1.13) In a proposed jet propulsion system for an automobile, air is drawn in vertically through a large intake in the roof at a rate of 3 kg/s, the velocity through this intake being small. Ambient pressure and temperature are 100 kPa and 30° C, respectively. This air is compressed and heated and then discharged horizontally out of a nozzle at the rear of the automobile at a velocity of 500 m/s and a pressure of 140 kPa. If the rate of heat addition to the air stream is 600 kW, find the nozzle discharge area and the thrust developed by the system.

Solution:

Given: air is drawn vertically, $\dot{m} = 3\,\mathrm{kg/s}$, $V_1 \approx 0$, $p_{\mathrm{amb}} = p_0 = 100\,\mathrm{kPa}$, $T_{\mathrm{amb}} = T_0 = 30°\,\mathrm{C} = 303\,\mathrm{K}$
air is compressed and heated and discharged horizontally,
$V_e = 500\,\mathrm{m/s}$, $p_e = 140\,\mathrm{kPa}$, $\dot{q} = 600\,\mathrm{kW}$.

To calculate: $A_e =?$, Thrust $=?$.

The schematic diagram of the problem description is shown in Fig. 1.

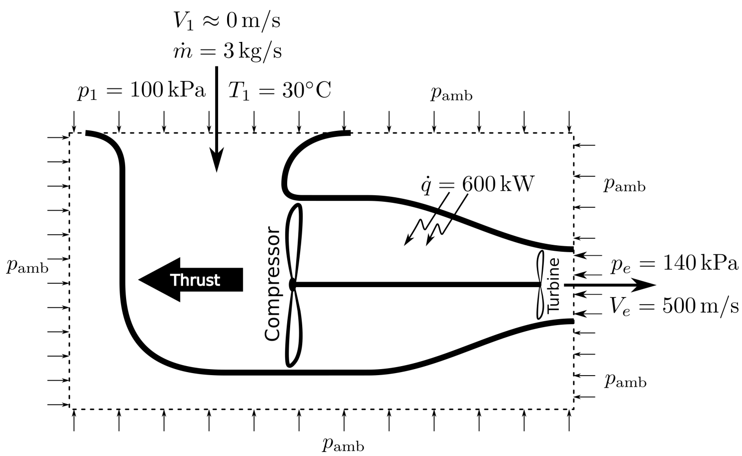

Fig. 1: Schematic diagram for problem description

Applying the conservation of energy through the system,

$$\text{Rate of energy entering} + \text{Rate of heat addition} = \text{Rate of energy leaving} + \text{Work done by the system}$$

$$\dot{m}\,c_p T_0 + \dot{q} = \dot{m}\,c_p T_e + \dot{m}\,\frac{V_e^2}{2} + W$$

$$3 \times 1005 \times 303 + 600 \times 10^3 = 3 \times 1005 \times T_e + 3 \times \frac{500^2}{2} + 0$$

$$T_e = \frac{3 \times 1005 \times 303 + 600 \times 10^3 - 3 \times \frac{500^2}{2}}{3 \times 1005} = 377.62686567\,\mathrm{K}$$

The density at the exit can be calculated using the ideal gas equation,

$$\rho_e = \frac{p_e}{R T_e} = \frac{140 \times 10^3}{287 \times 377.6} = 1.29176\,\mathrm{kg/m^3}$$

Applying the conservation of mass equation,

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{3}{1.29176 \times 500}$$

$$\boxed{A_e = 0.0046448\,\mathrm{m^2}}\,.$$

Applying the conservation of horizontal momentum,

$$\text{Thrust} = \dot{m}_e V_e + (p_e - p_{\mathrm{amb}})\,A_e$$

$$\text{Thrust} = 3 \times 500 + \left(140 \times 10^3 - 100 \times 10^3\right) \times 0.0046448$$

$$\boxed{\text{Thrust} = 1685.792\,\mathrm{N}}\,.$$