

MAT250
Final Exam

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Problem 1

$$\boxed{> p := \text{piecewise}\left(0 \leq x < 1, 2, x \geq 1, -\frac{2}{x}\right)}$$

$$p := \begin{cases} 2 & 0 \leq x < 1 \\ -\frac{2}{x} & 1 \leq x \end{cases} \quad (1)$$

$$\boxed{> sol := \text{diff}(y(x), x) + p \cdot y(x) = 4 \cdot x}$$

$$sol := \frac{d}{dx} y(x) + \begin{pmatrix} 2 & 0 \leq x < 1 \\ -\frac{2}{x} & 1 \leq x \end{pmatrix} y(x) = 4x \quad (2)$$

$$\boxed{> A := \text{dsolve}(\{y(0) = 3, sol\})}$$

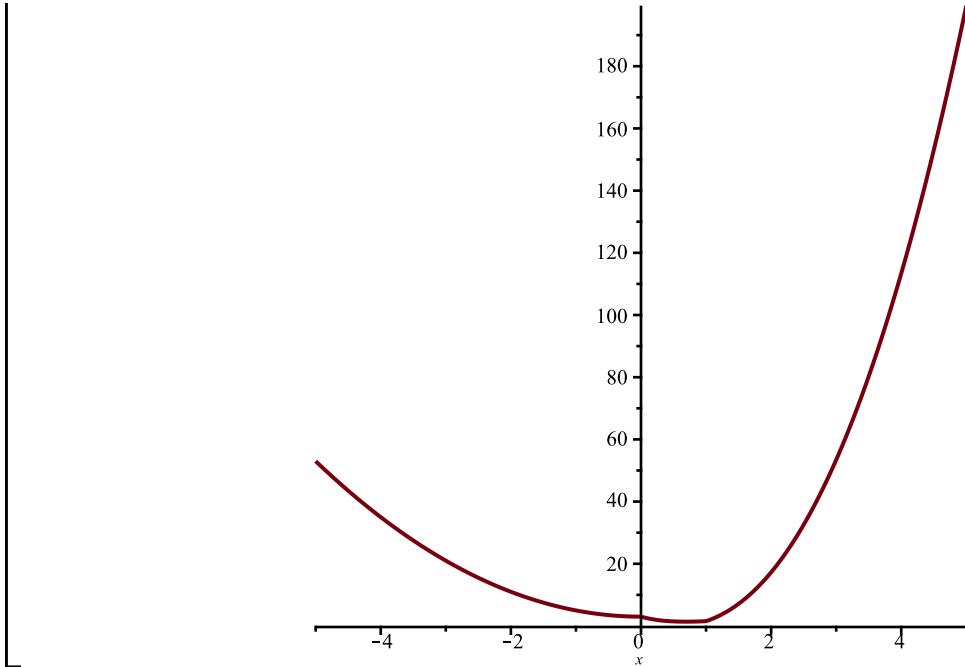
$$A := y(x) = \int_0^x \begin{pmatrix} 4_{-zI} & _{-zI} < 0 \\ 4_{-zI} e^{2_{-zI}} & _{-zI} < 1 \\ \frac{4 e^2}{-zI} & 1 \leq _{-zI} \end{pmatrix} d_{-zI} \begin{pmatrix} 1 & x < 1 \\ x^2 & 1 \leq x \end{pmatrix} \quad (3)$$

$$+ \begin{pmatrix} 3 & x < 1 \\ 3x^2 & 1 \leq x \end{pmatrix} - \int_0^x \begin{pmatrix} 4_{-zI} & _{-zI} < 0 \\ 4_{-zI} e^{2_{-zI}} & _{-zI} < 1 \\ \frac{4 e^2}{-zI} & 1 \leq _{-zI} \end{pmatrix}$$

$$d_{-zI} \begin{pmatrix} 1 & x < 1 \\ x^2 & 1 \leq x \end{pmatrix} \begin{pmatrix} 0 & x < 0 \\ 1 & 0 \leq x \end{pmatrix} + \begin{pmatrix} 0 & x < 0 \\ -3 & x < 1 \\ -3x^2 & 1 \leq x \end{pmatrix}$$

$$\begin{aligned}
& + \left(\begin{cases} 0 & x < 1 \\ -3 e^{-2x} x^2 & 1 \leq x \end{cases} \right) - e^{-2x} \left(\begin{cases} 0 & x < 1 \\ 1 & 1 \leq x \end{cases} \right) \\
& \quad \left. \begin{array}{l} \\ \end{array} \right\} \\
& \int_0^x \left(\begin{cases} 4 zI & zI < 0 \\ 4 zI e^{2-zI} & zI < 1 \\ \frac{4 e^2}{zI} & 1 \leq zI \end{cases} \right) dzI \left(\begin{cases} 1 & x < 1 \\ x^2 & 1 \leq x \end{cases} \right) + e^{-2x} \left(\begin{array}{l} \\ \end{array} \right. \\
& \quad \left. \begin{array}{l} \\ \end{array} \right\} \\
& \int_0^x \left(\begin{cases} 4 zI & zI < 0 \\ 4 zI e^{2-zI} & zI < 1 \\ \frac{4 e^2}{zI} & 1 \leq zI \end{cases} \right) dzI \left(\begin{cases} 1 & x < 1 \\ x^2 & 1 \leq x \end{cases} \right) \left(\begin{cases} 0 & x < 0 \\ 1 & 0 \leq x \end{cases} \right) \\
& + \left(\begin{cases} 0 & x < 1 \\ 3 e^{-2x} x^2 & 1 \leq x \end{cases} \right) + \left(\begin{cases} 0 & x < 0 \\ 3 e^{-2x} & x < 1 \\ 3 e^{-2x} x^2 & 1 \leq x \end{cases} \right) + e^{-2} \left(\begin{cases} 0 & x < 1 \\ 1 & 1 \leq x \end{cases} \right) \\
& \quad \left. \begin{array}{l} \\ \end{array} \right\} \\
& \int_0^x \left(\begin{cases} 4 zI & zI < 0 \\ 4 zI e^{2-zI} & zI < 1 \\ \frac{4 e^2}{zI} & 1 \leq zI \end{cases} \right) dzI \left(\begin{cases} 1 & x < 1 \\ x^2 & 1 \leq x \end{cases} \right)
\end{aligned}$$

> $\text{plot}(\text{rhs}(A), x=-5..5)$



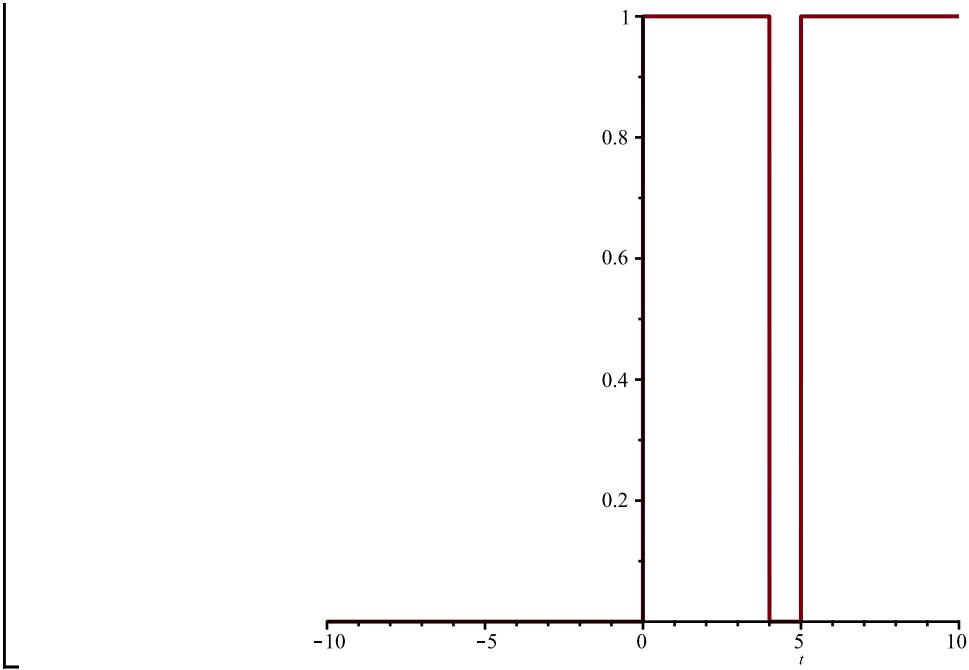
Problem 2

>

> $q := \text{piecewise}(0 \leq t < 4, 1, 4 \leq t < 5, 4, t \geq 5, 1)$

$$q := \begin{cases} 1 & 0 \leq t < 4 \\ 4 & 4 \leq t < 5 \\ 1 & 5 \leq t \end{cases} \quad (4)$$

> $\text{plot}(q)$



$$\begin{aligned}
 >& \text{with(inttrans)} : \\
 >& \text{laplace}(q, t, s) \\
 & \frac{1 - e^{-4s} + e^{-5s}}{s}
 \end{aligned} \tag{5}$$

Problem 3

$$\begin{aligned}
 >& \text{with(LinearAlgebra)} : \\
 >& B := \text{Matrix}([[4, 1, 4], [1, 7, 1], [4, 1, 4]]) \\
 & B := \begin{bmatrix} 4 & 1 & 4 \\ 1 & 7 & 1 \\ 4 & 1 & 4 \end{bmatrix}
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 >& \text{Eigenvalues}(B) \\
 & \begin{bmatrix} 0 \\ 9 \\ 6 \end{bmatrix}
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 >& \text{Eigenvectors}(B)
 \end{aligned} \tag{8}$$

$$\left[\begin{array}{c} \left[\begin{array}{c} 9 \\ 0 \\ 6 \end{array} \right], \left[\begin{array}{ccc} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{array} \right] \end{array} \right] \quad (8)$$

Problem 5

$$\left[\begin{array}{c} > C := Matrix([[1, 2, 3], [3, -2, 1], [4, 1, 1]]) \\ & C := \left[\begin{array}{ccc} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 1 & 1 \end{array} \right] \end{array} \right] \quad (9)$$

$$\left[\begin{array}{c} > MatrixInverse(C) \\ & \left[\begin{array}{ccc} -\frac{3}{32} & \frac{1}{32} & \frac{1}{4} \\ \frac{1}{32} & -\frac{11}{32} & \frac{1}{4} \\ \frac{11}{32} & \frac{7}{32} & -\frac{1}{4} \end{array} \right] \\ = \\ > \\ > \end{array} \right] \quad (10)$$

Problem 4

$$\left[\begin{array}{c} > X := diff(x1(t), t) = -3 \cdot x1(t) + 5 \cdot x2(t) - 5 \cdot x3(t) \\ & X := \frac{d}{dt} x1(t) = -3 x1(t) + 5 x2(t) - 5 x3(t) \end{array} \right] \quad (11)$$

$$\left[\begin{array}{c} > Y := diff(x2(t), t) = -7 \cdot x1(t) + 9 \cdot x2(t) - 5 \cdot x3(t) \\ & Y := \frac{d}{dt} x2(t) = -7 x1(t) + 9 x2(t) - 5 x3(t) \end{array} \right] \quad (12)$$

$$\left[\begin{array}{c} > Z := diff(x3(t), t) = -7 \cdot x1(t) + 7 \cdot x2(t) - 3 \cdot x3(t) \\ & Z := \frac{d}{dt} x3(t) = -7 x1(t) + 7 x2(t) - 3 x3(t) \end{array} \right] \quad (13)$$

$$\left[\begin{array}{c} > W := \{X, Y, Z\} : ics := \{x1(0) = 4, x2(0) = -5, x3(0) = -3\} \\ & \qquad \qquad \qquad ics := \{x1(0) = 4, x2(0) = -5, x3(0) = -3\} \end{array} \right] \quad (14)$$

$$\left[\begin{array}{c} > combine(dsolve(W \text{union} ics, \{x1(t), x2(t), x3(t)\})) \\ & \qquad \qquad \qquad \{x1(t) = -2 e^{2t} + 6 e^{-3t}, x2(t) = -2 e^{2t} + 6 e^{-3t} - 9 e^{4t}, x3(t) = 6 e^{-3t} - 9 e^{4t}\} \end{array} \right] \quad (15)$$