GR 11 MATHS

MEASUREMENT FORMULAE:

Total Surface Area & Volume

CONTENTS:

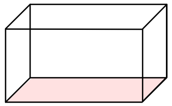
Compliments of

the ANSWER series... your key to exam success

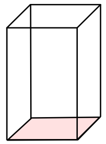
Right Prisms vs Right Pyramids

Compare right prisms to right pyramids

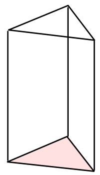
Right Prisms



rectangular base

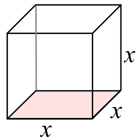


square base

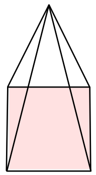


triangular base

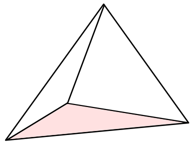
Special case: a cube



Right Pyramids



square base



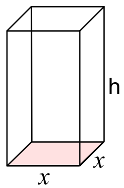
triangular base

• Surface Area

- Practically speaking, the total surface area (TSA) of any 3D solid is equal to the sum of the areas of all the faces

  e.g.

  A Prism with a square base

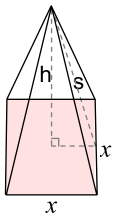


TSA of the prism (6 faces)

$= x^2 + x^2 + x\text{h} + x\text{h} + x\text{h} + x\text{h}$

$= 2x^2 + 4x\text{h}$

A Pyramid with a square base



TSA of the pyramid (5 faces)

$= x^2 + 4\left(\frac{x\text{s}}{2}\right)$ . . . $\frac{x\text{s}}{2} = \frac{1}{2} \times x \times \text{s}$

$= x^2 + 2x\text{s}$

- Using the formulae for prisms and pyramids in general:

Prism: TSA $=$ **2**(area of the base) + (perimeter of base × height)

$= 2(x^2) + (4x)(\text{h})$ . . . *same as above*

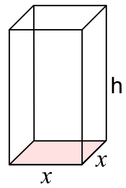
Pyramid: TSA $=$ area of the base + $\mathbf{\frac{1}{2}}$perimeter of base × **slant height**

$= x^2 + \frac{1}{2}(4x).\text{s}$ . . . *same as above*

$= x^2 + 2x\text{s}$

• Volume

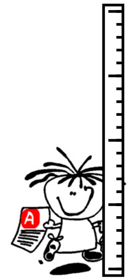
A Prism with a square base



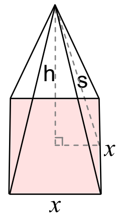
Volume of the prism

$=$ area of base × height

$= x^2\text{h}$



A Pyramid with a square base



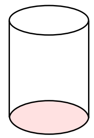
Volume of the pyramid

$= \mathbf{\frac{1}{3}}$ area of base × **perpendicular height**

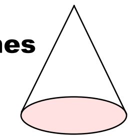
$= \frac{1}{3}x^2.\text{h}$

Cylinders vs Cones

Cylinders



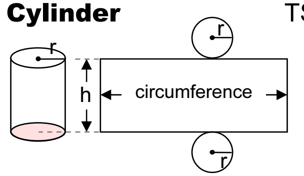
Cones



The base is a circle in both cases

• Surface Area

Cylinder



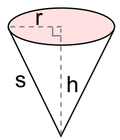
TSA $=$ **2**(area of base) + circumf. of base × height

$= 2(\pi\text{r}^2) + (2\pi\text{r})\text{h}$

$= 2\pi\text{r}(\text{r} + \text{h})$

same idea as for prisms

Cone



TSA $=$ area of base + $\mathbf{\frac{1}{2}}$ circumf. of base × **slant height**

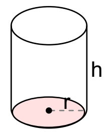
$= \pi\text{r}^2 + \frac{1}{2}(2\pi\text{r}).\text{s}$

$= \pi\text{r}^2 + \pi\text{rs}$

same idea as for pyramids

• Volume

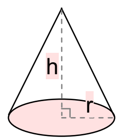
Cylinder



Volume $=$ area of base × height

$= \pi\text{r}^2.\text{h}$

Cone



Volume $= \frac{1}{3}$ area of base × **perpendicular height**

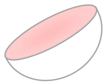
$= \frac{1}{3}\pi\text{r}^2.\text{h}$

Spheres and Hemispheres (Half-Spheres)

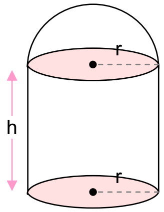
• Surface Area

TSA of a sphere $= 4 \times \pi r^2$
$= 4\pi r^2$

. . . $4 \times$ the area of a circle (also with radius, r)



TSA of a hemisphere $= \frac{1}{2}$(TSA of a sphere) + the area of a circle
$= \frac{1}{2}(4\pi r^2) + \pi r^2$
$= 3\pi r^2$



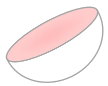
If a hemisphere is attached to a cylinder (see alongside),
then TSA of this object = area of base of cylinder **(πr²)**
+ area of curved surface of cylinder **(2πrh)**
+ area of hemisphere **(2πr²)**
$= 3\pi r^2 + 2\pi rh$

Note: The area of the base of the hemisphere is ignored.

• Volume

Volume of a sphere . . .
$= \frac{4}{3}\pi r^3$

Volume of a hemisphere . . .
$= \frac{1}{2} \times \frac{4}{3}(\pi r^3) = \frac{2}{3}\pi r^3$



SUMMARY OF FORMULAE: Pyramids/Cones/(Half)Spheres

• Total Surface Area (TSA)

A right pyramid: TSA = area of base + $\frac{1}{2}$perimeter of base $\times$ slant height

A right cone: TSA = area of base + $\frac{1}{2}$circumference of base $\times$ slant height

A sphere: TSA = $4\pi r^2$ . . . *4 × the area of a circle*

A hemisphere: TSA = area of base (πr^2) + curved surface area ($2\pi r^2$) = $3\pi r^2$

• Volume

A right pyramid: Volume = $\frac{1}{3}$area of base $\times$ perpendicular height

A cone: Volume = $\frac{1}{3}$area of base $\times$ perpendicular height

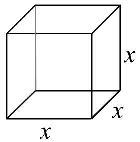
A sphere: Volume = $\frac{4}{3}\pi r^3$

A hemisphere: Volume = $\frac{2}{3}\pi r^3$

The effect on surface area and volume of a prism when multiplying the dimensions by a constant factor k.

Observe the effect on the total surface area (TSA) and volume of a cube of multiplying the dimensions by 2, and then, by 3.

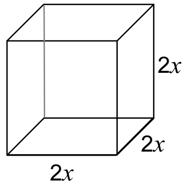
A Cube:



TSA $= 6(x \times x)$
$= 6x^2$

Vol $= x \times x \times x$
$= x^3$

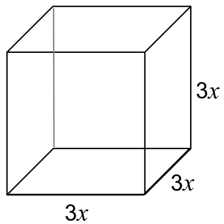
All dimensions **x2**:



TSA $= 6(2x \times 2x)$
$= 6.\mathbf{2^2}.x^2$
$= 24x^2$

Vol $= 2x \times 2x \times 2x$
$= \mathbf{2^3}.x^3$
$= 8x^3$

Or, all dimensions **x3**:



TSA $= 6(3x \times 3x)$
$= 6.\mathbf{3^2}.x^2$
$= 54x^2$

Vol $= 3x \times 3x \times 3x$
$= \mathbf{3^3}.x^3$
$= 27x^3$

Summary of the effects of scale factors 2, 3 and k:

All dimensions × 2 ➡ **TSA × 4** ➡ **Volume × 8**

All dimensions × 3 ➡ **TSA × 9** ➡ **Volume × 27**

In general:
All dimensions × k ➡ **TSA × k²** ➡ **Volume × k³**