

1.4) Two kilograms of air at an initial temperature and pressure of 30°C and 100 kPa undergoes an isentropic process, the final temperature attained being 850°C. Find the final pressure, the initial and final densities, and the initial and final volumes.

Solution:

Given: $m = 2$ kg, $T_1 = 30^\circ\text{C} = 303$ K, $p_1 = 100$ kPa, isentropic process, $T_2 = 850^\circ\text{C} = 1123$ K

To calculate: $p_2 = ?$, $\rho_1 = ?$, $\rho_2 = ?$, $v_1 = ?$, $v_2 = ?$.

Using isentropic relation, with $\gamma = 1.4$ for air,

$$\begin{aligned}\frac{p_2}{p_1} &= \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \\ p_2 &= p_1 \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \\ p_2 &= 100 \times 10^3 \times \left(\frac{1123}{303}\right)^{1.4/(1.4-1)} \\ \boxed{p_2 = 9801.216 \text{ kPa}}.\end{aligned}$$

The density ρ_1 can be calculated using the ideal gas equation as,

$$\begin{aligned}\rho_1 &= \frac{p_1}{R T_1} = \frac{100 \times 10^3}{287 \times 303} \\ \boxed{\rho_1 = 1.15 \text{ kg/m}^3}.\end{aligned}$$

The density ρ_2 can be calculated using the ideal gas equation as,

$$\begin{aligned}\rho_2 &= \frac{p_2}{R T_2} = \frac{9801.216 \times 10^3}{287 \times 1123} \\ \boxed{\rho_2 = 30.41 \text{ kg/m}^3}.\end{aligned}$$

Since the mass is constant $m = 2$ kg, the volumes can be calculated as,

$$\begin{aligned}\boxed{v_1 = \frac{m}{\rho_1} = \frac{2}{1.15} = 1.73913 \text{ m}^3} \\ \boxed{v_2 = \frac{m}{\rho_2} = \frac{2}{30.41} = 0.0657678 \text{ m}^3}\end{aligned}$$