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Analytical Approach of Tangency on Geometry Problem

(posted by Mr.Sourabh-Bhat)

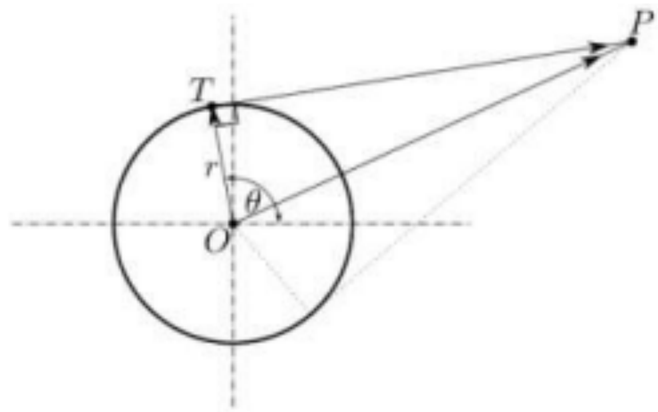

Figure 1. Schematic diagram of the problem[1]

Analytical solution:

From your final tangent line equation, it gives the following result:

$$(x_P - x_O)cos\theta + (y_P - y_O)sin\theta - r = 0$$

In behalf of algebraic manipulation, let define Δx and Δy as given below:

$$\Delta x \equiv x_p - x_O \; ; \; \Delta y \equiv y_p - y_O$$

then,

$\Delta x cos\theta + \Delta y sin\theta - r = 0$	(1)

Let's recall trigonometry identity as given below:

$$sin^2\theta + cos^2\theta = 1 \leftrightarrow \; cos^2\theta = 1 - sin^2\theta$$

Next, let's rearrange the form of (1) by taking square on both of sides as follows:

$(r - \Delta y sin\theta)^2 = (\Delta x cos\theta)^2 \; \leftrightarrow \; r^2 - 2r\Delta y sin\theta + \Delta y^2 sin^2\theta = \Delta x^2 cos^2\theta$	(1.1)

Now, let's substitute $cos^2\theta$ from trigonometry identity into (1.1) and by a little bit rearrangement of (1.1), it gives

$sin^2\theta - \frac{2r\Delta y}{\Delta x^2 + \Delta y^2} sin\theta + \frac{r^2 - \Delta x^2}{\Delta x^2 + \Delta y^2} = 0$	(2)

It can be viewed that (2) resembles the general quadratic equation in which $sin\theta$ as an unknown variable. Otherwise, I can apply "**abc formula**" as given below:

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} \; ; D = b^2 - 4ac$$

in this case, $a = 1; b = \frac{-2r\Delta y}{\Delta x^2+\Delta y^2}; c = \frac{r^2-\Delta x^2}{\Delta x^2+\Delta y^2}$

then,

$$sin\theta_{1,2} = \frac{-\left(\frac{-2r\Delta y}{\Delta x^2 + \Delta y^2}\right) \pm \sqrt{\left(-\frac{2r\Delta y}{\Delta x^2 + \Delta y^2}\right)^2 - 4(1)\left(\frac{r^2 - \Delta x^2}{\Delta x^2 + \Delta y^2}\right)}}{2(1)}$$

by a little bit simplification, it can be obtained that

$$sin\theta_{1,2} = \frac{r\Delta y \pm \Delta x\sqrt{\Delta x^2 + \Delta y^2 - r^2}}{\Delta x^2 + \Delta y^2}$$	(3)

From (3), It has two distinct solutions which are θ_1 and θ_2. Therefore, both of θ_1 and θ_2 are expressed below:

$$\theta_1 = \sin^{-1}\left(\frac{r\Delta y + \Delta x\sqrt{\Delta x^2 + \Delta y^2 - r^2}}{\Delta x^2 + \Delta y^2}\right)$$ $$\theta_2 = \sin^{-1}\left(\frac{r\Delta y - \Delta x\sqrt{\Delta x^2 + \Delta y^2 - r^2}}{\Delta x^2 + \Delta y^2}\right)$$	(3.1)

It's known that both of θ_1 and θ_2 are evaluated in radian unit (not in degree unit).

Moreover, the coordinate of tangency point (X_T, Y_T) associated with θ_1 and θ_2 is given below:

$$X_{T,1} = X_O + rcos\theta_1;\ Y_{T,1} = Y_O + rsin\theta_1$$
$$X_{T,2} = X_O + rcos\theta_2;\ Y_{T,2} = Y_O + rsin\theta_2$$

Hence,

The coordinate of 1st tangency point $(X_{T,1}, Y_{T,1})$:

$$X_{T,1} = X_O + rcos\theta_1 = X_O + rcos\left(\sin^{-1}\left(\frac{r\Delta y + \Delta x\sqrt{\Delta x^2 + \Delta y^2 - r^2}}{\Delta x^2 + \Delta y^2}\right)\right)$$

$$Y_{T,1} = Y_O + rsin\theta_1 = Y_O + rsin\left(\sin^{-1}\left(\frac{r\Delta y + \Delta x\sqrt{\Delta x^2 + \Delta y^2 - r^2}}{\Delta x^2 + \Delta y^2}\right)\right)$$

The coordinate of 2nd tangency point $(X_{T,2}, Y_{T,2})$:

$$X_{T,2} = X_O + rcos\theta_2 = X_O + rcos\left(\sin^{-1}\left(\frac{r\Delta y - \Delta x\sqrt{\Delta x^2 + \Delta y^2 - r^2}}{\Delta x^2 + \Delta y^2}\right)\right)$$

$$Y_{T,2} = Y_O + rsin\theta_2 = Y_O + rsin\left(\sin^{-1}\left(\frac{r\Delta y - \Delta x\sqrt{\Delta x^2 + \Delta y^2 - r^2}}{\Delta x^2 + \Delta y^2}\right)\right)$$

Reference:

[1]. https://spbhat.in/blogs/tangent/index.html, accessed on Friday, July 5, 2019 at 4.14 PM