

# Matrix-Conic

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## 2 Solution

- 1 Let  $\mathbf{X}$  be any point on the Locus formed by the midpoint joining the point  $\mathbf{P}$  and any point on the given locus say, point  $\mathbf{Q}$

- 2 Where,  $\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\mathbf{Q} = \begin{pmatrix} x' \\ y' \end{pmatrix}$  and  $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$

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## 1 Problem Statement

To find the locus of mid point of  $\overline{PQ}$  where  $\mathbf{P}$  is  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\mathbf{Q}$  is a point on the locus  $y^2 = 8x$ .

Symbol	Value	Description
$\mathbf{P}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	given point
$\mathbf{Q}$	$\begin{pmatrix} x' \\ y' \end{pmatrix}$	point on given locus
$\mathbf{X}$	$\begin{pmatrix} x \\ y \end{pmatrix}$	mid point of $\overline{PQ}$

Table 1: Parameters

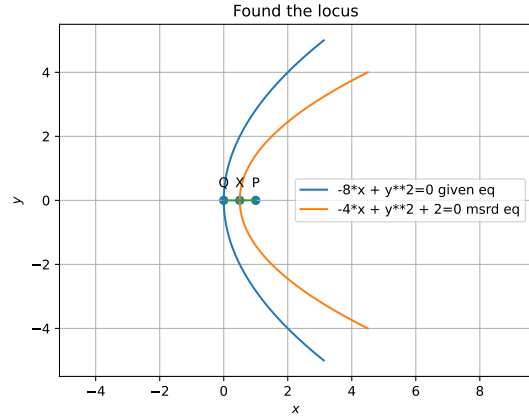


Figure 1: Found the locus equation

The given equation of parabola  $y^2 = 8x$  can be written in the general quadratic form as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (2) \quad 4\mathbf{X}^\top \mathbf{V} \mathbf{X} + 4\mathbf{u}^\top \mathbf{X} + 8 = 0 \quad (11)$$

$$\mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad (3) \quad \text{Therefore, required Locus equation of the mid point of given point } \mathbf{P} \text{ and } \mathbf{Q} \text{ is obtained}$$

$$f = 0 \quad (4) \quad \text{as:}$$

$$\mathbf{X}^\top \mathbf{V} \mathbf{X} + \mathbf{u}^\top \mathbf{X} + 2 = 0 \quad (12)$$

Substitute  $\mathbf{Q}$  and data in eq1 .

$$\mathbf{Q}^\top \mathbf{V} \mathbf{Q} + 2\mathbf{u}^\top \mathbf{Q} = 0 \quad (5)$$

By section formula mid point of line joining  $\mathbf{P}$  and  $\mathbf{Q}$  as  $\mathbf{X}$  is:

$$\mathbf{X} = \frac{\mathbf{Q} + \mathbf{P}}{2} \quad (6)$$

$$\mathbf{Q} = 2\mathbf{X} - \mathbf{P} \quad (7)$$

From eq5 and eq7 We get

$$(2\mathbf{X} - \mathbf{P})^\top \mathbf{V} (2\mathbf{X} - \mathbf{P}) + 2\mathbf{u}^\top (2\mathbf{X} - \mathbf{P}) = 0 \quad (8)$$

$$(2\mathbf{X}^\top \mathbf{V} - \mathbf{P}^\top \mathbf{V}) (2\mathbf{X} - \mathbf{P}) + 2\mathbf{u}^\top 2\mathbf{X} - 2\mathbf{u}^\top \mathbf{P} = 0 \quad (9)$$

$$(2\mathbf{X}^\top \mathbf{V} 2\mathbf{X} - 2\mathbf{X}^\top \mathbf{V} \mathbf{P}) + 2\mathbf{u}^\top 2\mathbf{X} - 2\mathbf{u}^\top \mathbf{P} = 0 \quad (10)$$

### 3 Software

Download the following code using,

```
svn co https://github.com/chanduputta/
FWC-Module1Assignments/blob/
main/conic/code/conic.py
```

and execute the code by using command

**cmd:**Python3 conic.py

### 4 Conclusion

We found the locus of mid point of  $\overline{PQ}$  where  $\mathbf{P}$  is  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\mathbf{Q}$  is a point on the locus  $y^2 = 8x$  as  $y^2 = 4x - 2$ .