

1.6) Two air streams are mixed in a chamber. One stream enters the chamber through a 5 cm diameter pipe at a velocity of 100 m/s with a pressure of 150 kPa and a temperature of 30°C. The other stream enters the chamber through a 1.5 cm diameter pipe at a velocity of 150 m/s with a pressure of 75 kPa and a temperature of 30°C. The air leaves the chamber through a 9 cm diameter pipe at a pressure of 90 kPa and a temperature of 30°C. Assuming that the flow is steady, find the velocity in the exit pipe.

Solution:

Given: Steady flow

- $T_1 = 30^\circ\text{C} = 303\text{ K}$, $p_1 = 150\text{ kPa}$, $D_1 = \varnothing 5\text{ cm}$, $V_1 = 100\text{ m/s}$ (Inlet)
- $T_2 = 30^\circ\text{C} = 303\text{ K}$, $p_2 = 75\text{ kPa}$, $D_2 = \varnothing 1.5\text{ cm}$, $V_2 = 150\text{ m/s}$ (Inlet)
- $T_3 = 30^\circ\text{C} = 303\text{ K}$, $p_3 = 90\text{ kPa}$, $D_3 = \varnothing 9\text{ cm}$, $V_3 = ?\text{ m/s}$ (Outlet)

The schematic diagram of the problem description is shown in Fig. 1.

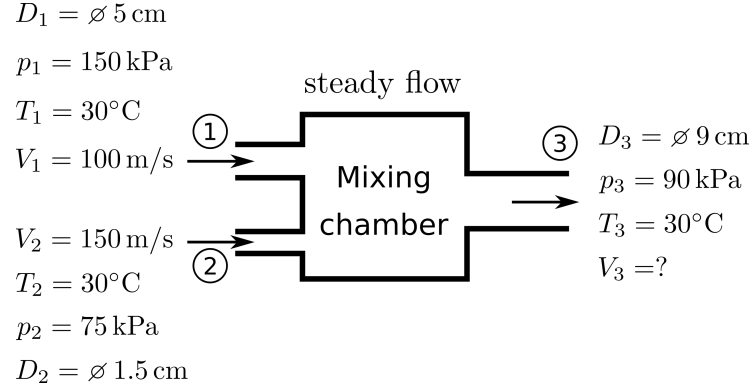


Fig. 1: Schematic diagram for problem description

The densities of air at the various stations may be calculated using the respective pressures, temperatures and using the ideal gas equation.

$$\rho_1 = \frac{p_1}{R T_1} = \frac{150 \times 10^3}{287 \times 303} = 1.7249\text{ kg/m}^3$$

$$\rho_2 = \frac{p_2}{R T_2} = \frac{75 \times 10^3}{287 \times 303} = 0.86246\text{ kg/m}^3$$

$$\rho_3 = \frac{p_3}{R T_3} = \frac{90 \times 10^3}{287 \times 303} = 1.0349\text{ kg/m}^3$$

Using the conservation of mass equation,

$$\begin{aligned} m_1 + m_2 &= m_3 \\ \rho_1 A_1 V_1 + \rho_2 A_2 V_2 &= \rho_3 A_3 V_3 \\ \rho_1 \times \frac{\pi}{4} D_1^2 \times V_1 + \rho_2 \times \frac{\pi}{4} D_2^2 \times V_2 &= \rho_3 \times \frac{\pi}{4} D_3^2 \times V_3 \\ 1.7249 \times \frac{\pi}{4} \times 0.05^2 \times 100 + 0.86246 \times \frac{\pi}{4} \times 0.015^2 \times 150 &= 1.0349 \times \frac{\pi}{4} \times 0.09^2 \times V_3 \\ V_3 &= \frac{1.7249 \times \frac{\pi}{4} \times 0.05^2 \times 100 + 0.86246 \times \frac{\pi}{4} \times 0.015^2 \times 150}{1.0349 \times \frac{\pi}{4} \times 0.09^2} \\ \boxed{V_3 = 54.915\text{ m/s}} \end{aligned}$$