Matrix-Lines

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CONTENTS

I. PROBLEM STATEMENT

Find equation of a line passing trough a point (2,2) and cutting off intercepts on the axes whose sum is 9.

Symbol	Value	Description
P	$\begin{pmatrix} a \\ 0 \end{pmatrix}$	Point on X-axis
Q	$\begin{pmatrix} 0 \\ b \end{pmatrix}$	Point on Y-axis
R	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	Given Point
a + b	9	Given Condition

TABLE I
PARAMETERS

II. CONSTRUCTION

Fig. 1. Equation of the Straight Line

III. SOLUTION

Given that resultant line passes through point(2,2) and intercepts on axes whose sum is 9 (let x intercept is a and y intercept is b therefore, a + b = 9)

so, b = 9 - a

Let $\mathbf{P} = \begin{pmatrix} a \\ 0 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 0 \\ 9-a \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Equation of line is $\mathbf{n}^\top \mathbf{X} = c$.

Now we have 3 points which lies on same line so,
The Equation of line through **P** is

$$\mathbf{n}^\top \begin{pmatrix} a \\ 0 \end{pmatrix} = c \tag{1}$$

Equation of line passing through **Q** is

$$\mathbf{n}^\top \begin{pmatrix} 0 \\ 9-a \end{pmatrix} = c \tag{2}$$

Now eq1 + eq2,

$$\mathbf{n}^\top \begin{pmatrix} a \\ 9-a \end{pmatrix} = 2c \tag{3}$$

Equation of line passing through **R** is

$$\mathbf{n}^\top \begin{pmatrix} 2 \\ 2 \end{pmatrix} = c \tag{4}$$

From eq3 and eq4 we can find normal vector **n**,

$$\mathbf{n}^\top \begin{pmatrix} a & 9-a \\ 2 & 2 \end{pmatrix} = c. \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{5}$$

Therefore,

$$\mathbf{n}^\top = \begin{pmatrix} a & 9-a \\ 2 & 2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} .c \qquad (6)$$

$$\mathbf{n}^\top = \begin{pmatrix} 3a-9 \\ -2 \end{pmatrix} \cdot \frac{c}{4a-18} \qquad (7)$$

Now eq4 can be expressed as,

$$\begin{pmatrix} 3a-9 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} \cdot \frac{c}{4a-18} = c \qquad (8)$$

Thus, we get **a = 2, b = 9-a = 7**

by substuting a in eq6, finally

$$\mathbf{n}^\top = \begin{pmatrix} 0.3 \\ 0.2 \end{pmatrix} .c \qquad (9)$$

The Resultant Equation of line is $\mathbf{n}^\top \mathbf{X} = c$

$$\begin{pmatrix} 0.3 \\ 0.2 \end{pmatrix} .\mathbf{X}.c = c \qquad (10)$$

i.e,

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} .\mathbf{X} = 10 \qquad (11)$$

Therefore equation of the line is,

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} .\mathbf{X} = 10 \qquad (12)$$

3x + 2y = 10

IV. SOFTWARE

Download the following code using,

svn co https://github.com/
mygit-sampath-govardhan/fwc-iith-assignments/blob/
5b65abbf8e5e3c803b1bff8cf4a95092e100de75/
Assignment-4(Matrices-line)/codes/Assignment4.py

and execute the code by using command

Python3 Assignment4.py

V. CONCLUSION

We found the equation of a line passing trough a point (2,2) and cutting off intercepts on the axes whose sum is 9.