A STUDY OF GOLD PRICE AND ITS RELATIONSHIP WITH CONSUMER PRICE INDEX OF CANADA DURING 1980 – 2020

Submitted by

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PROJECT TITLE:

A Study of Gold Price and Its Relationship with Consumer Price Index of Canada During 1980 – 2020

SUMMARY

We know that Gold has an important impact in the socio-economics of any country. If we can predict the future prices of gold that data can be used to make great strategies by the govt. to develop the country's economic status or we can use the data to smartly invest in Gold bonds or even invest directly in gold jewelries. So, my project is based on the study of study of Gold prices and Consumer Price Index(CPI) for the country Canada based on the data we have obtained from the observed values of the two variable in the years 1980 - 2020. We also want to check whether the gold prices are dependent on CPI, and the change in CPI also influences the Gold prices.

I have collected the gold price data of past 40 years (1980 – 2020) from www.kaggle.com and the consumer price index data of the same taking 2015 as the base year from https://fred.stlouisfed.org/ .

Firstly, I have tried to visualize the movement of gold prices and CPI separately w.r.t. time to find any similarities between them (if exists). Also I have drawn a scatter plot diagram of "gold price vs. CPI" directly to get a hint if they have a positive correlation between them. I tried to fit a Quadratic regression model, a Cubic regression model and a 4^{th} degree regression model having Y=Gold price as the dependent variable and X=CPI.

In the next step, I tried to fit a polynomial trend equation(T_t) for each data namely "Gold price – Time" and "CPI – Time". Then I adjusted the yearly trend equation to a monthly trend equation with the origin at month of January, 1980 and used ratio to trend method for calculating Seasonal ratios(S_t) and then the Cyclical variation(C_t) from it.

Finally, I have forecasted the gold prices and CPI for all the months of 2021 using the trend equations and seasonal ratios with the help of the multiplicative model $Y_t = T_t \times S_t \times C_t \times I_t$. I have also forecasted the gold prices of 2021 using the forecasted values of CPI and the regression equation of Gold price on CPI which I have got earlier. Then I compared both forecasts of gold prices and also took notice of the actual gold prices for 2021.

INTRODUCTION

Gold is one of the most highly desired and useful metals in the world. Arguably no other metal has in a higher esteem throughout history. In the ancient times when gold used to a medium of exchange, everyone wanted to own as much gold as they can and people would fight for it, even die for it. We have come a long way and gold has eventually become a metal mostly used to craft jewelries, medals, awards etc. But still gold affects the economy till date, a country is considered economically strong if it has more gold in its treasury.

In 21st century gold is used in various industries such as medical, technology, jewelry, chemical labs etc. From making wedding rings, Oscars, Olympic medals, crucifixes with gold to investing in Gold bonds, directly buying gold bricks or even gold jewelry as an investment strategy, gold affects a person's social life and financial life every day.

In the other hand Consumer price index (CPI) or Cost of living is constructed to study the effect of changes in price of a basket of goods and services on the purchasing power of a particular class of people during that period compared to some base period. Changes in the cost of living of an individual between two periods means the change in his money income which will be necessary for him to maintain the same standard of living in both periods.

Thus the consumer price index numbers are intended to measure the average increase in the cost of maintaining the same standard in a given year as in the base year.

Since the consumption habits of people differ widely from class to class (such as poor, low income, middle income, rich, etc.) and even within the same class from region to region, the changes in the level of prices affect different classes differently and consequently the general price index numbers usually fail to reflect the effects of changes in the general price level on the cost of living of different classes of people. Cost of living index numbers are, therefore compiled, to get a measure of the general price-movement of the people.

The Consumer Price Index(CPI) is given by the following relation:

$$CPI = \frac{\sum_j p_{ij} q_{0j}}{\sum_j p_{0j} q_{0j}} \times 100$$

$$= \frac{Total\ expenditure\ in\ current\ year\ with\ base\ year\ quantities\ as\ weights}{Total\ expenditure\ in\ base\ year} \times 100$$

Where, p_{ij} : $price\ of\ j^{th}\ commodity\ in\ the\ i^{th}\ year,$

q_{0j} : $quantity\ of\ the\ j^{th}\ commodity\ in\ the\ base\ year,$

p_{0j} : $price\ of\ the\ j^{th}\ commodity\ in\ the\ base\ year$

CPI is an indicator of inflation. It measures the percentage change in the price of a basket of goods and services consumed by households. Similarly, the Wholesale Price Index (WPI) measures changes at the wholesale price levels.

To measure inflation, we estimate how much CPI has increased in terms of percentage change over the same period the previous year. If prices have fallen, it is known as deflation.
Economists believe that low, stable, and predictable inflation is good for an economy.

Gold, due to its almost steady character as compared to currency, holds significant value and is used to hedge inflation. This is why investors prefer to hold gold rather than currency. As a result, when the inflation is high, the demand for gold increases and vice versa. The price of gold will then shoot up as a result of high demand from customers. This holds true for both international inflation as well as that which occurs in Canada.

In my project, I have considered to verify if there exist any dependence (interrelation) between the two variable: gold price and CPI, for the country Canada, using the monthly data from January 1980 to December 2020. We will also see that how the two variables have changed over the years by performing a time series analysis for both the variables.

OBJECTIVE OF THE PROJECT

In this Project, we will work with three variables namely Gold Prices per month, CPI per month, and Time which is obtained on a monthly basis. The objective of this Project is as follows:

- **Time series analysis of gold prices, in which we will fit a trend equation of Gold price vs. Time which will be a polynomial equation and forecast the values of gold prices in the future months and also calculate the seasonal variation, and comment on cyclical and irregular variation if there exists.**

- **Time series analysis of CPI, in which we will fit a trend equation of CPI vs. Time which will be a polynomial equation and forecast the values of CPI in the future months and also calculate the seasonal variation, and comment on cyclical and irregular variation if there exists.**

- **Fitting of a polynomial regression equation of Gold price vs. CPI, and see if the forecasts of gold prices, and the forecasts obtained by using the forecasts of CPI in the polynomial regression of Gold price vs. CPI are close to each other, i.e., if CPI predicts the gold prices quite accurately.**

OBTAINED DATA AND ITS DESCRIPTION

In this study, we have three variables namely: Gold Price, CPI, and Time. The Gold Price and CPI are observed simultaneously at a given time point; hence they are time series variables. The data we have obtained is given in months from January 1980 to December 2020. In this study, we have assumed each variable as a continuous variable.

The description of each obtained variable is given below:

- ***GOLD PRICES OF CANADA*:**

We have obtained the prices of gold in Canada on a monthly basis from January 1980 to December 2020. The prices of gold are given in the unit Canadian Dollars per unit ounce

(CAD / oz.). We have denoted gold price as 'Y' in this study.

- ***CONSUMER PRICE INDEX OF CANADA (CPI)*:**

We have obtained the CPI in Canada on a monthly basis from January 1980 to December 2020, taking 2015 as the base year, i.e., the CPI of the year 2015 is assumed to be 100. The CPI in given in Percentages (%). We have denoted CPI as 'X' in this study.

- ***Time*:**

The data on X and Y is obtained per month. We have obtained the data for X and Y for

492 months (or, time points) [$41(years) \times 12(months\ per\ year)$**]. We have denoted the time variable as 't' in this study.**

The obtained data on all the three variable is given below (only first 35 plots are mentioned here, find the complete data under the subheading: "OBTAINED DATA (COMPLETE)" (PAGE NO: 46)

Month	Year	Canada(CAD)	CPI
January	1980	774.1	33.6291452592247
February	1980	732.6	33.9494228331220
March	1980	599.5	34.3497698004938
April	1980	616.1	34.4298391939681
May	1980	615.4	34.8301861613398
June	1980	762.4	35.2305331287116
July	1980	718.1	35.6308800960833
August	1980	733.3	35.9511576699806
September	1980	783.9	36.2714352438780
October	1980	748	36.5917128177754
November	1980	738.7	37.1521985720958
December	1980	703.7	37.2322679655702
January	1981	605.7	37.7126843264162
February	1981	588.6	38.1130312937880
March	1981	609.5	38.5934476546340
April	1981	575.6	38.9937946220057
May	1981	576.1	39.2340028024288
June	1981	505.8	39.7944885567492
July	1981	500.3	40.0346967371722
August	1981	511.2	40.4350437045439
September	1981	514.4	40.7553212784413
October	1981	515.8	40.9955294588643
November	1981	488.9	41.4759458197104
December	1981	475.3	42.1165009675052
January	1982	464.5	42.1965703609795
February	1982	446.2	42.8371255087743
March	1982	394.1	43.1574030826716
April	1982	434.2	43.5577500500434
May	1982	404.8	44.1182358043638
June	1982	404.1	44.5986521652098
July	1982	429.7	44.9189297391072
August	1982	507.4	45.0790685260559
September	1982	487.5	45.3993460999533
October	1982	517.1	45.6395542803763
November	1982	530.3	45.9598318542737

Since our data is quite large, fitting of trend or applying a regression analysis with be quite cumbersome. Hence, we have to modify our data so that we can work with it easily and also obtain meaningful results.

So, we will use the values obtained by yearly averages of X and Y, by doing this we will eliminate the factor of seasonal variation, cyclical variation and irregular variation (random component), and then we can proceed to find the trend equations for X and Y.

Hence, the modified data on yearly averages are as follows:

Year	Avg. CPI	Avg. Gold price
1980	35.43737906	710.4833333
1981	39.8545406	538.9333333
1982	44.45185828	464.2583333
1983	46.90064723	517.125
1984	48.87569227	463.775
1985	51.03089344	438.0416667
1986	53.51971709	511.7416667
1987	55.90845399	592.5416667
1988	58.08367252	532.0416667
1989	60.64589311	448.6666667
1990	63.2481484	447.3916667
1991	67.90551812	411.7916667
1992	69.21331821	416.875
1993	70.20751318	471.0583333
1994	69.17328351	526.5083333
1995	70.41435911	526.4916667
1996	71.57536532	527.3166667
1997	72.56288784	457.4416667
1998	73.69720424	438.4083333
1999	74.8648829	415.1
2000	76.63975445	411.825
2001	66.48246024	473.7847368
2002	80.06939347	491.825
2003	81.99105892	510.9666667
2004	83.56575699	532.025
2005	85.44738774	541.8333333
2006	86.92867152	692.5583333

2007	88.29652365	747.5583333
2008	90.1514646	939.9416667
2009	90.8520718	1111.491667
2010	91.92633616	1274.766667
2011	94.51524655	1546.708333
2012	96.39020484	1675.983333
2013	97.17088143	1439.108333
2014	98.65216521	1389.433333
2015	100	1484.133333
2016	100.8073664	1648.35
2017	101.9617001	1641.525
2018	103.6831921	1643.35
2019	106.0318943	1857.083333
2020	107.0461066	2384.141667

Now, after the modification of the obtained data, we have 41 time points measured yearly. Let us consider a new time variable t_i such that,

$$t_i = i, \forall\, i = 0(1)40$$

Where t_i's represent the time points of the years, taking t_0 as the origin time point which correspond to the year 1980. Where t_i is the time point for the year $(1980 + i)$.

Similarly, for the variables X and Y, we have the following setup:

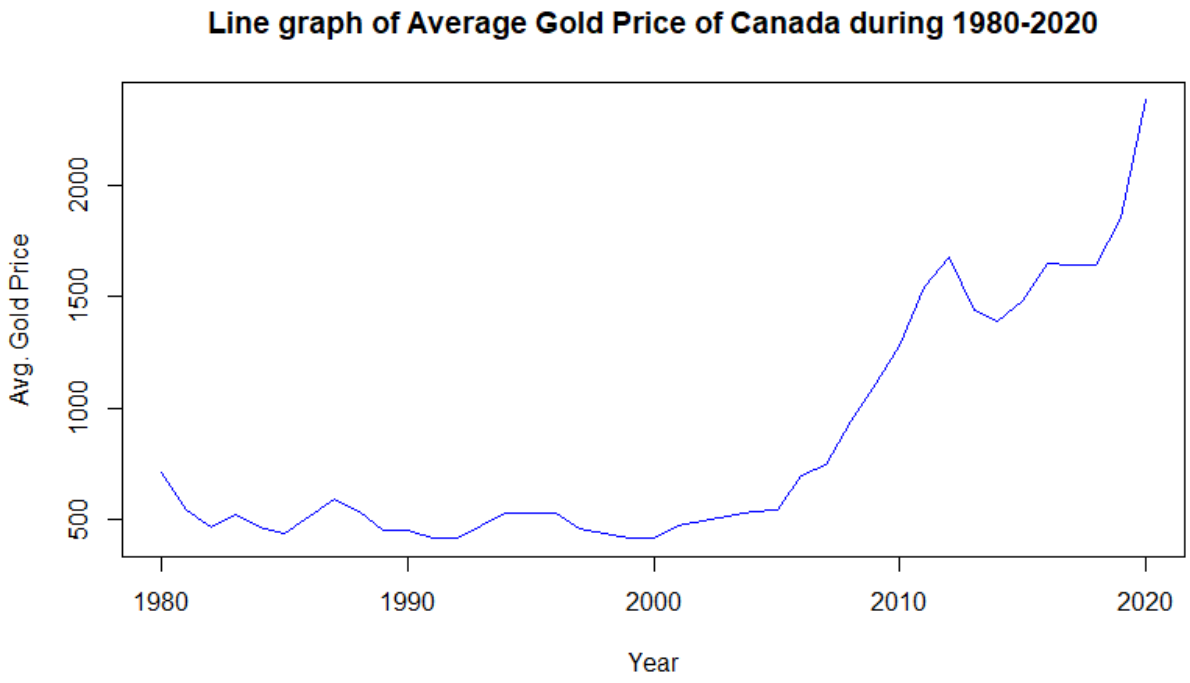
y_i : $Average\ Gold\ Price\ of\ Canada\ for\ the\ year\ t_i\ measured\ in\ ₹/oz$;

x_i : $Average\ CPI\ of\ Canada\ for\ the\ year\ t_i\ measured\ in\ \%$

Where, $i = 0(1)40.$

GRAPHICAL REPRESENTATION OF THE OBTAINED DATA

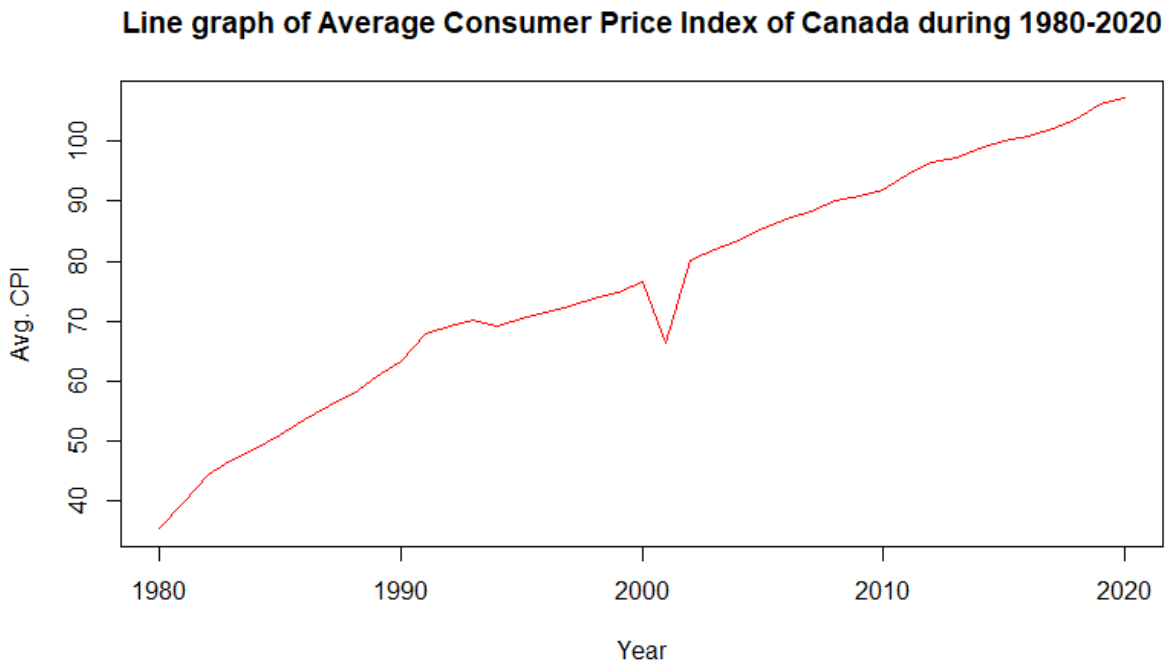
- **AVERAGE GOLD PRICES OF CANADA:**



- ***CONCLUSION:***

We see in the above graphical representation of Gold prices in Canada during 1980-2020, that the gold price seems to follow an increasing trend overall. Till 2005 the gold prices stay almost constant with an average of 500 CAD, after 2005 we see an upwards trend where the gold prices touch 2300 CAD ceiling even after having a price drop in 2013-1015.

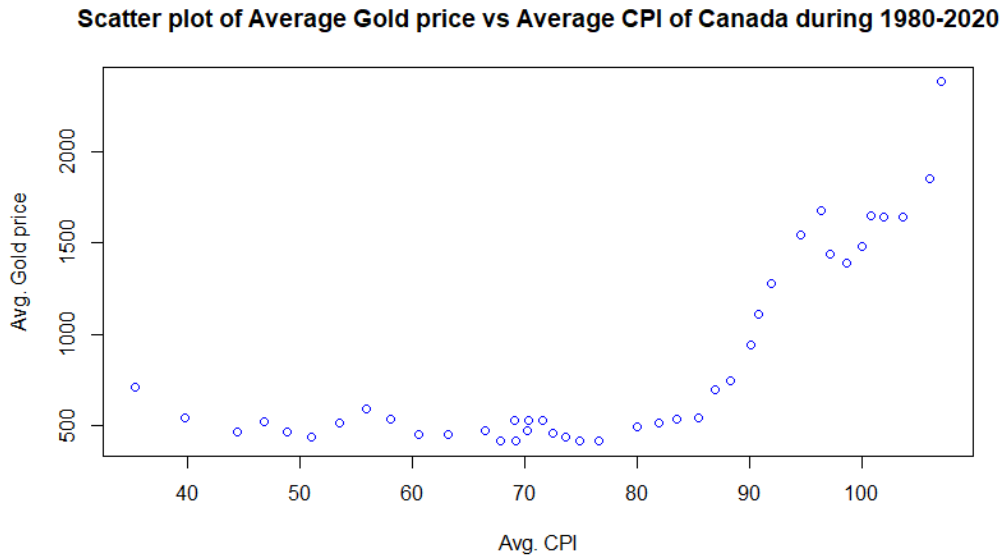
- **AVERAGE CONSUMER PRICE INDEX OF CANADA:**



- ***CONCLUSION:***

We see in the above graphical representation of Consumer Price Index (CPI) in Canada during 1980-2020, that the CPI seems to follow an increasing trend overall. We see an upwards trend where the CPI goes above 100 while having a sudden drop in 2001.

- **AVERAGE GOLD PRICE VS AVERAGE CPI OF CANADA:**



- ***CONCLUSION:***

We see that CPI is more or less directly proportional to the Gold price, i.e., an increase in CPI also results in an Increase in the Gold Prices.

METHODOLOGY

1. *TIME SERIES ANALYSIS OF THE DATA:*

A set of ordered observations of a quantitative variable taken at successive points in time is known as 'Time *Series*'. Such series have a unique important place in the field of Economic and Business Statistics since the data relating to prices, consumption and production of various commodities; money in circulation; bank deposits and bank clearings: sales and profits in departmental store, etc. A time series depicts the relationship between two variables, one of them being time, for example as in this project, the gold price (y_i) of Canada and the different years (t_i).

The various forces at work, affecting the values of a phenomenon in a time series, can be broadly classified into the following four categories, commonly known as the components of a time series, some or all of which are present (in a given time series) in varying degrees. The four components are as follows:

(a) Trend or Secular Trend (or, Long-term Movement)[T_t]

(b) Periodic Changes (or, Short-term Fluctuations) -

(i) Seasonal Variations[S_t](when period of the fluctuation is less than or equal to a year)

(ii) Cyclical Variations[C_t] (when the period of oscillation is more than a year)

(c) Random or Irregular movements[I_t].

In our study, we have analyzed assuming that the various components of the time series follow a multiplicative model, i.e., we can write a particular variable as:

$$y_t = T_t \times S_t \times C_t \times I_t$$

This assumption is appropriate for our study, since we have an economic data, and economic data tends to conform well with a multiplicative model. The same assumption is made for the variable X also.

2. ESTIMATION OF POLYNOMIAL REGRESSION EQUATION BY THE METHOD OF LEAST SQUARES:

Let there be two variables x and y such that they are related. We want to find an equation such that $y = f(x)$, i.e., we want to explain y as a function of x. Here, we consider the following $y : Dependent\ variable,\ \ x : Independent\ variable.$

We have obtained the following bivariate data : $(x_i, y_i)\ , i = 1(1)n,$

Where, $n : Number\ of\ paired\ observations$.

Since we are using Polynomial Regression, we would assume the following function ,i.e., we will assume y as a Polynomial function of x of degree p:

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_p x^p$$

Now, we write the Polynomial Regression model as:

$$y_i = a_0 + a_1 x_i + a_2 x_i^2 + \cdots + a_p x_i^p + \epsilon_i\ ,\ \ i = 1(1)n.$$

Where $\epsilon_i's$ are the error terms.

Now, to find the exact polynomial we will have to find the estimates of $a_0, a_1, a_2, \cdots, a_p$

Such that:

$$S^2 = \sum_{i=1}^{n} \epsilon_i^2\ , is\ minimized\ w.r.t.\ all\ possible\ real\ combinations\ of\ a_0, a_1, a_2, \cdots, a_p\ .$$

We can rewrite S^2 as :

$$S^2 = \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i - a_2 x_i^2 - \cdots - a_p x_i^p\right)^2$$

Now, we treat S^2 as a function of the coefficients $a_0, a_1, a_2, \cdots, a_p$, and minimise w.r.t each of them.

For a_0, we have $\frac{\partial S^2}{\partial a_0} = 0 \Rightarrow (-2)\sum_{i=1}^{n}\left(y_i - a_0 - a_1 x_i - a_2 x_i^2 - \cdots - a_p x_i^p\right) = 0$

$$\Rightarrow \sum_{i=1}^{n} y_i = n a_0 + a_1 \sum_{i=1}^{n} x_i + a_2 \sum_{i=1}^{n} x_i^2 + \cdots + a_p \sum_{i=1}^{n} x_i^p$$

Which is the first normal equation.

Similarly, on partial differentiation of S^2 with $a_k, k = 2(1)p$, we will have :

$$\frac{\partial S^2}{\partial a_k} = 0$$

$$\Rightarrow (-2)\sum_{i=1}^{n}\left(y_i - a_0 - a_1 x_i - a_2 x_i^2 - \cdots - a_p x_i^p\right)\cdot x_i^k = 0$$

$$\Rightarrow \quad a_0\sum_{i=1}^{n} x_i^k + a_1\sum_{i=1}^{n} x_i^{k+1} + \cdots + a_p\sum_{i=1}^{n} x_i^{k+p} = \sum_{i=1}^{n} x_i^k y_i \;; \quad \forall\, k = 2(1)p.$$

Where the above equation represents the k^{th} Normal Equation.

We now write the (p+1) normal equations in matrix form as:

$$\begin{pmatrix} n & \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 & \cdots & \sum_{i=1}^n x_i^{\,p} \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^{\,2} & \sum_{i=1}^n x_i^3 & \cdots & \sum_{i=1}^n x_i^{p+1} \\ \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i^3 & \sum_{i=1}^n x_i^4 & \cdots & \sum_{i=1}^n x_i^{p+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^n x_i^{\,p} & \sum_{i=1}^n x_i^{p+1} & \sum_{i=1}^n x_i^{p+2} & \cdots & \sum_{i=1}^n x_i^{2p} \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \\ \sum_{i=1}^n x_i^2 y_i \\ \vdots \\ \sum_{i=1}^n x_i^{\,p} y_i \end{pmatrix}$$

$$\Longrightarrow A \cdot b = c, (say)$$

Then, $b = A^{-1}.c, when\ A\ is\ non-singular, i.e., \det(A) \neq 0.$

Hence, we can say that the solution to the above matrix equation gives us the estimates of the coefficients. We write the estimates as $\widehat{a_0},\ \widehat{a_1},\ \widehat{a_2}, \cdots, \widehat{a_p}$.

These obtained estimates will be such that

$$S^2(a_0, a_1, a_2, \cdots, a_p) \geq S^2\left(\widehat{a_0},\ \widehat{a_1},\ \widehat{a_2}, \cdots, \widehat{a_p}\right)\ \forall\ (a_0, a_1, a_2, \cdots, a_p) \in \mathbb{R}^p$$

Note that, the $\overline{p+1} \times \overline{p+1}$ Hessian matrix :

$$\begin{pmatrix} \frac{\partial^2 S^2}{\partial a_0^2} & \frac{\partial^2 S^2}{\partial a_0 \partial a_1} & \cdots & \frac{\partial^2 S^2}{\partial a_0 \partial a_p} \\ \frac{\partial^2 S^2}{\partial a_1 \partial a_0} & \frac{\partial^2 S^2}{\partial a_1^2} & \cdots & \frac{\partial^2 S^2}{\partial a_1 \partial a_p} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 S^2}{\partial a_p \partial a_0} & \frac{\partial^2 S^2}{\partial a_p \partial a_1} & \cdots & \frac{\partial^2 S^2}{\partial a_p^2} \end{pmatrix} is\ a\ positive\ semi\ definite\ (p.s.d)\ matrix$$

Hence, the second order condition for the existence of minimum is automatically satisfied. Therefore, the estimates we get on solving the equation $A \cdot b = c$ are the estimates such that S^2 is minimised.

This exact method is used in the estimation of Trend equation of X and Y.

3. CALCULATION OF CORRELATION INDEX (R_p^2) OF A p^{th} DEGREE POLYNOMIAL REGRESSION:

In case of a regression based on a polynomial of p^{th} degree ($p \leq n-1$).

Where there are n observations. Then we can define a measure of association, similar to $|R|$, called the correlation index of the p^{th} order – say, R_p^2 – by:

$$R_p^2 = \frac{Var(Y_p)}{Var(y)} = \frac{\sum_i (Y_{pi} - \overline{Y_p})^2}{\sum_i (y_i - \overline{y})^2}$$

Where Y_{pi} is the predicted value of y as obtained from its p^{th} degree polynomial, its regression equation corresponding to $x = x_i$, and y_i are the obtained values of y.

Where the polynomial equation is:

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_p x^p$$

R_p^2 ranges from 0 to 1, the closer R_p^2 is to 1 the more closely the polynomial fits the given data.

4. REDUCTION OF YEARLY TREND EQUATION TO MONTHLY TREND EQUATION:

Suppose the yearly trend equation based on yearly totals is:

$T_t = a_0 + a_1 t + \cdots + a_p t^p$ ..... (i),
with origin of t at 1980 (for example) and unit 1 year.

To obtain the monthly trend equation, we are to divide the constants by 12 and replace

t by $t/12$. Then the trend equation for monthly values becomes:

$$T_t = \frac{a_0}{12} + \frac{a_1}{12} \cdot \frac{t}{12} + \cdots + \frac{a_p}{12} \cdot \left(\frac{t}{12}\right)^p \quad \cdots\cdots\cdots\cdots (ii)$$

Now, the origin of the above equation is in the middle of 1980. But monthly trend values should correspond to the midpoints of months. For proper centering, the origin should be shifted half a month to the right or to the left. If we want to the right, i.e., to the middle of July, 1980, we have to write $\left(t + \frac{1}{2}\right)$ for t and for left, i.e., to the middle of June, 1980, $\left(t - \frac{1}{2}\right)$ for t in (ii). Note that in this project we want the origin of the monthly trend equation to be in the middle of January, 1980, i.e., for the month of January, 1980, then we have to write $(t - 5.5)$ for t.

Also, note that this reduction is based on the trend equation of yearly totals, but in our project, we have calculated the trend equation for yearly average, hence to get the appropriate monthly trend equation we have to multiply the complete equation by 12.

Finally, we write the appropriate monthly trend equation:

$$T_t = 12 \times \left\{\frac{a_0}{12} + \frac{a_1}{12} \cdot \frac{1}{12} \cdot (t - 5.5) + \cdots + \frac{a_p}{12} \cdot \frac{1}{12^p} \cdot (t - 5.5)^p\right\}$$

$$\Rightarrow \; T_t = a_0 + a_1 \cdot \left(\frac{t - 5.5}{12}\right) + \cdots + a_p \cdot \left(\frac{t - 5.5}{12}\right)^p$$

$$with, origin : \; middle\ of\ January\ 1980, \;\; unit\ of\ t : 1\ month$$

5. *RATIO TO TREND METHOD FOR CALCULATION OF SEASONAL VARIATION:*

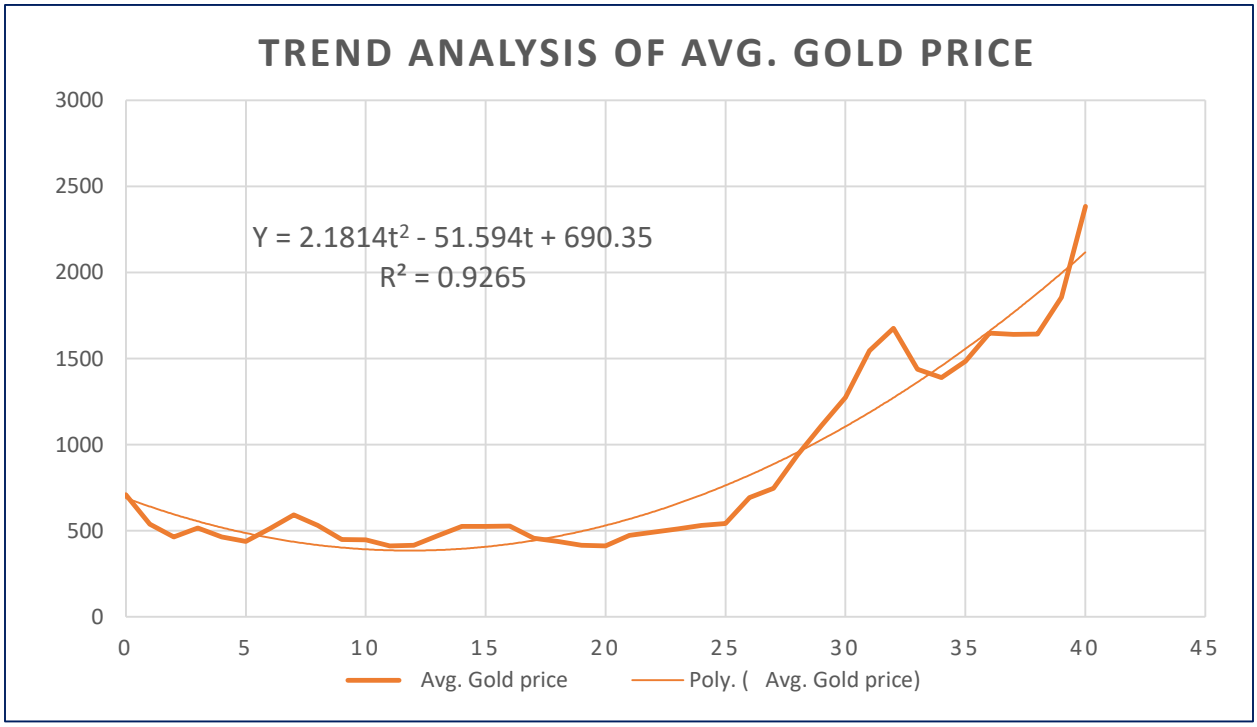
In this method, we first find an appropriate equation to determine trend values for various months. At the next step, we divide the original data month by month by the corresponding trend values and express them as percentages.

The different values for a month are then averaged, as in the previous method. And finally these averages are adjusted to a total of 1200. It may be noted that in this method we are trying to eliminate the irregular and cyclical variations by averaging. So this method is recommended for use either when cyclical variation is known to be absent or when it is not so pronounced even if present.

TIME SERIES ANALYSIS OF GOLD PRICES

- ***Secular Trend Analysis***:

For the Trend analysis of Gold Price, I have fitted a polynomial equation of $2nd$ degree. The graph below shows the trend equation of Y.

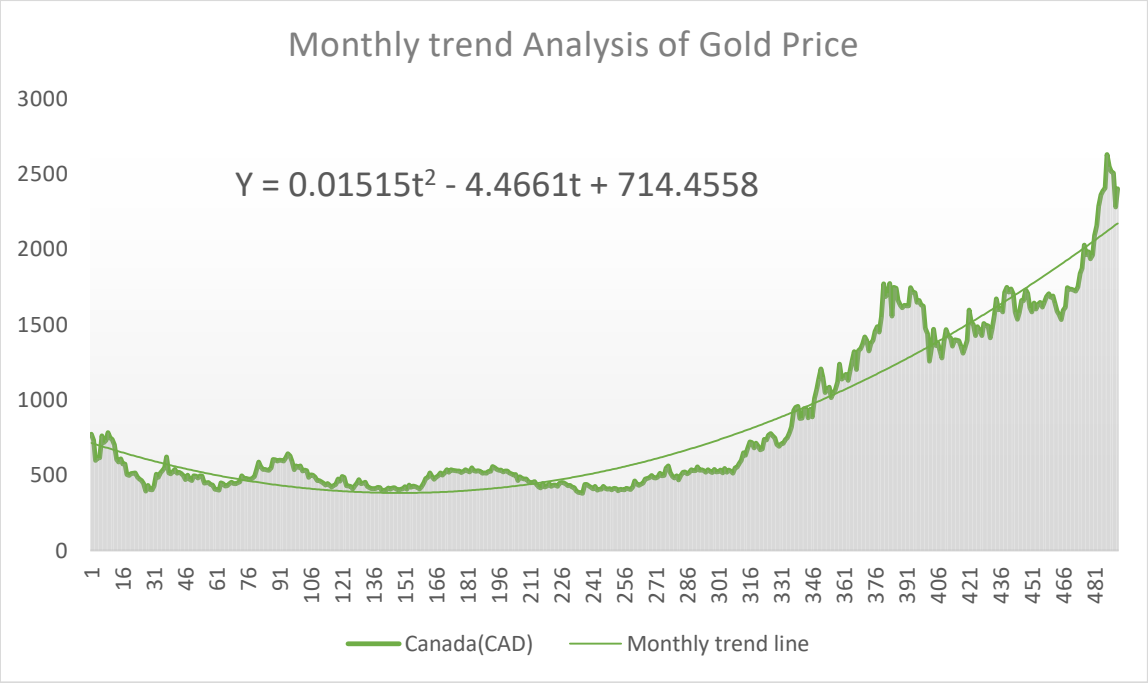


- ***CONCLUSION:***

We see that R^2 = 0.9265, which means the fit is quite appropriate. Though there are a few extreme values (like in year 2012, t=32). This maybe the influence of seasonal and cyclical variation. Overall the trend resembles 2nd degree polynomial quite well.

Now to calculate seasonal variation we need to find out the monthly trend equation for variable Y. Hence, we will apply appropriate adjustments to change the yearly trend equation of Y to monthly trend equation of Y.

The monthly trend equation of variable Y is given below:



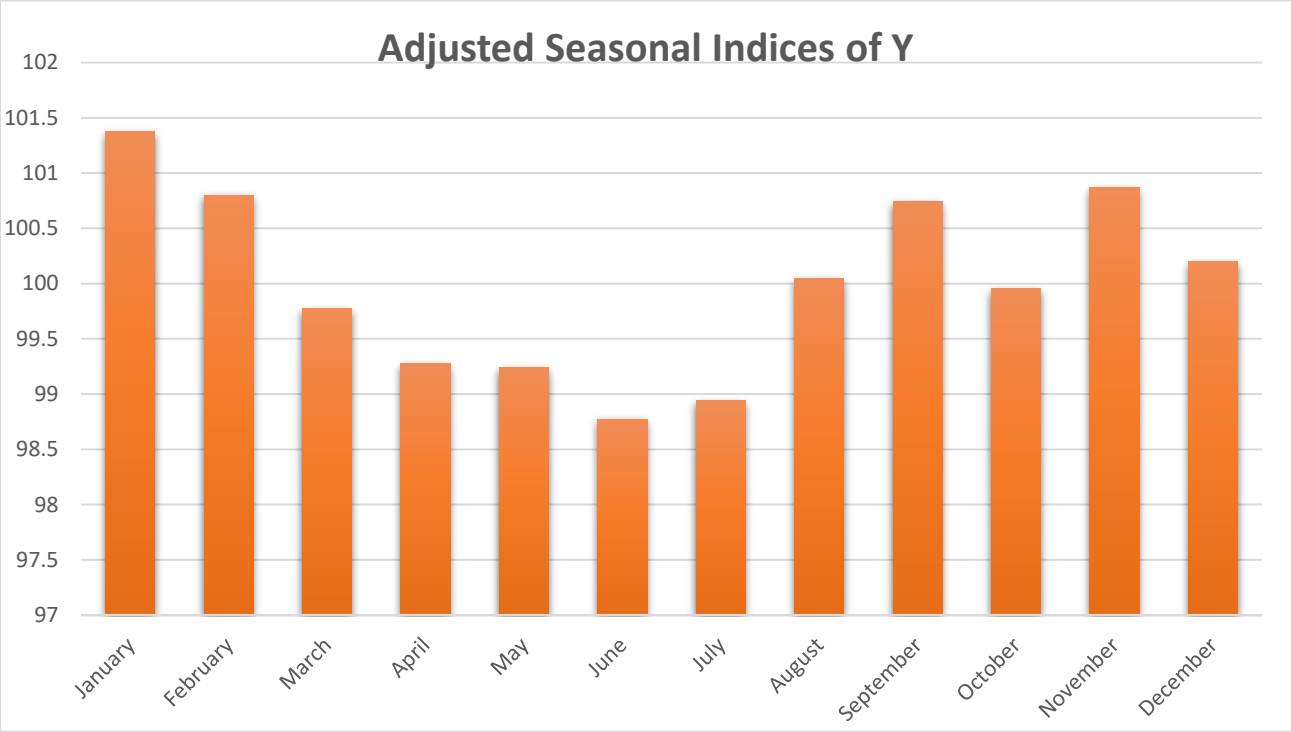
In the above graph, a particular value of $t = t_a$, corresponds to the $(t_a mod(12) + 1)\ th$ month of the year $\left(1980 + \frac{t_a - t_a mod(12)}{12}\right)$. Where the origin of t is at $t = 0$ corresponding to the month of January, 1980.

- ***Seasonal Variation Analysis***:

  Using the Ratio to Trend Method for calculating seasonal indices, we get the following seasonal indices of Gold Prices:

Month	Adjusted Seasonal Indices of Y	Seasonal ratios of Y
January	101.3744969	1.013744969
February	100.801401	1.00801401
March	99.77712686	0.997771269
April	99.27926285	0.992792628
May	99.23945503	0.99239455
June	98.77552071	0.987755207
July	98.94332926	0.989433293

August	100.043577	1.00043577
September	100.7445579	1.007445579
October	99.95583867	0.9995583867
November	100.8705598	1.008705598
December	100.194874	1.00194874

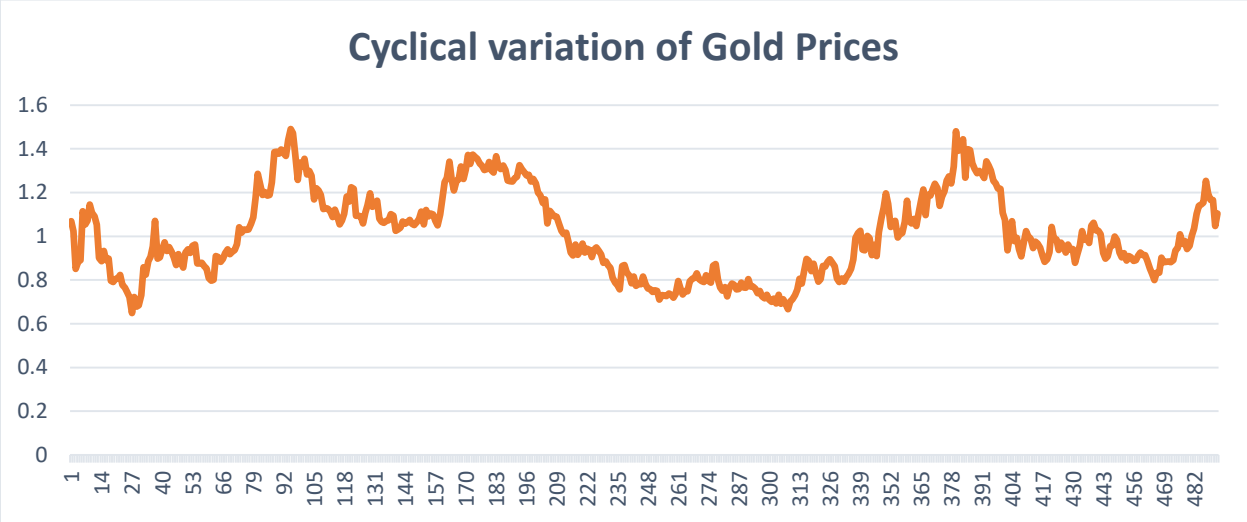


- ***CONCLUSION:***

From the above chart, we can say that gold prices seem to increase during Winter (November to February). This could be due to an increase in demand of gold items to the market, or due to some other factors like occurrence of some special occasions (for e.g., festivals, marriages, etc.). Also we see that the prices tend to be the highest in the months of November & January and lowest during the months of June & July.

- ***Cyclical and Random Variation Analysis:***

Now, we will analyze the Cyclical and Random component of Y graphically. First we have to detrend and remove the seasonal component from the original data, to obtain Cyclical and Random variation, the graph obtained is shown below:



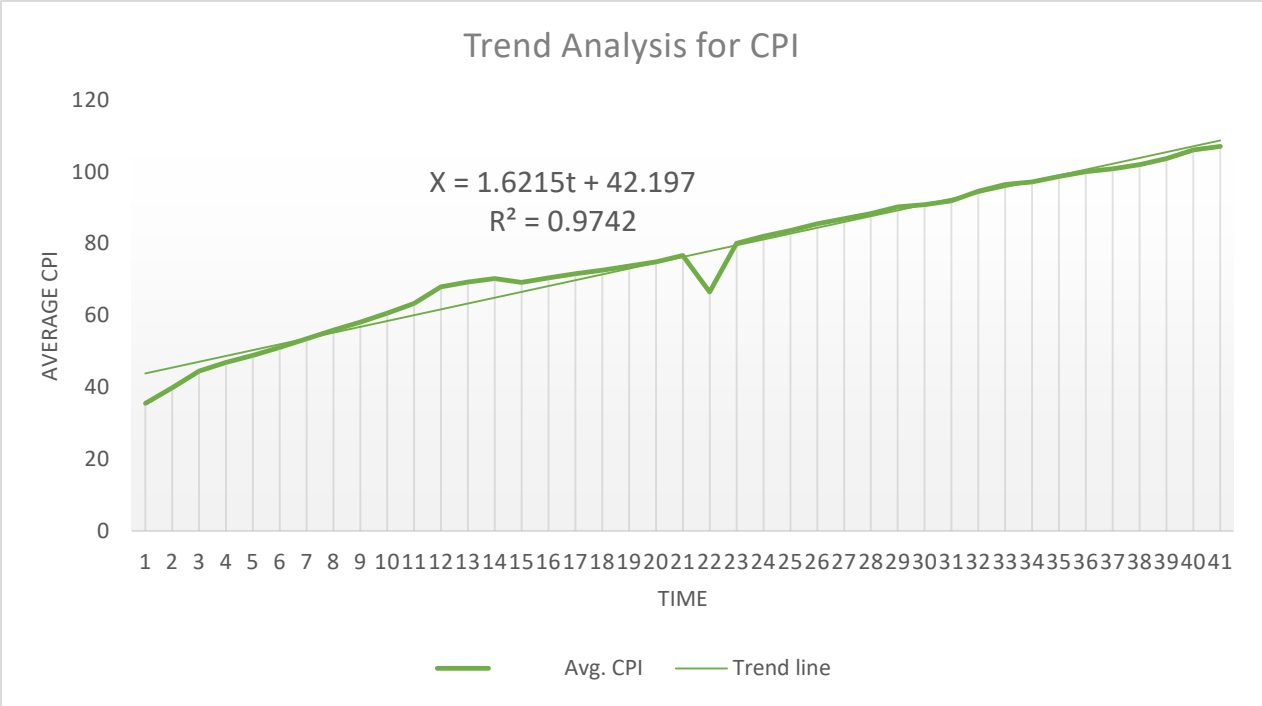
- ***CONCLUSION:***

We can see major Cyclical variation throughout the data of Gold prices. There is a major increase starting from t=29, till t=96. Again it falls at t=117 and remains almost constant up to t=159. Then we see a minor increase and a major decrease after that. This pattern continues till end t=491. We can say there is major random variations in gold prices of Canada.

TIME SERIES ANALYSIS OF CPI

- ***Secular Trend Analysis***:

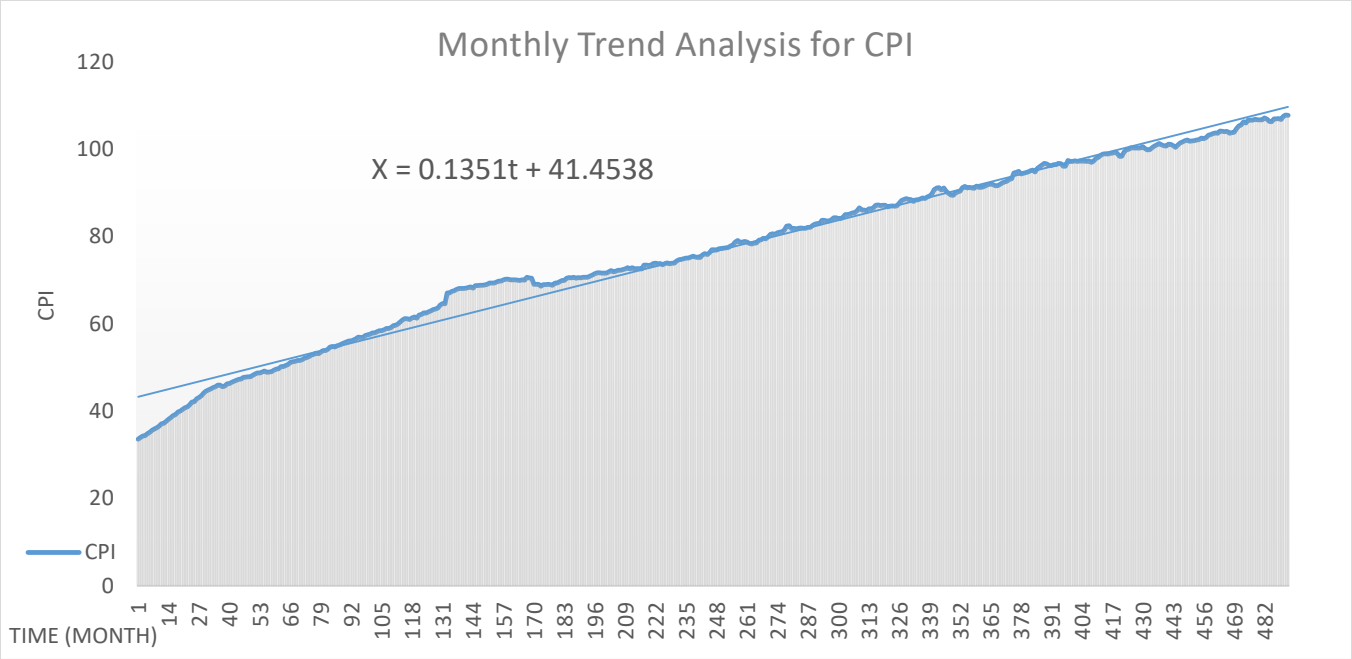
For the Trend analysis of Gold Price, I have fitted a polynomial equation of $1st$ degree. The graph below shows the trend equation of X.



- ***CONCLUSION:***

We see that R^2 = 0.9742, which means the fit is quite appropriate. Though there are a few extreme values (like in year 2002, t=22). This maybe the influence of seasonal and cyclical variation. Overall the trend resembles 1st degree polynomial quite well.

Now to calculate seasonal variation we need to find out the monthly trend equation for variable X. Hence, we will apply appropriate adjustments to change the yearly trend equation of X to monthly trend equation of X.



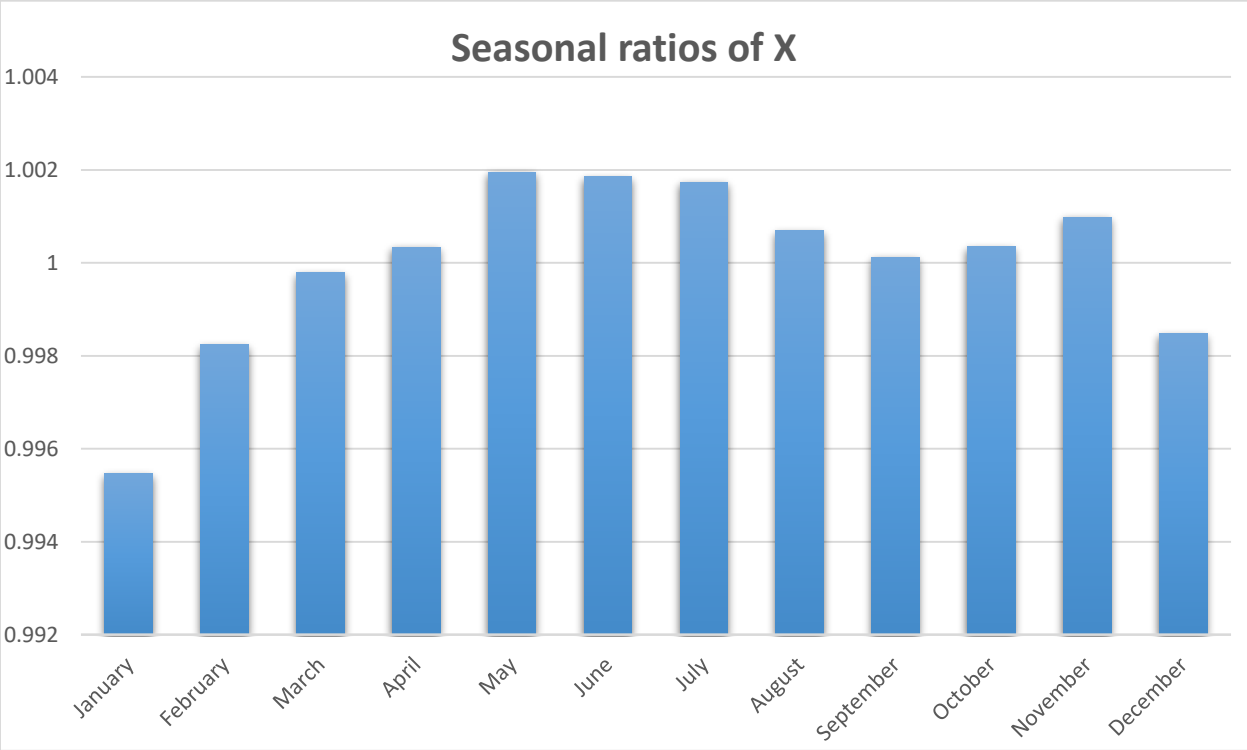
In the above graph, a particular value of $t = t_a$, corresponds to the $(t_a mod(12) + 1)\ th$ month of the year $\left(1980 + \frac{t_a - t_a mod(12)}{12}\right)$. Where the origin of t is at $t = 0$ corresponding to the month of January, 1980.

- ***Seasonal Variation Analysis***:

Using the Ratio to Trend Method for calculating seasonal indices, we get the following seasonal indices of CPI:

Month	Adjusted Seasonal Indices of X	Seasonal ratios of X
January	99.54660032	0.995466003
February	99.82398226	0.998239823
March	99.97975281	0.999797528
April	100.0337851	1.000337851
May	100.1949515	1.001949515
June	100.1852206	1.001852206
July	100.1736231	1.001736231
August	100.0686254	1.000686254
September	100.0122941	1.000122941
October	100.034557	1.00034557

November	100.0987082	1.000987082
December	99.84789967	0.998478997

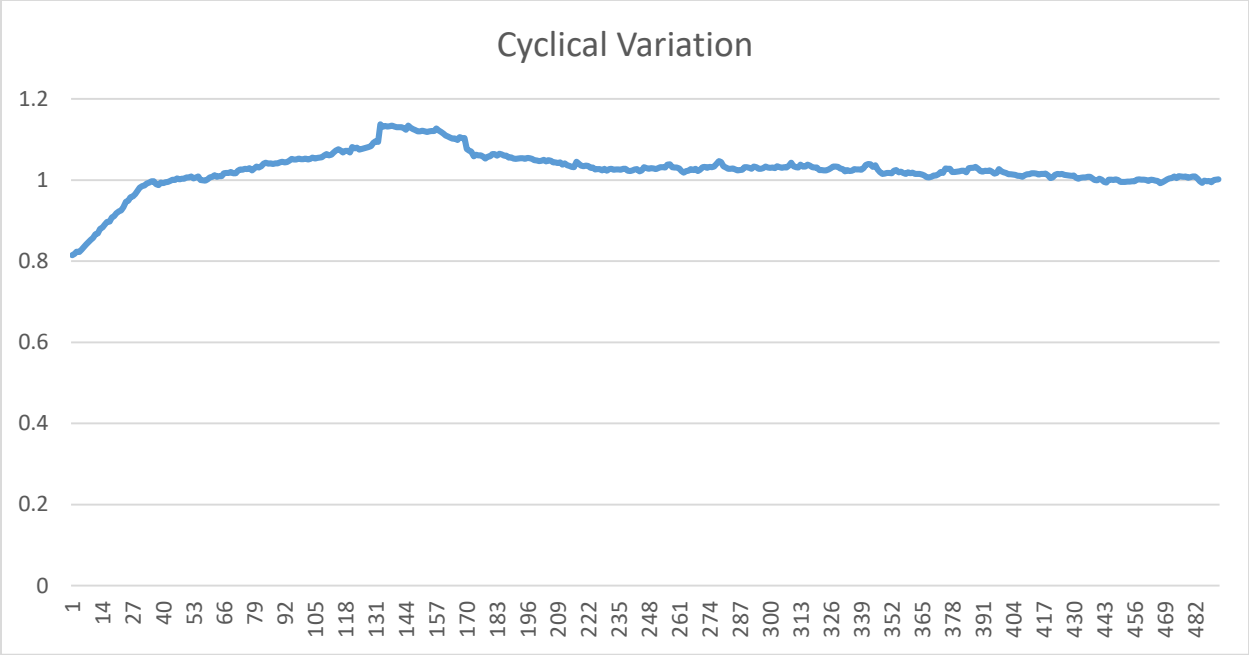


- ***CONCLUSION:***

From the above chart, we can say that CPI seem to increase during April to November. This could be due to the increase of different items in the latter part of the year especially during winter months, due to the increase in demand of some items that are only used in the latter part of the year (for example: winter essentials etc.). Also we see that the prices tend to be the highest in the months of May to July and lowest in the month of January.

- ***Cyclical and Random Variation Analysis:***

Now, we will analyze the Cyclical and Random component of X graphically. First we have to detrend and remove the seasonal component from the original data, to obtain Cyclical and Random variation, the graph obtained is shown below:

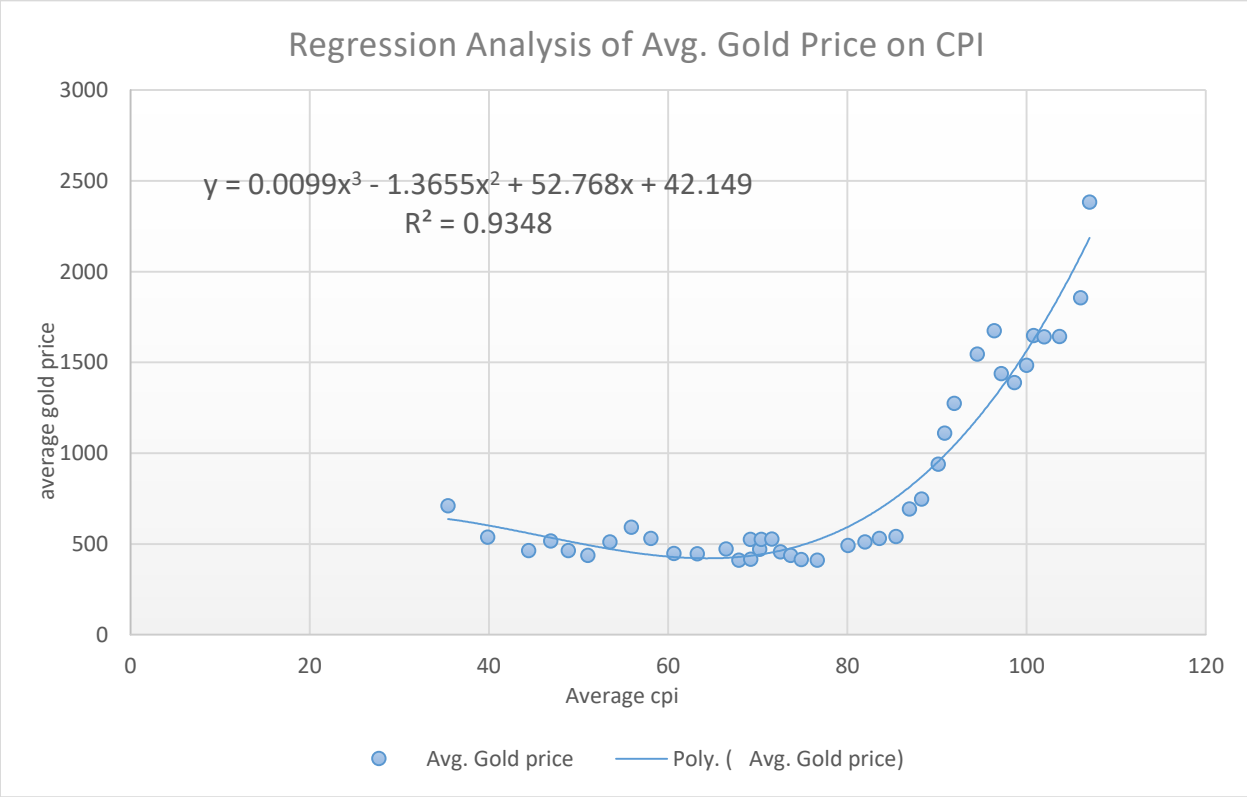


- ***CONCLUSION:***

We see that a minor cyclical variation starts from t=0 and increases up to t=136. Then it again slowly decreases till t=244 and maintains almost constant variation till the last point t=491.
That is, we don't see much fluctuation in Cyclical variation for CPI data.

REGRESSION OF GOLD PRICES ON CPI

For the Regression of Y on X, I have fitted a polynomial equation of 3rd degree. The graph of which is shown below:



- ***CONCLUSION:***

From the Polynomial Regression Equation of Y on X, given in the Fig 10, we can say that R^2 = 0.9348, which is very much close to 1. Also we can see from the graph that 3^{rd} degree polynomial fits our data quite well.

FORCASTS

We want to forecast the values of Gold Price and CPI for the next year, i.e., 2021 for all months.

Firstly, let us calculate forecasts based on our time series analysis. Hence, the table below shows the forecasts of Gold Price and CPI based on our time series analysis.

[Note that – the forecasts using the time series analysis will be calculated using the following equation: $Y_t = T_t \times S_t$.]

Date	Time Point (t)	Forecast of Gold Price Based on Time Series Analysis (y_t)	Forecast of CPI Based on Time Series Analysis (x_t)
January,2021	492	2214.428768	107.4336775
February,2021	493	2212.450486	107.8678986
March,2021	494	2200.432672	108.1712939
April,2021	495	2199.894487	108.3648988
Maay,2021	496	2209.479731	108.674851
June,2021	497	2209.598965	108.7996468
July,2021	498	2223.848883	108.9223866
August,2021	499	2259.221122	108.9434116
September,2021	500	2285.799117	109.0172011
October,2021	501	2278.598146	109.1766151
November,2021	502	2310.27293	109.3818624
December,2021	503	2305.578084	109.2426884

In the above table, the following trend equations are used:

1. $y_t = 0.01515t^2 - 6.2991t + 719.8922$
2. $x_t = 0.1351t + 41.4538$

Next let use the forecasts of CPI to predict the Gold Prices for the year 2021 for all months, using the polynomial regression equation we obtained. The table below shows the obtained forecasts:

Date	Time Point (t)	Forecast of CPI Based on Time Series Analysis (x_t)	Forecast of Gold Price Based on Time Series Analysis (y_t)	Forecast of Gold Price Based on Regression eqn. of Gold price on CPI (Y_x)
January,2021	492	107.4336775	2214.428768	2277.034176
February,2021	493	107.8678986	2212.450486	2291.112411
March,2021	494	108.1712939	2200.432672	2305.257953
April,2021	495	108.3648988	2199.894487	2319.470948
May,2021	496	108.674851	2209.479731	2333.751544
June,2021	497	108.7996468	2209.598965	2348.099886
July,2021	498	108.9223866	2223.848883	2362.51612
August,2021	499	108.9434116	2259.221122	2377.000395
September,2021	500	109.0172011	2285.799117	2391.552855
October,2021	501	109.1766151	2278.598146	2406.173647
November,2021	502	109.3818624	2310.27293	2420.862919
December,2021	503	109.2426884	2305.578084	2435.620815

In the above table, Y_x is calculated with the equation :

$$Y_x = 0.0099x^3 - 1.3655x^2 + 52.768x + 42.149$$

The table below shows a comparison between the forecasted gold price based on Time Series Analysis and forecasted gold price based on Regression eqn. of Gold price on CPI, with the observed Gold prices of every month of 2021:

Date	Forecast of Gold Price Based on Time Series Analysis (CAD)	Forecast of Gold Price Based on Regression eqn. of Gold price on CPI (CAD)	Observed Gold price (CAD)
January,2021	2214.428768	2226.604888	2404.8
February,2021	2212.450486	2271.311565	2391.3
March,2021	2200.432672	2302.959715	2194.9
April,2021	2199.894487	2323.332906	2170
May,2021	2209.479731	2356.238786	2168.9
June,2021	2209.598965	2369.588586	2282.8
July,2021	2223.848883	2382.775139	2207.5
August,2021	2259.221122	2385.039605	2262.4
September,2021	2285.799117	2393.000091	2282.6
October,2021	2278.598146	2410.267472	2227.2
November,2021	2310.27293	2432.639937	2216.2
December,2021	2305.578084	2417.452339	2281

- ***CONCLUSION:***

We can see that The forecast based on Time series Analysis is more accurate than the forecast based on Regression equation of Gold price on CPI. From the above data we can spot two major drawbacks of Regression equation of Gold price on CPI, they are:

(a) The regression equation $Y = 0.0099X^3 - 1.3655X^2 + 52.768X + 42.149$ is monotonically increasing, so the fact that "gold price gets higher during winters" is overlooked.

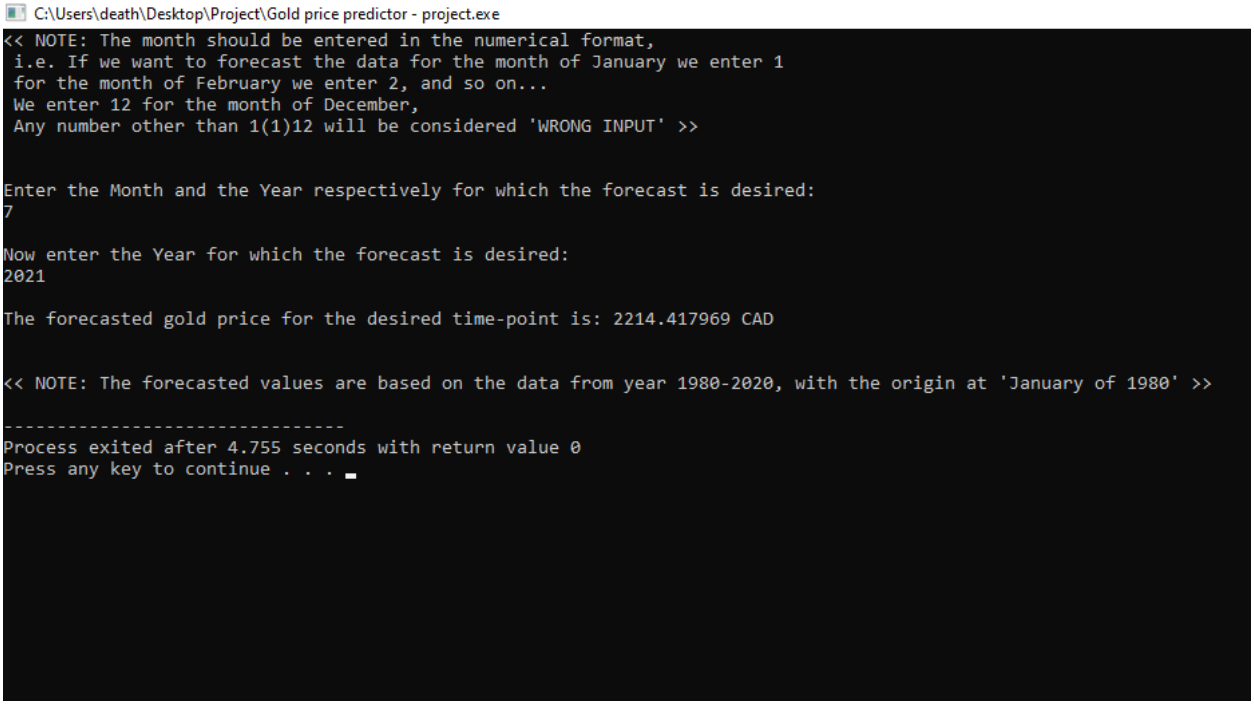
(b) The regression equation has X = CPI as its independent variable, which is also a forecasted value. So the total error is more due to the influence of error of forecast of CPI.

So, we use results got from the Time series Analysis of Gold price of Canada in further studies.

C++ PROGRAM: GOLD PRICE PREDICTOR (FOR CANADA)

A program is created to forecast the gold prices of Canada, using the trend line equation of Consumer Price Index and Regression equation of Gold price on CPI. The c code for the program can be found under the sub-heading "C CODE" (page no: 40)

The forecasted Gold price of "July 2021" is calculated using the C++ program as an example.



Here we can see the forecasted Gold price is 2214.417969 CAD.

CONCLUSION

From the forecasts we made we can say that CPI is a good predictor of Gold Price for Canada. Both the variables are of increasing nature in general. While CPI increases much smoothly Gold Price may show fluctuations due to the influence of some socio-economic phenomenon.

We also saw that Polynomial Equations predicts both the Gold Price and CPI quite well, since in all the cases the R^2 values obtained was quite close to 1, hence the predictions made in future forecasts should be quite accurate unless some calamity occurs due to which both the variables being economic in nature may show a significant variation.

We also see that the estimates y_t and Y_x are quite close to each other at the earlier part of the year 2021 but as the year rolls down, we see a slight drop in the estimates y_t due to seasonal variation, however the Y_x estimates keep increasing.

Also we see a difference between the observed values of 2021 and the estimates, this may happen due to the effect of cyclical and irregular variation. Due to these variations the time series forecasts of Gold Price becomes lower than the forecasts obtained from the regression equation of Gold Price on CPI.

ACKNOWLEDGEMENT

I would like to express my special thanks of gratitude to my college, **Maulana Azad College, Kolkata** for giving me golden opportunity to do this wonderful project on the topic 'A Study of Gold Price and Its Relationship with Consumer Price Index of Canada During 1980 – 2020', in the final semester of my Undergraduate. This project also helped me in doing a lot of research and I come to know about so many new things.

Next, I would like to thank our project supervisor **Mr. Debajit Chatterjee** (Assistant Professor in Department of Statistics, Maulana Azad College), who guided me in doing this project. He provided us with invaluable advice and helped us in difficult periods. His motivation and help contributed tremendously to the successful completion on the project. Finally, I would thank my parents and friends who helped me a lot in finishing this project within the limited time frame.

Chandrapaul Das

(CHANDRAPAUL DAS)

BIBLIOGRAPHY

I used the following websites to make the project:

- www.wikipedia.org
- www.mecmining.com
- www.geeksforgeeks.org
- www.statology.org
- www.stackexchange.org

The books used to develop the theories that are used in the project:

- **Fundamentals of Statistics Vol.1 – A.M. Gun, B.K. Gupta, B. Dasgupta**
- **Fundamentals of Statistics Vol.2 – A.M. Gun, B.K. Gupta, B. Dasgupta**
- **Statistical Tools and Techniques – P.K. Giri, J. Banerjee**
- **Fundamentals of Applied Statistics – S.C. Gupta, V.K. Kapoor**

The data used in this project was obtained from the following websites:

- www.kaggle.com
- https://fred.stlouisfed.org/

All the graphs and calculations done in this project is done using the software *Microsoft Excel* and *R-Studio*.

APPENDIX:

R CODE

```
#Input the variables X, Y and t

X<-c(35.43738,39.85454,44.45186,46.90065,48.87569,51.03089

,53.51972,55.90845,58.08367,60.64589,63.24815,67.90552,69.2133
2,70.20751,69.17328,70.41436,71.57537,72.56289,73.69720,74.864
88,76.63975,66.48246,80.06939,81.99106,83.56576,85.44739,86.92
867,88.29652,90.15146,90.85207,91.92634,94.51525,96.39020,97.1
7088,98.65217,100.00000,100.80737,101.96170,103.68319,106.031
89,107.04611)

Y<-c(710.4833,538.9333,464.2583,517.1250,463.7750,438.0417,

511.7417,592.5417,532.0417,448.6667,447.3917,411.7917,416.8750
,471.0583,526.5083,526.4917,527.3167,457.4417,438.4083,415.100
0,411.8250,473.7847,491.8250,510.9667,532.0250,541.8333,692.55
83,747.5583,939.9417,1111.4917,1274.7667,1546.7083,1675.9833,1
439.1083,1389.4333,1484.1333,1648.3500,1641.5250,1643.3500,18
57.0833,2384.1417)

t<-c(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,

24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41)

#Graphical representation of the data

Plot(t,Y,type="l",main="Line graph of Average Gold Price of Canada
during 1980-2020",xlab="Year",ylab="Avg. Gold price",col='blue')

Plot(t,X,type="l",main="Line graph of Average Consumer Price Index
of Canada during 1980-2020",xlab="Year",ylab="Avg. CPI",col='red')

plot(X,Y,main = "Scatter plot of Average Gold price vs Average CPI of
Canada during 1980-2020",xlab = "Avg. CPI",ylab = "Avg. Gold
price",col='blue')
```

```
#Fitting polynomial equation for Gold Price on CPI data
LM <- lm(formula = Y~X)
summary(LM)
Call:
lm(formula = y ~ x)

Residuals:
   Min     1Q  Median     3Q    Max
-484.40 -274.43  -26.25  240.89  912.31

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -736.628    216.673  -3.400  0.00157 **
x             20.631      2.754   7.492 4.56e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 342.7 on 39 degrees of freedom
Multiple R-squared:  0.5901,    Adjusted R-squared:  0.5795
F-statistic: 56.14 on 1 and 39 DF,  p-value: 4.562e-09
#The linear polynomial eqn. is Y = 20.631X – 736.628
X2 <- X*X
QM <- lm(formula = Y ~ X + X2)
summary(QM)
Call:
lm(formula = y ~ x + x2)
```

```
Residuals:
   Min     1Q  Median     3Q    Max
-289.87 -104.58    8.93   96.62  349.67

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 3225.81377 338.06701   9.542 1.24e-11 ***
x             -94.83445   9.50560  -9.977 3.64e-12 ***
x2              0.78195   0.06381  12.254 9.02e-15 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 156 on 38 degrees of freedom
Multiple R-squared:  0.9172,     Adjusted R-squared:  0.9129
F-statistic: 210.5 on 2 and 38 DF,  p-value: < 2.2e-16
#The 2nd degree polynomial eqn. is Y = 0.78195x2 - 94.8345x + 3225.8138
X3 <- X*X*X
CM <- lm(formula = Y ~ X + X2 + X3)
summary(CM)
Call:
lm(formula = y ~ x + x2 + x3)

Residuals:
   Min     1Q  Median     3Q    Max
-228.46  -78.61  -16.55   74.11  369.44

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)  4.2149e+01  1.053e+03   0.040  0.96827
x             5.2768e+01   4.749e+01   1.111 0.27370
x2           -1.3655e+00   6.821e-01  -2.002  0.05266 .
x3            9.9998e-03   3.133e-03   3.160  0.00314 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 140.3 on 37 degrees of freedom
Multiple R-squared:  0.9348,     Adjusted R-squared:  0.9295
F-statistic: 176.8 on 3 and 37 DF,  p-value: < 2.2e-16
```

#The 3rd degree polynomial eqn. is $Y = 0.0099X^3 - 1.3655X^2 + 52.768X + 42.149$

#The 3rd degree polynomial is the best fit for the data

C CODE

```
#include<stdio.h>
#include<math.h>

main()
{

int m,n,e,t;
float y,G;
printf("<< NOTE: The month should be entered in the numerical format, \n i.e. If we want to forecast the data for the month of January we enter 1 \n for the month of February we enter 2, and so on... \n We enter 12 for the month of December, \n Any number other than 1(1)12 will be considered 'WRONG INPUT' >> \n");
printf("\n \nEnter the Month and the Year respectively for which the forecast is desired: \n");
scanf("%d",&m);
if(m>0)
 {
    if(m<13)
 {
        printf("\nNow enter the Year for which the forecast is desired: \n");
  scanf("%d",&e);
  if(m=1)
  {
   n=m-1;
   t=12*e+n-23760;
   y=0.01515*t*t-4.4661*t+714.4558;
   G=y*1.01374;
```

```
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
  else if(m=2)
   {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*1.00801;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
  else if(m=3)
   {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*0.99777;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
  else if(m=4)
   {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*0.99279;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
```

```
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
  else if(m=5)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*0.99239;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
  else if(m=6)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*0.98776;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
   else if(m=7)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*0.98943;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
```

```
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
   else if(m=8)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*1.00044;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
   else if(m=9)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*1.00745;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
   else if(m=10)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*0.99956;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
```

```
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
   else if(m=11)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*1.00871;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
   else if(m=12)
  {
    n=m-1;
    t=12*e+n-23760;
    y=0.01515*t*t-4.4661*t+714.4558;
    G=y*1.00195;
    printf("\nThe forecasted gold price for the desired time-point is: %f CAD \n",G);
    printf("\n \n<< NOTE: The forecasted values are based on the data from year 1980-2020, with the origin at 'January of 1980' >> \n");
  }
 }
    else printf("\n \nThe entered Month doesn't exist \n");
}
 else printf("\n \nThe entered Month doesn't exist \n");

return 0;

}
```

OBTAINED DATA (COMPLETE)

Month	Year	Canada(CAD)	CPI
January	1980	774.1	33.6291452592247
February	1980	732.6	33.9494228331220
March	1980	599.5	34.3497698004938
April	1980	616.1	34.4298391939681
May	1980	615.4	34.8301861613398
June	1980	762.4	35.2305331287116
July	1980	718.1	35.6308800960833
August	1980	733.3	35.9511576699806
September	1980	783.9	36.2714352438780
October	1980	748	36.5917128177754
November	1980	738.7	37.1521985720958
December	1980	703.7	37.2322679655702
January	1981	605.7	37.7126843264162
February	1981	588.6	38.1130312937880
March	1981	609.5	38.5934476546340
April	1981	575.6	38.9937946220057
May	1981	576.1	39.2340028024288
June	1981	505.8	39.7944885567492
July	1981	500.3	40.0346967371722
August	1981	511.2	40.4350437045439
September	1981	514.4	40.7553212784413
October	1981	515.8	40.9955294588643
November	1981	488.9	41.4759458197104
December	1981	475.3	42.1165009675052
January	1982	464.5	42.1965703609795
February	1982	446.2	42.8371255087743
March	1982	394.1	43.1574030826716
April	1982	434.2	43.5577500500434
May	1982	404.8	44.1182358043638
June	1982	404.1	44.5986521652098
July	1982	429.7	44.9189297391072
August	1982	507.4	45.0790685260559
September	1982	487.5	45.3993460999533
October	1982	517.1	45.6395542803763
November	1982	530.3	45.9598318542737
December	1982	551.2	45.9598318542737

January	1983	622.1	45.6395542803763
February	1983	515.8	45.7996930673250
March	1983	510.7	46.2801094281711
April	1983	525.3	46.3601788216454
May	1983	540.2	46.6804563955428
June	1983	513.3	46.8405951824915
July	1983	521	47.0808033629145
August	1983	511.2	47.3210115433376
September	1983	497.8	47.4010809368119
October	1983	472.6	47.7213585107093
November	1983	501	47.8014279041836
December	1983	474.5	47.8814972976580
January	1984	464.5	47.8814972976580
February	1984	496.4	48.2818442650297
March	1984	496	48.5220524454527
April	1984	482.7	48.7622606258758
May	1984	496.2	48.7622606258758
June	1984	494.7	49.0024688062988
July	1984	448.7	49.2426769867218
August	1984	450.8	48.9223994128244
September	1984	453.2	49.0024688062988
October	1984	438.7	49.0825381997731
November	1984	435.2	49.4028157736705
December	1984	408.2	49.6430239540935
January	1985	404.8	49.7230933475679
February	1985	402.5	50.2035097084140
March	1985	449.7	50.2035097084140
April	1985	443.1	50.4437178888370
May	1985	430	50.6839260692600
June	1985	431.9	51.1643424301061
July	1985	443.5	51.3244812170548
August	1985	454.8	51.4045506105291
September	1985	445.2	51.6447587909522
October	1985	444	51.6447587909522
November	1985	449.8	51.8048975779008
December	1985	457.2	52.1251751517982
January	1986	497.6	52.2853139387469
February	1986	480.4	52.5255221191699
March	1986	480.1	52.8457996930673
April	1986	475	53.0059384800160

May	1986	473.5	53.3262160539134
June	1986	480.8	53.1660772669647
July	1986	492.5	53.5664242343364
August	1986	535.3	53.8867018082338
September	1986	587.9	53.8867018082338
October	1986	557.8	54.2069793821312
November	1986	540	54.6873957429772
December	1986	540	54.8475345299259
January	1987	536.4	54.6873957429772
February	1987	532.8	55.0076733168746
March	1987	549.9	55.1678121038233
April	1987	605.7	55.4080202842464
May	1987	603.9	55.6482284646694
June	1987	594.8	55.8884366450924
July	1987	601.2	56.1286448255154
August	1987	598.3	56.1286448255154
September	1987	595.1	56.2887836124641
October	1987	618.7	56.6090611863615
November	1987	644.2	57.0094081537332
December	1987	629.5	56.9293387602589
January	1988	593.5	56.9293387602589
February	1988	537.6	57.3296857276306
March	1988	563	57.4898245145793
April	1988	551.8	57.6499633015280
May	1988	563.6	57.9702408754254
June	1988	529.3	57.9702408754254
July	1988	535.5	58.2104490558484
August	1988	530.2	58.4506572362714
September	1988	487.2	58.4506572362714
October	1988	502.8	58.6908654166945
November	1988	501.5	58.9310735971175
December	1988	488.5	58.9310735971175
January	1989	466.3	59.1712817775405
February	1989	464.2	59.6516981383866
March	1989	456.5	59.7317675318610
April	1989	448.1	59.9719757122840
May	1989	436.5	60.5324614666044
June	1989	446.7	60.9328084339761
July	1989	433.6	61.2530860078735
August	1989	423	61.1730166143992
September	1989	432.3	61.0128778274505

October	1989	440.1	61.4132247948222
November	1989	475.1	61.5733635817709
December	1989	461.6	61.3331554013478
January	1990	492	62.0537799426169
February	1990	485.7	62.2139187295656
March	1990	431	62.5341963034630
April	1990	428.3	62.4541269099887
May	1990	426.2	62.7744044838860
June	1990	410.7	63.0146126643091
July	1990	429.8	63.2548208447321
August	1990	447.7	63.4149596316808
September	1990	471.4	63.6551678121038
October	1990	443.1	64.2957229598986
November	1990	449	64.6960699272703
December	1990	453.8	64.6160005337960
January	1991	426	67.0981517315006
February	1991	417.5	67.0981517315006
March	1991	411.3	67.4984986988724
April	1991	412.1	67.5785680923467
May	1991	412.8	67.8988456662441
June	1991	420.9	68.1390538466671
July	1991	418.5	68.1390538466671
August	1991	396.2	68.1390538466671
September	1991	401.2	68.2191232401415
October	1991	401.1	68.3792620270902
November	1991	415.4	68.4593314205645
December	1991	408.5	68.2191232401415
January	1992	415.5	68.7796089944619
February	1992	417.5	68.7796089944619
March	1992	406.8	68.8596783879362
April	1992	402	68.8596783879362
May	1992	406.9	68.9397477814106
June	1992	410.8	69.0998865683592
July	1992	424.6	69.3400947487823
August	1992	406.8	69.3400947487823
September	1992	435.5	69.3400947487823
October	1992	421	69.5803029292053
November	1992	429.8	69.8205111096284
December	1992	425.3	69.8205111096284
January	1993	419.5	70.1407886835257
February	1993	409.2	70.2208580770001

March	1993	424.5	70.2208580770001
April	1993	450.8	70.1407886835257
May	1993	479.9	70.1407886835257
June	1993	485.5	70.1407886835257
July	1993	515.4	70.0607192900514
August	1993	490.7	69.9806498965770
September	1993	474.8	70.1407886835257
October	1993	488.8	70.0607192900514
November	1993	496.5	70.7012744378461
December	1993	517.1	70.5411356508974
January	1994	501.2	70.4610662574231
February	1994	515.9	69.0998865683592
March	1994	538.1	69.0998865683592
April	1994	520.1	69.0998865683592
May	1994	537.4	68.6194702075132
June	1994	532.3	69.0198171748849
July	1994	530.7	69.0198171748849
August	1994	529.1	69.0998865683592
September	1994	529.6	69.0198171748849
October	1994	519.2	68.8596783879362
November	1994	526.9	69.3400947487823
December	1994	537.6	69.3400947487823
January	1995	529.9	69.6603723226796
February	1995	523.7	69.9806498965770
March	1995	549.9	69.9806498965770
April	1995	530.4	70.4610662574231
May	1995	526.1	70.6212050443718
June	1995	531.8	70.5411356508974
July	1995	526.1	70.7012744378461
August	1995	513.6	70.4610662574231
September	1995	517.3	70.6212050443718
October	1995	514.7	70.5411356508974
November	1995	526.6	70.7012744378461
December	1995	527.8	70.7012744378461
January	1996	558	70.7012744378461
February	1996	549.6	71.0215520117435
March	1996	540	71.1816907986922
April	1996	532.8	71.5019683725896
May	1996	535.5	71.7421765530126
June	1996	521.7	71.7421765530126
July	1996	529.6	71.6621071595383

August	1996	528.8	71.6621071595383
September	1996	516.3	71.6621071595383
October	1996	509	71.9023153399613
November	1996	500.4	72.2225929138587
December	1996	506.1	71.9023153399613
January	1997	465.8	72.0624541269100
February	1997	490.2	72.3026623073330
March	1997	481	72.3026623073330
April	1997	475.4	72.4628010942817
May	1997	476.7	72.6229398812304
June	1997	461.5	72.8631480616534
July	1997	451.1	72.6229398812304
August	1997	451.8	72.8631480616534
September	1997	459.2	72.6229398812304
October	1997	438.8	72.7030092747047
November	1997	422.5	72.7030092747047
December	1997	415.3	72.6229398812304
January	1998	445.6	73.5037032094482
February	1998	423.3	73.5037032094482
March	1998	428.1	73.4236338159738
April	1998	444.4	73.5037032094482
May	1998	427.2	73.8239807833456
June	1998	435.7	73.9040501768199
July	1998	435.4	73.7439113898712
August	1998	427.7	73.8239807833456
September	1998	448.5	73.5837726029225
October	1998	452.2	73.8239807833456
November	1998	450.7	73.9841195702942
December	1998	442.1	73.7439113898712
January	1999	431.2	73.9040501768199
February	1999	433.3	73.9040501768199
March	1999	422.8	74.4645359311403
April	1999	417.4	74.7047441115634
May	1999	396.6	74.7848135050377
June	1999	386.3	74.9449522919864
July	1999	383.6	75.1050910789351
August	1999	380.2	75.1050910789351
September	1999	439.4	75.3452992593581
October	1999	440	75.5054380463068
November	1999	429.2	75.3452992593581
December	1999	421.2	75.2652298658838

January	2000	410.5	75.2652298658838
February	2000	426.1	75.8257156202042
March	2000	401.8	76.1459931941016
April	2000	407.5	75.9057850136785
May	2000	407.8	76.3061319810502
June	2000	427	77.0267565223194
July	2000	411.2	77.0267565223194
August	2000	408.5	76.9466871288450
September	2000	411.7	77.1868953092680
October	2000	404	77.2669647027424
November	2000	413.6	77.3470340962167
December	2000	412.2	77.4271034896911
January	2001	397.2	77.5071728831654
February	2001	408.3	77.9075198505371
March	2001	405.8	78.0676586374858
April	2001	404.3	78.7882831787549
May	2001	413.6	79.1085607526523
June	2001	410.7	78.7082137852806
July	2001	406.4	78.7882831787549
August	2001	423.2	78.8683525722292
September	2001	463	78.7882831787549
October	2001	442.3	78.4680056048575
November	2001	433	78.3078668179089
December	2001	441.4	78.5480749983319
January	2002	448.9	78.5480749983319
February	2002	476.3	79.1886301461266
March	2002	480.9	79.2686995396010
April	2002	483.8	79.6690465069727
May	2002	499	79.5089077200240
June	2002	484.4	79.9092546873958
July	2002	482.7	80.5498098351905
August	2002	488	80.6298792286648
September	2002	513.5	80.5498098351905
October	2002	496.1	80.8700874090879
November	2002	499.8	81.0302261960366
December	2002	548.5	81.1102955895109
January	2003	562.9	81.5106425568826
February	2003	517.6	82.3914058851004
March	2003	492.6	82.4714752785748
April	2003	483.1	81.8309201307800
May	2003	496.3	81.9109895242543

June	2003	470.1	81.7508507373056
July	2003	499.6	81.9109895242543
August	2003	521	81.9910589177287
September	2003	523.6	81.9109895242543
October	2003	509.1	81.9109895242543
November	2003	517.8	82.1511977046774
December	2003	537.9	82.1511977046774
January	2004	531.8	82.5515446720491
February	2004	531.1	82.8718222459465
March	2004	556.9	83.0319610328952
April	2004	534.3	83.0319610328952
May	2004	536.6	83.7525855741643
June	2004	530.7	83.7525855741643
July	2004	520.2	83.5924467872156
August	2004	536.6	83.5924467872156
September	2004	526	83.8326549676386
October	2004	519.1	84.3130713284847
November	2004	539.1	84.3130713284847
December	2004	521.9	84.1529325415360
January	2005	523.8	84.1529325415360
February	2005	535.3	84.3931407219590
March	2005	517.2	85.0336958697538
April	2005	546.8	85.0336958697538
May	2005	519.8	85.1938346567025
June	2005	535.4	85.4340428371255
July	2005	525.9	85.5141122305999
August	2005	514.7	85.9144591979716
September	2005	549.3	86.6350837392407
October	2005	555.3	86.0745979849203
November	2005	579.2	86.0745979849203
December	2005	599.3	85.9144591979716
January	2006	650.3	86.3948755588176
February	2006	632.7	86.3948755588176
March	2006	679	86.6350837392407
April	2006	721.6	87.1955694935611
May	2006	718.9	87.2756388870354
June	2006	682.3	87.1155001000868
July	2006	714.9	87.1955694935611
August	2006	691.9	87.1955694935611
September	2006	668.6	86.7952225261894
October	2006	676.5	86.9553613131381

November	2006	738.5	87.0354307066124
December	2006	735.5	86.9553613131381
January	2007	767.9	87.0354307066124
February	2007	778.8	87.6759858544071
March	2007	763.4	88.3165410022019
April	2007	748.2	88.4766797891506
May	2007	704.6	88.7168879695736
June	2007	692	88.4766797891506
July	2007	710.6	88.4766797891506
August	2007	710.9	88.0763328217789
September	2007	738.3	88.3966103956762
October	2007	749.9	88.3966103956762
November	2007	783.2	88.6368185760993
December	2007	822.9	88.8770267565223
January	2008	930.8	88.7168879695736
February	2008	951.5	89.1172349369454
March	2008	957.8	89.2773737238940
April	2008	877.2	89.8378594782145
May	2008	880.2	90.7986921999066
June	2008	943.8	91.1189697738040
July	2008	941.5	91.1990391672783
August	2008	882.9	90.7186228064322
September	2008	940.4	91.1189697738040
October	2008	888.5	90.3983452325349
November	2008	1011	89.9979982651631
December	2008	1073.7	89.5175819043171
January	2009	1140	89.4375125108427
February	2009	1207.9	89.9979982651631
March	2009	1153.1	90.2382064455862
April	2009	1049.4	90.3983452325349
May	2009	1069	91.1990391672783
June	2009	1085.3	91.5193167411757
July	2009	1015.2	91.1189697738040
August	2009	1049.4	91.2791085607526
September	2009	1068.7	91.1189697738040
October	2009	1120.3	91.0389003803296
November	2009	1239.5	91.5993861346500
December	2009	1140.1	91.2791085607526
January	2010	1149.1	91.2791085607526
February	2010	1171.4	91.4392473477014
March	2010	1131.7	91.6794555281244

April	2010	1194.4	91.9196637085474
May	2010	1267.1	91.9997331020218
June	2010	1321.9	91.9196637085474
July	2010	1204	91.6794555281244
August	2010	1328.7	91.6794555281244
September	2010	1340.8	91.9196637085474
October	2010	1370.5	92.3200106759191
November	2010	1421	92.5602188563422
December	2010	1396.6	92.7203576432908
January	2011	1327.3	93.1207046106626
February	2011	1375.1	93.2808433976113
March	2011	1399.8	94.5619536932008
April	2011	1457	94.7220924801495
May	2011	1488.6	94.9623006605725
June	2011	1453	94.4018149062521
July	2011	1553.8	94.4818842997264
August	2011	1773.3	94.6420230866751
September	2011	1688.1	94.8021618736238
October	2011	1710.4	95.1224394475212
November	2011	1775.2	95.2825782344699
December	2011	1558.9	94.8021618736238
January	2012	1749	95.6028558083672
February	2012	1743.4	96.0832721692133
March	2012	1661.6	96.4035497431107
April	2012	1631.4	96.8038967104824
May	2012	1613.8	96.6437579235337
June	2012	1629.6	96.2434109561620
July	2012	1625.7	96.2434109561620
August	2012	1626.8	96.4836191365850
September	2012	1747.5	96.4836191365850
October	2012	1718.1	96.8038967104824
November	2012	1714.5	96.7238273170081
December	2012	1650.4	96.1633415626876
January	2013	1662.8	96.1633415626876
February	2013	1633.1	97.4444518582772
March	2013	1623.7	97.2042436778542
April	2013	1477.6	97.2042436778542
May	2013	1441.3	97.3643824648028
June	2013	1257.5	97.2042436778542
July	2013	1351.4	97.2843130713285
August	2013	1471.3	97.2843130713285

September	2013	1363.5	97.2843130713285
October	2013	1380.7	97.2042436778542
November	2013	1326.6	97.3643824648028
December	2013	1279.8	97.0441048909054
January	2014	1394.8	97.2843130713285
February	2014	1468.4	97.9248682191232
March	2014	1425.8	98.1650763995463
April	2014	1414.1	98.6454927603924
May	2014	1357.9	98.8857009408154
June	2014	1400.7	98.9657703342897
July	2014	1399.2	98.8857009408154
August	2014	1394.7	99.0458397277641
September	2014	1359.5	99.1259091212384
October	2014	1312.6	99.3661173016614
November	2014	1348.6	99.1259091212384
December	2014	1396.9	98.4052845799693
January	2015	1597.9	98.4052845799693
February	2015	1515.9	99.3661173016614
March	2015	1503.3	99.9266030559819
April	2015	1429.6	100.0066724494560
May	2015	1487.2	100.4070194168280
June	2015	1461.9	100.3269500233540
July	2015	1429	100.3269500233540
August	2015	1510.1	100.3269500233540
September	2015	1493.5	100.3269500233540
October	2015	1494.6	100.5671582037770
November	2015	1414.1	100.1668112364050
December	2015	1472.5	99.8465336625075
January	2016	1564.6	99.9266030559819
February	2016	1672.8	100.4070194168280
March	2016	1600	100.7272969907250
April	2016	1610.4	101.0475745646230
May	2016	1585.6	101.3678521385200
June	2016	1715.3	101.0475745646230
July	2016	1750	100.8073663842000
August	2016	1717.8	100.7272969907250
September	2016	1738.1	101.2077133515710
October	2016	1704.8	101.1276439580970
November	2016	1584.1	100.8073663842000
December	2016	1536.7	100.4870888103020
January	2017	1581.2	101.0475745646230

February	2017	1663	101.4479215319940
March	2017	1660.2	101.6881297124170
April	2017	1731.6	102.0084072863150
May	2017	1710.3	102.1685460732630
June	2017	1613.3	101.8482684993660
July	2017	1585.1	101.9283378928400
August	2017	1644.6	102.0084072863150
September	2017	1604.7	102.1685460732630
October	2017	1637.4	102.2486154667380
November	2017	1649.3	102.5688930406350
December	2017	1617.6	102.4087542536870
January	2018	1650.2	102.6489624341100
February	2018	1688.6	103.2094481884300
March	2018	1706.8	103.3695869753790
April	2018	1683.5	103.6097951558020
May	2018	1693.3	103.7699339427500
June	2018	1644.9	103.6898645492760
July	2018	1589.7	104.1702809101220
August	2018	1567.9	104.0902115166480
September	2018	1534.6	104.0101421231730
October	2018	1595.5	104.0902115166480
November	2018	1618.3	103.6898645492760
December	2018	1746.9	103.8500033362250
January	2019	1737.8	103.9300727296990
February	2019	1737.3	104.8108360579170
March	2019	1730.6	105.3713218122370
April	2019	1725	105.6115299926600
May	2019	1751.5	106.3321545339290
June	2019	1841.2	106.0118769600320
July	2019	1874.6	106.6524321078270
August	2019	2030.9	106.6524321078270
September	2019	1966.5	106.6524321078270
October	2019	1985.9	106.8926402882500
November	2019	1939.4	106.7325015013010
December	2019	1964.3	106.7325015013010
January	2020	2095.2	106.7325015013010
February	2020	2161	107.2129178621470
March	2020	2290.1	106.8926402882500
April	2020	2365	106.4122239274040
May	2020	2389.4	106.3321545339290
June	2020	2408.2	106.9727096817240

July	2020	2632	106.9727096817240
August	2020	2550.3	107.0527790751980
September	2020	2520.4	106.8125708947750
October	2020	2509.2	107.5331954360450
November	2020	2284.1	107.8534730099420
December	2020	2404.8	107.7734036164680