MAT250
Midterm

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Problem 1

```
> sol := diff(y(x), x) + p·y(x) = x
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$$sol := \frac{d}{dx}\,y(x) + \begin{cases} 1 & 0 \le x < 1 \\ -\frac{1}{x} & 1 \le x \end{cases} y(x) = x \tag{1}$$

```
> A := dsolve({y(0) = 1, sol})
```

$$A := y(x) = \begin{cases} 1 & x < 1 \\ x & 1 \le x \end{cases} \left(\int_0^x \begin{cases} \_z1 & \_z1 < 0 \\ e^{-\_z1}\,\_z1 & \_z1 < 1 \\ e & 1 \le \_z1 \end{cases} d\_z1 \right) + \begin{cases} 1 & x < 1 \\ x & 1 \le x \end{cases} \tag{2}$$

$$+ \begin{cases} 0 & x < 0 \\ -1 & x < 1 \\ -x & 1 \le x \end{cases} - \left(\int_0^x \begin{cases} \_z1 & \_z1 < 0 \\ e^{-\_z1}\,\_z1 & \_z1 < 1 \\ e & 1 \le \_z1 \end{cases} d\_z1 \right) \begin{cases} 1 & x < 1 \\ x & 1 \le x \end{cases} \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \end{cases} + \begin{cases} 0 & x < 1 \\ -e^{-x}x & 1 \le x \end{cases}$$

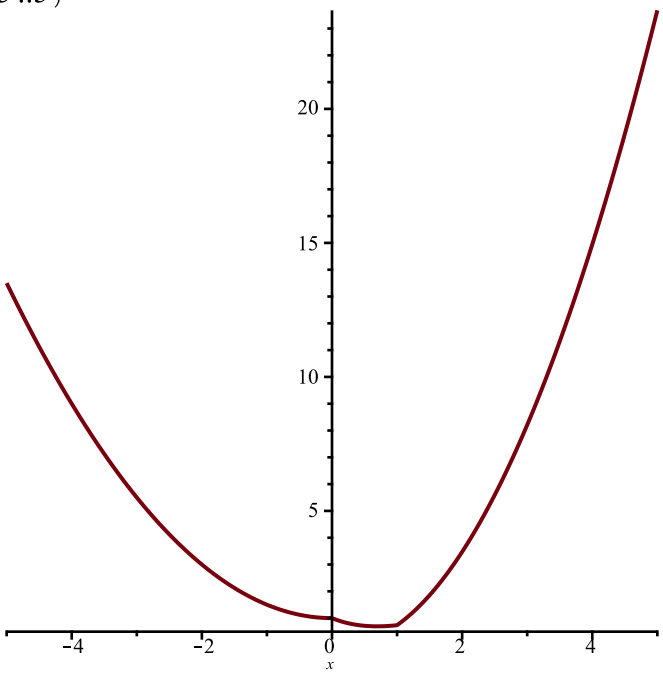
$$- e^{-x} \begin{cases} 0 & x < 1 \\ 1 & 1 \le x \end{cases} \left(\int_0^x \begin{cases} \_z1 & \_z1 < 0 \\ e^{-\_z1}\,\_z1 & \_z1 < 1 \\ e & 1 \le \_z1 \end{cases} d\_z1 \right) \begin{cases} 1 & x < 1 \\ x & 1 \le x \end{cases}$$

$$+ e^{-1} \begin{cases} 0 & x < 1 \\ 1 & 1 \le x \end{cases} \left(\int_0^x \begin{cases} \_z1 & \_z1 < 0 \\ e^{-\_z1}\,\_z1 & \_z1 < 1 \\ e & 1 \le \_z1 \end{cases} d\_z1 \right) \left(\begin{cases} 1 & x < 1 \\ x & 1 \le x \end{cases} + e^{-x} \right.$$

$$\int_0^x \left(\begin{cases} \_z1 & \_z1 < 0 \\ e^{-\_z1}\,\_z1 & \_z1 < 1 \\ e & 1 \le \_z1 \end{cases} d\_z1 \right) \begin{cases} 1 & x < 1 \\ x & 1 \le x \end{cases} \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \end{cases}$$

$$+ \begin{cases} 0 & x < 1 \\ e^{-1}\,x & 1 \le x \end{cases} + \begin{cases} 0 & x < 0 \\ e^{-x} & x < 1 \\ e^{-x}\,x & 1 \le x \end{cases}$$

```
> plot( rhs(A), x=-5..5)
```



Problem 2:

```
> with(LinearAlgebra) :
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```
> B := Matrix([[1, 0, 1], [2,-2,-1], [3, 0, 0]])
```

$$B := \begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{bmatrix} \quad \textbf{(3)}$$

```
> Eigenvalues(B)
```

(4)

$$\begin{bmatrix} -2 \\ \frac{1}{2} - \frac{\sqrt{13}}{2} \\ \frac{1}{2} + \frac{\sqrt{13}}{2} \end{bmatrix} \tag{4}$$

> $Eigenvectors(B)$

$$\left[\begin{bmatrix} -2 \\ \frac{1}{2} + \frac{\sqrt{13}}{2} \\ \frac{1}{2} - \frac{\sqrt{13}}{2} \end{bmatrix}, \begin{bmatrix} 0 & \frac{1}{-\frac{1}{2} + \frac{\sqrt{13}}{2}} & \frac{1}{-\frac{1}{2} - \frac{\sqrt{13}}{2}} \\ 1 & \frac{-2 + \sqrt{13}}{3\left(\frac{5}{2} + \frac{\sqrt{13}}{2}\right)} & \frac{-2 - \sqrt{13}}{3\left(\frac{5}{2} - \frac{\sqrt{13}}{2}\right)} \\ 0 & 1 & 1 \end{bmatrix}\right] \tag{5}$$

Problem 3:

> $with(inttrans)$:

> $laplace(\exp(-t)\cdot(5\cdot\cos(5\cdot t) + 7\cdot\sin(5\cdot t)), t, s)$

$$\frac{5\,(8+s)}{(s+1)^2 + 25} \tag{6}$$

Problem 4:

> $P := Matrix([[1, 2, 3], [3, -2, 1], [4, 1, 1]])$

$$P := \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 1 & 1 \end{bmatrix} \tag{7}$$

> $MatrixInverse(P)$

$$\begin{bmatrix} -\frac{3}{32} & \frac{1}{32} & \frac{1}{4} \\ \frac{1}{32} & -\frac{11}{32} & \frac{1}{4} \\ \frac{11}{32} & \frac{7}{32} & -\frac{1}{4} \end{bmatrix} \tag{8}$$