1.11) A solid fueled rocket is fitted with a convergent–divergent nozzle with an exit plane diameter of 30 cm. The pressure and velocity on this nozzle exit plane are 75 kPa and 750 m/s, respectively, and the mass flow rate through the nozzle is 350 kg/s. Find the thrust developed by this engine when the ambient pressure is (a) 100 kPa and (b) 20 kPa.

Solution:

Given: $\dot{m}_e = 350\,\text{kg/s}$, $V_e = 750\,\text{m/s}$, $p_e = 75\,\text{kPa}$, $D_e = \varnothing 0.3\,\text{m}$.

To calculate:

(a) Thrust when $p_{\text{amb}} = 100\,\text{kPa}$.

(b) Thrust when $p_{\text{amb}} = 20\,\text{kPa}$.

The schematic diagram of the problem description is shown in Fig. 1.

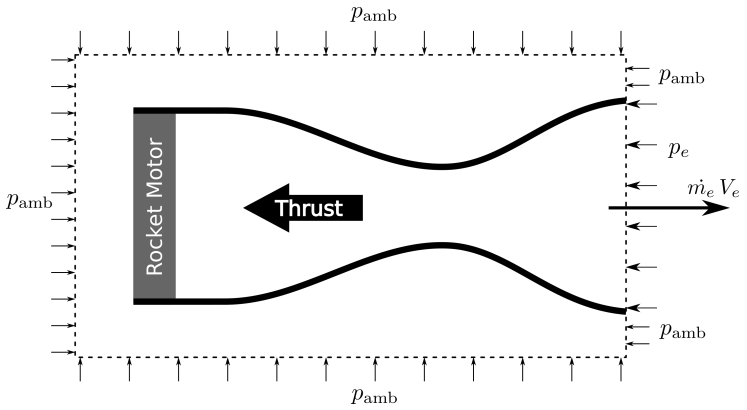

Fig. 1: Schematic diagram for problem description

(a) Thrust when $p_{\text{amb}} = 100\,\text{kPa}$.

Applying the conservation of momentum on the control-volume around the rocket,

$$\begin{aligned}
\text{Thrust} &= \text{rate of momentum exiting} - \text{rate of momentum entering} \\
&\quad + \text{pressure force at exit} - \text{pressure force at inlet} \\
\text{Thrust} &= \dot{m}_e V_e - 0 + (p_e - p_{\text{amb}})\, A_{\text{exit}} \\
\text{Thrust} &= 350 \times 750 - 0 + \left(75 \times 10^3 - 100 \times 10^3\right) \times \frac{\pi}{4} \times 0.3^2
\end{aligned}$$

$$\text{when } p_{\text{amb}} = 100\,\text{kPa} \quad \Longrightarrow \quad \boxed{\text{Thrust} = 260732.85\,\text{N}}\,.$$

(b) Thrust when $p_{\text{amb}} = 20\,\text{kPa}$.

Applying the conservation of momentum on the control-volume around the rocket,

$$\begin{aligned}
\text{Thrust} &= \text{rate of momentum exiting} - \text{rate of momentum entering} \\
&\quad + \text{pressure force at exit} - \text{pressure force at inlet} \\
\text{Thrust} &= \dot{m}_e V_e - 0 + (p_e - p_{\text{amb}})\, A_{\text{exit}} \\
\text{Thrust} &= 350 \times 750 - 0 + \left(75 \times 10^3 - 20 \times 10^3\right) \times \frac{\pi}{4} \times 0.3^2
\end{aligned}$$

$$\text{when } p_{\text{amb}} = 20\,\text{kPa} \quad \Longrightarrow \quad \boxed{\text{Thrust} = 266387.72\,\text{N}}\,.$$