Matrix-Circle

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1 Problem Statement

To find angle QPR of the triangle PQR which is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (-4,3) respectively .

Symbol	Value	Description
$\mathbf{Q}$	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	given point
$\mathbf{R}$	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$	given point
$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	center of given circle

Table 1: Parameters

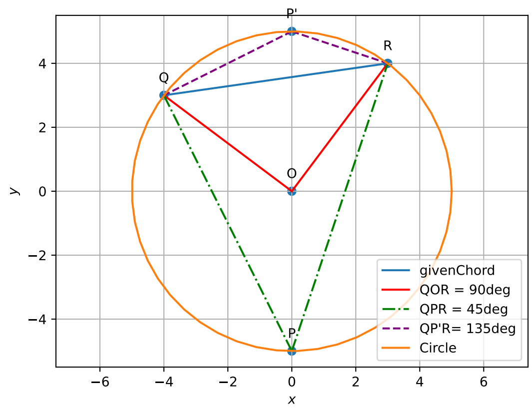

Figure 1: triangle inscribed in Circle and its angle QPR

2 Solution

Given that Points given are on the circle forms an inscribed triangle forms by joining the points (3,4),(-4,3) and any point on the given circle

Let $\mathbf{Q}=\begin{pmatrix} 3 \\ 4 \end{pmatrix}$,$\mathbf{R}=\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $\mathbf{O}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Given equation of circle is $\mathbf{x}^\top \mathbf{I}\ \mathbf{x} = 25$.

The direction vector of the line joining two points A, B is given by

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{1}$$

From eq1 we get dirction vectors of $\mathbf{Q}, \mathbf{O}$ and $\mathbf{R}, \mathbf{O}$ as m1 and m2 respectively

$$\mathbf{m1} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{2}$$

$$\mathbf{m2} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{3}$$

The normal vector to m is defined by:

$$\mathbf{m}^\top \mathbf{n} = 0. \tag{4}$$

From eq4 and eq2 weget

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{n1} = 0 \tag{5}$$

by eq5 we can cansider :

$$\mathbf{n1} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{6}$$

From eq4 and eq3 weget

$$\begin{pmatrix} -4 & 3 \end{pmatrix} \mathbf{n2} = 0 \tag{7}$$

by solving eq7 we consider :

$$\mathbf{n2} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{8}$$

The angle between two vectors is given by

$$\theta = cos^{-1} \frac{\mathbf{n1}^\top \mathbf{n2}}{\|\mathbf{n1}\| \|\mathbf{n2}\|} \tag{9}$$

By substituting $\mathbf{n1}\, and\, \mathbf{n2}$ we get

$$\measuredangle QOR = 90^\circ \tag{10}$$

The Central Angle Theorem states that the measure of inscribed angle is always half the measure of the central angle.

i.e

$$\angle QOR = 2\angle QPR \tag{11}$$

by eq11 and eq10

$$\angle QPR = 45^\circ \tag{12}$$

where,$\mathbf{P}$ is on major arc of the given circle

Additional:

Let $\mathbf{P'}$ is any point on minor arc of the given circle

Then,

$$\angle QP'R = 180^\circ - \angle QPR \tag{13}$$

From eq13 and eq12

$$\measuredangle QP'R = 135^\circ \tag{14}$$

3 Software

Download the following code using,

```
svn co https://github.com/chanduputta/
FWC-Module1Assignments/blob/
main/circle/circle.py
```

and execute the code by using command

Python3 circle.py

4 Conclusion

We found the $\measuredangle QPR$ of the $\triangle PQR$ which is inscribed in the circle $x^2 + y^2 = 25$. Where P is any point on the circle .