

**Johns Hopkins ■ CS600.424: Network Security ■ Spring 2012**  
**Homework 6 – Assignment**  
**Due Date – May 4 in class**

**Instructions:** Please do not submit handwritten assignment.

1. Use Fermat's Theorem to find

(a)  $3^{201} \bmod 11$

(b) a number  $x$  between 0 and 28 with  $x^{85} \equiv 6 \bmod 29$

2. (a) Determine  $\Phi(n)$  for (a)  $n=41$ , (b) 27, (c) 231 and (d) 440

(b) Find primitive roots of 25.

(c) Given 2 as the primitive root of 29, construct a table of discrete logarithms, and use it to solve the congruence  $17x^2 \equiv 10 \pmod{29}$

3. Users A and B use the Diffie-Hellman key exchange technique with a common prime  $q=71$  and a primitive root  $\alpha=7$ .

(a) If user A has private key  $X_A=5$ , what is A's public key  $Y_A$ ?

(b) If user B has the private key  $X_B=12$ , what is B's public key  $Y_B$ ?

(c) What is the shared secret key?

4. Consider a Diffie-Hellman scheme with a common prime  $q=11$  and a primitive root  $\alpha=2$ .

(a) Show that 2 is the primitive root of 11

(b) If user A has public key  $Y_A=9$ , What is A's private key  $X_A$ ?

(c) If user B has a public key  $Y_B=3$ , what is the secret key  $K$  shared with A?

5. Consider ElGamal scheme with a common prime  $q=71$  and a primitive root  $\alpha=7$ .

(a) If user B has a public key  $Y_B = 3$ , and A chose the random integer  $k = 2$ , what is the ciphertext of  $M = 30$ ?

(b) If A now chooses a different value of  $k$  so that the encoding of  $M = 30$  is  $C = (59, C_2)$ , what is the integer  $C_2$ ?