MAT250 Assignment 5
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1.

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> with(LinearAlgebra) :
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> p(x) := piecewise(0 ≤ x < 4, 1, 4 ≤ x < 5, 0, x ≥ 5, 1)
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$$p := x \mapsto \begin{cases} 1 & 0 \le x < 4 \\ 0 & 4 \le x < 5 \\ 1 & 5 \le x \end{cases} \quad \textbf{(1)}$$

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> plot(p(x))
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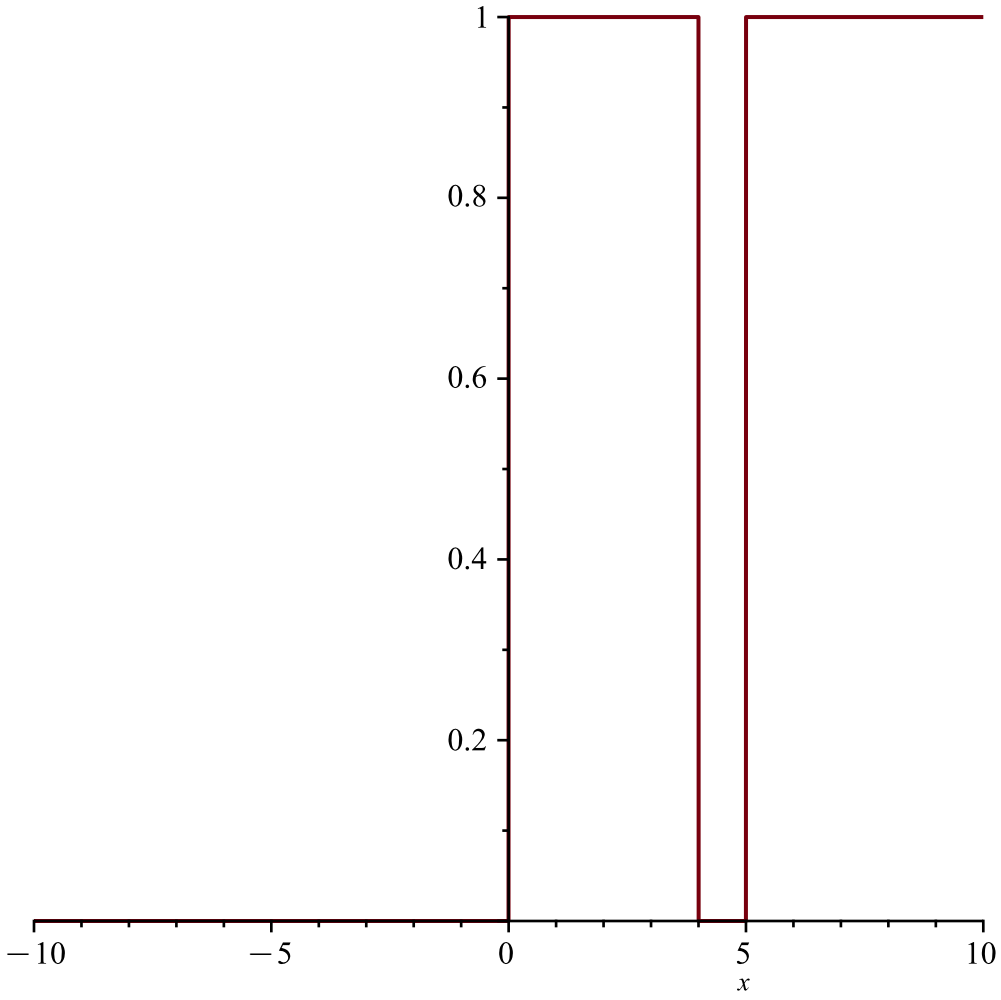


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> with(inttrans) :
> laplace(p(x), x, s)
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(2)

$$\frac{-e^{-4s}+1+e^{-5s}}{s} \tag{2}$$

2.

> $A := diff(x1(t),t) = 0.5\cdot x1(t)$

$$A := \frac{d}{dt}x1(t) = 0.5\,x1(t) \tag{3}$$

> $B := diff(x2(t),t) = x1(t) - 0.5\cdot x2(t)$

$$B := \frac{d}{dt}x2(t) = x1(t) - 0.5\,x2(t) \tag{4}$$

> $C := \{A,B\} : ics := \{x1(0)=3,\ x2(0)=5\}$

$$ics := \{x1(0)=3, x2(0)=5\} \tag{5}$$

> $combine(dsolve(C\ \mathbf{union}\ ics, \{x1(t),x2(t)\}))$

$$\left\{x1(t) = 3\,e^{\frac{t}{2}}, x2(t) = 3\,e^{\frac{t}{2}} + 2\,e^{-\frac{t}{2}}\right\} \tag{6}$$

3.

> $P := diff(x1(t),t) = x1(t) + x2(t) + 4\cdot x3(t)$

$$P := \frac{d}{dt}x1(t) = x1(t) + x2(t) + 4\,x3(t) \tag{7}$$

> $Q := diff(x2(t),t) = 2\cdot x2(t)$

$$Q := \frac{d}{dt}x2(t) = 2\,x2(t) \tag{8}$$

> $R := diff(x3(t),t) = x1(t) + x2(t) + x3(t)$

$$R := \frac{d}{dt}x3(t) = x1(t) + x2(t) + x3(t) \tag{9}$$

> $S := \{P,Q,R\} : ics := \{x1(0)=1, x2(0)=3, x3(0)=0\}$

$$ics := \{x1(0)=1, x2(0)=3, x3(0)=0\} \tag{10}$$

> $combine(dsolve(S\ \mathbf{union}\ ics, \{x1(t),x2(t),x3(t)\}))$

$$\left\{x1(t) = 5\,e^{3t} + e^{-t} - 5\,e^{2t}, x2(t) = 3\,e^{2t}, x3(t) = \frac{5\,e^{3t}}{2} - \frac{e^{-t}}{2} - 2\,e^{2t}\right\} \tag{11}$$

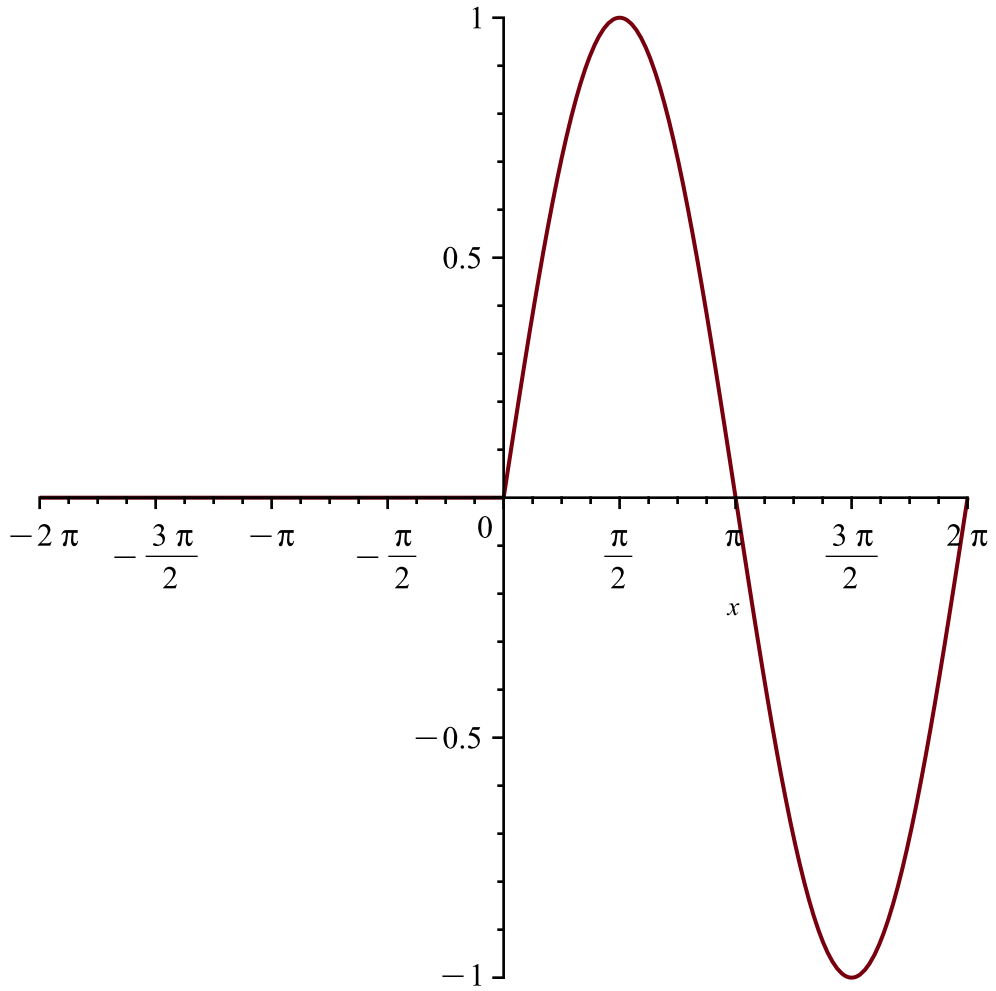
>

4.

> $p(x) := piecewise(0 \le x < 2\cdot\pi, sin(x), x \ge 2\cdot\pi, 0)$

$$p := x \mapsto \begin{cases} \sin(x) & 0 \le x < 2\cdot\pi \\ 0 & 2\cdot\pi \le x \end{cases} \tag{12}$$

> $plot(p(x), x)$



> $laplace(p(x), x, s)$

$$\frac{1 - e^{-2\pi s}}{s^2 + 1} \quad \textbf{(13)}$$

>

5.

> $X := diff(x1(t), t) = -3 \cdot x1(t) + 5 \cdot x2(t) - 5 \cdot x3(t)$

$$X := \frac{d}{dt} x1(t) = -3\, x1(t) + 5\, x2(t) - 5\, x3(t) \quad \textbf{(14)}$$

> $Y := diff(x2(t), t) = -7 \cdot x1(t) + 9 \cdot x2(t) - 5 \cdot x3(t)$

$$Y := \frac{d}{dt} x2(t) = -7\, x1(t) + 9\, x2(t) - 5\, x3(t) \quad \textbf{(15)}$$

> $Z := diff(x3(t), t) = -7 \cdot x1(t) + 7 \cdot x2(t) - 3 \cdot x3(t)$

$$Z := \frac{d}{dt} x3(t) = -7\, x1(t) + 7\, x2(t) - 3\, x3(t) \quad \textbf{(16)}$$

> $W := \{X, Y, Z\} : ics := \{x1(0) = 4, x2(0) = -5, x3(0) = -3\}$

$$ics := \{x1(0) = 4, x2(0) = -5, x3(0) = -3\} \quad \textbf{(17)}$$

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> combine(dsolve(W union ics, {x1(t), x2(t), x3(t)}))
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$$\{x1(t) = -2\,e^{2t} + 6\,e^{-3t},\ x2(t) = -2\,e^{2t} + 6\,e^{-3t} - 9\,e^{4t},\ x3(t) = 6\,e^{-3t} - 9\,e^{4t}\} \qquad \textbf{(18)}$$

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