Homework #1
Introduction to Algorithms/Algorithms 1
600.363/463

Due on: Friday, February 24th, 4pm
Where to submit: the mailbox accross 224NEB
(please ask Debbie DeFord in 224 if you cannot find it)
Late submissions: will NOT be accepted
Format: Please start each problem (1,2,3) on a new page.
Please type your answers (instead of handwriting) if you can.

February 10, 2012

1 Problem 1

For each statement below explain if it is true or false and prove your answer. Be as precise as you can.

1. $3n + 1 = O(\log(n))$
2. $100^{3409}(n + 34n^2) = o(n^{2.0000000001})$
3. $100^{3409}(n + 34n^2) = o(n^{1.999999999999})$
4. $e^n = \theta(e^{n^2})$
5. $e^n = \theta(e^{3n})$
6. $e^n = \theta(e^{n+3})$
7. $\log(n) = O(\log(n^{10000000000}))$
8. $\log(n) = O(\big(\log(n^{10000000000})\big)^{1.000001})$
9. $\log(n) = o(\big(\log(n^{10000000000})\big)^{1.000001})$
10. $\log(n) = \Omega(\big(\log(n^{10000000000})\big)^{1.000001})$

11. $\log(n) = \Theta(\left(\log(n^{10000000000})\right)^{1.000001})$
12. $\log(\log(n)) = \Omega(1)$

2 Problem 2

1. Compute $\sum_{i=0}^{n}(i^2 - 45i)$
2. Compute $\sum_{i=0}^{n} 3^i$
3. Bound from above the summation $\sum_{i=0}^{n}(\frac{1}{2i+1})$. Hint: use harmonic series.

3 Problem 3

1. Prove by induction that for every natural number n there exists a larger natural number m such that $n < m \leq 3n$ and m is a power of 3.
2. Prove that for any sets A, B, C, D it is true that

$$(A \cup B) \cap (C \cup D) = (A \cap C) \cup (B \cap C) \cup (A \cap D) \cup (B \cap D).$$