最佳記憶：理論、演算法與實踐

杜昌燁

學校老師要你學習 / 記憶，卻沒告訴你如何學習 / 記憶

什麼是最佳的學習 / 記憶方法？

記憶 — 傳奇、藝術與技術

Laurence Kim Peek (1951 — 2009)

利瑪竇（Matteo Ricci, 1552 — 1610）

西國記法

原本篇第一

人受造物主所賦之神魂，視萬物最爲靈悟，故遇萬類悉能記識，而區別以藏之，若庫藏之貯財貨然。及欲用時，則萬類各隨機而出，條理井井，絕無混雜。然人知能記憶，而不知所以藏貯、所以區別者從何而致，且翕受果在何處，其敷施之妙，卒莫能語諸人。此則造物主顯露密秘，運斡精蘊，人烏得而測之乎。吾西士間嘗論其概矣。玆再次第於左，以求同理。

記含有所，在腦囊，蓋顱顖後，枕骨下，爲記含之室。故人追憶所記之事，驟不可得，其手不覺搔腦後，若索物令之出者，雖兒童亦如是。或人腦後有患，則多遺忘。試觀人枕骨最堅硬，最豐厚，似乎造物主置重石以護記含之室，令之嚴密，猶庫藏之有扃鐍，取封閉鞏固之義也。

人之記含，有難有易，有多有寡，有久有暫，何故？蓋凡記識，必自目耳口鼻四體而入。當其入也，物必有物之象，事必有事之象，均似以印印腦。其腦剛柔得宜，豐潤完足，則受印深而明，藏象多而久。其腦反是者，其記亦反是。如幼稚，其腦大柔，譬若水，印之無跡，故難記。如成童，其腦稍剛，譬若泥，印之雖有

世界記憶錦標賽（World Memory Championships）

競賽項目	目前紀錄
1 小時數字位數	4620 位
5 分鐘數字位數	630 位
朗讀數字	547 位
30 分鐘二進位數字	7485 位
1 小時卡片	2530 張
15 分鐘隨機英文單字	335 個
15 分鐘名字與臉孔	224 對
5 分鐘虛擬事件與年份	154 對
15 分鐘圖片	975 張
競速撲克牌記憶	12.74 秒

Table 1: 世界記憶錦標賽比賽項目與目前紀錄

記憶技巧：精緻編碼 ＋ 位置記憶法

精緻編碼 (Elaborative Encoding)

圖像 ≫ 純文字

Von Restorff 效應："與衆不同" — 大小、形狀、顏色、動態…

人臉識別：眼熟之人戴口罩、墨鏡不影響

數字 ⟹ (生動的) 圖像 ⟹ 名人臉孔、特徵行動

Major 系統：

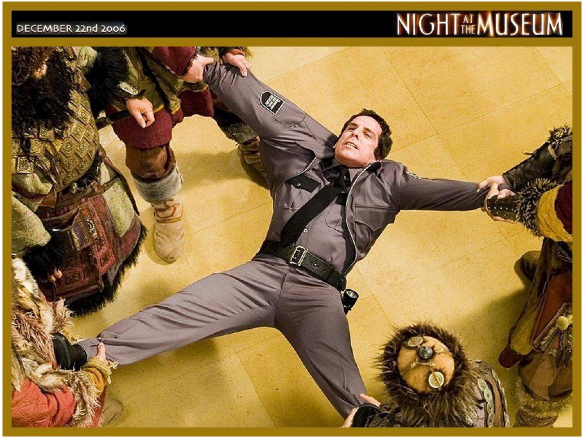
數字	1	2	3	4	5	6	7	8	9	0
發音	s	t, d	n	m	r	l	ch, sh	k, g	f, v	b, p

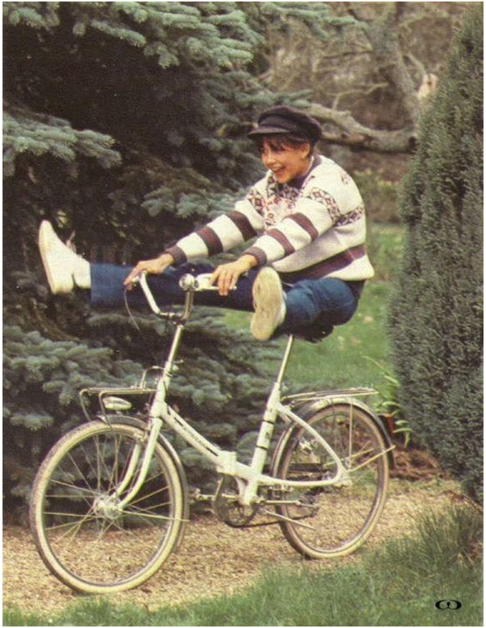
範例：08 — sofa，17 — dog，25 — nail，39 — map，46 — roach

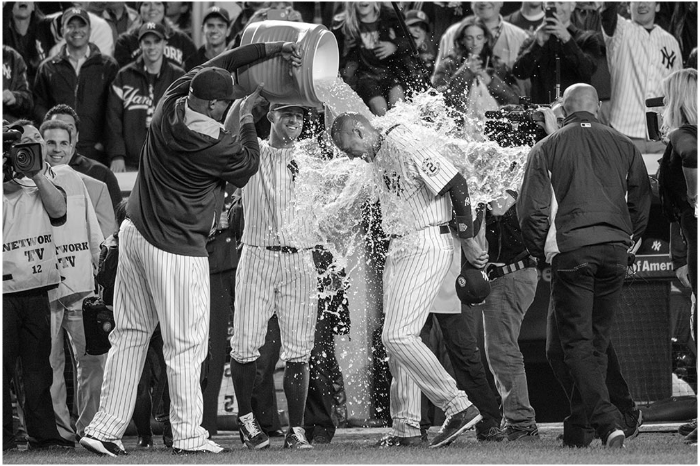
Dominic 系統：

數字	1	2	3	4	5	6	7	8	9	0
字母	A	B	C	D	E	S	G	H	N	O

範例：11 — Andre Agassi，15 — Albert Einstein，26 — Bill Gates，48 — Dustin Hoffman

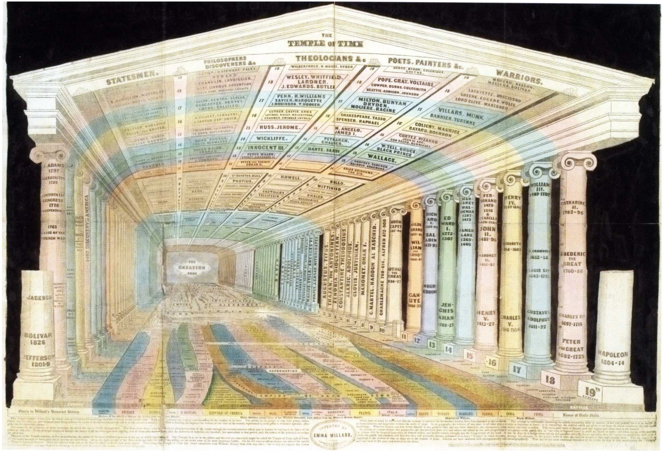

DECEMBER 22nd 2006
NIGHT AT THE MUSEUM






OPEL

位置記憶法（Method of Loci）




寵惠堂
Wang
Memorial
Hall


東吳校友會敬
SOOCHOW
UNIVERSITY
UNTO A FULL GROWN MAN
母校惠存


淡淡的晨光中
怡客閱讀時間 7:15am
Ikari Coffee


本校董事
吳舜文女士捐贈
Learning Commons


7-ELEVEN

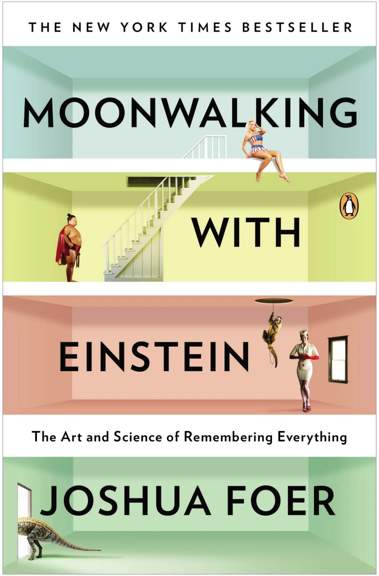

7-ELEVEn
CITY TEA
現萃茶
消防
招募


學生事務處
B222


第一閱覽室


中正圖書館
Soochow University Library


第一教學研究大樓


THE NEW YORK TIMES BESTSELLER
MOONWALKING
WITH
EINSTEIN
The Art and Science of Remembering Everything
JOSHUA FOER


大腦這樣記憶
什麼都學得會
精通所有技能的最高學習法，
比爾蓋茲、記憶冠軍、高績效人士
一生受用的記憶習慣
比爾蓋茲推薦兩次的書
「如果你想掌握一項新技能，就應該學習如何記憶。」
改變用腦習慣，就能突破潛能，打造專家表現。
天下財經
413
Moonwalking
with
Einstein
The Art and Science of
Remembering Everything
喬許・弗爾 Joshua Foer 著
謝佩妏 譯

記憶 — 觀察、理論與實證

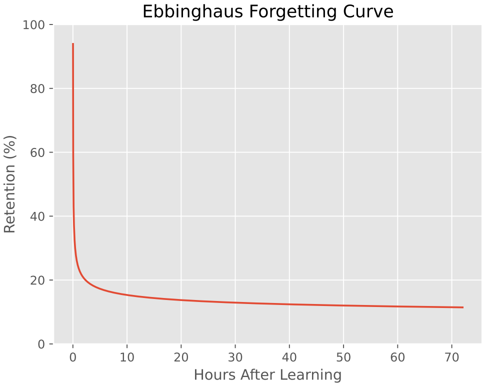
Memory: A Contribution to Experimental Psychology

Hermann Ebbinghaus (1885)

Translated by Henry A. Ruger & Clara E. Bussenius (1913)
Originally published in New York by Teachers College, Columbia University.

Hermann Ebbinghaus（1850 — 1909）遺忘曲線、複習間隔效應

$$\text{(記憶程度\%)}\;(t) = \frac{100\,k}{(\log t)^c + k}, \quad k = 1.84,\ c = 1.25,\ t\ \text{(單位：分)}$$



The Critical Importance of Retrieval for Learning

Jeffrey D. Karpicke[1]* and Henry L. Roediger III[2]

Learning is often considered complete when a student can produce the correct answer to a question. In our research, students in one condition learned foreign language vocabulary words in the standard paradigm of repeated study-test trials. In three other conditions, once a student had correctly produced the vocabulary item, it was repeatedly studied but dropped from further testing, repeatedly tested but dropped from further study, or dropped from both study and test. Repeated studying after learning had no effect on delayed recall, but repeated testing produced a large positive effect. In addition, students' predictions of their performance were uncorrelated with actual performance. The results demonstrate the critical role of retrieval practice in consolidating learning and show that even university students seem unaware of this fact.

The right time to learn: mechanisms and optimization of spaced learning

Paul Smolen, Yili Zhang and John H. Byrne

Abstract | For many types of learning, spaced training, which involves repeated long inter-trial intervals, leads to more robust memory formation than does massed training, which involves short or no intervals. Several cognitive theories have been proposed to explain this superiority, but only recently have data begun to delineate the underlying cellular and molecular mechanisms of spaced training, and we review these theories and data here. Computational models of the implicated signalling cascades have predicted that spaced training with irregular inter-trial intervals can enhance learning. This strategy of using models to predict optimal spaced training protocols, combined with pharmacotherapy, suggests novel ways to rescue impaired synaptic plasticity and learning.

Unbounded Human Learning: Optimal Scheduling for Spaced Repetition

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A Trainable Spaced Repetition Model for Language Learning

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[Settles and Meeder, 2016] 最佳複習間隔：半衰期迴歸最佳化解

$$x_{\text{學習}} \equiv (i_{\text{項次}}, c_{\text{特徵}}, r_{\text{是否記得}}, t_{\text{時間}})$$

(Ebbinghaus) p 為回憶率，Δ 為上次複習到現在時間長度，h 為「記憶半衰期」— 衡量記憶強度，則

$$p = 2^{-\frac{\Delta}{h}}$$

(機器學習) Θ 為待訂常數向量，回憶率為

$$p = 2^{\Theta \cdot x}$$

以 N 筆使用者學習資料求得最佳 Θ 值:

$$\Theta^\star = \operatorname*{argmin}_{\Theta} \sum_{i=1}^{N} \ell(\langle p, \Delta, x\rangle_i; \Theta), \quad \ell(\langle p, \Delta, x\rangle_i, \Theta) \equiv (p - \widehat{p_\Theta})^2 + \lambda \|\Theta\|_2^2$$

Enhancing human learning via spaced repetition optimization

Behzad Tabibian[a,b,1], Utkarsh Upadhyay[a], Abir De[a], Ali Zarezade[a], Bernhard Schölkopf[b], and Manuel Gomez-Rodriguez[a]

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$$\text{(learning event)} \equiv (i_{\text{item}}, \quad t_{\text{time}}, \quad r_{\text{recall}}), \quad r \in \{0,1\}$$

$$m_i(t) \equiv \mathsf{P}(r) = e^{-n_i(t)(t-t_r)}$$

$$N_i(t) \equiv (\#\text{ of the learner has reviewed item } i \text{ up to } t), \quad \mathsf{E}\{\mathrm{d}N_i(t)\} = u_i(t)\,\mathrm{d}t$$

$$n_i(t) = \begin{cases} (1-\alpha_i)n_i(t_r) & \text{if successful recall at } t_r;\ 0 \leqslant \alpha_i \leqslant 1 \\ (1+\beta_i)n_i(t_r) & \text{if unsuccessful recall at } t_r;\ 0 \leqslant \beta_i \end{cases}$$

$$\mathrm{d}n_i(t) = -\alpha_i\, n_i(t) r_i(t)\,\mathrm{d}N_i(t) + \beta_i\, n_i(t)(1-r_i(t))\,\mathrm{d}N_i(t)$$

$$\mathrm{d}m_i(t) = -n_i(t)m_i(t)\,\mathrm{d}t + (1-m_i(t))\,\mathrm{d}N_i(t)$$

$$
\begin{aligned}
\underset{u(t_0,t_f]}{\text{minimize}} \quad & \mathsf{E}\left\{\varphi\left(m(t_f),\, n(t_f)\right) + \int_{t_0}^{t_f} \ell\left(m(\tau),\, n(\tau),\, u(\tau)\right) \mathrm{d}\tau\right\} \\
\text{subject to} \quad & u(t) \geqslant 0 \quad \forall t \in (t_0, t_f)
\end{aligned}
$$

$$
J = \min_{u(t_0,t_f]} \mathsf{E}_{(N(s),r(s))\big|_{s=t}^{s=t_f}} \left\{\varphi(m(t_f), n(t_f)) + \int_{t}^{t_f} \ell(m(\tau), n(\tau), u(\tau))\, \mathrm{d}\tau\right\}
$$

記憶 — 工具、心得與實踐

有效學習 / 記憶方法:

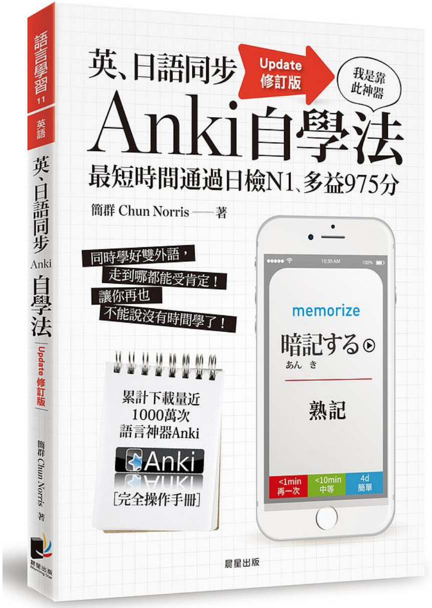
Flashcard + Spaced Repetition Algorithm

使用軟體推薦: Anki

如何有效製作卡片 / 整理知識:

- Twenty Rules of Formulating Knowledge
- Spaced Repetition for Mathematics
- How I Use Anki to Learn Mathematics

以「Anki 中文」搜尋創意使用法!



https://github.com/chang-ye-tu/ananda/

Spaced Repetition for Efficient Learning

Site
Me

Changes
News

support on Patreon

Efficient memorization using the spacing effect: literature review of widespread applicability, tips on use & what it's good for.

Haskell, nootropic, psychedelic, spaced repetition

2009-03-11–2019-05-17 · *finished* · certainty: *highly likely* · importance: *9* · backlinks · similar · bibliography

Spaced repetition is a centuries-old psychological technique for efficient memorization & practice of skills where instead of attempting to memorize by 'cramming', memorization can be done far more efficiently by instead spacing out each review, with increasing durations as one learns the item, with the scheduling done by software. Because of the greater efficiency of its slow but steady approach, spaced repetition can scale to memorizing hundreds of thousands of items (while crammed items are almost immediately forgotten) and is especially useful for foreign languages & medical studies.

I review what this technique is useful for, some of the large research literature on it and the testing effect (up to ~2013, primarily), the available software tools and use patterns, and miscellaneous ideas & observations on it.

One of the most fruitful areas of computing is making up for human frailties. They do arith-

Augmenting Long-term Memory

Michael Nielsen | Y Combinator Research | July 2018

One day in the mid-1920s, a Moscow newspaper reporter named Solomon Shereshevsky entered the laboratory of the psychologist Alexander Luria. Shereshevsky's boss at the newspaper had noticed that Shereshevsky never needed to take any notes, but somehow still remembered all he was told, and had suggested he get his memory checked by an expert.

Luria began testing Shereshevsky's memory. He began with simple tests, short strings of words and of numbers. Shereshevsky remembered these with ease, and so Luria gradually increased the length of the strings. But no matter how long they got, Shereshevsky could recite them back. Fascinated, Luria went on to study Shereshevsky's memory for the next 30 years. In a book summing up his research*, Luria reported that:

> [I]t appeared that there was no limit either to the *capacity* of S.'s memory or to the *durability of the traces he retained*. Experiments indicated that he had no difficulty reproducing any lengthy series of words whatever, even though these had originally been presented to him a week, a month, a year, or even many years earlier. In fact, some of these experiments designed to test his retention were performed (without his being given any warning) fifteen or sixteen years after the session in which he had originally recalled the words. Yet invariably they were successful.

Such stories are fascinating. Memory is fundamental to our thinking, and the notion of having a perfect memory is seductive. At the same time, many people feel ambivalent about their own memory. I've often heard people say “I don't have a very good memory”, sometimes sheepishly, sometimes apologetically, sometimes even defiantly.

Related Resources
Michael Nielsen on Twitter
Michael Nielsen's project announcement mailing list
cognitivemedium.com

By Michael Nielsen

* Alexander Luria, “The Mind of a Mnemonist”, Harvard University Press (1968).

Andy Matuschak, Michael Nielsen — "Tools for Thought"

Part of the origin myth of modern computing is the story of a golden age in the 1960s and 1970s. In this story, visionary pioneers pursued a dream in which computers enabled powerful tools for thought, that is, tools to augment human intelligence[1]. One of those pioneers, Alan Kay, summed up the optimism of this dream when he wrote of the potential of the personal computer: "the very use of it would actually change the thought patterns of an entire civilization"[2].

It's an inspiring dream, which helped lead to modern interactive graphics, windowing interfaces, word processors, and much else. But retrospectively it's difficult not to be disappointed, to feel that computers have not yet been nearly as transformative as far older tools for thought, such as language and writing. Today, it's common in technology circles to pay lip service to the pioneering dreams of the past. But nostalgia aside there is little determined effort to pursue the vision of transformative new tools for thought.

1. E.g., Douglas Engelbart, Augmenting Human Intellect: A Conceptual Framework (1962).

2. Alan Kay, User Interface: A Personal View (1989).

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Settles, B. and Meeder, B. (2016). A trainable spaced repetition model for language learning. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics*, pages 1848–1858. Association for Computational Linguistics.

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