1.16) The mass flow rate through the nozzle of a rocket engine is 200 kg/s. The areas of the nozzle inlet and exit planes are 0.7 and 2.4 m^2, respectively. On the nozzle inlet plane, the pressure and velocity are 1600 kPa and 150 m/s, respectively, whereas on the nozzle exit plane, the pressure and velocity are 80 kPa and 2300 m/s, respectively. Find the thrust force acting on the nozzle.

Solution:

Given: $\dot{m} = 200\,\mathrm{kg/s}$,

$A_1 = 0.7\,\mathrm{m}^2$, $p_1 = 1600\,\mathrm{kPa}$, $V_1 = 150\,\mathrm{m/s}$,

$A_e = 2.4\,\mathrm{m}^2$, $p_e = 80\,\mathrm{kPa}$, $V_e = 2300\,\mathrm{m/s}$.

To calculate: Thrust = ?.

The schematic diagram of the problem description is shown in Fig. 1.

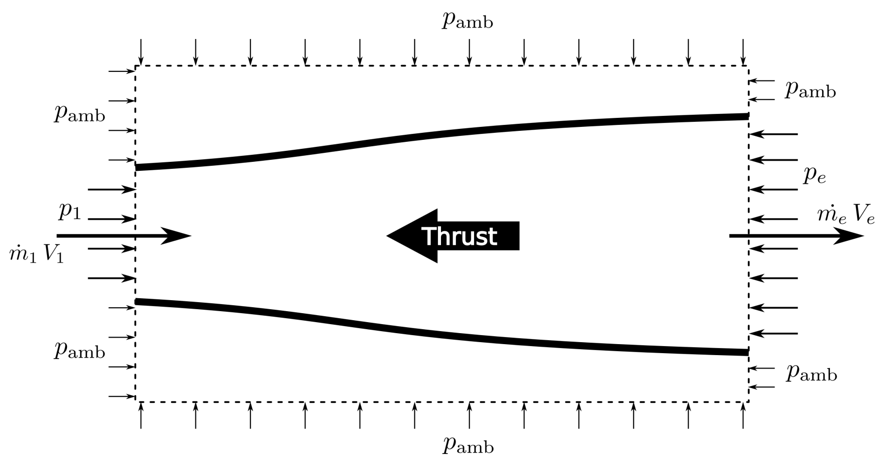

Fig. 1: Schematic diagram for problem description

Considering the control volume around the nozzle, and applying conservation of momentum in the horizontal direction, the thrust force acting on the nozzle can be written as,

$$\begin{aligned}\text{Thrust force on nozzle} &= \text{Rate of momentum leaving the nozzle} - \text{Rate on momentum entering the nozzle} \\ &\quad \text{pressure force on the exit plane} + \text{pressure force on the inlet plane}\end{aligned}$$

$$\text{Thrust force on nozzle} = \dot{m}_e V_e - \dot{m}_1 V_1 + p_e A_e + p_{\mathrm{amb}}\left(A - A_e\right) - p_1 A_1 - p_{\mathrm{amb}}\left(A - A_1\right)$$

where A is an arbitrary area of the vertical side of the control volume. Simplifying this equation gives,

$$\text{Thrust force on nozzle} = \dot{m}_e V_e - \dot{m}_1 V_1 + p_e A_e - p_1 A_1 - p_{\mathrm{amb}}\left(A_e - A_1\right)\ .$$

Since $\dot{m}_1 = \dot{m}_e = \dot{m}$,

$$\text{Thrust force on nozzle} = \dot{m}\left(V_e - V_1\right) + p_e A_e - p_1 A_1 - p_{\mathrm{amb}}\left(A_e - A_1\right)\ .$$

Case 1) Assuming that the inlet pressure is same as the ambient pressure, $p_1 = p_{\mathrm{amb}}$, the equation becomes,

$$\text{Thrust force on nozzle} = \dot{m}\left(V_e - V_1\right) + \left(p_e - p_1\right) A_e\ .$$

$$\text{Thrust force on nozzle} = 200 \times (2300 - 150) + \left(80 \times 10^3 - 1600 \times 10^3\right) \times 2.4\ .$$

$$\boxed{\text{Thrust force on nozzle} = -3218000\,\mathrm{N}}$$

Case 2) Assuming that the exit pressure is same as the ambient pressure, $p_e = p_{\mathrm{amb}}$, the equation becomes,

$$\text{Thrust force on nozzle} = \dot{m}\left(V_e - V_1\right) + \left(p_e - p_1\right) A_1\ .$$

$$\text{Thrust force on nozzle} = 200 \times (2300 - 150) + \left(80 \times 10^3 - 1600 \times 10^3\right) \times 0.7\ .$$

$$\boxed{\text{Thrust force on nozzle} = -634000.0\,\mathrm{N}}\ .$$