GR 10 MATHS – EXAM MEMOS

NATIONAL EXEMPLAR PAPER 1

1.1.1 $(m - 2n)(m^2 - 6mn - n^2)$

$= m^3 - 6m^2n - mn^2$

$\quad - 2m^2n + 12\,mn^2 + 2n^3$

$= m^3 - 8m^2n + 11\,mn^2 + 2n^3$ ➤

1.1.2 $\dfrac{x^3 + 1}{x^2 - x + 1} - \dfrac{4x^2 - 3x - 1}{4x + 1}$

$= \dfrac{(x + 1)\cancel{(x^2 - x + 1)}}{\cancel{(x^2 - x + 1)}} - \dfrac{\cancel{(4x + 1)}(x - 1)}{\cancel{(4x + 1)}}$

$= (x + 1) - (x - 1)$

$= x + 1 - x + 1$

$= 2$ ➤

1.2.1 $6x^2 - 7x - 20$

$= (2x - 5)(3x + 4)$ ➤

1.2.2 $a^2 + a - 2ab - 2b$

$= a(a + 1) - 2b(a + 1)$

$= (a + 1)(a - 2b)$ ➤

1.3 $49 < 51 < 64$ . . . *i.e. 51 lies between 49 and 64*

$\therefore\ 7 < \sqrt{51} < 8$ . . . *taking the square root*

i.e. $\sqrt{51}$ lies between 7 and 8 ➤

1.4 Let $x = 0,\dot{2}\dot{4}\dot{5}$

$\therefore\ x = 0,245\ 245\ldots$ . . . ❶

$\times 1\,000)$ $\therefore\ 1\,000x = 245,\ 245\ 245\ldots$ . . . ❷

❷ - ❶: $\therefore\ 999x = 245$

$\therefore\ x = \dfrac{245}{999}$

. . . *i.e. x can be expressed as $\frac{a}{b}$ where $a\ \&\ b \in \mathbb{Z}$*

$\therefore\ x$ is a rational number

2.1.1 $x^2 - 4x = 21$

$\therefore\ x^2 - 4x - 21 = 0$

$\therefore\ (x + 3)(x - 7) = 0$

$\therefore\ x + 3 = 0$ or $x - 7 = 0$

$\therefore\ x = -3$ ➤ $\quad \therefore\ x = 7$ ➤

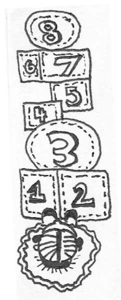
2.1.2 $3x^{\frac{5}{4}} = 96$

$\div 3)$ $\therefore\ x^{\frac{5}{4}} = 32$

$\therefore\ \left(x^{\frac{5}{4}}\right)^{\frac{4}{5}} = (2^5)^{\frac{4}{5}}$

$\therefore\ x = 2^4$

$\therefore\ x = 16$ ➤



2.1.3 $\dfrac{2\sqrt{x}}{3S} = R$

$\times 3S)$ $\therefore\ 2\sqrt{x} = 3SR$

$\div 2)$ $\therefore\ \sqrt{x} = \dfrac{3SR}{2}$

Square: $\therefore\ x = \dfrac{9S^2R^2}{4}$ ➤

2.2 $6q + 7p = 3$ . . . ❶

$2q + p = 5$ . . . ❷

❷ × 3: $6q + 3p = 15$ . . . ❸

❶ - ❸: $\therefore\ 4p = -12$

$\therefore\ p = -3$ ➤

❷: $\therefore\ 2q - 3 = 5$

$\therefore\ 2q = 8$

$\therefore\ q = 4$ ➤

3.1.1 The 1st 3 terms:

$3(3) + 1$; $2(3)$; $3(3) - 7$

$\therefore\ 10$; 6 ; 2 ➤

3.1.2 The difference is -4

$\therefore$ In $T_n = an + b$: $a = -4$

& $T_0 = b = 14$ . . . *the term before the 1st term*

$\therefore\ T_n = -4n + 14$ ➤

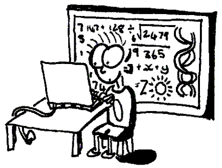
3.1.3 n? if $T_n < -31$

$\therefore\ -4n + 14 < -31$

$\therefore\ -4n < -45$

$\div(-4)\quad \therefore\ n > 11\frac{1}{4}$

$\therefore$ The 12th term ◄



3.2 The even numbers: 6 ; 12 ; 18 . . .

$\therefore$ The 13th even number $= 13 \times 6 = 78$ ◄

OR: The 13th even number
= the 26th term of the pattern
$= 26 \times 3$
$= 78$

4.1 $P = 4\,500;\ i = \frac{4{,}25}{100} = 0{,}0425;\ n = \frac{30}{12} = 2\frac{1}{2};\ A?$

$A = P(1+i)^n = 4\,500(1+0{,}0425)^{2{,}5} = R4\,993{,}47$ ◄

4.2.1 The loan amount = R5 999 - R600 = R5 399

The accumulated amount, $A = P(1 + in)$

where $P = 5\,399;\ i = 8\% = 0{,}08;\ n = 1\frac{1}{2}$ years; A?

$\therefore\ A = 5\,399\left[1 + (0{,}08)\left(\frac{3}{2}\right)\right]$

$= R6\,046{,}88$

$\therefore$ The monthly amount to be paid $= \frac{6\,046{,}88}{18}$

$= R335{,}94$ ◄

4.2.2 The amount of interest

= The total amount paid over the 18 months – the loan amount

= R6 046,88 – R5 399

= R647,88

4.3 28,35 g is worth \$978,34 $= R978{,}34 \times 8{,}79$
$= R8\,599{,}61$

$\therefore$ 1 g is worth $\frac{R8\,599{,}61}{28{,}35}$

$\therefore$ 1 kg is worth $R\frac{8\,599{,}61}{28{,}35} \times 1\,000$ . . . *1 kg = 1 000 g*

$\approx R303\,337{,}16$ ◄

5.1.1 $A \cap B$ ◄ OR: A and B ◄

5.1.2 A' ◄ OR: not A ◄

5.2 Set B ◄

5.3.1 Of the 40 learners, 7 are left-handed

$\therefore\ 40 - 7 = 33$ are right-handed

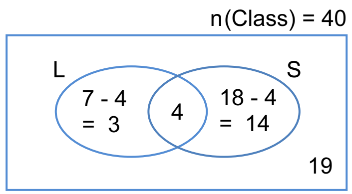
Of the 18 learners who play soccer, 4 are left-handed

$\therefore$ 14 learners who play soccer are right-handed

$\therefore$ The number of learners who are right-handed and DON'T play soccer

$= 33 - 14 = 19$ ◄

5.3.2



5.3.3 (a) n(L or S) = 3 + 4 + 14 = 21

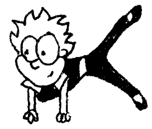
$\therefore$ P(L or S) $= \frac{21}{40}$ ◄

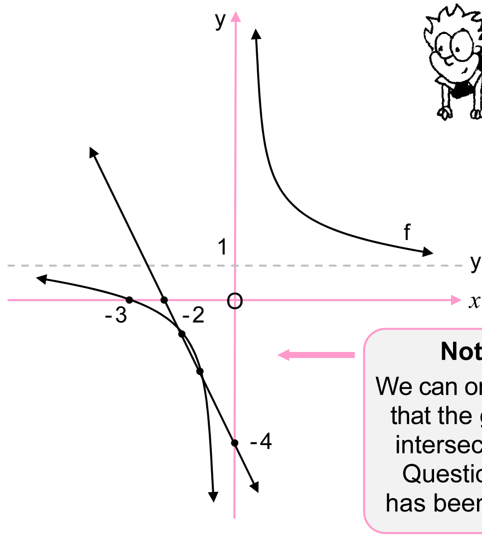
(b) n(R and S) = 14 . . . *where R is the set of all right-handed people*

$\therefore$ P(R and S) $= \frac{14}{40}$

$= \frac{7}{20}$ ◄

6.1





Note:
We can only know that the graphs intersect once Question 6.4 has been done!

f: $y = \frac{3}{x} + 1$

y-intercept ($x = 0$): none

x-intercept ($y = 0$): $\frac{3}{x} + 1 = 0$

$\therefore\ \frac{3}{x} = -1$

$\therefore\ x = -3$

g: $y = -2x - 4$

y-intercept ($x = 0$): $y = -4$

x-intercept ($y = 0$): $-2x - 4 = 0$

$\therefore\ -2x = 4$

$\therefore\ x = -2$

6.2 Asymptotes: $y = 1$ ✓

& $x = 0$ (the y-axis) ✓

6.3 Domain of f: $x \neq 0;\ x \in \mathbb{R}$ ✓

. . . *$(-\infty;\ 0) \cup (0;\ \infty)$*

6.4 $f(x) = g(x)$ ➡ $\frac{3}{x} + 1 = -2x - 4$

$\times x)$ $\therefore\ 3 + x = -2x^2 - 4x$

$\therefore\ 2x^2 + 5x + 3 = 0$

$\therefore\ (2x + 3)(x + 1) = 0$

$\therefore\ 2x + 3 = 0$ or $x + 1 = 0$

$\therefore\ 2x = -3$ $\therefore\ x = -1$ ✓

$\therefore\ x = -\frac{3}{2}$ ✓

Note: These are the x-coordinates of the points of intersection of f and g: $\left(-1\frac{1}{2};\ -1\right)$ & $(-1;\ -2)$

6.5 $-1 \leq g(x) < 3$

$\therefore\ -1 \leq -2x - 4 < 3$ . . . *$g(x) = -2x - 4$*

add 4: $\therefore\ 3 \leq -2x < 7$

$\div(-2)$: $\therefore\ -\frac{3}{2} \geq x > -\frac{7}{2}$ . . . *When one divides by a negative number, the direction of the 'inequality' changes.*

$\therefore\ -\frac{7}{2} < x \leq -\frac{3}{2}$ . . . *the inequality has been rewritten with the smaller value on the left*

i.e. $-3\frac{1}{2} < x \leq -1\frac{1}{2}$ ✓ OR: $\left(-3\frac{1}{2};\ -1\frac{1}{2}\right]$ ✓

(means excluding ;] means including

6.6 $k(x) = 2g(x) = 2(-2x - 4) = -4x - 8$

$\therefore$ The equation of k: $y = -4x - 8$

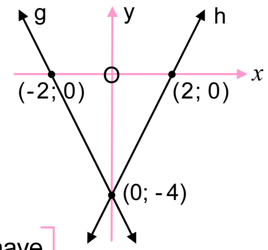
$\therefore$ The y-intercept of k: $(0;\ -8)$ ✓ . . . *substitute $x = 0$*

6.7 x-intercept of g: $(-2;\ 0)$

& x-intercept of h: $(2;\ 0)$ ✓

y-intercept of g: $(0;\ -4)$

& y-intercept of h: $(0;\ -4)$ ✓



Notice: The reflected points have the same y-coordinate, but the x-coordinates are opposite in sign.

7.1 C(-2; 0) ✓ . . . *symmetrical about the y-axis*

7.2 The equation of f: $y = a(x + 2)(x - 2)$ . . . *roots -2 & 2*

$\therefore\ y = a(x^2 - 4)$

Subst. $B\left(-3;\ \frac{5}{2}\right)$: $\therefore\ \frac{5}{2} = a\left[(-3)^2 - 4\right]$

$\therefore\ \frac{5}{2} = a(5)$

$\div 5)$ $\therefore\ a = \frac{1}{2}$

$\therefore$ The equation of f: $y = \frac{1}{2}(x^2 - 4)$

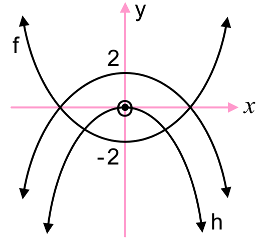
$\therefore\ y = \frac{1}{2}x^2 - 2$ ✓

7.3 The y-intercept of f is $(0;\ -2)$

$\therefore$ The range of f: $y \geq -2$ ✓ OR: $[-2;\ \infty)$ ✓

7.4 The graph of h is obtained by flipping f . . . *$-f(x)$*

then, shifting down 2 units . . . *-2*



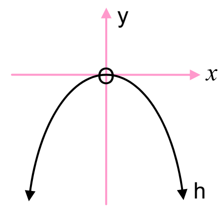
$\therefore$ The range of h: $y \leq 0$ ✓

OR: $(-\infty;\ 0]$ ✓

OR: $h(x) = -\left(\frac{1}{2}x^2 - 2\right) - 2$

$\therefore\ h(x) = -\frac{1}{2}x^2 + 2 - 2$

$\therefore\ h(x) = -\frac{1}{2}x^2$



$\therefore$ The range of h: $y \leq 0$ ✓

7.5 $q = -4$ . . . range $y > -4$ ➡ $y = -4$ is an asymptote

$\therefore$ Equation of g:

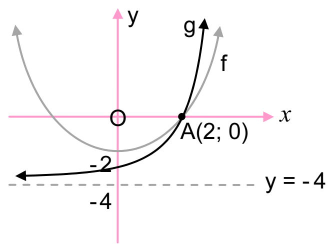
$y = b^x - 4;\ b > 0$

Substitute A(2; 0):

$0 = b^2 - 4$

$\therefore\ b^2 = 4$

$\therefore\ b = 2$ . . . *$b \neq -2$ $\because$ $b > 0$*



$\therefore$ Equation of g:

$y = 2^x - 4$ ✓

$\therefore$ means therefore

$\because$ means because

NATIONAL EXEMPLAR PAPER 2

1.1 The mean,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \ldots \frac{\textit{total number of scores}}{\textit{total number of days}}$$

$$= \frac{929}{19}$$

$$\approx 48{,}89 \;\blacktriangleleft$$

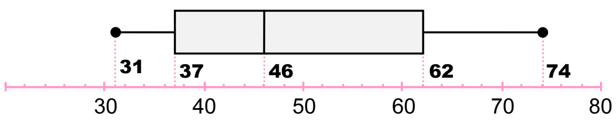
1.2 31; 31; 34; 36; 37 **(Q_1)**; 39; 40; 43; 46; 46 **(Q_2)**; 48;
52; 56; 60; 62 **(Q_3)**; 63; 65; 66; 74

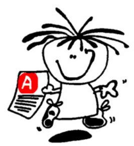
The median **(Q_2)** = 46 ◀

1.3 The lower quartile **(Q_1)** = 37 ◀

The upper quartile **(Q_3)** = 62 ◀

1.4 Min value = 31 & Max value = 74





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2.1 $2\,500 \leq x < 4\,500$

The **sum** of . . .
the **products** of the **frequency**
and the **mid-value** for each interval

2.2 Estimated mean, $\bar{X}$

$$= \frac{103 \times 3\,500 + 19 \times 5\,500 + 70 \times 7\,500 + 77 \times 9\,500 \ldots^*}{103 + 19 + 70 + 77 + 85 + 99}$$

The sum of the frequencies

$^*\ldots + 85 \times 11\,500 + 99 \times 13\,500$

$$= \frac{4\,035\,500}{453}$$

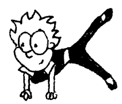
$$\approx 8\,908{,}39 \text{ kg} \;\blacktriangleleft$$

$$\bar{X} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

2.3 The estimated mean ◀

This value is at the centre of the set, whereas the modal class is an extreme situation in relation to the other intervals. ◀

3.1.1 $DE^2 = (3+3)^2 + (-5-3)^2$
$= 36 + 64$
$= 100$
$\therefore$ DE = 10 units ◀



3.1.2 Gradient of DE,

$$m_{DE} = \frac{-5-3}{3+3} = \frac{-8}{6} = -\frac{4}{3} \;\blacktriangleleft$$

3.1.3 $m_{EF} = \dfrac{k+5}{-1-3} = \dfrac{k+5}{-4}$

$D\hat{E}F = 90^\circ \Rightarrow m_{EF} = +\dfrac{3}{4}$ . . . *EF ⊥ DE*

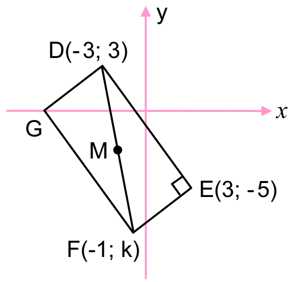
$\therefore \dfrac{k+5}{-4} = \dfrac{3}{4}$

$\times(-4)$ $\quad \therefore k + 5 = -3$

$\therefore k = -8$ ◀

3.1.4 $M\left(\dfrac{-3+(-1)}{2}\,;\ \dfrac{3+(-8)}{2}\right)$

$\therefore M\left(-2\,;\ -\dfrac{5}{2}\right)$ ◀



3.1.5 DEFG will be a $||^m$ if M is the midpoint of EG too.

& Since $D\hat{E}F = 90^\circ$,
DEFG will be a rectangle.
. . . *if one ∠ of a $||^m$ is a right ∠, then the $||^m$ is a rectangle.*

$\dfrac{x_G + 3}{2} = -2$ and $\dfrac{y_G + (-5)}{2} = -\dfrac{5}{2}$

$\times 2)$ $\quad \therefore x_G + 3 = -4 \qquad \therefore y_G - 5 = -5$

$\therefore x_G = -7 \qquad \therefore y_G = 0$

$\therefore$ G(-7; 0) ◀

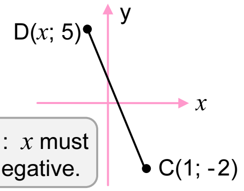
OR: The translation F to G equals that of E to D
$\therefore$ G(-1 - 6; -8 + 8)
$\therefore$ G(-7; 0) ◀

OR: The translation D to G equals that of E to F
$\therefore$ G(-3 - 4; 3 - 3)
$\therefore$ G(-7; 0) ◀

M

2

3.2 $\quad CD^2 = (x-1)^2 + (5+2)^2 = (\sqrt{53})^2$



Note: x must be negative.

$\therefore (x-1)^2 + 49 = 53$

$\therefore (x-1)^2 = 4$

$\therefore x - 1 = \pm 2$

$\therefore x = 3$ or -1

But $x < 0$ in the second quadrant

$\therefore x = -1$ ◀ . . . *only the neg. value of x is valid*

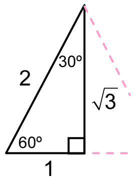
4.1.1 $\quad \sin C = \dfrac{AB}{\mathbf{AC}}$ ◀

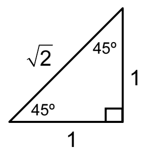
4.1.2 $\quad \mathbf{cot}\ A = \dfrac{AB}{BC}$

Note: $\tan A = \dfrac{BC}{AB}$; $\cot A = \dfrac{1}{\tan A}$

4.2 The expression

$= \dfrac{\dfrac{\cancel{\sqrt{3}}}{2} \times \dfrac{1}{\cancel{\sqrt{3}}}}{\sqrt{2}}$



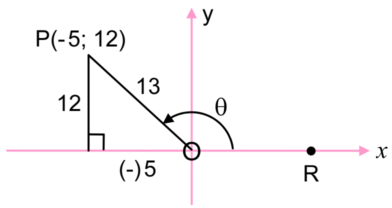


$= \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ . . . *The denominator must be rationalised*

$= \dfrac{\sqrt{2}}{4}$ ◀ . . . $\sqrt{2} \times \sqrt{2} = 2$

4.3.1 OP = 13 units . . . *5 : 12 : 13 Δ ; Pythagoras*



$\therefore \cos\theta = \dfrac{-5}{13} = -\dfrac{5}{13}$ ◀ . . . $\cos\theta = \dfrac{x}{r}$

4.3.2 $\quad \sin\theta = \dfrac{12}{13}$ ➡ $\operatorname{cosec}\theta = \dfrac{13}{12}$

$\therefore \operatorname{cosec}^2\theta + 1 = \left(\dfrac{13}{12}\right)^2 + 1 = \dfrac{169}{144} + 1$

$= \dfrac{169 + 144}{144} = \dfrac{313}{144}$ ◀ $\quad \left(= 2\dfrac{25}{144}\right)$ ◀

5.1.1 $\quad 5\cos x = 3$

$\div 5) \quad \therefore \cos x = \dfrac{3}{5} \quad (= 0{,}6)$

$\therefore x \approx 53{,}1^\circ$ ◀ . . . $\cos^{-1}\left(\dfrac{3}{5}\right) =$

5.1.2 $\quad \tan 2x = 1{,}19$

$\therefore 2x = 49{,}958\ldots^\circ$ . . . $\tan^{-1} 1{,}19 =$

$\div 2) \quad \therefore x \approx 25{,}0^\circ$ ◀

5.1.3 $\quad 4\sec x - 3 = 5$

$+ 3) \quad \therefore 4\sec x = 8$

$\div 4) \quad \therefore \sec x = 2$

$\therefore \cos x = \dfrac{1}{2}$

$x = 60^\circ$ ◀ . . . $\cos^{-1}\left(\dfrac{1}{2}\right) =$

5.2.1 $\quad J\hat{K}D = 8^\circ$ ◀ . . . *alternate ∠'s; || lines*

5.2.2 In ΔJDK: $\dfrac{DK}{5} = \cot 8^\circ$ . . . $= \dfrac{1}{\tan 8^\circ}$

$\times 5) \quad \therefore DK = \dfrac{5}{\tan 8^\circ}$

$= 35{,}5768\ldots$ km

$= 35\ 576{,}8$ metres

$\approx 35\ 577$ metres ◀

. . . *correct to the nearest metre*

5.2.3 $\quad DS = DK - SK$

$= 35{,}58$ km $- 8$ km

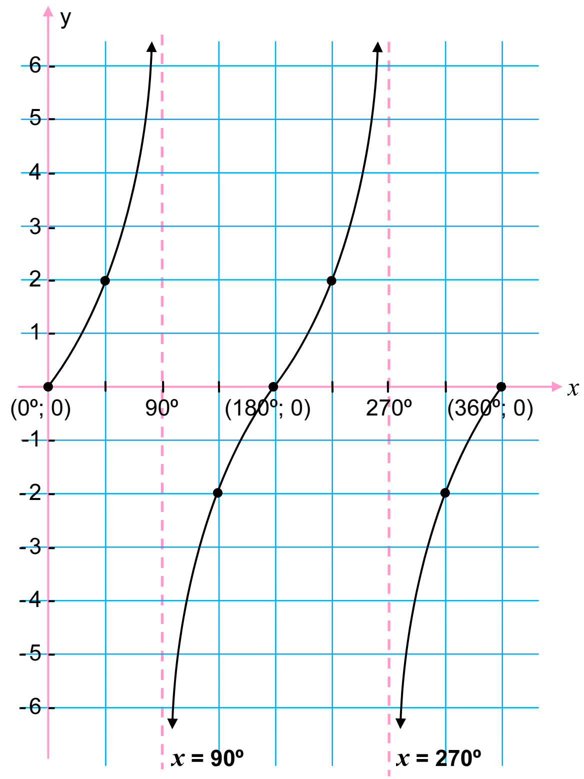
$= 27{,}58$ km ◀

5.2.4 $\quad \tan J\hat{S}D = \dfrac{5}{27{,}58}$

$\therefore J\hat{S}D \approx 10{,}3^\circ$ ◀ . . . $\tan^{-1}\left(\dfrac{5}{27{,}58}\right) =$

correct to 1 dec. place

6.1.1



6.1.2 $\quad y = -2\tan x$ ◀

6.2.1 $\quad a = 4$ ◀

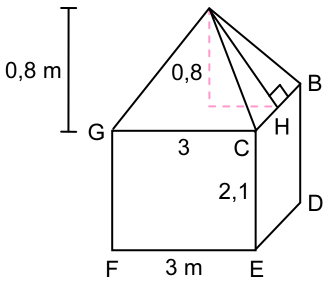
$g(x) = a\sin x$ ➡ $g(90^\circ) = a\sin 90^\circ$
➡ $4 = a$

6.2.2 The range of h:

$-2 \le y \le 6$ ◀ . . . *the values of y*

EXAM MEMOS: PAPER 2

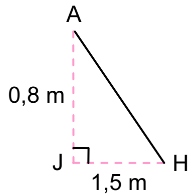
7.



7.1.1 $AH^2 = 0{,}8^2 + 1{,}5^2$

$= 2{,}89$

$\therefore$ AH $\approx$ 1,7 m ➤



OR: Pythag. triple: 8 : 15 : 17

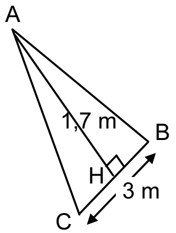
➡ 0,8 : 1,5 : **1,7** ➤

7.1.2 Surface area of roof

= 4 × area of ΔABC

$= 4 \times \frac{1}{2}(3)(1{,}7)$

$= 10{,}2\ m^2$ ➤



7.1.3 Surface area of the walls

= 4 × area of GFEC

= 4 × (3)(2,1)

$= 25{,}2\ m^2$ ➤

$\therefore$ The total surface area of the tent

= 10,2 + 25,2

$= 35{,}4\ m^2$ ➤

7.2.1 Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3 \approx 2\ 144{,}66\ mm^3$ ➤

7.2.2 $2^3 : 1$

= 8 : 1 ➤

$$\frac{\text{New volume}}{\text{Original volume}} = \frac{\cancel{\frac{4}{3}}\cancel{\pi}(2r)^3}{\cancel{\frac{4}{3}}\cancel{\pi}r^3} = \frac{2^3\cancel{r^3}}{\cancel{r^3}} = \frac{8}{1}$$

7.2.3 Volume of silver

$= \frac{4}{3}\pi(8 + 1)^3 - \frac{4}{3}\pi(8)^3$ . . . *The volume of silver covering the ball*

= 908,967...

$\approx 908{,}97\ mm^3$ ➤

Geometry is easier than you thought!
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8.1 OQ = 2 cm ➤ . . . *the longer diagonal of a kite bisects the shorter diagonal*

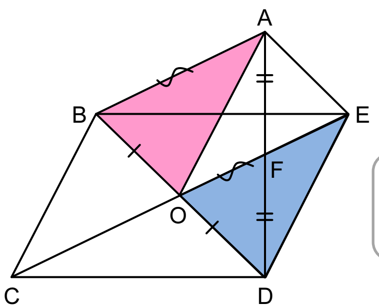
8.2 $P\hat{O}Q = 90°$ ➤ . . . *the diagonals of a kite intersect at right angles*

8.3 $Q\hat{P}O = 20°$ . . . *the longer diagonal of a kite bisects the (opposite) angles of a kite*

$\therefore Q\hat{P}S = 40°$ ➤

9.

Hint:
Use hiliters to mark the various $||^{ms}$ and Δ^s



The hilited Δ^s (and their sides) refer to Question 9.3.

9.1 In ΔDBA:

O is the midpt of BD . . . *diagonals of* $||^m$ *BCDE bisect each other*

& F is the midpt of AD . . . *diagonals of* $||^m$ *AODE bisect each other*

$\therefore$ OF || AB ➤ . . . *the line joining the midpoints of two sides of a Δ is || to the* 3^{rd} *side*

9.2 AE || OD . . . *opp. sides of* $||^m$ *AODE*

$\therefore$ AE || BO

and OF || AB . . . *proven above*

$\therefore$ OE || AB

$\therefore$ ABOE is a $||^m$ . . . *both pairs of opposite sides are parallel*

OR: In $||^m$ AODE: AE = and || OD . . . *opp. sides of* $||^m$

But OD = and || BO . . . *O proved midpt of BD in 9.1*

$\therefore$ AE = and || BO

$\therefore$ ABOE is a $||^m$ ➤ . . . *1 pr of opp. sides = and ||*

9.3 In Δ^s ABO and EOD

1) AB = EO . . . *opposite sides of* $||^m$ *ABOE*

2) BO = OD . . . *proved in 9.1*

3) AO = ED . . . *opposite sides of* $||^m$ *AODE*

$\therefore$ ΔABO ≡ ΔEOD ➤ . . . *SSS*