GR 12 MATHS

November 2015

Examination Papers 1 & 2

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GR 12 MATHS - EXAM QUESTION PAPERS

NATIONAL NOV 2015 PAPER 1

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

ALGEBRA AND EQUATIONS AND INEQUALITIES [26]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 9x + 20 = 0$ (3)

1.1.2 $3x^2 + 5x = 4$ (correct to TWO decimal places.) (4)

1.1.3 $2x^{\frac{-5}{3}} = 64$ . . . *No calculator!* (4)

1.1.4 $\sqrt{2 - x} = x - 2$ (4)

1.1.5 $x^2 + 7x < 0$ (3)

1.2 Given: $(3x - y)^2 + (x - 5)^2 = 0$
Solve for x and y. (4)

1.3 For which value(s) of k will the equation $x^2 + x = k$ have no real roots? (4) [26]

PATTERNS AND SEQUENCES [22]

QUESTION 2 . . . *Geometric Sequence*

The following geometric sequence is given:

10 ; 5 ; 2,5 ; 1,25 ; . . .

2.1 Calculate the value of the 5th term, T_5, of this sequence. (2)

2.2 Determine the nth term, T_n, in terms of n. (2)

2.3 Explain how you know that the infinite series $10 + 5 + 2{,}5 + 1{,}25 + \ldots$ converges. (2)

2.4 Determine $S_\infty - S_n$ in the form ab^n, where S_n is the sum of the first n terms of the sequence. (4) [10]

QUESTION 3 . . . *Arithmetic Sequence*

Consider the series: $S_n = -3 + 5 + 13 + 21 + \ldots$ to n terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write S_n in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4 Another sequence is defined as:

$Q_1 = -6$
$Q_2 = -6 - 3$
$Q_3 = -6 - 3 + 5$
$Q_4 = -6 - 3 + 5 + 13$
$Q_5 = -6 - 3 + 5 + 13 + 21$

3.4.1 Write down a numerical expression for Q_6. (2)

3.4.2 Calculate the **value** of Q_{129}. (3) [12]

FUNCTIONS AND GRAPHS [37]

QUESTION 4

Given: $f(x) = 2^{x+1} - 8$

4.1 Write down the equation of the asymptote of f. (1)

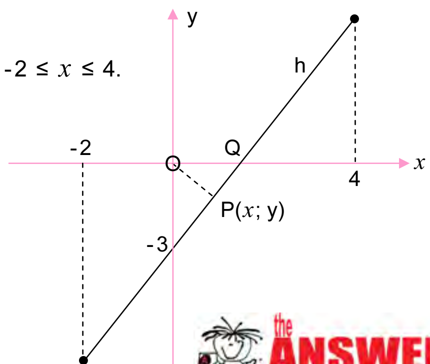
4.2 Sketch the graph of f. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

4.3 The graph of g is obtained by reflecting the graph of f in the y-axis. Write down the equation of g. (1) [6]

QUESTION 5

Given:
$h(x) = 2x - 3$ for $-2 \leq x \leq 4$.

The x-intercept of h is at Q.



5.1 Determine the coordinates of Q. (2)

5.2 Write down the domain of h^{-1}. (3)

5.3 Sketch the graph of h^{-1}, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x) = h^{-1}(x)$? (3)

5.5 $P(x; y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5)

5.6 Given: $h(x) = f'(x)$ where f is a function defined for $-2 \leq x \leq 4$.

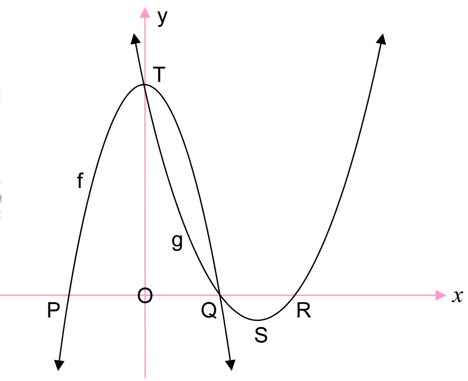
5.6.1 Explain why f has a local minimum. (2)

5.6.2 Write down the value of the maximum gradient of the tangent to the graph of f. (1) [19]

QUESTION 6

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

6.2 The function defined as $y = \frac{a}{x + p} + q$ has the following properties:

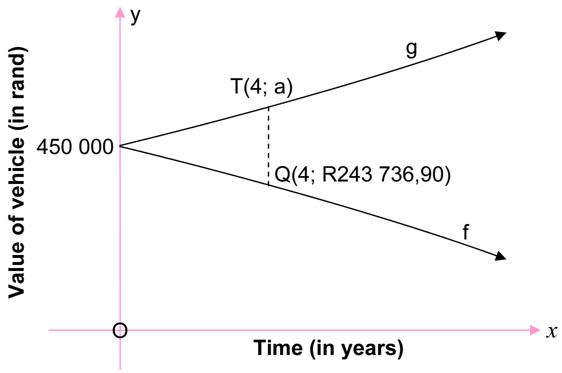
- The domain is $x \in \mathbb{R}$, $x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in \mathbb{R}$, $x \neq -2$.

Draw a neat sketch graph of this function.
Your sketch must include the asymptotes, if any. (4) [12]

FINANCE, GROWTH AND DECAY [13]

QUESTION 7

The graph of f shows the book value of a vehicle x years after the time Joe bought it. The graph of g shows the cost price of a similar new vehicle x years later.



7.1 How much did Joe pay for the vehicle? (1)

7.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought. (4)

7.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a. (3)

7.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13^{th} month and the last payment will be made at the end of the 48^{th} month. The sinking fund earns interest at a rate of 6,2% p.a., compounded monthly.

Calculate the monthly payment to the fund. (5) [13]

DIFFERENTIAL CALCULUS [35]

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ 8.2.2 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$ (3)(3)

[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$.
P and Q(2; 10) are the turning points of h.
The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

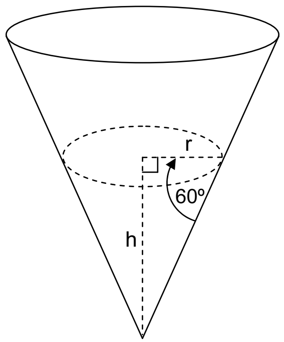
9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

9.4 Explain the significance of the change discussed in Question 9.3 with respect to the graph h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4) [17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60º, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5) [7]

PROBABILITY [17]

QUESTION 11

11.1 For two events, A and B, it is given that:

P(A) = 0,2
P(B) = 0,63
P(A and B) = 0,126

Are the events A and B independent?
Justify your answer with appropriate calculations. (3)

11.2 The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters.

How many different arrangements are possible if:

11.2.1 Letters may be repeated (2)

11.2.2 Letters may not be repeated (2)

11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)

11.3 There are t orange balls and 2 yellow balls in a bag.

Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to the bag.

It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6) [17]

TOTAL: 150

NATIONAL NOV 2015 PAPER 2

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

STATISTICS [20]

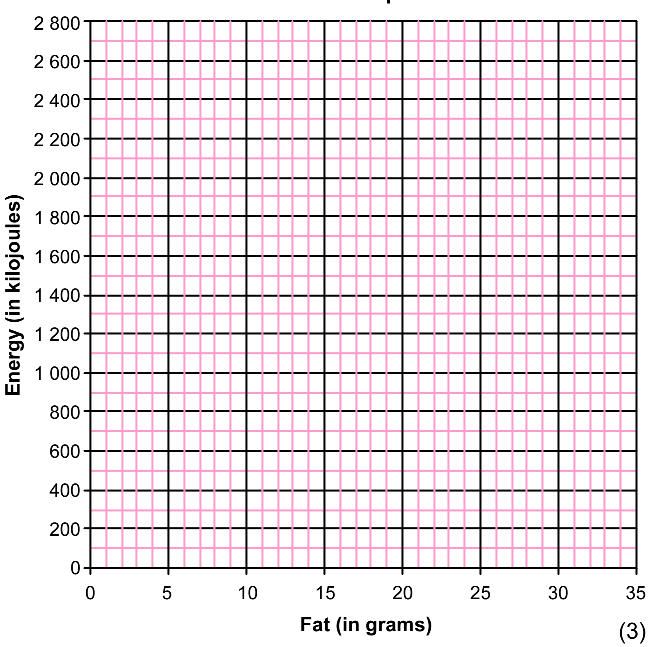
QUESTION 1

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in g)	9	14	25	8	12	31	28	14	29	20
Energy (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 Represent the information above in a scatter plot on the grid provided below.

Scatter plot



(3)

1.2 The equation of the least squares regression line is $\hat{y} = 154{,}60 + 77{,}13x$.

1.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide.

Give your answer rounded off to the nearest 100 kJ. (2)

1.2.2 Draw the least squares regression line on the scatter plot drawn for Question 1.1. (2)

1.3 Identify an outlier in the data set. (1)

1.4 Calculate the value of the correlation coefficient. (2)

1.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy. (1) [11]

QUESTION 2

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Calculate the mean of the data. (2)

2.2 Determine the median of the data. (2)

2.3 Determine the standard deviation of the data. (2)

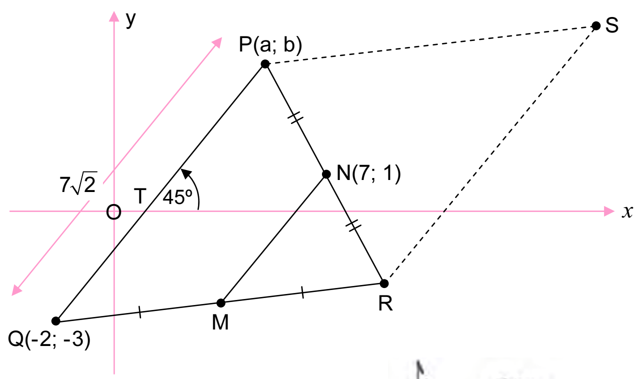
2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3) [9]

ANALYTICAL GEOMETRY [38]

QUESTION 3

In the diagram below, the line joining Q(-2; -3) and P(a; b), a and $b > 0$, makes an angle of 45° with the positive x-axis.

QP = $7\sqrt{2}$ units. N(7; 1) is the midpoint of PR and M is the midpoint of QR.



Determine:

3.1 The gradient of PQ. (2)

3.2 The equation of MN in the form $y = mx + c$ and give reasons. (4)

3.3 The length of MN. (2)

Given, furthermore that PQRS, in this order, is a parallelogram, determine:

3.4 The length of RS. (1)

3.5 The coordinates of S. (3)

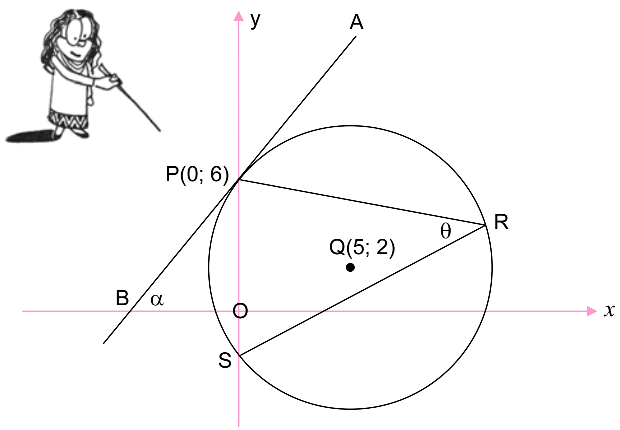
3.6 The coordinates of P. (6) [18]

Q 2

EXAM PAPERS: PAPER 2

QUESTION 4

In the diagram below, Q(5; 2) is the centre of a circle that intersects the y-axis at P(0; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

4.4 Calculate the size of α. (2)

4.5 Calculate, with reasons, the size of θ. (4)

4.6 Calculate the area of ΔPQS. (4) [20]

▸ TRIGONOMETRY [42]

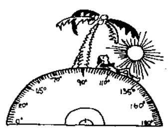
QUESTION 5

5.1 Given that $\sin 23^\circ = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203^\circ$ (2)

5.1.2 $\cos 23^\circ$ (3)

5.1.3 $\tan(-23^\circ)$ (2)



5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x)\,.\,\cos(90^\circ + x)}{\sin(30^\circ - x)\,.\,\cos x \;+\; \cos(30^\circ - x)\,.\,\sin x}$$ (6)

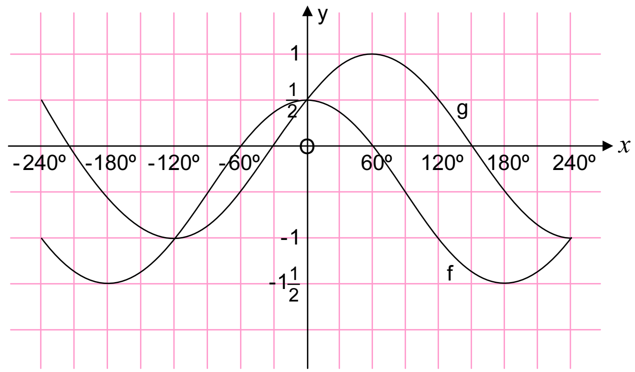
5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5) [24]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \le x \le 240^\circ$.

The graphs intersect at $\left(0^\circ; \frac{1}{2}\right)$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.



6.1 Determine the values of p and q. (4)

6.2 Determine the values of x in the interval $-240^\circ \le x \le 240^\circ$ for which $f(x) > g(x)$. (2)

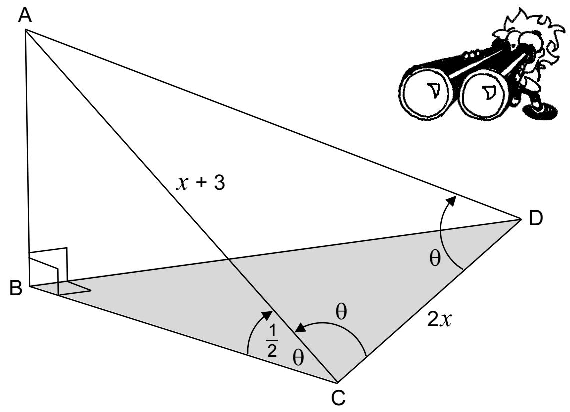
6.3 Describe a transformation that the graph of g has to undergo to form the graph of h, where $h(x) = -\cos x$. (2) [8]

QUESTION 7

A corner of a rectangular block of wood is cut off and shown in the diagram below.

The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$.

Also $A\hat{C}B = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.



7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

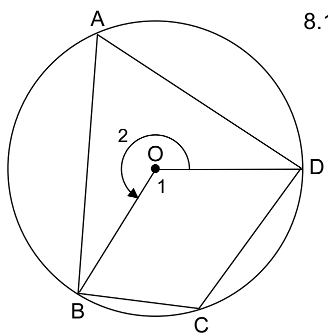
7.2 Prove that $\cos\theta = \dfrac{x}{x + 3}$. (4)

7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5) [10]

▸ EUCLIDEAN GEOMETRY AND MEASUREMENT [50]

QUESTION 8

8.1 In the diagram below, cyclic quadrilateral ABCD is drawn in the circle with centre O.



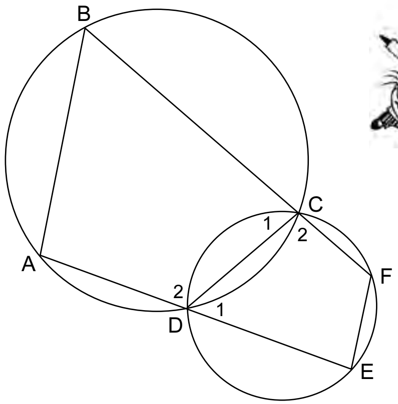
8.1.1 Complete the following statement:

The angle subtended by a chord at the centre of a circle is . . . . the angle subtended by the same chord at the circumference of the circle. (1)

8.1.2 Use Question 8.1.1 to prove that

$\hat{A} + \hat{C} = 180°$ (3)

8.2 In the diagram below, CD is a common chord of the two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.



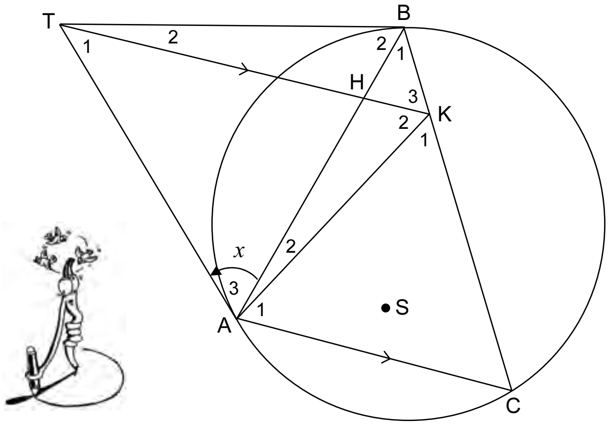
Prove that EF || AB. (5) [9]

QUESTION 9

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle.

The straight line THK is parallel to AC with H on BA and K on BC.

AK is drawn. Let $\hat{A}_3 = x$.



9.1 Prove that $\hat{K}_3 = x$. (4)

9.2 Prove that AKBT is a cyclic quadrilateral. (2)

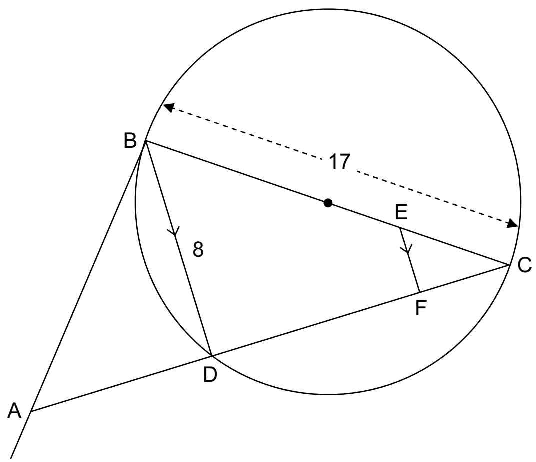
9.3 Prove that TK bisects $A\hat{K}B$. (4)

9.4 Prove that TA is a tangent to the circle passing through the points A, K and H. (2)

9.5 S is a point in the circle such that the points A, S, K and B are concyclic. Explain why A, S, B and T are also concyclic. (2) [14]

QUESTION 10

In the diagram below, BC = 17 units, where BC is a diameter of the circle. The length of chord BD is 8 units. The tangent at B meets CD produced at A.



10.1 Calculate, with reasons, the length of DC. (3)

10.2 E is a point on BC such that BE : EC = 3 : 1. EF is parallel to BD with F on DC.

10.2.1 Calculate, with reasons, the length of CF. (3)

10.2.2 Prove that ΔBAC ||| ΔFEC. (5)

10.2.3 Calculate the length of AC. (4)

10.2.4 Write down, giving reasons, the radius of the circle passing through points A, B and C. (2) [17]

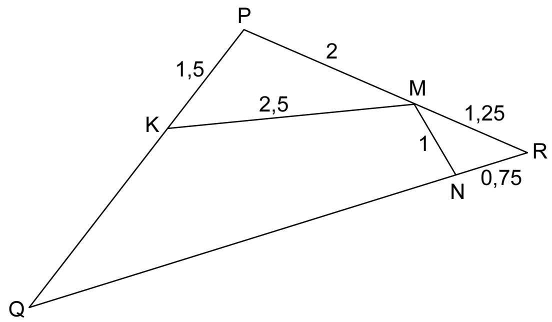
QUESTION 11

11.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are . . . . (1)

11.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ΔPQR.

KP = 1,5 ; PM = 2 ; KM = 2,5 ; MN = 1 ; MR = 1,25 and NR = 0,75.



11.2.1 Prove that ΔKPM ||| ΔRNM. (3)

11.2.2 Determine the length of NQ. (6) [10]

TOTAL: 150

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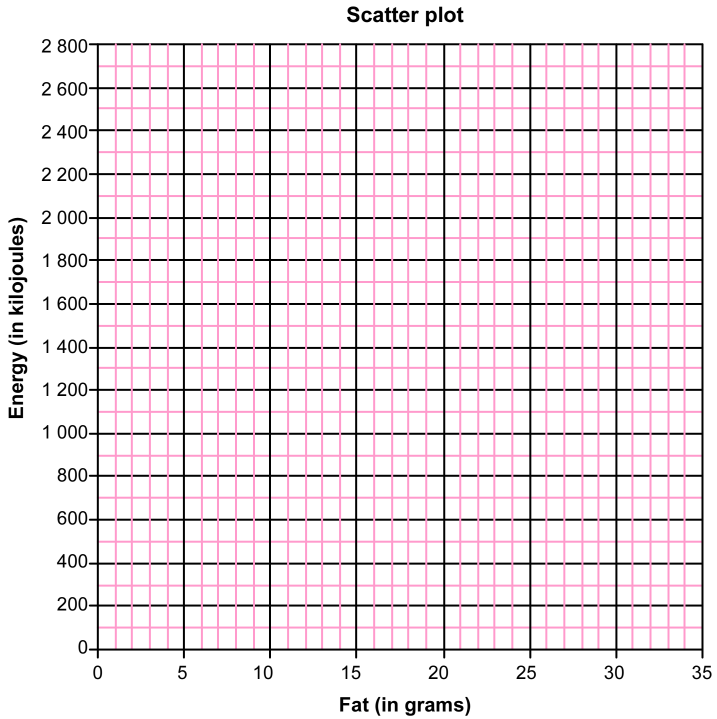
STATISTICS [20]

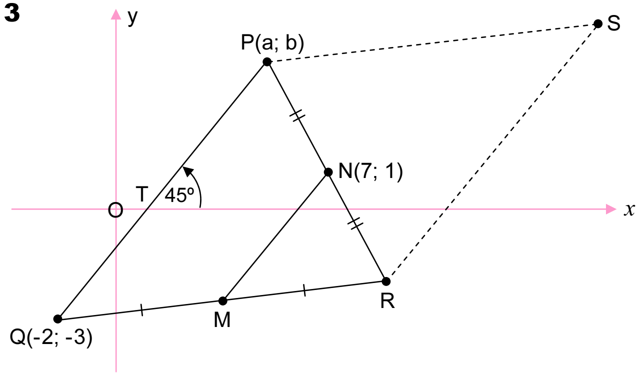
QUESTION 1

1.1 & 1.2.

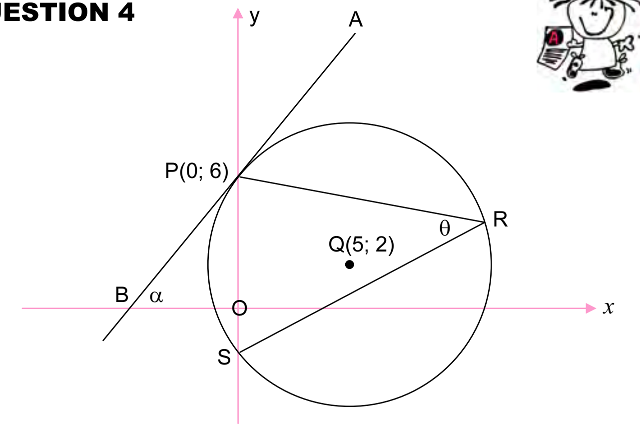


ANALYTICAL GEOMETRY [38]

QUESTION 3

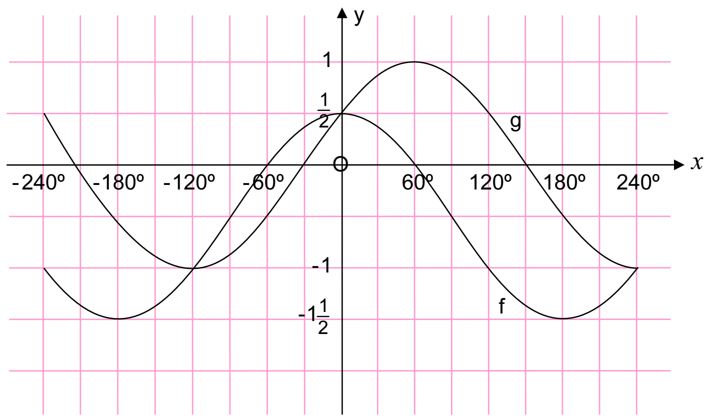


QUESTION 4

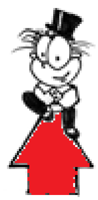


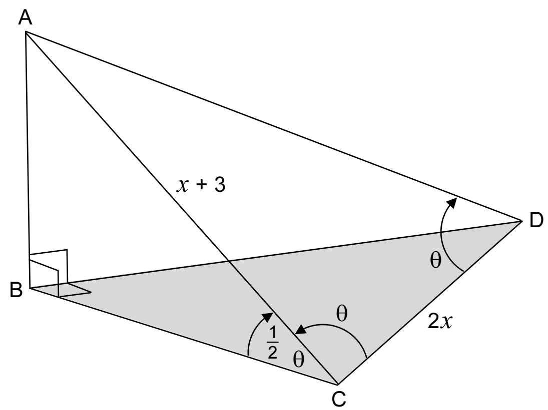
TRIGONOMETRY [42]

QUESTION 6



QUESTION 7

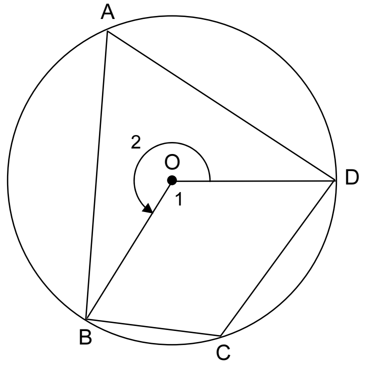




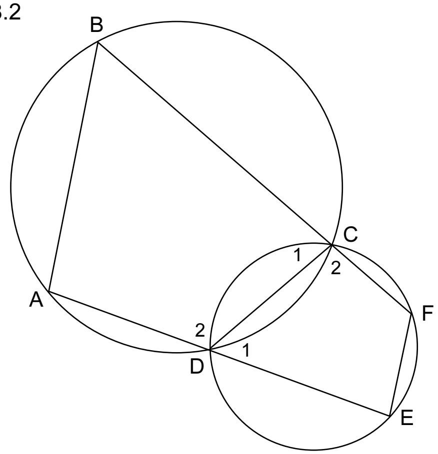
▸ EUCLIDEAN GEOMETRY AND MEASUREMENT [50]

QUESTION 8

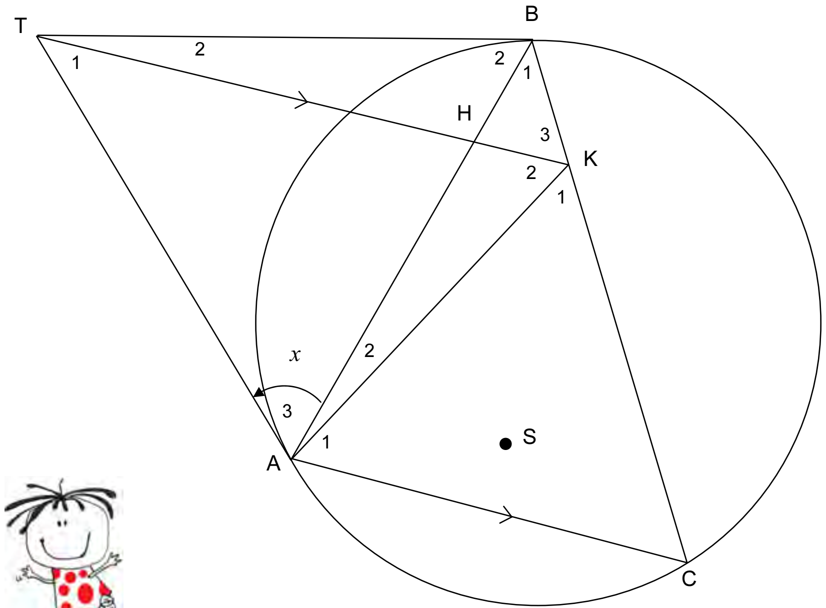
8.1



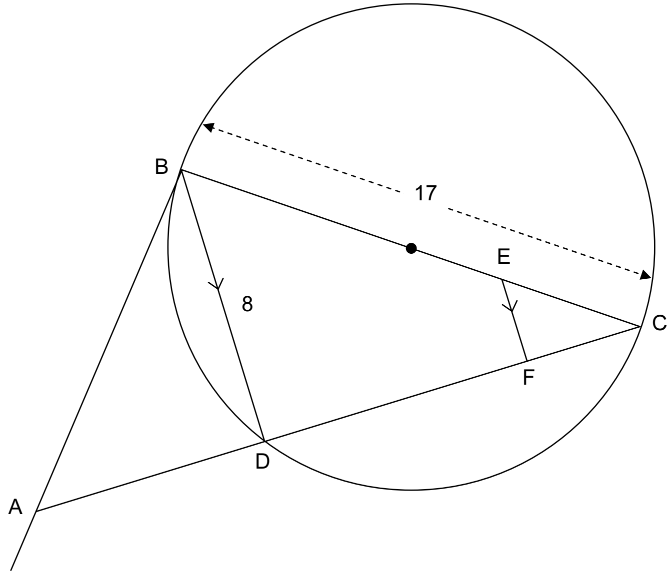
8.2



QUESTION 9

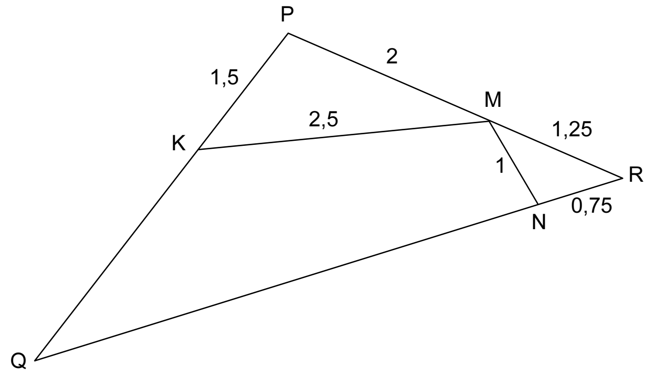


QUESTION 10



QUESTION 11

11.2



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November 2015

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GR 12 MATHS – EXAM MEMOS

NATIONAL NOV 2015 PAPER 1

ALGEBRA AND EQUATIONS AND INEQUALITIES [26]

1.1.1 $x^2 - 9x + 20 = 0$

$\therefore (x-4)(x-5) = 0$

$\therefore x - 4 = 0$ **or** $x - 5 = 0$

$\therefore \boldsymbol{x = 4}$ ◄ $\therefore \boldsymbol{x = 5}$ ◄

1.1.2 $3x^2 + 5x - 4 = \mathbf{0}$

$\therefore x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-4)}}{2(3)}$

$= \frac{-5 \pm \sqrt{73}}{6}$

$\approx$ **0,59** or **-2,26** ◄

1.1.3 $2x^{\frac{-5}{3}} = 64$

$\div 2)$ $\therefore x^{\frac{-5}{3}} = 32$

$\left(x^{\frac{-5}{3}}\right)^{-\frac{3}{5}} = (2^5)^{-\frac{3}{5}}$ ... $32 = 2^5$

$\therefore x = 2^{-3}$

$\therefore \boldsymbol{x} = \frac{\mathbf{1}}{\mathbf{8}}$ ◄ ... $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

1.1.4 $+\sqrt{2-x} = x - 2$

Square: $(\sqrt{2-x})^2 = (x-2)^2$

$\therefore 2 - x = x^2 - \mathbf{4x} + 4$ *Note middle term!*

$\therefore 0 = x^2 - 3x + 2$

$\therefore (x-2)(x-1) = 0$

$\therefore x = 2$ or 1

Note: The solutions must be checked . . . ➡

Check $\boldsymbol{x = 2}$: LHS = RHS = 0 ✓ . . . **in line 1**

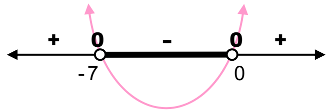
Check $\boldsymbol{x = 1}$: LHS = $\sqrt{1}$ = 1 & RHS = 1 - 2 = -1

$\therefore$ LHS $\neq$ RHS ✗

$\therefore$ Solution: $\boldsymbol{x = 2}$ **only** ◄

1.1.5 $x^2 + 7x < 0$

$\therefore x(x+7) < 0$



$\therefore$ **-7 < $\boldsymbol{x}$ < 0** ◄

1.2

AN IMPORTANT FACT:

A square is always only ever positive or 0.

$\therefore$ **The SUM of 2 squares CAN ONLY EQUAL 0 IF both squares are zero.**

$3x - y = 0$ . . . ❶ and $x - 5 = 0$

$x = 5$ . . . ❷

❷ in ❶: $\therefore 3(5) - y = 0$

$\therefore y = 15$

$\therefore$ **$\boldsymbol{x}$ = 5 and y = 15** ◄

1.3 $x^2 + x - k = 0$

$\Delta = b^2 - 4ac = 1^2 - 4(1)(-k) = 1 + 4k$

For no real roots: $1 + 4k < 0$ . . . $\Delta < 0$

$\therefore 4k < -1$

$\therefore k < -\frac{1}{4}$

OR: Solve the equation, using the formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1)(-k)}}{2(1)}$

$= \frac{-1 \pm \sqrt{\mathbf{1 + 4k}}}{2}$

So, 1 + 4k cannot be negative!

. . . *Then proceed as above . . .*

PATTERNS AND SEQUENCES [22]

2. **G.S.:** 10 ; 5 ; 2,5 ; 1,25 ; . . .

with a = 10 and r = $\frac{1}{2}$

2.1 $\mathbf{T_n = ar^{n-1}}$ ➡ $T_5 = 10 . \left(\frac{1}{2}\right)^{5-1}$

$= \frac{10}{16}$

$= \frac{\mathbf{5}}{\mathbf{8}}$ ◄ . . . *or: 0,625*

2.2 $\mathbf{T_n = 10 . \left(\frac{1}{2}\right)^{n-1}}$ ◄

$= 10 . (2^{-1})^{n-1}$ . . . *or:* $\frac{20}{2^n}$ ◄

$= 2^1 . 5 . 2^{-n+1}$

$= \mathbf{5 . 2^{2-n}}$ ◄

2.3 $r = \frac{1}{2}$; so, **-1 < r < 1** ◄

2.4 $\mathbf{S_\infty} = \dfrac{\mathbf{a}}{\mathbf{1 - r}} = \dfrac{10}{1 - \frac{1}{2}} = \dfrac{10}{\frac{1}{2}} = 10 \times 2 = 20$

& $\mathbf{S_n} = \dfrac{\mathbf{a(1 - r^n)}}{\mathbf{1 - r}} = \dfrac{10\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}} = 20\left[1 - \left(\frac{1}{2}\right)^n\right]$

$\therefore\ S_\infty - S_n = 20 - 20\left[1 - \left(\frac{1}{2}\right)^n\right]$

$= 20 - 20 + 20 \cdot \left(\frac{1}{2}\right)^n$

$= \mathbf{20 \cdot \left(\frac{1}{2}\right)^n}$ ◄

3. **A.S.:** $S_n = -3 + 5 + 13 + 21 + \ldots$ to n terms

where $a = -3$ and $d = 8$

3.1 T_k ? ; $n = k$

$\mathbf{T_n = a + (n - 1)d}$ ➡ $T_k = a + (k - 1)d$ . . . *use k for n*

$= -3 + (k - 1)(8)$

$= -3 + 8k - 8$

$= \mathbf{8k - 11}$ ◄

3.2 $\mathbf{S_n} = \displaystyle\sum_{k=1}^{n} (8k - 11)$ ◄ Tk

3.3 $\mathbf{S_n} = \mathbf{\frac{n}{2}[2a + (n - 1)d]}$

$= \frac{n}{2}[2(-3) + (n - 1)(8)]$

$= \frac{n}{2}[-6 + 8n - 8]$

$= \frac{n}{2}(8n - 14)$

$= n(4n - 7)$

$= \mathbf{4n^2 - 7n}$ ◄

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3.4

Go down the information below and see that:

$Q_1 = -6$

$Q_2 = -6$ + the first term of the given series

$Q_3 = -6$ + the sum of the first **2** terms of the given series

⋮

$\therefore\ Q_6 = -6$ + the sum of the first **5** terms of the given series

⋮

So, $Q_{129} = -6$ + the sum of the first **128** terms of the given series

$\therefore\ \mathbf{Q_{129}} = -6 + (-3 + 5 + 13 + 21 + \ldots$ up to **128** terms)

$= -6 + \mathbf{S_{128}}$ of the GIVEN series

3.4.1 $\mathbf{Q_6 = -6 - 3 + 5 + 13 + 21 + 29}$ ◄ . . . *This is a 'numerical expression'*

3.4.2 $Q_{129} = -6 + \mathbf{S_{128}}$ . . . *n = 128 in the given series*

$= -6 + 4(128)^2 - 7(128)$ . . . *$S_n = 4n^2 - 7n$ in 3.3 above*

$= \mathbf{64\ 634}$ ◄

▶ FUNCTIONS AND GRAPHS [37]

4.1 $\mathbf{y = -8}$ ◄

4.2 f: $y = 2^{x+1} - 8$

Y-intercept: Put $\mathbf{x = 0}$

$\therefore\ y = 2^{\mathbf{0}+1} - 8$

$= 2 - 8$

$= -6$

X-intercept: Put $\mathbf{y = 0}$

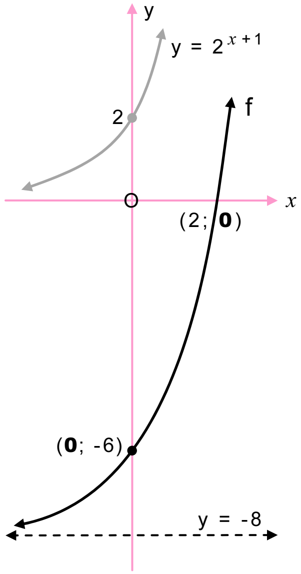
$\therefore\ \mathbf{0} = 2^{x+1} - 8$

$\therefore\ 8 = 2^{x+1}$

$\therefore\ 2^3 = 2^{x+1}$

$\therefore\ x + 1 = 3$

$\therefore\ x = 2$

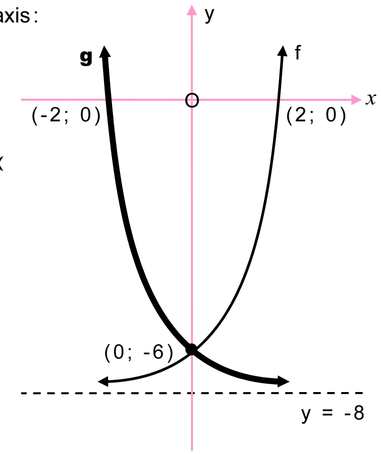


4.3 Reflection in the y-axis:

$(x;\ y)$ ➡ $(-x;\ y)$

$\therefore$ Equation of g is:

$\mathbf{y = 2^{-x+1} - 8}$ ◄



5.1 At Q, $y = 0$ and $y = 2x - 3$

$\therefore\ 2x - 3 = 0$

$\therefore\ 2x = 3$

$\therefore\ x = \frac{3}{2}$

$\therefore\ \mathbf{Q\left(\frac{3}{2};\ 0\right)}$ ◄

5.2 The domain of h^{-1} = the range of h . . . *h and h^{-1} are inverse functions*

$h(x) = 2x - 3$ ➡ $h(-2) = 2(-2) - 3 = -7$

& $h(4) = 2(4) - 3 = 5$

$\therefore$ The domain of h^{-1}: $\mathbf{-7 \leq x \leq 5}$ ◄

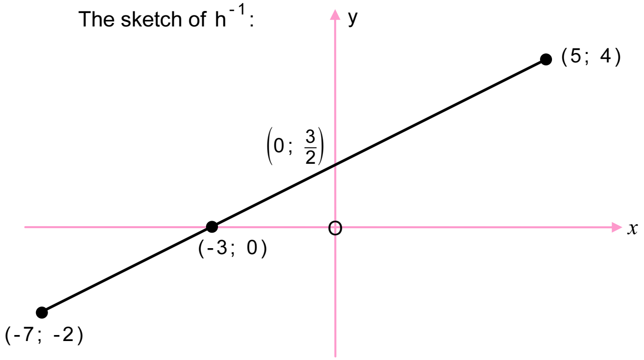
5.3
- To sketch h^{-1}, reverse x and y:

  $\therefore\ Q\left(\frac{3}{2};\ 0\right)$ on h becomes $\left(0;\ \frac{3}{2}\right)$ on h^{-1}
- The y-int. of h: (0; 3) ➡ (-3; 0), the x-int. on h^{-1}
- The end points of h: (-2; -7) and (4; 5)

  ➡ (-7; -2) and (5; 4), the end points of h^{-1}

EXAM MEMOS: PAPER 1

The sketch of h^{-1}:



5.4 Equation of h: $y = 2x - 3$

Equation of h^{-1}: $x = 2y - 3$

$\therefore\ x + 3 = 2y$

$\therefore\ y = \frac{1}{2}x + \frac{3}{2}$

At the point of intersection:

$2x - 3 = \frac{1}{2}x + \frac{3}{2}$ ... *$h(x) = h^{-1}(x)$*

$\times 2)\quad \therefore\ 4x - 6 = x + 3$

$\therefore\ 3x = 9$

$\therefore\ \boldsymbol{x = 3}$ ✓

5.5 $OP^2 = x^2 + (2x - 3)^2$

$= x^2 + 4x^2 - 12x + 9$

$= 5x^2 - 12x + 9$

Min values of OP^2 occurs when $x = -\frac{b}{2a} = -\frac{-12}{2(5)} = \frac{6}{5}$

or, when the derivative, $10x - 12 = 0$

$\therefore\ 10x = 12$

$\therefore\ x = \frac{6}{5}$

$\therefore$ Min $OP^2 = 5\left(\frac{6}{5}\right)^2 - 12\left(\frac{6}{5}\right) + 9$

$= 1{,}8$

$\therefore$ Min OP $\approx$ 1,34 units

5.6.1 If $f'(x) = h(x)$, a linear function then $f(x)$ is a quadratic function

5.6.2 The gradient of the tangent to f is $f'(x)$

$\therefore$ The minimum of the tangent to f is $f'(4)$

$= h(4)$

$= 2(4) - 3$

$= \mathbf{5}$ ✓

6.1.1 **T(0; 18)** ✓

6.1.2 At Q (and P): $\boldsymbol{f(x) = 0}$ ... *At the x-intercepts, y = 0*

$\therefore\ -2x^2 + 18 = 0$

$\therefore\ -2x^2 = -18$

$\div(-2)$: $\therefore\ x^2 = 9$

$\therefore\ x^2 = 9$

$\therefore\ x = \pm 3$

$\therefore$ **Q(3; 0)** ✓ ... *$x > 0$ at Q*

6.1.3 $x_R = 3 + 1{,}5 + 1{,}5 = 6$ ... *Q and R symmetrical about the axis of symmetry.*

$\therefore$ **R(6; 0)** ✓ ... *$y = 0$ on the x-axis*

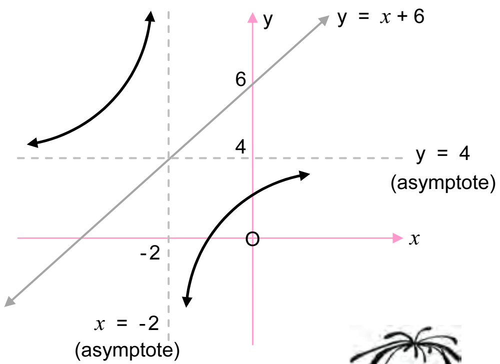
6.1.4 $\boldsymbol{x \in \mathbb{R}}$ ✓

$g'(x) = 2ax + b$

$g''(x) = 2a$

$g''(x) > 0$ 'always' because $a > 0$ ... *g:*

6.2



Axis of symmetry

$y = (x + \mathbf{2}) + \mathbf{4}$

2 units left

4 units up

▶ FINANCE, GROWTH AND DECAY [13]

7.1 **R450 000** ✓

7.2 $\mathbf{A = P(1 - \boldsymbol{i})^n}$ ➡ where **A** = 243 736,90 ; **P** = 450 000 ; **n** = 4 ; ***i*** ?

➡ $243\,736{,}90 = 450\,000(1 - i)^4$

$\therefore\ \frac{243\,736{,}90}{450\,000} = (1 - i)^4$

$\therefore\ (1 - i) = \sqrt[4]{0{,}5416\ldots}$

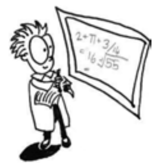
$= 0{,}857\ldots$

$\therefore\ 1 - 0{,}857\ldots = i$

$\therefore\ i = 0{,}1421\ldots$

$\therefore$ % annual rate of depreciation = 14,21%

7.3 $\mathbf{A = P(1 + \boldsymbol{i})^n}$



$\therefore\ a = 450\,000\left(1 + \frac{8{,}1}{100}\right)^4$

$\approx$ **R614 490,66** ✓

7.4 The sinking fund = R614 490,66 - R243 736,90

= R370 753,76

$\mathbf{F} = \frac{x\left[(1 + i)^n - 1\right]}{i} = 370\,753{,}76$

$\therefore\ \frac{x\left[\left(1 + \frac{0{,}062}{12}\right)^{36} - 1\right]}{\frac{0{,}062}{12}} = 370\,753{,}76$

$\therefore\ x =$ **R9 397,11** ✓

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M 2

EXAM MEMOS: PAPER 1

M 2

DIFFERENTIAL CALCULUS [35]

8.1

NB : The **DEFINITION** of the derivative, $\mathbf{f'(x)}$, of the function/graph **f** is :

$$f'(x) = \lim_{h \to 0} \boxed{\frac{f(x+h) - f(x)}{h}}$$

. . . *See the formula sheet !*

A good idea is to **FIRST** work out this part :

Build up the expression $\boxed{\dfrac{\mathbf{f(x+h) - f(x)}}{\mathbf{h}}}$ *gradually :*

$f(x) = x^2 - 3x$

❶ **Replace x by $x + h$:** $f(x + h) = (x + h)^2 - 3(x + h)$

$= {}^*x^2 + \mathbf{2xh} + h^2 - 3x - 3h$

❷ **Now subtract :**

$\therefore\ f(x + h) - f(x) = 2xh + h^2 - 3h$

❸ **Finally, divide by h :**

$\therefore\ \boxed{\dfrac{f(x + h) - f(x)}{h}} = \dfrac{2xh + h^2 - 3h}{h}$

$= \dfrac{2xh}{h} + \dfrac{h^2}{h} - \dfrac{3h}{h}$ . . . *Put each term over h*

$= 2x + h - 3$

Now apply the DEFINITION :

$\mathbf{f'(x)} = \lim\limits_{\mathbf{h \to 0}} (2x + h - 3)$ . . . *The* **limit** *is the* **TARGET** *value reached* **as h tends to the value 0.**

$= \mathbf{2x - 3}$ ➤

* **To square a binomial :**

$(a + b)^2 = (a + b)(a + b) = a^2 + \mathbf{ab + ab} + b^2$

$= a^2 + \mathbf{2ab} + b^2$

Note the middle term ! ! !

8.2.1

There are **3 different notations** for the derivative :

$\mathbf{f'(x)}$ or $\dfrac{\mathbf{dy}}{\mathbf{dy}}$ or $\mathbf{D}_x(\quad)$

Now, in this question, **y** is the function

and $\dfrac{\mathbf{dy}}{\mathbf{dx}}$ is the derivative !

$y = \left(x^2 - \dfrac{1}{x^2}\right)^2$ . . . *Again, we need to SQUARE a BINOMIAL !*

$= \left(x^2 - \dfrac{1}{x^2}\right)\left(x^2 - \dfrac{1}{x^2}\right)$

$= x^4 - \mathbf{1} - \mathbf{1} - \dfrac{1}{x^4}$

$= x^4 - \mathbf{2} - \dfrac{1}{x^4}$ . . . *Note the middle term*

$= x^4 - 2 - x^{-4}$

Now that we have separate terms, we can find the derivative BY RULES :

$\therefore\ \dfrac{\mathbf{dy}}{\mathbf{dx}} = 4x^3 + 0 - (-4x^{-5})$ . . .

If $f(x) = x^n$
then $f'(x) = nx^{n-1}$

$= 4x^3 + 4x^{-5}$

$= \mathbf{4x^3 + \dfrac{4}{x^5}}$ ➤

Note : $(4x)^{-5} = \dfrac{1}{(4x)^5}$

But $4 \cdot x^{-5} = 4 \times \dfrac{1}{x^5}$

8.2.2 The function : $\dfrac{x^3 - 1}{(x - 1)} = \dfrac{(x - 1)(x^2 + x + 1)}{(x - 1)}$ *

$= x^2 + x + 1$

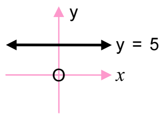
* Factorisation of difference between cubes.

The **derivative** of the function, $\mathbf{D}_x = \mathbf{D}_x(x^2 + x + 1)$

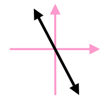
$= \mathbf{2x + 1}$ ➤

Note :
If $f(x)$ = a number, e.g. $y = 5$, then $f'(x) = 0$

The **gradient** of this line is **0**.
$\therefore$ The **derivative** is **0**.



9. h is a cubic graph shaped :  . . . $a < 0$

& g is a straight line graph shaped :  . . . $m < 0$

$h(x) = -x^3 + ax^2 + bx \quad (= y)$

9.1 **Point** Q(2; 10) **on the graph** (so, substitute)

➡ $h(2) = 10$

i.e. $\mathbf{x = 2}$ & $\mathbf{y = 10}$ satisfy the eqn.

➡ $-(2)^3 + a(2)^2 + b(2) = 10$ . . . $\boxed{h(2) = 10}$

$\therefore\ -8 + 4a + 2b = 10$

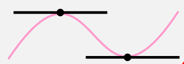
$\therefore\ 4a + 2b = 18$

÷ 2) $\therefore\ 2a + b = 9$ . . . ❶

Turning point at Q(2; 10)

➡ The **derivative equals 0** at $x = 2$

The **derivative** of a function is the **GRADIENT** of a function at a point. At the turning points, the tangent to the graph is horizontal and so has zero gradient.



➡ **The gradient of h :**

$\mathbf{h'x} = -3x^2 + 2ax + b = \mathbf{0}$ . . . $\boxed{h'(2) = \mathbf{0}}$

and this occurs at $x = 2$

$\therefore\ -3(2)^2 + 2a(2) + b = 0$

$\therefore\ -12 + 4a + b = 0$

$\therefore\ 4a + b = 12$ . . . ❷

❷ - ❶ : $\therefore\ 2a = 3$

$\therefore\ \mathbf{a} = \mathbf{\dfrac{3}{2}}$ ➤

Substitute $a = \dfrac{3}{2}$ in ❶ : $\therefore\ 2\left(\dfrac{3}{2}\right) + b = 9$

$\therefore\ 3 + b = 9$

$\therefore\ \mathbf{b = 6}$ ➤

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EXAM MEMOS : PAPER 1

9.2 We now know that :

$h(x) = -x^3 + \frac{3}{2}x^2 + 6x \quad (= y)$

At point P : $x = -1$ . . . *so, substitute to find y*

$\therefore h(-1) = -(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1)$

$= -(-1) + \frac{3}{2} - 6$

$= -3\frac{1}{2}$

$\therefore$ Point P is $\left(-1; -3\frac{1}{2}\right)$

The AVERAGE GRADIENT between P and Q

$= \frac{y_Q - y_P}{x_Q - x_P}$ . . . $\frac{\textit{Change in } y}{\textit{Change in } x}$ . . . *Point Q is (2; 10)*

$= \frac{10 - \left(-3\frac{1}{2}\right)}{2 - (-1)}$ . . . **Note :** The AVERAGE GRADIENT is NOT the derivative !

$= \frac{13\frac{1}{2}}{3}$

$=$ **4,5** ✓

9.3 The concavity changes where the **double derivative** equals **0**.

$h'x = -3x^2 + 2\left(\frac{3}{2}\right)x + 6$ . . . $a = \frac{3}{2}$ & $b = 6$ *in 9.1*

$\therefore h'x = -3x^2 + 3x + 6$

$\therefore h''x = -6x + 3$

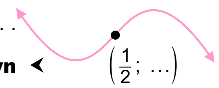
Refer to our note on concavity in the Gr 12 Maths 2 in 1 on page 1.31.

So : $h''(x) = 0 \Rightarrow -6x + 3 = 0$

$\therefore -6x = -3$

$\therefore x = \frac{1}{2}$ ✓

9.4 At $x = \frac{1}{2}$, the graph changes . . .
from **concave up** to **concave down** ✓



9.5 The **gradient of the tangent** to h (at any point) = the **derivative** of h

If it is parallel to g, the gradient = -12

$\therefore h'(x) = -3x^2 + 3x + 6 = -12$

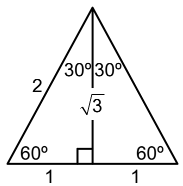
$\therefore -3x^2 + 3x + 18 = 0$

$\div(-3) \quad \therefore x^2 - x - 6 = 0$

$\therefore (x - 3)(x + 2) = 0$

$\therefore$ $\boldsymbol{x = -2}$ **only** ✓ . . . *Note : $x < 0$!*

10.1 Use :



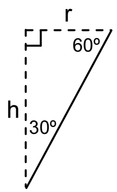
The '30° - 60° sketch'

In the given triangle :

$\Rightarrow \frac{\mathbf{r}}{h} = \tan 30°$ . . . *so that* **r** *is on top !*

$\therefore \frac{r}{h} = \frac{1}{\sqrt{3}}$

$\times h) \quad \therefore \mathbf{r} = \frac{\mathbf{h}}{\sqrt{3}}$ ✓



OR : $\frac{\mathbf{h}}{r} = \tan 60°$

$\therefore \frac{h}{r} = \sqrt{3}$

$\therefore \frac{r}{h} = \frac{1}{\sqrt{3}}$, etc

To write **r** *in terms of* **h** means :

Work out **r** = ...
and the answer must have only one unknown, namely **h**.

$\therefore$ The Theorem of Pythagoras won't work !

10.2 Formulae have been provided. You must **choose** the correct one (for the volume of a **cone**).

Step 1 : Determine the volume of the water :

$\mathbf{V} = \frac{1}{3}\pi r^2 h$ ← **V** is in terms of **r** & **h**

Step 2 : Replace r in terms of h (from 10.1) :

$\therefore \mathbf{V} = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 . h$ ← Now, **V** is in terms of **h** only.

Step 3 : Simplify the expression :

$\therefore \mathbf{V} = \frac{1}{3}\pi\left(\frac{h^2}{3}\right) . h$

$\therefore \mathbf{V} = \frac{1}{9}\pi h^3$ ← Now, the expression is ready to differentiate **with respect to h**.

Step 4 : Determine the derivative
(*of* **V with respect to h**) :

$\therefore \frac{\mathbf{dV}}{\mathbf{dh}} = \frac{1}{9}\pi . 3h^2$

$= \frac{1}{3}\pi h^2$

Step 5 : Only now, substitute h = 9 :

$\therefore$ **The derivative of V** w.r.t. **h**, **when h = 9 cm**

$= \frac{1}{3}\pi(9)^2$

$= 27\pi$ or $84{,}82 \text{ cm}^3$ per cm

The Application . . .

In Step 4 : The **derivative** is the **RATE OF CHANGE** of the volume of water (in cm^3) with respect to the height of the water (in cm), as the water flows into the rain gauge.

In Step 5 : We find that when the height of the water reaches 9 cm, the volume of the water is increasing at a **rate** of 84,32 cm^3 (in volume) per cm (in height) - a positive rate of change.

M 2

PROBABILITY [17]

11.1 **Independent events:**

For 2 events A and B to be INDEPENDENT:

P(A and B) must be equal to **P(A) X P(B)**

. . . called the **PRODUCT** *Rule*

So, calculate the value of each of these expressions to determine whether they are equal or not:

P(A) x P(B) = $0{,}2 \times 0{,}63$ = 0,126

But, **P(A and B)** = 0,126 also *. . . given*

$\therefore$ **P(A) x P(B) = P(A and B)**

$\therefore$ The 2 events A and B <u>are</u> independent

Note the layout of the PROOF;
i.e. the answer must be JUSTIFIED!

NB: Do **<u>not</u>** confuse **independent** events with **mutually exclusive** events

Mutually exclusive events:

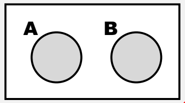
For 2 events to be MUTUALLY EXCLUSIVE:

P(A or B) must be equal to **P(A) + P(B)**

. . . called the **SUM** *rule*

So, necessarily: **P(A∩B) = 0** . . .

(A and B do <u>not</u> overlap)



Fundamental Counting Principle:

11.2 Letters of a word: A good method . . .

- Create 'slots' for each position
- Determine the number of possibilities that there are for each slot.

11.2.1 There are 7 (different) letters in the word DECIMAL

__ __ __ __ __ __ __

and, if they MAY be repeated, then there are 7 possibilities for each slot

<u>7</u> <u>7</u> <u>7</u> <u>7</u> <u>7</u> <u>7</u> <u>7</u>

$\therefore$ The number of ways in which the letters could be arranged is $7 \times 7 \times 7 \times \ldots$ 7 times

$= 7^7$

= **823 543** ✓ *. . . the power button on your calculator*

11.2.2 If the letters MAY NOT be repeated, then, each time, there is one less to use.

<u>7</u> <u>6</u> <u>5</u> <u>4</u> <u>3</u> <u>2</u> <u>1</u>

Now, the number of arrangements is

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

= 7!

= **5 040** ✓ *. . . the factorial (x!) button on your calculator*

11.2.3 There are 3 vowels: E I A
& 4 consonants: D C M L

So: **<u>3 ways</u>** __ __ __ __ __ **<u>4 ways</u>**

Then, after choosing **1 vowel for the 1st slot and 1 consonant for the last slot**, there are **5 letters left** to be used for the 'middle' 5 slots, with NO REPEATS, so, one less each time.

<u>3</u> **<u>5</u>** **<u>4</u>** **<u>3</u>** **<u>2</u>** **<u>1</u>** <u>4</u>

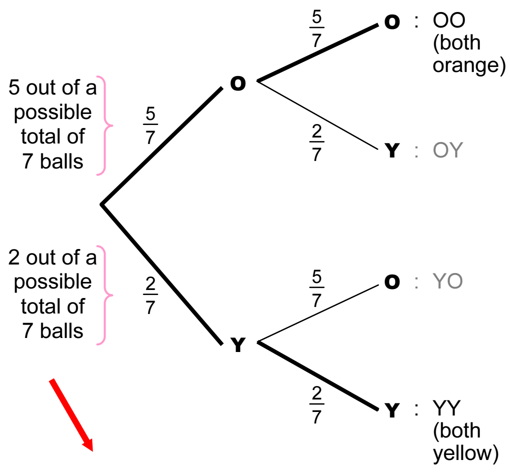
$\therefore$ The number of ways is

3 × **5!** × 4 *. . . 5! means $5 \times 4 \times 3 \times 2 \times 1$*

= **1 440** ✓

11.3 Try doing this question using **5** orange balls instead of **t** orange balls. The method and concepts will be easier to follow.

If there were **5** orange and 2 yellow balls:



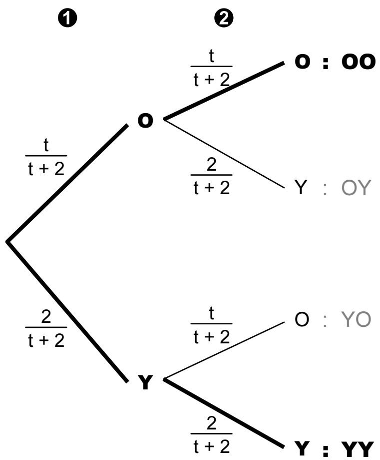
P(O and O)
$= \frac{5}{7} \times \frac{5}{7}$
$= \mathbf{\frac{25}{49}}$

P(Y and Y)
$= \frac{2}{7} \times \frac{2}{7}$
$= \mathbf{\frac{4}{49}}$

The same for the 2nd selection, since the 1st ball was returned to the bag.

P(2 balls of the same colour) = P(OO or PYY)
= P(OO) + P(YY)
$= \frac{25}{49} + \frac{4}{49}$
$= \frac{29}{49}$

If there are **t** orange and 2 yellow balls



P(O and O)
$= \frac{t}{t+2} \times \frac{t}{t+2}$
$= \frac{t^2}{(t+2)^2}$

P(Y and Y)
$= \frac{2}{t+2} \times \frac{2}{t+2}$
$= \frac{4}{(t+2)^2}$

the ANSWER series ... your key to exam success

EXAM MEMOS: PAPER 1

$$P(\text{2 balls of the same colour}) = 52\%$$

As above: $\therefore \dfrac{t^2}{(t+2)^2} + \dfrac{4}{(t+2)^2} = \dfrac{13}{25}$

$\times 25(t+2)^2$: $\therefore 25t^2 + 100 = 13(t+2)^2$

$$\therefore -13(t^2 \mathbf{+ 4t} + 4) + 25t^2 + 100 = 0$$

$$\therefore -13t^2 - 52t - 52 + 25t^2 + 100 = 0$$

$$\therefore 12t^2 - 52t + 48 = 0$$

$\div 4)$ $\therefore 3t^2 - 13t + 12 = 0$

3	1
4	3

$$\therefore (3t - 4)(t - 3) = 0$$

$$\therefore t = 3 \quad \ldots t \neq \frac{4}{3} \quad \because t \in \mathbb{N}$$

$\therefore$ **There are 3 orange balls in the bag** ✓

We trust that working through these exam papers and following our detailed answers and comments will help you prepare thoroughly for your final exam.

The Answer Series study guides offer a key to exam success in several major subjects.

In particular, Gr 12 Maths 2 in 1 offers 'spot-on' exam practice in separate topics and on CAPS-constructed Maths exam papers.

NOTES

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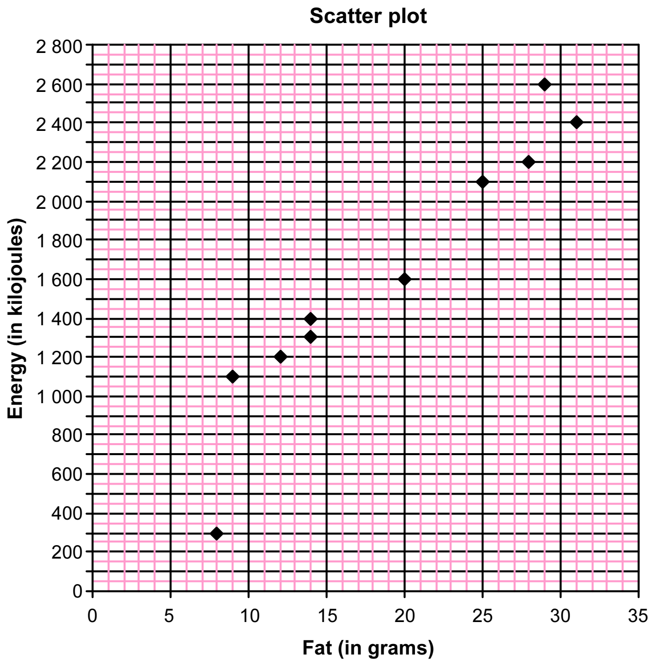
M 2

EXAM MEMOS: PAPER 1

NATIONAL NOV 2015 PAPER 2

▶ STATISTICS [20]

1.1



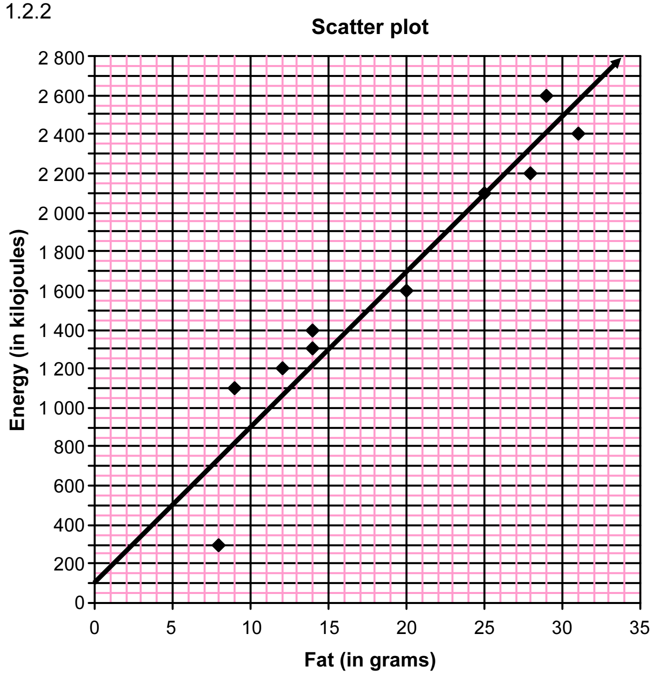
1.2.1 Amount of fat in grams : $x = 18$

Number of kJ of energy, $\hat{y} = 154{,}60 + 77{,}13\,(18)$

$= 1\ 542{,}94$

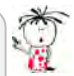
$\approx$ **1 500 kJ** ✓

1.2.2



1.3 **(8; 300)** ✓

1.4 **r = 0,95** ✓ . . .



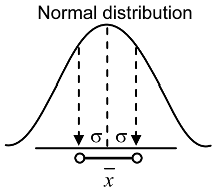
Please note :
When determining the correlation coefficient, DO NOT REMOVE ANY OUTLIERS.

1.5 **Very strong positive correlation** ✓

2.1 The mean, $\bar{x} = \frac{202}{30} = 6\frac{11}{15} \approx$ **6,73** ✓

2.2 The median = **7** ✓

2.3 The standard deviation, $\sigma \approx 2{,}26$



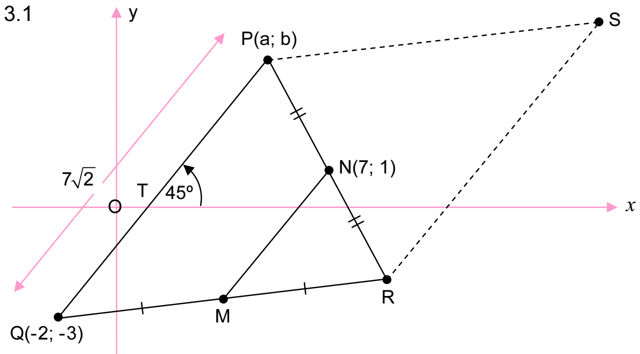
2.4 $\bar{x}$ **- 1σ** = 6,73 **- 2,26** = 4,47

& $\bar{x}$ **+ 1σ** = 6,73 **+ 2,26** = 8,99

∴ The sum of the values must lie between 4 and 9
The number of times this happens, i.e. the frequency
= **4 + 4 + 8 + 3 = 19 times** ✓

▶ ANALYTICAL GEOMETRY [38]

3.1



The gradient of PQ = tan 45° . . . *The gradient of a line = the tan of the ∠ of inclination.*

= **1** ✓

3.2 In ΔRQP : M & N are the midpoints of RQ & RP

∴ MN || QP . . . *The midpoint theorem*

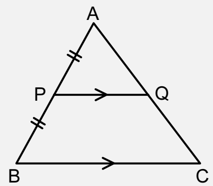
The following **Gr 10 theorems - THE MIDPOINT THEOREMS -** are frequently required in a Grade 12 maths paper :

THE MIDPOINT THEOREMS

❶ **The line segment through the midpoint of one side of a triangle, parallel to a second side, bisects the third side.**

If P is the midpoint of AB
& PQ || BC,

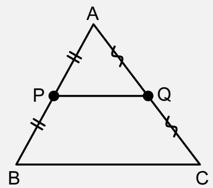
then : Q is the midpoint of AC
and PQ = $\frac{1}{2}$BC



❷ **The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half of the third side.**

If P & Q are the
midpoints of AB & AC,

then : PQ || BC
and PQ = $\frac{1}{2}$BC



. . . ➡

$\therefore$ $m_{MN} = m_{QP}$ ... *parallel lines have equal gradients*

$= 1$ ... *in 3.1*

Subst. $m = 1$ & pt. N(7; 1) in:

$\mathbf{y - y_1 = m(x - x_1)}$

$\therefore$ $y - 1 = 1(x - 7)$

$\therefore$ $y = x - 7 + 1$

$\therefore$ $\mathbf{y = x - 6}$ ✓

OR: $\mathbf{y = mx + c}$

$\therefore$ $1 = (1)(7) + c$

$\therefore$ $-6 = c$, etc.

3.3 The length of MN $= \frac{1}{2}$ the length of QP ... *Same theorem(s) indicated in Q3.2.*

$= \frac{1}{2}(7\sqrt{2})$

$\approx$ **4,95 units** ✓

3.4 The length of RS = the length of QP ... *opposite sides of a $||^m$*

= $\mathbf{7\sqrt{2}}$ **(≈ 9,90) units** ✓

3.5 The co-ordinates of S:

Hint: Draw in diagonal QNS ... *diagonals bisect at their midpoints*

N(7; 1) midpoint of QS where Q(-2; -3)

➡ **S(16; 5)** ✓ ...

By projection: $-2 \xrightarrow{+9} 7 \xrightarrow{+9} ?$

$-3 \xrightarrow{+4} 1 \xrightarrow{+4} ?$

3.6

NB:

We need to find the values of **a** and **b**, i.e. **2 unknowns**.
So, we need to determine and solve **2 equations** in **a** and **b**.

- **The GRADIENT formula:**

$m_{QP} = \frac{b - (-3)}{a - (-2)} = 1$ ... *see 3.1*

$\therefore$ $\frac{b + 3}{a + 2} = 1$

$\therefore$ $b + 3 = a + 2$

$\therefore$ $b = a - 1$ ... ❶

- **The DISTANCE formula:**

$QP^2 = [a - (-2)^2]^2 + [b - (-3)]^2 = (7\sqrt{2})^2$

$\therefore$ $(a + 2)^2 + (b + 3)^2 = 98$ ... ❷

Substitute ❶ in ❷:

$\therefore$ $(a + 2)^2 + (a - 1 + 3)^2 = 98$

$\therefore$ $2(a + 2)^2 = 98$

$\therefore$ $(a + 2)^2 = 49$

$\therefore$ $a + 2 = \pm 7$

$\therefore$ $a = \pm 7 - 2$

$= 5$ or -9

❶: For $a = 5$: $b = 5 - 1 = 4$

& For $a = -9$: $b = -9 - 1 = -10$

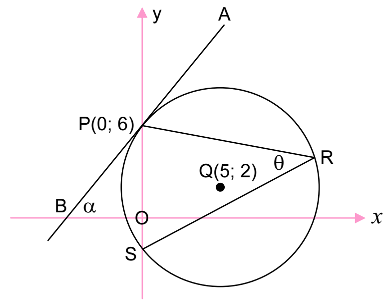
$\therefore$ **Either** (a; b) is (5; 4) **or** (-9; -10)

But a & b are both positive ... *given, and in the diagram*

$\therefore$ **Point P(5; 4)** ✓

4.1

To determine the equation of a circle, we need:
- the centre ... Q(5; 2)
- the radius



The radius, PQ:

$PQ^2 = (5 - 0)^2 + (2 - 6)^2$ ... **DISTANCE FORMULA**

$= 25 + 16$

$= 41$ $(= r^2)$

The standard equation of a ⊙, centre (a; b) and radius, r:

$(x - a)^2 + (y - b)^2 = r^2$

$\therefore$ $\mathbf{(x - 5)^2 + (y - 2)^2 = 41}$ ✓

OR: Substitute (0; 6) in $(x - 5)^2 + (y - 2)^2 = r^2$ to determine r^2.

4.2 At S, $\mathbf{x = 0}$ ... $\mathbf{x = 0}$ **on the y-axis**

$\therefore$ Substitute $x = 0$ in the equation in 4.1

$\therefore$ $(\mathbf{0} - 5)^2 + (y - 2)^2 = 41$

$\therefore$ $25 + (y - 2)^2 = 41$

$\therefore$ $(y - 2)^2 = 16$

$\therefore$ $y - 2 = \pm 4$

$\therefore$ $y = 2 \pm 4$

$\therefore$ $y = 6$ or -2, but $y < 0$ ....

$\therefore$ **S(0; -2)** ✓

4.3 The **y-intercept** is (0; 6) ➡ **c = 6** ... $y = \mathbf{m}x + \mathbf{c}$

The **gradient, m:**

The gradient of the radius, QP $= \frac{6 - 2}{0 - 5}$ ... $\mathbf{m_2 = \frac{y_2 - y_1}{x_2 - x_1}}$

$= \frac{4}{-5}$

$\therefore$ The gradient of the tangent, APB, $m = +\frac{5}{4}$

... **tangent ⊥ radius**
➡ **product of gradients equals -1**

$\therefore$ The equation of tangent APB: $\mathbf{y = \frac{5}{4}x + 6}$ ✓

4.4 $\tan\alpha = \frac{5}{4}$

➡ $\alpha = \tan^{-1}\frac{5}{4}$

$= \mathbf{51{,}34°}$ ✓

4.5 From Euclidean Geometry, we know that:

$\theta = B\hat{P}S$ ... *∠ between a tangent and a chord = ∠ subtended by the chord in the alternate segment*

$\therefore$ $\theta = 90° - 51{,}34°$ ... *∠s of ΔBPO*

$= \mathbf{38{,}66°}$ ✓

4.6 Area of ΔPQS $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} PS \,.\, x_Q$ . . . *h = distance from θ to the y-axis*

$= \frac{1}{2}(6 + 2)\,.\,5$ . . . $PS = PO + OS = 6 + 2$

$= 4\,.\,5$

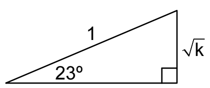
$=$ **20 square units** ✓

▸ TRIGONOMETRY [42]

5.1 $\sin 23^\circ = \sqrt{k}$

5.1.1 $\sin 203^\circ = \sin(180^\circ + 23^\circ) = -\sin 23^\circ = \mathbf{-\sqrt{k}}$ ✓ . . . *3^rd Quad.*

5.1.2 $\cos 23^\circ$? $\sin 23^\circ = \frac{\sqrt{k}}{1}$:



$1 - (\sqrt{k})^2$

$= \sqrt{1-k}$

$\therefore \cos 23^\circ = \frac{\sqrt{1-k}}{1} = \mathbf{\sqrt{1-k}}$ ✓

OR: $\sin^2\theta + \cos^2\theta = 1$

➡ $\cos^2\theta = 1 - \sin^2\theta$

$= 1 - (\sqrt{k})^2$

$= 1 - k$

$\therefore \cos\theta = \mathbf{\sqrt{1-k}}$ ✓

5.1.3 $\tan(-23^\circ) = -\tan 23^\circ = -\frac{\sqrt{k}}{\sqrt{1-k}} = \mathbf{-\frac{\sqrt{k}}{\sqrt{1-k}}}$ ✓

5.2 Exp. $= \frac{4\cos x\,.\,(-\sin x)}{\sin[(30^\circ - x) + x]}$ . . . *cos is neg. in the 2^nd Quad.*

. . . ***NB:*** *RECOGNISE compound ∠ formula for sin(A + B)*

$= \frac{-4\cos x\,\sin x}{\sin 30^\circ}$

$= \frac{-2(2\sin x\,\cos x)}{\frac{1}{2}}$

$= \mathbf{-4\,.\,\sin 2x}$ ✓ . . . *sin 2x = 2 sin x cos x* . . . *double ∠ formula*

5.3

> $\mathbf{\cos 2x = \cos^2 x - \sin^2 x}$
>
> **or:** $\mathbf{1 - 2\sin^2 x}$
>
> **or:** $\mathbf{2\cos^2 x - 1}$ * . . . $\cos^2 x - (1 - \cos^2 x)$
>
> * We choose this formula because then we have an equation in cos x.

$\mathbf{(2\cos^2 x - 1)} - 7\cos x - 3 = 0$

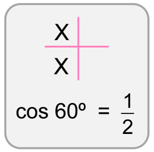
$\therefore\ 2\cos^2 x - 7\cos x - 4 = 0$

$\therefore\ (2\cos x + 1)(\cos x - 4) = 0$

$\therefore\ 2\cos x + 1 = 0$ or $\cos x = 4$, but . . .

$\therefore\ 2\cos x = -1$ $\quad -1 \le \cos \le 1$ for all x

$\therefore\ \cos x = -\frac{1}{2}$



$\therefore\ x = 180^\circ - 60^\circ + n(360^\circ)$

$= 120^\circ + n(360^\circ)\,;$ $\mathbf{n \in \mathbb{Z}}$ ✓

or $x = 180^\circ + 60^\circ + n(360^\circ)$

$= 240^\circ + n(360^\circ)\,;$ $\mathbf{n \in \mathbb{Z}}$ ✓

OR: $x = \pm 120^\circ + n(360^\circ)$; $\mathbf{n \in \mathbb{Z}}$ ✓

5.4 $\sin 3\theta = \sin(2\theta + \theta)$ . . . *This is a level 4 question which 'comes with practice'!*

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ **Convert to sin θ**

$= 2\sin\theta\,.\,\cos\theta\,.\,\cos\theta + (1 - 2\sin^2\theta)\,.\,\sin\theta$

$= 2\sin\theta\,.\,\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

$= 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$

$= 1 - 4\left(\frac{1}{27}\right)$

$= 1 - \frac{4}{27}$

$= \mathbf{\frac{23}{27}}$ ✓

6.1 $g(x) = \sin(x + 30^\circ)$ & $f(x) = \cos x - \frac{1}{2}$

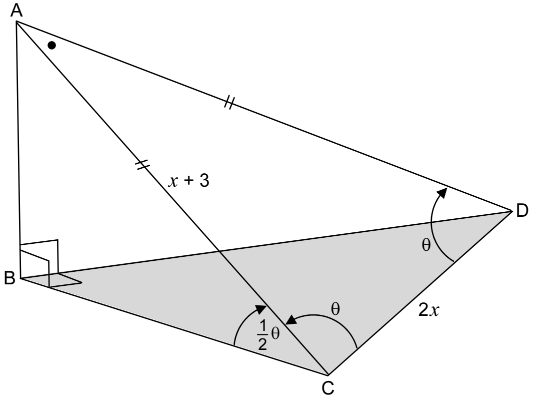
$\therefore$ $\mathbf{p = 30^\circ}$ ✓ . . . *30° left of the graph y = sin x*

& $\mathbf{q = -\frac{1}{2}}$ ✓ . . . *$\frac{1}{2}$ unit down from the graph y = cos x*

6.2 $\mathbf{-120^\circ < x < 0^\circ}$ ✓ . . . *f is above g for these values* . . . *read from the graph*

6.3 **translation of 60° left**
and, a reflection in the x-axis ✓ (or **vice versa**) ✓

7.1 In ΔCAD: **$C\hat{A}D = 180^\circ - 2\theta$** ✓



7.2 In ΔCAD: $\dfrac{\sin C\hat{A}D}{2x} = \dfrac{\sin\theta}{x+3}$ . . .

You have to apply either the sine or the cosine formula, the former when there are no squares!

where $\sin C\hat{A}D = \sin(180^\circ - 2\theta)$

$= \sin 2\theta$

$= 2\sin\theta\cos\theta$

$\therefore \dfrac{2\sin\theta\cos\theta}{2x} = \dfrac{\sin\theta}{x+3}$

$\times \dfrac{2x}{2\sin\theta}\Big)$ $\therefore \cos\theta = \dfrac{\cancel{\sin\theta}}{x+3} \times \dfrac{\cancel{2x}}{2\cancel{\sin\theta}}$

$= \boldsymbol{\dfrac{x}{x+3}}$ ✓

7.3 In ΔABC: $\dfrac{AB}{x+3} = \sin\frac{1}{2}\theta$

But, $x = 2$

$\therefore \dfrac{AB}{5} = \sin\frac{1}{2}\theta$

$\therefore AB = 5\sin\frac{1}{2}\theta$

Now: $\cos\theta = \dfrac{x}{x+3}$ in 7.2

$= \dfrac{2}{5}$ . . . *$x = 2$*

$\therefore \theta = 66{,}42^\circ$

$\therefore AB = 5\sin\frac{1}{2}(66{,}42^\circ) = 5\sin 33^\circ, 21^\circ$

$\approx$ **2,74 units** ✓

▶ EUCLIDEAN GEOMETRY AND MEASUREMENT [50]

8.1.1 . . . double . . .

See Bookwork in Gr 12 Maths 2 in 1 for theorem statements & proofs.

8.1.2 $\hat{A} = \text{half of } \hat{O}_1$ . . . *Theorem in 8.1.1*

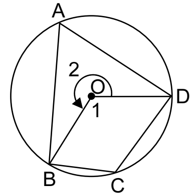
& $\hat{C} = \text{half of } \hat{O}_2$ . . . *Theorem in 8.1.1*

$\therefore \hat{A} + \hat{C} = \frac{1}{2} \times \hat{O}_1 + \frac{1}{2} \times \hat{O}_2$

$= \frac{1}{2}(\hat{O}_1 + \hat{O}_2)$

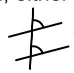
$= \frac{1}{2} \times 360^\circ$ . . . *$\angle^s$ round a point*

$= 180^\circ$



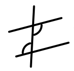
8.2 To prove that 2 lines are parallel, either prove that:

corresponding $\angle^s$ are **equal**,



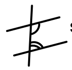
or

alternate $\angle^s$ are **equal**,



or

co-interior $\angle^s$ are **supplementary**



(See Summary of Gr 8 & 9 Geometry)

Let $\hat{A} = x$

then $\hat{C}_2 = x$ . . . *ext. $\angle$ of c.q. ABCD = int. opp. $\angle$*

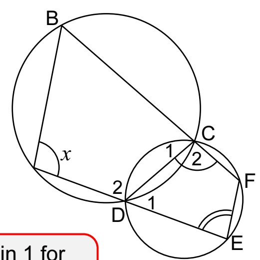
then $\hat{E} = 180^\circ - x$ . . . *opp. $\angle^s$ of c.q. CDEF*

$\therefore \hat{A} + \hat{E} = x + (180^\circ - x)$

$= 180^\circ$

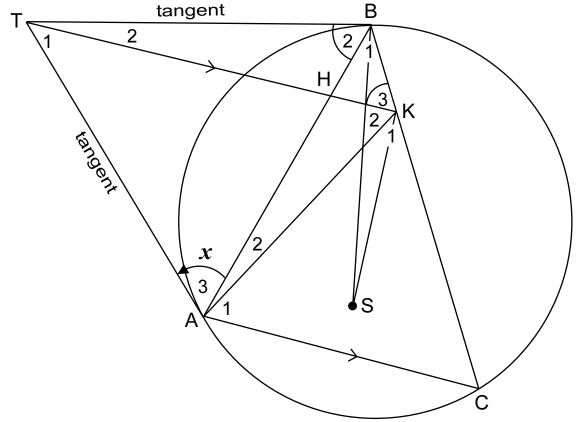
i.e. Co-interior $\angle^s$ are supplementary

$\therefore$ **EF || AB** ✓



See Bookwork in Gr 12 Maths 2 in 1 for statements & proofs on cyclic quadrilaterals.

9.1



$\hat{K}_3 = \hat{C}$ . . . *corresponding $\angle^s$; TK || AC*

$= \hat{B}_2$ . . . *tangent (TB) / chord (AB) theorem*

$= \hat{A}_3$ . . . *equal tangents; TA = TB; base $\angle^s$ equal in ΔTAB*

$= \boldsymbol{x}$ ✓

9.2 $\hat{K}_3 = \hat{A}_3$ (= x in 9.1)

i.e. BT subtends equal $\angle^s$ at K & A

$\therefore$ **AKBT is a c.q.** ✓ . . . *converse of '$\angle^s$ in same segment' theorem*

9.3 $\hat{K}_2 = \hat{B}_2$ . . . *AT subtends in c.q. AKBT, proved in 9.2*

$= \hat{K}_3$ . . . *both equal to x in 9.1*

i.e. **TK bisects $A\hat{K}B$** ✓

'bisects' means 'halves'

9.4 $\hat{A}_3 = \hat{K}_3$ . . . *both equal to x in 9.1*

$= \hat{K}_2$ . . . *proved in 9.3*

i.e. The $\angle$ between AT and (chord) AH equals the $\angle$ subtended by AH at K.

$\therefore$ **TA is a tangent to the circle passing through the points A, K and H** ✓

M 2

9.5 $A\hat{S}B = A\hat{K}B$ ... *AB subtends in c.q. ASKB*

$= 2x$ ... $K_2 + K_3$

& $A\hat{T}B = 180° - 2x$ ... *sum of* $\angle^s$ *in isosceles* ΔATB

$\therefore$ $A\hat{S}B + A\hat{T}B = 180°$

$\therefore$ **ASBT is a c.q.** ◄ ... *opp* $\angle^s$ *are supplementary*

10.1 In ΔBDC: $B\hat{D}C = 90°$ ... $\angle$ *in semi-*⊙

$\therefore$ $DC^2 = BC^2 - BD^2$ ... *Theorem of Pythagoras*

$= 289 - 64$

$= 225$

$\therefore$ **DC = 15 units** ◄

OR: Recognise the 'Pythagorean **TRIPLE**' 8 : **15** : 17

10.2.1 In ΔCBD: EF || BD

$\therefore$ $\frac{CF}{CD} = \frac{CE}{CB}$ ... *proportion theorem*

$\therefore$ $\frac{CF}{15} = \frac{1}{4}$ ... *CE : EB = 1 : 3*

$\times 15)$ $\therefore$ $CF = \frac{15}{4}$

$\therefore$ **CF = 3,75 units** ◄

10.2.2 $A\hat{B}C = 90°$ ... *tangent* $\perp$ *diameter*

$\therefore$ $E\hat{F}C = B\hat{D}C\ (= 90°)$ ... *corresponding* $\angle^s$*; EF || BD*

$\therefore$ In Δ^s BAC and FEC

(1) $\hat{C}$ is common

(2) $A\hat{B}C = E\hat{F}C$ ... *both = 90°*

$\therefore$ **ΔBAC ||| ΔFEC** ◄ ... $\angle\angle\angle$

10.2.3 $\therefore$ $\frac{AC}{EC} = \frac{BC}{FC}$

$\therefore$ $\frac{AC}{\frac{1}{4}(17)} = \frac{17}{\frac{15}{4}}$

$\times \frac{17}{4})$ $\therefore$ $AC = \frac{17}{1} \times \frac{\cancel{4}}{15} \times \frac{17}{\cancel{4}}$

$=$ **19,27 units** ◄

10.2.4 ⊙ABC has AC as diameter ... $A\hat{B}C = 90°$

$\therefore$ the radius $= \frac{1}{2}AC$

$=$ **9,63 units** ◄ ... *or, 9,64 units*

11.1 ... **equiangular (according to the converse theorem) and therefore similar** ◄

11.2.1 In ΔKPM and ΔRNM:

$\frac{KP}{RN} = \frac{1,5}{0,75} = 2$; $\frac{PM}{NM} = \frac{2}{1} = 2$ and

$\frac{KM}{RM} = \frac{2,5}{1,25} = 2$

$\therefore$ **ΔKPM ||| ΔRNM** ◄ ... *corresponding sides of the triangle are in the same proportion - theorem in 11.1*

11.2.2 ΔKPM and ΔRNM are **equiangular** ... *see 11.1*

$\therefore$ $P\hat{K}M = \hat{R}$ $(= x$, say$)$ ... $\angle^s$ *opposite middle length side*

In ΔPKM and ΔPQR

(1) $\hat{P}$ is common

(2) $P\hat{K}M = \hat{R}$ $(= x)$

[(3) Therefore the 3rd angles are equal]

$\therefore$ ΔPKM ||| ΔPRQ ... $\angle\angle\angle$

$\therefore$ $\frac{QR}{MK} = \frac{PR}{PK}$ ... *proportional sides*

$\therefore$ $\frac{QR}{2,5} = \frac{3,25}{1,5}$

$\times 2,5)$ $\therefore$ $QR = \frac{3,25 \times 2,5}{1,5}$

$= 5,42$ units

$\therefore$ $NQ = 5,42 - 0,75$

$=$ **4,67 units** ◄

We wish you the very best of luck.

From Anne Eadie, Gretel Lampe and the whole Answer team.

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