MAT250
Quiz 1
Abeda Zahid Chandrica
ID 22341061

1.

```
> p := piecewise(0 < x < 2, 1, x ≥ 2, -4/x)
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$$p := \begin{cases} 1 & 0 < x < 2 \\ -\frac{4}{x} & 2 \le x \end{cases} \tag{1}$$

```
> sol := diff(y(x), x) + p·y(x) = 5·x
```

$$sol := \frac{d}{dx} y(x) + \left(\begin{cases} 1 & 0 < x < 2 \\ -\frac{4}{x} & 2 \le x \end{cases} \right) y(x) = 5\,x \tag{2}$$

```
> B := dsolve({y(0) = 2, sol})
```

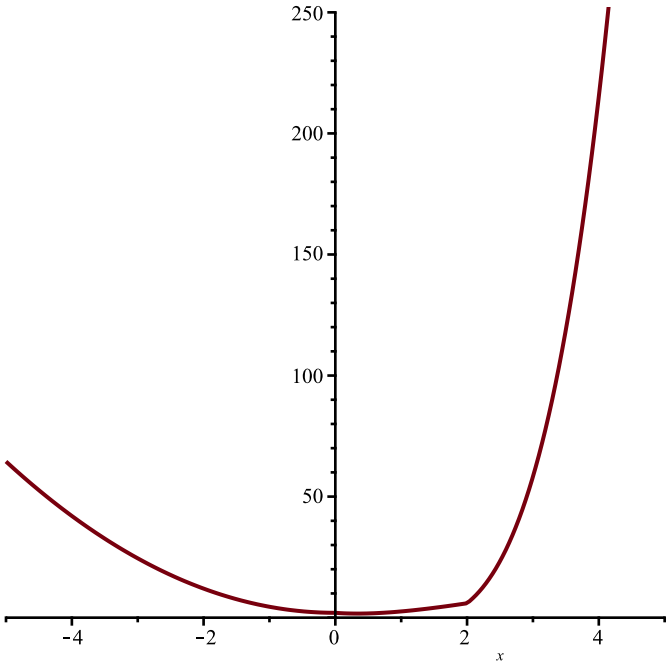
$$B := y(x) = \left(\begin{cases} 0 & x < 0 \\ -2 & x < 2 \\ -\frac{x^4}{8} & 2 \le x \end{cases} \right) + e^{-2} \left(\begin{cases} 0 & x < 2 \\ 1 & 2 \le x \end{cases} \right) \int_0^x \left(\begin{cases} 5\,\_z1 & \_z1 < 0 \\ 5\,\_z1\, e^{-\_z1} & \_z1 < 2 \\ \frac{80\, e^2}{\_z1^3} & 2 \le \_z1 \end{cases} \right) d\_z1 \left(\begin{cases} 1 & x < 2 \\ \frac{1}{16} & 2 \le x \end{cases} \right) \left(\begin{cases} 1 & x < 2 \\ x^4 & 2 \le x \end{cases} \right) + \int_0^x \left(\begin{cases} 5\,\_z1 & \_z1 < 0 \\ 5\,\_z1\, e^{-\_z1} & \_z1 < 2 \\ \frac{80\, e^2}{\_z1^3} & 2 \le \_z1 \end{cases} \right) d\_z1 \left(\begin{cases} 1 & x < 2 \\ \frac{1}{16} & 2 \le x \end{cases} \right) \left(\begin{cases} 1 & x < 2 \\ x^4 & 2 \le x \end{cases} \right) + \left(\begin{cases} 2 & x < 2 \\ \frac{x^4}{8} & 2 \le x \end{cases} \right) \tag{3}$$

$$-e^{-x}\left(\begin{cases} 0 & x<2 \\ 1 & 2\le x \end{cases}\right)\left(\int_0^x \left(\begin{cases} 5\,\_z1 & \_z1<0 \\ 5\,\_z1\,e^{-\_z1} & \_z1<2 \\ \dfrac{80\,e^2}{\_z1^3} & 2\le \_z1 \end{cases}\right) d\_z1\right)\left(\begin{cases} 1 & x<2 \\ \dfrac{1}{16} & 2\le x \end{cases}\right)\left(\begin{cases} 1 & x<2 \\ x^4 & 2\le x \end{cases}\right) - \left(\int_0^x \left(\begin{cases} 5\,\_z1 & \_z1<0 \\ 5\,\_z1\,e^{-\_z1} & \_z1<2 \\ \dfrac{80\,e^2}{\_z1^3} & 2\le \_z1 \end{cases}\right) d\_z1\right)\left(\begin{cases} 1 & x<2 \\ x^4 & 2\le x \end{cases}\right)\left(\begin{cases} 1 & x<2 \\ \dfrac{1}{16} & 2\le x \end{cases}\right)\left(\begin{cases} 0 & x<0 \\ 1 & 0\le x \end{cases}\right)$$

$$+\left(\begin{cases} 0 & x<2 \\ -\dfrac{e^{-x}x^4}{8} & 2\le x \end{cases}\right)+e^{-x}\left(\int_0^x \left(\begin{cases} 5\,\_z1 & \_z1<0 \\ 5\,\_z1\,e^{-\_z1} & \_z1<2 \\ \dfrac{80\,e^2}{\_z1^3} & 2\le \_z1 \end{cases}\right) d\_z1\right)\left(\begin{cases} 1 & x<2 \\ x^4 & 2\le x \end{cases}\right)\left(\begin{cases} 1 & x<2 \\ \dfrac{1}{16} & 2\le x \end{cases}\right)\left(\begin{cases} 0 & x<0 \\ 1 & 0\le x \end{cases}\right)$$

$$+\left(\begin{cases} 0 & x<2 \\ \dfrac{e^{-2}x^4}{8} & 2\le x \end{cases}\right)+\left(\begin{cases} 0 & x<0 \\ 2\,e^{-x} & x<2 \\ \dfrac{e^{-x}x^4}{8} & 2\le x \end{cases}\right)$$

```
> plot( rhs(B), x=-5..5)
```



2.

(4)

```
> with(LinearAlgebra) :
> A := Matrix([[1, 1, -1], [1, 14, 1], [1, 1, -1]])
```

$$A := \begin{bmatrix} 1 & 1 & -1 \\ 1 & 14 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad \textbf{(5)}$$

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> Eigenvalues(A)
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$$\begin{bmatrix} 0 \\ 7+\sqrt{51} \\ 7-\sqrt{51} \end{bmatrix} \quad \textbf{(6)}$$

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> Eigenvectors(A)
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$$\begin{bmatrix} 0 \\ 7+\sqrt{51} \\ 7-\sqrt{51} \end{bmatrix}, \quad \textbf{(7)}$$

$$\begin{bmatrix} \frac{15}{13} & -\frac{15\left(-8+\sqrt{51}\right)}{\left(-99+14\sqrt{51}\right)\left(6+\sqrt{51}\right)} & -\frac{15\left(-8-\sqrt{51}\right)}{\left(-99-14\sqrt{51}\right)\left(6-\sqrt{51}\right)} \\ -\frac{2}{13} & -\frac{-21+\sqrt{51}}{-99+14\sqrt{51}} & -\frac{-21-\sqrt{51}}{-99-14\sqrt{51}} \\ 1 & 1 & 1 \end{bmatrix}$$

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