Matrix-Circle

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1 Problem Statement

To find angle QPR of the triangle PQR which is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (-4,3) respectively .

2 Construction

Symbol	Value	Description
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Center point
r	5	Radius
Q	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	given point
R	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$	given point
P	$\begin{pmatrix} rcos(\alpha) \\ rsin(\alpha) \end{pmatrix}$	user defined point

Table 1: Parameters

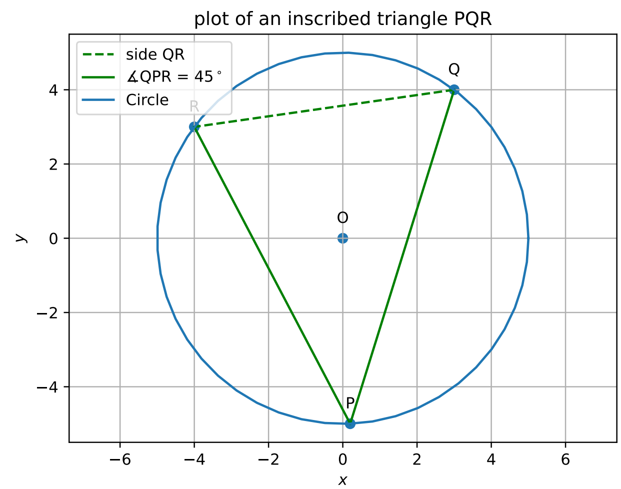

Figure 1: triangle inscribed in Circle and its angle QPR

3 Solution

Given that Points given are on the circle forms an inscribed triangle the points **Q**, **R** and any point**P** on the given circle

The circle given,

$$\mathbf{x}^\top \mathbf{x} = 25 \tag{1}$$

From General Equation:

$$\mathbf{x}^\top \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

where

$$\mathbf{u} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{3}$$

$$f = -25 \tag{4}$$

Centre and Radius of given circle are :

$$\mathbf{O} = -\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

$$r = \sqrt{\mathbf{u}^\top \mathbf{u} - f} = 5 \tag{6}$$

Point on the circle should satisfy the circle equation so, we take

$$\mathbf{P} = \begin{pmatrix} rcos(\alpha) \\ rsin(\alpha) \end{pmatrix} \tag{7}$$

$$\mathbf{Q} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{8}$$

$$\mathbf{R} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{9}$$

Considering $\mathbf{P}$,$\mathbf{Q}$and $\mathbf{R}$ as the Coordinates of the Inscribed Triangle

We know, The direction vector of the line joining two points $\mathbf{A}$, $\mathbf{B}$ is given by

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{10}$$

From eq11 we get dirction vectors of $\overline{\mathbf{PQ}}$ as $\mathbf{m1}$ i.e

$$\mathbf{m1} = \mathbf{Q} - \mathbf{P} \tag{11}$$

From eq11 we get dirction vector of $\overline{\mathbf{PR}}$ as $\mathbf{m2}$ i.e

$$\mathbf{m2} = \mathbf{R} - \mathbf{P} \tag{12}$$

The angle between two vectors is given by

$$\theta = cos^{-1}\frac{\mathbf{A}^\top \mathbf{B}}{\|\mathbf{A}\|\|\mathbf{B}\|} \tag{13}$$

By substituting $\mathbf{m1}\ and\ \mathbf{m2}$ in eq13 we get,

3.1 Case 1: Major Arc

If $\mathbf{P}$ lies on major Arc of the circle for given $\mathbf{Q}$ and $\mathbf{R}$ points

$$\measuredangle\mathbf{QPR} = 45^\circ \tag{14}$$

3.2 Case 2: Minor Arc

If $\mathbf{P}$ lies on minor Arc of the circle for given $\mathbf{Q}$ and $\mathbf{R}$ points.

$$\measuredangle\mathbf{QPR} = 135^\circ \tag{15}$$

where,$\mathbf{P}$ is a random point on the given circle for its $\boldsymbol{\alpha}$

4 Software

Download the following code using, and execute

https://github.com/chanduputta/FWC-Module1Assignments/blob/main/circle/circle.py

the code by using command

cmd1:Python3 circle.py
cmd2:Input your $\boldsymbol{\alpha}$ value (0 to 360°)

5 Conclusion

We found the $\measuredangle\mathbf{QPR}$ of the $\triangle\mathbf{PQR}$ which is inscribed in the circle $\mathbf{x^2 + y^2 = 25}$.
Where $\mathbf{P}$ is point on the circle as
i) $\mathbf{45^\circ}$, If $\mathbf{P}$ lies on Major Arc of Given Circle
ii) $\mathbf{135^\circ}$, If $\mathbf{P}$ lies on Minor Arc of Given Circle