Matrix-Circle

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1 Problem Statement

To find angle QPR of the triangle PQR which is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (-4,3) respectively .

Symbol	Value	Description
Q	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	given point
R	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$	given point
P	$\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	A point on circle

Table 1: Parameters

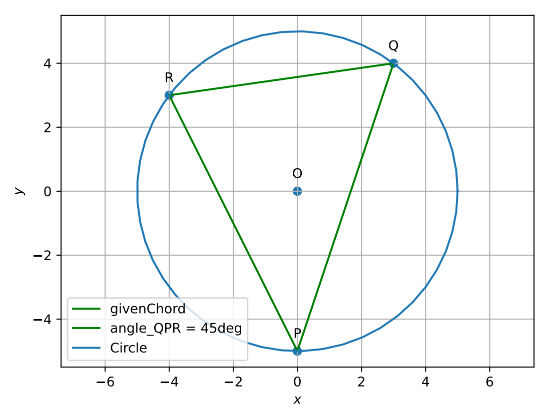

Figure 1: triangle inscribed in Circle and its angle QPR

2 Solution

Given that Points given are on the circle forms an inscribed triangle forms by joining the points (3,4),(-4,3) and any point on the given circle

Let $\mathbf{Q}=\begin{pmatrix} 3 \\ 4 \end{pmatrix}$,$\mathbf{R}=\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $\mathbf{P}=\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

The direction vector of the line joining two points A, B is given by

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{1}$$

From eq1 we get dirction vectors of $\mathbf{Q}, \mathbf{O}$ and $\mathbf{R}, \mathbf{O}$ as m1 and m2 respectively

$$\mathbf{m1} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \tag{2}$$

$$\mathbf{m2} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ -5 \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \end{pmatrix} \tag{3}$$

We know, The normal vector to direction vector is defined by:

$$\mathbf{m}^\top \mathbf{n} = 0. \tag{4}$$

From eq4 and eq2 weget

$$\begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{n1} = 0 \tag{5}$$

by eq5 we can cansider :

$$\mathbf{n1} = \begin{pmatrix} -9 \\ 3 \end{pmatrix} \tag{6}$$

From eq4 and eq3 weget

$$\begin{pmatrix} -4 & 8 \end{pmatrix} \mathbf{n2} = 0 \tag{7}$$

by solving eq7 we consider :

$$\mathbf{n2} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \tag{8}$$

The angle between two vectors is given by

$$\theta = cos^{-1} \frac{\mathbf{n1}^\top \mathbf{n2}}{\|\mathbf{n1}\| \|\mathbf{n2}\|} \tag{9}$$

By substituting $\mathbf{n1}\, and\, \mathbf{n2}$ we get

$$\measuredangle QPR = 45° \tag{10}$$

where,$\mathbf{P}$ is a random point on the given circle

3 Software

Download the following code using,

svn co https://github.com/chanduputta/FWC-Module1Assignments/blob/main/circle/circle.py

and execute the code by using command

Python3 circle.py

4 Conclusion

We found the $\measuredangle QPR$ of the $\triangle PQR$ which is inscribed in the circle $x^2 + y^2 = 25$. Where P is any point on the circle as 45°.