1.6) Two air streams are mixed in a chamber. One stream enters the chamber through a 5 cm diameter pipe at a velocity of 100 m/s with a pressure of 150 kPa and a temperature of 30°C. The other stream enters the chamber through a 1.5 cm diameter pipe at a velocity of 150 m/s with a pressure of 75 kPa and a temperature of 30°C. The air leaves the chamber through a 9 cm diameter pipe at a pressure of 90 kPa and a temperature of 30°C. Assuming that the flow is steady, find the velocity in the exit pipe.

Solution:

Given: Steady flow

- $T_1 = 30°\text{C} = 303\,\text{K}$, $p_1 = 150\,\text{kPa}$, $D_1 = \varnothing 5\,\text{cm}$, $V_1 = 100\,\text{m/s}$ (Inlet)
- $T_2 = 30°\text{C} = 303\,\text{K}$, $p_2 = 75\,\text{kPa}$, $D_2 = \varnothing 1.5\,\text{cm}$, $V_2 = 150\,\text{m/s}$ (Inlet)
- $T_3 = 30°\text{C} = 303\,\text{K}$, $p_3 = 90\,\text{kPa}$, $D_3 = \varnothing 9\,\text{cm}$, $V_3 = ?\,\text{m/s}$ (Outlet)

The schematic diagram of the problem description is shown in Fig. 1.

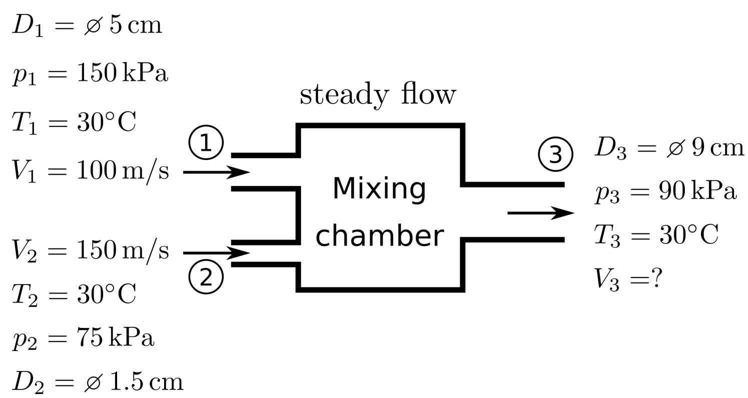

Fig. 1: Schematic diagram for problem description

The densities of air at the various stations may be calculated using the respective pressures, temperatures and using the ideal gas equation.

$$\rho_1 = \frac{p_1}{R\,T_1} = \frac{150 \times 10^3}{287 \times 303} = 1.7249\,\text{kg/m}^3$$

$$\rho_2 = \frac{p_2}{R\,T_2} = \frac{75 \times 10^3}{287 \times 303} = 0.86246\,\text{kg/m}^3$$

$$\rho_3 = \frac{p_1}{R\,T_1} = \frac{90 \times 10^3}{287 \times 303} = 1.0349\,\text{kg/m}^3$$

Using the conservation of mass equation,

$$\begin{aligned}
m_1 + m_2 &= m_3 \\
\rho_1\,A_1\,V_1 + \rho_2\,A_2\,V_2 &= \rho_3\,A_3\,V_3 \\
\rho_1 \times \frac{\pi}{4} D_1^2 \times V_1 + \rho_2 \times \frac{\pi}{4} D_2^2 \times V_2 &= \rho_3 \times \frac{\pi}{4} D_3^2 \times V_3 \\
1.7249 \times \frac{\pi}{4} \times 0.05^2 \times 100 + 0.86246 \times \frac{\pi}{4} \times 0.015^2 \times 150 &= 1.0349 \times \frac{\pi}{4} \times 0.09^2 \times V_3
\end{aligned}$$

$$V_3 = \frac{1.7249 \times \frac{\pi}{4} \times 0.05^2 \times 100 + 0.86246 \times \frac{\pi}{4} \times 0.015^2 \times 150}{1.0349 \times \frac{\pi}{4} \times 0.09^2}$$

$$\boxed{V_3 = 54.915\,\text{m/s}}\,.$$