1.4) Two kilograms of air at an initial temperature and pressure of 30°C and 100 kPa undergoes an isentropic process, the final temperature attained being 850°C. Find the final pressure, the initial and final densities, and the initial and final volumes.

Solution:

Given: $m = 2\,\text{kg}$, $T_1 = 30°\text{C} = 303\,\text{K}$, $p_1 = 100\,\text{kPa}$, isentropic process, $T_2 = 850°\text{C} = 1123\,\text{K}$

To calculate: $p_2 = ?$, $\rho_1 = ?$, $\rho_2 = ?$, $v_1 = ?$, $v_2 = ?$.

Using isentropic relation, with $\gamma = 1.4$ for air,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$p_2 = 100 \times 10^3 \times \left(\frac{1123}{303}\right)^{1.4/(1.4-1)}$$

$$\boxed{p_2 = 9801.216\,\text{kPa}}\,.$$

The density ρ_1 can be calculated using the ideal gas equation as,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{100 \times 10^3}{287 \times 303}$$

$$\boxed{\rho_1 = 1.15\,\text{kg/m}^3}\,.$$

The density ρ_2 can be calculated using the ideal gas equation as,

$$\rho_2 = \frac{p_2}{RT_2} = \frac{9801.216 \times 10^3}{287 \times 1123}$$

$$\boxed{\rho_2 = 30.41\,\text{kg/m}^3}\,.$$

Since the mass is constant $m = 2\,\text{kg}$, the volumes can be calculated as,

$$\boxed{v_1 = \frac{m}{\rho_1} = \frac{2}{1.15} = 1.73913\,\text{m}^3}$$

$$\boxed{v_2 = \frac{m}{\rho_2} = \frac{2}{30.41} = 0.0657678\,\text{m}^3}$$