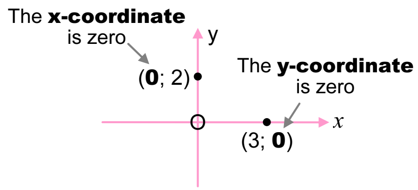
GR 11 MATHS – FUNCTIONS & GRAPHS

Graphs in general: Background Required

3 Basic facts about graphs in general

❶: Axis intercepts

Every point on the y-axis has $x = 0$.
Every point on the x-axis has y = 0.



❷: The equation

The **equation** of a graph is true for all points on the graph.

e.g. The **equation** of the y-axis is **x = 0**;
the **equation** of the x-axis is **y = 0**.

❸: Types of graph

Different **types/patterns** are indicated by various equations.

It is critical to successfully identify whether an equation is going to give you a straight line, a parabola, a hyperbola or an exponential graph.

4 types of graphs

- **Straight Lines**
- **Parabolas**
- **Hyperbolas**
- **Exponential Graphs**

The Straight Line

All forms of straight line equations:

y = mx + c ; y = mx ; y = c ; x = k

How to draw a straight line

Dual intercept method

Find the y-intercept by putting $x = 0$
Find the x-intercept by putting y = 0

Gradient-intercept method

Convert the equation into standard form y = mx + c and read off the gradient (m) and y-intercept (c)

How to determine the equation of a straight line

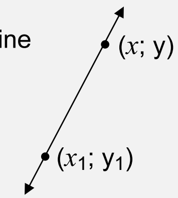
To find the equation of a straight line you will need to determine **m** and **c** in the equation y = **m**x + **c** or **a** and **q** in the equation y = **a**x + **q** *(unless the line is a 'grid line' i.e. $y = c$ or $x = k$!).*

Where does $y - y_1 = m(x - x_1)$ come from?

If $(x_1; y_1)$ is a specific (given) point and $(x; y)$ represents ***any*** other point on the line, then

$$\frac{y - y_1}{x - x_1} = m, \text{ the gradient of the line}$$

$$\times (x - x_1): \quad \therefore\ y - y_1 = m(x - x_1)$$



Apply this equation:

e.g. Given point (2; -3) and m = 4:

$$y - (-3) = 4(x - 2)$$
$$\therefore\ y + 3 = 4x - 8$$
$$\therefore\ y = 4x - 11$$

A very quick and easy method!

The Parabola

The 3 General forms are:

Turning point form	Root form	Standard form
$\mathbf{y = a(x - p)^2 + q}$	$\mathbf{y = a(x - A)(x - B)}$	$\mathbf{y = ax^2 + bx + c}$
t.p. (p; q)	*roots A & B*	*y-intercept*

How to draw a sketch of a parabola

❶ **Decide on the shape** ➨ **a > 0:** or **a < 0:**

Then, in any order, determine:

❷ **the turning point** ➨ the **axis of symmetry** & the **min/max value** of y

❸ **the axis intercepts** ➨ Put **x = 0** to find the **y-intercept**
Put **y = 0** to find the **x-intercept(s)**, *if any*

The form of the equation will determine the order: ❷ or ❸ first.

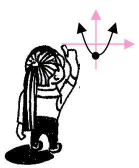
How to determine the equation of a parabola

Case 1: Given the turning point, (p; q), use $\mathbf{y = a(x - p)^2 + q}$

- Substitute **(p; q)** in the **turning point form** of the equation;
- Then substitute any other point to find a

Case 2: Given the roots, A and B, use y = a(x – A)(x – B)

- Substitute **A** and **B** in the **root form** of the equation;
- Then substitute any other point to find a.



The Hyperbola

Equation in standard form:

$$\mathbf{y = \frac{a}{x - p} + q}$$

How to draw a sketch of a hyperbola

❶ Draw the **asymptotes: x = p & y = q** *. . . note the 'new grid'*

Decide on **the quadrants on the grid**. **a > 0:** or **a < 0:**

❷ Calculate **the axis intercepts**. Put **x = 0** to find the **y-intercept**
Put **y = 0** to find the **x-intercept**

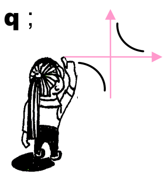
❸ Determine **the axes of symmetry**. y = x ➨ y = (x - **p**) + **q** and
y = -x ➨ y = -(x - **p**) + **q**

How to determine the equation of a hyperbola

❶: Determine p and q

Write down the equations of the asymptotes: **x = p** and **y = q** ;
then, substitute the values of **p** and **q** into the equation:

$$y = \frac{\mathbf{a}}{x - \mathbf{p}} + \mathbf{q}$$



❷: Determine a

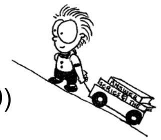
Substitute any point, (x; y), on the graph to determine the value of **a**.

Exponential Graph

Equation in standard form:

$$y = ab^{x-p} + q$$

How to sketch an exponential graph, given the equation



❶ Determine the equation of the asymptote: $\mathbf{y = q}$

❷ Determine **the axis intercepts**, i.e. y-int (x = 0) & x-int (y = 0)

❸ Interpret **the parameters**: **a** shifts up and down
p shifts left and right
b determines direction of graph

How to determine the equation of an exponential graph, given a sketch.

There are 4 parameters to be considered:

❶ Write down what you can, e.g. **q** = . . . , if the asymptote has been shown.

❷ Find the remaining parameters by substituting points.

Note: The **'zero value' of x, i.e. when x = p, eliminates b because b^0 = 1,** so always substitute this point first.

NOTES

FUNCTIONS

QUESTIONS

CHARACTERISTICS OF GRAPHS & FUNCTIONS

Examinable in both Grade 11 and Grade 12.

Identifying different types of graphs is very important!

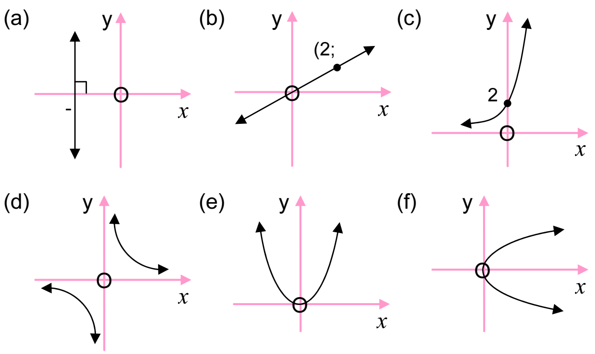
1. On a separate set of axes, for each, draw graphs of:

   1.1 $y = 4 - x^2$ 1.2 $y = \frac{1}{x - 4}$ 1.3 $y = 4 - x$

   1.4 $y = -\frac{4}{x}$ 1.5 $y = \frac{x}{4}$ 1.6 $y = 4^x$

   $(6 \times 3 = 18)$

2.1 Six graphs named (a) → (f) are sketched below.
They are followed by 10 equations. Match the graphs with the equations. Write down (a) → (f) and alongside these, the number selected from (1) → (10) that is the equation of the graph.



List of possible equations

(1) $xy = 2$ (2) $xy = -2$

(3) $y = -2$ (4) $x = -2$

(5) $y = x^2$ (6) $x = y^2$

(7) $y = 2x$ (8) $y = \frac{x}{2}$

(9) $y = 2^{x+1}$ (10) $y = 2^{x-1}$ (10)

2.2 Write down (a) → (f) and say whether the graph represents a one-to-one, a many-to-one or a one-to-many relationship between the values of x (the domain) and the values of y (the range). (6)

2.3 Which of the graphs (a) → (f) are not functions? Why not? (4)

> **Hint:** If a vertical line cuts a graph more than once, it is not a function.
> If all vertical lines will cut a graph once (only), then the graph is a function.

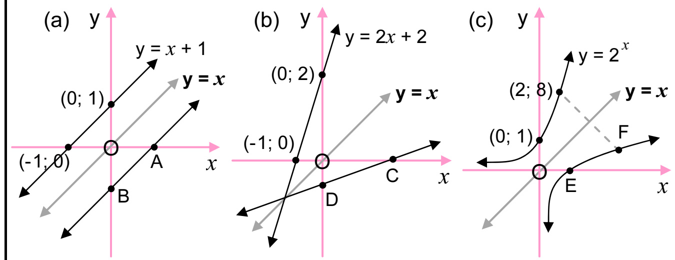
2.4 Write down the domain and range of graphs (a) → (f). (12)

2.5 Write down the equations of the asymptotes in (c) and (d). (3)

3.1 Draw sketches to show the reflections of point P(5; 2)
(a) in the y-axis (b) in the x-axis (c) in the line $y = x$ (6)

3.2 Describe the change in the coordinates in each case. (3)

3.3 Note the reflections of the graphs in the line $y = x$ in the following cases:

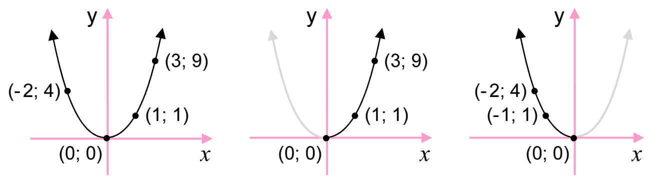


3.3.1 Write down the coordinates of the points A to F which are reflections of the given points in the line $y = x$. (6)

3.3.2 Determine the equations of the reflected graphs in (a), (b) and (c) by inspection. (3)

4.1 Draw the reflections of the following graphs in the line $y = x$. (6)

(a) $\mathbf{y = x^2}$ (b) $\mathbf{y = x^2 \,;\, x \geq 0}$ (c) $\mathbf{y = x^2 \,;\, x \leq 0}$



4.2 Are the reflections drawn in 4.1 functions? (3)

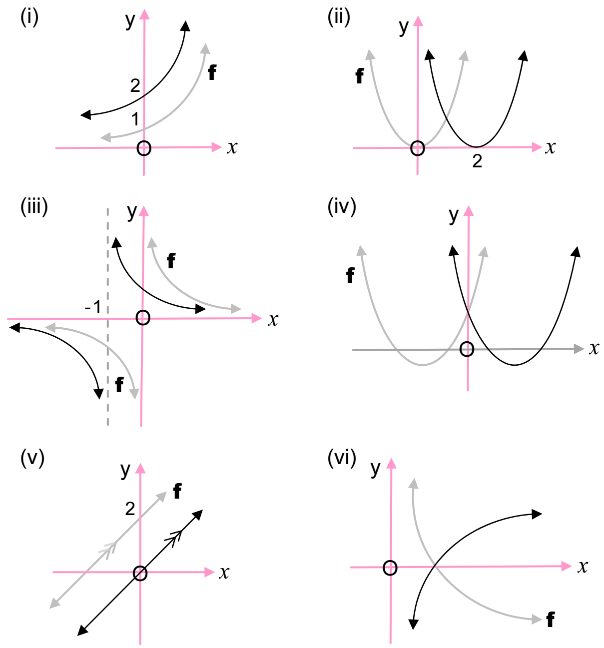
4.3 Determine the equations of the reflections drawn in 4.1. (6)

5.1 Given any function, $y = \mathbf{f}(x)$, line, hyperbola, parabola or exponential, describe the transformation required for the following images of $\mathbf{f}$ to be obtained:

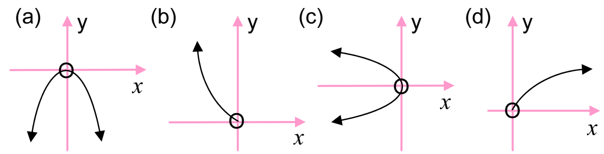
A $y = \mathbf{f}(x) + 1$ B $y = \mathbf{f}(x) - 2$ C $y = \mathbf{f}(x + 1)$

D $y = \mathbf{f}(x - 2)$ E $y = \mathbf{f}(-x)$ F $y = -\mathbf{f}(x)$ (6)

5.2 Match the black graph in each of these sketches to the equations A, B, C, . . . in 5.1. (The grey graph is the original graph $\mathbf{f}$ in each case.) (6)



6.1 Four graphs (a) → (d) are sketched below. Are any of these graphs functions? Give reasons. (2)



6.2 Match graphs (a) → (d) with the equations (1) → (6) below. Write down (a) → (d) and alongside these the number selected from (1) to (6) that is the equation of the graph.

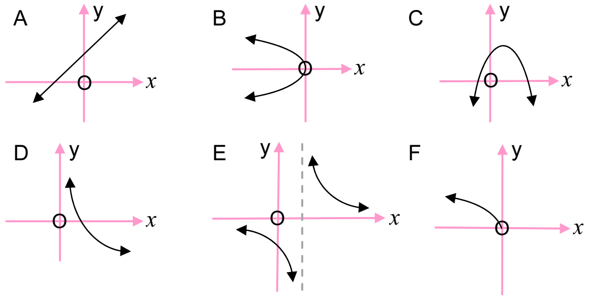
(1) $y = x^2$ (2) $y = x^2 \,;\, x \leq 0$

(3) $y = -x^2$ (4) $y = -x^2 \,;\, x \geq 0$

(5) $x = -y^2$ (6) $y = \sqrt{x} \,;\, x \geq 0$ (4)

6.3 Draw the graph defined by $y = \pm\sqrt{x}$. (2)

7. Which of the following graphs (if any) are not functions? Why? (2)



8.1 If point P(1; 8) lies on the graph of the function $\mathbf{f}(x) = 2^{x+a}$, determine the value of a. (2)

8.2 If **h** is the graph **f** moved 2 units to the right and 5 units down, write down the equation of the graph in this new position, i.e. the equation of **h**. (2)

8.3 Write down the equation of the asymptote of the shifted graph, **h**, in Question 8.2. (1)

9.1 Do the points (-1; 2) and (-1; -2) lie on a function? Why (not)? (2)

9.2 For which value(s) of x would the points $P(2x; x+6)$ and $Q(x+5; x)$ NOT lie on a function? (2)

10.1 Determine the domain of the function $y = (x-4)^{-1}$. (2)

10.2 Write down the equations of the asymptotes of this function. (4)

10.3 If the graph in 10.1 is moved 2 units left and 1 unit up, write down:

(a) the equation of the graph in this new position, and (2)

(b) the equations of the asymptotes. (2)

11. Given: $\mathbf{f}(x) = x^2 - 4x - 5$, calculate the

11.1 x-intercepts of **f**. 11.2 y-intercept of **f**. (3)(1)

11.3 coordinates of the turning point. (5)

11.4 Draw a neat sketch graph of **f**, showing clearly all intercepts on the axes and the coordinates of the turning point. (5)

11.5 What is the largest value of c for which $x^2 - 4x - 5 \geq c$ for every value of x? (2)

11.6 Use the graph to solve for x if $x^2 - 4x - 5 \geq 0$. (2)

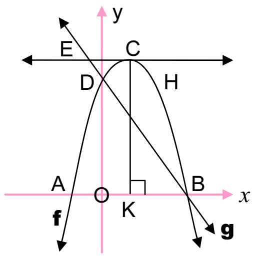
11.7 Without any further calculations, sketch the graph of $g(x) = -x^2 + 4x + 5$. (2)

11.8 For which values of x does $g(x)$ decrease as x increases? (1)

12. The accompanying sketch, not drawn to scale, shows the graphs of the functions defined by:

$\mathbf{f}(x) = -x^2 + 2x + 3$ and

$\mathbf{g}(x) = mx + c$.



12.1 Find the coordinates of C, the turning point of the curve of **f**. (4)

12.2 Determine the length of KC. (1)

12.3 If A and B are the x-intercepts of the curve of **f**, find the length of AB. (4)

12.4 Determine the equation of **g**. (2)

12.5 If H and D are mirror images of each other with respect to KC, determine the coordinates of H. (D is the y-intercept of **f** and **g**.) (2)

12.6 Through C a tangent is drawn to the curve of **f**.

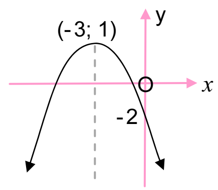
12.6.1 What is the gradient of this tangent? (1)

12.6.2 What are the coordinates of the point E where this tangent and the graph of **g** intersect? (4)

12.7 Explain how you would shift the graph of **f** so that it represents the function defined by $y = -x^2 + 2x + 5$. (2)

12.8 Use the graph and 12.3 to write down the values of x for which $f(x) > 0$. (2)

13. In this figure the graph is defined by the equation $y = ax^2 + bx + c$.



13.1 Determine the range of the graph. (2)

13.2 For what values of x is y increasing? (1)

13.3 How must the graph be shifted to:

13.3.1 just touch the x-axis? (1)

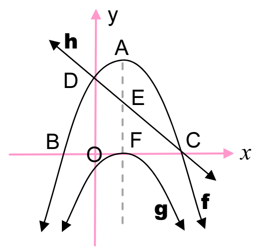
13.3.2 satisfy the equation $y = ax^2 + bx$? (1)

14. The diagram, which is not drawn to scale, shows the functions defined by

$\mathbf{f}(x) = -x^2 + 3x + 10$,

$\mathbf{g}(x) = -x^2 + 3x + r$ and

$\mathbf{h}(x) = mx + k$.



D, B, F and C are the intercepts of **f**, **g** and **h**. A is the turning point of **f**. Determine:

14.1 the equation of the axis of symmetry of **f** and **g**. (2)

14.2 the maximum value of $\mathbf{f}(x)$. (2)

14.3 the coordinates of points B, C and D. (5)

14.4 the value of r if the curve of **g** touches the x-axis. (2)

14.5 the nature of the roots of the equation $\mathbf{g}(x) = 0$ by making use of the graph. (2)

14.6 the values of m and k. (3)

14.7 the length of AE. (E is the point of intersection of **h** with the axis of symmetry of **f** and **g**.) (4)

15. Given: $\mathbf{f}(x) = 2x^2 + 4x - 6$; $\mathbf{g}(x) = 4x - 4$

15.1 On the same system of axes, draw neat sketch graphs of the functions **f** and **g**. Indicate all intercepts with the axes, as well as the coordinates of the axis-intercepts and of the turning point. (8)

15.2.1 Now, use your answer in 15.1 to write down the x- and y-intercepts and the turning point of the following functions:

$\mathbf{h}(x) = x^2 + 2x - 3$ & $\mathbf{p}(x) = -2x^2 - 4x + 6$. (6)

15.2.2 Describe how **h** and **p** relate to **f** as far as shape is concerned. (2)

15.3.1 Does the turning point of **f** lie on graph **g**? (1)

15.3.2 Use your sketch to write down the points of intersection of **f** and **g**. (2)

15.3.3 Confirm your answers in 15.3.2 by determining the points of intersection algebraically. (5)

15.3.4 Write down the solution of the equation $\mathbf{f}(x) = \mathbf{g}(x)$. (2)

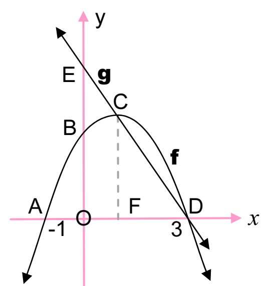
15.4 By using only your graph, determine the value(s) of d for which the equation $2x^2 + 4x + d = 0$ will have real roots. (2)

15.5 By using your graph and one other, determine the value(s) of k for which the equation $2x^2 + 4x - 6 = k$ will have no real roots. (2)

16. $\mathbf{f}(x) = ax^2 + bx + c$ is a parabola that passes through the points A(-1; 0), B(0; 6), C and D(3; 0).

C is the turning point.

$g(x) = dx + e$ is a straight line that passes through the points D, C and E.



Determine by calculation:

16.1 the values of a, b and c (6)

16.2 the length of OF and FC (6)

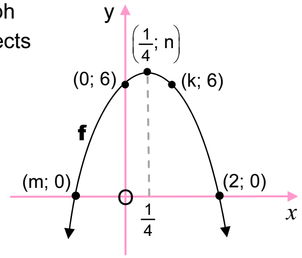
16.3 the values of d and e (4)

16.4 the length of BE (2)

16.5 the range of **f** (2)

17. The sketch represents the graph of the parabola **f**, which intersects the x-axis at (m; 0) and (2; 0).

It is further given that $\left(\frac{1}{4};\ n\right)$ is the turning point of the parabola while (0; 6) and (k; 6) are also on the curve of **f**.



Determine:

17.1 the value of k (1)

17.2 the value of m (2)

17.3 the value of n (show all the necessary calculations) (6)

18. Determine the equation of the parabola which passes through (1; 13) and has a turning point at (-1; 5). (5)

19. Given: $\mathbf{f}(x) = (x - 2)^2 - 9$

19.1 Write down the coordinates of the turning point of the graph of **f**. (1)

19.2 Calculate the x- and y-intercepts of the graph of **f**. (5)

19.3 Draw a neat sketch graph of **f** and show the intercepts of the axes as well as the coordinates of the turning point clearly. (5)

19.4 Hence write down the range of the function. (2)

19.5 For which values of x is $\mathbf{f}(x)$ decreasing? (1)

19.6 Use your graph to solve the inequality: $\mathbf{f}(x) \le 0$. (2)

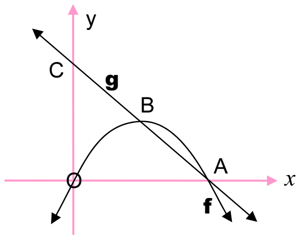
19.7 Write down the equations (in turning point form) of the graphs obtained by:

(a) shifting **f** 2 units left and 9 units up. (2)

(b) reflecting **f** in the y-axis (2)

(c) reflecting **f** in the x-axis (2)

20. Sketched (not drawn to scale) are the graphs of $\mathbf{f}(x) = ax^2 + bx + c$ and $\mathbf{g}(x) = -4x + 16$ intersecting at A and B, where O and A are x-intercepts and B is the turning point of the parabola.



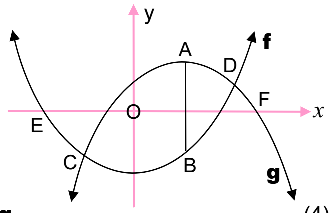
20.1 Determine the coordinates of A. (2)

20.2 Write down the equation of the axis of symmetry of **f**. (2)

20.3 Show that the coordinates of B are (2; 8). (2)

20.4 Find the values of a, b and c. (6)

21. Sketched (not drawn to scale) are the graphs of $\mathbf{f}(x) = x^2 - 9$ and $\mathbf{g}(x) = -x^2 + 4x + 5$



21.1 Determine the coordinates of A, the turning point of **g**. (4)

21.2 AB is parallel to the y-axis, with B on **f**. Calculate the length of AB. (3)

21.3 Calculate the length of EF. (6)

21.4.1 Determine an expression for the length of AB if AB || y-axis and A lies on **g** between C and D. (2)

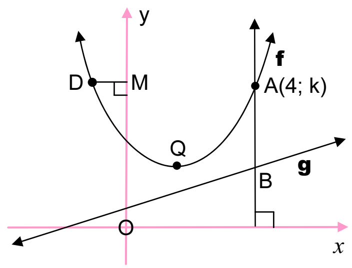
21.4.2 Hence determine the maximum possible length of AB. (4)

22. The graph is not drawn according to scale and represents the functions of **f** and **g** where:

$\mathbf{f}(x) = x^2 - 2x + 4$;

$\mathbf{g}(x) = x + 1$

DM $\perp$ y-axis and AB $\perp$ x-axis.



22.1 Write $x^2 - 2x + 4$ in the form $y = (x - p)^2 + q$ and write down the coordinates of Q, the turning point of **f**. (4)

22.2 Calculate the coordinates of D if OM = 12. (4)

22.3.1 Write down the value of k. (1)

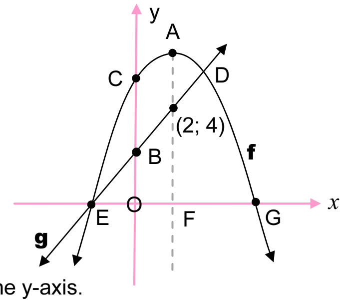
22.3.2 Determine the length of AB. (2)

22.4.1 If A were any point (x; y) on **f**, and B on **g** such that AB $\perp$ x-axis, determine, in terms of x, an expression for the vertical length AB. (2)

22.4.2 Hence, calculate the minimum length of AB. (4)

23. The sketch represents the graphs of $\mathbf{f}(x) = -x^2 + bx + c$ and $\mathbf{g}(x) = x + k$.

The point (2; 4) lies on the axis of symmetry of **f**. A is the turning point of **f** and B and C are respectively the points where **g** and **f** intersect the y-axis. The two graphs intersect at D and E.



23.1 Determine the values of k, b and c. (8)

23.2 If $\mathbf{f}(x) = -x^2 + 4x + 12$ and $\mathbf{g}(x) = x + 2$, determine

(a) the coordinates of A by completing the square. (4)

(b) the length of BC. (2)

(c) the coordinates of D, the intersection of **f** and **g**. (5)

24.1 Draw a sketch graph of the curve of $\mathbf{f}(x) = x^2 - x - 6$. Show the intercepts with the axes and the coordinates of the turning point clearly on your graph. (6)

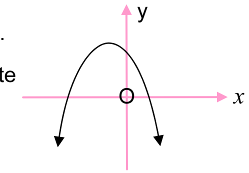
24.2 Draw on the same system of axes the straight line defined by $\mathbf{g}(x) = mx + c$ which intersects the parabola at (-2; p) and (4; q). (2)

24.3 Calculate the values of p, q, m and c. (7)

24.4 Deduce from the graph for what values of k, the equation $x^2 - x - 6 = k$ will have one negative and one positive real root. (4)

25. The graph of the parabola given by $y = ax^2 + bx + c$ is drawn alongside.

Write down the letter with appropriate values of a, b and c.



A $a < 0,\ b < 0,\ c < 0$

B $a < 0,\ b > 0,\ c < 0$

C $a < 0,\ b > 0,\ c > 0$

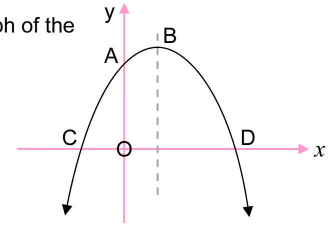
D $a < 0,\ b < 0,\ c > 0$ (2)

26. Draw a neat sketch graph of the curve of $\mathbf{f}(x) = ax^2 + bx + c$ from the following information:

- The roots of $\mathbf{f}(x) = 0$ differ by 4.
- The line of symmetry of the graph is $x = -2$.
- The range of **f** is $y \ge -2$.

Indicate the coordinates of the turning point of **f**, its axis of symmetry and the x-intercepts. (6)

27. The sketch shows the graph of the parabola given by
$y = -x^2 + 3x + 10$.
B is the turning point and A, C and D are the intercepts on the axes.



27.1 Determine the coordinates of B, C and D. (7)

27.2 Use the graph to determine the values of x for which $x^2 - 3x - 10 \geq 0$.

27.3 Explain how you would shift the graph so that $-x^2 + 3x + k = 0$ will have only one solution. (2)

27.4 Give the equation of the parabola obtained by the shifting in 27.3. (2)

27.5 For which values of p will the equation $-x^2 + 3x + 10 = p$ have 1 negative and 1 positive root? (3)

27.6 What is the average gradient of the curve between C and A? (3)

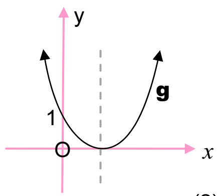
28. Given: $\mathbf{f}(x) = -x^2 - 4x - 5$

28.1 Does **f** have a maximum or a minimum value? (1)

28.2 Determine the coordinates of the turning point of **f** by completing the square. (4)

28.3 Without any further working out, decide whether or not **f** cuts the x-axis. (1)

29.1 Given: $\mathbf{g}(x) = (x - 1)^2$



29.1.1 If **g** is moved 4 units down, what will be the equation of the parabola in its new position?
Write the equation in the form $y = ax^2 + bx + c$. (2)

29.1.2 Determine the equation (in any form) of the parabola if **g** is moved 2 units left. (2)

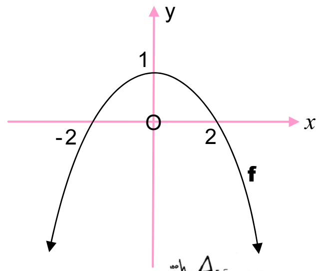
29.2 Given the function $\mathbf{f}(x) = x^2$.
Write down the equations of the functions one would get if you moved:

(a) **f** . . . . 2 units to the right
(b) **f** . . . . 1 unit to the left
(c) **f** . . . . 3 units up
(d) **f** . . . . 3 units down (4)

30. The function $y = f(x)$ is illustrated alongside.
Sketch graphs of the functions

(a) $y = \mathbf{f}(x) - 1$ (1)
(b) $y = \mathbf{f}(x - 1)$ (1)
(c) $y = -\mathbf{f}(x)$ (1)
(d) $y = \mathbf{f}(-x)$ (1)

Label each graph clearly.



(4)

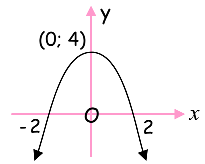
NOTES

FUNCTIONS

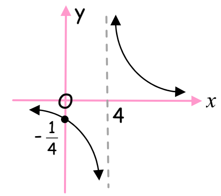
ANSWERS

CHARACTERISTICS OF GRAPHS & FUNCTIONS

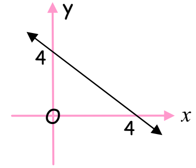
1.1



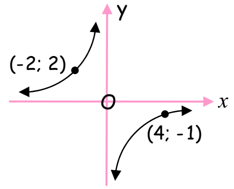
1.2



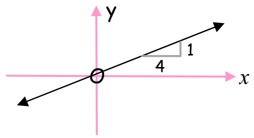
1.3



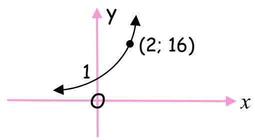
1.4



1.5



1.6



2.1 (a) (4) (b) (8) (c) (9)
(d) (1) (e) (5) (f) (6)

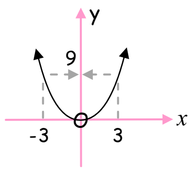
2.2 (a) **one-to-many** ◄

For one value of x (x = -2), y can have 'many' values.

(b) **one-to-one** ◄
(c) **one-to-one** ◄
(d) **one-to-one** ◄

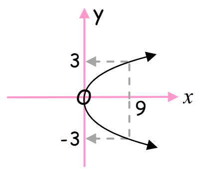
For each value of x there is only 1 y-value.

(e) **many-to-one** ◄



2 x-values give one y-value

(f) **one-to-many** ◄



1 x-value gives 2 y-values

2.3 (a) and (f) are not functions;
A graph is only a function if for each x-value there is only one y-value. In the case of (a) and (f), each x-value has more than one y-value.

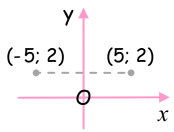
The vertical line test: A vertical line would cut these graphs more than once.

2.4

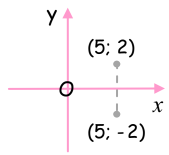
	Domain	Range
(a)	$x = -2$	$y \in \mathbb{R}$
(b)	$x \in \mathbb{R}$	$y \in \mathbb{R}$
(c)	$x \in \mathbb{R}$	$y > 0\ ;\ y \in \mathbb{R}$
(d)	$x \neq 0\ ;\ x \in \mathbb{R}$	$y \neq 0\ ;\ y \in \mathbb{R}$
(e)	$x \in \mathbb{R}$	$y \geq 0\ ;\ y \in \mathbb{R}$
(f)	$x \geq 0\ ;\ x \in \mathbb{R}$	$y \in \mathbb{R}$

2.5 (c) **y = 0** ◄ . . . *the x-axis*
(d) **y = 0** ◄ . . . *the x-axis*
& **x = 0** ◄ . . . *the y-axis*

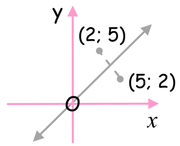
3.1 (a) reflection in the **y-axis**



(b) reflection in the **x-axis**



(c) reflection in the line **y = x**



3.2 (a) $x \to -x$, $y \to y$ (b) $x \to x$, $y \to -y$ (c) $x \to y$, $y \to x$ — i.e. x & y swop

3.3.1 A(1; 0), B(0; -1), C(2; 0), D(0; -1), E(1; 0), F(8; 2)

3.3.2 (a) $\mathbf{y = x - 1}$ ◄ (b) $\mathbf{y = \frac{1}{2}x - 1}$ ◄ (c) $\mathbf{x = 2^y}$ ◄

Note:
- In (c), understandably, x and y are swopped in the equation to get the reflection in y = x.
- Now swop x & y in the given equations in (a) & (b):

Given: (a) $y = x + 1$ (b) $y = 2x + 2$

The reflection: $x = y + 1$ $x = 2y + 2$

Now make y the subject:

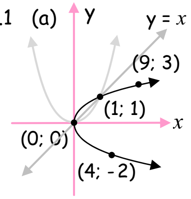
$y + 1 = x$ $\therefore\ 2y + 2 = x$

$\therefore\ \mathbf{y = x - 1}$ ◄ $\therefore\ 2y = x - 2$

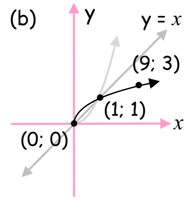
$\therefore\ \mathbf{y = \frac{x}{2} - 1}$ ◄

These are the equations determined by inspection above.

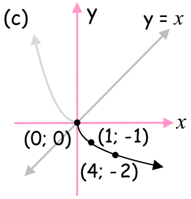
4.1 (a)



(b)



(c)



Locate the 'critical points' by swopping x and y.

4.2 (b) and (c) are, but (a) is not.

Note: Restricting the domains ($x \geq 0$ or $x \leq 0$) ensured that the reflections are functions.

4.3 (a) $x = y^2$
$\therefore\ y^2 = x$
$\therefore\ \mathbf{y = \pm\sqrt{x}}$ ◄

(b) $\mathbf{y = (+)\sqrt{x}}$ ◄ (c) $\mathbf{y = -\sqrt{x}}$ ◄

Note: The graph $y = \pm\sqrt{x}$ is split into 2 graphs:
$y = +\sqrt{x}$ and $y = -\sqrt{x}$

5.1 A translated up 1 unit
B translated down 2 units
C translated 1 unit to the left
D translated 2 units to the right
E reflection in the y-axis
F reflection in the x-axis

5.2 (i) A (ii) D (iii) C (iv) E (v) B (vi) F

6.1 All except (c), because in (c), there are 2 values of y for each x-value (except for x = 0).

Note: This graph will be cut twice by a vertical line. (All other graphs will only be cut once.)

6.2 (a) (3) (b) (2) (c) (5) *(d) (6)

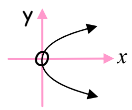
*Note: (d)
- One has to have $x \geq 0$ in $y = \sqrt{x}$ . . . $\sqrt{\text{a negative number}}$ is imaginary
- $y = +\sqrt{x}$ ➔ $y \geq 0$
- $y = \sqrt{x}$ ➔ $y^2 = x$

. . . $\therefore$ Only the 'top arm' of the parabola.

i.e. $x = y^2$, **but y ≥ 0** (& x > 0)

6.3 $y = \pm\sqrt{x}$ ➔ $y^2 = x$ and y can be + or -.

$\therefore$ The sketch:



. . . 'Both arms' of the parabola.

$x = y^2$ or $y = \pm\sqrt{x}$

7. **B is not a function** ◄

For each value of x (in the domain) there is not only one y-value. (A vertical line would cut this graph twice.)

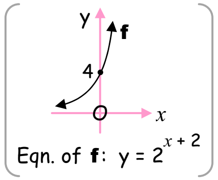
8.1 Equation of **f**: $y = 2^{x+a}$

If a point lies on a graph, its co-ords make the eqn. true!

Subst. pt (1; 8): $8 = 2^{1+a}$

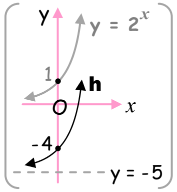
$\therefore\ 1 + a = 3$

$\therefore\ a = 2$ ◄



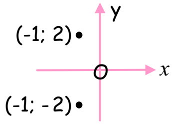
8.2 **h**: $y = 2^{x+2-2} - 5$ . . .

$\therefore\ y = 2^{x} - 5$ ◄



8.3 $y = -5$ ◄

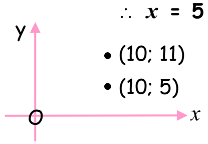
9.1 No ;
For $x = -1$,
y can be 2 or -2.



9.2 Not a function if $x_P = x_Q$

i.e. if $2x = x + 5$

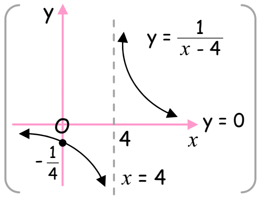
$\therefore\ x = 5$ ◄



A vertical line will cut the graph more than once.

10.1 $x \neq 4$; $x \in \mathbb{R}$ ◄

In $y = \frac{1}{x-4}$: $x - 4 \neq 0$ $\therefore\ x \neq 4$

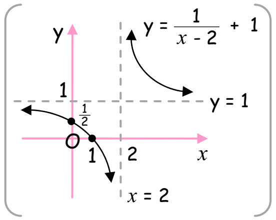


10.2 $x = 4$ & $y = 0$ ◄

10.3 (a) $y = \frac{1}{x - 4 + 2} + 1$

$\therefore\ y = \frac{1}{x-2} + 1$ ◄

(b) $x = 2$ ◄ and $y = 1$ ◄



11.1 **x-intercepts**: $f(x) = 0$ ➡ $x^2 - 4x - 5 = 0$

$\therefore\ (x - 5)(x + 1) = 0$

$\therefore\ x = 5$ or -1 ◄

11.2 **y-intercept: f(0) = -5** ◄

11.3 At the t. pt., $x = 2$

& $y = f(2)$

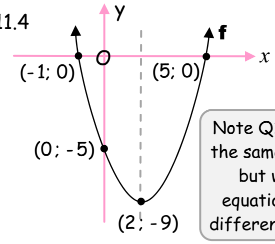
$= (2)^2 - 4(2) - 5$

$= 4 - 8 - 5$

$= -9$

$\therefore$ Turning point is **(2; -9)** ◄

11.4

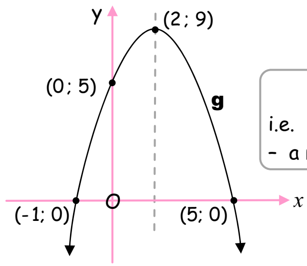


Note Q19 with the same graph but with equation in a different form.

11.5 **-9** . . . *y is ≥ -9 for all x* ◄

11.6 $x \leq -1$ or $x \geq 5$ ◄ . . . For these x's **f** lies above or on the x-axis.

11.7



$y = -x^2 + 4x + 5$

i.e. $y = -(x^2 - 4x - 5)$

– a reflection of **f** in the x-axis

11.8 **g(x) decreases for $x > 2$** ◄

12.1 At C, $x = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$

& Max. $y = -(1)^2 + 2(1) + 3$

$= -1 + 5 = 4$

$\therefore$ **C(1; 4)** ◄

OR:

$f(x) = -(x^2 - 2x + 1 - 1 - 3)$

$= -[(x-1)^2 - 4]$

$= -(x-1)^2 + 4$

12.2 **KC = 4 units** ◄

12.3 At A & B, $y = 0$

$\therefore\ -x^2 + 2x + 3 = 0$

$\therefore\ x^2 - 2x - 3 = 0$

$\therefore\ (x + 1)(x - 3) = 0$

$\therefore\ x = -1$ (at A) & $x = 3$ (at B)

$\therefore$ **AB = 4 units** ◄

12.4 Point D is (0; 3) (D on f)

$\therefore$ For g, y-intercept, $c = 3$

& gradient, $m = -\frac{3}{3} = -1$ . . . *B(3; 0)*

$\therefore$ Equation of **g**: $y = -x + 3$ ◄

12.5 **H(2; 3)** ◄

12.6.1 **0** ◄

12.6.2 At E, $y = 4$ (the equation of the tangent)

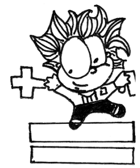
& $y = -x + 3$ (the equation of **g**)

$\therefore\ -x + 3 = 4$

$\therefore\ -x = 1$

$\therefore\ x = -1$

$\therefore$ **E(-1; 4)** ◄



12.7 You would shift it **up 2 units** ◄

12.8 $-1 < x < 3$ ◄

13.1 $y \leq 1$; $y \in \mathbb{R}$ ◄

13.2 $x < -3$ ◄

13.3.1 **down 1 unit** ◄

13.3.2 **up 2 units** ◄

14.1 $x = -\frac{b}{2a} = -\frac{3}{2(-1)} = +\frac{3}{2}$

$\therefore\ x = 1\frac{1}{2}$ ◄

14.2 **Maximum f(x)**:

$f\left(\frac{3}{2}\right) = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 10 = -\frac{9}{4} + \frac{9}{2} + 10 = 12\frac{1}{4}$ ◄

14.3 At B & C, $f(x) = 0$

$\therefore\ -x^2 + 3x + 10 = 0$

$\therefore\ x^2 - 3x - 10 = 0$

$\therefore\ (x + 2)(x - 5) = 0$

$\therefore\ x = -2$ or 5

$\therefore$ **B(-2; 0) & C(5; 0)** ◄

& at D, $x = 0$ & $f(0) = 10$

$\therefore$ **D(0; 10)** ◄

14.4 $r = -2\frac{1}{4}$ ◄ . . .

g is: **f** moved down $12\frac{1}{4}$ units

. . . *see point A to point F*

$\therefore$ y-intercept D down $12\frac{1}{4}$ units

14.5 **The roots are real, equal and rational** ◄

(and equal to $1\frac{1}{2}$!!!)

14.6 Gradient, $m = -\frac{10}{5} = -2$ ◄ ; $k = 10$ ◄

14.7 At E, $x = \frac{3}{2}$ & $h(x) = -2x + 10$

$\therefore\ h\left(\frac{3}{2}\right) = -2\left(\frac{3}{2}\right) + 10$

$= 7$ $(= y_E)$

$\therefore$ AE $= 12\frac{1}{4} - 7 = 5\frac{1}{4}$ units ◄

15.1 **f**: $y = 2x^2 + 4x - 6$

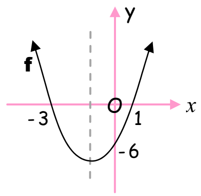
y-intercept: $y = -6$
(put $x = 0$)

x-intercepts: $2x^2 + 4x - 6 = 0$
(put $y = 0$)

$(\div 2)$ $\therefore x^2 + 2x - 3 = 0$

$\therefore (x + 3)(x - 1) = 0$

$\therefore$ **$x = -3$ or 1** ◄



Turning pt.: $x = -1$ . . . *halfway between roots*

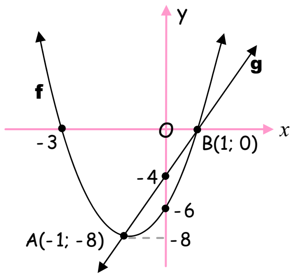
& **minimum** $y = 2(-1)^2 + 4(-1) - 6$
$= 2 - 4 - 6$
$= -8$

$\therefore$ Turning point $(-1; -8)$

g: $y = 4x - 4$

y-int.: $y = -4$
(put $x = 0$)

x-int.: $4x - 4 = 0$
(put $y = 0$) $\therefore 4x = 4$
$\therefore x = 1$



15.2.1 $f(x) = 2x^2 + 4x - 6$

$h(x) = x^2 + 2x - 3 = \frac{1}{2} f(x)$

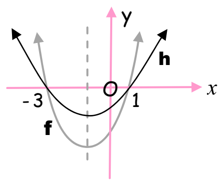
x-intercepts:	$x = -3$ and 1	. . . *the same!*
y-intercept:	$y = -3$	. . . *different*
Turning point:	$(-1; -4)$	. . . *same x; different y*

$p(x) = -2x^2 - 4x + 6 = -f(x)$

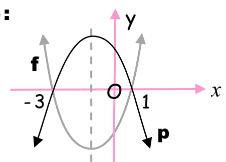
x-intercepts:	$x = -3$ and 1	. . . *the same!*
y-intercept:	$y = 6$	. . . *different*
Turning point:	$(-1; 8)$	. . . *same x; different y*

15.2.2 Sketch **h** & **p** yourself to understand these answers.

h is a wider graph ◄



p is the reflection of f in the x-axis ◄



15.3.1 **Yes** ◄ ; $g(-1) = 4(-1) - 4 = -8$

15.3.2 **A(-1; -8) and B(1; 0)** ◄

15.3.3 At the points of intersection: $2x^2 + 4x - 6 = 4x - 4$

$\therefore 2x^2 - 2 = 0$

$\therefore x^2 = 1$

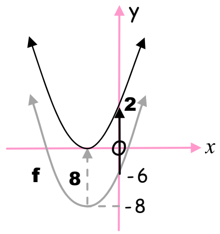
$\therefore x = \pm 1$

$\therefore$ **(-1; 8) & (1; 0)** ◄

The same as those read off the graph!

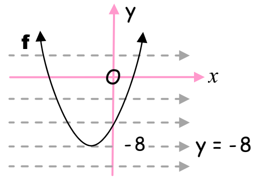
15.3.4 **$x = -1$ or $x = 1$** ◄ . . . *read at A & B, the points of intersection of f and g*

15.4 **y-int., $d \leq 2$** ◄



A translation 8 units up is the highest f can go while still having (real) roots.

15.5 **$k < -8$** ◄



Lines $y = k$ (|| to the x-axis) will not cut f at all if drawn lower than $y = -8$.
$\therefore$ No (real) roots.

16.1 Equation of parabola: $y = a(x + 1)(x - 3)$

Substitute (0; 6): $6 = a(1)(-3)$

$\therefore 6 = -3a$

$\div(-3)$ $\therefore a = -2$

$\therefore$ Equation is $y = -2(x^2 - 2x - 3)$

$\therefore y = -2x^2 + 4x + 6$

$\therefore$ **$a = -2$; $b = 4$ & $c = 6$** ◄

16.2 At C, $x = 1$ . . . *halfway between -1 and 3*

and $y = -2 + 4 + 6 = 8$

$\therefore$ Point C is (1; 8)

OF = 1 unit & FC = 8 units ◄

16.3 d = gradient of DE $= -\frac{8}{2}$

$\therefore$ **$d = -4$** ◄

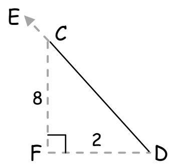
Substitute (3; 0) & $d = -4$

in $y = dx + e$:

$\therefore 0 = (-4)(3) + e$

$\therefore 12 = e$

$\therefore$ **$e = 12$** ◄



16.4 **BE = OE - OB = 12 - 6 = 6 units** ◄

16.5 Range of **f**: **$y \leq 8$; $y \in \mathbb{R}$** ◄

17.1 **$k = \frac{1}{2}$** ◄ (by symmetry) 17.2 **$m = -1\frac{1}{2}$** ◄ (by symmetry)

17.3 Equation of **f**: $y = a\left(x + 1\frac{1}{2}\right)(x - 2)$

Substitute (0; 6): $\therefore 6 = a\left(\frac{3}{2}\right)(-2)$

$\therefore 6 = -3a$

$\div(-3)$ $\therefore -2 = a$

$\therefore$ The equation is $y = -2\left(x^2 - \frac{1}{2}x - 3\right)$

$\therefore y = -2x^2 + x + 6$

$\left(\frac{1}{4}; n\right)$ on **f**: $\therefore$ **n** $= -2\left(\frac{1}{4}\right)^2 + \frac{1}{4} + 6$

$= -2\left(\frac{1}{16}\right) + 6\frac{1}{4}$

$= 6\frac{1}{8}$ ◄

18. Equation of parabola: $y = a(x + 1)^2 + 5$

Substitute (1; 13): $13 = a(1 + 1)^2 + 5$

$\therefore 8 = 4a$

$\therefore a = 2$

$\therefore$ Equation is: $y = 2(x^2 + 2x + 1) + 5$

$\therefore y = 2x^2 + 4x + 2 + 5$

$\therefore$ **$y = 2x^2 + 4x + 7$** ◄

19.1 **(2; -9)** ◄

19.2 $y = (x - 2)^2 - 9$

x-intercepts:

$(x - 2)^2 - 9 = 0$ . . . *$y = 0$*

$\therefore (x - 2)^2 = 9$

$\therefore x - 2 = \pm 3$

$\therefore x = 2 \pm 3$

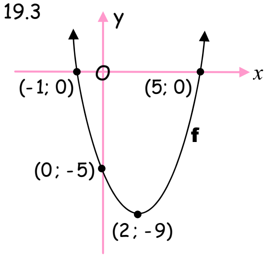
$\therefore$ **$x = 5$ or -1** ◄

y-intercept:

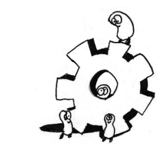
$y = (0 - 2)^2 - 9$ . . . *$x = 0$*

$\therefore$ **$y = -5$** ◄

19.3



Q11 has the same graph but has the equation in a different form.
Note the difference.



19.4 **$y \geq -9$** ◄

19.5 **$x < 2$** ◄

19.6 **f**$(x) = 0$ on the x-axis and is positive above the x-axis.

$\therefore$ **f**$(x) \leq 0$ ➡ **$-1 \leq x \leq 5$** ◄

19.7 (a) $y = (x - 2 \mathbf{+ 2})^2 - 9 \mathbf{+ 9}$

$\therefore \; \mathbf{y = x^2}$ ◄

(b) $y = (-x - 2)^2 - 9$ . . . reflection in the y-axis ➡ $x \to -x$; y unchanged

$\therefore \; \mathbf{y = (x + 2)^2 - 9}$ ◄

(c) $-y = (x - 2)^2 - 9$ . . . reflection in the x-axis ➡ $y \to -y$; x unchanged

$\therefore \; \mathbf{y = -(x - 2)^2 + 9}$ ◄

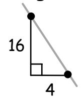
20.1 At A, $g(x) = 0$, i.e. $-4x + 16 = 0$

$\therefore \; -4x = -16$

$\therefore \; x = 4$

$\therefore$ **A(4; 0)** ◄ (y = 0 on the x-axis)

or by inspection, using the y-intercept & the gradient



20.2 $\mathbf{x = 2}$ ◄ . . . *halfway between 0 & 4*

20.3 B on graph g ➡ $y_B = -4(2) + 16$ . . . ($x_B = 2$ – see 20.2)

$= 8$

$\therefore$ **B(2; 8)** ◄

20.4 Roots 0 & 4 ➡ equation is:

$y = ax(x - 4)$ . . . $y = a(x - 0)(x - 4)$

Subst. (2; 8) $\therefore \; 8 = a(2)(-2)$

$\therefore \; 8 = -4a$

$\therefore \; -2 = a$

$\therefore \; y = -2x^2 + 8x$

$\therefore$ **a = -2; b = 8; c = 0** ◄

OR: Turning point (2; 8) ➡ eqn. is: $y = a(x - 2)^2 + 8$

Substitute (0; 0): $0 = a(-2)^2 + 8$

$\therefore \; -8 = 4a$

$\therefore \; a = -2$

$\therefore$ equation is $y = -2(x^2 - 4x + 4) + 8$

$\therefore \; y = -2x^2 + 8x - 8 + 8$

$\therefore \; y = -2x^2 + 8x$, etc.

21.1 At A, $x = -\dfrac{b}{2a}$

$= -\dfrac{4}{2(-1)}$

$= 2$

OR:
$g(x) = -(x^2 - 4x + 4 - 4 - 5)$
$= -[(x - 2)^2 - 9]$
$= -(x - 2)^2 + 9$

& $\mathbf{g}(2) = -(2)^2 + 4(2) + 5 = -4 + 8 + 5 = 9$

$\therefore$ **A(2; 9)** ◄

21.2 $x_B = x_A = 2$ . . . *AB ⊥ x-axis*

& $y_B = \mathbf{f}(2) = (2)^2 - 9 = -5$ . . . *B on* **f**

$\therefore$ **AB** $= y_A - y_B = 9 - (-5) =$ **14 units** ◄

21.3 At E, $\mathbf{f}(x) = 0$, i.e. $x^2 - 9 = 0$

$\therefore \; x^2 = 9$

$\therefore \; x = -3$ . . . *$x < 0$ at E!*

& At F, $\mathbf{g}(x) = 0$, i.e. $-x^2 + 4x + 5 = 0$

$\therefore \; x^2 - 4x - 5 = 0$

$\therefore \; (x - 5)(x + 1) = 0$

$\therefore \; x = 5$ . . . *$x > 0$ at F!*

$\therefore$ **EF** $= 3 + 5 =$ **8 units** ◄ $\quad x_F - x_E = 5 - (-3) = 8$

21.4.1 AB $= \mathbf{g}(x) - \mathbf{f}(x)$

$= (-x^2 + 4x + 5) - (x^2 - 9)$

$= \mathbf{-2x^2 + 4x + 14}$ ◄

21.4.2 Maximum AB occurs when $x = -\dfrac{b}{2a} = -\dfrac{4}{2(-2)} = 1$

$\therefore$ Maximum AB $= -2(1)^2 + 4(1) + 14$

$=$ **16 units** ◄

[OR: Complete the square]

Later (in Topic 8) you also have the choice to use Calculus.

22.1 **METHOD 1:**

By completing the square

$y = x^2 - 2x + 4$

$= \underbrace{x^2 - 2x \mathbf{+ 1}} + 4 \mathbf{- 1}$

$= (x - 1)^2 + 3$

$\therefore$ **Q(1; 3)** ◄

METHOD 2:

Formula for axis of symmetry

At Q, $x = -\dfrac{b}{2a} = -\dfrac{-2}{2(1)} = 1$

& $y = x^2 - 2x + 4$

$= 1 - 2 + 4 = 3$

$\therefore$ **Q(1; 3)** ◄

YOU MUST KNOW BOTH METHODS!

22.2 OM = 12 ➡ $y_D = 12$

$\therefore \; x^2 - 2x + 4 = 12$

$\therefore \; x^2 - 2x - 8 = 0$

$\therefore \; (x - 4)(x + 2) = 0$

$\therefore \; x = -2$ at D . . . *$x = 4$ at A!!!*

$\therefore$ **D(-2; 12)** ◄

22.3.1 $\therefore$ A(4; 12) . . . *$x = 4$ at A!!!*

$\therefore$ **k = 12** ◄

22.3.2 $y_B = 4 + 1 = 5$ . . . *$y = x + 1$ at B!*

$\therefore$ **AB** $=$ $\mathbf{y_A - y_B}$ . . . **VERTICAL LENGTH FORMULA!**

$= 12 - 5 =$ **7 units** ◄

22.4.1 For any x if A is the point (x; y):

AB $= y_A - y_B$

$= (x^2 - 2x + 4) - (x + 1)$

$= x^2 - 3x + 3$

22.4.2 **METHOD 1:**

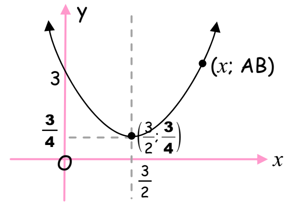
Minimum value of $x^2 - 3x + 3$ occurs when

$x = -\dfrac{b}{2a} = -\dfrac{-3}{2(1)} = \dfrac{3}{2}$

$\therefore$ Minimum value of $x^2 - 3x + 3$, i.e. of AB

$= \left(\dfrac{3}{2}\right)^2 - 3\left(\dfrac{3}{2}\right) + 3 = \dfrac{9}{4} - \dfrac{9}{2} + 3 = -2\dfrac{1}{4} + 3$

$= \dfrac{3}{4}$ **unit** ◄



Note: The length of AB is represented by this parabola.

This 'picture' shows all the possible values that AB can have – the 'lowest' value (i.e. the lowest y-coordinate) is $\frac{3}{2}$.

METHOD 2: *By completion of square . . .*

$x^2 - 3x + 3$

$= x^2 - 3x + \left(\mathbf{\dfrac{3}{2}}\right)^{\mathbf{2}} + 3 - \mathbf{\dfrac{9}{4}}$

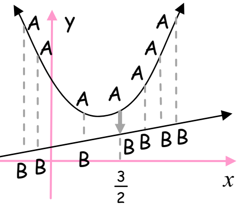
$= \left(x - \dfrac{3}{2}\right)^2 + \dfrac{3}{4}$

$\therefore$ **Minimum value** $= \mathbf{\dfrac{3}{4}}$ ◄ . . . *when $x = \frac{3}{2}$*

YOU MUST KNOW BOTH METHODS!

This 'picture' shows the varying lengths of the vertical length AB on the original sketch – it is clear that its shortest length is at $x = \frac{3}{2}$.

not at the t. pt. of **f**!



This minimum length is then $\frac{3}{4}$ unit.

23.1 Equation of ED: $y = x + k$

Subst. (2; 4) ➡ $4 = 2 + k$

$\therefore$ **k = 2** ◄

$\therefore$ Point E(-2; 0), *by inspection* & Point G(6; 0), *by symmetry*

$\therefore$ Equation of **f**: $y = a(x + 2)(x - 6)$ . . . *roots -2 and 6*

But $a = -1$ . . . *given*

$\therefore \; y = -(x^2 - 4x - 12)$

$\therefore \; y = -x^2 + 4x + 12$

$\therefore$ **b = 4 & c = 12** ◄

23.2 (a) $\mathbf{f}(x) = -x^2 + 4x + 12$
$= -[x^2 - 4x - 12]$
$= -[\underbrace{x^2 - 4x + 2^2}_{} - 4 - 12]$
$= -[\ (x-2)^2\ - 16]$
$= -(x-2)^2 + 16$ ... *the turning point form of the quadratic expression*

$\therefore$ **A(2; 16)** ◄

(b) BC $= y_C - y_B = 12 - 2 =$ **10 units** ◄

(c) At D: $y = -x^2 + 4x + 12$ and $y = x + 2$
$\therefore\ x + 2 = -x^2 + 4x + 12$
$\therefore\ x^2 - 3x - 10 = 0$
$\therefore\ (x-5)(x+2) = 0$
$\therefore\ x = 5$ at D ... *$x = -2$ at E (we already have!)*
& $y = 5 + 2 = 7$ ... *D on* **g**

$\therefore$ **D(5; 7)** ◄

24.1 & 24.2 $\mathbf{f}(x) = x^2 - x - 6$

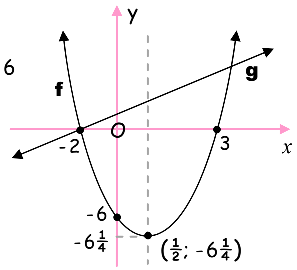
y-int.: $y = -6$
$(x = 0)$

x-ints.: $x^2 - x - 6 = 0$
$(y = 0)$ $\therefore (x-3)(x+2) = 0$
$\therefore\ x = 3$ or -2

Turning point:
$x = \frac{-2+3}{2}$ ... *ave. of the roots!*
$= \frac{1}{2}$

& **minimum:** $y = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 6$
$= -6\frac{1}{4}$

$\therefore$ Turning point: $\left(\frac{1}{2}; -6\frac{1}{4}\right)$

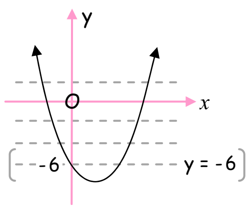


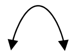
24.3 (-2; p) on **f** ➡ **p = 0** ◄ ... *see (-2; 0)!*
(4; q) on **f** ➡ **q** $= 4^2 - 4 - 6 =$ **6** ◄
$\therefore$ Gradient, **m** $= \frac{6-0}{4-(-2)} = \frac{6}{6} =$ **1** ◄

$\therefore$ y-intercept: **c = 2** ◄ ... *by inspection*

24.4 **k > -6** ◄

All lines $y = k$, parallel to the x-axis, will cut f for one negative and one positive value of x, IF $k > -6$!

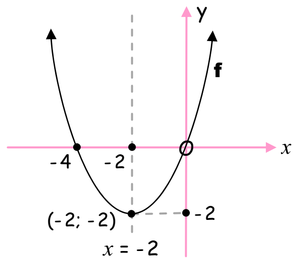


25. a < 0 ➡ 

a < 0 & b < 0 ➡ $x = -\frac{b}{2a} = -\frac{-}{-} = -$

$\therefore$ Axis of symmetry left of y-axis:
c > 0 ➡ y-intercept is positive $\therefore$ **D** ◄

26.



27.1 At C & D, $y = 0$
$\therefore\ 0 = -x^2 + 3x + 10$
$\therefore\ x^2 - 3x - 10 = 0$
$\therefore\ (x-5)(x+2) = 0$
$\therefore$ **C(-2; 0) & D(5; 0)** ◄

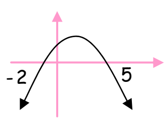
At B, $x = \frac{-2+5}{2}$ ... 'average of roots'
or: $x = -\frac{b}{2a} = -\frac{3}{2(-1)}$

$\therefore\ x = \frac{3}{2}$

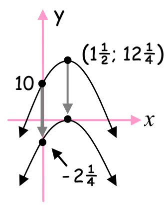
& $y = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 10$
$= -\frac{9}{4} + \frac{9}{2} + 10$
$= -2\frac{1}{4} + 4\frac{1}{2} + 10$
$= 12\frac{1}{4}$

$\therefore$ B$\left(\frac{3}{2}; 12\frac{1}{4}\right)$ ◄

27.2 $x^2 - 3x - 10 \geq 0$
➡ $-x^2 + 3x + 10 \leq 0$
$\therefore$ **$x \leq -2$ or $x \geq 5$** ◄

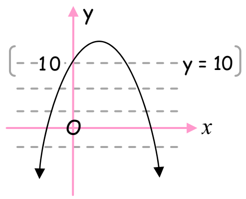


27.3 You would shift the graph **down $12\frac{1}{4}$ units** ◄
... $\left(\text{then } k = -2\frac{1}{4}\right)$



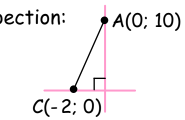
27.4 **$y = -x^2 + 3x - 2\frac{1}{4}$** ◄

27.5 **p < 10** ◄



27.6 Ave. gradient between C & A
$= \frac{y_A - y_C}{x_A - x_C} = \frac{10-0}{0-(-2)} = \frac{10}{2}$
$=$ **5** ◄

or, by inspection:



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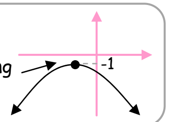
28. $\mathbf{f}(x) = -x^2 - 4x - 5$

28.1 **a maximum** ◄

28.2 $\mathbf{f}(x) = -[x^2 + 4x + 4 - 4 + 5]$
$= -[(x+2)^2 + 1]$
$= -(x+2)^2 - 1$

28.3 **No, it does not** ◄

The **maximum** *value is negative $\therefore$ whole graph under the x-axis*



29.1.1 Then $g(x) = (x-1)^2 - 4$
$\therefore\ y = x^2 - 2x + 1 - 4$
$\therefore$ **$y = x^2 - 2x - 3$** ◄

29.1.2 $y = (x - 1 \mathbf{+ 2})^2$
$\therefore$ **$y = (x+1)^2$** ◄ ... [OR: **$y = x^2 + 2x + 1$** ◄]

29.2 (a) **$y = (x-2)^2$** ◄ (b) **$y = (x+1)^2$** ◄
(c) **$y = x^2 + 3$** ◄ (d) **$y = x^2 - 3$** ◄

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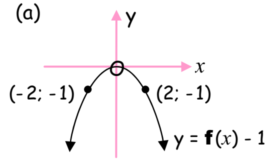
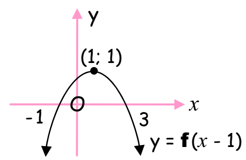
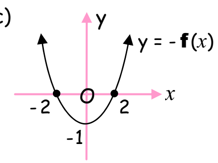
30. (a)



(b)



(c)



(d) Same as the given graph.
(Reflection in the y-axis).