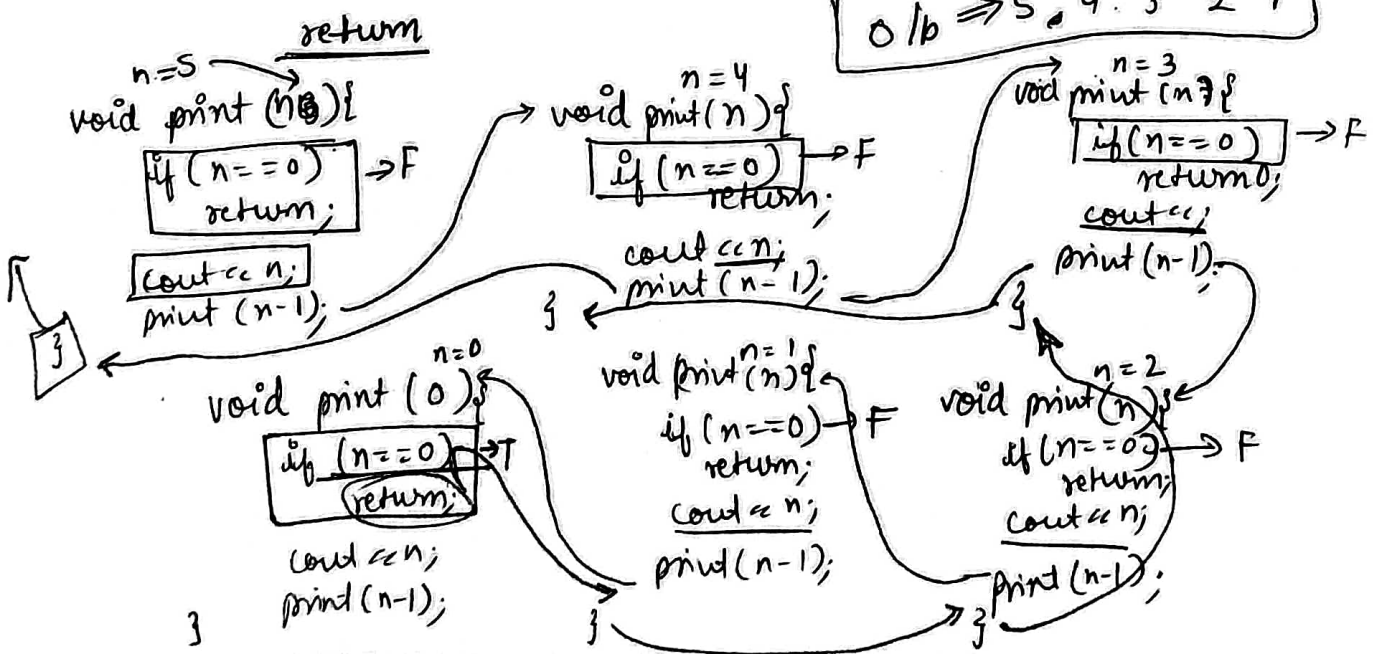
$n = 3$   
 $3 == 0 \rightarrow F$   
 $cout << 3$   
 func call for 2

$n = 2$   
 $2 == 0 \rightarrow F$   
 $cout << 2$   
 func call for 1

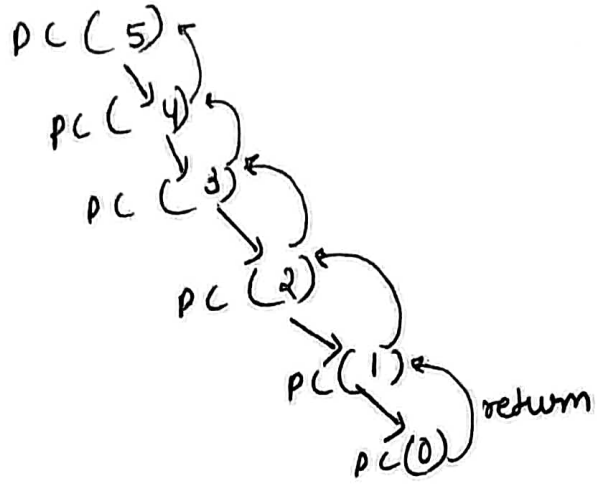
$n = 1$   
 $1 == 0 \rightarrow F$   
 $cout << 1$   
 func call for 0

$n = 0$   
 $0 == 0 \rightarrow T$

o/p  $\Rightarrow 5 \ 4 \ 3 \ 2 \ 1$



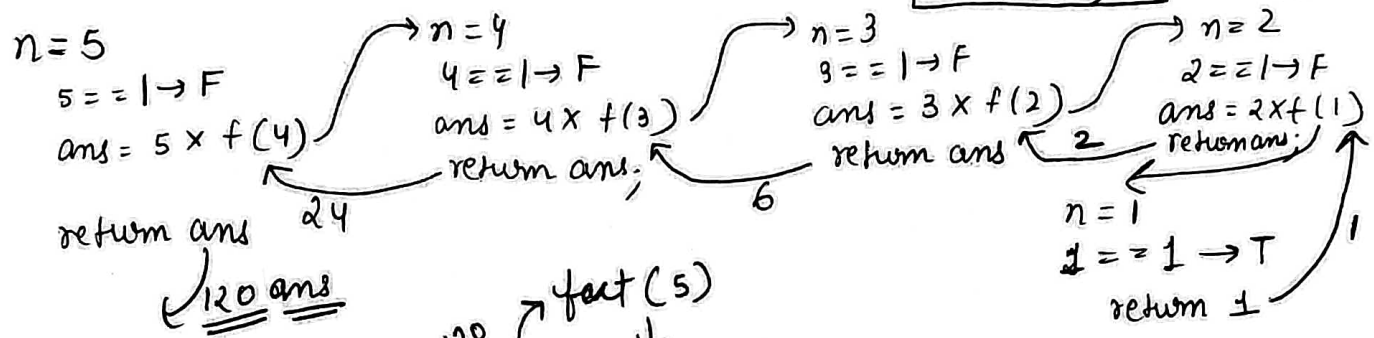
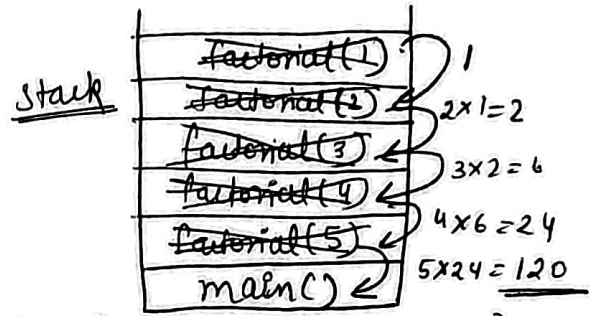
Recurs Tree → n=5



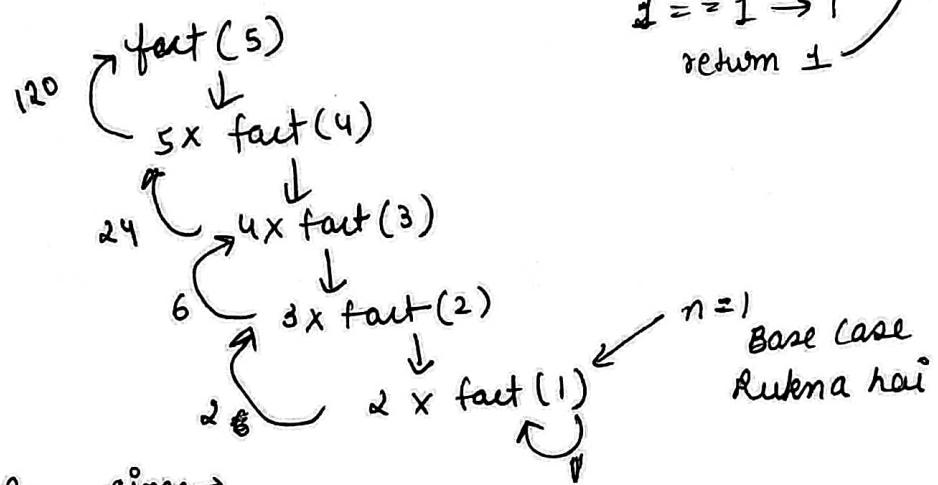
① Factorial →

```

int factorial(int n){
    if(n==1) return 1;
    int ans = n * factorial(n-1);
    return ans;
}
  
```



Recurs Tree → n=5



n=1 Base Case  
Rukna hai

② Head And Tail Recursion →

```

void solve() {
    // B.C
    // Processing
    // R.R
}
  
```

Tail Recursion  
↙  
Recursive relation at the last.

```

void solve() {
    // B.C
    // R.R
    // Processing
}
  
```

Head Recursion  
↓  
Recursive relation before processing.

# ① Counting 1 to n:->

```
void print (int n){
    if (n==0) return;
    print (n-1);
    cout << n;
}
```

i/p → 5

n = 5

5 == 0 → F  
print (n-1)  
cout << n;

n = 4

4 == 0 → F  
print (n-1)  
cout << n

n = 3

3 == 0 → F  
print (n-1)  
cout << n

n = 2

2 == 0 → F  
print (n-1)  
cout << n

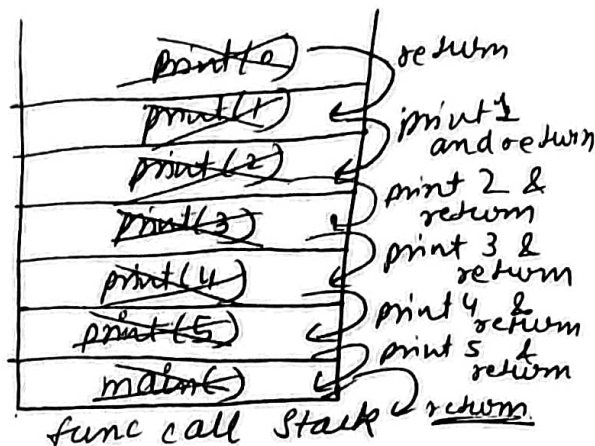
n = 1

1 == 0 → F  
print (n-1)  
cout << n

n = 0

0 == 0 → T  
return

o/p → 1 2 3 4 5



## ② Fibonacci Series

This rule can't be applicable on the first two numbers in the series.

$$n^{th} \text{ term} = (n-1)^{th} \text{ term} + (n-2)^{th} \text{ term}$$

0, 1, 1, 2, 3, 5, 8, ...

Base cases

har ek no. pickle 2 no. ch sum 2 2+1 = 3

```
int fib (int n){
```

```
    if (n==1)
```

```
        return 0;
```

```
    if (n==2)
```

```
        return 1;
```

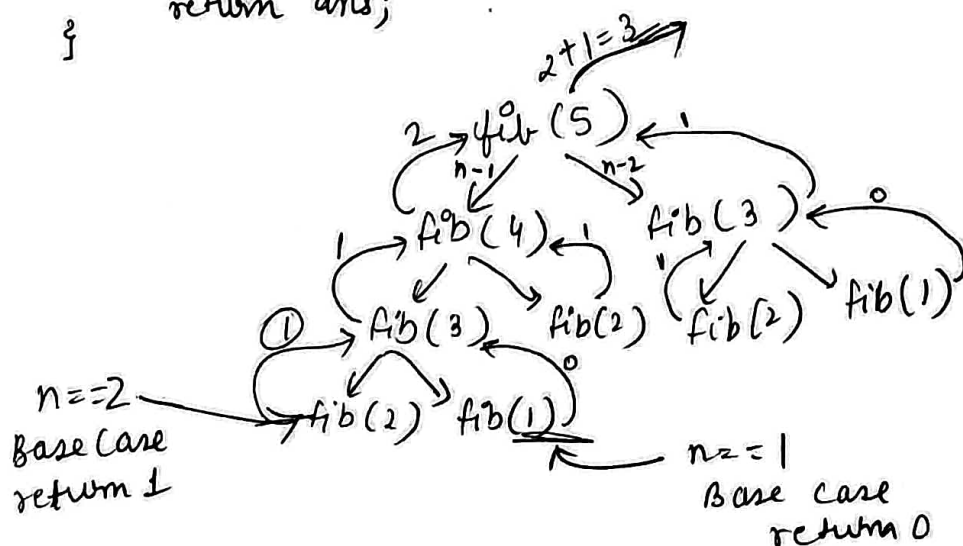
base cases  
(Here considering 0 is the first term and 1 is the second term)

// RR →  $f(n) = f(n-1) + f(n-2)$

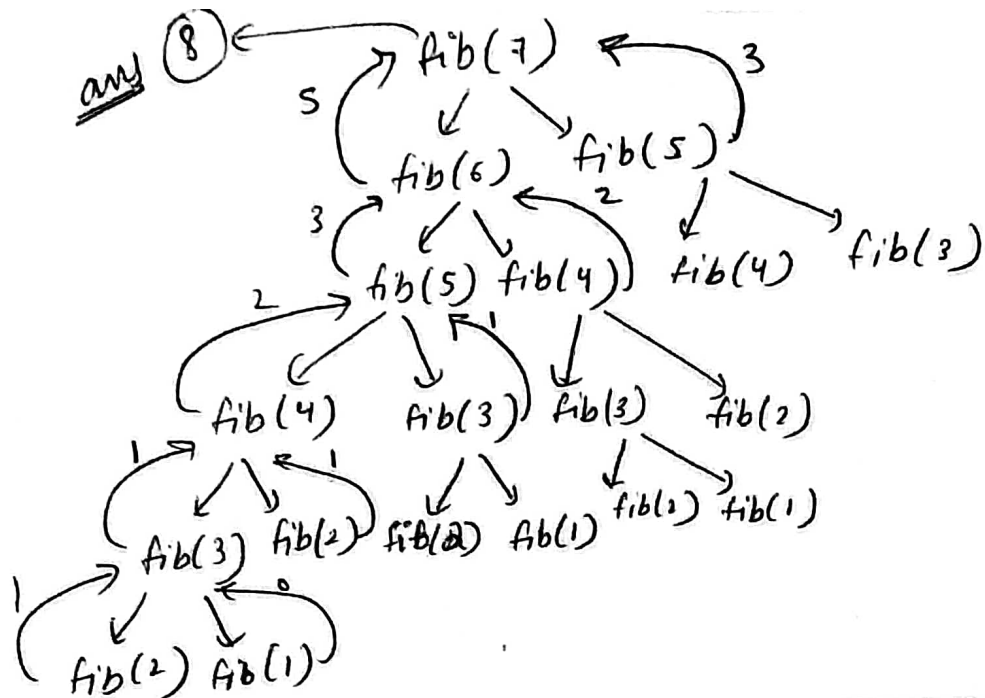
```
int ans = fib(n-1) + fib(n-2);
```

```
return ans;
```

```
}
```



n=7



Magical line  $\rightarrow$  1 case solve karlo baki recursion sambhal lega

for eg  $\rightarrow$  5!

$$5! = \underbrace{5}_{\substack{\downarrow \\ \text{1 case solve} \\ \text{(case will solve)}}} \times \underbrace{4!}_{\text{recursion will solve this.}}$$

$$\text{fib}(n) = \frac{\text{fib}(n-1) + \text{fib}(n-2)}{\substack{\uparrow \text{ we only did this addition} \\ \downarrow \text{ this is done by recursion.}}}$$

Questions  $\rightarrow$

- ① Factorial
- ② Counting (forward and backward)
- ③ Power of 2
- ④ Fibonacci
- ⑤ Climb Stairs (leetcode ques.)
- ⑥ Print all digit of a number.
- ⑦ Max element in a array.
- ⑧ Min element " " "
- ⑨ Find key in " " "
- ⑩ Find Index of key " " "
- ⑪ Find the no. of times key is existing in the array.