GR 10 MATHS – EUCLIDEAN GEOMETRY

Geometry is a fun topic in which we work with

lines • angles • triangles • quadrilaterals • polygons

while we
- investigate and discover as much as we can for ourselves;
- explain what we've found, make conjectures, and then
- proceed to confirm these various properties by proof.

TOPIC OUTLINE

As important as the **facts** which we discover is the **language** we use to state these facts. So, be sure to study the vocabulary as well as the facts which you gather.

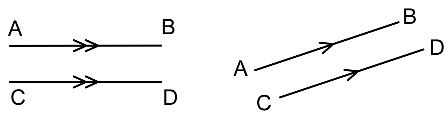
REVISION – Lines, Angles & Triangles

❑ The Language of Geometry (Vocabulary)

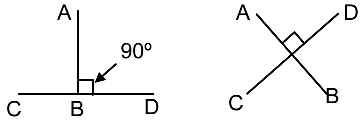
Make sure you know the meanings of all the **WORDS** we use in geometry.

■ Lines – Parallel and perpendicular

Parallel lines: AB || CD

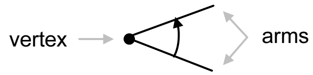


Perpendicular lines: AB ⊥ CD

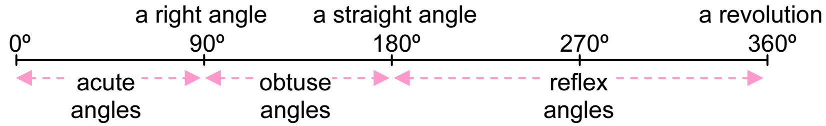


■ Angles – their names

▸ **Angles (single):**



The **ANGLE** is the amount of rotation about the vertex.

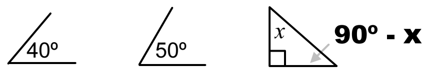


Angles can be **acute, obtuse** or **reflex**; a **right** angle, a **straight** angle or a **revolution**.

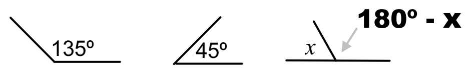
<u>*NOTE:*</u> *The plural of vertex is vertices!*

▸ **Angles (pairs):**

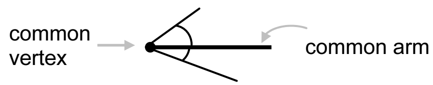
Complementary $\angle^s$ add up to 90º
e.g. 40º and 50º; x and 90º - x



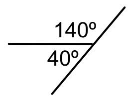
Supplementary $\angle^s$ add up to 180º
e.g. 135º and 45º; x and 180º - x



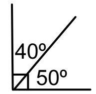
Adjacent $\angle^s$ have a common vertex and a common arm and lie on opposite sides of the common arm (side)



e.g. A pair of **adjacent supplementary** $\angle^s$:



A pair of **adjacent complementary** $\angle^s$:



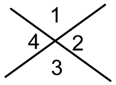
When 2 lines intersect, we have . . .

➥ **adjacent supplementary** $\angle^s$

$\hat{1}$ and $\hat{4}$ and $\hat{4}$ and $\hat{3}$, etc.

➥ **vertically opposite** $\angle^s$,

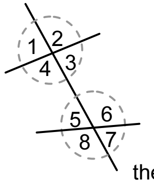
$\hat{1}$ and $\hat{3}$ or $\hat{2}$ and $\hat{4}$



NOTE: We are just talking about the naming, not the relationships (i.e. whether $\angle^s$ are equal or supplementary, etc.).

or, when 2 lines are cut by a transversal . . .

2 **'families'** of 4 angles are formed 1 2 3 4 : 5 6 7 8



the transversal

Taking one from each family, we have the following pairs:

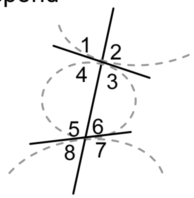
- Pairs of **corresponding** $\angle^s$: 1 & 5 ; 2 & 6 ; 3 & 7 ; 4 & 8
- Pairs of **alternate** $\angle^s$: 3 & 5 and 4 & 6
- Pairs of **co-interior** $\angle^s$: 4 & 5 and 3 & 6

➥ **Corresponding** means their positions correspond

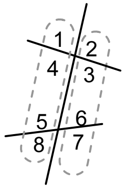
1 & 5 ; 2 & 6 ; 3 & 7; 4 & 8

➥ **Interior** $\angle^s$: 3, 4, 5 & 6 on the *inside*

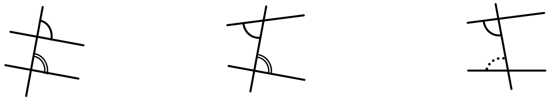
& **Exterior** $\angle^s$: 1, 2, 7 & 8 on the *outside*



➥ We say 1, 4, 5 and 8 are **alternate** to 2, 3, 6 and 7, i.e. the two groups lie on *opposite sides* of the transversal whereas, **'co-'** means: 'on the *same side* of the transversal'.



e.g. corresponding $\angle^s$ interior alternate $\angle^s$ co-interior $\angle^s$



Often the transversal doesn't go right across both || lines.

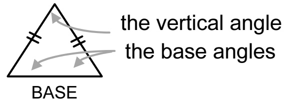
FUN!

■ Types of Triangles

▶ Classification according to . . .

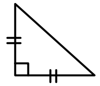
SIDES	**ANGLES**
◆ **Scalene** Δ *(all 3 sides different in length)*	◆ **Acute** $\angle^d$ Δ *(all 3 $\angle^s$ are acute)*
◆ **Isosceles** Δ *(2 sides equal in length)*	◆ **Right** $\angle^d$ Δ *(one $\angle$ = 90°)*
◆ **Equilateral** Δ *(all 3 sides equal in length)*	◆ **Obtuse** $\angle^d$ Δ *(one $\angle$ is obtuse)*

In an isosceles triangle:



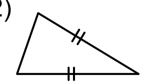
We can classify Δ^s (triangles) according to sides and $\angle^s$ (angles) simultaneously.

(1)



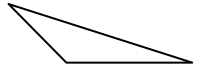
This is a/an . . . . -angled Δ

(2)



This is a/an . . . . -angled Δ

(3)



This is a/an . . . . -angled Δ

Answers:

(1) An isosceles right-$\angle^d$ Δ (2) An isosceles acute-$\angle^d$ Δ (3) A scalene obtuse-$\angle^d$ Δ

❑ The Facts – Lines & Angles

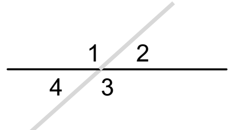
Now that you know the meanings of the words, let's revise the **FACTS**.

▶ Intersecting Lines

FACT 1

2 intersecting lines form four angles

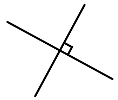
When two lines intersect, any pair of adjacent angles is supplementary.



FACT 2

When 2 lines intersect, the vertically opposite angles are equal.

Special case: perpendicular lines

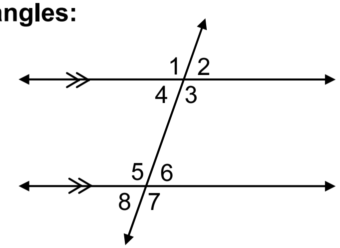


The sizes of the other angles?

▶ Parallel Lines

3 types of pairs of angles:

- corresponding ;
- alternate ; and
- co-interior

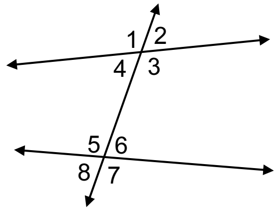


FACT 3

If 2 **parallel** lines are cut by a transversal,

- corresponding angles are equal ;
- alternate angles are equal ; and
- co-interior angles are supplementary.

When 2 lines are cut by a transversal, 2 'families' of four angles are formed:

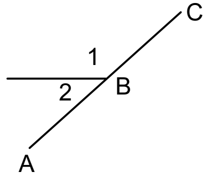


whether the lines are parallel or not!

Converse Facts

e.g. 1 The original statement (see **FACT 1**) is:

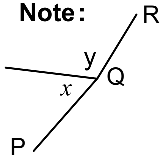
If ABC is a straight line, then $\hat{1} + \hat{2} = 180°$



Converse statement:

If $\hat{1} + \hat{2} = 180°$, then ABC is a straight line.

Note:



Given: $x = 40°$ and $y = 130°$

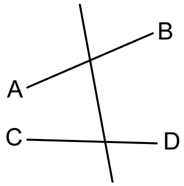
Question: Is PQR a straight line?

Answer: $x + y = 40° + 130° = 170° \neq 180°$

$\therefore$ No, PQR is not a straight line . . . *Whether it looks like it or not!*

e.g. 2 If the original statement is **FACT 3**, then the

Converse statement:



IF corresponding angles are equal; OR

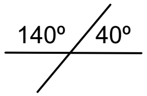
IF alternate angles are equal; OR

IF co-interior angles are supplementary,

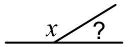
THEN, line AB is parallel to line CD . . . *whether it looks like it or not!*

EXERCISE 7.1 – Revision: Lines and Angles
QUESTIONS

1. 140° and 40° are adjacent supplementary angles:

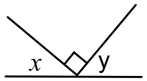


The angle supplementary to x is:



(1) ....

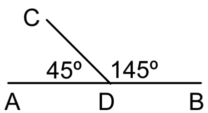
2. In this figure



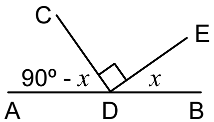
: $x + 90° + y = 180°$ . . . *$\angle^s$ on a straight line*

$\therefore$ $x + y =$ **(2.1)** .... and so they are called **(2.2)** .... angles.

3. Is ADB a straight line? Give a reason for your answer.

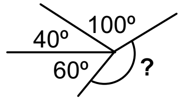


(3.1) ....

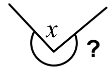


(3.2) ....

4.



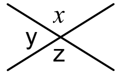
(4.1) ....



(4.2) ....

The sum of adjacent angles about a point is 360°.

5. *When two lines intersect, the vertically opposite $\angle^s$ are equal. Why?*

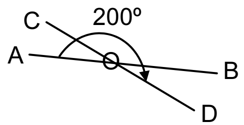


$x + y = 180°$ . . . $\angle^s$ *on a straight line*

and $z + y = 180°$ . . . $\angle^s$ *on a straight line*

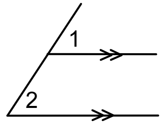
$\therefore$ **(5)** ....

6. Given: reflex $A\hat{O}D = 200°$ *(see figure below)*.



		Reasons:
Obtuse $A\hat{O}D$ =	**(6.1)**	**(6.2)** ....
$B\hat{O}D$ =	**(6.3)**	**(6.4)** ....
$A\hat{O}C$ =	**(6.5)**	**(6.6)** ....

7.

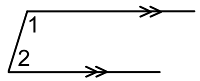


$\hat{1}$ and $\hat{2}$ are

(7.1) .... $\angle^s$ (NAME)

RELATIONSHIP:

(7.4) ....

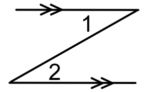


$\hat{1}$ and $\hat{2}$ are

(7.2) .... $\angle^s$ (NAME)

RELATIONSHIP:

(7.5) ....



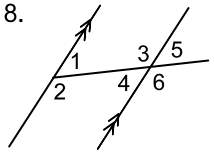
$\hat{1}$ and $\hat{2}$ are

(7.3) .... $\angle^s$ (NAME)

RELATIONSHIP:

(7.6) ....

8.



$\hat{1}$ and **(8.1)** .... are corresponding $\angle^s$ and they are **(8.2)** ....

$\hat{1}$ and **(8.3)** .... are alternate $\angle^s$ and they are **(8.4)** ....

$\hat{1}$ and **(8.5)** .... are co-interior $\angle^s$ and they are **(8.6)** ....

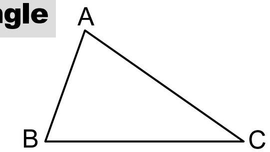
9. **NB:** It is **ONLY BECAUSE THE LINES ARE (9)** ...., that the **corresponding** and **alternate** $\angle^s$ **ARE EQUAL** and the **co-interior** $\angle^s$ are **SUPPLEMENTARY !!!**

❑ The Facts – Triangles

FACT 1: Sum of the interior angles of a triangle

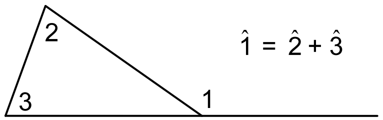
The sum of the (interior) angles of a triangle is 180º.

$\hat{A} + \hat{B} + \hat{C} = 180°$



FACT 2: The exterior angle of a triangle

The exterior angle of a triangle equals the sum of the interior opposite angles.



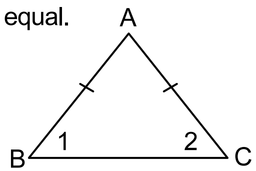
$\hat{1} = \hat{2} + \hat{3}$

FACT 3: An isosceles triangle

In an isosceles triangle, the base angles are equal.

The converse states:

If 2 angles of a triangle are equal, then the sides opposite them are equal.



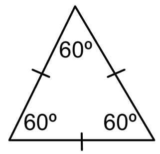
If AB = AC, then $\hat{1} = \hat{2}$

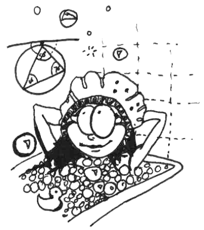
Converse:

If $\hat{1} = \hat{2}$, then AB = AC

FACT 4: An equilateral triangle

The angles of an equilateral triangle all equal 60º.



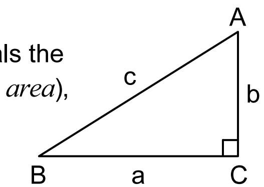


FACT 5: Theorem of Pythagoras

The Theorem of Pythagoras states:
The square on the hypotenuse of a right-angled triangle equals the sum of the squares on the other two sides (*in area*).

The converse states:

If the square on one side of a triangle equals the sum of the squares on the other two sides (*in area*), then the triangle is right-angled.



If $\hat{C} = 90°$, then $c^2 = a^2 + b^2$

Converse:

If $c^2 = a^2 + b^2$, then $\hat{C} = 90°$

Note:
- Only one angle can be 90º
- The side opposite the right-angle is called the hypotenuse.

FACT 6: The area of a triangle

The area of a triangle = $\dfrac{\text{base} \times \text{height}}{2}$

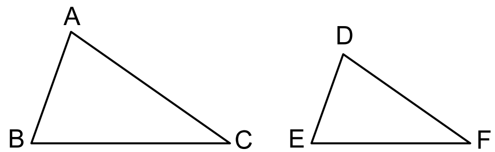
FACT 7: Similar triangles

If triangles are equiangular, they are similar.
Their respective sides will also be in proportion.

If $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$,

then ΔABC ||| ΔDEF, and

$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.

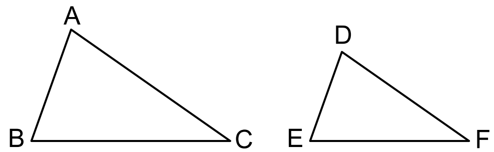


If the sides of two triangles are in proportion, then the triangles are similar.
These triangles will also be equiangular.

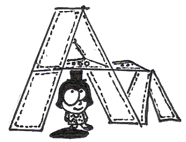
If $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$,

then ΔABC ||| ΔDEF, and

$\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



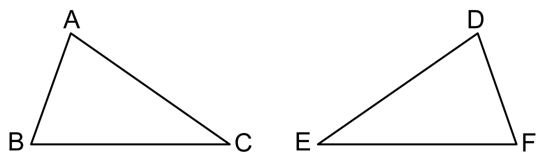
Similar Δ^s have the same SHAPE, but not necessarily the same SIZE.



FACT 8: Congruent triangles

Two triangles are congruent if they have

- 3 sides the same length . . . **SSS**
- 2 sides & an included angle equal . . . **S∠S**
- a right angle, hypotenuse & a side equal . . . **RHS** or **SS90º**
- 2 angles and a side equal . . . **∠∠S**

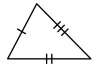


If we can prove ΔABC ≡ ΔDEF, then we can conclude that the sides and angles not yet mentioned, are equal.

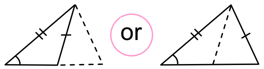
Congruent Δ^s have the same SHAPE, and the same SIZE.

Congruency can be understood best by constructing triangles, even casually! A triangle has 6 parts which can be measured, 3 sides and 3 angles. However, we can only use 3 measurements at a time to construct a triangle. The possibilities are:

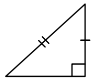
❶ **SSS** – only one size (& shape) of triangle could be constructed.



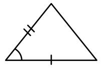
❷ SS∠ – the case where the angle is NOT INCLUDED between the sides. This is the ambiguous case because there are 2 possible Δ^s which we could draw



❸ **SS90°** or **RHS** – the angle is not included, but it is a right angle, so only 1 option is possible:

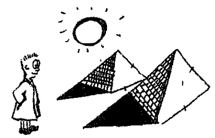


❹ **S∠S** – the angle is included. Only 1 option is possible.



❺ ∠∠**S** – given 2 angles, we actually have 3 angles (*The sum of the angles must be 180°!*). The side restricts the size of the triangle. However, when comparing triangles, the given equal sides must correspond in relation to the angles.

❻ ∠∠∠ – A side is required!

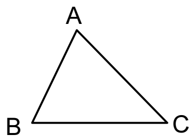


■ Proofs of Facts 1 and 2

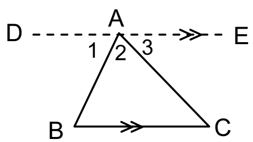
The sum of the (interior) $\angle^s$ of a triangle is 180°. Why?

Why is the sum of the angles of a triangle 180°?

See ΔABC:



Draw line DAE through A, parallel to BC.



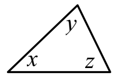
We know: $\hat{1} + \hat{2} + \hat{3} = 180°$ . . . (straight angle, $D\hat{A}E$)

But $\hat{1}$ = alternate $\hat{B}$ & $\hat{3}$ = alternate $\hat{C}$ . . . (DAE || BC)

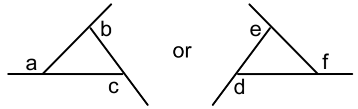
$\therefore \hat{B} + \hat{2} + \hat{C} = 180°$

Interior & Exterior angles of Triangles

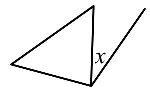
We have **interior** angles:



and **exterior** angles:



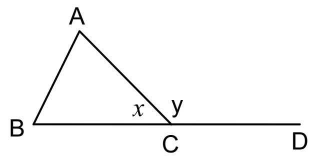
An exterior angle is formed between a side of the triangle and the **produced** (extension) of an adjacent side of the triangle.



x is not an exterior ∠ . . . the side is not 'produced'.

The exterior angle of a triangle equals the SUM of the two interior opposite angles. Why?

Why is the exterior angle of a triangle equal to the sum of the two interior opposite angles?



$\hat{x} + \hat{y} = 180°$ . . . *see fact 1*

but $\hat{x} + (\hat{A} + \hat{B}) = 180°$ . . . *see fact 5*

$\therefore \hat{y} = \hat{A} + \hat{B}$ **Logical!**

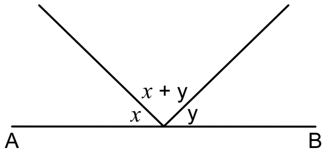
■ Comparing Triangles – Congruence vs. Similarity

➥ If 2 triangles are **equal in every respect** (i.e. all 3 sides and all 3 angles), we say they are **congruent**. We write **ΔABC ≡ ΔPQR**. (ΔABC is congruent to ΔPQR)

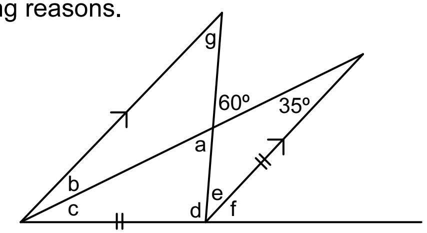
➥ Enlarging or reducing a triangle, as on a copier, will produce a triangle **similar** to the original one. i.e. It will have the **same shape** (all angles will be the same size as before) but the respective sides will not be the same length. They will be proportional. We write **ΔABC ||| ΔPQR**. (ΔABC is similar to ΔPQR)

Exercise 7.2
Lines, Angles & Triangles
QUESTIONS

1. In the sketch, AB is a straight line. If x - y = 10º, find the values of x and y.

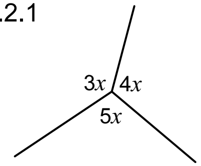


2.1 Find the size of angles a to g (in that order), giving reasons.

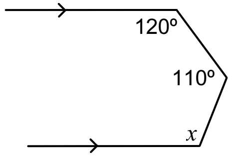


2.2 Calculate, with reasons, the values of x.

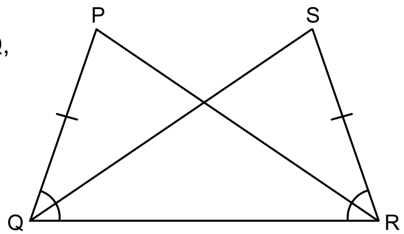
2.2.1



2.2.2

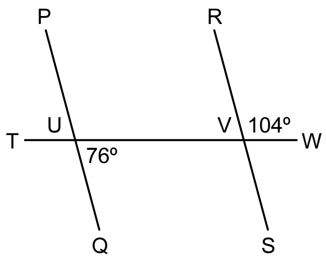


2.3 If PQ = SR and ∠PQR = ∠SRQ, prove that PR = SQ.



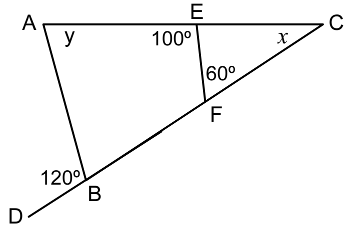
(Hint: first prove that ΔPQR ≡ ΔSRQ)

2.4 State, giving reasons, whether PQ || RS.

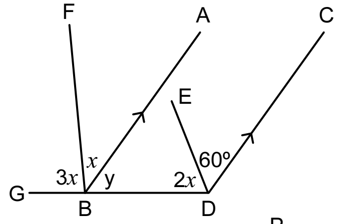


3. Determine the values of x and y in the following diagrams. Give reasons for your answers.

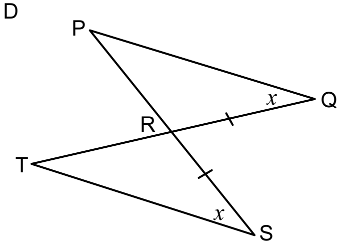
3.1



3.2

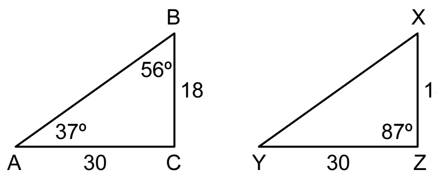


4. Prove that $\hat{P} = \hat{T}$ by first proving that the 2 triangles are congruent.

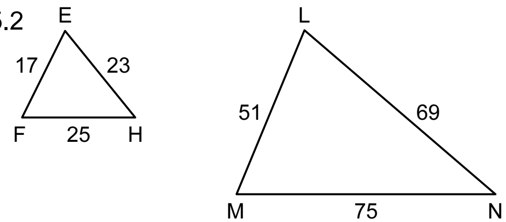


5. State whether the following pairs of triangles are congruent or similar, giving reasons for your choice.

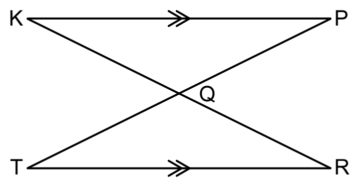
5.1



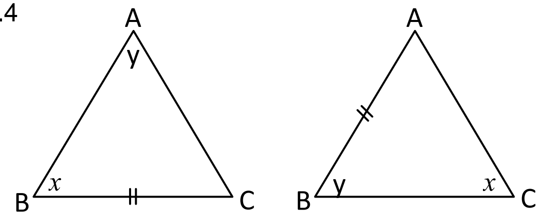
5.2



5.3

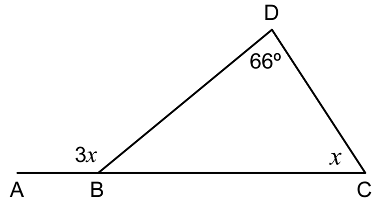


5.4

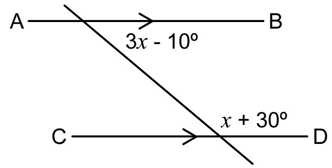


6.1 Find the value of x, by **forming an equation first** and then solving for x.
Show all your working and give reasons.

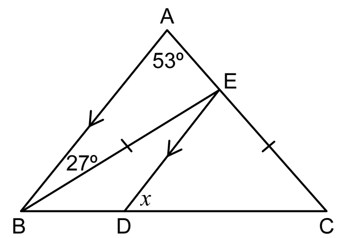
6.1.1



6.1.2



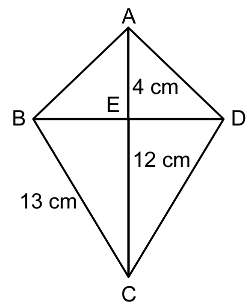
6.2 In the diagram, AB || ED and BE = EC.
Also, $A\hat{B}E$ = 27º and $B\hat{A}C$ = 53º.



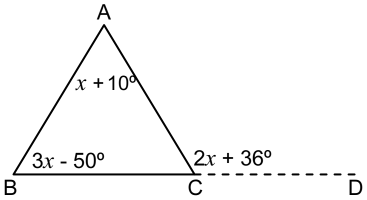
6.2.1 Write down the sizes of $B\hat{E}D$ and $C\hat{E}D$, **giving reasons.**

6.2.2 Hence, or otherwise, calculate the value of x, **showing all working and giving reasons.**

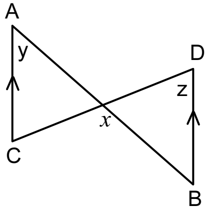
7. Calculate the area of the kite alongside.



8. Calculate the value of x giving reasons.



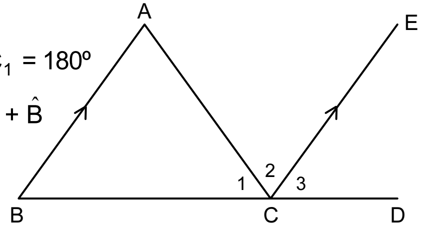
9. If AC || DB, prove with geometric reasons that x = y + z.



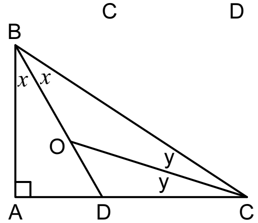
10. In ΔABC, BC is produced to D and CE || BA. Prove that:

10.1 $\hat{A} + \hat{B} + \hat{C}_1 = 180°$

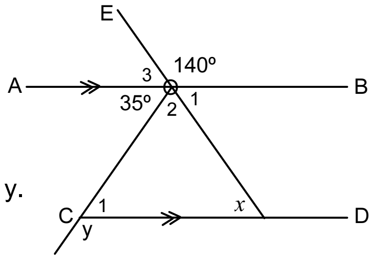
10.2 $A\hat{C}D = \hat{A} + \hat{B}$



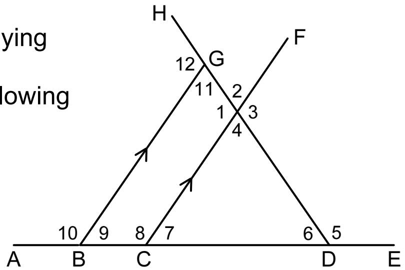
11. Prove that $D\hat{O}C = 45°$.



12. If AB || CD, $B\hat{O}E = 140°$ and $A\hat{O}C = 35°$, determine stating reasons, the values of x and y.



13. In the accompanying figure BG || CF. Complete the following statements:



13.1 $\hat{1} + \hat{2} + \hat{3} + \hat{4}$ = ... (degrees)

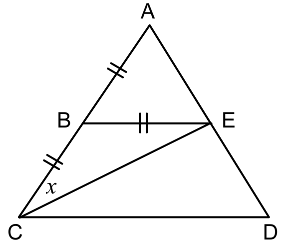
13.2 $\hat{1} + \widehat{11}$ = ... (degrees)

13.3 $\hat{6} + \hat{7} + \hat{2}$ = ... (degrees)

13.4 $\hat{9} + \hat{6}$ = . . . (the no. of one angle)

13.5 $\hat{7}$ = ... (list only one angle)

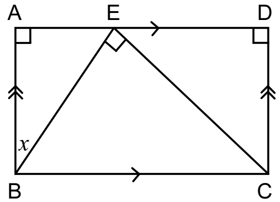
13.6 If $\widehat{12} = \widehat{10}$, then GD = ...



14.1 Express $C\hat{E}B$, $A\hat{B}E$ and $A\hat{E}B$ in terms of x.

14.2 What is the relationship between CE and AD? Explain.

14.3 If CE bisects $A\hat{C}D$ what is the relationship between BE and CD? Explain.



15.1 Make a neat copy of this sketch and fill in all the other angles in terms of x. **Reasons are not required.**

15.2 Complete the following statement:
ΔABE ||| Δ . . . ||| Δ . . .

15.3 If BC = 18 cm and BE = 12 cm, calculate the length of

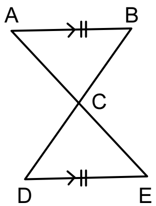
15.3.1 AE

15.3.2 AB correct to two decimals.

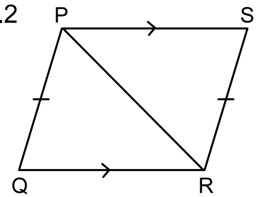
15.4 Hence calculate the area of rectangle ABCD to the nearest cm^2.

16. In each of the following, **state** whether the given triangles are congruent or not, and in each case give a reason for your answer. **Do not prove** the triangles congruent, but name each congruent pair correctly.

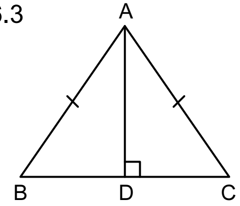
16.1



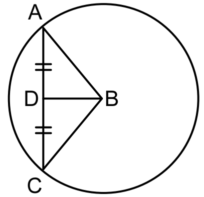
16.2



16.3

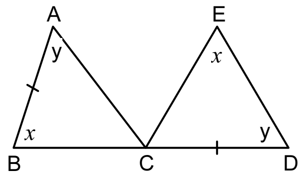


16.4

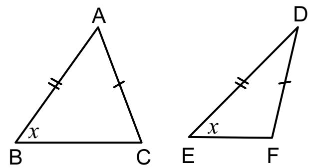


B is the centre of the circle

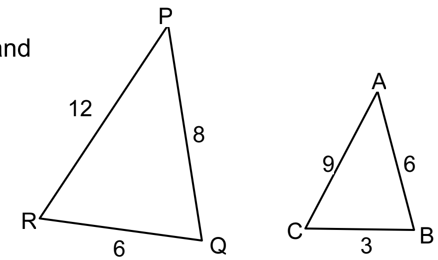
16.5



16.6



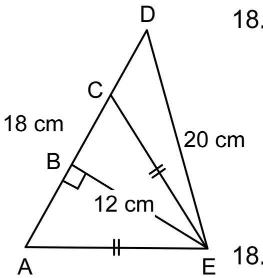
17. Given ΔPQR and ΔABC.



17.1 Show that ΔPQR is NOT similar to ΔABC. Clearly show all relevant calculations and reasons.

17.2 Prove that ΔPQR is not right-angled.

18.



18.1 Calculate the length of

18.1.1 BD

18.1.2 BC

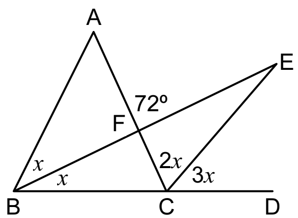
18.1.3 CD

18.1.4 AE

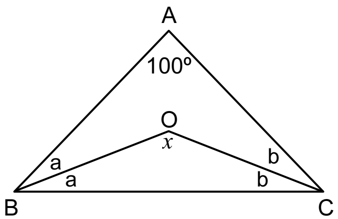
18.2 Prove that $A\hat{E}D = 90°$

19. Calculate the value of x in each of the following figures:

19.1



19.2



EXERCISE 7.1
Revision: Lines and Angles
ANSWERS

1. $180° - x$

2.1 $90°$ 2.2 complementary

3.1 No, $45° + 145° \neq 180°$

3.2 Yes, $90° - x + x + 90° = 180°$

4.1 $160°$ 4.2 $360° - x$

5. $\therefore\ x = z$

6.1 $160°$ 6.2 revolution / $\angle^s$ about a point

6.3 $20°$ 6.4 (straight) $A\hat{O}B = 180°$

6.5 $20°$

6.6 (straight) $C\hat{O}D = 180°$ or vertically opposite to $B\hat{O}D$

7.1 corresponding   7.2 co-interior   7.3 alternate

7.4 equal   7.5 supplementary   7.6 equal

8.1 $\hat{5}$ 8.2 equal   8.3 $\hat{4}$

8.4 equal   8.5 $\hat{3}$

8.6 supplementary   9. parallel

the ANSWER series ... your key to exam success

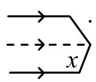
Exercise 7.2
Lines, Angles & Triangles
ANSWERS

1. $2x + 2y = 180°$ ... *$\angle^s$ on a straight line*
$\therefore\ x + y = 90$
But $x - y = 10°$ ... *given*
$\therefore\ x = 50°$ and $y = 40°$ ... *by inspection*

2.1 $a = 60°$ ... *vertically opposite angles*
$b = 35°$ ... *alternate $\angle^s$; || lines*
$c = 35°$ ... *isosceles Δ*
$d = 85°$ ... *sum of interior $\angle^s$ of Δ*
$e = 60° - 35° = 25°$ ... *ext. $\angle$ of Δ*
$f = 35° + 35° = 70°$ ... *either ext. $\angle$ of Δ or corresp. $\angle^s$; || lines*
$g = 25°$ ... *alternate $\angle^s$; || lines*

2.2.1 $12x = 360°$ ... *revolution*
$\therefore\ x = 30°$

2.2.2 $120° + 110° + x = 2(180°)$ ... *2 prs. co-int. $\angle^s$*
$\therefore\ 230° + x = 360°$
$\therefore\ x = 130°$



2.3 In Δ^s PQR & SRQ   *[Note: order of letters !]*
(1) PQ = SR ... *given*
(2) $P\hat{Q}R = S\hat{R}Q$ ... *given*
(3) QR is common
$\therefore\ \Delta PQR \equiv \Delta SRQ$ ... *S∠S*
$\therefore$ PR = SQ

2.4 $P\hat{U}V = 180° - 76° = 104°$ ... *str. line*
$\therefore\ P\hat{U}V = R\hat{V}W$
$\therefore$ Yes, PQ || RS ... *corresponding $\angle^s$ are equal*

3.1 $x = 100° - 60° = 40°$ ... *ext. $\angle$ of ΔEFC*
$y = 120° - 40° = 80°$ ... *ext. $\angle$ of ΔABC*

3.2 $4x = 2x + 60°$ ... *corr. $\angle^s$; BA || DC*
$\therefore\ 2x = 60°$
$\therefore\ x = 30°$
$y = 180° - 4x = 60°$ ... *$\angle^s$ on straight line*

4. In Δ^s QRP and SRT
(1) $\hat{Q} = \hat{S}\ (= x)$
(2) $Q\hat{R}P = S\hat{R}T$ ... *vertically opposite $\angle^s$*
(3) QR = SR ... *given*
$\therefore\ \Delta QRP \equiv \Delta SRT$ ... *∠∠S*
$\therefore\ \hat{P} = \hat{T}$

5.1 Congruent ; S∠S ... *[$\hat{C} = 87°$, sum of interior $\angle^s$ of Δ]*

5.2 Similar ; prop. sides ... *[17 : 23 : 25 = 51 : 69 : 75]*

5.3 Similar ; equiangular (alt. $\angle^s$ & vert. opp. $\angle^s$) ... *[no sides]*

LOOKS MAY NOT DECEIVE!

5.4 Similar; 2 equal $\angle^s$ ... *Note: not congruent because the equal sides don't correspond*

6.1.1 $3x = 66° + x$ ... *ext. $\angle$ of Δ*
$\therefore\ 2x = 66°$
$\therefore\ x = 33°$

6.1.2 $(3x - 10°) + (x + 30°) = 180°$ ... *co-int. $\angle^s$; || lines*
$\therefore\ 4x + 20° = 180°$
$\therefore\ 4x = 160°$
$\therefore\ x = 40°$

6.2.1 $B\hat{E}D = 27°$ ... *alt. $\angle^s$; AB || ED*
$C\hat{E}D = 53°$ ... *corresp. $\angle^s$; AB || ED*

6.2.2 $E\hat{B}C = E\hat{C}B = \dfrac{180° - (27° + 53°)}{2}$ ... *isosceles Δ & sum of int. $\angle^s$ of Δ*
$= 50°$
$\therefore\ x = 50° + 27°$ ... *ext. $\angle$ of ΔBED*
$= 77°$

7. $BE \perp AC$ ... *diagonals of a kite*
$BE = 5$ cm ... *5 : 12 : 13 Δ ; Pythag.*
Area of kite $= 2.\frac{1}{2}(12 + 4).5$ ... $2 \times \Delta ACD$
$= 80 \text{ cm}^2$
[OR: $\frac{1}{2}$ product of diagonals $= \frac{1}{2}(10)(16)\text{cm}^2$ ... *why?*]

8. $2x + 36° = 3x - 50° + x + 10°$ ... *ext. ∠ of Δ*
$\therefore -2x = -76°$
$\therefore x = 38°$ A VERY IMPORTANT THEOREM!

9. $\hat{C} = z$ ... *alt. ∠^s ; AC || DB*
$\therefore x = y + z$ ... *ext. ∠ of Δ*

10.1 $\hat{C}_3 + \hat{C}_2 + \hat{C}_1 = 180°$ ... *∠^s on str. line*
But $\hat{C}_2 = \hat{A}$ ... *alternate ∠^s ; CE || BA*
& $\hat{C}_3 = \hat{B}$ ... *corresp. ∠^s ; CE || BA*
$\therefore \hat{B} + \hat{A} + \hat{C}_1 = 180°$

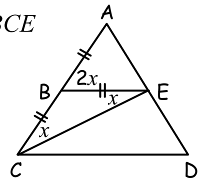
10.2 $A\hat{C}D = 180° - \hat{C}_1$ ... *∠^s on str. line*
& $\hat{A} + \hat{B} = 180° - \hat{C}_1$ ... *sum of int. ∠^s of Δ*
$\therefore A\hat{C}D = \hat{A} + \hat{B}$

11. $D\hat{O}C = x + y$ ... *ext. ∠ of Δ*
But $2x + 2y = 90°$ ... *∠ sum of rt. ∠^d ΔABC*
$\therefore x + y = 45°$
$\therefore D\hat{O}C = 45°$

12. $x = \hat{O}_1$ or $\hat{O}_3$ ... *alt. or corresp. ∠^s ; AB || CD*
$= 40°$ ... *∠^s on str. line*
$\hat{C}_1 = 35°$ ... *alt. ∠^s ; AB || CD*
$\therefore y = 145°$ ... *∠^s on str. line*

13.1 360° ... *revolution*
13.2 180° ... *co-interior ∠^s supplementary ; BG || CF*
13.3 180° ... *vert. opp. ∠^s ; sum of int. ∠^s of Δ*
13.4 $\hat{12}$ ... *ext. ∠ of Δ*
13.5 $\hat{9}$ ... *corresp. ∠^s ; BG || CF*
13.6 BD ... $\hat{12} = \hat{10}$ ➜ $\hat{11} = \hat{9}$; ∴ *isosceles Δ*

14.1 $C\hat{E}B = x$ ... *base ∠^s of isos. ΔBCE*
$A\hat{B}E = 2x$ ... *ext. ∠ of Δ*

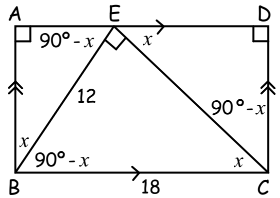


$A\hat{E}B = \frac{1}{2}(180° - 2x)$ ... *∠^s of isosceles ΔABE*
$= 90° - x$

14.2 They are perpendicular to each other; i.e. $CE \perp AD$;
Explanation: $C\hat{E}B$ & $A\hat{E}B$ are comp. ... $x + (90° - x) = ?$

14.3 They are parallel; i.e. BE || CD ;
Explanation: $E\hat{C}D = x$ ... *CE bisects $A\hat{C}D$*
$\therefore E\hat{C}D$ = alternate angle, $C\hat{E}B$

15.1



15.2 ΔABE ||| ΔECB ||| ΔDEC   *NB: The order of the letters!*

15.3.1 ΔABE ||| ΔECB
➜ $\frac{AE}{BE} = \frac{BE}{BC}$ ... *sides in proportion*
×BE) $\therefore AE = \frac{BE^2}{BC} = \frac{12^2}{18} = 8$ cm

15.3.2 $AB^2 = 12^2 - 8^2 = 80$ ... *Theorem of Pythagoras*
$\therefore AB = \sqrt{80}$
$\simeq 8{,}94$ cm

15.4 Area of rect. ABCD $= 8{,}94 \times 18 \simeq 161 \text{ cm}^2$

16.1 Yes, ΔABC ≡ ΔEDC ... *∠∠S*
16.2 No ... *2 sides and an angle, but the angle isn't included*
16.3 Yes, ΔABD ≡ ΔACD ... *SS90°*
[The angle is not included, but it is a right angle.]
16.4 Yes, ΔADB ≡ ΔCDB ... *SSS [Note: equal radii]*
16.5 No ... *2 angles and a side, but they don't correspond*
16.6 No ... *the angle is not included.* ...

Note: This is the ambiguous case: SS∠.
A triangle could be acute ∠^d or obtuse ∠^d.

17.1 $\frac{12}{9} = \frac{4}{3}$; $\frac{8}{6} = \frac{4}{3}$ but $\frac{6}{3} = 2$
∴ The sides are not in proportion
∴ ΔPQR $\not|||$ ΔABC

17.2 $12^2 = 144$ and $6^2 + 8^2 = 36 + 64 = 100$
$\therefore 12^2 \neq 6^2 + 8^2$
$\therefore \hat{Q} \neq 90°$, i.e. ΔPQR is not rt. ∠^d ... *CONV. of Thm of Pythag.*

18.1.1 BD = 16 cm ... *3 : 4 : 5 = 12 : 16 : 20 ; Pythag.*
18.1.2 AC = 18 cm
∴ BC = 9 cm ... *In isos. Δ, perpendicular bisects base, or*
BC = AB in congruent Δ^s EAB & ECB
18.1.3 CD = 16 - 9 = 7 cm
18.1.4 In ΔEBC:
CE = 15 cm ... *3 : 4 : 5 = 9 : 12 : 15 ; Pythag.*
∴ AE = 15 cm

18.2 AD = 18 + 7 = 25 cm
$\therefore AD^2 = 25^2 = 625$
& $AE^2 + DE^2 = 15^2 + 20^2 = 225 + 400 = 625$
$\therefore AD^2 = AE^2 + DE^2$
∴ AED = 90° ... *converse of the Theorem of Pythag.*

Also note: 15 : 20 : 25 = 3 : 4 : 5 - a Pythag. Triple
(An acceptable proof)

19.1 $\hat{E} = 3x - x$ ... *ext ∠ of ΔECB*
$= 2x$
$\therefore 2x + 2x = 72°$ ... *ext ∠ of ΔEFC*
$\therefore 4x = 72°$
$\therefore x = 18°$
OR
$B\hat{F}C = 72°$ ... *vert. opp. ∠s =*
$\therefore x + 72° = 2x + 3x$ ... *ext. ∠ of ΔBFC*
$\therefore 4x = 72°$
$\therefore x = 18°$

19.2 $2a + 2b + 100° = 180°$ ... *sum of int. ∠^s of ΔABC*
$\therefore 2a + 2b = 80°$
÷2) $\therefore a + b = 40°$
∴ In $\Delta O\hat{B}C$: $x = 180° - (a + b)$ ... *sum of int. ∠^s of ΔOBC*
$= 180° - 40°$
$= 140°$