GR 12 MATHS

FUNCTIONS

QUESTIONS and ANSWERS

Work through the Grade 11 Functions downloads first to ensure your foundation is solid before attempting inverse Functions.

We wish you the best of luck for your exams.

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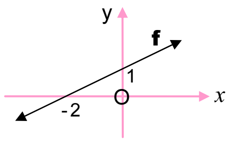
INVERSE FUNCTIONS

(Gr 12 only)

QUESTIONS

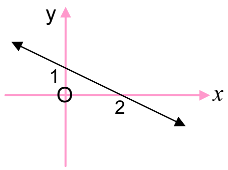
- Concepts and techniques involving the general characteristics of functions (first part of this topic) should be thoroughly mastered before this section on inverse functions. In particular, work through Questions 2 → 7.
- **NB:** The INVERSE of a function REVERSES the process of a function.

1. The graph of **f** is . . .



1.1 If the inverse of **f** is the reflection of **f** in the line y = x, then the graph of the inverse is:

A.



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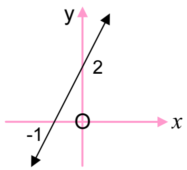

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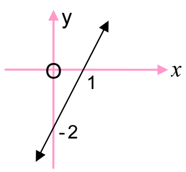
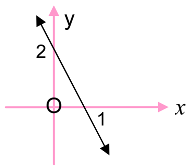


D.



(2)

1.2 Consider the graph of **f** given above.

1.2.1 Write down the equation of the given function and of the inverse function in the form y = . . . (2)

1.2.2 Hence complete: (a) $\mathbf{f}(x) = \ldots$ (b) $\mathbf{f}^{-1}(x) = \ldots$ (2)

1.2.3 Show how the equation of $\mathbf{f}^{-1}$ could have been calculated from the equation of **f**. (2)

1.2.4 Explain why **f** and $\mathbf{f}^{-1}$ are both one-to-one relations. (2)

2. Consider the function $\mathbf{f}(x) = -3x + 6$.

2.1 Write down the domain and range of **f**. (2)

2.2 Determine the equation of the inverse of **f** in the form $\mathbf{f}^{-1}(x) = \ldots$ (2)

2.3 Sketch the graphs of the functions **f**, $\mathbf{f}^{-1}$ and $y = x$ on the same set of axes. What do you notice? (3)

2.4 If $\mathbf{f}(1) = 3$, then $\mathbf{f}^{-1}(3) = \ldots$? (1)

2.5 $\mathbf{f}(2) = 0$ ➡ (;) lies on **f** and (;) lies on $\mathbf{f}^{-1}$. (2)

3.1 Write down the coordinates of the x- and y-intercepts of the function $\mathbf{f}(x) = 2x + 6$ and of $\mathbf{f}^{-1}$, the inverse function of **f**. (4)

3.2 Sketch the graphs of **f** and $\mathbf{f}^{-1}$ on the same set of axes, indicating also the line $y = x$. (4)

3.3 Write down the equation of $\mathbf{f}^{-1}$ in the form $\mathbf{f}^{-1}(x) = \ldots.$ (2)

3.4 Are **f** and $\mathbf{f}^{-1}$ both functions? Why (not)? (2)

4. Given: $\mathbf{g}(x) = 3x - 2$. Determine each of the following:

4.1 $\mathbf{g}^{-1}(x)$ 4.2 $\dfrac{1}{\mathbf{g}(x)}$ 4.3 $\mathbf{g}\left(\dfrac{1}{x}\right)$ (6)

5.1 Sketch the graph $y = 2^x$, indicating the coordinates of any three points on the graph. (3)

5.2 Use the three points on the sketch to write down the coordinates of three points on the inverse function of $y = 2^x$. (3)

5.3 On the same system of axes, sketch the inverse function of $y = 2^x$ and the line $y = x$. (3)

5.4 Describe the transformation from the graph $y = 2^x$ to its inverse in words and give the rule for this transformation. (2)

5.5 Write down the equation of the inverse function in the form $x = \ldots$
[In Topic 4, you will convert this equation to y = . . .] (2)

5.6 Are both the above graphs functions? Why (not)? (2)

5.7 Write down the domain and the range of the graphs of:
(a) $y = 2^x$ (b) $x = 2^y$ (4)

6. Consider the function **f** where $\mathbf{f}(x) = 2x^2$

6.1 Write down the domain and range of **f**. (2)

6.2 Sketch the graph of **f** and **g** on the same set of axes where **g** is the reflection of **f** in the line $y = x$.
Draw the line $y = x$ on the sketch. (3)

6.3 Determine the equation of **g** in the form y = . . . (2)

6.4 Is **g** the inverse function of **f**? Why (not)? (2)

6.5 (a) Name 2 ways in which the domain of **f** could be restricted to ensure that the inverse is a function. Sketch the 2 cases. (4)

(b) Determine $\mathbf{f}^{-1}(x)$ in each case. Sketch the 2 cases. Include the line $y = x$. (4)

(c) Determine the domain and the range of $\mathbf{f}^{-1}$ in each case. (4)

7. Consider the function **f** where $\mathbf{f}(x) = -x^2$ and $x \geq 0$.

7.1 Write down the domain and range of **f**. (2)

7.2 Determine the inverse function $\mathbf{f}^{-1}$ in the form $\mathbf{f}^{-1}(x) = \ldots$ (2)

7.3 Sketch the graphs of the functions **f**, $\mathbf{f}^{-1}$ and the line $y = x$ on the same set of axes. What do you notice? (4)

7.4 Is $\mathbf{f}^{-1}$ an increasing or a decreasing function? (1)

8.1 Write down the coordinates of the reflections of the following points in the line $y = x$:
P(0; 0), Q(-1;1), R(2; 4) and S(3; 9).
Let the images be P′, Q′, R′ and S′ respectively. (4)

8.2 Draw a sketch of the graph **f** which has equation $y = x^2$ for $x \geq 0$. (2)

8.3 Which of the points in Question 8.1 lie on the graph of **f**? (2)

8.4 **f** and its inverse function, $\mathbf{f}^{-1}$, are reflections in the line. . .? (1)

8.5 Draw the graph $\mathbf{f}^{-1}$, the inverse function of **f**, on the same system of axes. (2)

8.6 Which of the points in Question 8.1 (question or answer) lie on the graph of $\mathbf{f}^{-1}$? (2)

8.7 Complete: (a) $\mathbf{f}(3) = \ldots$

(b) $\mathbf{f}^{-1}(9) = \ldots$

(c) $\mathbf{f}(2) = 4$ ➡ (;) lies on **f**

(d) $\mathbf{f}^{-1}(4) = 2$ ➡ (;) lies on $\mathbf{f}^{-1}$ (4)

8.8 Determine the equation of $\mathbf{f}^{-1}$ in the form $\mathbf{f}^{-1}(x) = \ldots$ (2)

8.9 Is $\mathbf{f}^{-1}$ a function? Give a reason for your answer. (2)

9. If $\mathbf{f}(x) = (x + 2)^2$; $x \leq -2$, then $\mathbf{f}^{-1}(x)$ is equal to

A. $x^2 - 2$ B. $\pm\sqrt{x} - 2$ C. $\sqrt{x} - 2$ D. $-\sqrt{x} - 2$ (2)

10. Given $\mathbf{h}^{-1}(x) = -\sqrt{x}$. Then the equation of **h** is y = . . .

A. x^2; $x \leq 0$ B. x^2; $x \geq 0$ C. x^2 D. $\sqrt{x}$ (2)

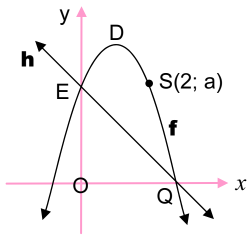
11. Given: $\mathbf{g}(x) = -1 + \sqrt{x}$.

Determine the inverse of $\mathbf{g}(x)$ in the form $\mathbf{g}^{-1}(x) = \ldots$ (4)

12. $\mathbf{f}(x) = -2(x - 3)(x + 1)$ and
$\mathbf{h}(x) = mx + c$

The graphs of **f** and **h** have a common x-intercept at Q and a common y-intercept at E.

The turning point of **f** is at D and S(2; a) is a point on **f**.



12.1 Calculate the coordinates of E and D. (4)

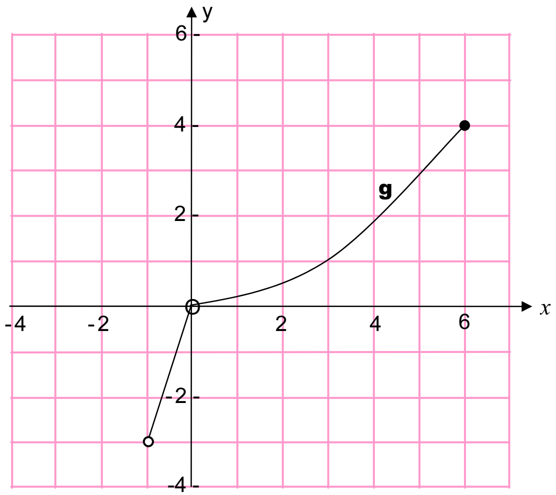
12.2 Write down the coordinates of Q. (1)

12.3 Determine the numerical values of:
(a) m (b) a (2)(2)

12.4 Write down the coordinates of the turning point of $\mathbf{f}^{-1}$, the inverse of **f**. (2)

12.5 Is $\mathbf{f}^{-1}$ a function? Why (not)? (2)

13. The graph of the function **g** is shown below.



13.1 Determine the domain and range of the function. (2)

13.2 On this set of axes, draw the graph of the inverse function of **g**. (4)

13.3 Explain why this inverse is a function. (1)

- *Now do Paper E1 Q10 in Section 2 of this book.*
- *The Topic Guide indicates the examples in all the papers.*
- *See the end of Topic 4 for mixed questions (including exponential and log functions) on inverse functions.*

NOTES

INVERSE FUNCTIONS
(Gr 12 only)
ANSWERS

1. The given graph has points (-2; 0) & (0; 1)
   ∴ The graph of the inverse has points (0; -2) & (1; 0)
   ∴ **C** ◄

1.2.1 Given: $y = \frac{1}{2}x + 1$ ◄ C: $y = 2x - 2$ ◄

1.2.2 (a) $\mathbf{f}(x) = \frac{1}{2}x + 1$ ◄ (b) $\mathbf{f}^{-1}(x) = 2x - 2$ ◄

1.2.3 Equation of **f**: $y = \frac{1}{2}x + 1$

∴ Equation of $\mathbf{f}^{-1}$: $x = \frac{1}{2}y + 1$ . . . *swop x & y*

$\times 2)$ ∴ $2x = y + 2$

∴ $\mathbf{y = 2x - 2}$ ◄ . . . *make y the subject*

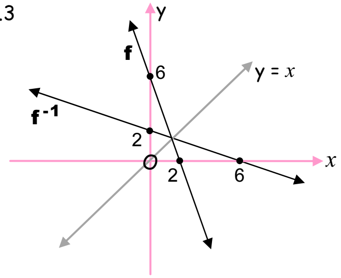
1.2.4 For each value of x, there is only one y-value for both graphs.

2.1 Domain: $x \in \mathbb{R}$ ◄ & Range: $y \in \mathbb{R}$ ◄

2.2 Equation of **f**: $y = -3x + 6$
∴ Equation of $\mathbf{f}^{-1}$: $x = -3y + 6$
∴ $3y = -x + 6$
∴ $y = -\frac{x}{3} + 2$

∴ $\mathbf{f^{-1}(x) = -\frac{x}{3} + 2}$ ◄ . . . $f(x) = -\frac{1}{3}x + 2$

2.3



We notice that **f** and $\mathbf{f}^{-1}$ are reflections in the line $y = x$.

2.4 $\mathbf{f}(1) = 3$ ➜ $\mathbf{f}^{-1}(3) = 1$ ◄

2.5 $\mathbf{f}(2) = 0$ ➜ (2 ; 0) lies on **f** and (0 ; 2) lies on $\mathbf{f}^{-1}$.

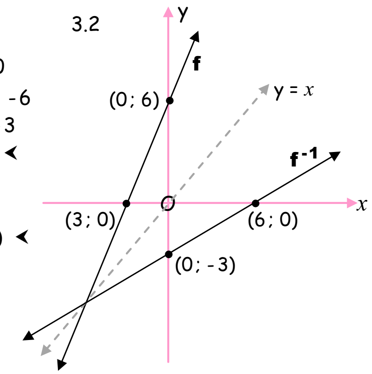
NB: The **INVERSE** of a function reverses the process of a function.
So: • **f** maps $x = 1$ onto $y = 3$ whereas $\mathbf{f}^{-1}$ maps $x = 3$ onto $y = 1$
• (2; 0) lies on **f** and (0; 2) lies on $\mathbf{f}^{-1}$
That is why x and y are swopped to determine the graph (and the equation) of the inverse of a function.

3.1 **f:** $y = 2x + 6$

x-int.: $2x + 6 = 0$
(put y = 0) ∴ $2x = -6$
∴ $x = -3$
∴ **(-3; 0)** ◄

y-int.: $y = 6$
(put x = 0) ∴ **(0; 6)** ◄

3.2



∴ For $\mathbf{f}^{-1}$ (the inverse function)
x-int.: (6; 0) ◄
y-int.: (0; -3) ◄ . . .

f *and* $\mathbf{f}^{-1}$ *are reflections in the line y = x. So, swop x and y.*

3.3 Equation of $\mathbf{f}^{-1}$: $y = mx + c$ where $m = +\frac{1}{2}$ & $c = -3$

∴ $y = \frac{1}{2}x - 3$ *by inspection on the sketch*

∴ $\mathbf{f^{-1}(x) = \frac{1}{2}x - 3}$ ◄

OR: Swop x and y in $y = 2x + 6$ and then make y the subject.

3.4 **Yes** ◄ ; for both **f** and $\mathbf{f}^{-1}$, each x-value is associated with only one y-value. ◄

4.1 Equation of **g**: $y = 3x - 2$
∴ Equation of $\mathbf{g}^{-1}$: $x = 3y - 2$
∴ $x + 2 = 3y$
$\div 3)$ ∴ $y = \frac{x}{3} + \frac{2}{3}$

∴ $\mathbf{g}^{-1}(x) = \frac{x}{3} + \frac{2}{3}$ ◄ . . . *a straight line* $[g^{-1}(x) = \frac{1}{3}x + \frac{2}{3}]$

4.2 $\frac{1}{\mathbf{g}(x)} = \frac{1}{3x - 2}$ ◄ . . . *a hyperbola!*

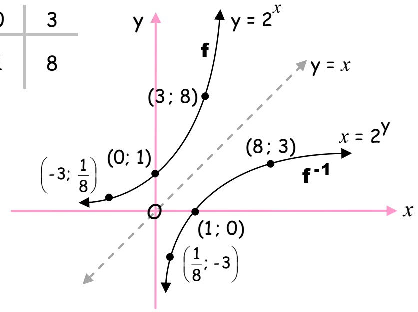
NB: $\mathbf{f}^{-1}(x) \neq \frac{1}{\mathbf{f}(x)}$
$\mathbf{f}^{-1}$ *is the* **inverse** *of* **f** *(not of* $\mathbf{f}(x)$*)*.

4.3 $\mathbf{g}\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right) - 2$

∴ $\mathbf{g}\left(\frac{1}{x}\right) = \frac{3}{x} - 2$ . . . *a hyperbola!*

5.1 & 5.3

x	-3	0	3
$y = 2^x$	$\frac{1}{8}$	1	8



5.2 (8; 3), (1; 0) and $\left(\frac{1}{8}; -3\right)$ ◄

5.4 **f** is reflected in the line $y = x$ to produce the image $\mathbf{f}^{-1}$ ◄
The rule: **(x; y) → (y; x)** ◄

5.5 $x = 2^y$ . . . *In Topic 4, you will convert this to $y = \log_2 x$, i.e. $\mathbf{f}^{-1}(x) = \log_2 x$, by using the definition of a log.*

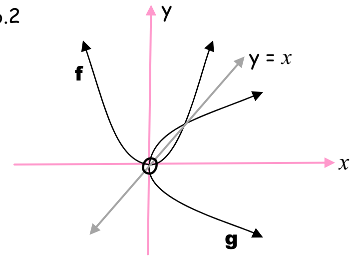
5.6 Yes; for every x-value, there is only one y-value.
[They are both one-to-one relations.]

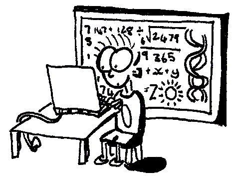
5.7 (a) Domain: $x \in \mathbb{R}$ & Range: $y > 0$; $y \in \mathbb{R}$ ◄
(b) Domain: $x > 0$; $x \in \mathbb{R}$ & Range: $y \in \mathbb{R}$ ◄

Note: The domains and ranges are swopped.

6.1 Domain: $x \in \mathbb{R}$ ◄ ; Range: $y \geq 0$; $y \in \mathbb{R}$ ◄

6.2





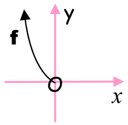
6.3 Equation of **f**: $y = 2x^2$
∴ Equation of **g**: $x = 2y^2$
∴ $2y^2 = x$
∴ $y^2 = \frac{x}{2}$
∴ $y = \pm\sqrt{\frac{x}{2}}$ ◄

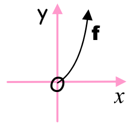
6.4 No; **g** is the inverse, but **not** the inverse **function**, because it is not a function. It is a one-to-many relation. ◄

Note: A vertical line could cut **g** more than once.

6.5 (a) See the 2 ways in (1) and (2) below.

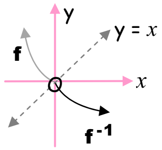
(1) Consider $y = 2x^2$; $x \leq 0$ (2) Consider $y = 2x^2$; $x \geq 0$

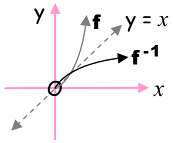




(b) Eqn. of **f**: $y = 2x^2$; $x \leq 0$ $y = 2x^2$; $x \geq 0$

$\therefore$ $\mathbf{f^{-1}}(x) = -\sqrt{\frac{x}{2}}$ ➤ $\therefore$ $\mathbf{f^{-1}}(x) = +\sqrt{\frac{x}{2}}$ ➤





(c) For $\mathbf{f^{-1}}$:   For $\mathbf{f^{-1}}$:

Domain: $x \geq 0$; $x \in \mathbb{R}$ ➤   Domain: $x \geq 0$; $x \in \mathbb{R}$ ➤

Range: $y \leq 0$; $y \in \mathbb{R}$ ➤   Range: $y \geq 0$; $y \in \mathbb{R}$ ➤

7.1 Domain of **f**: $x \geq 0$; $x \in \mathbb{R}$ ➤

Range of **f**: $y \leq 0$; $y \in \mathbb{R}$ ➤

7.2 The equation of **f**: $y = -x^2$; $x \geq 0$

$\therefore$ The eqn. of the inverse of **f**: $x = -y^2$ . . . *swop x & y*

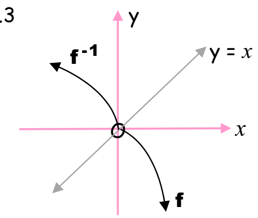
$\therefore$ $y^2 = -x$ . . . *make y the subject*

$\therefore$ $y = \pm\sqrt{-x}$; but $y \geq 0$

$\therefore$ The equation of $\mathbf{f^{-1}}$ (the inverse, which is a function):

$y = +\sqrt{-x}$ ➤

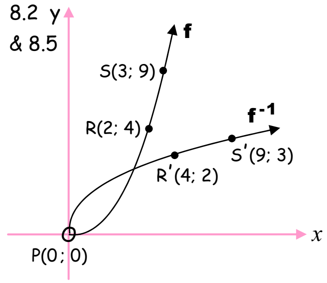
7.3



7.4 A decreasing function.

8.1 P′(0 ; 0), Q′(1 ; -1), R′(4 ; 2) and S′(9 ; 3) ➤

8.2 & 8.5



8.3 **P(0; 0), R(2; 4) & S(3; 9)** ➤

8.4 **y = *x*** ➤

8.5 See sketch.

8.6 **P′(0; 0), R′(4; 2) & S′(9; 3)** ➤

8.7 (a) $\mathbf{f}(3) = 9$ ➤   (c) $\mathbf{f}(2) = 4$ ➡ (2 ; 4) lies on **f** ➤

(b) $\mathbf{f^{-1}}(9) = 3$ ➤   (d) $\mathbf{f^{-1}}(4) = 2$ ➡ (4 ; 2) lies on $\mathbf{f^{-1}}$ ➤

8.8 Equation of **f**: $y = x^2$; $x \geq 0$

$\therefore$ Equation of $\mathbf{f^{-1}}$: $x = y^2$

$\therefore$ $y^2 = x$

$\therefore$ $y = \pm\sqrt{x}$; but $y \geq 0$

$\therefore$ $y = \sqrt{x}$ . . . i.e. $y = +\sqrt{x}$

$\therefore$ $\mathbf{f^{-1}}(x) = \sqrt{x}$ ➤

8.9 Yes, $\mathbf{f^{-1}}$ is a function because for every $x \geq 0$ (the given domain) there is only 1 value of y.

Note: $y = \pm\sqrt{x}$, the inverse of $y = x^2$, is not a function. The restriction on the values of x for **f**, i.e. $x \geq 0$, maps onto the values of y for $\mathbf{f^{-1}}$. (See the sketch above).

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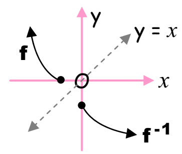
9. Equation of **f**: $y = (x + 2)^2$; $x \leq -2$

$\therefore$ Equation of $\mathbf{f^{-1}}$: $x = (y + 2)^2$

$\therefore$ $y + 2 = \pm\sqrt{x}$

$\therefore$ $y = \pm\sqrt{x} - 2$

But $y \leq -2$: $\therefore$ $y = -\sqrt{x} - 2$ $\therefore$ **D** ➤

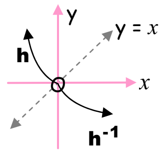


10. Equation of $\mathbf{h^{-1}}$: $y = -\sqrt{x}$; $y \leq 0$

$\therefore$ Equation of **h**: $x = -\sqrt{y}$

$\therefore$ $x^2 = y$

$\therefore$ $y = x^2$; $x \leq 0$ $\therefore$ **A** ➤



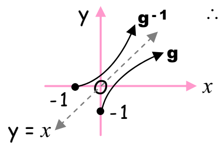
11. Equation of **g**: $y = -1 + \sqrt{x}$; $x \geq 0$

Note: $y \geq -1$ because $\sqrt{x} \geq 0$

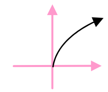
$\therefore$ Equation of $\mathbf{g^{-1}}$: $x = -1 + \sqrt{y}$

$\therefore$ $\sqrt{y} = x + 1$

$\therefore$ $\mathbf{y = (x + 1)^2}$; $\mathbf{x \geq -1}$ ➤

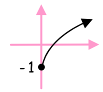


Note:

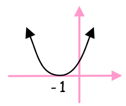


Graph of $y = \sqrt{x}$ &

g:

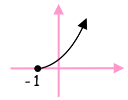


Graph of $y = \mathbf{-1} + \sqrt{x}$



Graph of $y = (x + 1)^2$ &

$\mathbf{g^{-1}}$:



Graph of $y = (x + 1)^2$; $\mathbf{x \geq -1}$

12.1 At E, $x = 0$

& $\mathbf{f}(0) = -2(-3)(1) = 6$

$\therefore$ **E(0; 6)** ➤

At D, $x = \frac{-1 + 3}{2}$ . . . *ave. of roots*

$= 1$

& $\mathbf{f}(1) = -2(-2)(2) = 8$

$\therefore$ **D(1; 8)** ➤

12.2 **Q(3; 0)** ➤ . . . $\boxed{\mathbf{f}(3) = 0}$

12.3 (a) $\mathbf{m} = -\frac{6}{3} = \mathbf{-2}$ ➤ . . . $\boxed{\text{Gradient of h}}$

(b) $\mathbf{a} = \mathbf{f}(2) = -2(2 - 3)(2 + 1)$

$\therefore$ $a = (-2)(-1)(3)$

$\therefore$ $\mathbf{a = 6}$ ➤

12.4 **(8; 1)** ➤ . . . *turning point of* **f** *is D(1; 8)*

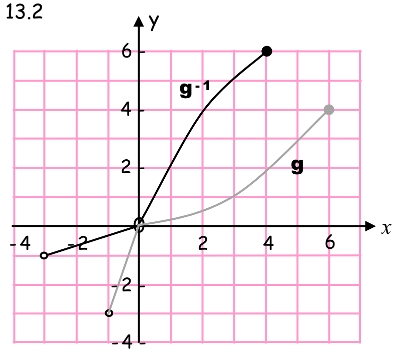
12.5 No; for each x-value in the domain there is not only one y-value.

e.g. (6; 2) and (6; 0) both lie on $\mathbf{f^{-1}}$
i.e. For $x = 6$, $y = 2$ or $y = 0$

13.1 Domain: $\mathbf{-1 < x \leq 6}$ ➤

Range: $\mathbf{-3 < y \leq 4}$ ➤

13.2



Find $\mathbf{g^{-1}}$ by swopping the coordinates of points on **g**.

Note: **g** and $\mathbf{g^{-1}}$ are symmetrical about the line $y = x$.

13.3 $\mathbf{g^{-1}}$ is a function because for every value of x, there is a unique value of y. ➤