GR 10 MATHS – TRIG RATIOS & FUNCTIONS

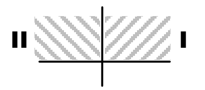
Summary of TRIG RATIOS

The SIGNS of the trig ratios IN A FLASH!

$\sin\theta = \dfrac{y}{r}$ ← and **y is positive** in **I** & **II**

∴ **sin θ is POSITIVE** in quadrants 1 & 2
(and negative in 3 & 4)

sin θ

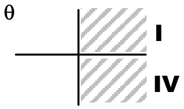


'SUNRISE FOR SINE'

$\cos\theta = \dfrac{x}{r}$ ← and **x is positive** in **I** & **IV**

∴ **cos θ is POSITIVE** in quadrants 1 & 4
(and negative in 2 & 3)

cos θ



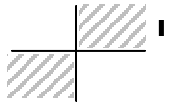
'PULL A CURTAIN FOR COS'

$\tan\theta = \dfrac{y}{x}$ ← ←

and **x & y** have the same sign in **I** & **III**

∴ **tan θ is POSITIVE** in quadrants 1 & 3
(and negative in 2 & 4)

tan θ



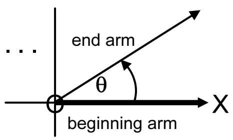
Apparently, this is **THE BMW SYMBOL**!!!

Learn these easy PICTURES so that you know the SIGNS of your trig ratios IN A FLASH!

NO MORE CAST RULE!!!

The 4 steps to find the trig ratios of <u>any</u> angle:

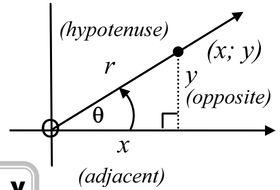
1. **Place the ∠ in STANDARD POSITION** (starting at $\overrightarrow{OX}$) . . .



2. **Pick a point (x; y) on the end arm of the ∠**
   – we'll call its distance from the origin **r**

3. **Write down x = y = r =**

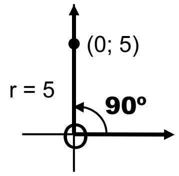
4. **Apply the DEFINITIONS**



$\boxed{\sin\theta = \dfrac{y}{r}}$ $\boxed{\cos\theta = \dfrac{x}{r}}$ $\boxed{\tan\theta = \dfrac{y}{x}}$

The trig ratios of 90° and multiples of 90°

Use this procedure to find the trig ratios of 90° ; 180° ; 270° & 360° (& 0°)

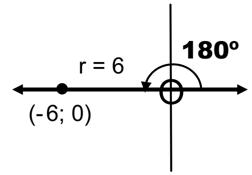


$\sin 90° = \frac{y}{r} = \frac{5}{5} = 1$

$\cos 90° = \frac{x}{r} = \frac{0}{5} = 0$

$\tan 90° = \frac{y}{x} = \frac{5}{0} = \infty$

$x = 0$; $y = 5$; $r = 5$

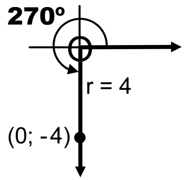


$\sin 180° = \frac{y}{r} = \frac{0}{6} = 0$

$\cos 180° = \frac{x}{r} = \frac{-6}{6} = -1$

$\tan 180° = \frac{y}{x} = \frac{0}{-6} = 0$

$x = -6$; $y = 0$; $r = 6$

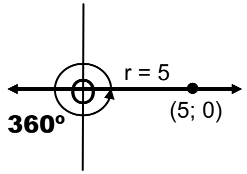


$\sin 270° = \frac{y}{r} = \frac{-4}{4} = -1$

$\cos 270° = \frac{x}{r} = \frac{0}{4} = 0$

$\tan 270° = \frac{y}{x} = \frac{-4}{0} \Rightarrow \infty$

$x = 0$; $y = -4$; $r = 4$



$\sin 360° = \frac{y}{r} = \frac{0}{5} = 0$

$\cos 360° = \frac{x}{r} = \frac{5}{5} = 1$

$\tan 360° = \frac{y}{x} = \frac{0}{5} = 0$

$x = 5$; $y = 0$; $r = 5$

Note: The results for 0° and 360° are the same.

SUMMARY

θ:	0°	⟶	90°	⟶	180°	⟶	270°	⟶	360°
sin θ:	0	- - - →	1	- - - →	0	- - - →	-1	- - - →	0
cos θ:	1	- - - →	0	- - - →	-1	- - - →	0	- - - →	1
tan θ:	0	- - - →	± ∞	- - - →	0	- - - →	± ∞	- - - →	0

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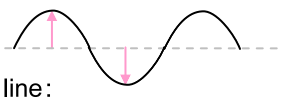
TRIG FUNCTIONS

Trigonometric graphs

We will learn how to sketch the graphs $y = \sin\theta$, $y = \cos\theta$ and $\tan\theta$ for $0° \leq \theta \leq 360°$.
We will use the critical values of these ratios to make it easy. But first, some terminology . . .

Terminology

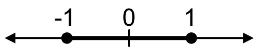
The sine and cosine graphs are WAVE-shaped.



- The **amplitude** of a WAVE is the deviation from its centre line:
- The **period** of a graph is the number of degrees spanning a FULL WAVE.
- The **range** is the set of all the possible y-values.
  Our investigations of the trig ratios have shown us that the range of values of sines and cosines is very small - only between **-1** and **1**.

  We write: $\mathbf{-1} \leq \mathbf{\sin}\,\theta \leq \mathbf{1}$ and $\mathbf{-1} \leq \mathbf{\cos}\,\theta \leq \mathbf{1}$ for all values of θ!

  By contrast, the range of tan values is from $-\infty$ to $+\infty$!

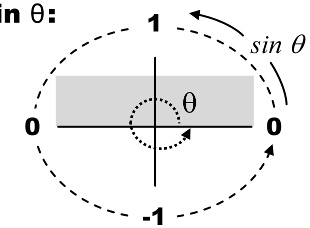


  Before drawing the graphs, we will depict the 'critical values' of the ratios as the angle increases from 0° to 360° as:

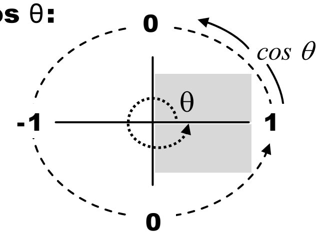
➤ 'Wheels' of values

As θ : 0° → 360°

sin θ:



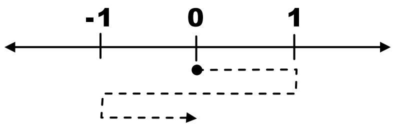
cos θ:



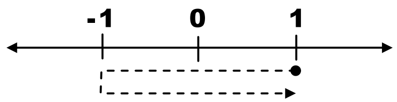
In these 2 'wheels' of values we are considering angles **from 0° to 360°**, going anticlockwise from the line $\overline{OX}$. We read the 'critical values'; i.e. the sine and cosine values of multiples of 90° accordingly, as indicated on the wheels.

➤ and now, these values on a number line:

sin θ:



cos θ:

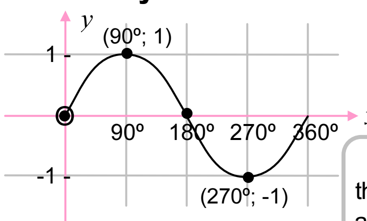


It is clear that sin θ and cos θ can only be PROPER FRACTIONS or equal to ±1 or 0.

The graphs of y = sin θ & y = cos θ for θ ∈ [0°; 360°]

Use the wheels to plot the 'critical points' before drawing the waves.

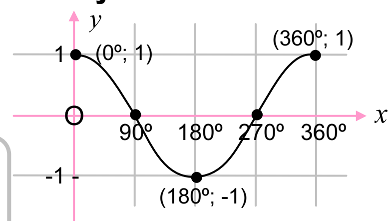
y = sin θ



Note the TURNING POINTS

For both graphs:
the **amplitude = 1 unit**
and the **period = 360°.**

y = cos θ



The range: $-1 \leq \sin\theta \leq 1$

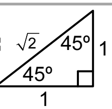
The range: $-1 \leq \cos\theta \leq 1$

➤ The Critical Values of y = tan θ

The **range** of tan values is **$(-\infty\,;\,\infty)$** !

So, we need tan values 'more often' than for sine and cosine.

Remember:

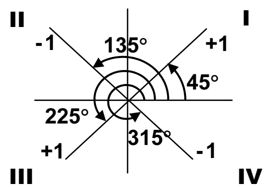


tan 45° = 1

tan 135° = -1 . . . *quadrant II*

tan 225° = +1 . . . *quadrant III*

tan 315° = -1 . . . *quadrant IV*



Check these values on your calculator.
Also, confirm them by placing each angle in standard position. . . .

Use the definition $\tan\theta = \dfrac{y}{x}$

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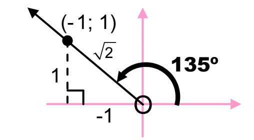
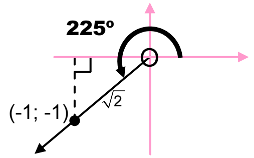


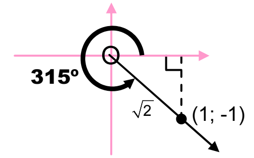
$x = -1$; $y = 1$

$\therefore$ tan 135º = $\dfrac{1}{-1}$ = -1



$x = -1$; $y = -1$

$\therefore$ tan 225º = $\dfrac{1}{-1}$ = 1

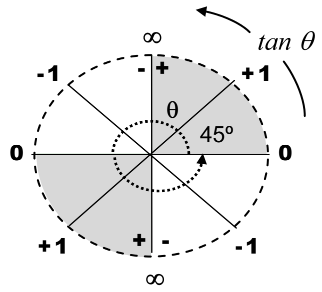


$x = 1$; $y = -1$

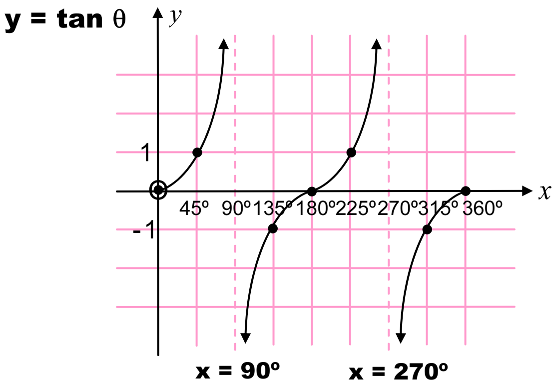
$\therefore$ tan 315º = $\dfrac{-1}{1}$ = -1

As θ : 0º → 360º

tan θ:



The graph of y = tan θ for θ ∈ [0º; 360º]



The dashed lines,
x = 90º and **x = 270º**, are called **asymptotes**

The range: (-∞ ; ∞)

There is **no amplitude**, but the **period** of this graph is **180º**.

An asymptote is a line which a curve approaches but will never touch or cut.

The Quadrants

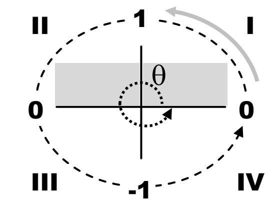
We have observed the relationship between angles (0º to 360º) and their trigonometric ratios (sin θ, cos θ, tan θ), both on the 'wheels' and on the graphs.

The wheels	The graphs
• We observe the angles, and • write down the ratio values.	• We write down the values of the angles on the x-axis, and • observe the values of the ratios.

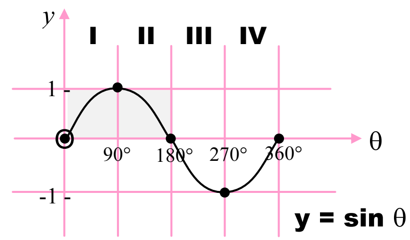
See where the quadrants lie in both cases.

The quadrants :	**I**	**II**	**III**	**IV**
The intervals :	0º → 90º	90º → 180º	180º → 270º	270º → 360º

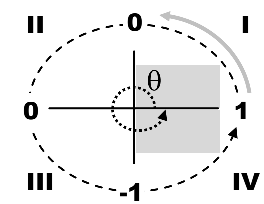
sin θ:



Note where sin θ > 0
(and where it is negative)



cos θ:



Note where cos θ > 0
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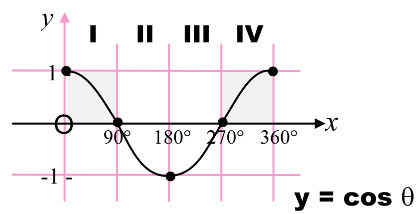
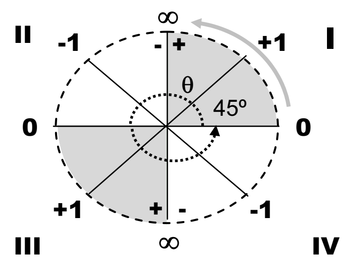
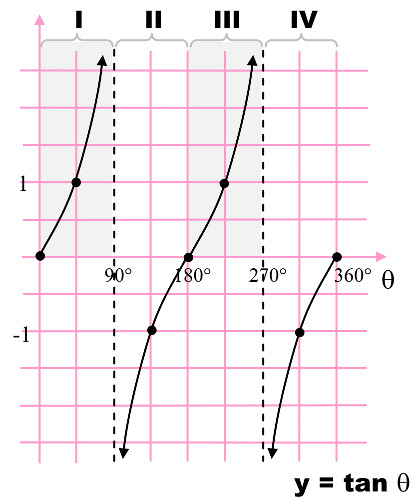


tan θ:



Note where tan θ > 0
(and where it is negative)



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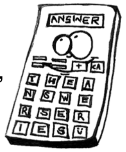
An investigation



Use your calculator to observe various values of sin θ, cos θ and tan θ.
Get to know how each of these three ratios behave as an angle increases, through each quadrant, from 0º to 360º.

▸ Investigating the values of sin θ for θ ∈ [0º; 360º]

1.1 Fill in the sin θ values (correct to 2 decimal digits) of these angles

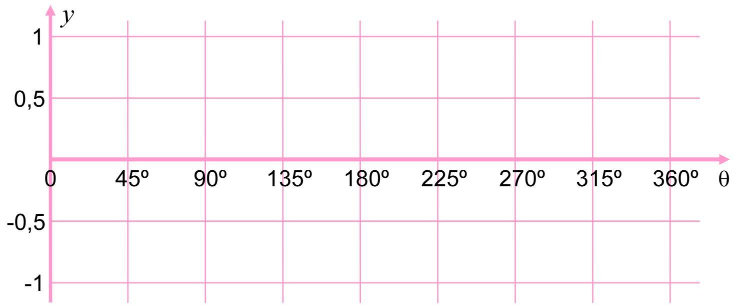
θ	0º	20º	45º	70º	90º	110º	135º	160º	180º
sin θ									

θ	180º	200º	225º	250º	270º	290º	315º	340º	360º
sin θ									

Compare the values – what do you notice?

Use the accompanying table to plot points, even approximately, on the set of axes.

1.2 Draw the graph of **y = sin** θ on the following set of axes:



▸ Investigating the values of cos θ for θ ∈ [0º; 360º]

2.1 Fill in the cos θ values (correct to 2 decimal digits) of these angles

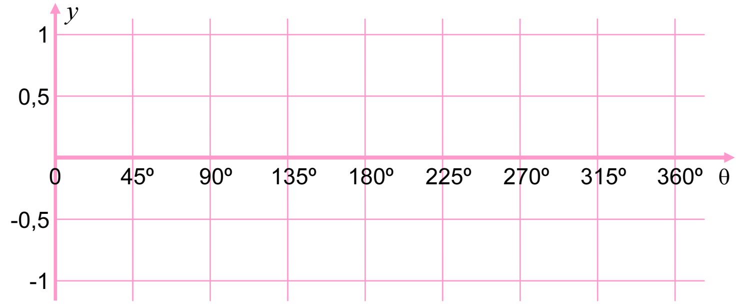
θ	0º	20º	45º	70º	90º	110º	135º	160º	180º
cos θ									

θ	180º	200º	225º	250º	270º	290º	315º	340º	360º
cos θ									

Compare the values – what do you notice? (Also compare the values of 1.1 vs 2.1

Use the accompanying table to plot points, even approximately, on the set of axes.

2.2 Draw the graph of **y = cos** θ on the following set of axes:



▸ Investigating the values of tan θ for θ ∈ [0º; 360º]

3.1 Fill in the tan θ values (correct to 2 decimal digits) of these angles

θ	0º	20º	45º	70º	90º	110º	135º	160º	180º
tan θ									

θ	180º	200º	225º	250º	270º	290º	315º	340º	360º
tan θ									

Again, compare the values – what do you notice?

3.2 What is happening at 90º? And at 270º?

We maybe need to investigate more values, namely those just before and after 90º and 270º:

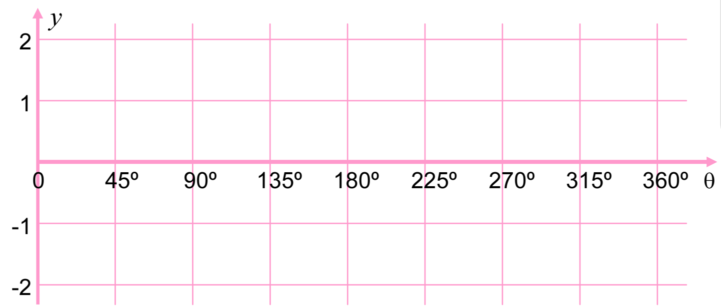
Round off to the nearest whole number

θ	80º	85º	89º	89,9º	90º	90,1º	91º	95º	100º
tan θ	6	11	57	572	?	-572	-57	-11	-6

θ	260º	265º	269º	269,9º	270º	270,1º	271º	275º	280º
tan θ	6	11	57	572	?	-572	-57	-11	-6

3.3 Draw the graph of **y = tan** θ on the following set of axes:

Use the accompanying table to plot points, even approximately, on the set of axes.



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4. **Increase / Decrease**

Now, use the tables and/or the graphs to fill in the spaces below and circle the correct word.

	Quadrant number	sin θ	cos θ	tan θ
θ: 0° ➡ 90°		increases / decreases from . . . . . to	increases / decreases from . . . . . to	increases / decreases from . . . . . to . . . . .
θ: 90° ➡ 180°		increases / decreases from . . . . . to	increases / decreases from . . . . . to	increases / decreases from . . . . . to . . . . .
θ: 180° ➡ 270°		increases / decreases from . . . . . to	increases / decreases from . . . . . to	increases / decreases from . . . . . to . . . . .
θ: 270° ➡ 360°		increases / decreases from . . . . . to	increases / decreases from . . . . . to	increases / decreases from . . . . . to . . . . .

5. **Positive / Negative**

	I	II	III	IV
sin θ positive				
cos θ positive				
tan θ positive				

	I	II	III	IV
sin θ negative				
cos θ negative				
tan θ negative				

6. **Maximum / Minimum**

	sin θ	cos θ	tan θ
Maximum value			
Minimum value			

7. **Features of the graphs**

	Amplitude	Period	Range
y = sin θ			
y = cos θ			
y = tan θ			

8. **Asymptotes**

The equations of the asymptotes of the graph y = tan θ:

....

9. **Function notation**

If f(x) = sin x ; g(x) = cos x and h(x) = tan x

then f(0°) = ... ; g(0°) = ... and h(0°) = ...

f(90°) = ... ; g(180°) = ... and h(315°) = ...

10. **Solving Equations**

Solve the following equations where 0° ≤ θ ≤ 360°, correct to the nearest whole number.

Remember:
Use the tables and graphs in 1, 2 and 3

10.1 Solve for θ:

(a) sin θ = 0 (b) sin θ = 1 (c) sin θ = -1
(d) sin θ = 0,34 (e) sin θ = -0,34 (f) sin θ = 1,9
(g) sin θ = 0,94 (h) sin θ = -0,94 (i) sin θ = -1,3

10.2 Solve for θ:

(a) cos θ = 0 (b) cos θ = 1 (c) cos θ = -1
(d) cos θ = 0,34 (e) cos θ = -0,34 (f) cos θ = 1,9
(g) cos θ = 0,94 (h) cos θ = -0,94 (i) cos θ = -1,3

10.3 Solve for θ:

(a) tan θ = 0 (b) tan θ = 1 (c) tan θ = -1
(d) tan θ = 0,36 (e) tan θ = -0,36 (f) tan θ = 2,75
(g) tan θ = -2,75 (h) tan θ = 572 (i) tan θ = -572
(j) tan θ is undefined when θ = ?

(h) & (i): correct to 1 decimal digit

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EXERCISE 6.8
Exploring the role of a and q in trigonometric functions
QUESTIONS

See figures 1 to 6 alongside where the following graphs are drawn for θ: $0° \to 360°$:

Figure 1 & 2 both show the graph **y = sin θ**;

Figure 3 & 4 both show the graph **y = cos θ** and

Figure 5 & 6 both show the graph **y = tan θ**.

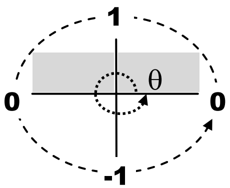
1. Sketch the following graphs on the given sets of axes:

A. $y = 2\sin\theta$ **C.** $y = -\cos\theta$ **E.** $y = \frac{1}{2}\tan\theta$

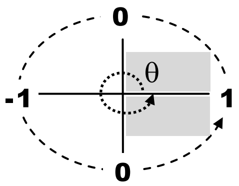
B. $y = \sin\theta + 1$ **D.** $y = \cos\theta - 1$ **F.** $y = \tan\theta + 1$

Plot each point using the sin, cos & tan 'wheels' and <u>not</u> a calculator!

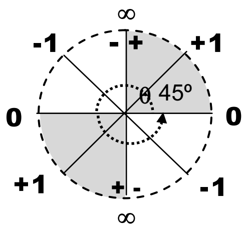
sin θ:



cos θ:



tan θ:



2. Compare: **A** and **B** to the given graph of $y = \sin\theta$ in figures 1 & 2;

C and **D** to the given graph of $y = \cos\theta$ in figures 3 & 4; and

E and **F** to the given graph of $y = \tan\theta$ in figures 5 & 6

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Establish for each graph: ▪ **the amplitude** ▪ **the range** ▪ **the period**

Figure 1 (for **A**)

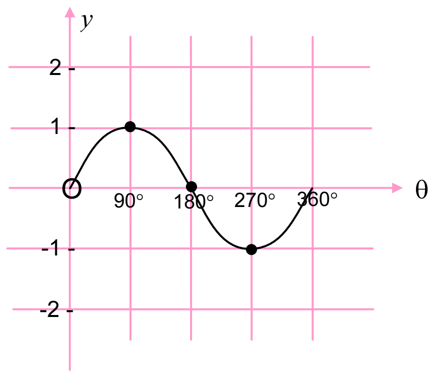


Figure 2 (for **B**)

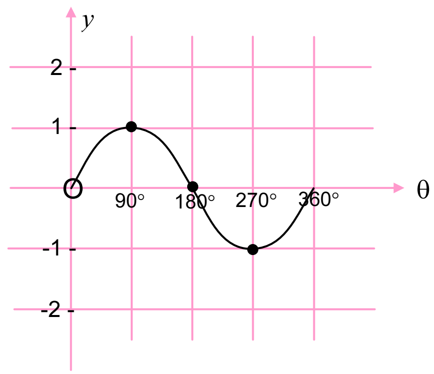


Figure 3 (for **C**)

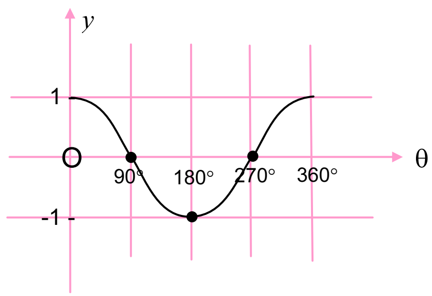
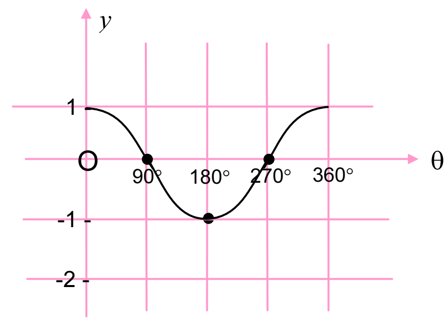


Figure 4 (for **D**)



For A & B

	y = 2 sin θ	y = sin θ	y = sin θ + 1
Amplitude			
Range			
Period			

For C & D

	y = -cos θ	y = cos θ	y = cos θ - 1
Amplitude			
Range			
Period			

For E & F

		y = tan θ	y = tan θ + 1
Amplitude			
Range			
Period			

Figure 5 (for **E**)

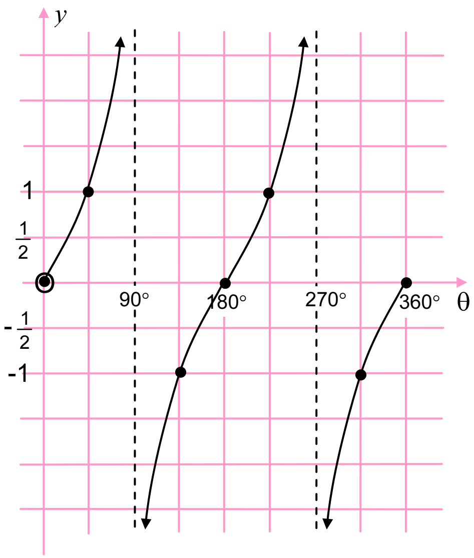
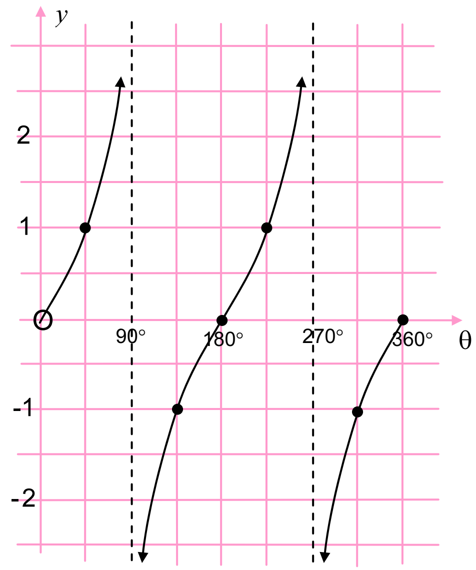


Figure 6 (for **F**)



Study the 'variations' carefully once you've drawn the graphs *yourself* by plotting points.

In each case, notice how the graph was different to the basic graph,

i.e. $y = \mathbf{2}\sin\theta$ vs. $y = \sin\theta$, etc.

The aim of the exercise has been for you to understand the effect of the values of **a** and **q** - **the parameters** - in the graphs:

$y = \mathbf{a}\sin x + \mathbf{q}$ • $y = \mathbf{a}\cos x + \mathbf{q}$ • $y = \mathbf{a}\tan x + \mathbf{q}$

The following table will help you make conjectures about the variations:

	amplitude	range	max value	min value	period	asymptotes
$y = 3\sin\theta$	3	$-3 \le y \le 3$	3	-3	360°	
$y = -2\sin\theta$	2	$-2 \le y \le 2$	2	-2	360°	
$y = \sin\theta - 1$	1	$-2 \le y \le 0$	0	-2	360°	
$y = 2\cos\theta$	2	$-2 \le y \le 2$	2	-2	360°	
$y = \cos\theta + 2$	1	$1 \le y \le 3$	3	1	360°	
$y = -\cos\theta$	1	$-1 \le y \le 1$	1	-1	360°	
$y = 2\tan\theta$		$y \in \mathbb{R}$			180°	y = 90° & y = 270°
$y = -\tan\theta$		$y \in \mathbb{R}$			180°	y = 90° & y = 270°
$y = \tan\theta - 1$		$y \in \mathbb{R}$			180°	y = 90° & y = 270°

This package is an extract from our Gr 10 Maths 3 in 1 study guide.

We trust that this will help you to grow in confidence as you prepare for your exams.

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