

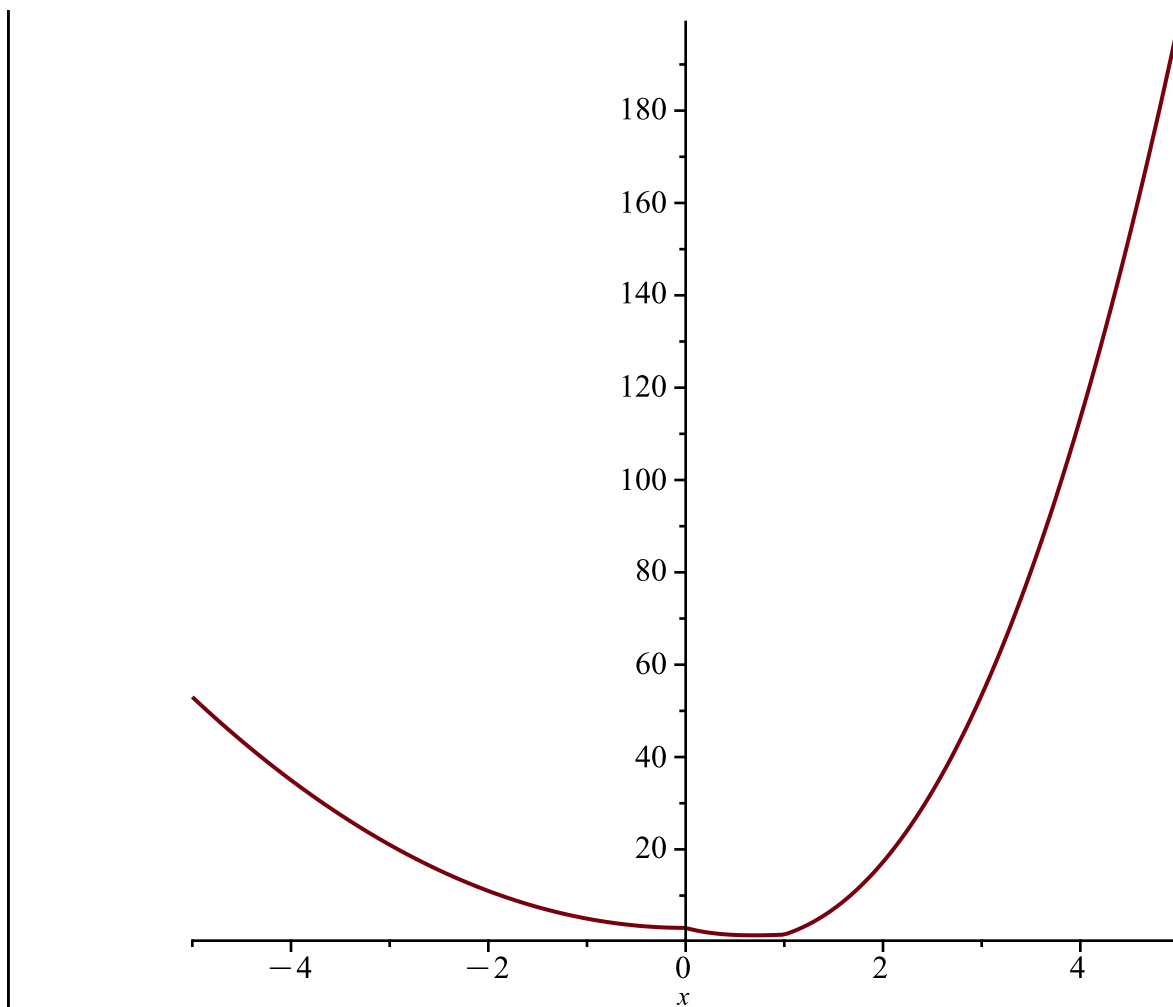
1.

$$\left[\begin{array}{l} \text{> } p := \text{piecewise}\left(0 \leq x \leq 1, 2, x \geq 1, -\frac{2}{x}\right) \\ \\ p := \begin{cases} 2 & 0 \leq x \leq 1 \\ -\frac{2}{x} & 1 \leq x \end{cases} \end{array} \right. \quad (1)$$

$$\left[\begin{array}{l} \text{> } sol := \text{diff}(y(x), x) + p \cdot y(x) = 4 \cdot x \\ \\ sol := \frac{d}{dx} y(x) + \left(\begin{cases} 2 & 0 \leq x \leq 1 \\ -\frac{2}{x} & 1 \leq x \end{cases} \right) y(x) = 4x \end{array} \right. \quad (2)$$

$$\left[\begin{array}{l} \text{> } A := \text{dsolve}(\{y(0) = 3, sol\}) \\ \\ A := y(x) = \begin{cases} 2x^2 + 3 & x < 0 \\ 2x + 4e^{-2x} - 1 & x < 1 \\ x^2(1 + 4\ln(x) + 4e^{-2}) & 1 \leq x \end{cases} \end{array} \right. \quad (3)$$

$$\left[\begin{array}{l} \text{> } \text{plot}(rhs(A), x = -5..5) \end{array} \right.$$



2.

$$\text{sol2} := x \cdot \text{diff}(y(x), x, x) - (x + 10) \cdot \text{diff}(y(x), x) + 10 y(x) = 0 \quad (4)$$

$$\text{sol2} := x \left(\frac{d^2}{dx^2} y(x) \right) - (x + 10) \left(\frac{d}{dx} y(x) \right) + 10 y(x) = 0 \quad (5)$$

$$B := \text{dsolve}(\text{sol2})$$

$$B := y(x) = c_1 e^x + c_2 (x^{10} + 10 x^9 + 90 x^8 + 720 x^7 + 5040 x^6 + 30240 x^5 + 151200 x^4 + 604800 x^3 + 1814400 x^2 + 3628800 x + 3628800) \quad (6)$$

$$\text{eval}((6), [c_1 = 1, c_2 = 0]) \quad y(x) = e^x \quad (7)$$

therefore, shown

3.

$$\left[\begin{array}{l} \text{> } sol3 := diff(y(x), x, x) + diff(y(x), x) = x \\ \qquad \qquad \qquad sol3 := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = x \end{array} \right. \quad (8)$$

$$\left[\begin{array}{l} \text{> } C := dsolve(\{sol3, y(0) = 1, D(y)(0) = 2\}) \\ \qquad \qquad \qquad C := y(x) = \frac{x^2}{2} - 3 e^{-x} - x + 4 \end{array} \right. \quad (9)$$

4.

$$\left[\begin{array}{l} \text{> } sol4 := (2 \cdot y(x) + 2 \cdot x^2) \cdot diff(y(x), x) + (4 \cdot x \cdot y(x) + 3 \cdot x^2) = 0 \\ \qquad \qquad \qquad sol4 := (2 y(x) + 2 x^2) \left(\frac{d}{dx} y(x) \right) + 4 x y(x) + 3 x^2 = 0 \end{array} \right. \quad (10)$$

$$\left[\begin{array}{l} \text{> } E := (2 * y(x) + 2 * x^2) * diff(y(x), x) + 4 * x * y(x) + 3 * x^2 = 0; \\ E0 := (E) + (-3 * x^2); \\ E1 := (E0) + (-4 * x * y(x)); \\ E2 := (E1) * (1 / (2 * y(x) + 2 * x^2)); \\ \qquad \qquad \qquad E := (2 y(x) + 2 x^2) \left(\frac{d}{dx} y(x) \right) + 4 x y(x) + 3 x^2 = 0 \\ \qquad \qquad \qquad E0 := (2 y(x) + 2 x^2) \left(\frac{d}{dx} y(x) \right) + 4 x y(x) = -3 x^2 \\ \qquad \qquad \qquad E1 := (2 y(x) + 2 x^2) \left(\frac{d}{dx} y(x) \right) = -3 x^2 - 4 x y(x) \\ \qquad \qquad \qquad E2 := \frac{d}{dx} y(x) = \frac{-3 x^2 - 4 x y(x)}{2 y(x) + 2 x^2} \end{array} \right. \quad (11)$$

$$\left[\begin{array}{l} \text{> } E := x^3 + 2 x^2 y + y^2 = c \\ \qquad \qquad \qquad E := x^3 + 2 x^2 y + y^2 = c \end{array} \right. \quad (12)$$

$$\left[\begin{array}{l} \text{> } isolate(diff(subs(y=y(x), E), x), diff(y(x), x)) \\ \qquad \qquad \qquad \frac{d}{dx} y(x) = \frac{-3 x^2 - 4 x y(x)}{2 y(x) + 2 x^2} \end{array} \right. \quad (13)$$

$$\left[\begin{array}{l} \text{> } eval(E, [x=1, y=1]) \\ \qquad \qquad \qquad 4 = c \end{array} \right. \quad (14)$$

$$\left[\begin{array}{l} \text{> } eval(E, [x=0, y=-2]) \\ \qquad \qquad \qquad 4 = c \end{array} \right. \quad (15)$$

> $F := dsolve(\{y(0) = -2, sol4\})$

$$F := y(x) = -x^2 - \sqrt{x^4 - x^3 + 4}$$

(16)

> $plot(rhs(F), x = -10..10)$

