1.14) Carbon dioxide flows through a constant area duct. At the inlet to the duct, the velocity is 120 m/s and the temperature and pressure are 200° C and 700 kPa, respectively. Heat is added to the flow in the duct and at the exit of the duct the velocity is 240 m/s and the temperature is 450° C. Find the amount of heat being added to the carbon dioxide per unit mass of gas and the mass flow rate through the duct per unit cross-sectional area of the duct. Assume that for carbon dioxide, $\gamma = 1.3$.

Solution:

Given: $\gamma = 1.3$

$A_1 = A_e = A$, $V_1 = 120\,\text{m/s}$, $T_1 = 200°\,\text{C} = 473\,\text{K}$, $p_1 = 700\,\text{kPa}$,

$V_e = 240\,\text{m/s}$, $T_e = 450°\,\text{C} = 723\,\text{K}$.

To calculate: $\dot{q}/\dot{m} =?$, $\dot{m}/A =?$.

The schematic diagram of the problem description is shown in Fig. 1.

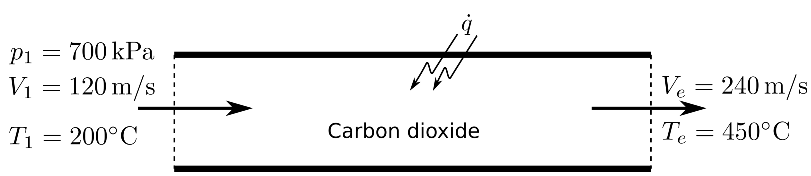

Fig. 1: Schematic diagram for problem description

Using the molar mass of cardon-dioxide (CO_2) as $12 \times 1_\text{C} + 16 \times 2_\text{O} = 44\,\text{kg/kmol}$ and the universal gas constant as 8314 J/kmol-K, the gas constant can be written as,

$$R = \frac{R_u}{\hat{m}} = \frac{8314}{44} = 188.95\,\text{J/kg-K}$$

The heat capacity at constant pressure can be calculated as,

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.3 \times 188.95}{1.3 - 1} = 818.78\,\text{J/kg-K}$$

Applying the conservation of energy principle,

$$c_p T_1 + \frac{V_1^2}{2} + \dot{q}/\dot{m} = c_p T_e + \frac{V_e^2}{2}$$

$$818.78 \times 473 + \frac{120^2}{2} + \dot{q}/\dot{m} = 818.78 \times 723 + \frac{240^2}{2}$$

We can solve for the heat added per unit mass to be,

$$\dot{q}/\dot{m} = 818.78 \times 723 + \frac{240^2}{2} - 818.78 \times 473 - \frac{120^2}{2}$$

$$\boxed{\dot{q}/\dot{m} = 226295\,\text{J/kg}}\,.$$

The density at the inlet can be calculated using the ideal gas equation to be,

$$\rho_1 = \frac{p_1}{R T_1} = \frac{700 \times 10^3}{188.95 \times 473} = 7.8323\,\text{kg/m}^3$$

The mass flow rate,

$$\dot{m} = \rho_1 A_1 V_1$$

which gives the mass flow rate per unit area as,

$$\dot{m}/A = \rho_1 V_1 = 7.8323 \times 120$$

$$\boxed{\dot{m}/A = 939.876\,\text{kg/s-m}^2}\,.$$