

confidence interval - data problem

$$n = 44$$

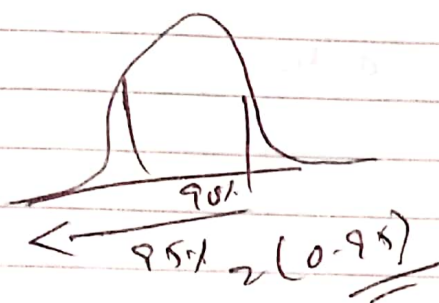
$$\text{mean} = 10.455$$

$$\sigma_{\text{pop}} = 2.2$$

$$\mu_{\text{pop}} \in [\mu_s \pm 2SE]$$

$$= \left[ 10.455 \pm 2 \frac{\sigma_{\text{pop}}}{\sqrt{n}} \right]$$

$$= 10.455 \pm 2 \times \frac{2.2}{\sqrt{44}} = 9.57.$$



for 90% confidence interval

$$\mu_s \pm 1.65 SE$$

Confidence Interval Assignment

2)  $\sigma = 2.6 \text{ min/hr}$   
 $n = 120$

1)  $n_s = 1000$   
 $n_{\text{pop}} = 1,000,000$   
 $\mu = 180$   
 $\sigma = 30$

$$\mu_s \pm 2\sigma$$

$$180 \pm 2(30)$$

$$180 \pm 60$$

$$95\% = (120, 240)$$

$$\mu_s \pm 2 \frac{s}{\sqrt{n}}$$

$$180 \pm 2 \frac{30}{\sqrt{1000}}$$

$$180 \pm 2 \cdot \frac{30}{10\sqrt{10}}$$

$$180 \pm \frac{6}{\sqrt{10}}$$

2)  $\mu_p = 3.6 \text{ m.}$

a)  $n = 120$   $\mu = 16.2$

$$\mu_s \pm 2 \frac{s}{\sqrt{n}}$$

$$16.2 \pm 2 \cdot \frac{s}{\sqrt{n}}$$

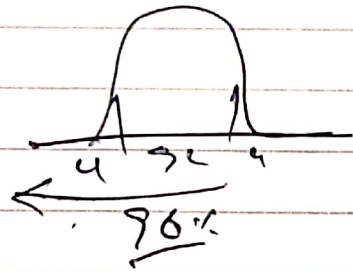
$$16.2 \pm 2 \cdot \frac{3.6}{\sqrt{120}}$$

$$\pm 2 \cdot \frac{3.6}{10.95}$$

$$16.2 \pm 6.575$$

to 9.625, 22.775

92%  $16.2 \pm (1.26) \cdot \frac{3.6}{\sqrt{120}}$   
 $16.2 \pm 5.286$   
 $(10.914, 21.486)$





b)  $\pm 15$  sec. (margin of error)

margin of error:  $\pm 15$  sec

$$\pm 15 = \pm 2 \times SE$$

$$= \pm 1.95 \times \frac{\sigma_{pop}}{\sqrt{n}}$$

$$\pm 15 = \pm 1.95 \times \frac{36}{\sqrt{n}}$$

$$\sqrt{n} = \frac{1.95 \times 36}{15}$$

$$n = \left( \frac{1.95 \times 36}{15} \right)^2$$

$$n = \underline{\underline{635}}$$

proportion given instead of frequency

$$Z = \frac{\hat{p} + \frac{0.5}{n} - p_0}{\sqrt{p_0 \left( \frac{1-p_0}{n} \right)}}$$

SE of  
Sample  
proportion  
 $\sqrt{\frac{p_0}{n}}$

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}}$$

$$SE = \sqrt{\frac{p_0 q_0}{n}}$$

Sample = 950

predicted: 49% =  $p_1$

$q_1 = 95\%$

margin of error:  $2 \times SE = 1\%$

$$ME = 1\% = 2 \times SE$$

$$0.01 = 1.96 \sqrt{\frac{p_0 q_0}{n}}$$

$$= 1.96 \times \sqrt{\frac{0.49 \times 0.51}{n}}$$

$$\frac{n}{0.49 \times 0.51} = \left( \frac{1.96}{0.01} \right)^2$$

$$n = \left( \frac{1.96}{0.01} \right)^2 \times 0.49 \times 0.51$$

$$n = 9600$$



What happens to Confidence Interval as  
Confidence level changes

$$CI = \mu \pm z \cdot SE$$

$$99\% = 3$$

$$95\% = 2$$

$$90\% = 1$$

Confidence level increases

$$\left( \begin{array}{c} \text{Sample} \\ 30-120 \end{array} \right) 99\%$$

$$(80-85) \rightarrow 80\%$$

happens Confidence interval of Sample size changes

$$CI = \mu \pm z \cdot \sqrt{\frac{p \cdot q}{n}} \rightarrow \text{decreases}$$

size decreases

Sample size increases  
decreases

When CI decreases

then CI becomes less accurate

Shortcuts for calculating Confidence interval

normal  
you know  $\sigma^2$   
 $n$  is large & small

$\bar{x}$  is the Sample mean

variance  $\sigma^2$

$n$  is large ( $> 30$ )

$\bar{x}$  is the Sample mean

$$CI = \bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} \pm 2 \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} \pm 2 \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} \pm 2 \cdot \frac{\sigma}{\sqrt{n}}$$

3a)  $p = 0.5$

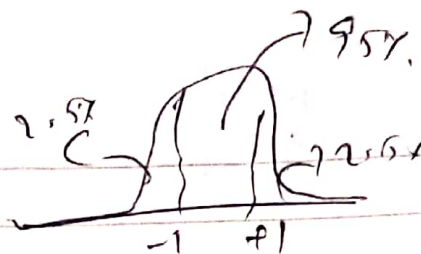
import  
 using Stata  
 of Stata  
 Stata user's guide (196)  
 Stata user's guide (196)  
 = 1.96

$$0.8413$$

$$0.8413$$

$$0.1587$$

$$\underline{0.6826}$$



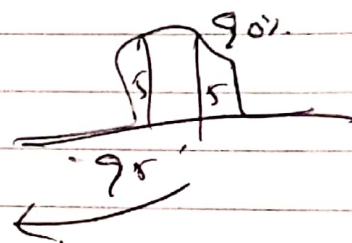
$$\underline{1.96}$$

$$1.96$$

$$ME = 2 \times SE$$

$$= PP(0.95)$$

$$0.02 = 1.65 \times \sqrt{\frac{0.5 \times 0.5}{n}}$$



$$0.02 = 1.65 \sqrt{\frac{0.25}{n}}$$

$$\frac{0.02}{1.65} = \left( \frac{0.02}{1.65} \right)^2$$

$$\frac{n}{0.25} = \left( \frac{1.65}{0.02} \right)^2$$

$$n = \left( \frac{1.65}{0.02} \right)^2 \times 0.25$$

$$PS \pm 2 \sqrt{\frac{PS(1-PS)}{n}}$$

$$= 0.5 \pm 1.96 \sqrt{\frac{0.5 \times 0.5}{1000}}$$