MAT250
Final Exam

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Problem 1

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> p := piecewise(0 ≤ x < 1, 2, x ≥ 1, -2/x)
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$$p := \begin{cases} 2 & 0 \le x < 1 \\ -\frac{2}{x} & 1 \le x \end{cases} \tag{1}$$

```
> sol := diff(y(x), x) + p·y(x) = 4·x
```

$$sol := \frac{d}{dx} y(x) + \left(\begin{cases} 2 & 0 \le x < 1 \\ -\frac{2}{x} & 1 \le x \end{cases} \right) y(x) = 4x \tag{2}$$

```
> A := dsolve({y(0) = 3, sol})
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$$A := y(x) = \left(\int_0^x \left(\begin{cases} 4\_z1 & \_z1 < 0 \\ 4\_z1\, e^{2\_z1} & \_z1 < 1 \\ \frac{4e^2}{\_z1} & 1 \le \_z1 \end{cases} d\_z1 \right) \left(\begin{cases} 1 & x < 1 \\ x^2 & 1 \le x \end{cases} \right) \right.$$

$$+ \left(\begin{cases} 3 & x < 1 \\ 3x^2 & 1 \le x \end{cases} \right) - \left(\int_0^x \left(\begin{cases} 4\_z1 & \_z1 < 0 \\ 4\_z1\, e^{2\_z1} & \_z1 < 1 \\ \frac{4e^2}{\_z1} & 1 \le \_z1 \end{cases} \right) \right.$$

$$\left. \left. d\_z1 \right) \left(\begin{cases} 1 & x < 1 \\ x^2 & 1 \le x \end{cases} \right) \left(\begin{cases} 0 & x < 0 \\ 1 & 0 \le x \end{cases} \right) + \left(\begin{cases} 0 & x < 0 \\ -3 & x < 1 \\ -3x^2 & 1 \le x \end{cases} \right) \right. \tag{3}$$

$$+\left(\begin{cases} 0 & x<1 \\ -3\,e^{-2x}x^2 & 1\le x \end{cases}\right)-e^{-2x}\left(\begin{cases} 0 & x<1 \\ 1 & 1\le x \end{cases}\right)\left(\right.$$

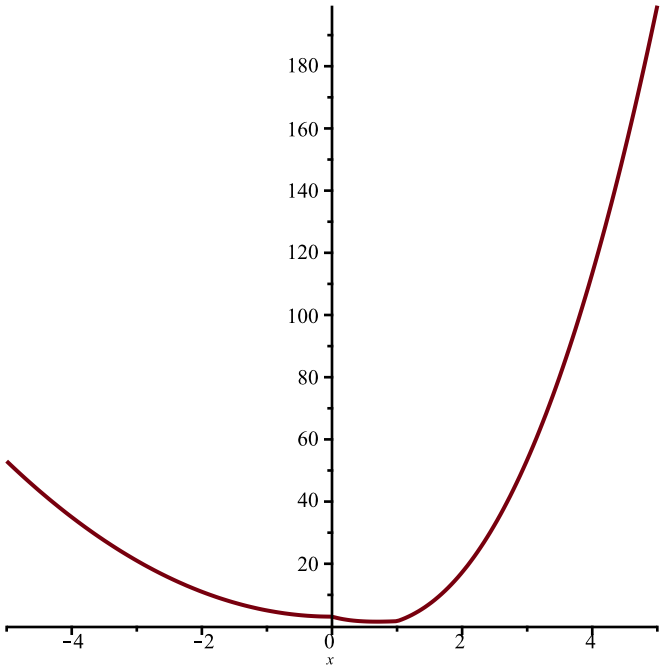
$$\int_0^x\left(\begin{cases} 4\_z1 & \_z1<0 \\ 4\,\_z1\,e^{2\,\_z1} & \_z1<1 \\ \frac{4\,e^2}{\_z1} & 1\le \_z1 \end{cases}\right)d\_z1\left(\begin{cases} 1 & x<1 \\ x^2 & 1\le x \end{cases}\right)+e^{-2x}\left(\right.$$

$$\int_0^x\left(\begin{cases} 4\_z1 & \_z1<0 \\ 4\,\_z1\,e^{2\,\_z1} & \_z1<1 \\ \frac{4\,e^2}{\_z1} & 1\le \_z1 \end{cases}\right)d\_z1\left(\begin{cases} 1 & x<1 \\ x^2 & 1\le x \end{cases}\right)\left(\begin{cases} 0 & x<0 \\ 1 & 0\le x \end{cases}\right)$$

$$+\left(\begin{cases} 0 & x<1 \\ 3\,e^{-2}x^2 & 1\le x \end{cases}\right)+\left(\begin{cases} 0 & x<0 \\ 3\,e^{-2x} & x<1 \\ 3\,e^{-2x}x^2 & 1\le x \end{cases}\right)+e^{-2}\left(\begin{cases} 0 & x<1 \\ 1 & 1\le x \end{cases}\right)\left(\right.$$

$$\int_0^x\left(\begin{cases} 4\_z1 & \_z1<0 \\ 4\,\_z1\,e^{2\,\_z1} & \_z1<1 \\ \frac{4\,e^2}{\_z1} & 1\le \_z1 \end{cases}\right)d\_z1\left(\begin{cases} 1 & x<1 \\ x^2 & 1\le x \end{cases}\right)$$

> $plot(rhs(A), x=-5..5)$



Problem 2

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>
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> q := piecewise(0 ≤ t < 4, 1, 4 ≤ t < 5, 0, t ≥ 5, 1)
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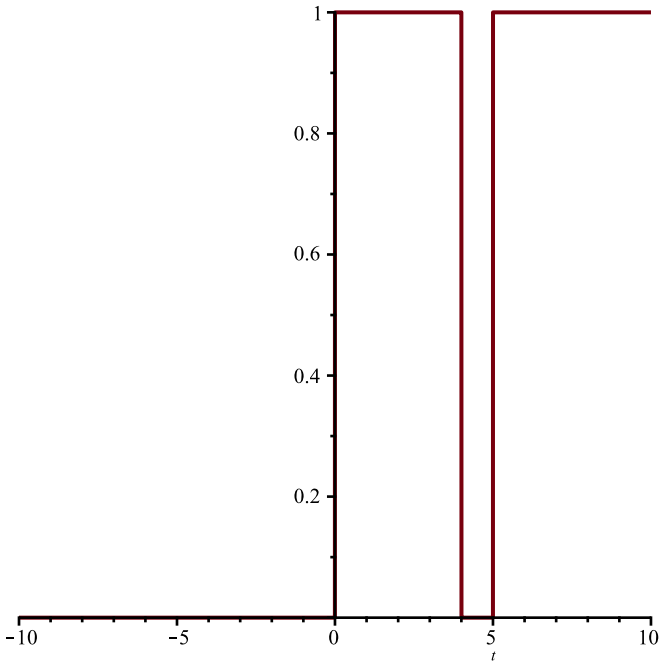
$$q := \begin{cases} 1 & 0 \le t < 4 \\ 0 & 4 \le t < 5 \\ 1 & 5 \le t \end{cases} \quad \textbf{(4)}$$

```
> plot(q)
```




```
> with(inttrans) :
> laplace(q, t, s)
```

$$\frac{1 - e^{-4s} + e^{-5s}}{s} \quad \textbf{(5)}$$

Problem 3

```
> with(LinearAlgebra) :
> B := Matrix([[4, 1, 4], [1, 7, 1], [4, 1, 4]])
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$$B := \begin{bmatrix} 4 & 1 & 4 \\ 1 & 7 & 1 \\ 4 & 1 & 4 \end{bmatrix} \quad \textbf{(6)}$$

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> Eigenvalues(B)
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$$\begin{bmatrix} 0 \\ 9 \\ 6 \end{bmatrix} \quad \textbf{(7)}$$

```
> Eigenvectors(B)
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(8)

$$\left[\begin{bmatrix} 9 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}\right] \quad (8)$$

Problem 5

> $C := Matrix([[1, 2, 3], [3, -2, 1], [4, 1, 1]])$

$$C := \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 1 & 1 \end{bmatrix} \quad (9)$$

> $MatrixInverse(C)$

$$\begin{bmatrix} -\frac{3}{32} & \frac{1}{32} & \frac{1}{4} \\ \frac{1}{32} & -\frac{11}{32} & \frac{1}{4} \\ \frac{11}{32} & \frac{7}{32} & -\frac{1}{4} \end{bmatrix} \quad (10)$$

>

>

Problem 4

> $X := diff(x1(t), t) = -3 \cdot x1(t) + 5 \cdot x2(t) - 5 \cdot x3(t)$

$$X := \frac{d}{dt} x1(t) = -3\, x1(t) + 5\, x2(t) - 5\, x3(t) \quad (11)$$

> $Y := diff(x2(t), t) = -7 \cdot x1(t) + 9 \cdot x2(t) - 5 \cdot x3(t)$

$$Y := \frac{d}{dt} x2(t) = -7\, x1(t) + 9\, x2(t) - 5\, x3(t) \quad (12)$$

> $Z := diff(x3(t), t) = -7 \cdot x1(t) + 7 \cdot x2(t) - 3 \cdot x3(t)$

$$Z := \frac{d}{dt} x3(t) = -7\, x1(t) + 7\, x2(t) - 3\, x3(t) \quad (13)$$

> $W := \{X, Y, Z\} : ics := \{x1(0) = 4, x2(0) = -5, x3(0) = -3\}$

$$ics := \{x1(0) = 4, x2(0) = -5, x3(0) = -3\} \quad (14)$$

> $combine(dsolve(W \textbf{ union } ics, \{x1(t), x2(t), x3(t)\}))$

$$\{x1(t) = -2\, e^{2t} + 6\, e^{-3t}, x2(t) = -2\, e^{2t} + 6\, e^{-3t} - 9\, e^{4t}, x3(t) = 6\, e^{-3t} - 9\, e^{4t}\} \quad (15)$$

>