1.8) The engine of a small jet aircraft develops a thrust of $18\,\mathrm{kN}$ when the aircraft is flying at a speed of $900\,\mathrm{km/h}$ at an altitude where the ambient pressure is $50\,\mathrm{kPa}$. The air flow rate through the engine is $75\,\mathrm{kg/s}$ and the engine uses fuel at a rate of $3\,\mathrm{kg/s}$. The pressure on the engine discharge plane is $55\,\mathrm{kPa}$ and the area of the engine exit is $0.2\,\mathrm{m^2}$. Find the jet efflux velocity.

Solution:

Given:

Thrust=18 kN, fuel rate=3 kg/s Inlet $\rightarrow V_1 = 900 \,\mathrm{km/h} = 900/3.6 = 250 \,\mathrm{m/s}, \, p_1 = 50 \,\mathrm{kPa}, \, \dot{m}_1 = 75 \,\mathrm{kg/s}.$ Outlet $\rightarrow p_2 = 55 \,\mathrm{kPa}, \, A_{\mathrm{exit}} = 0.2 \,\mathrm{m}^2, \, V_2 = ? \,\mathrm{m/s}.$

The schematic diagram of the problem description is shown in Fig. 1.

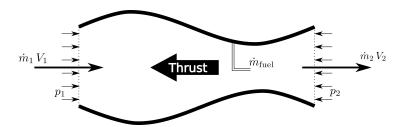


Fig. 1: Schematic diagram for problem description

Using the mass conservation equation,

$$\dot{m}_2 = \dot{m}_1 + \dot{m}_{\text{fuel}}$$
 $\dot{m}_2 = 75 + 3 = 78 \,\text{kg/s}$

Using the conservation of momentum along horizontal direction,

$$\begin{array}{rcl} \text{Thrust} &=& \text{rate of momentum exiting} - \text{rate of momentum entering} \\ &+& \text{pressure force at exit} - \text{pressure force at inlet} \\ \text{Thrust} &=& \dot{m}_2 V_2 - \dot{m}_1 V_1 + (p_2 - p_1) \, A_{\text{exit}} \\ 18 \times 10^3 &=& 78 \times V_2 - 75 \times 250 + \left(55 \times 10^3 - 50 \times 10^3\right) \times 0.2 \\ \\ V_2 &=& \frac{18 \times 10^3 + 75 \times 250 - \left(55 \times 10^3 - 50 \times 10^3\right) \times 0.2}{78} \\ \hline V_2 &=& 458.3333 \, \text{m/s} \, \right]. \end{array}$$