# GR 12 MATHS - EXAM QUESTION PAPERS



## NATIONAL NOV 2014 PAPER 1

*You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.*

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

## ALGEBRA AND EQUATIONS AND INEQUALITIES [21]

### QUESTION 1

1.1 Solve for $x$:

1.1.1 $(x - 2)(4 + x) = 0$ (2)

1.1.2 $3x^2 - 2x = 14$ (correct to TWO decimal places.) (4)

1.1.3 $2^{x+2} + 2^x = 20$ (3)

1.2 Solve the following equations simultaneously:

$x = 2y + 3$

$3x^2 - 5xy = 24 + 16y$ (6)

1.3 Solve for $x$: $(x - 1)(x - 2) < 6$ (4)

1.4 The roots of a quadratic equation are: $x = \dfrac{3 \pm \sqrt{-k-4}}{2}$

For which values of k are the roots real? (2) [21]

## PATTERNS AND SEQUENCES [31]

### QUESTION 2

Given the arithmetic series: 2 + 9 + 16 + . . . (to 251 terms).

2.1 Write down the fourth term of the series. (1)

2.2 Calculate the 251st term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms in the series are divisible by 4? (4) [12]

### QUESTION 3

3.1 Given the quadratic sequence: - 1 ; - 7 ; - 11 ; p ; . . .

3.1.1 Write down the value of p. (2)

3.1.2 Determine the nth term of the sequence. (4)

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)

3.2 The first three terms of a geometric sequence are: 16 ; 4 ; 1

3.2.1 Calculate the value of the 12th term. (Leave your answer in simplified exponential form.) (3)

3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)

3.3 Determine the value of:

$$\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \ldots$$

up to 98 factors. (4) [19]

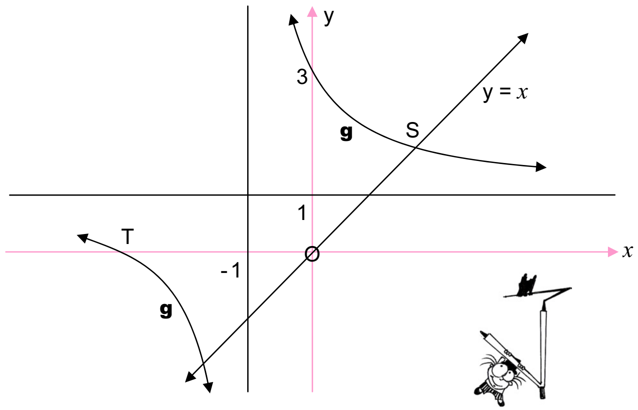
## FUNCTIONS AND GRAPHS [33]

### QUESTION 4

The diagram below shows the hyperbola g defined by $g(x) = \dfrac{2}{x + p} + q$ with asymptotes $y = 1$ and $x = -1$.

The graph of g intersects the $x$-axis at T and the y-axis at (0; 3).

The line $y = x$ intersects the hyperbola in the first quadrant at S.




4.1 Write down the values of p and q. (2)

4.2 Calculate the $x$-coordinate of T. (2)

4.3 Write down the equation of the vertical asymptote of the graph of h, if $h(x) = g(x + 5)$ (1)

4.4 Calculate the length of OS. (5)

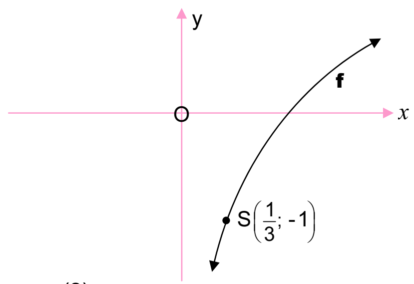
4.5 For which values of k will the equation $g(x) = x + k$ have two real roots that are of opposite signs? (1) [11]

### QUESTION 5

Given:

$f(x) = \log_a x$ where $a > 0$.

$S\left(\frac{1}{3}; -1\right)$ is a point on the graph of f.



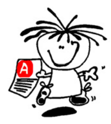

5.1 Prove that $a = 3$. (2)

5.2 Write down the equation of h, the inverse of f, in the form y = . . . (2)

5.3 If $g(x) = -f(x)$, determine the equation of g. (1)

5.4 Write down the domain of g. (1)

5.5 Determine the values of $x$ for which $f(x) \geq -3$. (3) [9]



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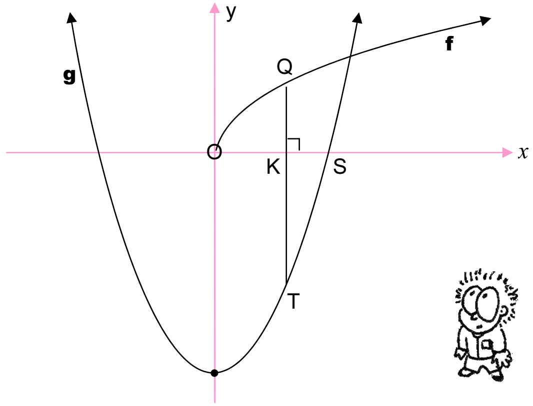
## QUESTION 6

Given: $g(x) = 4x^2 - 6$ and $f(x) = 2\sqrt{x}$.

The graphs of g and f are sketched below.
S is an $x$-intercept of g and K is a point between O and S.
The straight line QKT with Q on the graph of f and T on the graph of g, is parallel to the y-axis.




6.1 Determine the $x$-coordinate of S, correct to TWO decimal places. (2)

6.2 Write down the coordinates of the turning point of g. (2)

6.3.1 Write down the length of QKT in terms of $x$ (where $x$ is the $x$-coordinate of K). (3)

6.3.2 Calculate the maximum length of QT. (6) [13]

## ▸ FINANCE, GROWTH AND DECAY [13]

## QUESTION 7

7.1 Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the rate of depreciation. (3)

7.2 Samuel took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

7.2.1 Calculate the value of Samuel's monthly instalment. (4)

7.2.2 Melissa took out a loan for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan. (4)

7.2.3 Who pays more interest, Samuel or Melissa? Use calculations to justify your answer. (2) [13]

## ▸ DIFFERENTIAL CALCULUS [36]

## QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = x^3$. (5)

8.2 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.3 If $y = (x^6 - 1)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$ or $x < -1$. (3)

8.4 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up. (4) [14]

## QUESTION 9

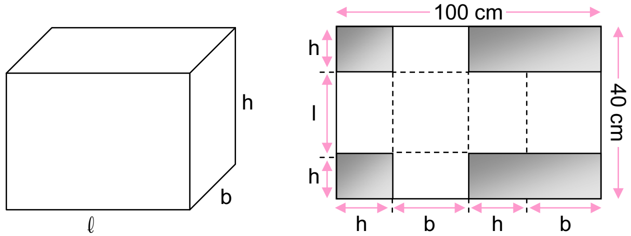
Given: $f(x) = (x + 2)(x^2 - 6x + 9)$
$= x^3 - 4x^2 - 3x + 18$

9.1 Calculate the coordinates of the turning points of the graph of f. (6)

9.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

9.3 For which value(s) of $x$ will $x.f'(x) < 0$? (3) [13]

## QUESTION 10




A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

10.1 Express the length $\ell$ in terms of the height h. (1)

10.2 Hence prove that the volume of the box is given by $V = h(50 - h)(40 - 2h)$. (3)

10.3 For which value of h will the volume of the box be a maximum? (5) [9]

## ▸ PROBABILITY [16]

## QUESTION 11

A survey concerning their holiday preferences was done with 180 staff members. The options they could choose from were to:

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

| | Coast | Game Park | Home | Total |
|---|---|---|---|---|
| **Male** | 46 | 24 | 13 | 83 |
| **Female** | 52 | 38 | 7 | 97 |
| **Total** | 98 | 62 | 20 | 180 |

11.1 Determine the probability that a randomly selected staff member:

11.1.1 is male (1)

11.1.2 does not prefer visiting a game park (2)

11.2 Are the events 'being a male' and 'staying at home' independent events? Motivate your answer with relevant calculations. (4) [7]

## QUESTION 12

12.1 A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

12.1.1 all the letters of the alphabet can be used (2)

12.1.2 the password must start with a 'D' and end with an 'L' (2)

12.2 Seven cars of different makes, of which 3 are silver, are to be parked next to each other.

12.2.1 In how many different ways can ALL the cars be parked next to each other? (2)

12.2.2 If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked. (3) [9]

**TOTAL: 150**

# NATIONAL NOV 2014 PAPER 2

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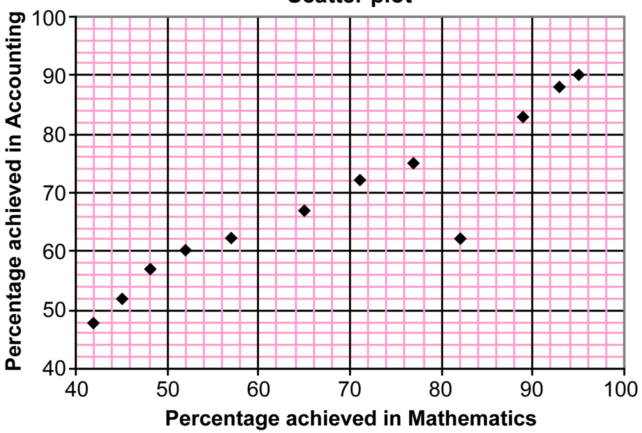
## ▶ STATISTICS [20]

### QUESTION 1

At a certain school, only 12 candidates take Mathematics and Accounting. The marks, as a percentage, scored by these candidates in the preparatory examinations for Mathematics and Accounting, are shown in the table and scatter plot below.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Mathematics** | 52 | 82 | 93 | 95 | 71 | 65 | 77 | 42 | 89 | 48 | 45 | 57 |
| **Accounting** | 60 | 62 | 88 | 90 | 72 | 67 | 75 | 48 | 83 | 57 | 52 | 62 |

**Scatter plot**




1.1 Calculate the mean percentage of the Mathematics data. (2)

1.2 Calculate the standard deviation of the Mathematics data. (1)

1.3 Determine the number of candidates whose percentages in Mathematics lie within ONE standard deviation of the mean. (3)

1.4 Determine an equation for the least squares regression line (line of best fit) for the data. (3)

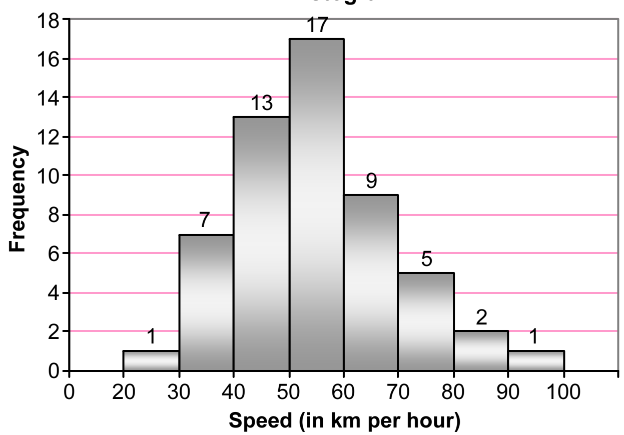
1.5 If a candidate from this group scored 60% in the Mathematics examination but was absent for the Accounting examination, predict the percentage that this candidate would have scored in the Accounting examination, using your equation in Question 1.4 (Round off your answer to the nearest integer.) (2)

1.6 Use the scatter plot and identify any possible outlier(s) in the data. (1) [12]

### QUESTION 2

The speeds of 55 cars passing through a certain section of a road are monitored for one hour. The speed limit on this section of road is 60 km per hour. A histogram is drawn to represent this data.

**Histogram**




2.1 Identify the modal class of the data. (1)

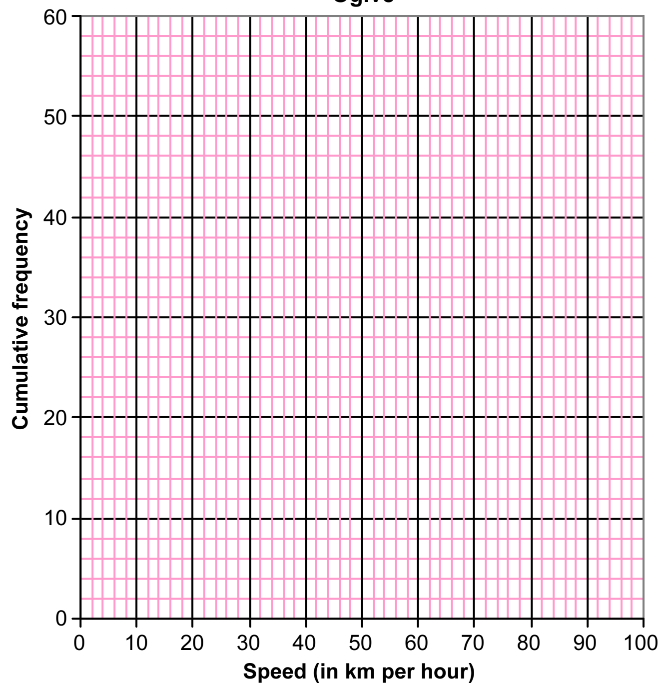
2.2 Use the histogram to:

2.2.1 Complete the cumulative frequency column in the table below. (2)

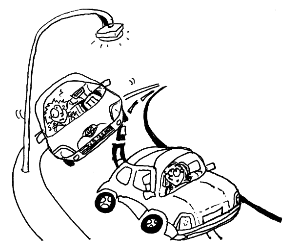
| Class | Frequency | Cumulative frequency |
|---|---|---|
| $20 < x \leq 30$ | 1 | |
| $30 < x \leq 40$ | 7 | |
| $40 < x \leq 50$ | 13 | |
| $50 < x \leq 60$ | 17 | |
| $60 < x \leq 70$ | 9 | |
| $70 < x \leq 80$ | 5 | |
| $80 < x \leq 90$ | 2 | |
| $90 < x \leq 100$ | 1 | |

2.2.2 Draw an ogive (cumulative frequency graph) of the data in Question 2.2.1 on the following grid. (3)

**Ogive**




2.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66 km per hour. Estimate the number of motorists who will receive a speeding fine. (2) [8]



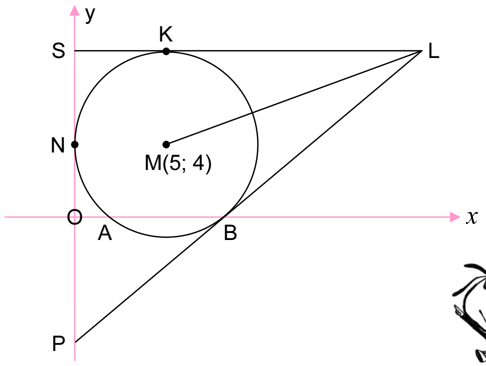
## ANALYTICAL GEOMETRY [40]

### QUESTION 3

In the diagram below, a circle with centre M(5; 4) touches the y-axis at N and intersects the $x$-axis at A and B.

PBL and SKL are tangents to the circle where SKL is parallel to the $x$-axis and P and S are points on the y-axis.

LM is drawn.




3.1 Write down the length of the radius of the circle having centre M. (1)

3.2 Write down the equation of the circle having centre M, in the form $(x - a)^2 + (y - b)^2 = r^2$. (1)

3.3 Calculate the coordinates of A. (3)

3.4 If the coordinates of B are (8; 0), calculate:

3.4.1 The gradient of MB (2)

3.4.2 The equation of the tangent PB in the form $y = mx + c$ (3)

3.5 Write down the equation of tangent SKL. (2)

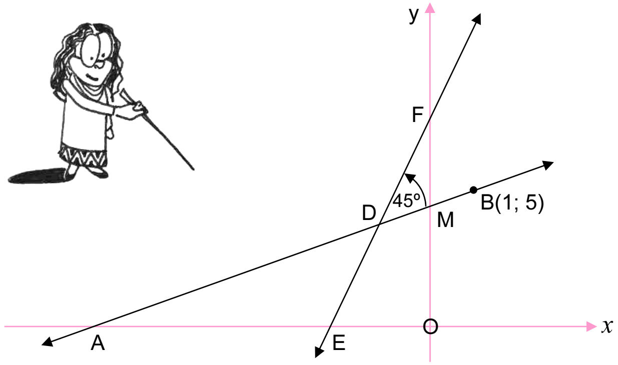
3.6 Show that L is the point (20; 9). (2)

3.7 Calculate the length of ML in surd form. (2)

3.8 Determine the equation of the circle passing through points K, L and M in the form $(x - p)^2 + (y - q)^2 = c^2$ (5) [21]

### QUESTION 4

In the diagram below, E and F respectively are the $x$- and y-intercepts of the line having equation $y = 3x + 8$. The line through B(1; 5) making an angle of 45° with EF, as shown below, has $x$- and y-intercepts A and M respectively.




4.1 Determine the coordinates of E. (2)

4.2 Calculate the size of $D\hat{A}E$. (3)

4.3 Determine the equation of AB in the form $y = mx + c$. (4)

4.4 If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D. (4)

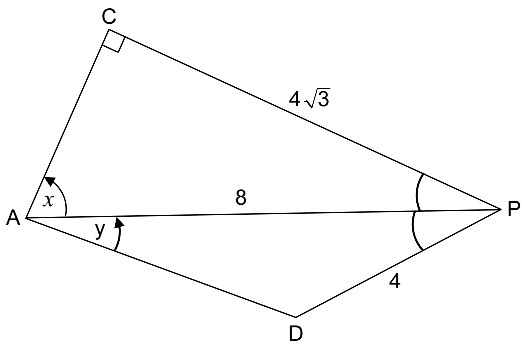
4.5 Calculate the area of quadrilateral DMOE. (6) [19]

## TRIGONOMETRY [40]

### QUESTION 5

In the figure below, ACP and ADP are triangles with $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$.

PA bisects $D\hat{P}C$. Let $C\hat{A}P = x$ and $D\hat{A}P = y$.




5.1 Show, by calculation, that $x = 60°$. (2)

5.2 Calculate the length of AD. (4)

5.3 Determine y. (3) [9]

### QUESTION 6

6.1 Prove the identity:

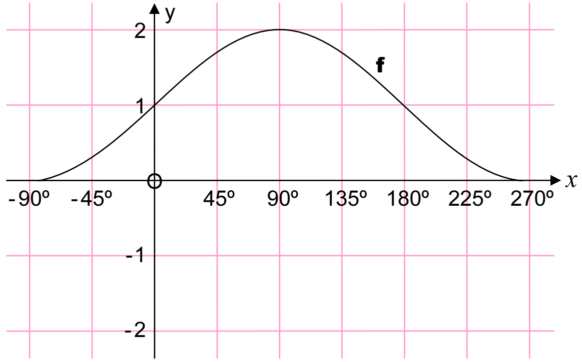
$\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x = \cos 2x$ (5)

6.2 Use $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

6.3 If $\sin 76° = x$ and $\cos 76° = y$, show that $x^2 - y^2 = \sin 62°$, without using a calculator. (4) [12]

### QUESTION 7

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90° \leq x \leq 270°$.




7.1 Write down the range of f. (2)

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

7.4 Use the grid above to draw the graph of $g(x) = \cos 2x$ for $-90° \leq x \leq 270°$. (3)

7.5 Determine the value(s) of $x$ for which $f(x + 30°) = g(x + 30°)$ in the interval $-90° \leq x \leq 270°$. (3)

7.6 Consider the following geometric series:

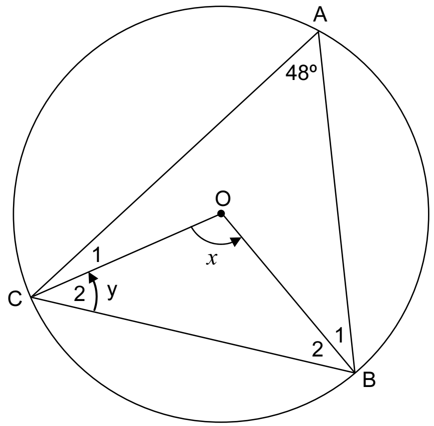
$1 + 2\cos 2x + 4\cos^2 2x + \ldots$

Refer to the graph of g to determine the value(s) of $x$ in the interval $0° \leq x \leq 90°$ for which this series will converge. (5) [19]

## ▸ EUCLIDEAN GEOMETRY AND MEASUREMENT [50]

### QUESTION 8

8.1 In the diagram, O is the centre of the circle passing through A, B and C. $\hat{CAB} = 48°$, $\hat{COB} = x$ and $\hat{C}_2 = y$.

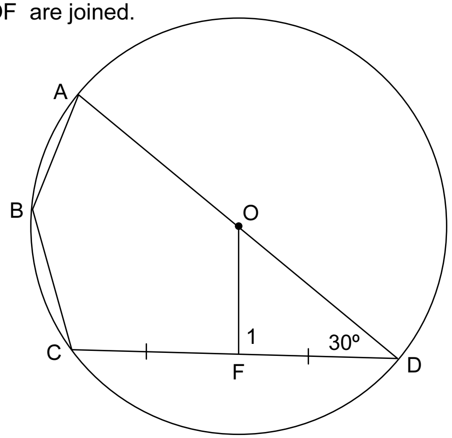



Determine, with reasons, the size of:

8.1.1 $x$ (2)

8.1.2 y (2)

8.2 In the diagram, O is the centre of the circle passing through A, B, C and D. AOD is a straight line and F is the midpoint of chord CD. $\hat{ODF} = 30°$ and OF are joined.

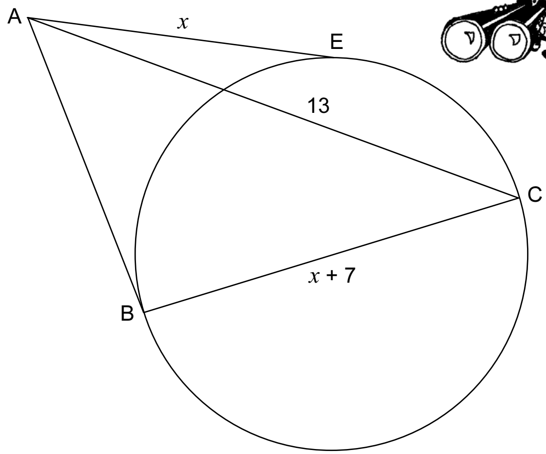



Determine, with reasons, the size of:

8.2.1 $\hat{F}_1$ (2)

8.2.2 $\hat{ABC}$ (2)

8.3 In the diagram, AB and AE are tangents to the circle at B and E respectively. BC is a diameter of the circle. AC = 13, AE = $x$ and BC = $x + 7$.




8.3.1 Give reasons for the statement below.

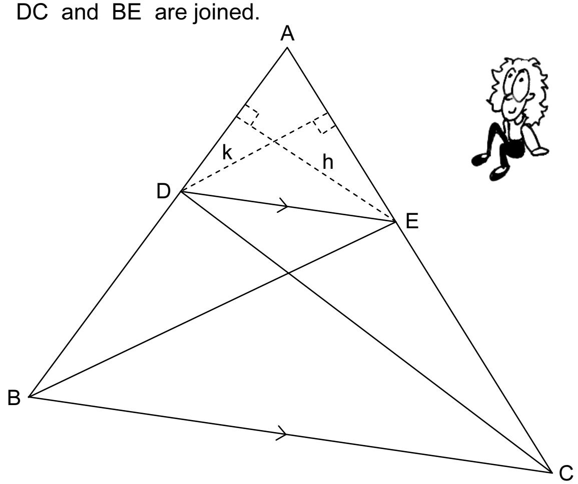
| | Statement | Reason |
|---|---|---|
| (a) | $\hat{ABC} = 90°$ | |
| (b) | AB = $x$ | |

(2)

8.3.2 Calculate the length of AB. (4) [14]

### QUESTION 9

9.1 In the diagram, points D and E lie on sides AB and AC of ΔABC respectively such that DE || BC. DC and BE are joined.




9.1.1 Explain why the areas of ΔDEB and ΔDEC are equal. (1)

9.1.2 Given below is the partially completed proof of the theorem that states that if in any ΔABC the line DE || BC then $\frac{AD}{DB} = \frac{AE}{EC}$.

Using the above diagram, complete the proof of the theorem.

Construction: Construct the altitudes (heights) h and k in ΔADE.

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(DB)(h)} \qquad = \ldots\ldots$$

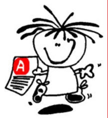
$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \ldots\ldots\ldots\ldots\ldots\ldots \qquad = \frac{AE}{EC}$$

But area ΔDEB = ................ (reason: ..................)

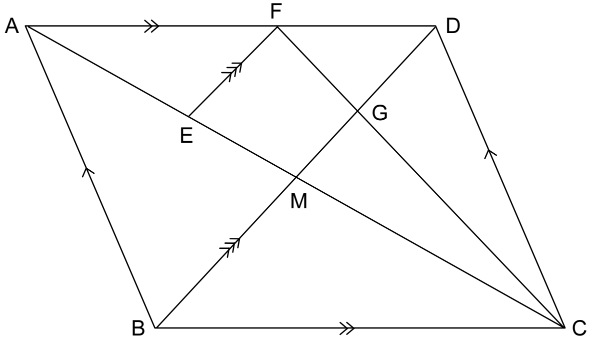
$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)



9.2 In the diagram, ABCD is a parallelogram.
The diagonals of ABCD intersect in M.
F is a point on AD such that AF : FD = 4 : 3.
E is a point on AM such that EF || BD.
FC and MD intersect in G.




Calculate, giving reasons, the following ratios:

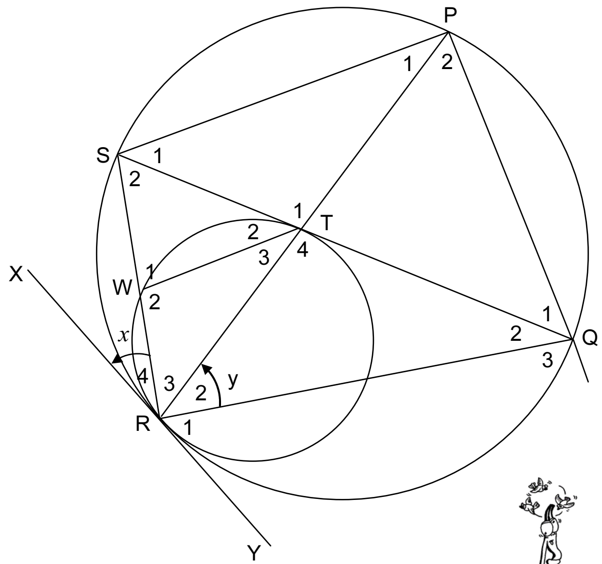
9.2.1 $\frac{EM}{AM}$ (3)

9.2.2 $\frac{CM}{ME}$ (3)

9.2.3 $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC}$ (4) [16]

## QUESTION 10

The two circles in the diagram have a common tangent XRY at R. W is any point on the small circle. The straight line RWS meets the large circle at S. The chord STQ is a tangent to the small circle, where T is the point of contact. Chord RTP is drawn. Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$




10.1 Give reasons for the statements below.
Complete the table.

| Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$ | | |
|---|---|---|
| | **Statement** | **Reason** |
| 10.1.1 | $\hat{T}_3 = x$ | |
| 10.1.2 | $\hat{P}_1 = x$ | |
| 10.1.3 | WT \|\| SP | |
| 10.1.4 | $\hat{S}_1 = y$ | |
| 10.1.5 | $\hat{T}_2 = y$ | |

(5)

10.2 Prove that $RT = \frac{WR.RP}{RS}$ (2)

10.3 Identify, with reasons, another TWO angles equal to y. (4)

10.4 Prove that $\hat{Q}_3 = \hat{W}_2$. (3)

10.5 Prove that $\Delta RTS \,|||\, \Delta RQP$. (3)

10.6 Hence, prove that $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3) [20]

**TOTAL: 150**

**NOTES**

# GR 12 MATHS – EXAM MEMOS



## NATIONAL NOV 2014 PAPER 1

### ALGEBRA AND EQUATIONS AND INEQUALITIES [21]

1.1.1 $(x-2)(4+x) = 0$

$\therefore x - 2 = 0$ or $4 + x = 0$

$\therefore \mathbf{x = 2}$ ◄ $\quad \therefore \mathbf{x = -4}$ ◄

1.1.2 $3x^2 - 2x - 14 = 0$

$\therefore x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-14)}}{2(3)}$

$\therefore x = \dfrac{2 \pm \sqrt{172}}{6}$

$\therefore \approx$ **2,52 or -1,85** ◄

1.1.3 $2^{x+2} + 2^x = 20$

$\therefore 2^x . 2^2 + 2^x = 20$

$\therefore 2^x (4 + 1) = 20$

$\div 5)$ $\quad \therefore 2^x = 14$

$\therefore \mathbf{x = 2}$ ◄

1.2 $x = 2y + 3 \quad \ldots$ ❶

$3x^2 - 5xy = 24 + 16y \quad \ldots$ ❷

❶ in ❷: $\therefore 3(2y+3)^2 - 5y(2y+3) = 24 + 16y$

$\therefore 3(4y^2 + 12y + 9) - 10y^2 - 15y - 24 - 16y = 0$

$\therefore 12y^2 + 36y + 27 - 10y^2 - 15y - 24 - 16y = 0$

$\therefore 2y^2 + 5y + 3 = 0$

$\therefore (2y + 3)(y + 1) = 0$

$\therefore 2y + 3 = 0$ or $y + 1 = 0$

$\therefore 2y = -3 \quad \therefore y = -1$

$\therefore y = -\dfrac{3}{2}$

❶: For $y = -\dfrac{3}{2}$: $x = 2\left(-\dfrac{3}{2}\right) + 3 = -3 + 3 = 0$

& For $y = -1$: $x = 2(-1) + 3 = -2 + 3 = 1$

$\therefore$ The solution: $\left(\mathbf{0; -\frac{3}{2}}\right)$ **or (1; -1)** ◄

1.3 $(x-1)(x-2) < 6$

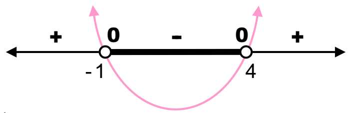
$\therefore x^2 - 3x + 2 - 6 < 0$

$\therefore x^2 - 3x - 4 < 0$

$\therefore (x+1)(x-4) < 0$

**The expression:**

$x$:




$\therefore \mathbf{-1 < x < 4}$ ◄

1.4 The roots are real when

$-k - 4 \geq 0 \quad \ldots \Delta \geq 0$

$\therefore -k \geq 4$

$\times(-1) \quad \therefore \mathbf{k \leq -4}$ ◄

### PATTERNS AND SEQUENCES [31]

2. $2 + 9 + 16 + \ldots$ (to 251 terms)

2.1 $\mathbf{T_4 = 23}$ ◄

2.2 **A.S.:** $T_{251}$? ; $n = 251$ ; $a = 2$ ; $d = 7$

$\mathbf{T_n = a(n-1)d}$ ➡ $T_{251} = 2 + (251 - 1)(7)$

$= \mathbf{1\ 752}$ ◄

OR: It is a linear sequence: $\mathbf{T_n = an + b}$

where $\mathbf{a} = 7$ and $\mathbf{b = T_0} = -5$

$\therefore T_n = 7n - 5$

$\therefore T_{251} = 7(251) - 5$

$= \mathbf{1\ 752}$ ◄

2.3 $\displaystyle\sum_{n=1}^{251}\mathbf{(7n - 5)}$ ◄ $\quad \ldots$ *if you found $T_n$ in 2.2*

OR: $\displaystyle\sum_{i=1}^{251}\mathbf{(7i - 5)}$ ◄

OR: Find $\mathbf{T_n}$ using the A.S. formula

$\mathbf{T_n}$**?** ; $\mathbf{a} = 2$ ; $\mathbf{d} = 7$

$\mathbf{T_n = a + (n-1)d}$ ➡ $T_n = 2 + (n-1)(7)$

$= 2 + 7n - 7$

$= 7n - 5$

Then, sigma notation, as above.

2.4 $\mathbf{S_n = \frac{n}{2}(a + T_n)}$, were $\mathbf{n} = 251$ ; $\mathbf{a} = 2$ ; $\mathbf{T_n} = 1\ 752$

$\therefore S_{251} = \dfrac{251}{2}(2 + 1\ 752)$

$= \mathbf{220\ 127}$ ◄

OR:

$\mathbf{S_n = \frac{n}{2}[2a + (n-1)d]}$, were $\mathbf{n} = 251$ ; $\mathbf{a} = 2$ ; $\mathbf{d} = 7$

$\therefore S_{251} = \dfrac{251}{2}[2(2) + (251 - 1)(7)]$

$= \mathbf{220\ 127}$ ◄

2.5 $2 + 9 + 16 + 23 + 30 + 37 + 44 + 51 + 58 + 65 + 72 + \ldots$

Consider the series: 16 ; 44 ; 72 ; . . .

$\mathbf{n}$**?** ; $\mathbf{T_n} = 1\ 752$ (divisible by 4) ; $\mathbf{a} = 16$ ; $\mathbf{d} = 28$

$\mathbf{a + (n-1)d = T_n}$ ➡ $16 + (n-1)(28) = 1\ 752$

$\therefore (n-1)(28) = 1\ 736$

$\therefore n - 1 = 62$

$\therefore n = 63$

$\therefore$ **63 terms** ◄

OR: By inspection, after 16 every 4th term is divisible by 4.
So, imagining $T_0$ to be inserted at the start, determine the number of groups of 4 terms up to $T_{251}$.

The number of groups $= \dfrac{252}{4} \quad \ldots$ *1 + 251*

$= 63$

$\therefore$ 63 terms divisible by 4 ◄

3.1 The sequence: 7 ; -1 ; -7 ; -11 ; P

$1^{st}$ difference: -8 ; -6 ; -4 ; -2

$2^{nd}$ difference: 2 ; 2 ; 2

3.1.1 **p = -13** ✓ . . . *see pattern above*

3.1.2 $\mathbf{T_n = an^2 + bn + c}$ . . . *the general term of a quadratic sequence*

- Common $2^{nd}$ difference, $2a = 2$
  $\therefore a = 1$
- $1^{st}$ term of $1^{st}$ differences: $3a + b = -6$
  $\therefore 3(1) + b = -6$
  $\therefore b = -9$
- $T_0 = c = 7$

$\therefore \mathbf{T_n = n^2 - 9n + 7}$ ✓

3.1.3 The $1^{st}$ differences from the **linear** sequence:
-6 ; -4 ; -2 ; . . . .

**n?** ; $T_n = 96$ ; **a** = -6 ; **d** = 2

$\mathbf{a + (n - 1)d = T_n}$ ➡ $-6 + (n - 1)(2) = 96$

$\therefore 2(n - 1) = 102$

$\therefore n - 1 = 51$

$\therefore n = 52$

$\therefore$ The 2 terms are:

$T_{52} = 52^2 - 9(52) + 7 = \mathbf{2\ 243}$ ✓

& $T_{53} = 53^2 - 9(53) + 7 = \mathbf{2\ 339}$ ✓ . . . $T_n = n^2 - 9n + 7$

OR: Consider $T_{n+1} - T_n = 96$ to find n:

$\therefore (n + 1)^2 - 9(n + 1) + 7 - (n^2 - 9n + 7) = 96$

$\therefore \cancel{n^2} + 2n + 1 - \cancel{9n} - 9 + \cancel{7} - \cancel{n^2} + \cancel{9n} - \cancel{7} = 96$

$\therefore 2n = 104$

$\therefore n = 52$

$\therefore$ The 2 terms are $T_{52}$ and $T_{53}$, etc.

3.2 **G.S.:** 16 ; 4 ; 1

3.2.1 $\mathbf{T_{12}?}$ ; **n** = 12 ; **a** = 16 ; **r** = $\frac{1}{4}$

$\mathbf{T_n = ar^{n-1}}$ ➡ $T_{12} = 16 . \left(\frac{1}{4}\right)^{12-1}$

*NB: No calculator*

$= 2^4 . (2^{-2})^{11}$ . . . $\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$

$= 2^4 . 2^{-22}$

$= 2^{-18}$ ✓ . . . $2^4 . 2^{-22} = 2^{4 + (-22)}$

$= \frac{1}{2^{18}}$ or $\left(\frac{1}{2}\right)^{18}$ ✓

OR: $T_{12} = 4^2 . (4^{-1})^{11}$
$= 4^2 . 4^{-11}$
$= 4^{-9}$ ✓

3.2.2 $\mathbf{S_{10}?}$ ; **n** = 10 ; **a** = 16 ; **r** = $\frac{1}{4}$

$\mathbf{S_n = \frac{a(1 - r^n)}{1 - r}}$ ➡ $S_{10} = \frac{16\left[1 - \left(\frac{1}{4}\right)^{10}\right]}{1 - \frac{1}{4}}$ . . . *Calculator allowed!*

$\approx 21,33$ ✓

3.3 $\left(\frac{\cancel{3}}{2}\right)\left(\frac{\cancel{4}}{\cancel{3}}\right)\left(\frac{\cancel{5}}{\cancel{4}}\right)\left(\frac{\cancel{6}}{\cancel{5}}\right) \ldots\ldots \left(\frac{100}{\cancel{99}}\right)$ . . . . $98 + 2$ / . . . . $98 + 1$ *by inspection*

$= \frac{100}{2}$

$= \mathbf{50}$ ✓

## ▶ FUNCTIONS AND GRAPHS [33]

4.1 p = 1 ✓ q = 1 ✓

4.2 At T, $g(x) = 0$ ➡ $\frac{2}{x + 1} + 1 = 0$ . . . *y = 0 on the x-axis*

$\therefore \frac{2}{x + 1} = -1$

$\times (x + 1)$ $\therefore 2 = -(x + 1)$

$\therefore x + 1 = -2$

$\therefore x = -3$

$\therefore \mathbf{x_T = -3}$ ✓

4.3 $x = -6$ ✓ . . . *graph g shifts 5 units to the left*

OR: $h(x) = g(x + 5) = \frac{2}{(x + 5) + 1} + 1 = \frac{2}{x + 6} + 1$

$\therefore$ Equation of h: $y = \frac{2}{x + 6} + 1$

$\therefore$ The vertical asymptote is: $\mathbf{x = -6}$ ✓

4.4 At S: $y = x$ and $y = g(x)$

$\therefore x = \frac{2}{x + 1} + 1$

$\times (x + 1)$ $\therefore x(x + 1) = 2 + (x + 1)$

$\therefore x^2 + \cancel{x} = 2 + \cancel{x} + 1$

$\therefore x^2 = 3$

$\therefore x = +\sqrt{3}$ . . . *S in the $1^{st}$ Quadrant* ➡ $x > 0$

$\therefore$ Point $S(\sqrt{3}; \sqrt{3})$ . . . $y = x$

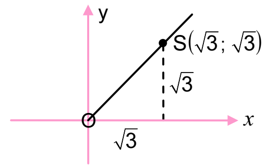
$\therefore OS^2 = (\sqrt{3})^2 + (\sqrt{3})^2$ . . . *Pythagoras*

$= 3 + 3$

$= 6$

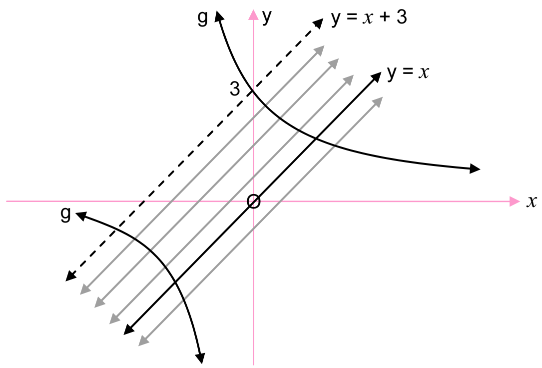
$\therefore OS = \sqrt{6}$

$\approx$ **2,45 units** ✓




4.5 Consider the given graph g and the line $y = x + k$ and the $x$-coordinates of their points of intersection.

See the dotted line $y = x + k$ where k = 3 and the lines below it where k < 3.




**k < 3** ✓

For k = 3, one root is zero
For k > 3, both roots are negative
(Sketch these cases on the graph)

5.1 Equation of f: $y = \log_a x$

i.e. $x = a^y$

$S\left(\frac{1}{3}; -1\right)$ on f: $\therefore\ \frac{1}{3} = a^{-1}$

$\therefore$ **a = 3** ◀ ... $3^{-1} = \frac{1}{3}$

5.2 Equation of h (which is $f^{-1}$): $\mathbf{y = 3^x}$ ◀

5.3 $g(x) = -f(x) = -\log_3 x$

$\therefore$ Equation of g: $y = -\log_3 x$ ◀

5.4 $\boldsymbol{x > 0\ ;\ x \in \mathbb{R}}$ ◀ ... **Note:** *In log x, x cannot be negative or zero.*

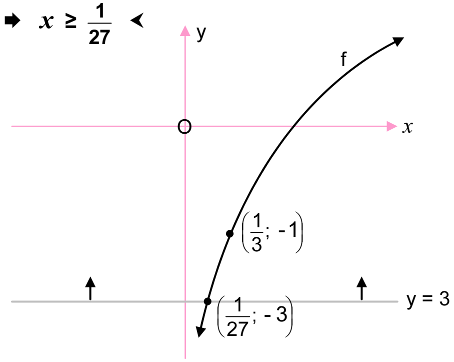
OR: $(\mathbf{0; \infty})$ ◀

5.5 $f(x) = -3 \Rightarrow \log_3 x = -3$

$\therefore\ x = 3^{-3}$

$= \frac{1}{27}$ ... $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

$\mathbf{f(x) \geq -3 \Rightarrow x \geq \frac{1}{27}}$ ◀




6.1 At S, $g(x) = 0$: $\therefore\ 4x^2 - 6 = 0$

$\therefore\ 4x^2 = 6$

$\therefore\ x^2 = \frac{6}{4}$

$\therefore\ x \approx 1{,}22$ ◀ ... $x > 0$

6.2 $g(\mathbf{0}) = -6$ ... $\boldsymbol{x = 0}$ *on the y-axis*

$\therefore$ **(0; -6)** ◀

6.3.1 $QKT = f(x) - g(x)$

$= 2\sqrt{x} - (4x^2 - 6)$

$= \mathbf{2x^{\frac{1}{2}} - 4x^2 + 6}$ ◀

6.3.2 Max QT when the derivative = 0

$\therefore\ 2 \cdot \frac{1}{2}x^{-\frac{1}{2}} - 8x = 0$

$\times\, x^{\frac{1}{2}}$) $\therefore\ 1 - 8x^{\frac{3}{2}} = 0$

$\therefore\ 1 = 8x^{\frac{3}{2}}$

$\therefore\ \frac{1}{8} = x^{\frac{3}{2}}$

Take $\sqrt[3]{\ }$: $\therefore\ \frac{1}{2} = x^{\frac{1}{2}}$

Square: $\therefore\ x = \frac{1}{4}$

$\therefore$ Maximum length $= 2\left(\frac{1}{4}\right)^{\frac{1}{2}} - 4\left(\frac{1}{4}\right)^2 + 6$

$= 2 \times \frac{1}{2} - 4 \times \frac{1}{16} + 6$

$= 1 - \frac{1}{4} + 6$

$= 6\frac{3}{4}$ units ◀

## FINANCE, GROWTH AND DECAY [13]

7.1 **n** = 5 ; **P** = R145 000 ; **A** = R72 500 ; ***i*** **?**

$\mathbf{A = P(1 - i)^n} \Rightarrow 145\,000(1 - i)^5 = 72\,500$

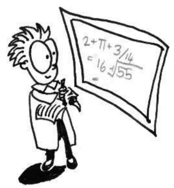
$\therefore\ (1 - i)^5 = 0{,}5$

Take 5$^{\text{th}}$ root: $\therefore\ \sqrt[5]{0{,}5} = 1 - i$

$\therefore\ 0{,}87055\ldots = 1 - i$

$\therefore\ 0{,}12944\ldots = i$

$\therefore\ \boldsymbol{i} = \mathbf{12{,}94\%}$ ◀



7.2.1 $\mathbf{P_v}$ = R500 000 ; $\boldsymbol{i} = \frac{12\%}{12} = 1\% = 0{,}01$ ;

**n** = 20 × 12 = 240 ; $\boldsymbol{x}$**?**

$\mathbf{P_v} = \dfrac{x\left[\mathbf{1 - (1 + i)^{-n}}\right]}{\boldsymbol{i}}$ ... **PRESENT VALUE FORMULA**

$\therefore\ 500\,000 = \dfrac{x\left[1 - 1{,}01^{-240}\right]}{0{,}01}$

$\therefore\ 500\,000 = x \cdot A$ ... $A = 90{,}819\ldots$

$\therefore\ \boldsymbol{x}$ = **R5 505,43** ◀

7.2.2 $\mathbf{P_v}$ = 500 000 ; $\boldsymbol{i}$ = 0,01 ; **n?** ; $\boldsymbol{x}$ = R6 000

Using the present value formula again:

$500\,000 = \dfrac{6\,000\left[1 - 1{,}01^{-n}\right]}{0{,}01}$

$\times \frac{0{,}01}{6\,000}$) $\therefore\ \left[1 - 1{,}01^{-n}\right] = 0{,}8\dot{3}$

$\therefore\ 0{,}1\dot{6} = 1{,}01^{-n}$

$\therefore\ -n = \log_{1{,}01} 0{,}1\dot{6}$

$= \dfrac{\log 0{,}1\dot{6}}{\log 1{,}01}$

$\approx -180{,}07$

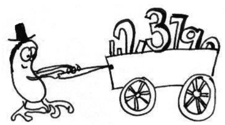
$\therefore$ **n** ≈ **180,07 months** ◀ ... *15 years*

7.2.3 Samuel pays: R5 505,43 × 240 = R1 321 303,20

Melissa pays: R6 000 × 180 = R1 080 000

$\therefore$ **Samuel pays (R241 303,20) more!** ◀

**Note:** *It was really a good decision that Melissa made!*



## DIFFERENTIAL CALCULUS [36]

8.1 $f(x) = x^3$

$\therefore\ f(x+h) = (x+h)^3$

$= x^3 + 3x^2h + 3xh^2 + h^3 \quad \ldots$

**Pascal's $\Delta$:**

1 1
1 2 1
**1 3 3 1**

$\therefore\ f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3$

$\therefore\ \dfrac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (3x^2 + 3xh + h^2)$

$= \mathbf{3x^2}$ ◀

8.2 $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$

$\therefore\ f'(x) = 2.2x + \frac{1}{2}.4x^3$

$= \mathbf{4x + 2x^3}$ ◀

8.3 $y = (x^6 - 1)^2$

$= x^{12} - 2x^6 + 1$

$\therefore\ \dfrac{dy}{dx} = 12x^{11} - 12x^5$

$= 12x^5(x^6 - 1)$

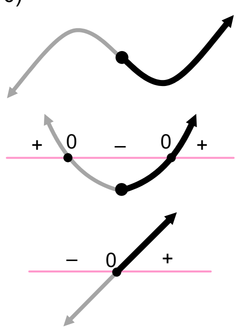
$= \mathbf{12x^5\sqrt{y}}$ ◀

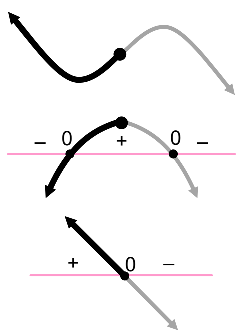
**Note:** If $x < -1$ or $x > 1$, then $y < 0$ $\therefore\ \sqrt{y}$ non-real

8.4 Shape of f:

(a > 0)

If a < 0:



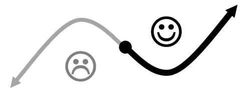

f is concave up for $f''(x) > 0$

Again: $f''(x) > 0$ for 'concave up'

$f(x) = 2x^3 - 2x^2 + 4x - 1$

$\therefore\ f'(x) = 6x^2 - 4x + 4$

$\therefore\ f''(x) = 12x - 4$



Concave up: $f''(x) > 0$ ➡ $12x - 4 > 0$

$\therefore\ 12x > 4$

$\therefore\ x > \frac{4}{12}$

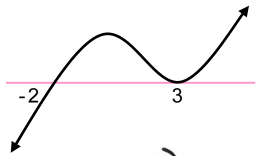
$\therefore\ x > \frac{1}{3}$

$\therefore$ The interval: $\left(\mathbf{\frac{1}{3}}; \infty\right)$ ◀

9. $f(x) = (x+2)(x^2 - 6x + 9)$

$= (x+2)(x-3)^2$ ➡

$= x^3 - 4x^2 - 3x + 18$




9.1 **Note:** 2 equal factors ➡ 2 equal roots ➡ a turning point at 3

$f'(x) = 3x^2 - 8x - 3 = 0$ ... *derivative = 0 at the turning points*

$\therefore\ (3x+1)(x-3) = 0$

$\therefore\ x = -\frac{1}{3}$ or $3$

$f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 18 \approx 18{,}52$

& $f(3) = 0$ ... *see sketch above*

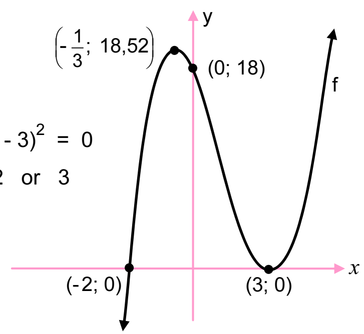
$\therefore$ The turning points: $\left(\mathbf{-\frac{1}{3}; 18{,}52}\right)$ & **(3; 0)** ◀

9.2 **Y-int.:** (0; 18)

(Put $x = 0$)

**X-ints:** $(x+2)(x-3)^2 = 0$

(Put y = 0) $\therefore\ x = -2$ or $3$




9.3 $x < -\frac{1}{3}$ or $0 < x < 3$ ◀

These are the values of $x$ for which the product of $x$ and the gradient of f will be negative (< 0)

10.1 $\ell + 2h = 40$

$\therefore\ \boldsymbol{\ell = 40 - 2h}$ ◀

10.2 Similarly: $2b + 2h = 100$

$\therefore\ b + h = 50$

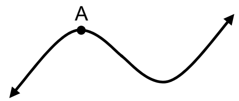
$\therefore\ b = 50 - h$

Volume $= \ell bh = (40 - 2h)(50 - h)h$

$\therefore\ \mathbf{V = h(50 - h)(40 - 2h)}$ ◀

10.3 $V = h(2\,000 - 140h + 2h^2)$

$V = 2\,000h - 140h^2 + 2h^3$



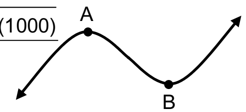

$\dfrac{dV}{dh} = 2\,000 - 280h + 6h^2 = 0$ ... *at the turning points*

÷ 2) $\therefore\ 3h^2 - 140h + 1\,000 = 0$

$\therefore\ h = \dfrac{-(-140) \pm \sqrt{(-140)^2 - 4(3)(1000)}}{2(3)}$

$\therefore\ h = \dfrac{140 - \sqrt{7\,600}}{6}$

$\approx 8{,}80$ cm ... *see maximum turning point A on the sketch*




We ignore h ≈ 37,86 because this greater value applies to the minimum turning point B

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## PROBABILITY [16]

11.1.1 P(male) $= \mathbf{\frac{83}{180}}$ ✓

> **NB:** $P(E) = \frac{n(E)}{n(S)}$

11.1.2 P(not game park) $= \frac{98 + 20}{180}$

$= \frac{118}{180}$

$= \mathbf{\frac{59}{90}}$ ✓ . . . *P(coast or at home)*

11.2 P(M and at home) $= \frac{13}{180} = 0{,}07\dot{2}$ ✓

P(at home) $= \frac{20}{180}$

$\therefore$ P(male) $\times$ P(at home) $= \frac{83}{180} \times \frac{20}{180}$

$= 0{,}051\ldots$

$\therefore$ **The events are not independent** ✓

12.1.1 $\underline{26}$ $\underline{25}$ $\underline{24}$ $\underline{23}$ $\underline{22}$

The number of ways the 5 different letters can be chosen

$= 26 \times 25 \times 24 \times 23 \times 22$

$=$ **7 893 600** ✓

12.1.2 $\underset{\mathbf{D}\uparrow}{\underline{1}}$ $\underline{24}$ $\underline{23}$ $\underline{22}$ $\underset{\mathbf{L}\uparrow}{\underline{1}}$

When determining the 1st and 5th positions, the no. of ways

$= 24 \times 23 \times 22$ . . . *2 letters are allocated to fixed positions*

$=$ **12 144** ✓

12.2.1 The number of ways the 7 cars can be parked

$= 7!$

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ . . . *or, use your calculator*

$=$ **5 040** ✓

12.2.2 _ _ _ (_ _ _) _

*The 3 silver cars together occupy 1 slot.*

$\therefore$ *There are 5 slots altogether. But the 2 silver cars can be arranged in 3! ways.*

5! 3!

$=$ **720** ✓

> **NB:** If in 12.2.2, the question had been:
> What is the probability that the 3 silver cars would be parked together?
>
> Then, the answer would be $\frac{720}{5\,040}$ $\begin{matrix}\textit{. . . 12.2.2}\\ \textit{. . . 12.2.1}\end{matrix}$
>
> $= \mathbf{\frac{1}{7}}$ ✓
>
> $P(E) = \frac{n(E)}{n(S)}$

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## NATIONAL NOV 2014 PAPER 2

## STATISTICS [20]

1.1 The mean, $\bar{x}$ **= 68** ✓ . . . *for maths*

1.2 The standard deviation, $\sigma \approx$ **18,42** ✓ . . . *for maths*

1.3 $\bar{x} + \sigma = 86{,}42$

& $\bar{x} - \sigma = 49{,}58$

**6 candidates scored between these values** ✓

. . . . 52 ; 82 ; 71 ; 65 ; 77 ; 57

1.4 In $y = \mathbf{A} + \mathbf{B}x$, $\mathbf{A} \approx 22{,}83$ & $\mathbf{B} \approx 0{,}66$

$\therefore$ $\mathbf{y = 22{,}83 + 0{,}66}x$ ✓

> **Note:**
> $x$: Mathematics
> y: Accounting

1.5 Approximate accounting mark $= 22{,}83 + 0{,}66\,(60)$

$\approx$ **62%** ✓

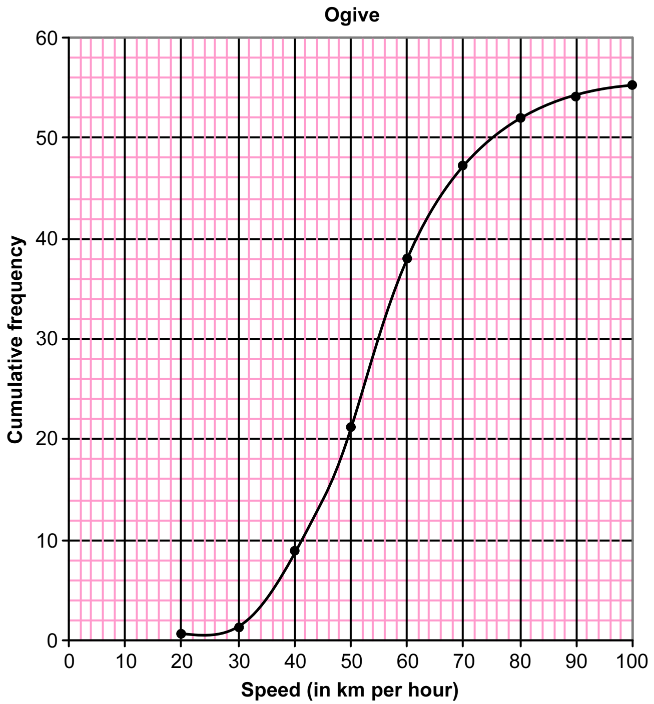
1.6 **(82; 62)** ✓

2.1 $\mathbf{50 < x \leq 60}$ ✓

2.2.1

| Class | Frequency | Cumulative frequency |
|---|---|---|
| $20 < x \leq 30$ | 1 | 1 |
| $30 < x \leq 40$ | 7 | 8 |
| $40 < x \leq 50$ | 13 | 21 |
| $50 < x \leq 60$ | 17 | 38 |
| $60 < x \leq 70$ | 9 | 47 |
| $70 < x \leq 80$ | 5 | 52 |
| $80 < x \leq 90$ | 2 | 54 |
| $90 < x \leq 100$ | 1 | 55 |

2.2.2




2.3 44 motorists travel @ 66 km/h or less

$\therefore$ **The number receiving fines = 55 - 44 = 11** ◄

## ► ANALYTICAL GEOMETRY [40]

3.1 **radius = 5 units** ◄ . . . $NM = 5$

3.2 $\mathbf{(x - 5)^2 + (y - 4)^2 = 25}$ ◄

3.3 At A (& B): y = 0

$\therefore\ (x - 5)^2 + 16 = 25$

$\therefore\ (x - 5)^2 = 9$

$\therefore\ x - 5 = \pm 3$

$\therefore\ x = 5 \pm 3$

$\therefore\ x = 2$ . . . $x = 8$ at B

$\therefore$ **A(2; 0)** ◄

3.4.1 $m_{MB} = \frac{4-0}{5-8} = \frac{4}{-3} = \mathbf{-\frac{4}{3}}$ ◄

3.4.2 $m_{PB} = +\frac{3}{4}$ . . . *tangent PB ⊥ radius MB*

Substitute $m = \frac{3}{4}$ & B(8; 0) in

$y - y_1 = m(x - x_1)$ OR: $y = mx + c$

$\therefore\ y - 0 = \frac{3}{4}(x - 8)$

$\therefore\ \mathbf{y = \frac{3}{4}x - 6}$ ◄

3.5 At S (and K), y = 4 + 5 . . . $y_M$ + *radius*

= 9

$\therefore$ Equation of tangent SKL: y = 9

3.6 At L: y = 9 and $y = \frac{3}{4}x - 6$ . . . *point of intersection*

$\therefore\ \frac{3}{4}x - 6 = 9$

$\times 4)$ $\therefore\ 3x - 24 = 36$

$\therefore\ 3x = 60$

$\therefore\ x = 20$

$\therefore$ **L(20; 9)** ◄

3.7 In ΔMKL: MK = 5 and KL = 20 - 5 = 15

$\therefore\ ML^2 = 5^2 + 15^2$ . . . $M\hat{K}L = 90°$; *Pythagoras*

$= 25 + 225$

$= 250$

$\therefore$ **ML** $= \sqrt{250} = \sqrt{25}\sqrt{10} = \mathbf{5\sqrt{10}}$ **units** ◄

3.8 ⊙KLM has ML as diameter . . . $M\hat{K}L = 90°$; ∠ *in semi-⊙*

**Note:** This reason must be given.

Centre is midpoint ML: $\left(\frac{5+20}{2};\ \frac{4+9}{2}\right)$

$= \left(\frac{25}{2};\ \frac{13}{2}\right)$

& radius $= \frac{1}{2}$ML $= \frac{5}{2}\sqrt{10}$

$\therefore$ Equation: $(x - 12{,}5)^2 + (y - 6{,}5)^2 = \mathbf{62{,}5}$ ◄

. . . $\left(\frac{5}{2}\sqrt{10}\right)^2 = \frac{25}{4}(10) = 62{,}5$

4.1 At E:

y = 0

$\therefore\ 3x + 8 = 0$

$\therefore\ 3x = -8$

$\therefore\ x = -\frac{8}{3}$

$\therefore$ $\mathbf{E\left(-\frac{8}{3};\ 0\right)}$ ◄

4.2 $\tan F\hat{E}O = 3$

$\therefore\ F\hat{E}O = 71{,}57°$

& $A\hat{D}E = 45°$ . . . *vertically opp.* ∠*s*

$\therefore\ D\hat{A}E = F\hat{E}O - 45°$

$= \mathbf{26{,}57°}$ ◄

4.3 Gradient, $m = \tan 26{,}57° = \frac{1}{2}$

$\therefore$ Substitute $m = \frac{1}{2}$ and B(1; 5) in

$y - y_1 = m(x - x_1)$ OR: $y = mx + c$

$\therefore\ y - 5 = \frac{1}{2}(x - 1)$

$\therefore\ y - 5 = \frac{1}{2}x - \frac{1}{2}$

$\therefore\ \mathbf{y = \frac{1}{2}x + 4\frac{1}{2}}$ ◄

4.4 Equation of AB: $x - 2y + 9 = 0$

At D: $y = 3x + 8$ also

$\therefore\ x - 2(3x + 8) + 9 = 0$

$\therefore\ x - 6x - 16 + 9 = 0$

$\therefore\ -5x = 7$

$\therefore\ x = -\frac{7}{5}$

& $y = 3\left(-\frac{7}{5}\right) + 8 = 3\frac{4}{5}$

$\therefore$ $\mathbf{D\left(-\frac{7}{5};\ 3\frac{4}{5}\right)}$ ◄

4.5 Area of Quad DMOE = Area ΔAMO - Area ΔADE

Area ΔAMO $= \frac{1}{2}$AO . OM

$= \frac{1}{2}(9)\left(4\frac{1}{2}\right)$

$= 20{,}25$

& Area ΔADE $= \frac{1}{2}$AE . $y_D$

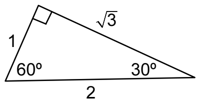
$= \frac{1}{2}\left(9 - \frac{8}{3}\right) . 3\frac{4}{5}$

$= 12{,}03$

$\therefore$ **Area of quad DMOE** $\approx$ $\mathbf{8{,}22}$ **units$^2$** ◄

## TRIGONOMETRY [40]

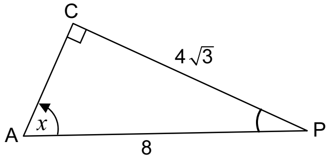
5.1 In right-angled $\Delta$ACP :




$\sin x = \dfrac{4\sqrt{3}}{8}$

$\therefore \sin x = \dfrac{\sqrt{3}}{2}$

$\therefore$ **$x$ = 60°** ✓




5.2 $A\hat{P}C$ = 30° . . . *sum of $\angle^s$ of $\Delta$ACP*

$\therefore$ $A\hat{P}D$ = 30° . . . *PA bisects $D\hat{P}C$*

In $\Delta$ADP : $AD^2 = 4^2 + 8^2 - 2(4)(8)\cos 30°$ . . . *cos-rule*

$\therefore$ **AD ≈ 4,96 units** ✓

5.3 In $\Delta$ADP : $\dfrac{\sin y}{4} = \dfrac{\sin 30°}{4{,}96}$

$\therefore \sin y = \dfrac{4\left(\frac{1}{2}\right)}{4{,}96}$ (= 0,40322...)

$\therefore$ **y = 23,78°** ✓

---

6.1 LHS $= (-\cos x)^2 + (+\tan x)(-\sin x)\cos x$

$= \cos^2 x - \left(\dfrac{\sin x}{\cancel{\cos x}}\right)\left(\dfrac{\sin x}{1}\right)\left(\dfrac{\cancel{\cos x}}{1}\right)$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

= **RHS** ✓

6.2 $\sin(\alpha - \beta) = \cos[90° - (\alpha - \beta)]$

$= \cos[90° - \alpha + \beta]$

$= \cos[(90° - \alpha) + \beta]$

$= \cos(90° - \alpha)\cos\beta - \sin(90° - \alpha)\sin\beta$ ... *from the formula provided*

$=$ **$\sin\alpha\cos\beta - \cos\alpha\sin\beta$** ✓

6.3 $x^2 - y^2$

$= \sin^2 76° - \cos^2 76°$

$= \cos^2 14° - \sin^2 14°$ . . . OR $= -(\cos^2 76° - \sin^2 76°)$

$= \cos 2(14°)$ $= -\cos 2(76°)$

$= \cos 28°$ $= -\cos 152°$

$= \sin(90° - 28°)$ $= -(-\cos 28°)$

$=$ **sin 62°** ✓ $= \cos 28°$, etc.

---

7.1 **$0 \le y \le 2$ ; $y \in \mathbb{R}$** ✓

7.2 $\sin x + \cancel{1} = \cancel{1} - 2\sin^2 x$ . . . *$\cos 2x = 1 - 2\sin^2 x$*

$\therefore 2\sin^2 x + \sin x = 0$

$\therefore$ **$\sin x(2\sin x + 1) = 0$** ✓

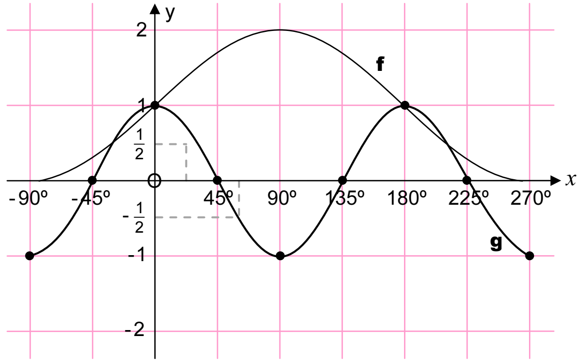
7.3 $\therefore \sin x = 0$ or $\sin x = -\dfrac{1}{2}$

**Note :** $\sin 30° = \dfrac{1}{2}$

$\therefore$ **$x$ = 0° + n(180°), n $\in \mathbb{Z}$** ✓ $\therefore$ **$x$ = 210° + n(360°)** ✓

or $\therefore$ **$x$ = 330° + n(360°), n $\in \mathbb{Z}$** ✓

7.4




7.5 In the given domain :

f($x$) = g($x$) ➡ $x$ = -30° ; 0° ; 180° or 210°

. . . *from the general solution in 7.3*

$\therefore$ f($x$ + 30°) = g($x$ + 30°) ➡ **$x$ = -60° ; -30° ; 150° or 180°** ✓

. . . *both graphs move 30° to the left $\therefore$ The solutions too*

7.6 **G.S. :** r = 2 cos 2$x$

The series will converge for -1 < r < 1

$\therefore -1 < 2\cos 2x < 1$

÷ 2) $\therefore -\dfrac{1}{2} < \cos 2x < \dfrac{1}{2}$

$\therefore$ **30° < $x$ < 60°** ✓ . . . *in the required interval*

***Note :*** *$\cos 2(30°) = \dfrac{1}{2}$ & $\cos 2(60°) = \cos 120° = -\dfrac{1}{2}$*

*See the graph where y lies between $-\dfrac{1}{2}$ and $\dfrac{1}{2}$ for values of x between A & B.*

## EUCLIDEAN GEOMETRY AND MEASUREMENT [50]

8.1.1 **$x$ = 96°** ✓ . . . *$\angle$ at centre = 2 × $\angle$ at circumference*

8.1.2 In $\Delta$OCB : y $= \dfrac{1}{2}(180° - 96°)$ . . . *base $\angle$ of isosceles $\Delta$; equal radii*

= **42°** ✓

8.2.1 **$\hat{F}_1$ = 90°** ✓ . . . *$\angle$ in semi-⊙*

8.2.2 **$A\hat{B}C$ = 150°** ✓ . . . *opp $\angle^s$ of cyclic quadrilateral*

8.3.1 (a) **tangent ⊥ diameter** ✓

(b) **tangents from a common point are equal** ✓

8.3.2 In right-angled $\Delta$ABC :

$AB^2 = AC^2 - BC^2$ . . . *theorem of Pythagoras*

$\therefore x^2 = 13^2 - (x + 7)^2$ . . . *AB = x above*

$\therefore x^2 = 169 - (x^2 + 14x + 49)$

$\therefore x^2 = 169 - x^2 - 14x - 49$

$\therefore 2x^2 + 14x - 120 = 0$

÷ 2) $\therefore x^2 + 7x - 60 = 0$

$\therefore (x + 12)(x - 5) = 0$

$\therefore x = 5$ . . . $x \ne -12$ $\because x > 0$

$\therefore$ **AB = 5 units** ✓

---

9.1.1 They lie on the same base DE and between the same || lines, DE and BC

9.1.2 $\dfrac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \dfrac{\cancel{\frac{1}{2}}(AD)(\cancel{h})}{\cancel{\frac{1}{2}}(DB)(\cancel{h})} = \dfrac{AD}{DB}$

$\dfrac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \dfrac{\frac{1}{2}(AE)(k)}{\frac{1}{2}(EC)(k)} = \dfrac{AE}{EC}$

But area $\Delta$DEB = area $\Delta$DEC (reason : 9.1.1)

$\therefore \dfrac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \dfrac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$

$\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC}$

M
2
EXAM MEMOS: PAPER 2

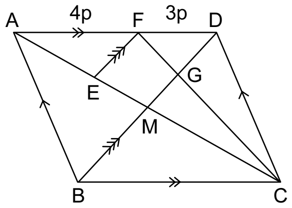
**M**
**2**

9.2.1 Let AF = 4p; then FD = 3p

In $\Delta$AMD: $\dfrac{EM}{AM} = \dfrac{FD}{AD}$ . . . *prop theorem; EF || MD*

$= \dfrac{3p}{7p}$

$= \dfrac{\mathbf{3}}{\mathbf{7}}$ ◄




9.2.2 CM = AM . . . *diagonals of a* $||^m$ *bisect one another*

$\therefore \dfrac{CM}{ME} = \dfrac{AM}{ME}$

$= \dfrac{\mathbf{7}}{\mathbf{3}}$ ◄ . . . *see 9.2.1*

9.2.3 $\dfrac{\text{area } \Delta FDC}{\text{area } \Delta ADC} = \dfrac{3}{7}$ . . . *common height* $\therefore$ *ratio of areas = ratio of bases*

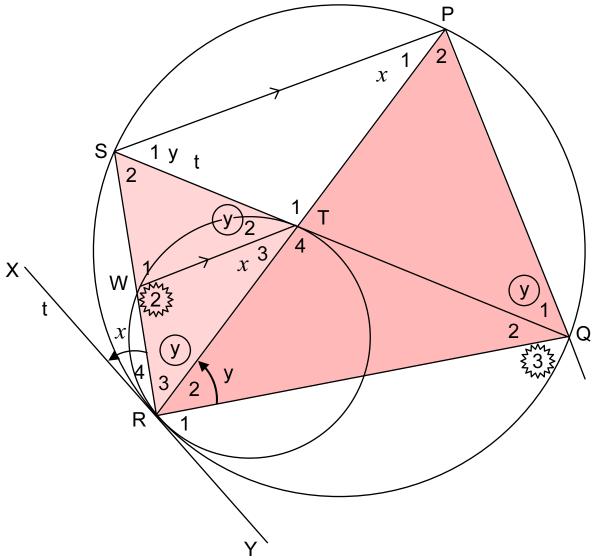
But area $\Delta$ADC = area of $\Delta$BDC . . . *the diagonals of a* $||^m$ *bisect the area of the* $||^m$

$\therefore \dfrac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \dfrac{\mathbf{3}}{\mathbf{7}}$ ◄

*or: same base DC and between same || lines AB & DC*

| | Statement | Reason |
|---|---|---|
| 10.1.1 | $\hat{T}_3 = x$ | tan-chord theorem; small ⊙ |
| 10.1.2 | $\hat{P}_1 = x$ | tan-chord theorem; large ⊙ |
| 10.1.3 | WT \|\| SP | corresponding $\angle^s$ are equal |
| 10.1.4 | $\hat{S}_1 = y$ | $\angle^s$ in the same segment; chord PQ |
| 10.1.5 | $\hat{T}_2 = y$ | alternate $\angle^s$; WT \|\| SP |

10.2 ***Hint:*** *Mark the sides on the drawing. It will then be clear what to do!*




In $\Delta$SRP: $\dfrac{RT}{RP} = \dfrac{WR}{RS}$ . . . *prop. theorem; WT || SP*

$\times$ RP) $\therefore$ $\mathbf{RT} = \dfrac{\mathbf{WR \cdot RP}}{\mathbf{RS}}$ ◄

10.3 $\hat{\mathbf{T}}_2 = \mathbf{y}$ ◄ . . . *alternate* $\angle^s$*; WT || SP* $(\hat{T}_2 = \hat{S}_1)$

$\therefore$ $\hat{\mathbf{R}}_3 = \mathbf{y}$ ◄ . . . *tan-chord theorem; tangent ST/small* ⊙

$\therefore$ $\hat{\mathbf{Q}}_1 = \mathbf{y}$ ◄ . . . *same segment theorem; chord SP*

[Only TWO required!]

10.4 $\hat{Q}_3 = R\hat{S}P$ . . . *exterior* $\angle$ *of cyclic quadrilateral*

$= \hat{\mathbf{W}}_2$ ◄ . . . *corresponding* $\angle^s$*; WT || SP*



10.5 In $\Delta^s$ RTS and RQP

(1) $\hat{R}_3 = \hat{R}_2$ . . . *both = y*

(2) $\hat{S}_2 = \hat{P}_2$ . . . *same segment theorem ; chord RQ*

It can also be proved that $\hat{Q}_2 \left(= \hat{P}_1\right) = x$

$\therefore$ that $R\hat{T}S = R\hat{Q}P = x + y$

but 2 $\angle^s$ are sufficient.

$\therefore$ **$\Delta$RTS ||| $\Delta$RQP** ◄ . . . $\angle\angle\angle$

10.6 From 10.5:

$\dfrac{RS}{RP} = \dfrac{\mathbf{RT}}{RQ}$ . . . *prop. sides of similar* $\Delta^s$ . . . ❶

& From 10.2: $\mathbf{RT} = \dfrac{WR \cdot RP}{RS}$ . . . ❷

❷ in ❶: $\therefore \dfrac{RS}{RP} = \dfrac{\mathbf{WR \cdot RP}}{\mathbf{RS} \cdot RQ}$

$\times \dfrac{RS}{RP}$ $\therefore \dfrac{RS^2}{RP^2} = \dfrac{WR \cdot \cancel{RP}}{\cancel{RS} \cdot RQ} \times \dfrac{\cancel{RS}}{\cancel{RP}}$

$= \dfrac{\mathbf{WR}}{\mathbf{RQ}}$ ◄

OR: From 10.5:

$\dfrac{RS}{RP} = \dfrac{RT}{RQ}$ . . . *prop. sides of similar* $\Delta^s$ . . . ❶

& From 10.2: $\dfrac{RT}{WR} = \dfrac{RP}{RS}$

$\therefore \dfrac{WR}{RT} = \dfrac{RS}{RP}$ . . . ❷

❶ $\times$ ❷: $\therefore \dfrac{RS}{RP} \times \dfrac{RS}{RP} = \dfrac{\cancel{RT}}{RQ} \times \dfrac{WR}{\cancel{RT}}$

$\therefore \dfrac{RS^2}{RP^2} = \dfrac{\mathbf{WR}}{\mathbf{RQ}}$ ◄

**EXAM MEMOS: PAPER 2**