

Per Sentence.

$$S_t = \max[0, (uX_t + wS_{t-1})]$$

$$\Rightarrow S_t = \tanh(uX_t + wS_{t-1})$$

$$\frac{d \tanh x}{dx} = 1 - \tanh^2 x$$

$$\frac{d \max(0, x)}{dx} = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$\frac{\partial S_1}{\partial w} = \frac{\partial (a + wS_0)}{\partial w} = S_0$$

$$\frac{\partial S_1}{\partial w} = \frac{\partial \tanh(a + wS_0)}{\partial w} = (1 - S_1^2) \cdot \frac{\partial (a + wS_0)}{\partial w} = (1 - S_1^2) \cdot S_0$$

$$\begin{aligned} \frac{\partial S_2}{\partial w} &= \cancel{\phi} = (1 - S_2^2) \cdot \frac{\partial (a + wS_1)}{\partial w} \\ &= (1 - S_2^2) \cdot \left( S_1 + \underbrace{\frac{\partial S_1}{\partial w} \cdot w}_{\text{chain rule}} \right) \\ &= (1 - S_2^2) \cdot \left( S_1 + \frac{\partial S_1}{\partial w} \cdot w \right) \end{aligned}$$

$$\begin{aligned} \frac{\partial S_3}{\partial w} &= (1 - S_3^2) \cdot \frac{\partial (a + wS_2)}{\partial w} \\ &= (1 - S_3^2) \cdot \left( S_2 + \frac{\partial S_2}{\partial w} \cdot w \right) \\ &= \end{aligned}$$

$$\frac{\partial E_3}{\partial w} = \frac{\partial E_3}{\partial \hat{y}_3} \cdot \frac{\partial \hat{y}_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial w}$$

$\downarrow$   
 $(\hat{y}_3 - y_3) \cdot \sqrt{\phantom{x}}$

in wild ML.

$$\text{delta}_t = (\hat{y}_3 - y_3) \cdot V \cdot (1 - S_3^2)$$

$$dw = \text{delta}_t \oplus S_2$$