# Algorithm

# Homework 1

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### Problem 1

1. 

This statement is FALSE.

To show that , we assume there exist constant c, n­0 > 0 such that

 for all .

For the left part,



For the right part,



But no constants is greater than all , and so the assumption leads to a contradiction.

2. 

This statement is TRUE.

To show that , we must proof that :

Given

,



Thus



Which meets the definition of little o, thus the statement is TRUE.

3. 

This statement is FALSE

To show that , we must proof that :

Given

,



Thus



Which does not meet the definition of little o, thus the statement is FALSE.

4. 

This statement is FALSE

To show that the statement is FALSE, we assume there exist constants c1, c2, n0 > 0 such that:

 for all n>=n0

Then 

For the left part



But no constant is greater than all , and so the assumption leads to a contradiction.

5. 

This statement is FALSE

To show that the statement is FALSE, we assume there exist constants c1, c2, n0 > 0 such that:

 for all n>=n0

Then 

For the left part



But no constant is greater than all , and so the assumption leads to a contradiction.

6. 

This statement is TRUE.

To show that the statement is TRUE, we must find there exist constants c1, c2, n0 > 0 such that:

 for all n>=n0

Then 

For all n, we can satisfy the definition with 

Thus the statement is TRUE.

7. 

This statement is TRUE.

To show the statement is TRUE, we must find constants c, n0 > 0 such that:

 for all n>=n0

For the left part, 

For the right part,



We can satisfy the right part with c = 1 and n0 = 2.

Thus for c = 1 and n0 = 2, we can prove the statement is TRUE.

8. 

The statement is TRUE.

To show that , we need to find constants c, n0 > 0 such that

 for all n >= n0

For the left part, 

For the right part,

We can satisfy the statement with c = 100, n0 = 1. Thus the statement is TRUE.

9. 

The statement is TRUE.

We got





Thus



Which is meet the definition of little o, thus the statement is TRUE.

10. 

The statement is FALSE.

To show that , we can assume that there exist constants c, n0 > 0 such that

 for all n > =n0

For the left part,



For the right part,



But no constant of  is greater than all , and so the assumption leads to a contradiction.

11. 

This statement is FALSE.

To show that , assume there exist c1, c2, n0 > 0 such that:



For the left part,



But no constant of  is greater than all , and so the assumption leads to a contradiction.

12. 

The statement is TRUE.

We must find the constants c, n0 > 0 such that

 for all n >=n0

For the left part, we have 

For the right part,



We can satisfy the definition with c = 0, n0 = 2

Thus the statement is TRUE.

### Problem 2

1. 



2. 



3. 

As we know 

Thus 

Apply harmonic series:



We got:



### Problem 3

1. Prove by induction that for every natural number n there exists a larger natural number m such that n < m <= 3n and m is a power of 3.

n=1: , thus m = 3 = 31.

Assume n=k holds: 

Show n=k+1 holds: 

Start from the left side:

1. If  

Then





Given , we have



1. If  

Then



Given , we have



From (1) and (2), we know



2. 



Apply Distributive law:



Apply Distributive law again:



Apply Associative lay:



QED