Compressible Fluid Flows : Introduction

Let us denote the compressibility by "τ".

$$\tau = -\frac{dV/V}{dp} = -\frac{1}{V}\frac{dV}{dp} \, .$$

Writing in terms of specific volume, $v = V/m$,

$$\tau = -\frac{1}{V}\frac{dV}{dp}\,\frac{m}{m} = -\frac{1}{v}\frac{dv}{dp} \, . \tag{1}$$

Also,

$$v = \frac{1}{\rho} = \frac{m}{V} \, .$$

$$\frac{dv}{dp} = -\frac{1}{\rho^2}\frac{d\rho}{dp} \, .$$

Substituting in equation (1),

$$\tau = \frac{1}{\rho}\frac{d\rho}{dp} \, . \tag{2}$$

Depending on the compression process, compressibility can be defined as,

1. Isothermal compressibility
2. Isentropic compressibility

Isothermal compressibility: Fluid temperature, T, is constant.

$$\tau_T = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right)_T \, .$$

Isentropic compressibility: Fluid entropy, S, is constant.

$$\tau_S = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right)_S \, .$$

Ideal gas obeys the equation,

$$p\,V = m\,R\,T \, . \tag{3}$$

p : pressure.
V : volume of the gas.
m : mass of the gas.
T : temperature.
R : gas constant, 287 J/kg-K for air.

$$R = \frac{R_u}{\hat{m}} \, .$$

R_u : universal gas constant, 8314 J/kmol-K.
$\hat{m}$: molecular mass of gas measured in, mole/gram.

Dividing equation (3) by volume, V, we can write,

$$p = \rho\,R\,T \, . \tag{4}$$

ρ : density of gas $= m/V$.
Equation (4) is called the "equation of state".

Perfect gas: Intermolecular forces are neglected.

- "Perfect gas" is an "Ideal gas" with its specific heats, c_p and c_v, assumed to be constant.
- Air and many other gases can be assumed to be "perfect gas" in many practical applications.

Problem 1:
Calculate isothermal compressibility of air at 1 atm pressure.

$$1\,\text{atm} = 101325\,\text{N/m}^2\,.$$

The isothermal compressibility is given by,

$$\tau_T = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right)_T\,.$$

Using the equation of state we can obtain the density as,

$$\rho = \frac{p}{R\,T}\,.$$

Differentiating w.r.t. pressure with constant temperature (isothermal) we get,

$$\left(\frac{d\rho}{dp}\right)_T = \frac{1}{R\,T}\,.$$

Substituting in compressibility equation,

$$\tau_T = \frac{R\,T}{p}\cdot\frac{1}{R\,T}$$

$$\tau_T = \frac{1}{p}$$

$$\tau_T = \frac{1}{101325} \approx 1\times 10^{-5}\,\text{m}^2/\text{N}\,.$$

Compare this with compressibility of water, $5\times 10^{-10}\,\text{m}^2/\text{N}$.