# GR 11 MATHS – ANALYTICAL GEOMETRY

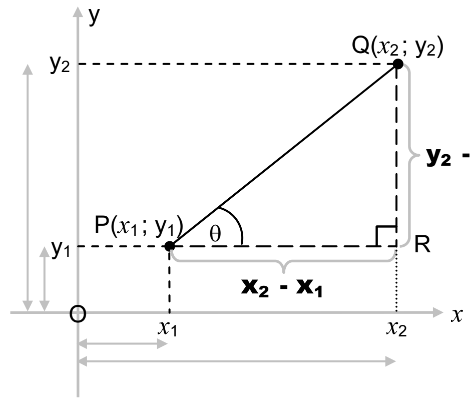
## Checklist: The Drawers of Tools

Consider 4 'drawers' of tools - all BASIC FACTS. Use these to analyse the sketches, to reason, calculate, prove . . . .

### Distance, Midpoint & Gradient

**NB:** Bear in mind **Case 1**, **Case 2**, and **Case 3** on page 5.6 & 5.7

For any two fixed points, P($x_1$; $y_1$) & Q($x_2$; $y_2$)




**Note:**

**Vertical** length QR = $\mathbf{y_2 - y_1}$

**Horizontal** length PR = $\mathbf{x_2 - x_1}$

θ is the angle of inclination of the line PQ

$$\tan\theta = \frac{\textbf{opposite}}{\textbf{adjacent}} = \frac{y_2 - y_1}{x_2 - x_1}$$

**1 Distance** PQ . . .

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \quad \Rightarrow \quad PQ = \sqrt{(\quad)^2 + (\quad)^2}$$

*the sum of the squares! (Pythag.)*

**2 Gradient** PQ . . .

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} \quad (= \tan\theta)$$

*change in y / change in x*

**3 Midpoint** of PQ . . .

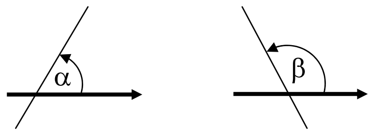
$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

*Average of the x's & of the y's*

### Parallel lines, Perpendicular lines & Collinearity

- AB || CD $\Leftrightarrow$ $m_{AB} = m_{CD}$
- AB ⊥ CD $\Leftrightarrow$ $m_{AB} = -\dfrac{1}{m_{CD}}$ . . . $m_{AB} = -\dfrac{1}{m_{CD}}$ also means: $m_{AB} \times m_{CD} = -1$
- A, B and C are collinear points $\Leftrightarrow$ $m_{AB} = m_{AC}$ ; $m_{AB} = m_{BC}$ ; $m_{AC} = m_{BC}$

### The Angle of Inclination of a line




The ∠ of inclination of a line is the ∠ which the line makes with the positive direction of the $x$-axis.

NB: If α or β is the angle of inclination (measured in degrees), then the gradient of the line = tan α or tan β (which is a ratio or number).

∴ Given α or β, one can find **the gradient**: . . . **a number**

Or, given the gradient, one can find α or β: . . . **an angle** (measured in degrees)

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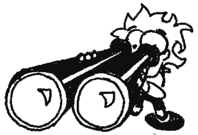
# Equations of lines

**NB:** Bear in mind **Case 1**, **Case 2,** and **Case 3** on p. 5.9.

- **Standard forms:**
  - General: $\mathbf{y = mx + c}$ or $\mathbf{y - y_1 = m(x - x_1)}$
  - $\mathbf{y = mx}$ . . . when c = 0 . . . lines through the origin
  - $\mathbf{y = c}$ . . . when m = 0 . . . lines || $x$-axis
  - $\mathbf{x = k}$ . . . lines || y-axis



Finding the equation of a line: Special focus
- through 2 given points . . . find m first
- through 1 point and || or ⊥ to a given line . . . substitute m and the point.

**the gradient**

$y = \mathbf{m}x + c$

the point

we need
- **the gradient** &
- **a point**

**the gradient**

$y - y_1 = \mathbf{m}(x - x_1)$

the point

- **Y-cuts and X-cuts:** Put $x = 0$ and $y = 0$, respectively.

- **Point of intersection of 2 graphs:**
  Solve the equations of the graphs simultaneously.

- **If a point lies on a line, the equation is true for it,**
  and, vice versa . . .

  **If a point satisfies the equation of a line, the point lies on the line.**

  e.g. If a line has the equation $y = x + 1$, then all points on the line can be represented by $(x; x + 1)$

## NOTES

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# ANALYTICAL GEOMETRY

## QUESTIONS

**Grade 10 & 11 (essential for Grade 12!)**

**FORMULAE (distance, midpoint, gradient), ∠ OF INCLINATION & STRAIGHT LINES**

**An extract from our Gr 12 Maths 2 in 1**

1. A(-1; 3), B(7; 1) and C($x$; 2) are points in a Cartesian plane. Calculate $x$ if:

   1.1 BC = $\sqrt{2}$ units   1.2 the gradient of BC is $\frac{1}{2}$. (6)(3)

   1.3 C is the midpoint of AB.   1.4 CB ⊥ $x$-axis (2)(1)

2. P(4; 3), Q(4; -1) and R(8; -1) – and the origin, O – are points on the Cartesian plane. Write down the following:

   **A SKETCH IS ESSENTIAL!**

   2.1 (a) the length of OP   2.2 (a) the midpoint of OP
   (b) the length of PQ   (b) the midpoint of PQ
   (c) the length of QR   (c) the midpoint of QR

   2.3 (a) the gradient of OP   2.4 (a) the equation of OP
   (b) the gradient of PQ   (b) the equation of PQ
   (c) the gradient of QR   (c) the equation of QR (12)

3.1 If M(2; -3) is the midpoint of PQ and the coordinates of point P are (3; 8), then determine the coordinates of Q. (5)

3.2 A(4; 8) and B(-3; 6) are points in a Cartesian plane. Determine:

3.2.1 the gradient of AB (2)

3.2.2 the gradient of CD if CD || AB (2)

3.2.3 the gradient of MN if MN ⊥ AB (2)

3.3 Prove that M(0; 1), N(1; -2) and P(2; -5) lie on a straight line (are collinear). (3)

3.4 Draw simple sketches of the following graphs:

(a) y = 2   (b) $x = 3$   (c) $y = x$

(d) $y = -2x$   (e) $y = x + 1$   (f) $y = -x + 2$

(g) $x + y + 1 = 0$   (h) $2x + 3y = 6$   (i) $\frac{x}{2} - \frac{y}{5} = 1$ (18)

4. A(-4; -1), R(2; 3) and M(6; -3) are the vertices of a triangle.

   4.1.1 Calculate the coordinates of S, the midpoint of AM. (2)

   4.1.2 Determine the equation of line RS. (3)

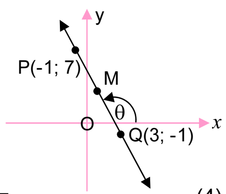
   4.1.3 Calculate the length of RA. (2)

   4.2.1 Show that ΔARM is right-angled. (4)

   4.2.2 Show that R$\hat{A}$M = 45°; giving reasons. (5)

   4.3 Calculate the area of ΔARM. (2)

5. P(1; 7) and Q(3; -1) are two points in a Cartesian plane. Determine:




   5.1 the length of PQ *(leave the answer in simplified surd form).* (3)

   5.2 the coordinates of M, the midpoint of PQ. (2)

   5.3 the equation of PQ, in the form y = . . . (4)

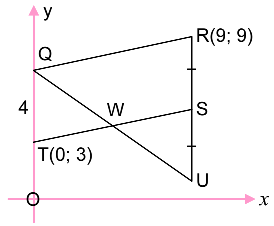
   5.4 the size of θ, the angle between PQ and the positive $x$-axis. (3)

   5.5 the equation of the line which is parallel to PQ and passes through the point (-5; 1). The equation must be in the form y = . . . (3)

6.1 Determine the angle that $2x + 3y = 5$ makes with the positive $x$-axis. *(Rounded off to one decimal digit.)* (4)

6.2 Determine the numerical value of p if the straight line defined by $2y = px + 1$ has an angle of inclination 135° with respect to the positive $x$-axis. (4)

7. In the figure, QRST is a parallelogram with vertices Q and T lying on the y-axis.




   The side RS is produced to U such that RS = SU.

   The length of QT is 4 units and the coordinates of R and T are (9; 9) and (0; 3) respectively.

   The line segment QU intersects TS at W.

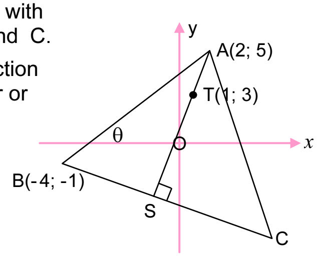
   7.1 Determine the coordinates of:

   7.1.1 Q   7.1.2 U (1)(2)

   7.2 Determine the equation of line OR. (2)

   7.3 Now, if W is the midpoint of UQ, determine whether W lies on line OR (i.e. whether O, W and R are *collinear*). (4)

8. In the figure ABC is a triangle with vertices A(2; 5), B(-4; -1) and C.




   T(1; 3) is the point of intersection of the altitudes (perpendicular or heights) from A, B and C.

   The inclination of AB to the $x$-axis is θ. The equation of the line passing through C and T is given by $y = -x + 4$.

   8.1 Determine the length of AB. *(Leave the answer in simplest surd form.)* (2)

   8.2 Calculate θ. (2)

   8.3 Write down the gradient of AS. (1)

Learners must know the definitions and properties of geometric figures.
Revise 'Quadrilaterals' at the back of the book.

   8.4 Show that the equation of BC is given by $x + 2y + 6 = 0$. (3)

   8.5 Hence, determine the coordinates of point C. (4)

9. Determine the gradient of a straight line if it was

   9.1 parallel to the line $3x + 2y + 6 = 0$, or

   9.2 perpendicular to the same line. (2)

10. A(1; 4), B(-2; -2) and C(4; 1) are the vertices of triangle ABC in a Cartesian plane.

   10.1 Determine the coordinates of the midpoint D of AB. (2)

   10.2 Find the equation of the perpendicular bisector of AB. (3)

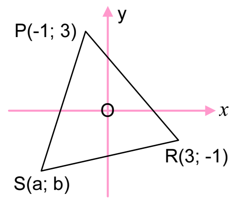
   10.3 If E is the midpoint of AC, determine the equation of BE. (2)

   10.4 Show that DE || BC. (3)

   10.5 Calculate the magnitude of A$\hat{B}$C . (5)

   10.6 Determine the coordinates of M so that AMBC, in this order, is a parallelogram. (2)

11. Triangle PRS has vertices P(-1; 3), R(3; -1) and S(a; b), as shown in the accompanying sketch.




   11.1 Show that T(1; 1) is the midpoint of PR is. (1)

   11.2 If the perpendicular bisector of PR passes through S, show that a = b. (4)

   11.3 If a < 0, b < 0 and the area of ΔPRS 12 square units, find the coordinates of S. (8)

   11.4 If Q(4; 4) lies on the perpendicular bisector, explain why PQRS is a rhombus. (4)

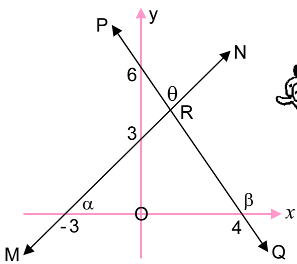
12. The vertices of ΔABE is A(0; 4), B(5; 3) and E(2; 1).

   12.1 Prove that $\hat{E}$ = 90°. (3)

   12.2 If ABCD is a rhombus with diagonals AC and BD intersecting at E, determine the coordinates of C and D. (4)

   12.3 Prove that ABCD is a square, giving reasons. (4)

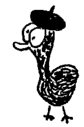
13. Determine:




   13.1 the size of α. (2)

   13.2 the size of β. (2)

   13.3 the size of θ. (1)



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# ANALYTICAL GEOMETRY

## ANSWERS

### Gr 10 & 11 (essential for Grade 12!)

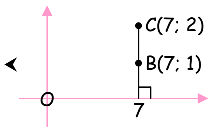
### FORMULAE (distance, midpoint, gradient), ∠ OF INCLINATION & STRAIGHT LINES

1.1 $BC^2 = (x-7)^2 + (2-1)^2 = 2$
$\therefore\ x^2 - 14x + 49 + 1 = 2$
$\therefore\ x^2 - 14x + 48 = 0$
$\therefore\ (x-8)(x-6) = 0$
$\therefore\ x = 8$ or $6$ ◄

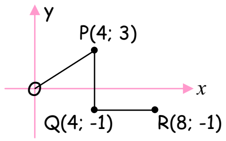
1.2 $m_{BC} = \dfrac{2-1}{x-7} = \dfrac{1}{2}$
$\therefore\ x - 7 = 2$
$\therefore\ x = 9$ ◄

1.3 $x = \dfrac{-1+7}{2} = \dfrac{6}{2} = 3$ ◄

1.4 $x = 7$ ◄




2.




You don't need any formulae!

2.1 (a) 5 units (b) 4 units (c) 4 units

2.2 (a) $\left(2; 1\frac{1}{2}\right)$ (b) (4; 1) (c) (6; -1)

2.3 (a) $\frac{3}{4}$ (b) undefined (c) 0

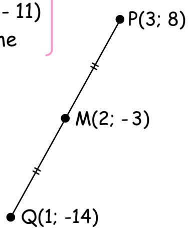
2.4 (a) $y = \frac{3}{4}x$ (b) $x = 4$ (c) $y = -1$

**Notice:**
- OP is from the origin
- P & Q have the same $x$-coordinates
  ➨ PQ ⊥ $x$-axis (or || y-axis)
- Q & R have the same y-coordinates
  ➨ QR || $x$-axis (or ⊥ y-axis)

3.1 By inspection:
**Q(1; -14)** ◄

The translation
P → M is $(x-1; y-11)$
∴ M → Q is the same

OR: $\dfrac{3+x_Q}{2} = 2$ & $\dfrac{8+y_Q}{2} = -3$
$\therefore\ 3 + x_Q = 4$ $\therefore\ 8 + y_Q = -6$
$\therefore\ x_Q = 1$ $\therefore\ y_Q = -14$
$\therefore$ **Q(1; -14)** ◄



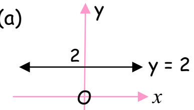

3.2.1 $m_{AB} = \dfrac{6-8}{-3-4} = \dfrac{-2}{-7} = \dfrac{2}{7}$

3.2.2 $m_{CD} = \dfrac{2}{7}$

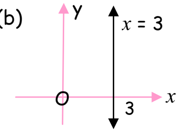
3.2.3 $m_{MN} = -\dfrac{7}{2}$

3.3 $m_{MN} = \dfrac{-2-1}{1-0} = \dfrac{-3}{1} = -3$ & $m_{MP} = \dfrac{-5-1}{2-0} = \dfrac{-6}{2} = -3$ as well
∴ M, N & P are collinear

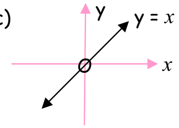
3.4 (a)



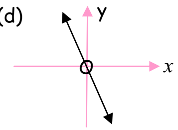

(b)



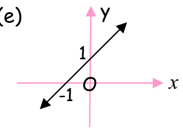

(c)



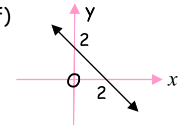

(d)




(e)



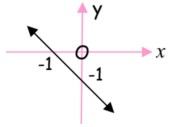

(f)



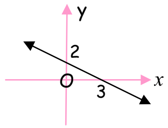

**(g) & (h): There is no need to write these equations in standard form!**

(g) $x + y + 1 = 0$
y-intercept: *(when $x = 0$)* $y = -1$
$x$-intercept: *(when $y = 0$)* $x = -1$

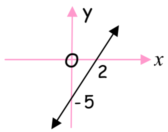
**Use the "dual-intercept" method**



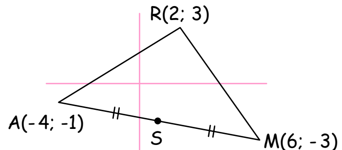

(h) $2x + 3y = 6$
y-intercept: *(when $x = 0$)* $y = 2$
$x$-intercept: *(when $y = 0$)* $x = 3$




(i) $\dfrac{x}{2} - \dfrac{y}{5} = 1$
y-intercept: *(when $x = 0$)* $y = -5$
$x$-intercept: *(when $y = 0$)* $x = 2$




4.1.1 $S\left(\dfrac{-4+6}{2}; \dfrac{-1-3}{2}\right)$
∴ S(1; -2)




4.1.2 $m_{RS} = \dfrac{3-(-2)}{2-1} = \dfrac{5}{1} = 5$

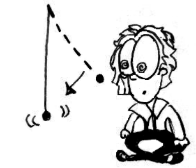
∴ Equation of RS: Substitute m = 5 & point (2; 3) in

**y = mx + c**
$\therefore\ 3 = (5)(2) + c$
$\therefore\ -7 = c$
$\therefore$ Eqn.: $y = 5x - 7$ ◄

OR

**$y - y_1 = m(x - x_1)$**
$y - 3 = 5(x-2)$
$\therefore\ y - 3 = 5x - 10$
$\therefore\ y = 5x - 7$ ◄

4.1.3 $RA^2 = (2+4)^2 + (3+1)^2$
$= 6^2 + 4^2$
$= 36 + 16$
$= 52$
$\therefore\ RA = \sqrt{52} \simeq 7{,}21$ **units** ◄



4.2.1 $m_{AR} = \dfrac{3-(-1)}{2-(-4)} = \dfrac{4}{6} = \dfrac{2}{3}$
& $m_{RM} = \dfrac{3-(-3)}{2-6} = \dfrac{6}{-4} = -\dfrac{3}{2}$
$\therefore\ m_{AR} \times m_{RM} = -1$
$\therefore\ A\hat{R}M = 90°$
i.e. **ΔARM is right-angled** ◄

4.2.2 We have $RA = \sqrt{52}$ *. . . in 4.1.3*
& $A\hat{R}M = 90°$ *. . . in 4.2.1*
Now, $RM^2 = (2-6)^2 + (3+3)^2$
$= 16 + 36$
$= 52$
$\therefore\ RM = \sqrt{52}$ (= RA!!)
∴ ΔARM is an isosceles right-angled Δ
$\therefore$ **$R\hat{A}M = 45°$** ◄

4.3 Area $= \frac{1}{2}$ AR.RM . . . $\left[\frac{1}{2}bh\right]$
$= \frac{1}{2}\sqrt{52}\sqrt{52}$
$=$ **26 square units** ◄

5.1 $PQ^2 = (3+1)^2 + (1-7)^2$
$= 16 + 64$
$= 80$
$\therefore\ PQ = \sqrt{80} = \sqrt{16 \times 5} = \sqrt{16}\sqrt{5} = 4\sqrt{5}$ **units** ◄

5.2 $M\left(\dfrac{-1+3}{2}; \dfrac{7+(-1)}{2}\right)$ $\therefore$ **M(1; 3)** ◄

5.3 $m_{PQ} = \dfrac{-1-7}{3-(-1)} = \dfrac{-8}{4} = -2$

Subst. P(-1; 7) & m = -2 in **y = mx + c**
$\therefore\ 7 = (-2)(-1) + c$
$\therefore\ 7 = 2 + c$
$\therefore\ 5 = c$

**OR:**
**$y - y_1 = m(x - x_1)$**

∴ Equation of PQ: $y = -2x + 5$ ◄

5.4 $\tan\theta = m_{PQ} = -2$
$\therefore\ \theta = 180° - 63{,}4°$
$=$ **116,6°** ◄

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5.5 Substitute $m = -2$ & $(-5; 1)$ in

$\mathbf{y - y_1 = m(x - x_1)}$ . . . or, $\mathbf{y = mx + c}$

$\therefore\ y - 1 = -2(x + 5)$

$\therefore\ y - 1 = -2x - 10$

$\therefore\ y = -2x - 9$

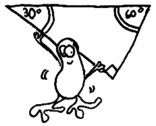
6.1 $2x + 3y = 5$

➡ $\therefore\ 3y = -2x + 5$

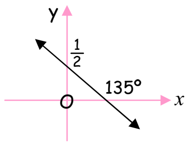
$\therefore\ y = -\frac{2}{3}x + \frac{5}{3}$

$\therefore$ Gradient $= \tan\theta = -\frac{2}{3}$

$\therefore\ \theta \simeq 180° - 33{,}7° = \mathbf{146{,}3°}$ ◄



6.2



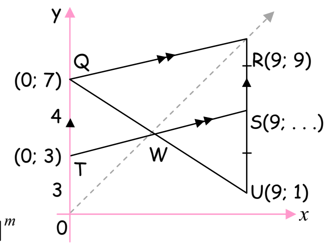

$2y = px + 1$

$\therefore\ y = \frac{p}{2}x + \frac{1}{2}$

$\therefore\ \frac{p}{2} = \tan 135° = -1$

$\therefore\ \mathbf{p = -2}$ ◄

7.1.1 **Q(0; 7)** ◄




7.1.2 SU = RS . . . *given*

= (QT =) 4 . . . *opp. sides of* $||^m$

$\therefore$ UR = 8 units

& UR || y-axis . . . *opp. sides of* $||^m$

$\therefore\ x_U = x_R$

$\therefore$ **U(9; 1)** ◄

7.2 $y = x$ ◄ . . . it goes through (0; 0) and (9; 9) i.e $y = x$!

or "m" $= \frac{9}{9} = 1$ & "c" = 0

7.3 The gradient of OW $= \frac{4}{4\frac{1}{2}} \simeq 0{,}9$

whereas the gradient of OR = 1

$m_{OR} \neq m_{OW}$ ➡ O, W & R are **not** collinear

$\therefore$ **W does not lie on OR** ◄

**OR,** For W, $y \neq x$ . . . $y = 4$ *and* $x = 4\frac{1}{2}$

$\therefore$ **W does not lie on OR** ◄ . . . *its coordinates don't satisfy the equation of OR*

8.1 $AB^2 = (2 + 4)^2 + (5 + 1)^2$

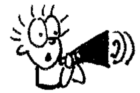
$= 36 + 36 \quad (= 72)$

$\therefore\ AB = \sqrt{36 \times 2} = \sqrt{36}\sqrt{2} = \mathbf{6\sqrt{2}\ units}$ ◄

8.2 $m_{AB} = \frac{5 - (-1)}{2 - (-4)} = \frac{6}{6} = 1 \quad (= \tan\theta)$

$\therefore\ \theta = \mathbf{45°}$ ◄

8.3 $m_{AS} = 2$ . . . $\frac{5 - 3}{2 - 1}$



8.4 Substitute $m_{BC} = -\frac{1}{2}$ & point B(- 4; -1)

in $\mathbf{y = mx + c}$ or $\mathbf{y - y_1 = m(x - x_1)}$

$\therefore\ -1 = \left(-\frac{1}{2}\right)(-4) + c$ $\qquad$ $\therefore\ y + 1 = -\frac{1}{2}(x + 4)$

$\therefore\ -1 = 2 + c$ $\qquad$ $\therefore\ y + 1 = \frac{1}{2}x - 2$

$\therefore\ -3 = c$ $\qquad$ $\therefore\ y = -\frac{1}{2}x - 3$, etc.

$\therefore$ Equation of BC is $y = -\frac{1}{2}x - 3$

$(\times 2) \quad \therefore\ 2y = -x - 6$

$\therefore\ \mathbf{x + 2y + 6 = 0}$ ◄

8.5 At C, $y = -\frac{1}{2}x - 3$ . . . *equation of BC*

& $y = -x + 4$ . . . *equation of CT*

$\therefore\ -\frac{1}{2}x - 3 = -x + 4$

$\therefore\ \frac{1}{2}x = 7$

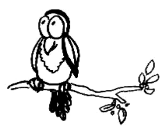
$\therefore\ x = 14$ & $y = -14 + 4 = -10$

$\therefore$ **C(14; -10)** ◄

9. $3x + 2y + 6 = 0$ has y-intercept, -3

& x-intercept, -2

$\therefore$ Gradient $= -\frac{3}{2}$



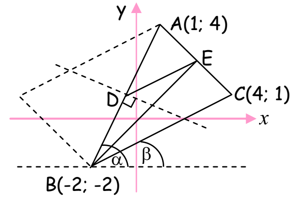
**OR:** Standard form: $2y = -3x - 6$

$\therefore\ y = -\frac{3}{2}x - 3$

9.1 $\therefore$ Gradient of parallel line $= \mathbf{-\frac{3}{2}}$ ◄

9.2 & Gradient of perpendicular line $= \mathbf{+\frac{2}{3}}$ ◄

10.1 $D\left(-\frac{1}{2}; 1\right)$ ◄




10.2 $m_{AB} = \frac{4 - (-2)}{1 - (-2)} = \frac{6}{3} = 2$

$\therefore\ m_{\text{perpendicular bisector}} = -\frac{1}{2}$

$\therefore\ D\left(-\frac{1}{2}; 1\right)$ & $m = -\frac{1}{2}$ in $\mathbf{y - y_1 = m(x - x_1)}$:

$y - 1 = -\frac{1}{2}\left(x + \frac{1}{2}\right)$

$\therefore\ y - 1 = -\frac{1}{2}x - \frac{1}{4}$

$\therefore\ y = -\frac{1}{2}x + \frac{3}{4}$

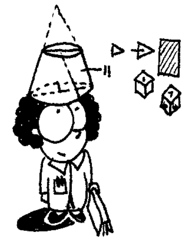
10.3 Equation of BE:

E midpoint AC ➡ $E\left(2\frac{1}{2}; 2\frac{1}{2}\right)$

$\therefore\ m_{BE} = \frac{2\frac{1}{2} + 2}{2\frac{1}{2} + 2} = \frac{4\frac{1}{2}}{4\frac{1}{2}} = 1$

& (-2; -2): $\quad \therefore\ y + 2 = 1(x + 2)$

$\therefore\ y = x$



10.4 $m_{DE} = \frac{2\frac{1}{2} - 1}{2\frac{1}{2} - \left(-\frac{1}{2}\right)} = \frac{1\frac{1}{2}}{3} = \frac{1}{2}$

& $m_{BC} = \frac{1 - (-2)}{4 - (-2)} = \frac{3}{6} = \frac{1}{2}$

$\therefore$ **DE || BC** ◄ . . . *equal gradients*

10.5 $m_{BC} = \frac{1 + 2}{4 + 2} = \frac{3}{6} = \frac{1}{2}$ ➡ $\beta = 26{,}57°$

& $m_{AB} = 2$ ➡ $\alpha = 63{,}43°$

$\therefore\ A\hat{B}C = \alpha - \beta = 63{,}43° - 26{,}57° = \mathbf{36{,}86°}$ ◄

10.6 **M(-5; 1)** ◄

The translation, $C \to A$ is: $(x; y) \to (x - 3; y + 3)$

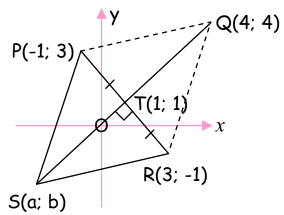
$\therefore$ Also $B \to M$ . . . *opposite sides of a parallelogram are equal and parallel*

$\therefore$ M(-2 -3; -2 + 3)

11.1 Midpoint of PR is

$T\left(\frac{-1 + 3}{2}; \frac{3 + (-1)}{2}\right)$

$\therefore$ **T(1; 1)** ◄

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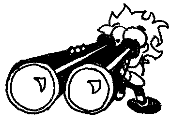
11.2 $m_{PR} = \frac{-1-3}{3-(-1)}$

$= \frac{-4}{4}$

$= -1$



$\therefore\ m_{\text{perpendicular bisector}} = 1$

Substitute $m = 1$ & point (1; 1) in $\mathbf{y - y_1 = m(x - x_1)}$

$\therefore\ y - 1 = 1(x - 1)$

$\therefore\ y = x$

At S, **a = b** ◄ . . . *S on line y = x*

11.3 Area of $\Delta PRS = \frac{1}{2}PR\,.\,ST = 12$

Now, $PR = \sqrt{(3+1)^2 + (-1-3)^2}$

$= \sqrt{16+16}$

$= \sqrt{16 \times 2}$

$= 4\sqrt{2}$

& $ST = \sqrt{(a-1)^2 + (a-1)^2}$ . . . *(b = a in 11.2)*

$= \sqrt{2(a-1)^2}$ . . . $\boxed{= \sqrt{2}\,.\sqrt{(a-1)^2}\ !!}$

$\therefore\ \frac{1}{2}.4\sqrt{2}\,.\sqrt{2}\,\sqrt{(a-1)^2} = 12$

$\therefore\ 4\sqrt{(a-1)^2} = 12$

$\therefore\ \sqrt{(a-1)^2} = 3$

$\therefore\ (a-1)^2 = 9$

$\therefore\ a - 1 = \pm 3$

$\therefore\ a = 1 \pm 3$

$\therefore\ a = -2$ . . . $(\because a < 0)$

$\therefore$ **S(-2 ; -2)** ◄ *(a = b)*

11.4 The other solution to "a" in 11.3 is $a = 1 + 3 = 4$ (& $b = a = 4$)

$\therefore$ For Q(4; 4), also on the $\perp^r$ bisector of PR (like S), ΔPRQ also has an area of 12 square units.

$\therefore$ TQ = TS

**OR:** $ST = \sqrt{(-2-1)^2 + (-2-1)^2} = \sqrt{18}$

& $TQ = \sqrt{(4-1)^2 + (4-1)^2} = \sqrt{18}$

$\therefore$ ST = TQ

**OR:** The midpoint of SQ is $\left(\frac{-2+4}{2}; \frac{-2+4}{2}\right)$ $\therefore$ T(1; 1)

$\therefore$ The diagonals bisect one another ($\therefore\ ||^m$)

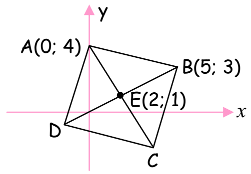
& they do so perpendicularly $\therefore$ PQRS is a rhombus

12.1 $m_{AE} = \frac{1-4}{2-0} = \frac{-3}{2}$

& $m_{EB} = \frac{3-1}{5-2} = \frac{2}{3}$

$\therefore\ m_{AE} \times m_{EB} = -1$

$\therefore\ AE \perp EB$, i.e. $\hat{E}$ = **90°** ◄




12.2 C(4; - 2) . . . *E midpoint AC*

& D(-1; -1) . . . *E midpoint BD*

12.3 We just need to prove 1 angle = 90° – it's already a rhombus!

$m_{DA} = \frac{4-(-1)}{0-(-1)} = \frac{5}{1} = 5$

& $m_{AB} = \frac{3-4}{5-0} = \frac{-1}{5} = -\frac{1}{5}$

$\therefore\ m_{DA} \times m_{AB} = -1$

$\therefore\ DA \perp AB$, i.e. $D\hat{A}B = 90°$

$\therefore$ **ABCD is a square** ◄ . . . A rhombus with one angle of 90°

13.1 $m_{MN} = +1$

$\therefore\ \alpha$ = **45°** ◄

13.2 $m_{PQ} = -\frac{6}{4} = -1{,}5$

$\therefore\ \beta = 180° - 56{,}3°$ = **123,7°** ◄

13.3 $\theta = M\hat{R}Q$ . . . *vertically opposite* $\angle^s$

$= \beta - \alpha$ . . . *exterior* $\angle$ *of* $\Delta$

= **78,7°** ◄

**NOTES**