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DRAFT
The GRGEN User Manual

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ABSTRACT

This is the abstract. TODO!

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CHAPTER 1

SYSTEM OVERVIEW

1.1 What is GRGEN?

GRGEN (Graph Rewrite GENarator) is a generative programming system for graph rewriting, which applies heuristic optimizations to the matching problem. According to Varró's benchmark it is at least one order of magnitude faster than any other tool known to us.

For the rewriting step, our tool implements the well-founded single-pushout approach (SPO, for explanation see [**?**]). In order to accelerating the matching step, we define the notion of search plans to represent different matching strategies and equip these search plans with a cost model, taking the present host graph into account. The task of selecting a good search plan is then viewed as an optimization problem.

For the ease of use, GRGEN features an expressive specification language and generates program code with a convenient interface. In contrast to systems like Fujaba [**?**] our pattern matching algorithm is fully automatic and does not need be tuned or partly be implemented by hand.

1.2 Features of GRGEN

The process of graph rewriting can be divided into four step: Representing a graph according to a model, searching a pattern aka. finding a match, performing changes to the found match and, finally, selecting the next rule(s) for application. We have organized the features of GRGEN according to this breakdown of graph rewriting.

- Graph model (Meta-Model) supports:

  - Directed graphs
  - Typed nodes and edges, with multiple inheritance on types
  - Typed attributes can be assigned to node and edge types
  - Multigraphs including multiple edges of the same type
  - Connection assertions to restrict the "shape" of graphs

- Patten language supports:

  - Plain isomorphic subgraph matching (injective mapping)
  - Homomorphic matching for a declarable set of nodes/edges, so that the matching is not injective
  - Attribute conditions (including arithmetic operations on the attributes)
  - Type conditions (including powerful instanceof-like type expressions)
  - Parameter passing to rules
  - Dynamic patterns with iterative or recursive paths and graphs (yet to be implemented)

- Rewrites language supports:

  - Attribute re-calculation (using arithmetic operations on the attributes)
  - Retyping of nodes/edges (a stronger version of casts of common programming languages)
  - Creation of new nodes/edges with static as well as dynamically computed types (some kind of generic templates)
  - A rule can alternatively express changes to be made to the match or replace the whole match (the semantics is always mapped to SPO)
  - Returning certain edges/nodes for further computations

- Rule applications language (GRSHELL) supports:

  - Composing several rules together with logical and iterative sequence control (called regular graph rewrite sequences, RGS)
  - Various methods for creation/deletion/input/output of graphs/nodes/edges
  - Stepwise and graphic debugging of rule application
  - Regular graph rewrite sequences can optionally contain nested transactions (yet to be implemented)

- Alternatively, you can access the matching and rewrite facility through LIBGR, so you can build your own algorithmic rule applications in a .NET language of your choice.

1.3   System Overview

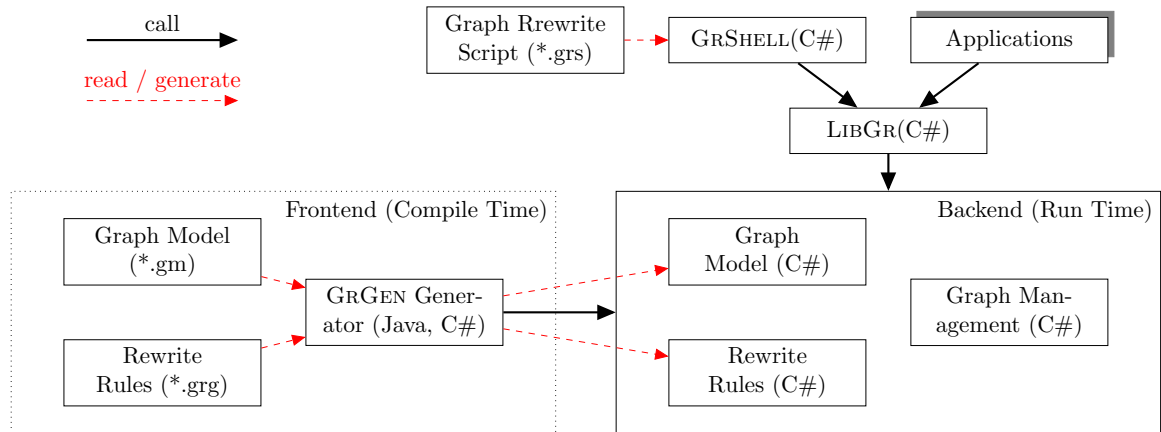Figure 1.1 gives an overview of the GRGEN system components, whereas table 1.1 shows the GRGEN directory structure.

Figure 1.1: GRGEN system components [2]

bin	Contains the .NET assemblies, in particular GrGen.exe (the graph rewriting system generator), LGSPBackend.dll (a GRGEN backend) and the shell `GrShell.exe`.
lib	Contains the GRGEN generated assemblies (*.dll).
specs	Contains the graph rewriting system source documents (*.gm and *.grg).

Table 1.1: GRGEN directory structure

A graph rewriting system[1] is defined by a rule set file (*.grg) and zero or more graph model description files (*.gm). Such a graph rewrite system is generated from these specifications

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[1]System, in this context, is not a CH0-like grammar rewriting system, but rather a set of interacting software components.

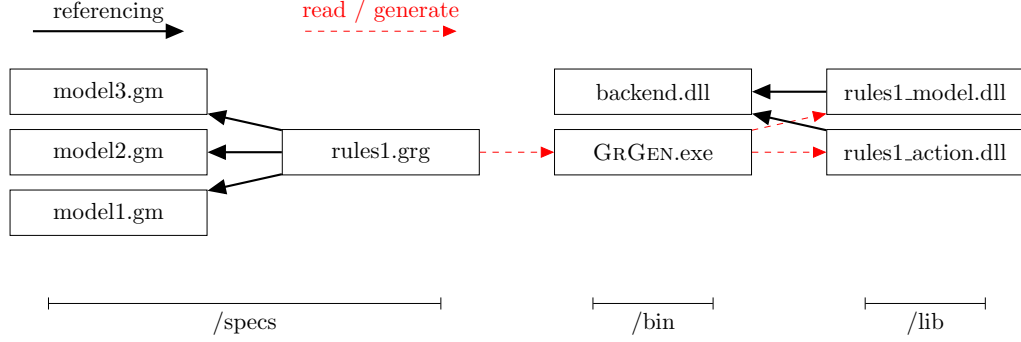by GrGen.exe and can be used by applications such as GRSHELL. Figure 1.2 shows the generation process.

Figure 1.2: Generating a graph rewriting system

In general you have to distinguish carefully between a graph model (meta level), a host graph, a pattern graph and a rewriting rule. In GRGEN pattern graphs are implicitly defined by rules, i.e. each rule defines its pattern. On the technical side, specification documents for a graph rewriting system can be available as source documents for graph models and rule sets (plain text *.gm and *.grg files) or as their translated .NET modules, either C# source files or their compiled assemblies (*.dll).

1.4 What is Graph Rewriting?

The notion of graph rewriting as understood by GRGEN is a method for declaratively specifying "changes" to a graph. This is comparable to term rewriting. Normally you use one or more *graph rewrite rules* to accomplish a certain task. GRGEN implements an SPO-based approach. In the simplest case such a graph rewrite rules consists of a tuple $L \to R$, whereas L the left hand side of the rule is called *pattern graph* and R the right hand side of the rule is the *replacement graph*.
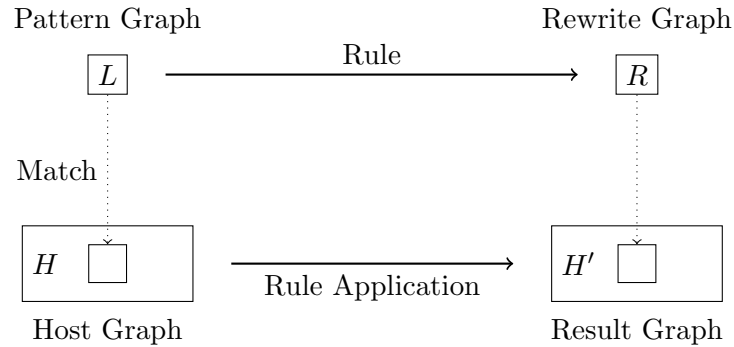

Figure 1.3: Basic Idea of Graph Rewriting

Moreover we need to identify graph elements (node or edges) of L and R for preserving them during rewrite. This is done by a *preservation morphism r* mapping elements from L to R; the morphism r is injective, but neither needs to be surjective nor total. Together with a rule name p we have $p : L \xrightarrow{r} R$.

The Transformation is done by *application* of a rule to a *host graph H*. To perform a rule application, we have to find a suitable location in the host graph with respect to the pattern graph. Mathematically speaking, such a *match* is a isomorphic subgraph of the L in H. This morphism may not be unique, meaning there may be several matches. Afterwards we change the matched spot of the host graph, such that it becomes a isomorphic subgraph

of the replacement graph R. Elements of L not mapped by r are deleted from $m(L)$ during rewrite. Elements of R not in the image of r are inserted to H, all others (elements that are mapped by r) are retained. The outcome of this steps is the resulting graph H'.

1.5   An Example

We'll have a look at a small example. First we use a special case to construct our host graph: an empty pattern always produces exactly one match (independently of the host graph). So starting with an empty host graph H we construct an apple using

$$p_0 : \quad \emptyset \quad \longrightarrow$$
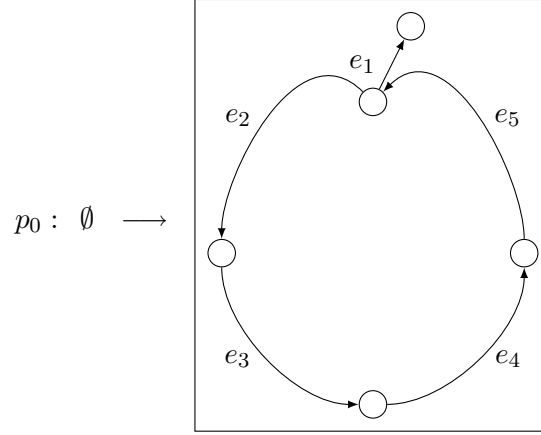
applied to H. We'll get the apple as new host graph H_1. Now we want to rewrite our apple with stem to an apple with a leaflet. We use

$$p_1 :$$
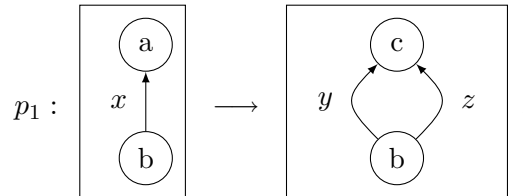
apply p_1 to H_1 and get the new host graph H_2, something like this:
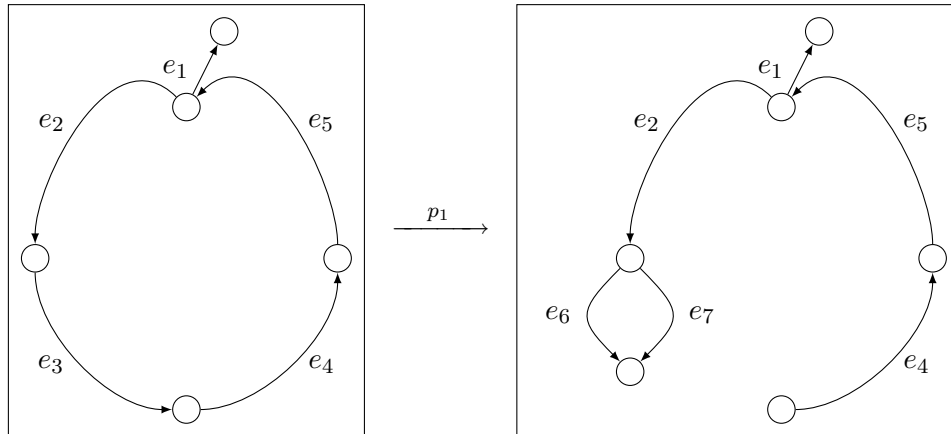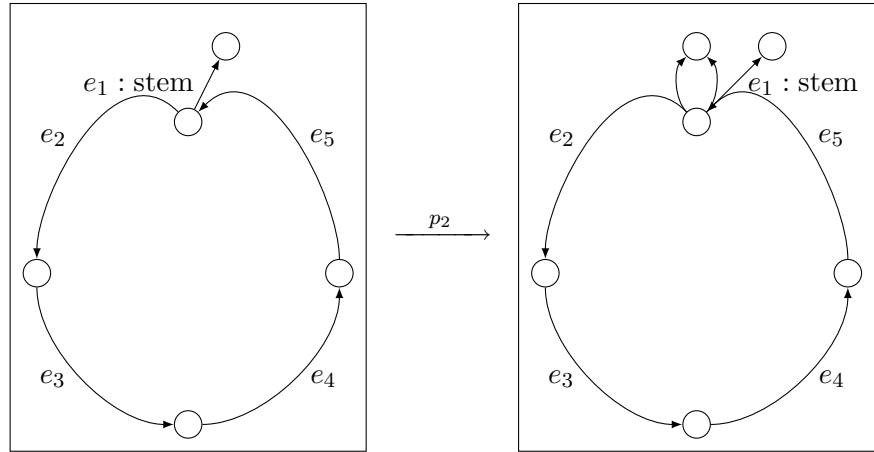
What happened? GRGEN has arbitrary chosen one match among the set of possible matches, because x matches edge $e3$ as well as $e1$. A correct solution could make use of edge type information. We have to change rule p_0 to generate the edge e_1 with a special type "stem".

And this time we'll even keep the stem. So let now be
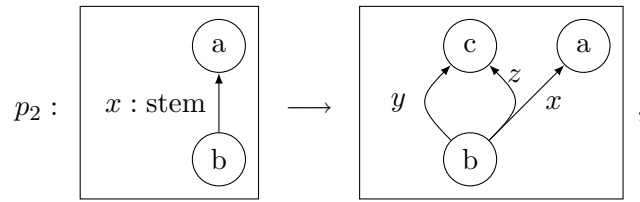
If we apply p_2 to the modified H_1 this leads to

Note: If we had applied p_1 to H_1 and consecutively to its resulting graph, this would not lead to a fix point, because each rewrite produces one new candidate (one deleted, two added) for matching.

1.6 The Tools

TODO Command line parameters, etc. GRGEN, GRSHELL, LIBGR, Ycomp,...

CHAPTER 2

GRAPH MODEL LANGUAGE

The key features of GRGEN graph models from [1]:

Types.
Nodes and edges can have types (classes). This is similar to common programming languages, except GRGEN types have no concept of methods.

Attributes.
Nodes and edges can possess attributes. The set of attributes assigned to a node or edge is determined by its type. The attributes themselfs are typed, too.

Inheritance.
Types (classes) can be composed by multiple inheritance. *Node* and *Edge* are built-in root types of node and edge types, respectively. Inheritance eases the specification of attributes, because subtypes inherit the attributes of their super types. Note that GRGEN lacks a concept of overwriting. On a path in the type hierarchy graph from a type up to the built-in root type there must be exactly one declaration for each attribute identifier.

Connection Assertions.
To specify that certain edge types should only connect specific nodes, we include connection assertions. Furthermore the number of outgoing and incoming edges can be constrained.

In this chapter as well as in the GRSHELL chapter 5 we use pieces of the example 1 (the `Map` model) for further descriptions.

2.1 Building Blocks

NOTE (1)

The following syntax specifications make heavy use of syntax diagrams (also known as rail diagrams). Syntax diagrams provide a visualization of EBNF grammars. Follow a path along the arrows from left to right through a diagram to get a valid sentence (or sub sentence) of the language. Ellipses are terminals whereas rectangles are non-terminals. For further information on syntax diagrams see [7].

Basic elements of the GRGEN graph model language are numbers and identifiers to denominate types, fields and the model itself. The GRGEN graph model language is case sensitive.

Ident, IdentDecl
A character sequence of arbitrary length consisting of letters, digits or underscores. The first character must not be a digit. *Ident* and *IdentDecl* differ in their role: while *IdentDecl* is a

EXAMPLE (1)

The following micro model of street maps gives a rough picture of the language:

```
1  model Map;
2
3  enum resident {village = 500, town = 5000, city = 50000}
4
5  node class sight;
6
7  node class city {
8      size: resident;
9  }
10
11 const node class metropolis extends city {
12   river: string;
13 }
14
15 abstract node class abandoned_city extends city;
16 node class ghost_town extends abandoned_city;
17
18 edge class street;
19 edge class trail extends street;
20 edge class highway extends street
21     connect metropolis [+] -> metropolis [+]
22 {
23     jam: boolean;
24 }
```

defining occurrence of an identifier, *Ident* is a *using* occurence. An *IdentDecl* non-terminal can be annotated. See 4.4 for annotations on declarations.
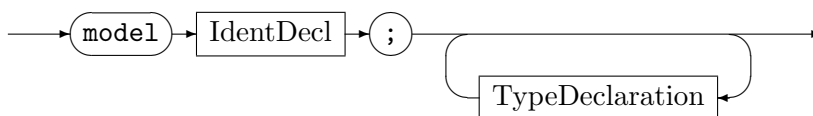
NodeType, EdgeType, EnumType
These are (semantic) specializations of Ident to restrict an identifier to a specific type.
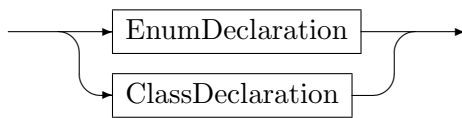
Number
A sequence of digits. The sequence has to form a non-negative integer in the decimal system and will be internally stored in 32 bit two's complement representation.

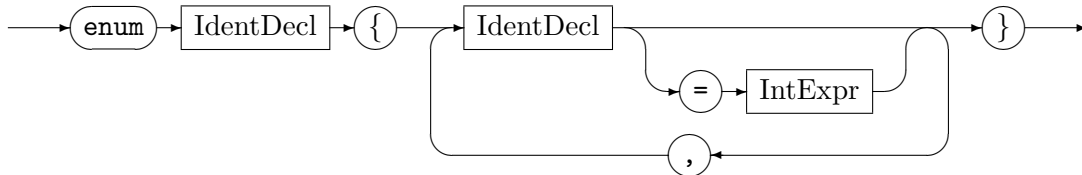## 2.2 Type Declarations

GraphModel

The graph model consists of its name *IdentDecl* and type declarations defining specific node and edge types as well as enums.

TypeDeclaration

ClassDeclaration defines a node or an edge. *EnumDeclaration* defines an enum type for use as attribute of nodes or edges. Types do not need to be declared before they are used.
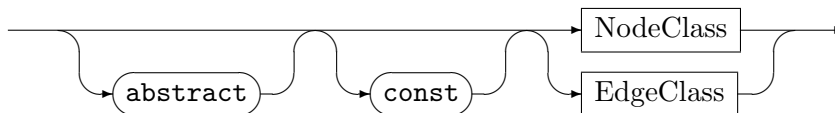
EnumDeclaration

Defines an enum type.

EXAMPLE (2)

```
1  enum Color {red, green, blue}
2  enum Resident {village = 500, town = 5000, city = 50000}
3  enum AsInC {a = 2, b, c = 1, d, e = Resident::village + c}
```

The semantics is as it is in C [8]. So the examples hold red $= 0$, $a = d = 2$, $b = 3$, $c = 1$ and $e = 501$.

ClassDeclaration

Defines a new node type or edge type.
The keyword `abstract` indicates that you can't instantiate graph elements of this type but rather you have to derive non-abstract types for graph elements. The abstract-property will not be inherited by subclasses.

EXAMPLE (3)

We adjust our map model and make `city` abstract:

```
1  abstract node class city {
2      size: int;
3  }
4  abstract node class abandoned_city extends city;
5  node class ghost_town extends abandoned_city;
```

You will be able to create nodes of type `ghost_town`, but not of type `city` or `abandoned_city`. However, nodes of type `ghost_town` are also nodes of type `abandoned_city` and of type `city` and they have got the attribute `size`.

The keyword `const` indicates that rules may not write to attributes. See also 3.4, `eval`. However, attributes are writable by LibGr and GrShell. This property will not be inherited by subclasses. If you want a subclass to have constant attributes, you have to set the `const` modifier explicitly.

NodeClass

node class IdentDecl

extends NodeType

,

;

{ FieldDeclarations }
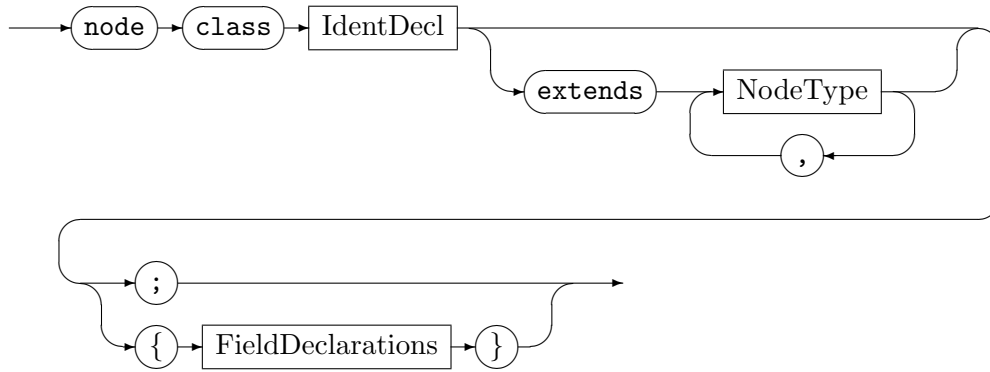
Defines a new node type. Node types can inherit from other node types defined within the same file. If the extends clause is omitted, *NodeType* will inherit from the built-in type Node. Optionally nodes can possess attributes (fields).

EdgeClass

edge class IdentDecl

extends EdgeType

,

;

ConnectAssertions { FieldDeclarations }
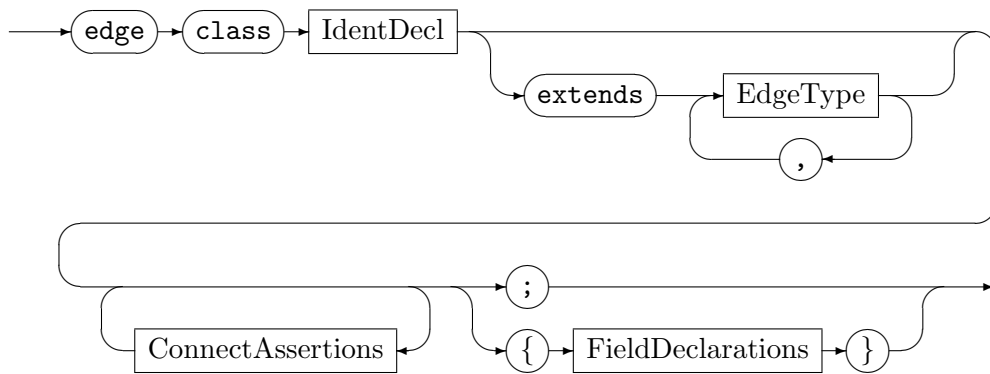
Defines a new edge type. Edge types can inherit from other edge types defined within the same file. If the extends clause is omitted, *EdgeType* will inherit from the built-in type Edge. Optionally edges can possess attributes (fields). A *connection assertion* specifies that certain edge types should only connect specific nodes and, moreover, the number of outgoing and incoming edges can be constrained.

ConnectAssertions

connect NodeConstraint -> NodeConstraint

,

NodeConstraint

NodeType

[*]

+

Number
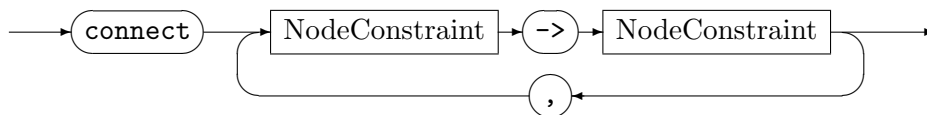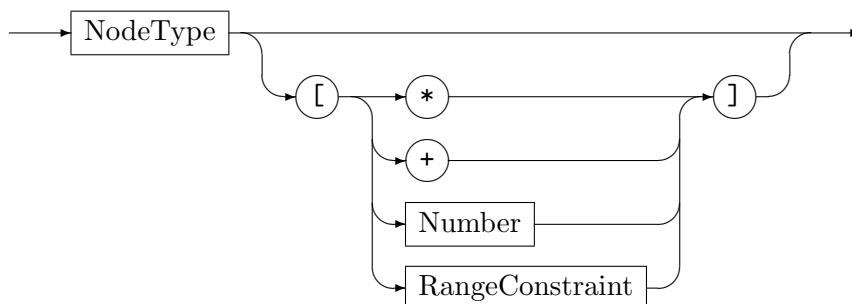
RangeConstraint

RangeConstraint
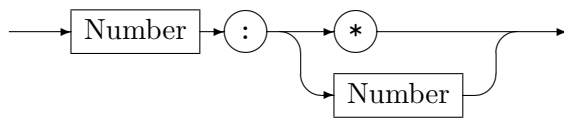
A connection assertion is described as a pair of node types, optionally together with a multiplicity. A corresponding edge may connect a node of the first node type or one of its subtypes (source) with a node of the second node type or one of its subtypes (destination). The multiplicity is a constraint on the out-degree and in-degree of the source and destination node type respectively. See 5.2.2, `validate`, for an example. Table 2.1 describes the multiplicity definitions.

`[*]`	Number of edges the node is adjacent to is unbounded. The node must not be connected to any edge. This is the **default**.
`[+]`	Number of edges the node is adjacent to is unbounded. At least one edge must be adjacent to nodes of that type.
`[n:*]`	Number of edges the node is adjacent to is unbounded. At least n edges must be adjacent to nodes of that type.
`[n:m]`	At least n edges must be adjacent to the nodes of that type, but at most m edges may be adjacent to the nodes of that type ($m \geq n$ holds).
`[n]`	Abbreviation for `[n:n]`.

Table 2.1: GRGEN node constraint multiplicities concerning a specific pair of an edge type and a node type.

FieldDeclarations
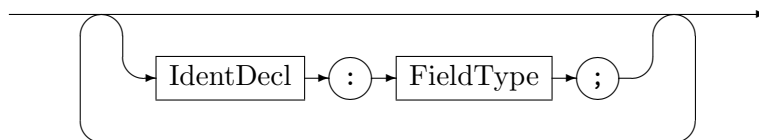
FieldType
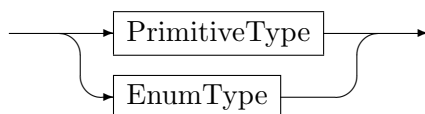
Defines a node or edge attribute. Possible types are `enum` and primitive types. See 4.1 for a list of built-in primitive types.

RULE SET LANGUAGE

The rule set language forms the core of GRGEN. Rule files refer to zero[1] or more graph models and specify a set of (usually coherent) rewriting rules. The rule language covers the pattern specification and the replace / modify specification. Graph element's attributes can be re-evaluated during a rule application. The following rewriting rule from [1] gives a rough picture of the language: In this chapter we use pieces of example 4 (`SomeRule`) in further

EXAMPLE (4)

```
1  actions SomeActions using SomeModel;
2
3  rule SomeRule {
4    pattern {
5      n1 : NodeTypeA;
6      n2 : NodeTypeA;
7      hom(n1, n2);
8      n1 --> n2;
9      n3: NodeTypeB;
10     negative {
11       n3 -e1:EdgeTypeA-> n1;
12       if {n3.a1 == 42*n2.a1;}
13     }
14     negative {
15       n4: Node \ (NodeTypeB);
16       n3 -e1:EdgeTypeB->n4;
17       if {typeof(e1) >= EdgeTypeA;}
18     }
19   }
20   replace {
21     n5: NodeTypeC<n1>;
22     n3 -e1:EdgeTypeB-> n5;
23     eval {
24       n5.a3 = n3.a1*n1.a2;
25     }
26   }
27 }
```

descriptions.

---

[1]TODO no unsing means that....

3.1   Building Blocks

The GRGEN rule set language is case sensitive. The language makes use of a couple of identifier specializations in order to denominate all the GRGEN entities.

Ident, IdentDecl
A character sequence of arbitrary length consisting of letters, digits or underscores. The first character must not be a digit. *Ident* may be an identifier defined in a graph model (see 2.1). *Ident* and *IdentDecl* differ in their role: while *IdentDecl* is a *defining* occurrence of an identifier, *Ident* is a *using* occurrence. An *IdentDecl* non-terminal can be annotated. See 4.4 for annotations on declarations.
ModelIdent, TypeIdent, NodeType, EdgeType
These are (semantic) specializations of Ident. *TypeIdent* matches every type identifier, i.e. a node type, an edge type, an enum type or a primitive type. All the type identifiers are actually type *expressions.* See 4.3 for the use of type expressions.
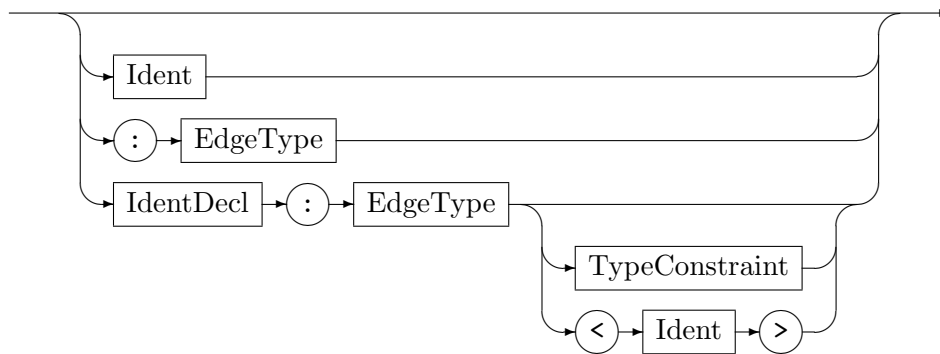
ForwardEdge

ReverseEdge

EdgeRefinement

In general edges are specified by `-->` or `<--`. Those edges are called *anonymous.* For a more detailed specification use an edge refinement clause between the arrow dashes. Type constraints are allowed in the pattern part only. See 4.3, *TypeConstraint.* The `<>` operator retypes an edge. Retyping is allowed in the replace / modify part only. See 3.4, *Retyping.*

3.2 Declarations
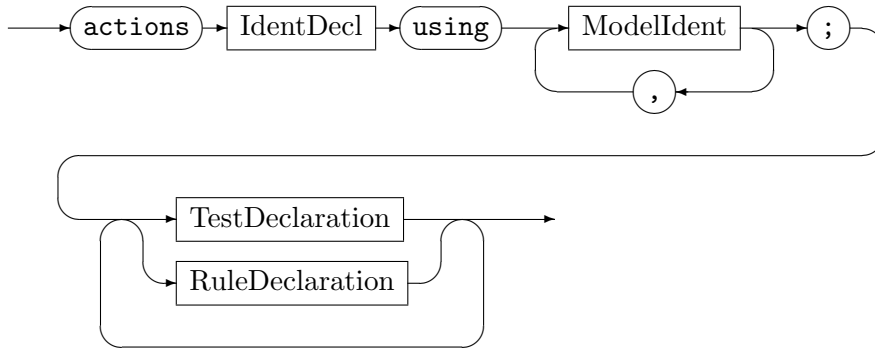
A rule set consists of the underlying graph models and several rewriting rules. In case of multiple graph models GRGEN use the union of the models. In this case beware of conflicting declarations.

TestDeclaration

RuleDeclaration
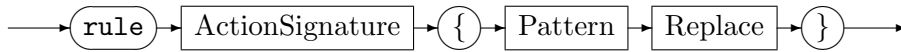
Declares a single rewriting rule such as `SomeRule`. It consists of a pattern part (see 3.3) together with its rewrite / modify part (see 3.4). A test rule has no rewrite specification. It's intended to test whether (and maybe how many times) a pattern occurs.

EXAMPLE (5)

We define a test rule `SomeCond`

```
1  test SomeCond {
2    pattern {
3      n: SeldomNodeType;
4    }
5  }
```
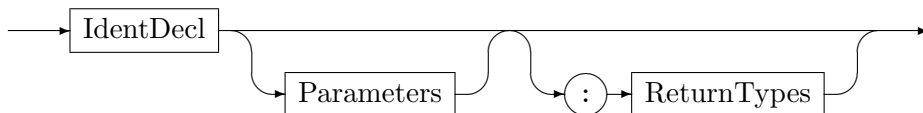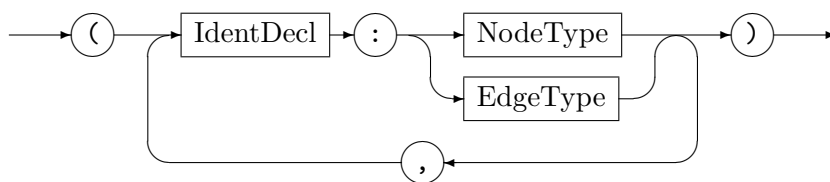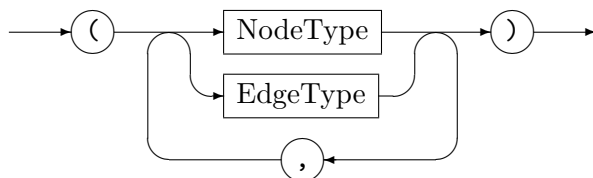
and execute in GRSHELL:

```
1    grs SomeCond & SomeRule
```

SomeRule will only be executed, if a node of type `SeldomNodeType` exists. For regular graph rewriting sequences in GRSHELL see 5.2.6.

ActionSignature

The signature sets the name of a rewriting rule to *IdentDecl*. Additionally you can provide parameters to the rule and specify return types.

Parameters

ReturnTypes

Parameters are treated as predefined graph elements for the pattern. Even if a supplied parameter value is undefined, it is treated as valid node or edge definition. So in any case a graph element of the specified type has to be matched.

The return types specify edge and node types of graph elements that are returned by the replace / modify part. If return types are specified, the `return` statement is mandatory. Otherwise no `return` statement may occur. If no pattern is found, the return values are undefined. See also 3.4, `return`.

EXAMPLE (6)

We extend `SomeRule` with a variable node to find, and we want it to return the rewritten graph elements `n5` and `e1`.

```
1   rule SomeRuleExt(varnode: Node): (Node, EdgeTypeB) {
2     pattern{
3       n1: NodeTypeA;
4       ...
5     }
6     replace {
7       varnode;
8       ...
9       return(n5, e1);
10      eval {
11        ...
```

We don't define `varnode` within the pattern part, because this is already covered by the parameter specification itself.

3.3 Pattern Part

Pattern
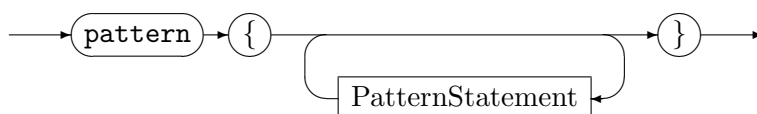
A pattern consists of zero, one or several pattern statements. All of the pattern statements must be fulfilled by a subgraph of the host graph, in order to form a match. Even stronger – a graph element of the host graph, that is matched by a statement, is "bound", i.e. it can not be part of another pattern statement, unless you use the `hom` operator. An empty pattern

always produces exactly one (empty) match.

Pattern statements may define variables for use by other pattern statements or replace statements. Such variables may be used before declaration.

> **NOTE (2)**
>
> The application of a rule is not deterministic, specifically there may be more than one sub graph that matches the pattern and any of them may be selected.

PatternStatement
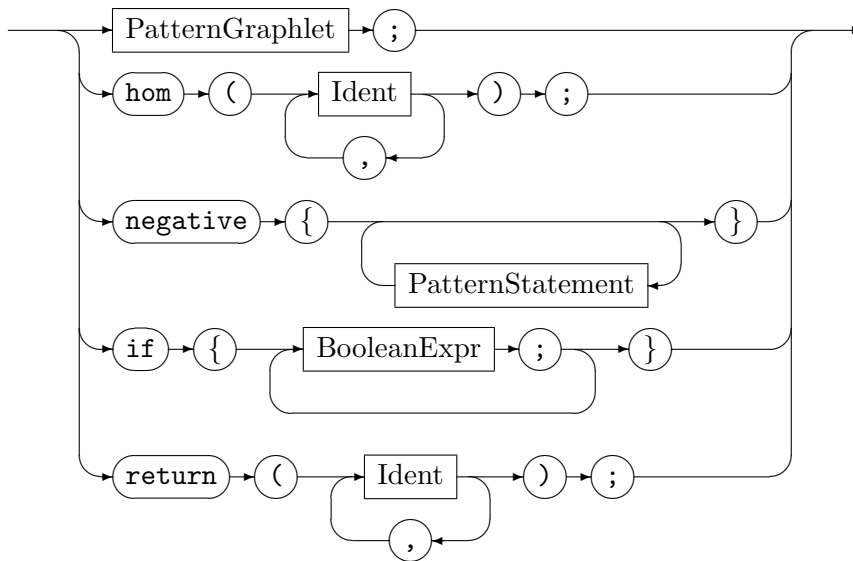
The semantics of the various pattern statements:

Pattern Graphlet.
> Graphlets specify a connected subgraph, i.e. certain node types connected by certain edge types.

Isomorphic/Homomorphic Matching.
> The `hom` operator specify the nodes or edges, that may be matched homomorphically. In contrast to the default isomorphic matching, the specified graph elements *may* be matched to the same graph element in the host graph. Note that the graph elements shall have a common subtype. If the match is not isomorphic, than the elements will be mapped to a graph element of a common subtype.
>
> In our example `n1` and `n2` may be the same node. This is possible because they are of the same type (`NodeTypeA`).

Negative Application Conditions (NACs).
> With negative application conditions (keyword `negative`) we can specify graph patterns which forbid the application of a rule if any of them is present in the host graph (cf. [5]). NACs may not be nested. Variables defined within a NAC are not alive outside the NAC. Identifiers from surrounding scopes may be overwritten.
>
> In our example the second negative condition uses `n3` from the surrounding scope and defines `n4` and `e1`. We may safely reuse the variable name `e1` in the replace part.

Attribute Conditions.
> The Java-like attribute conditions (keyword `if`) in the pattern part allows for further restriction of the applicability of a rule.

Return values.

> The return statement is only allowed for `test` rules. Otherwise the `return` statement belongs to the replace part. See 3.4, *Return Values.*

Keep in mind that using type constraints or the `typeof` operator might be helpful. See 4.3 for further information.

PatternGraphlet
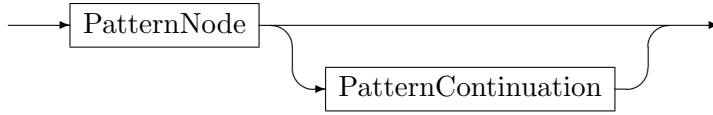
PatternContinuation
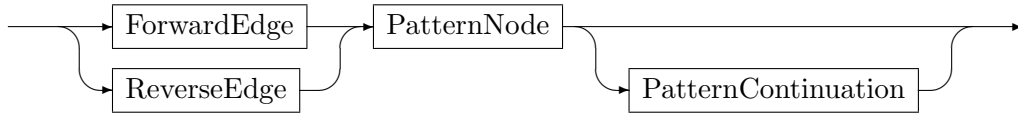
PatternNode
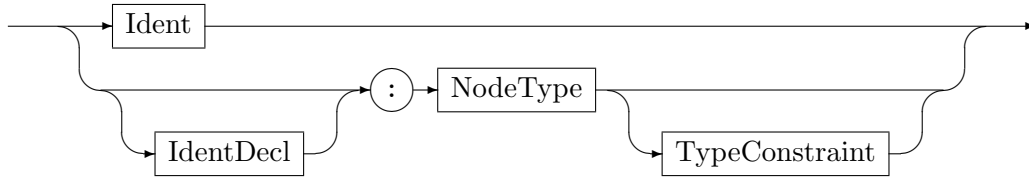
A pattern graphlet is specified piecewise. Start with a *PatternNode* in order to specify a node of type *NodeType* (with respect to *TypeConstraint*) and optionally construct a larger subgraph with several pattern continuations.

In our example the statement `n1 --> n2` is the node identifier `n1` followed by the pattern continuation `--> n2`.

3.4   Replace / Modify Part

For the task of rewriting GRGEN provides two different modes: a replace mode and a modify mode.

Replace mode.

> The semantics of this mode is to delete every graph element of the pattern that is not used (denoted) in the replace part, keep every graph element that is used and create every additionally defined graph elements.
>
> In our example `SomeRuleExt` the nodes `varnode` and `n3` will be kept. The node `n1` is replaced by the node `n5` preserving `n1`'s edges. The anonymous edge instance between `n1` and `n2` only occurs in the pattern and therefore gets deleted.

Modify mode.

> The modify mode can be regarded as a replace part in replace mode, where every pattern graph element is added (denoted) before the first replace statement. Additionally this mode supports the `delete` operator, that deletes every element given as an argument. Deletion takes place after all rewrite operations. Multiple deletion of the same graph element is allowed (but senseless) as well as deletion of just created elements (senseless, too).
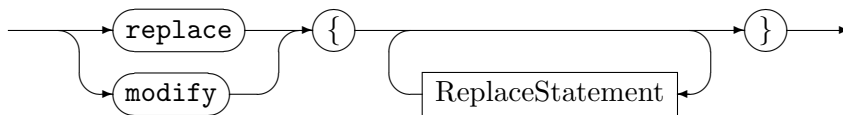
EXAMPLE (7)

How might our example look in modify mode? We have to denominate the anonymous edge between **n1** and **n2** in order to delete it. The node **varnode** should be omitted. So we have
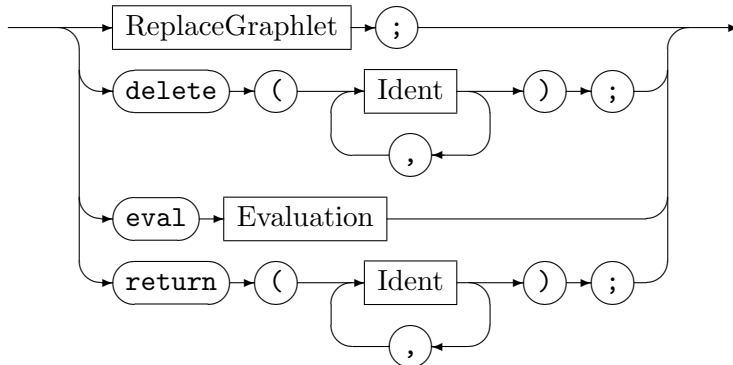
```
1  rule SomeRuleExtMod(varnode: Node): (Node, EdgeTypeB) {
2    pattern {
3      ...
4      n1 -e0:Edge-> n2;
5      ...
6    }
7    modify {
8      n5 : NodeTypeC<n1>;
9      n3 -e1:EdgeTypeB-> n5;
10     delete(e0);
11     eval {
12       ...
```

Replace

Selects whether the replace mode or the modify mode is used. Several replace statements describe the transformation from the pattern subgraph to the destination subgraph.

ReplaceStatement

The semantics of the various pattern statements:

Replace Graphlet.
Analogous to a pattern graphlet, a specification of a connected subgraph. Its graph elements are either kept because they are elements of the pattern or added otherwise.

Deletion.
The **delete** operator is only supported by the modify mode. It deletes the specified pattern graph elements. Multiple occurrences of **delete** statements are allowed. Deletion statements are executed after all other replace statements. Multiple deletion of the same graph element is allowed (but senseless) as well as deletion of just created elements (senseless, too).

Attribute Evaluation.
If a rule is applied, then the attributes of matched and inserted graph elements will be recalculated.

Return Values.

> Graph elements of the replace part can be returned according to the return types in the signature (see 3.2, `ActionSignature`). The `return` statement may not occur multiple times. The values have to be in the same order as the corresponding return types in the signature.
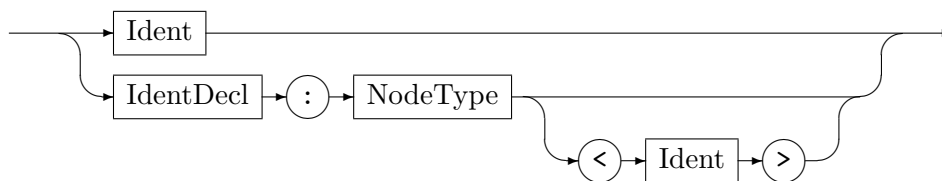
Retyping.

> Retyping enables us to keep all adjacent nodes and all attributes stemming from common super types of a graph element while changing its type. Retyping differs from a type cast: During replacement both of the graph elements are alive. Specifically both of them are available for evaluation. Furthermore the source and destination types must not be on a path in the directed type hierarchy tree, but are arbitrary.
> The edge specification as well as *ReplaceNode* supports retyping. In our example node `n5` is a retyped node stemming from node `n1`.

ReplaceGraphlet
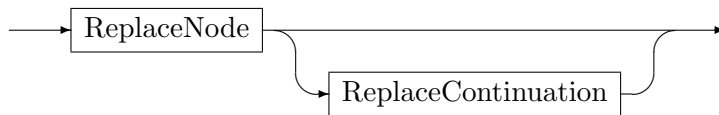
ReplaceContinuation
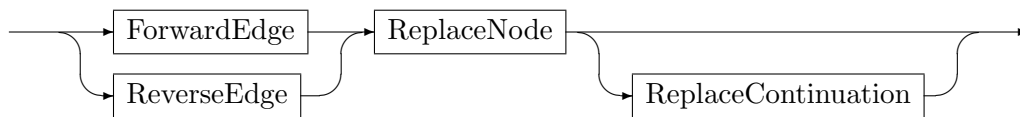
The same as *PatternGraphlet*, but with *ReplaceNode* instead of *PatternNode*. The retyping operator for edges `<>` is allowed. Identifiers defined in the pattern part may not be redefined in the replace graphlet. See 3.3, *PatternGraphlet* and *PatternContinuations*, for a description of the graphlet / continuation semantics.

ReplaceNode

A single node that is either kept because it's a node of the pattern or added otherwise. GRGEN supports *retyping* of nodes by the `<>` operator.

Evaluation
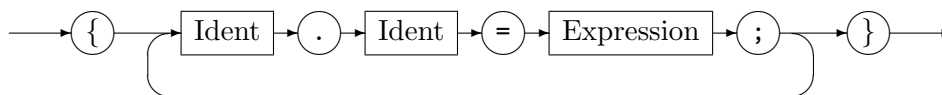
Several Evaluation parts are allowed within the replace part. Multiple evaluation statements will be internally concatenated, preserving their order. Evaluation statements have an imperative semantics. GRGEN does not care about data dependencies. Evaluation takes place before any graph elements get deleted. You may read (and write, although this doesn't make sense) attributes of deleted graph elements.

EXAMPLE (8)

```
1  ...
2  replace{
3    ...
4    eval {y.i = 40;}
5    eval {y.j = 0;}
6    x: IJNode;
7    y: IJNode;
8    delete(x);
9    eval {
10     x.i = 1;
11     y.j = x.i;
12     x.i = x.i + i;
13     y.i = y.i + x.i;
14   }
```

This nonsense example yields to $y.i = 42$, $y.j = 1$.

TYPES AND EXPRESSIONS

In the following sections *Ident* refers to a identifier of the graph model language (see 2.1) or the rule set language (see 3.1). By analogy *TypeIdent* is a identifier of a type.

4.1 Built-In Types

Besides user-defined node types, edge types and enum types, GrGen supports the built-in primitive types in table 4.1. Table 4.2 lists GRGEN's implicit type casts and the allowed

`boolean`	Covers the values `true` and `false`.
`int`	A 32-bit signed integer, in two's complement representation.
`float`, `double`	A floating-point number in IEEE 754 format with single precision or double precision respectively.
`string`	A sequence of digits, letters, underscores and white spaces. Strings are of arbitrary length and may be enclosed by double quotes. If the string contains white spaces, double quotes are mandatory.

Table 4.1: GRGEN built-in types

explicit type casts. Of course you are free to express an implicit type cast by an explicit type cast as well as "cast" a type to itself.

EXAMPLE (9)

`myfloat = myint; mydouble = (float)myint; mystring = (string)mybool` is allowed, `myenum = (myenum)int; myfloat = mydouble; myint = (int)mybool` is forbidden.

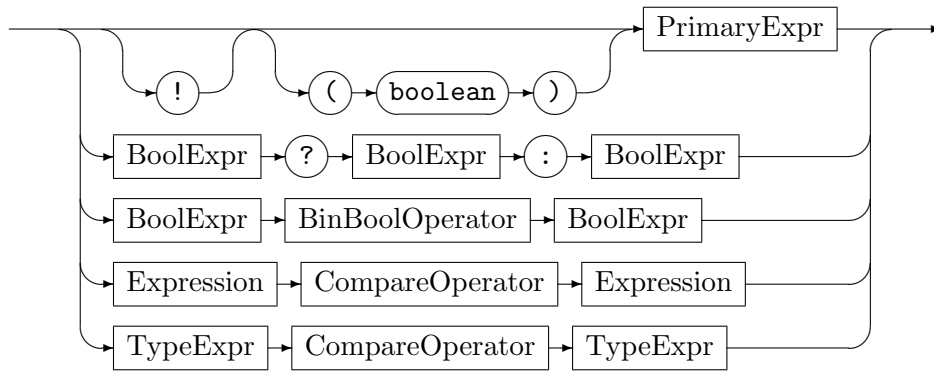| from to	enum	boolean	int	float	double	string
enum	—					
boolean		—				
int	`(int)`		—	`(int)`	`(int)`	
float	`(float)`		implicit	—	`(float)`	
double	`(double)`		implicit	implicit	—	
string		`(string)`	`(string)`	`(string)`	`(string)`	—

Table 4.2: GRGEN type casts.

4.2 Expressions

Expression
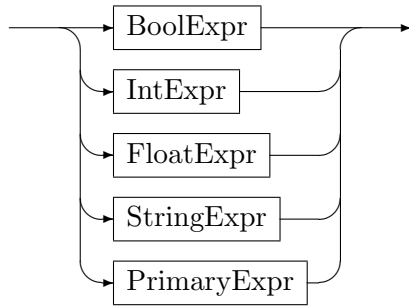
BoolExpr

As in C [8], ! negates a Boolean. Table 4.4 lists the binary operators for Boolean expressions. The ? operator is a simple if-then-else: if the first *BoolExpr* is evaluated to true, the operator returns the second *BoolExpr*, otherwise it returns the third *BoolExpr*.
The *CompareOperator* is one of the following operators:

$$< \quad <= \quad == \quad != \quad >= \quad >$$

Table 4.2 describes the semantics of compare operators on type expressions.
The *BinBoolOperator* is one of the operators in table 4.4.

|	
A == B	True, iff A and B are identical. Different types in a type hierarchy are *not* identical.
A != B	True, iff A and B are not identical.
A < B	True, iff A is a supertype of B, but A and B are not identical.
A > B	True, iff A is a subtype of B, but A and B are not identical.
A <= B	True, iff A is a supertype of B or A and B are identical.
A >= B	True, iff A is a subtype of B or A and B are identical.

NOTE (3)

A < B corresponds to the direction of the arrow in an UML class diagram.

Table 4.3: Compare operators on type expressions

| ^	Logical XOR. True, iff either the first or the second Boolean expression is true.
&& | |	Logical AND an OR. Lazy evaluation.
& |	Logical AND an OR. Complete evaluation.

Table 4.4: Binary Boolean operators, in ascending order of precedence
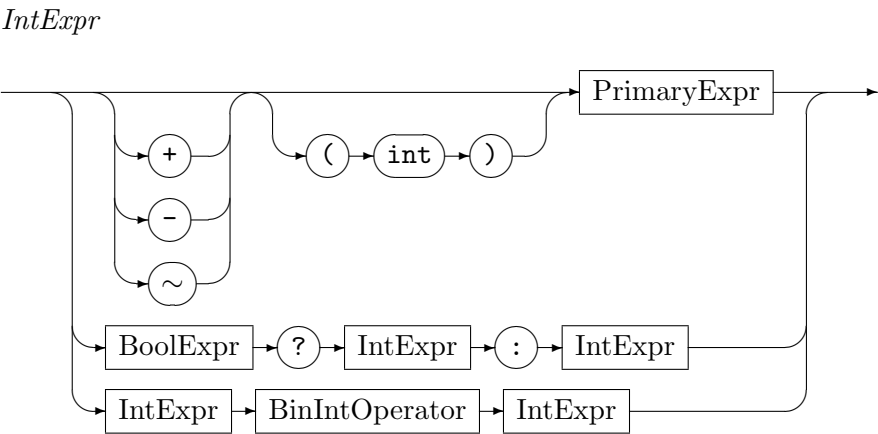
IntExpr

The ∼ operator is a bitwise complement. That means every bit of an integer value will be flipped. The `?` operator is a simple if-then-else: if the *BoolExpr* is evaluated to `true`, the operator returns the first *IntExpr*, otherwise it returns the second *IntExpr*.

The *BinIntOperator* is one of the operators in table 4.5.

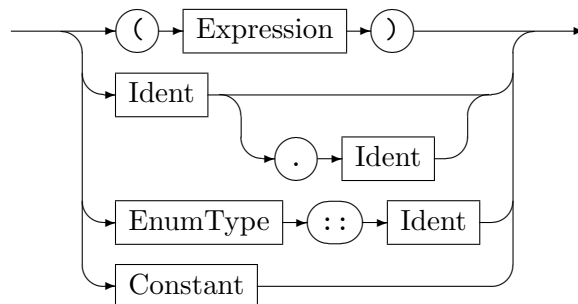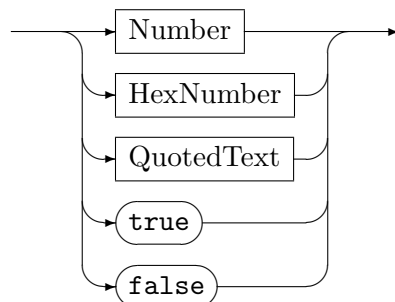| ^ & |	Bitwise XOR, AND and OR
<< >> >>>	Bitwise shift left, bitwise shift right and bitwise shift right preserving the sign
+ −	Addition and subtraction
* / %	Multiplication, integer division and modulo

Table 4.5: Binary integer operators, in ascending order of precedence

FloatExpr
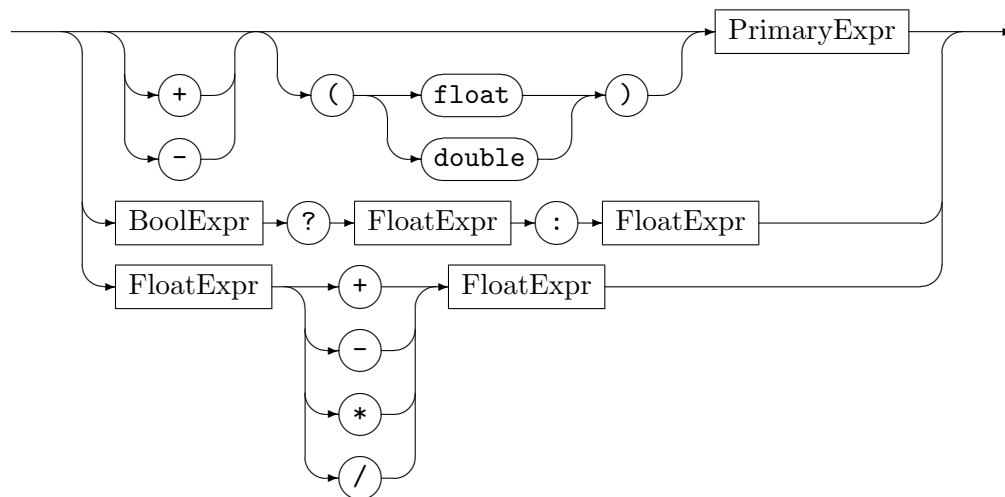
PrimaryExpr

+

−

(float)

double

BoolExpr ? *FloatExpr* : *FloatExpr*

FloatExpr + *FloatExpr*

−

*

/

The `?` operator is a simple if-then-else: if the *BoolExpr* is evaluated to `true`, the operator returns the first *FloatExpr*, otherwise it returns the second *FloatExpr*.

StringExpr

PrimaryExpr

StringExpr + *StringExpr*
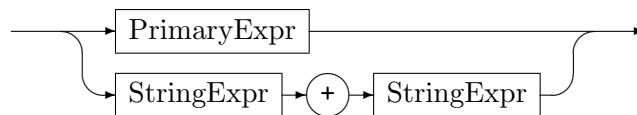
The operator `+` concatenates two strings.

PrimaryExpr

(*Expression*)

Ident

. *Ident*

EnumType :: *Ident*

Constant

Constant

Number

HexNumber

QuotedText

true

false

Number.
    Is an `int`, `float` or `double` constant in decimal notation.
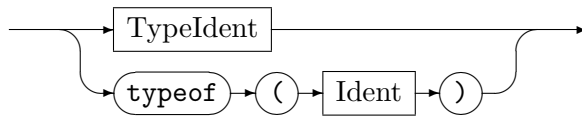
HexNumber.
    Is an `int` constant in hexadecimal notation starting with `0x`.

QuotedText.
    Is a string constant. A sequence of characters, enclosed by double quotes.

4.3   Type Related Conditions

TypeExpr

A type expression identifies a type (and – in terms of matching – also its subtypes). A type expression is either a type identifier itself or the type of a graph element. So *Ident* has to be a node or edge identifier.
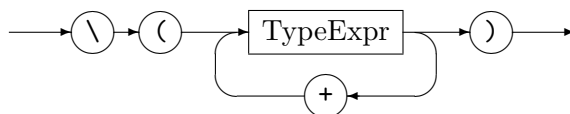
EXAMPLE (10)

The following rule will add a reverse edge to an one-way street.

```
1  rule oneway {
2      pattern{
3          a:Node -x:street-> y:Node;
4          negative{
5              y -:typeof(x)-> a;
6          }
7      }
8      replace{
9        a -x-> y;
10       y -:typeof(x)-> a;
11     }
12 }
```

Remember that we have several subtypes of `street`. By the aid of the `typeof` operator, the reverse edge will be automatically typed correctly (the same type as the one-way edge). This behavior is not possible without the `typeof` operator.

TypeConstraint

A type constraint is used to exclude parts of the type hierarchy. The operator + is used to identify a union of types.
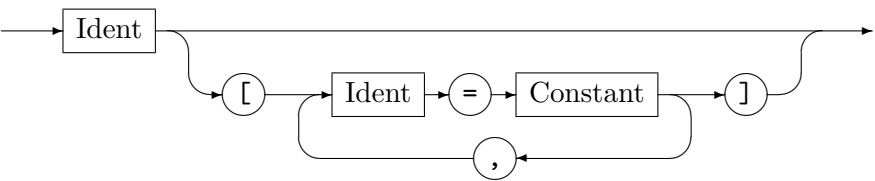
EXAMPLE (11)
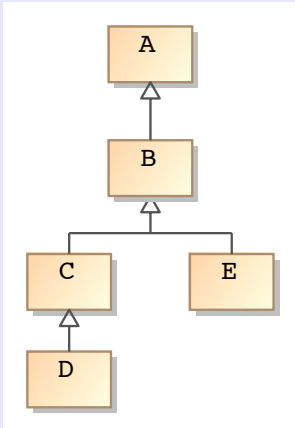
The expression `A\(C+E)` applied to the type hierarchy on the left site covers *A* an

4.4   Annotations

Identifier definitions can be annotated by pragmas. Annotations are key-value pairs.

IdentDecl

Although you can use any key-value pairs between the brackets, only the identifier `prio` has an effect so far.

| Key	Value Type	Applies to	Meaning
prio	int	node, edge	Changes the ranking of a graph element for search plans. The default is `prio=1000`. Graph elements with high values are likely to appear prior to graph elements with low values in search plans. **Example:** We search the pattern `v:NodeTypeA -e:EdgeType-> w:NodeTypeB`. We have a host graph with about 100 nodes of `NodeTypeA`, 1,000 nodes of `NodeTypeB` and 10,000 edges of `EdgeType`. Furthermore we know that between each pair of `NodeTypeA` and `NodeTypeB` there exists at most one edge of `EdgeType`. GRGEN can use this information to improve the initial search plan, if we adjust the pattern like `v[prio=10000]:NodeTypeA -e[prio=5000]:EdgeType-> w:NodeTypeB`.

Table 4.6: Annotations

CHAPTER 5

GRSHELL LANGUAGE

The GRSHELL is a shell application of the LIBGR. It belongs to GRGEN's standard equipment. GRSHELL is capable of creating, manipulating and dumping graphs as well as performing graph rewriting and debugging graph rewriting.

The GRSHELL language is a line oriented scripting language. It is structured by simple statements separated by line breaks.
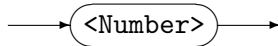
5.1 Building Blocks

The GRSHELL is case sensitive. A comment starts with a # and is terminated by end-of-line or end-of-file. Any text left of the # will be treated as a statement.
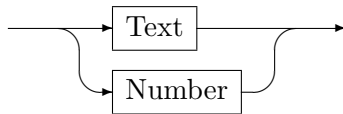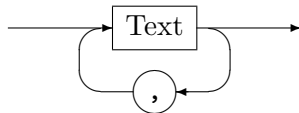
Text

```
        ┌──<DoubleQuotedText>──┐
────────┼──<SingleQuotedText>──┼────────▶
        └──────<Word>──────────┘
```
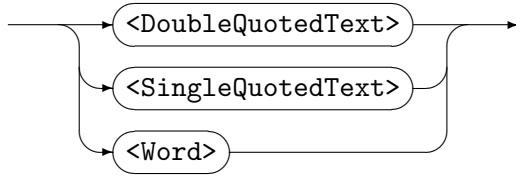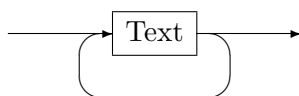
Number

```
────▶──<Number>──────▶
```

TextOrNumber

```
    ┌──│ Text │──┐
────┤             ├────▶
    └──│Number│──┘
```

Parameters

```
    ┌───│ Text │───┐
────┤              ├────▶
    └──────,───────┘
```

SpacedParameters

```
    ┌───│ Text │───┐
────┤              ├────▶
    └──────────────┘
```

Those items are required for representing text, numbers and parameters within rules. The tokens `<...>` are defined as follows:

| `<$Word$>`: | A character sequence consisting of letters, digits and underscores. The first character must not be a digit. |
`<DoubleQuotedText>`:	Arbitrary text enclosed by double quotes (" ").
`<SingleQuotedText>`:	Arbitrary text enclosed by single quotes (' ').
`<Number>`:	A sequence of digits.

In order to describe the possible input to some of the commands more precisely, the following (semantic) specializations of *Text* are defined:

Filename:	A file path without spaces (e.g. /Users/Bob/amazing_file.txt) or a single quoted or double quoted file path that may contain spaces ("/Users/Bob/amazing_file.txt").
Variable:	Identifier of a variable that contains a graph element.
NodeType:	Identifier of a node type within the model of the current graph.
AttributeName:	Identifier of an attribute.
Graph:	Identifies a graph by its name.
Action:	Identifies a rule by its name.
Color:	One of the following color identifiers: Black, Blue, Green, Cyan, Red, Purple, Brown, Grey, LightGrey, LightBlue, LightGreen, LightCyan, LightRed, LightPurple, Yellow, White, DarkBlue, DarkRed, DarkGreen, DarkYellow, DarkMagenta, DarkCyan, Gold, Lilac, Turquoise, Aquamarine, Khaki, Pink, Orange, Orchid. These are the same color identifiers as in VCG/yComp files.

The elements of a graph (nodes and edges) can be accessed both by their variable identifier and by their persistent name specified through a constructor (see 5.2.3):

GraphElement
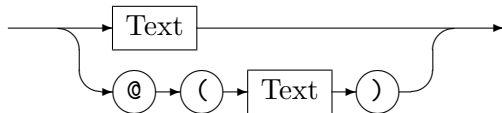
The specializations *Node* and *Edge* of *GraphElement* requires the corresponding graph element to be a node or an edge respectively.

EXAMPLE (12)

We insert a node, anonymously and with a constructor:

```
 1 > select backend lgspBackend.dll
 2 Backend selected successfully.
 3 > new graph "../lib/lgsp-TuringModel.dll" G
 4 New graph "G" of model "Turing" created.
 5
 6 # insert an anonymous node...
 7 # it will get a persistent pseudo name
 8 > new :State
 9 New node "$0" of type "State" has been created.
10 > delete node @("$0")
11
12 # and now with constructor
13 > new v:State($=start)
14 new node "start" of type "State" has been created.
15 # Variable v is now assigned to start
16 > new :State($=end)
17 new node "end" of type "State" has been created.
18
19 # actually we want v to be "end":
20 >  v = @(end)
21 >
```

NOTE (4)

Persistent names belong to a specific graph, whereas variables belong to the current GRSHELL environment. Persistent names will be saved (`save graph...`, see 5.2.5) and, if you visualize a graph (`dump graph...`, see 5.2.5), graph elements will be labeled with their persistent names. Persistent names have to be unique for a graph.
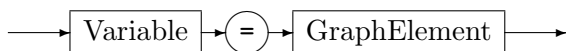

Assigns the variable or persistent name *GraphElement* to *Variable*. If *Variable* has not been defined yet, it will be defined implicitly. As usual for scripting languages, variables have neither types nor declarations.
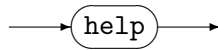
5.2 GRSHELL Commands

This section describes the GRSHELL commands. Commands are assembled from basic elements. As stated before commands are terminated by a line breaks. Alternatively commands can be terminated by the `;;` symbol.
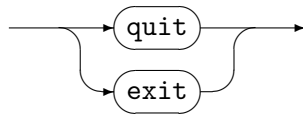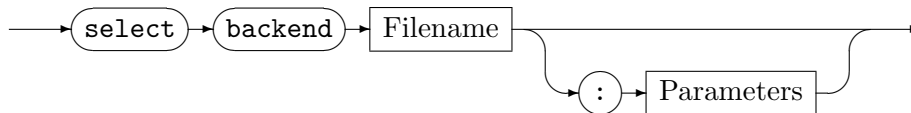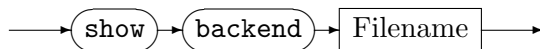
Script
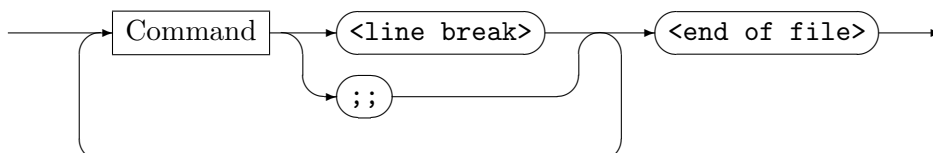
5.2.1 Common Commands

help

Displays an information message describing supported commands.

quit
exit

Quits GRSHELL. If GRSHELL is opened in debug mode, currently active graph displayers (such as YCOMP) will be closed as well.

select backend Filename : Parameters

Selects a backend that handles graph and rules representation. *Filename* has to be a .NET assembly (e.g. "lgspBackend.dll"). Comma-separated parameters can be supplied optionally. If so, the backend must support these parameters.

show backend Filename

List all the parameters supported by the backend *Filename*, that can be provided to the `select backend` command.

include Filename

Executes the GRSHELL script *Filename*. A GRSHELL script is just a plain text file containing GRSHELL commands. They are treated as they would be entered interactively, except for parser errors. If an parser error occurs, execution of the script will stop immediately.
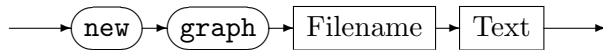
debug enable
disable
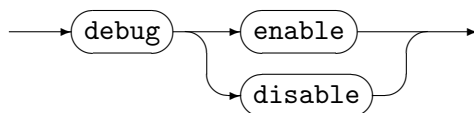
Enables and disables the debug mode. The debug mode shows the current working graph in a YCOMP window. All changes to the working graph are tracked by YCOMP immediately.
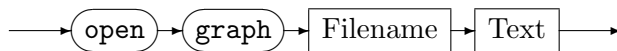
echo Text
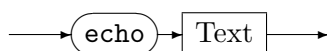
Prints *Text* onto the GRSHELL command prompt.

5.2.2 Graph Commands

Creates a new graph with the model specified in *Filename*. Its name is set to *Text*. The model file can be either source code (e.g. "turing_machine.cs") or a .NET assembly (e.g. "turing_machine.dll").
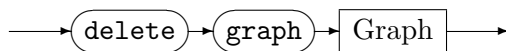

Opens the graph *Text* stored in the backend. Its model is specified in *Filename*. However, the *LGSPBackend* doesn't support persistent graphs. *LGSPBackend* is the only backend so far. Therefore this command is useless at the moment.
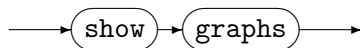
Displays a list of currently available graphs.
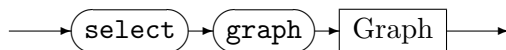
Selects the current working graph.

Deletes the graph *Graph* from the backend storage.
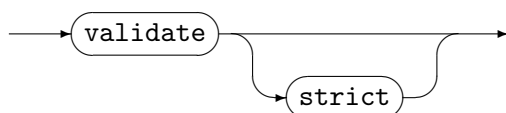
Validates if the current working graph fulfills the edge constraints specified in the corresponding graph model. The *strict* mode additionally requires all the edges of the working graph to be specified in order to get a "valid". Otherwise edges between nodes without specified constraints are ignored.
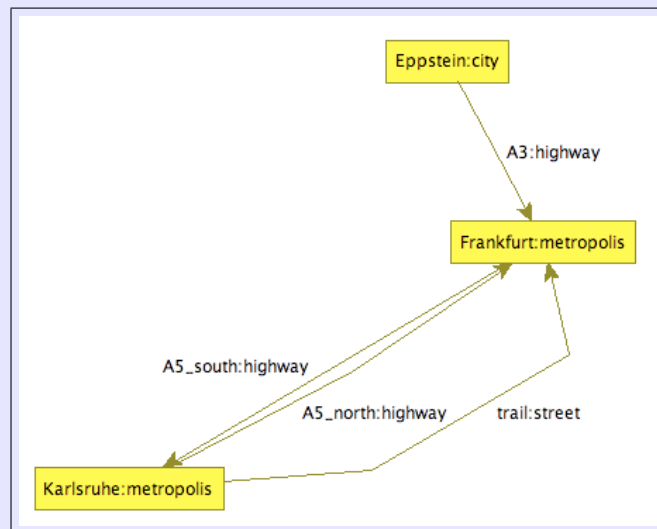
EXAMPLE (13)

We reuse a simplified version of the street map model from chapter 2:

```
1  model Map;
2
3  node class city;
4  node class metropolis;
5
6  edge class street;
7  edge class highway
8         connect metropolis [+] -> metropolis [+];
```

The node constraint on *highway* requires all the metropolises to be connected by highways. Now have a look at the following graph:
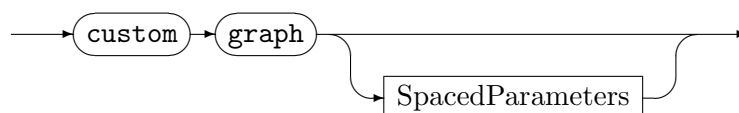

This graph is valid, but not strict valid.

```
1  > validate
2  The graph is valid.
3  > validate strict
4  The graph is NOT valid:
5    CAE: city "Eppstein" -- highway "A3" --> metropolis "Frankfurt" not specified
6    CAE: metropolis "Karlsruhe" -- street "trail" --> metropolis "Frankfurt" not specified
7  >
```
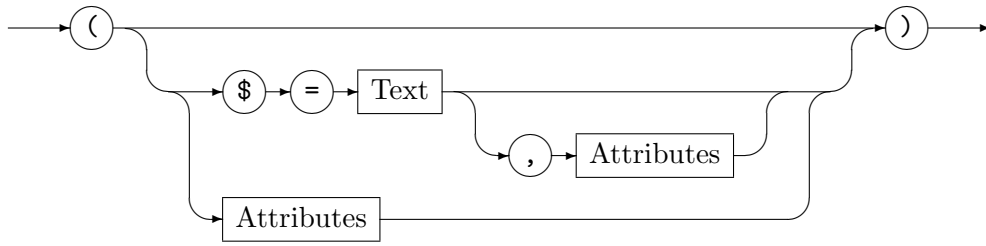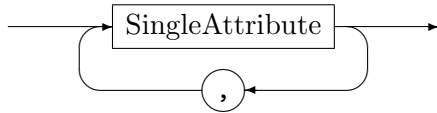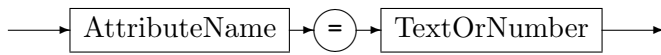

Executes a command specific to the current backend. If *SpacedParameters* is omitted, a list of available commands will be displayed (for the LGSP backend see 5.3).

5.2.3   Graph Manipulation Commands

In order to create graph elements an optional constructor can be used.

Constructor

Attributes

SingleAttribute

A comma separated list of attribute declarations is supplied to the constructor. Available attribute names are specified by the graph model of the current working graph. All the undeclared attributes will be initialized with default values, depending on their type (int, enum ← 0; boolean ← false; float, double 0.0; string ← "").

The $ marks a special attribute: an unique identifier of the new graph element. This identifier also is denoted as *persistent name* (see 12, *GraphElement*). This name can be specified by a constructor only.
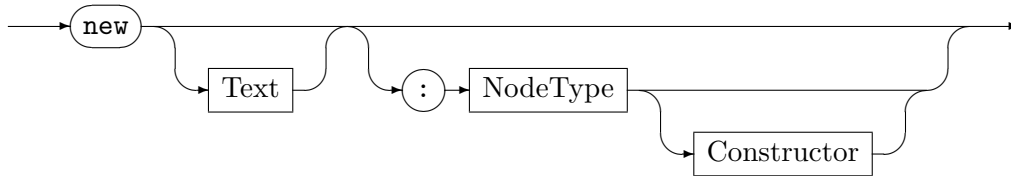
Creates a new node within the current graph. Optionally a variable *Text* is assigned to the new node. If *NodeType* is supplied, the new node will be of type *NodeType* and attributes can be initialized by a constructor. Otherwise the node will be of the base node class type *Node*.
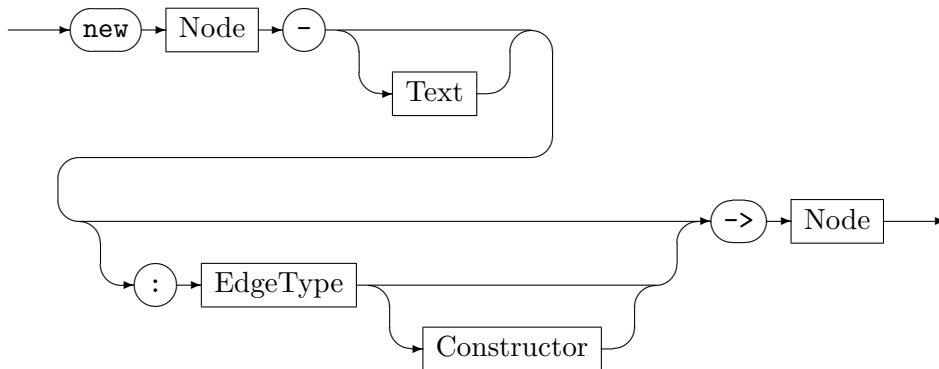
Creates a new edge within the current graph between the specified nodes, directed towards the second *Node*. Optionally a variable *Text* is assigned to the new edge. If *EdgeType* is supplied, the new edge will be of type *EdgeType* and attributes can be initialized by a constructor. Otherwise the edge will be of the base edge class type *Edge*.

```
────▶│ GraphElement │─▶(.)─▶│ AttributeName │─▶(=)─▶│ TextOrNumber │────▶
```
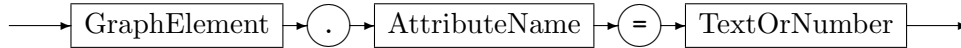
Set the attribute *AttributeName* of the graph element *GraphElement* to the value of *TextOrNumber*.

```
────▶(delete)─▶(node)─▶│ Node │────▶
```
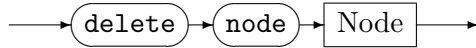
Deletes the node *Node* from the current graph. Incident edges will be deleted as well.

```
────▶(delete)─▶(edge)─▶│ Edge │────▶
```

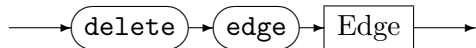Deletes the edge *Edge* from the current graph.

5.2.4 Graph Query Commands

```
───▶(show)──┬──────────┬──┬─(nodes)──────┬────────────▶
            └─▶(num)───┘  │        └▶│ NodeType │┘
                          └─(edges)──────────────┐
                                   └▶│ EdgeType │┘
```
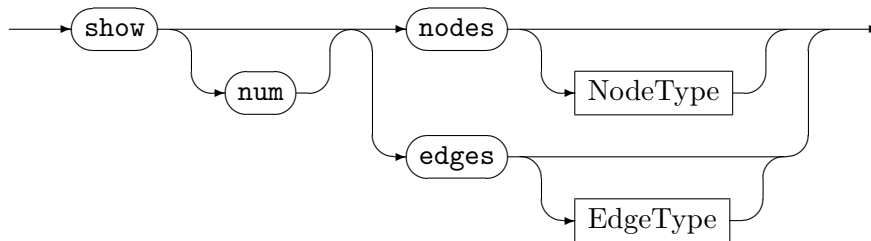
Gets the persistent names and the types of all the nodes / edges of the current graph. If a node type or edge type is supplied, only elements compatible to this type are considered. Nodes / edges without persistent names are shown with a pseudo-name.
If the command is specified with `num`, only the number of nodes/edges will be displayed.

```
────▶(show)──▶(node)──▶(types)────▶
             └▶(edge)┘
```
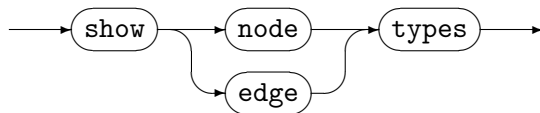
Gets the node / edge types of the current graph model.

```
────▶(show)──┬─(node)──┬─(super)─┬─▶(types)─▶│ NodeType │┐
             │         └─(sub)──┘                        │
             └─(edge)──┬─(super)─┬─▶(types)─▶│ EdgeType │┘
                       └─(sub)──┘
```
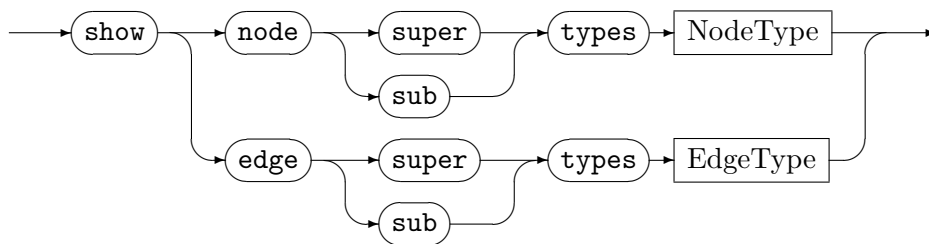
Gets the inherited / descended types of *NodeType* / *EdgeType*.

```
────▶(show)──┬─(node)─▶(attributes)──────────────────────────────▶
             │              └──┬───────────▶│ NodeType │┐
             │                 └▶(only)──┘             │
             └─(edge)─▶(attributes)─────────────────────┤
                            └──┬───────────▶│ EdgeType │┘
                               └▶(only)──┘
```
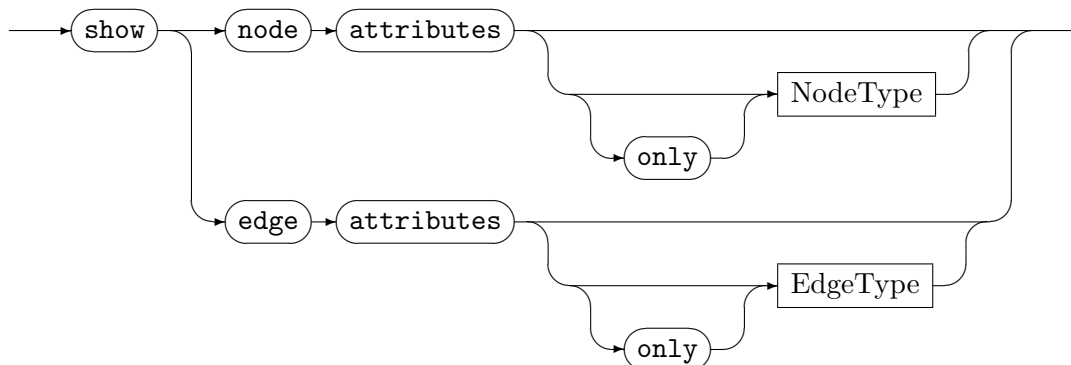
Gets the available node / edge attribute types. If *NodeType* / *EdgeType* is supplied, only attributes defined in *NodeType* / *EdgeType*. The `only` keyword excludes inherited attributes.

NOTE (5)

This is in contrast to the `show num...`, `show nodes...` and `show edges...` commands where types and *sub*types are specified.
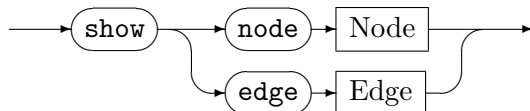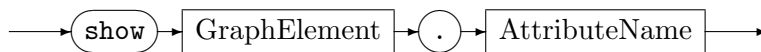

Gets the attribute types and values of a specific graph element.

Gets the value of a specific attribute.
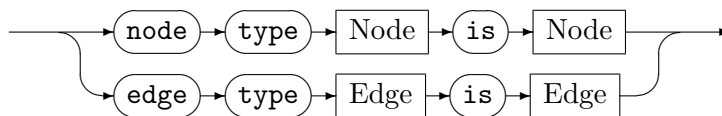
Gets the information, whether the first element is type-compatible to the second element.

5.2.5 Graph Output Commands
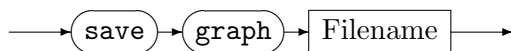
Dumps the current graph as GRSHELL script into *Filename*. The created script includes

- selecting the backend

- creating all nodes and edges

- restoring the persistent names (see 5.2.3),

but not necessarily using the same commands you typed in during construction.
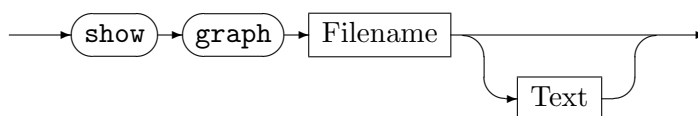
Dumps the current graph as VCG formatted file into a temporary file. *Filename* specifies an executable. The temporary VCG file will be passed to *Filename* as last parameter. Additional parameters, such as program options, can be specified by *Text*. If you use YCOMP as executable, this may look like
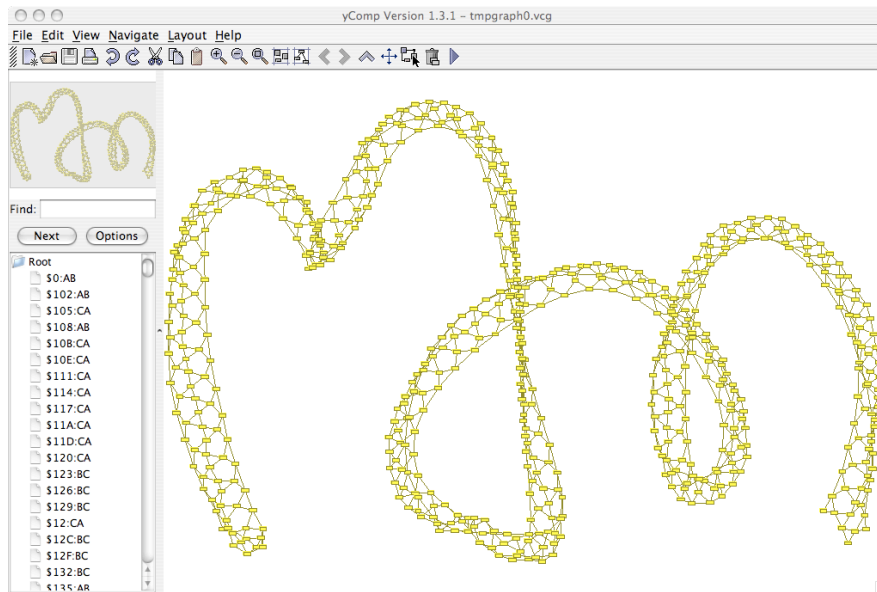
The temporary file will be deleted, when *Filename* is terminated, if GRSHELL is still running at this time.

`dump` → `graph` → Filename
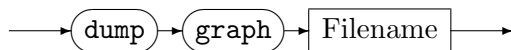
Dumps the current graph as VCG formatted file into *Filename*.

The following commands control the style of the VCG output. This affects `dump graph`, `show graph` and `enable debug`.

`dump` → `set` → `color` / `textcolor` / `bordercolor` → `node` → NodeType → `=` → Color
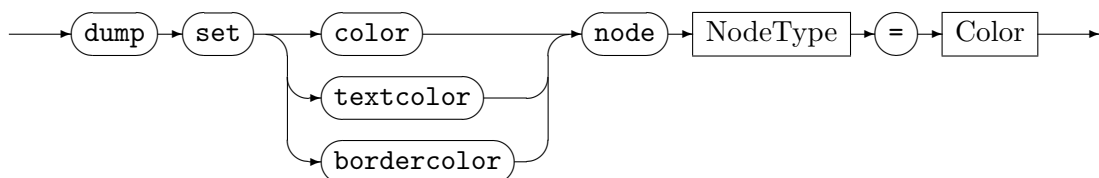
Sets the color / text color / border color of the nodes of type *NodeType*. This doesn't include sub types of *NodeType*.

`dump` → `set` → `color` / `textcolor` → `edge` → EdgeType → `=` → Color
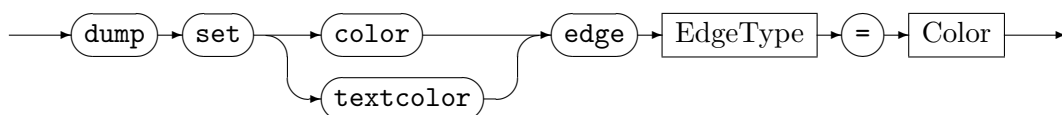
Sets the color / text color of the edges of type *EdgeType*. This doesn't include sub types of *NodeType*.

`dump` → `add` → `exclude` → `node` → NodeType
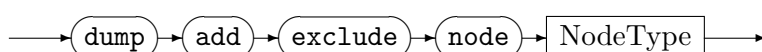
Excludes nodes of type *NodeType* (or sub type of *NodeType*) as well as their incident edges from output.

`dump` → `add` → `group` → `node` → NodeType
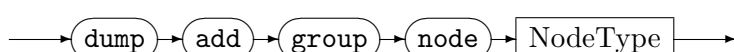
Declares *NodeType* (or sub type of *NodeType*) as group node type. All the different typed nodes that points to a node of type *NodeType* (i.e. there is a directed edge between such nodes) will be grouped and visibly enclosed by the *NodeType*-node. The following example shows *metropolis* ungrouped and grouped:

right side: dumped with `dump add group node metropolis`
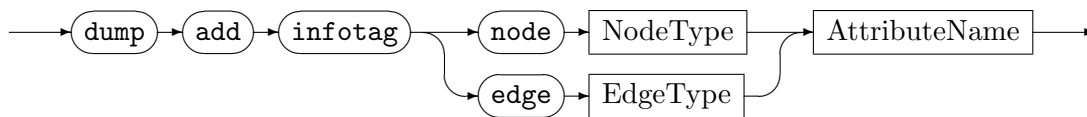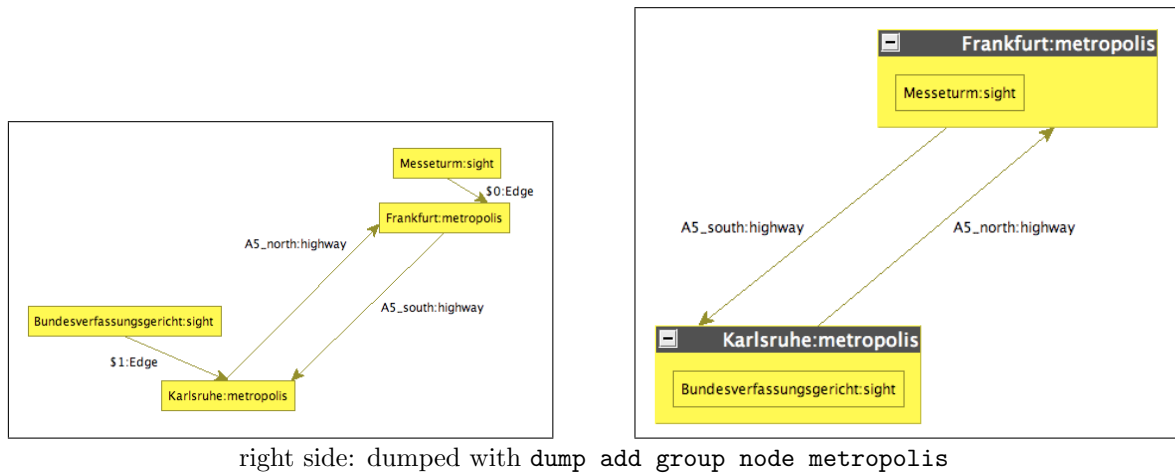
Declares the attribute *AttributeName* to be an "info tag". Info tags are displayed like additional node / edge labels. In the following example *river* and *jam* are info tags:
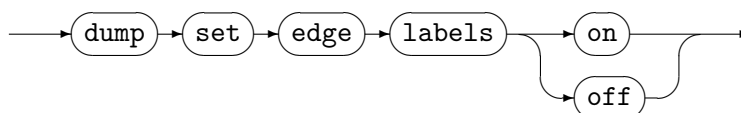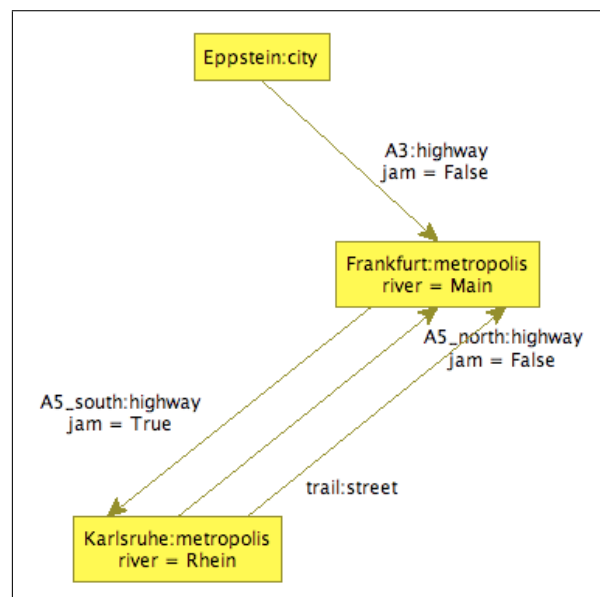
Specifies, whether edge labels will be displayed or not (default to "on").
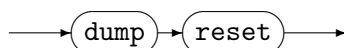
Reset all style options (`dump set...`) to their default values.

5.2.6 Action Commands (GRS)

An *action* denotes a graph rewriting rule.

```
──▶( select )─▶( actions )─▶│ Filename │──▶
```
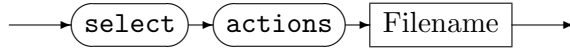
Selects a rule set. *Filename* can be either a .NET assembly (e.g. "rules.dll") or a source file ("rules.cs"). Only one rule set can be loaded at once.

```
──▶( show )─▶( actions )──▶
```
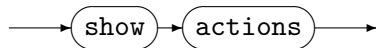
Lists all the rules of the loaded rule set, their parameters and their return values. Rules can return a set of graph elements.

```
──▶( custom )─▶( actions )──────────────▶
                        └▶│ SpacedParameters │┘
```
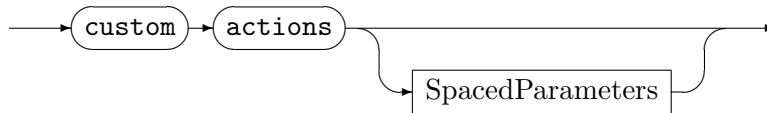
Executes an action specific to the current backend. If *SpacedParameters* is omitted, a list of available commands will be displayed (for the LGSPBackend see section 5.3).

Regular Graph Rewrite Sequences

Basically a graph rewriting command looks like this:

```
──┬──────────▶( grs )─▶│ SimpleRewriteSequence │──▶
  └▶( debug )┘
```
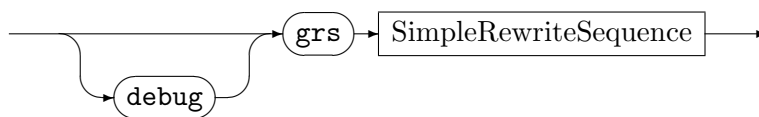
SimpleRewriteSequence

```
──┬─▶│ SimpleTerm │──────────────────────────▶
  │              ├──▶( * )──────┤
  │              └▶( { )─▶│ Number │─▶( } )┘
  ├─▶( ; )◀┤
  ├─▶( | )◀┤  └▶( $ )◀┘
  └▶( & )◀┘
```
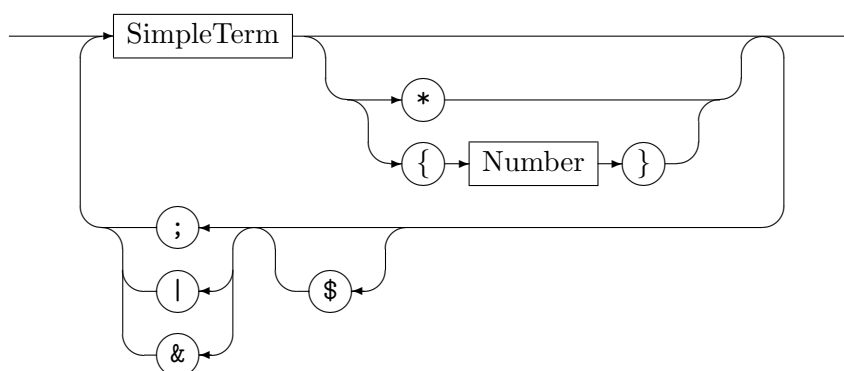
SimpleTerm
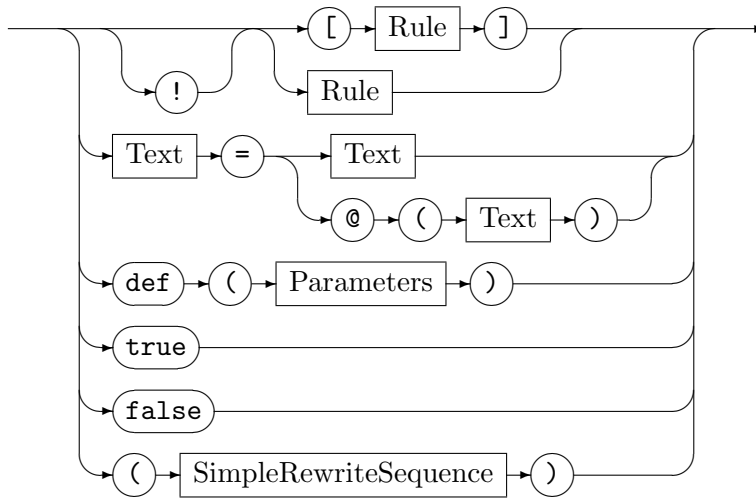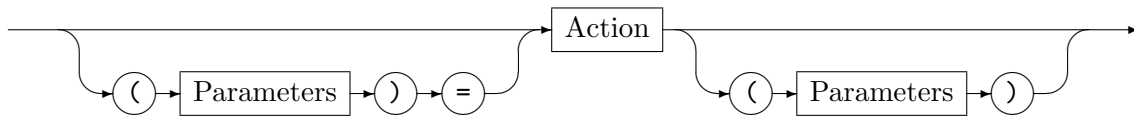
Rule

Table 5.1 lists graph rewriting expressions at a glance. The operators hold the following order of precedence, starting with the lowest precedence:

$$; \qquad | \qquad \&$$

Variables can be assigned to graph elements returned by rules using (**Parameters**) = **Action**. The desired variable identifiers have to be listed in *Parameters*. Graph elements required by rules must be provided using **Action** (**Parameters**), where *Parameters* is a list of variable identifiers. For undefined variables the specific element constraint of *Action* will be ignored (every element matches).

Use the `debug` option to trace the rewriting process step-by-step. During execution yCOMP will display every single step[1]. The debugger can be controlled by GRSHELL. The debug commands are shown in table 5.3.

Figure 5.1: Delayed step rule application.

---

[1]Make sure, that the path to your `yComp.jar` package is set correctly in the `ycomp` shell script within GRGEN's `/bin` directory.

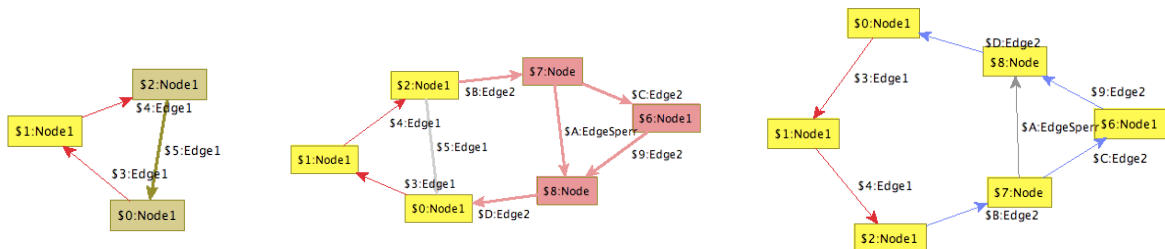|	
s ; t	**Concatenation.** First, s is executed, afterwards t is executed. The sequence s ; t is *successfully* executed iff s or t is successfully executed.
s | t	**XOR.** First, s is executed. Only if s fails, t is executed. The sequence s | t is successfully executed iff s or t is successfully executed.
s & t	**Transactional AND.** First, s is executed, afterwards t is executed. If s or t fails, the action will be terminated and a rollback to the state before s & t is performed.
$<op>	Flags the operator <op> as commutative. Usually operands are executed / evaluated from left to right with respect to bracketing (left-associative). But the sequences s, t, u in s $<op> t $<op> u are executed / evaluated in arbitrary order.
s *	Executes s repeatedly as long as its execution does not fail.
s {n}	Executes s repeatedly as long as its execution does not fail, but anyway n times at most.
!	Found matches are dumped into VCG formatted files. Every match produces three files within the current directoy[a]: 1. The complete graph that has the matched graph elements marked 2. The complete graph with additional information about matching details 3. A subgraph containing only the matched graph elements
Rule	Only the first pattern match produced by the action *Rule* will be rewritten.
[*Rule*]	Every pattern match produced by the action *Rule* will be rewritten. **Note:** This operator is mainly added for benchmark purposes. Its semantic is not equal to Rule*. Instead this operator collects all the matches first before starting rewritings. In particular one needs to avoid deleting a graph element that is bound by another match.
v = w	The variable v is assigned to w. If w is undefined, v will be undefined, too.
v = @(x)	The variable v is assigned to the graph element identified by x. If x is not defined any more, v will be undefined, too.
def(*Parameters*)	Gets *successful* if all the graph elements in *Parameters* exist, i.e. if all the variables are defined.
true	A constant acting as a successful match.
false	A constant acting as a failed match.

[a]Make sure, that the path to your yComp.jar package is set correctly in the ycomp shell script within GRGEN's /bin directory.

Let s, t, u be graph rewriting sequences, v, w variable identifiers, x an identifier of a graph element,

$$<op> \in \{;, |, \&\} \text{ and } n \in \mathbb{N}_0.$$

Table 5.1: Graph rewriting expressions

s(tep)	Executes the next rewriting rule (match and rewrite)
d(elaied step)	Executes a rewriting rule in a three-step procedure: matching, highlighting elements to be changed, doing rewriting
n(ext)	Ascends one level up within the Kantorowitsch tree of the current rewrite sequence
(step) **o**(ut)	Continues a rewriting sequence until the end of the current loop. If the execution is not in a loop at this moment, the complete sequence will be executed
r(un)	Continues execution without further stops
a(bort)	Cancels the execution immediately

Table 5.2: GRSHELL debug commands

Command	Active rule
s	makeFlake1
o	beautify
s	doNothing
s	beautify
n	beautify
o	makeFlake2
r	—

Table 5.3: Debug commands on `grs (makeFlake1* ; (beautify ; doNothing)* ; makeFlake2* ; beautify*)`

EXAMPLE (14)

We demonstrate the debug commands with a slightly adjusted script for the Koch snowflake from GRGEN's examples (see also 6.1). The graph rewriting sequence is

```
1  debug grs (makeFlake1* ; (beautify ; doNothing)* ; makeFlake2* ; beautify*){1}
```

YCOMP will be opened with an initial graph (resulting from `grs init`):
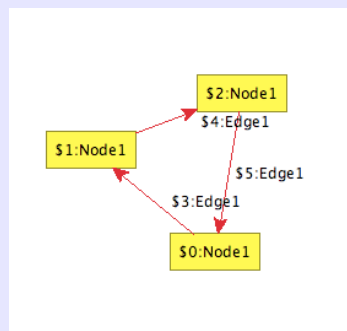
We type **d**(elayed step) to apply `makeFlake1` step by step resulting in the graphs of figure 5.1. Table 5.3 shows the effect of further debug commands.

5.3 LGSPBackend Custom Commands

The LGSPBackend supports the following custom commands:

5.3.1 Graph Related Commands

```
⟶( custom )⟶( graph )⟶( analyze_graph )⟶
```
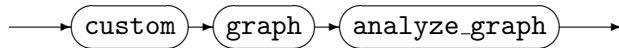
Analyzes the current working graph. The analysis data provides vital information for efficient search plans. Analysis data are available as long as GRSHELL is running, i.e. when the working graph changes the analysis data is still available but maybe obsolete.

5.3.2 Action Related Commands

```
⟶( custom )⟶( actions )⟶( gen_searchplan )⟶[ Action ]⟶
```
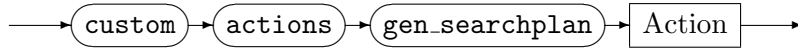
Creates a search plan for the rewriting rule *Action* using a heuristic method and the analyze data (if the graph has been analyzed by `custom graph analyze_graph`). Otherwise a default search plan is used. For efficiency reasons it is recommended to do analyzing and search plan creation during the rewriting procedure. Therefore the host graph should be in a stage similar to the final result. This is kind of a rough specification. In deed there might be some trial-and-error steps necessary to get a efficient search plan. A search plan is available as long as the current rule set remains loaded.

```
⟶( custom )⟶( actions )⟶( set_max_matches )⟶[ Number ]⟶
```
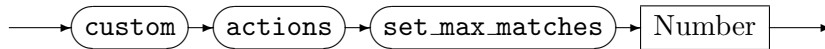
Sets the maximum amount of possible pattern matches to *Number*. This command affects the expression `[Rule]`. For *Number* less or equal to zero, the constraint is reset.

CHAPTER 6

EXAMPLES

6.1 Fractals

The GRGEN package ships with samples for fractal generation. We will construct the Sierpinski triangle and the Koch snowflake. First of all we have to compile the model and rule set files. So execute in GRGEN's `bin` directory

```
GrGen.exe ..\specs\sierpinski.grg
GrGen.exe ..\specs\snowflake.grg
```

or

```
mono GrGen.exe ../specs/sierpinski.grg
mono GrGen.exe ../specs/snowflake.grg
```

respectively. If you are on a Unix-like system you have to adjust the path separators of the GRSHELL scripts. Just edit the first three lines of `/test/Sierpinski.grs` and `/test/Snowflake.grs`. And as we have the file `Sierpinski.grs` already opened, we can increase the number of iterations to get even more beautiful graphs. Just follow the comments. Be careful when increasing the number of iterations of Koch's snowflake – YCOMP's layout algorithm might need some time and attempts to layout it nicely.

We execute the Sierpinski script by

```
grShell.exe ..\test\Sierpinski.grs
```

or

```
mono grShell.exe ../test/Sierpinski.grs
```

respectively. Because both of the scripts are using the debug mode, we complete execution by typing `r`(un). See 5.2.6 for further information. The resulting graphs should look like this:

6.2 Busy Beaver

We want GRGEN to work as hard as a busy beaver [2, 9]. Our busy beaver is a Turing machine, that has got five states, writes 1,471 bars onto the tape and terminates [10]. So first of all we design a Turing machine as graph model. Besides this example shows that GRGEN is Turing complete.

6.2.1 Graph Model

Let's start the model file *TuringModel.gm* with

```
1  model TuringModel;
```

Figure 6.1: Sierpinski triangle

Figure 6.2: Koch snowflake

The tape will be a chain of *TapePosition* nodes connected by right edges. A cell value is modeled by a reflexive *value* edge, attached to a *TapePosition* node. The leftmost and the rightmost cell (*TapePosition*) does not have an incoming and outgoing edge respectively. Therefore we have the node constraint [0 : 1].

```
2  node class TapePosition;
3  edge class right
4    connect TapePosition[0:1] -> TapePosition[0:1];
5
6  edge class value
7    connect TapePosition[1] -> TapePosition[1];
8  edge class zero extends value;
9  edge class one extends value;
10 edge class empty extends value;
```

Finally we need states and transitions. The current configuration is modeled with a *RWHead* edge pointing to a *TapePosition* node. *State* nodes are connected with *WriteValue* nodes via *value* edges and from a *WriteValue* node a *move. . .* edge leads to the next state.

```
11 node class RWHead;
12
13 node class WriteValue;
14 node class WriteZero extends WriteValue;
15 node class WriteOne extends WriteValue;
16 node class WriteEmpty extends WriteValue;
17
18 edge class moveLeft;
19 edge class moveRight;
20 edge class dontMove;
```

6.2.2   Rule Set

Now the rule set: we begin the rule set file *Turing.grg* with

```
1  actions Turing using TuringModel;
```

We need rewrite rules for the following steps of the Turing machine:

1. Read the value of the current tape cell and select an outgoing edge of the current state.

2. Write a new value into the current cell, according to the sub type of the *WriteValue* node.

3. Move the read-write-head along the tape and propagate a new state as current state.

As you can see a transition of the Turing machine is split into two graph rewriting steps: Writing the new value onto the tape and performing the state transition. We need eleven rules, three rules for each step (for "zero", "one" and "empty") and two rules for extending the tape to the left and the right, respectively.

```
2  rule readZeroRule {
3    pattern {
4      s:State -:RWHead-> tp:TapePosition -zv:zero-> tp;
5      s -zr:zero-> wv:WriteValue;
6    }
7    replace {
8      s -zr-> wv;
9      tp -zv-> tp;
10     wv -:RWHead->tp;
11   }
12 }
```

We take the current state *s* and the current cell *tp* implicitly given by the unique *RWHead* edge and check, whether the cell value is zero. Furthermore we check if the state has a transition for zero. The replacement part deletes the *RWHead* edge between *s* and *tp* and adds it between *wv* and *tp*. The remaining rules are analogous:

```
13 │ rule readOneRule {
14 │    pattern {
15 │        s:State -:RWHead-> tp:TapePosition -ov:one-> tp;
16 │        s -or:one-> wv:WriteValue;
17 │    }
18 │    replace {
19 │        s -or-> wv;
20 │        tp -ov-> tp;
21 │        wv -:RWHead-> tp;
22 │    }
23 │ }
24 │
25 │ rule readEmptyRule {
26 │    pattern {
27 │        s:State -:RWHead-> tp:TapePosition -ev:empty-> tp;
28 │        s -er:empty-> wv:WriteValue;
29 │    }
30 │    replace {
31 │        s -er-> wv;
32 │        tp -ev-> tp;
33 │        wv -:RWHead-> tp;
34 │    }
35 │ }
36 │
37 │ rule writeZeroRule {
38 │    pattern {
39 │        wv:WriteZero -rw:RWHead-> tp:TapePosition -:value-> tp;
40 │    }
41 │    replace {
42 │        wv -rw-> tp -:zero-> tp;
43 │    }
44 │ }
45 │
46 │ rule writeOneRule {
47 │    pattern {
48 │        wv:WriteOne -rw:RWHead-> tp:TapePosition -:value-> tp;
49 │    }
50 │    replace {
51 │        wv -rw-> tp -:one-> tp;
52 │    }
53 │ }
54 │
55 │ rule writeEmptyRule {
56 │    pattern {
57 │        wv:WriteEmpty -rw:RWHead-> tp:TapePosition -:value-> tp;
58 │    }
59 │    replace {
60 │        wv -rw-> tp -:empty-> tp;
61 │    }
62 │ }
63 │
64 │ rule moveLeftRule {
65 │    pattern {
66 │        wv:WriteValue -m:moveLeft-> s:State;
67 │        wv -:RWHead-> tp:TapePosition <-r:right- ltp:TapePosition;
```

```
68      }
69      replace {
70          wv -m-> s;
71          s -:RWHead-> ltp -r-> tp;
72      }
73 }
74
75 rule moveRightRule {
76      pattern {
77          wv:WriteValue -m:moveRight-> s:State;
78          wv -:RWHead-> tp:TapePosition -r:right-> rtp:TapePosition;
79      }
80      replace {
81          wv -m-> s;
82          s -:RWHead-> rtp <-r- tp;
83      }
84 }
85
86 rule dontMoveRule {
87      pattern {
88          wv:WriteValue -m:dontMove-> s:State;
89          wv -:RWHead-> tp:TapePosition;
90      }
91      replace {
92          tp;
93          wv -m-> s;
94          s -:RWHead-> tp;
95      }
96 }
97
98 rule ensureMoveLeftValidRule {
99      pattern {
100         wv:WriteValue -m:moveLeft-> s:State;
101         wv -rw:RWHead-> tp:TapePosition;
102         negative {
103             tp <-:right- ltp:TapePosition;
104         }
105     }
106     replace {
107         wv -m-> s;
108         wv -rw-> tp <-:right- ltp:TapePosition -:empty-> ltp;
109     }
110 }
111
112 rule ensureMoveRightValidRule {
113     pattern {
114         wv:WriteValue -m:moveRight-> s:State;
115         wv -rw:RWHead-> tp:TapePosition;
116         negative {
117             tp -:right-> rtp:TapePosition;
118         }
119     }
120     replace {
121         wv -m-> s;
122         wv -rw-> tp -:right-> rtp:TapePosition -:empty-> rtp;
123     }
124 }
```

Have a look at the negative conditions within the *ensureMove...* rules. They ensure, that the current cell is indeed at the end of the tape: an edge to a right / left neighbor cell must not exist. Now don't forget to compile your model and the rule set with `GrGen.exe` (see 6.1).

6.2.3   Rule Execution with GRSHELL

Finally we construct the busy beaver and let it work with GRSHELL:

```
1  select backend "lgspBackend.dll"
2  new graph "../lib/lgsp-TuringModel.dll" "Busy␣Beaver"
3  select actions "../lib/lgsp-TuringActions.dll"
4
5  # Initialize tape
6  new tp:TapePosition($="Startposition")
7
8  # States
9  new sA:State($="A")
10 new sB:State($="B")
11 new sC:State($="C")
12 new sD:State($="D")
13 new sE:State($="E")
14 new sH:State($ = "Halt")
15
16 new sA -:RWHead-> tp
17
18 # Transitions: three lines per state for
19 #   - updating cell value
20 #   - moving read-write-head
21 # respectively
22
23 new sA_0: WriteOne
24 new sA -:empty-> sA_0
25 new sA_0 -:moveLeft-> sB
26
27 new sA_1: WriteOne
28 new sA -:one-> sA_1
29 new sA_1 -:moveLeft-> sD
30
31 new sB_0: WriteOne
32 new sB -:empty-> sB_0
33 new sB_0 -:moveRight-> sC
34
35 new sB_1: WriteEmpty
36 new sB -:one-> sB_1
37 new sB_1 -:moveRight-> sE
38
39 new sC_0: WriteEmpty
40 new sC -:empty-> sC_0
41 new sC_0 -:moveLeft-> sA
42
43 new sC_1: WriteEmpty
44 new sC -:one-> sC_1
45 new sC_1 -:moveRight-> sB
46
47 new sD_0: WriteOne
48 new sD -:empty-> sD_0
49 new sD_0 -:moveLeft->sE
50
51 new sD_1: WriteOne
52 new sD -:one-> sD_1
```

```
53  new sD_1 -:moveLeft-> sH
54
55  new sE_0: WriteOne
56  new sE -:empty-> sE_0
57  new sE_0 -:moveRight-> sC
58
59  new sE_1: WriteOne
60  new sE -:one-> sE_1
61  new sE_1 -:moveLeft-> sC
62  }
```

Our busy beaver looks like this:
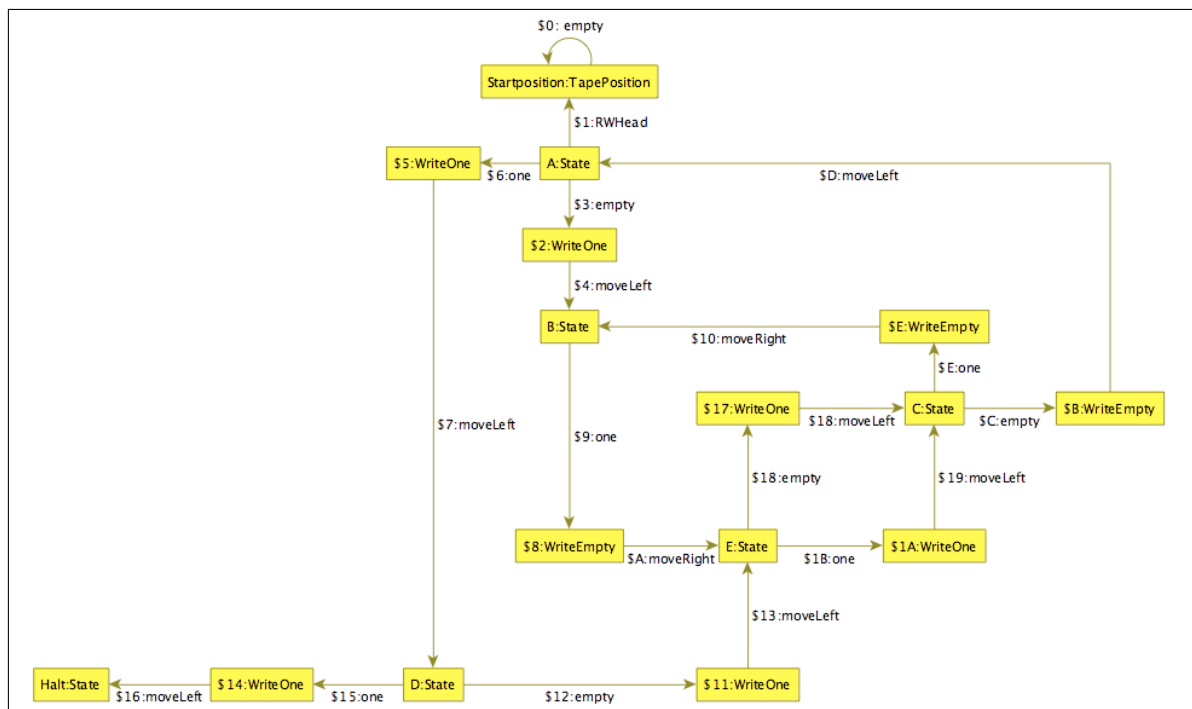
The graph rewriting sequence is quite straight forward and generic to the Turing graph model. Note that for each state the "...Empty... — ...One..." selection is unambiguous.

```
63  grs ((readOneRule | readEmptyRule) ; (writeOneRule | writeEmptyRule) ;
        (ensureMoveLeftValidRule | ensureMoveRightValidRule) ; (moveLeftRule |
        moveRightRule)){32}
```

We intercept the machine after 32 iterations and look at the result so far:
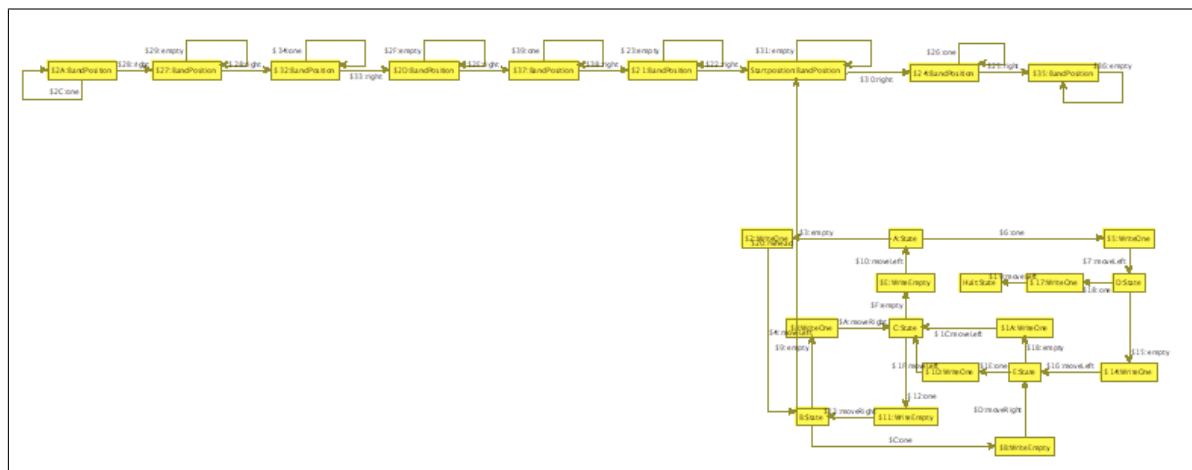
In order to improve the performance we generate better search plans. This is a crucial step for execution time: with the initial search plans the beaver runs for 9 minutes. With improved search plans after the first 32 steps he takes about 170 seconds.

```
64  custom graph analyze_graph
65  custom actions gen_searchplan readOneRule readEmptyRule writeOneRule writeEmptyRule
        ensureMoveLeftValidRule ensureMoveRightValidRule moveLeftRule moveRightRule
```

Let the beaver run:

```
66    grs ((readOneRule | readEmptyRule) ; (writeOneRule | writeEmptyRule) ;
        (ensureMoveLeftValidRule | ensureMoveRightValidRule) ; (moveLeftRule |
        moveRightRule))*
```

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