Matrix-Lines

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1 Problem Statement

A line perpendicular to the line segement joining the points (1,0) and (2,3) divides it in the ratio 1:n. find the equation of the line? (note: we are taking n as user Input).

Symbol	Value	Description
P	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	given point
Q	$\binom{2}{3}$	given point
R	$\begin{pmatrix} \frac{2+n}{1+n} \\ \frac{3}{1+n} \end{pmatrix}$	intersecting point

Table 1: Parameters

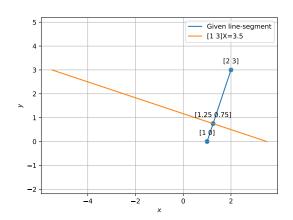


Figure 1: Equation of the required Straight Line

2 Solution

Given that resultant will divide the equation of line in the ratio 1:n and the line is perpendicular to line segment joining the points (1,0) and (2,3)

Let
$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
 and $\mathbf{Q} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Equation of line is $\mathbf{n}^{\top}\mathbf{X} = c$.

We know if 2 points of the linesegment is given then,

Direction vector of line joining two points \mathbf{P} \mathbf{Q} is \mathbf{M} = \mathbf{Q} - \mathbf{P}

$$\mathbf{M} = \begin{pmatrix} 2\\3 \end{pmatrix} - \begin{pmatrix} 1\\0 \end{pmatrix} \tag{1}$$

$$\mathbf{M} = \begin{pmatrix} 1\\3 \end{pmatrix} \tag{2}$$

We know, that position or directional vector of points P and Q line segement used as the normal vector

The general equation of the required perpendicular line is $\mathbf{M}^{\top}\mathbf{X} = c$.

The perpendicular line cutting a line segment P and Q in ratio 1:n is passes through the point R.

$$\mathbf{R} = \frac{Q + nP}{1 + n} \tag{3}$$

Equation of line passing through \mathbf{R} is

$$\mathbf{M}^{\top}(X-R) = 0 \tag{4}$$

$$\mathbf{M}^{\mathsf{T}}\mathbf{X} - \mathbf{M}^{\mathsf{T}}\mathbf{R} = 0 \tag{5}$$

From eq3, eq5 and eq2 we can find the required Perpenducular line equation.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix}^{\top} \mathbf{X} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}^{\top} \begin{pmatrix} \frac{2+n}{1+n} \\ \frac{3}{1+n} \end{pmatrix} \tag{6}$$

Therefore, equation of the line is:

$$x+nx+3y+3ny = n+11$$

3 Software

Download the following code using, and execute the code by using command svn co https://github.com/chanduputta/ FWC-Module1Assignments/blob/ main/assignment4/line/lines3.py

Python3 lines3.py

4 Conclusion

We found the equation of a line perpendicular to the line segement joining the points (1,0) and (2,3) divides it in the ratio 1:n .