# Financial Markets & Instruments

# What is Finance?

- A possible definition:

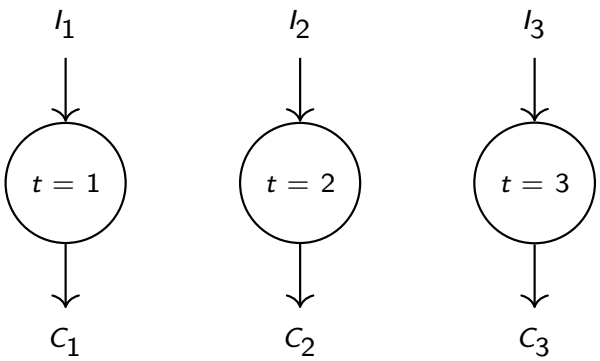
  *Finance is the study of how people and organizations allocate scarce resources over time, subject to uncertainty.*

- Two essential ingredients:
  - **Time**: Reflected in interest rates, borrowing, and investing
  - **Uncertainty**: Future asset values, returns, and economic conditions
- These two dimensions are intertwined throughout all financial concepts and models
- Financial markets facilitate:
  - Consumption timing (saving and borrowing)
  - Risk transfer between market participants

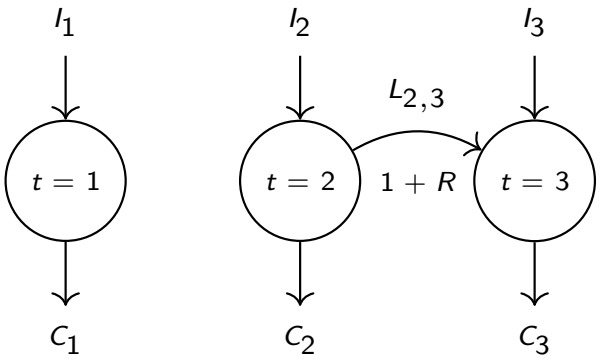
# Consumption Timing

- Imagine a world without "storage" of money:
  - We would have to consume all income immediately
  - No saving for future needs, no borrowing against future income
- Financial markets allow shifting consumption across time
  - Money markets: Short-term borrowing and lending
  - Capital markets: Long-term investment and financing
- Time value of money: \$1 now is worth more than \$1 in the future
  - Investment opportunities
  - Precautionary cushion against unforeseen needs
- Shifting consumption comes with compensation in the form of interest rates

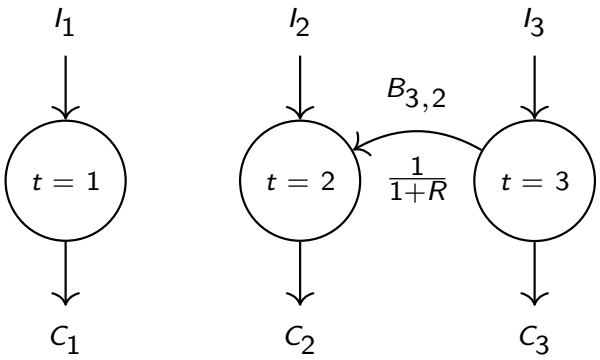
# Shifting Consumption in Time




(a) Income equals consumption




(b) Shifting consumption forward by saving




(c) Shifting consumption backward by borrowing

Figure: Shifting consumption forward and backward in time

# Flow Balance Equations

- When saving (shifting consumption forward):

$$C_2 = I_2 - L_{2,3}$$
$$C_3 = I_3 + L_{2,3}(1 + R)$$

  where $L_{2,3}$ is the amount saved/lent and $R$ is the interest rate
- When borrowing (shifting consumption backward):

$$C_2 = I_2 + \frac{B_{3,2}}{1+R}$$
$$C_3 = I_3 - B_{3,2}$$

  where $B_{3,2}$ is the amount to be repaid at $t = 3$
- Alternative expression when borrowing at $t = 2$:

$$C_2 = I_2 + B^*_{3,2}$$
$$C_3 = I_3 - B^*_{3,2}(1 + R)$$

  where $B^*_{3,2}$ is the amount borrowed at $t = 2$

# Uncertainty and Risk

- Financial markets not only deal with time, but also uncertainty
- Sources of financial risk:
  - Market risk: Fluctuations in stock prices, interest rates, etc.
  - Credit risk: Possibility of default on debt obligations
  - Currency risk: Exchange rate fluctuations
  - Inflation risk: Loss of purchasing power
  - Operational risk: Failures in processes, systems, or people
- Risk vs. uncertainty: Risk can be quantified with probabilities
- Risk transfer: Insurance markets, derivatives markets
- Risk management: Identifying, measuring, and controlling risk exposures

# Holding Period Return and Gain

- **Definition 1.1 (Holding period return)** Let us consider a holding period $[0, T]$, where the initial asset price is $S(0)$ and the terminal random asset price is $S(T, \omega)$. We define the holding period return as

$$R(\omega) \doteq \frac{S(T,\omega) - S(0)}{S(0)}$$

and the holding period gain as

$$G(\omega) \doteq \frac{S(T,\omega)}{S(0)} = 1 + R(\omega)$$

- A return of 10% means the stock price was multiplied by a gain factor of 1.10
- Gain (or gross return) vs. return (or net return)
- Term "rate of return" typically reserved for annual returns

# Different Types of Returns

- Consider a holding period of two consecutive years:
  - Year 1: Return is +10%
  - Year 2: Return is -10%
  - What is the "average" return?
- Arithmetic mean: $R_a = \frac{0.10+(-0.10)}{2} = 0$
- But actual gain over two years:

$$\begin{aligned} G &= (1 + 0.10) \times (1 - 0.10) \\ &= 0.99 \quad \text{(a loss of 1\%)} \end{aligned}$$

- Geometric average (annualized):

$$\begin{aligned} (1 + 0.10) \times (1 - 0.10) &= (1 + R_g)^2 \\ \Rightarrow R_g &= -0.5013\% \end{aligned}$$

- The return depends on the measurement approach

# Risk-Free Asset

- A risk-free asset has a predetermined return with certainty
- Example: Safe bank account or high-quality government bonds
- For a risk-free asset:

$$B(T, \omega) = B(0) \cdot (1 + R_f)$$

  for every state of the world $\omega \in \Omega$
- $R_f$ is the risk-free return; if $T$ is one year, referred to as risk-free rate
- Why invest in risky assets if risk-free ones exist?
  - Risk premium: Compensation for bearing risk
  - Higher expected returns for risky assets
- Risky and risk-free assets form the foundation of many financial models

# Market Scenario Trees

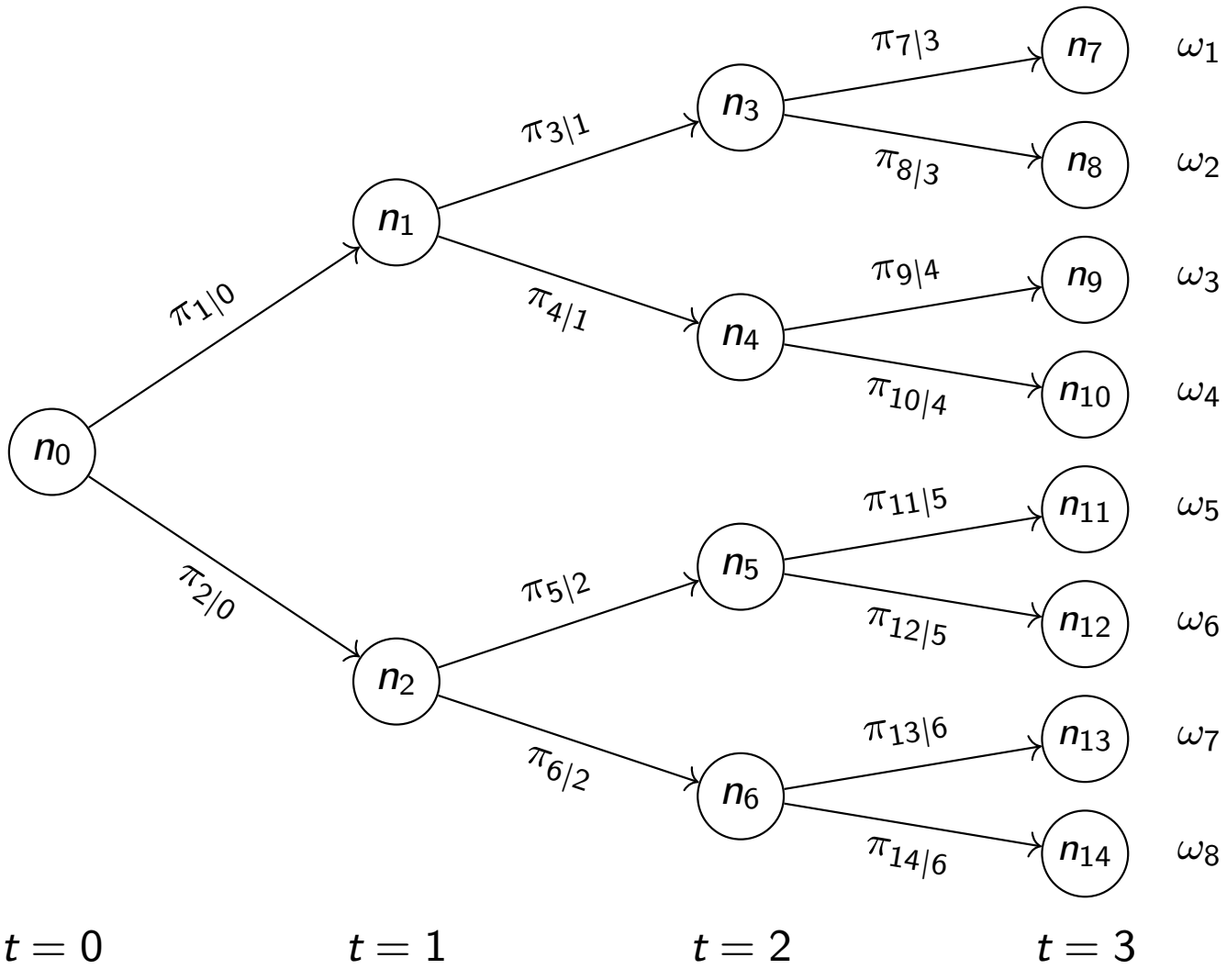



Figure: A scenario tree: uncertainty unfolding progressively over time

# Types of Assets

- An **asset** is anything that can be transformed into money by its owner
- Key dimensions for classifying assets:
  - **Real vs. financial**: Physical assets vs. securities/claims
  - **Risky vs. risk-free**: Uncertain vs. certain future value
  - **Liquid vs. illiquid**: Ease of selling at fair price
  - **Tradable vs. non-tradable**: Can be bought/sold in markets
  - **Exchange-traded vs. over-the-counter (OTC)**: Standardized vs. customized
- Main types of financial assets:
  - Stock shares (equity)
  - Bonds and other debt securities
  - Foreign currencies
  - Derivatives: Forwards/futures, options, swaps
  - Hybrid securities

# Assets vs. Securities

- **Securities** are tradable financial assets
  - Readily purchased or sold on financial markets
  - Examples: Stocks, bonds, exchange-traded derivatives
- Not all assets are securities:
  - Insurance policies: Assets but not tradable
  - Human capital: Valuable but not marketable
  - Bank loans: Assets for the bank but not readily tradable
- **Securitization**: Transforming illiquid assets into tradable securities
  - Example: Mortgages $\rightarrow$ Mortgage-backed securities (MBS)
  - Pools cash flows from similar assets
  - Creates liquid securities from illiquid assets
- Liquidity depends on both asset type and market conditions

# Equity

- Stock shares represent residual claims on the equity of a firm
  - If a firm is liquidated, assets are sold to pay liabilities
  - Any remaining equity is distributed to stockholders
  - This is why stocks are referred to as "equity" investments
- Holding period return for a stock share: $R(\omega) = \frac{S(T,\omega)+D-S(0)}{S(0)}$ where $D$ is dividend paid during the holding period $(0, T)$
  - Dividend timing affects valuation when reinvested
  - Forward-projected dividends should be considered in calculations
- Key features of stock shares:
  - Limited liability assets: stockholders not responsible for firm's illegal behavior
  - Stock prices cannot be negative and worst-case return is $-100\%$
  - Common vs. preferred stock shares with different voting rights and dividend guarantees
  - No defined maturity, unlike bonds or options
  - Stock shares can be created and destroyed (unlike energy in physics)
- Extraordinary events affecting stock prices:
  - Stock splits and reverse splits: create multiple shares from one or vice versa
  - Spinoffs: firm separates into two with corresponding shares
  - Mergers and acquisitions: firms combine with share conversion
  - These events affect price but not market capitalization

# Fixed Income

- A bond is a security defined by:
  - Face value $F$ (nominal or par value): amount issuer promises to pay
  - Maturity $T$ (time when face value is paid back)
  - Coupon rate $c$ (interest rate applied to face value for periodic payments)
  - Coupons historically were physical pieces of paper detached for payment
- Types of bonds:
  - Zero-coupon bonds ($c = 0$): no periodic payments, only face value at maturity
  - Coupon-bearing bonds ($c > 0$): periodic interest payments plus face value
  - Coupon frequency typically twice a year, but can vary
- US Treasury bonds classified as:
  - T-bills: zeros, maturities up to one year
  - T-notes: coupon-bearing, maturities up to ten years
  - T-bonds: coupon-bearing, longer maturities
  - Some long-term zeros created by stripping coupons (unbundling cash flows)
- Bond risk factors:
  - Default risk: issuer may fail to pay coupons or principal
  - Inflation risk: relevant for long-term bonds, erodes purchasing power
  - Foreign-exchange risk: for bonds denominated in foreign currencies
  - Interest rate risk: inverse relationship between rates and bond prices

# FOREX Markets

- Foreign exchange (FOREX) markets facilitate currency trading
  - Important for international equity and fixed-income portfolios
  - Also critical for non-financial firms with international operations
- Risk factor: exchange rate between currency pairs
  - Affects international trade and investments
  - Source of both risk and speculative opportunity
- Quotation conventions:
  - Currency pair notation (e.g., EUR/USD = 1.1166)
  - Base currency (left of slash): fixed amount (e.g., EUR 1)
  - Quoted currency (right of slash): variable amount (e.g., USD 1.1166)
  - Means 1 euro can buy 1.1166 US dollars
- Direct vs. indirect quotation:
  - Direct: domestic currency is quoted currency (variable amount)
  - Indirect: domestic currency is base currency (fixed amount)
  - Also called "uncertain for certain" vs. "certain for uncertain"
  - Choice depends on perspective, convenience, and local conventions
- Bid-ask spreads: difference between buying and selling prices
  - Example: EUR/USD 1.1165/67
  - Bank buys base currency (EUR) at bid price (1.1165)
  - Bank sells base currency at ask price (1.1167)
  - "Three Bs rule": market maker Buys the Base currency at the Bid price

# Derivatives

- Derivative: financial asset deriving its value from an underlying variable
  - Defined by explicit formula written in contract
  - Typical payoff is $f(S_T)$ for some function $f(\cdot)$ at maturity $T$
  - More complex derivatives depend on price path until maturity
- Types of underlying variables:
  - Primary financial assets (stocks, bonds)
  - Nonfinancial assets (commodities like gold or oil)
  - Risk factors (interest rates, market indexes, volatility)
  - Weather or other non-price variables
  - Other derivatives (compound options, swaptions)
- Key issues in derivatives:
  - Pricing: determining fair value based on underlying factors
  - Hedging: managing risk exposure of writing derivatives
  - Portfolio management: changing exposure to risk factors
  - Regulatory and accounting issues beyond scope of quantitative models
- Basic derivative families:
  - Forward contracts: obligation to buy/sell at future date
  - Futures contracts: standardized, exchange-traded forwards
  - Options: right but not obligation to buy/sell
  - Plain vanilla vs. exotic derivatives with complex payoffs

# Forward Contracts

- Agreement between two counterparties to buy/sell an asset at a prespecified forward price $F(t_0, T)$ at a later date $T$
  - A private arrangement between counterparties (OTC)
  - Both parties are obligated to fulfill the contract terms
- Long position: agrees to buy the asset at maturity
- Short position: agrees to sell the asset at maturity
- At maturity, spot-forward convergence applies: $S(T) = F(T, T)$
  - Forward price at maturity equals spot price (law of one price)
  - Otherwise would create arbitrage opportunity
- Payoff for long position: $S(T) - F(t_0, T)$
  - Positive when spot price exceeds delivery price
  - Negative when delivery price exceeds spot price
- Payoff for short position: $F(t_0, T) - S(T)$
  - Zero-sum game: one party's gain is other's loss
- Forward contracts:
  - Tailored to specific needs (OTC)
  - Less liquid and standardized than futures
  - Subject to counterparty risk (default risk)
  - Initial value is zero, allowing leverage without initial investment
  - Used for both hedging and speculation

# Futures Contracts

- Exchange-traded equivalents of forward contracts
  - Delivery of underlying or cash settlement at maturity
- Key features to address issues with forwards:
  - Clearinghouse to reduce counterparty risk
  - Daily marking-to-market instead of single settlement
- Standardization:
  - Limited underlying assets and delivery dates
  - Deep market with easier entry/exit
  - Standard contract sizes (e.g., 100 oz gold)
  - Sometimes allows range of deliverable assets (e.g., bond futures)
- Clearinghouse:
  - Steps between long and short positions as counterparty
  - Assumes counterparty risk from both sides
  - Manages margin accounts for all positions
  - Nets positions to reduce exposure (proved effective in 1987 crash)
- Daily marking-to-market:
  - Daily cash flow settlements based on price changes
  - Margin accounts with initial and maintenance margins
  - Margin calls when account drops below threshold
  - Position liquidated if margin call not met
  - Sum of daily cash flows equals forward contract payoff

# Vanilla Options I

- Asymmetric contracts with nonlinear payoffs
  - Two distinct roles: option writer and option holder
  - Option writer creates and sells the option
  - Option holder purchases the right but not obligation
- Call option: right (not obligation) to buy the underlying asset at strike price $K$
  - Profitable when asset price exceeds strike price
  - Maximum loss limited to premium paid
  - Unlimited profit potential as asset price rises
- Put option: right (not obligation) to sell the underlying asset at strike price $K$
  - Profitable when asset price falls below strike price
  - Maximum loss limited to premium paid
  - Substantial profit potential in market downturns

# Vanilla Options II

- Option styles:
  - European-style: exercise only at maturity $T$
  - American-style: exercise at any time before or at expiration date $T$
  - Bermudan-style: exercise at specific dates before maturity
- Payoffs:
  - European call: $\max\{S(T) - K, 0\}$
  - European put: $\max\{K - S(T), 0\}$
  - Profit $=$ payoff - premium paid
- Option writer (short position) is compensated by option premium
  - May face unlimited losses with written calls
  - Requires risk management strategies
- Option holder (long position) pays premium for potential upside
  - Uses include hedging, speculation, and portfolio enhancement

# Hybrid Securities and Securitization

- Financial engineering creates complex assets through:
  - Cash flow bundling and unbundling
  - Adding option-like features to traditional assets
  - Securitization of illiquid assets
- Hybrid securities examples:
  - Convertible bonds: corporate bonds with option to convert to shares
    - Combines debt security with equity option
    - Appeal to issuers when stock price perceived as low
    - Offers investors upside potential with downside protection
  - Callable bonds: bonds that issuer can repurchase
    - Issuer holds call option on their own bonds
    - Advantageous when interest rates fall (refinancing)
    - Cheaper than non-callable bonds due to embedded option
  - Structured bonds: bonds with coupons linked to market indexes
    - Principal repayment guaranteed, coupons linked to index
    - Often used to circumvent derivative restrictions
- Securitization:
  - Creates liquid securities from illiquid assets
  - Example: mortgage-backed securities (MBS)
  - Can involve tranching by risk level
  - Both benefits (enhanced returns, risk distribution) and risks (complexity, correlation)
  - 2008 financial crisis revealed dangers of securitized products

# Market Participants and Their Roles

- Commercial vs. investment banks
  - Deposit-taking vs. non-deposit taking institutions
  - Commercial banks: retail and business services, deposit protection
  - Investment banks: underwriting, M&A, capital markets
  - Different capital raising mechanisms (deposits vs. equity/debt)
  - Regulatory separation (Glass–Steagall) and blurring boundaries
  - Leverage ratios impact both returns and bankruptcy risk
  - Systemic risk concerns with interconnected institutions
- Investment funds and insurance companies
  - Mutual funds: active vs. passive management
    - Active: stock-picking and market-timing
    - Passive: index tracking with lower fees
  - Hedge funds: high-risk strategies
    - Partners rather than clients
    - Sophisticated investors only
    - Complex strategies and illiquid assets
  - ETFs: exchange-traded passive funds
    - Lower costs than mutual funds
    - Traded throughout day like stocks
  - Insurance companies and pension funds: asset-liability management
    - Defined-benefit vs. defined-contribution pension plans
    - Longevity and inflation risks

# Dealers and Brokers

- Market liquidity providers
  - Essential for functioning financial markets
  - Allow continuous buying and selling of assets
  - Different functions but sometimes within same institution
- Brokers:
  - Similar to real estate agents connecting buyers and sellers
  - Compensated by commission on trades
  - No inventory risk exposure
  - Primary brokers serve specialized clients like hedge funds
- Dealers:
  - Maintain inventory of assets they trade
  - Buy and sell from own account
  - Face inventory risk from price fluctuations
  - Compensated by bid-ask spread; reflects liquidity and market conditions
  - Bid price: price at which dealer buys from clients
  - Ask price: price at which dealer sells to clients
- Market frictions:
  - Bid-ask spreads represent transaction costs
  - Taxes and fees are additional frictions
  - Information technology has generally reduced frictions
  - But high-frequency trading raises questions about stability

# Market Participants' Strategies

- Hedgers
  - Exposed to risk factors in normal business
  - Aim to reduce or eliminate risk exposure
  - Example: firms hedging currency risk on international contracts
  - Example: bond issuers hedging interest rate risk
  - May use derivatives to offset existing exposures
  - Willing to sacrifice some return for reduced risk
- Speculators
  - Have views about market direction
  - Willing to take risk for potential returns
  - Example: directional bets on asset prices
  - May use derivatives for leverage
  - Provide liquidity by assuming risk from hedgers
  - Often blamed for market volatility
- Arbitrageurs
  - Exploit price inconsistencies between related assets
  - Help keep prices in line with fundamental values
  - Example: ensuring ETF prices match underlying index
  - Example: options pricing vs. theoretical models
  - Vital role in market efficiency
  - Face practical limitations from transaction costs and liquidity

# Market Structure and Trading Strategies

- Primary vs. secondary markets
  - Primary: where securities are first traded
    - IPOs (initial public offerings) for stocks
    - Seasoned offerings for additional equity
    - Auctions for government bonds
    - Underwritten by investment banks
  - Secondary: where securities trade after issuance
    - Majority of trading volume
    - Provides liquidity to investors
    - Continuous price discovery
- OTC vs. exchange-traded derivatives
  - OTC: tailored but less liquid
    - Customized to specific hedging needs
    - Negotiated directly between counterparties
    - Harder to unwind positions
    - Pricing transparency issues
  - Exchange-traded: standardized and liquid
    - Easily bought and sold
    - Price transparency
    - Reduced counterparty risk

# Auction Mechanisms and the Limit Order Book

- Limit order book structure
  - Electronic system recording all pending orders
  - Two columns: buy orders (left) and sell orders (right)
  - Buy orders sorted by decreasing price
  - Sell orders sorted by increasing price
  - Top entries are "inside quotes" (highest bid, lowest ask)
- Order types:
  - Limit orders: specify maximum buy or minimum sell price
  - Market orders: execute at best available price
  - Trade occurs when limit prices cross
- Price-contingent orders:
  - Stop-loss order: sell when price falls below threshold
  - Limit-sell order: sell when price rises above threshold
  - Limit-buy order: buy when price falls below threshold
  - Stop-buy order: buy when price rises above threshold
- Market liquidity indicators:
  - Bid-ask spread size
  - Order book depth (quantity at each price level)
  - Trading volume and frequency

# Margin Trading and Leverage

- Buying on margin: borrowing money to buy assets
  - Assets serve as collateral plus cash buffer
  - Balance sheet view: assets, liabilities, and equity
  - Initial margin requirement (e.g., 50% or 60%)
  - Maintenance margin requirement (e.g., 25% or 30%)
  - Margin ratio: equity divided by asset value
  - Margin calls when ratio falls below maintenance threshold
  - Position may be liquidated if margin call not met
- Effect of leverage:
  - Boosts both profit and loss potential
  - ROE (return on equity) amplified compared to ROA (return on assets)
  - Interest on borrowed funds reduces net return
  - Example: 50% leverage can nearly double returns
  - Increased bankruptcy risk with excessive leverage
  - LTCM collapse demonstrated dangers of extreme leverage
- Feedback effects in illiquid markets
  - Margin calls can force asset sales
  - Price drops may trigger more margin calls
  - Asset liquidation in thin markets depresses prices further
  - Vicious cycle can lead to market crashes
  - Particularly dangerous with illiquid assets

# Example of Margin Trading

Initial scenario:

- Boom Corp stock price: \$100 per share
- Buy 100 shares (\$10,000 total)
- Borrow \$4,000 from broker
- Initial margin ratio: $\frac{\$6,000}{\$10,000} = 60\%$
- Maintenance margin: 30%

**Balance sheet representation:**

| **Assets** | |
|---|---|
| Stock | \$10,000 |

| **Liabilities** | |
|---|---|
| Loan from broker | \$4,000 |
| **Equity** | \$6,000 |

**If price falls to \$70:**

| **Assets** | |
|---|---|
| Stock | \$7,000 |

| **Liabilities** | |
|---|---|
| Loan from broker | \$4,000 |
| **Equity** | \$3,000 |

Margin ratio: $\frac{\$3,000}{\$7,000} = 43\%$

# Example 1.14: Margin Trading (cont.)

**Limit price calculation:**

- Margin ratio: $\frac{100P - \$4{,}000}{100P}$
- Setting equal to maintenance margin (30%):

$$\begin{aligned} \frac{100P - \$4{,}000}{100P} &= 30\% \\ 100P - \$4{,}000 &= 0.3 \times 100P \\ 100P - 30P &= \$4{,}000 \\ 70P &= \$4{,}000 \\ P &= \$57.14 \end{aligned}$$

**Leverage effect on returns:**

- Asset rises 30%: ROE $= \frac{\$10{,}000 \times 0.30 - \$4{,}000 \times 0.03}{\$6{,}000} = 48\%$
- 50% leverage (borrow \$5,000): ROE $= \frac{\$10{,}000 \times 0.30 - \$5{,}000 \times 0.03}{\$5{,}000} = 57\%$
- If price falls 30%: ROE $= \frac{-\$10{,}000 \times 0.30 - \$5{,}000 \times 0.03}{\$5{,}000} = -63\%$

Leverage magnifies both gains and losses

# Short-Selling

- Mechanics of short-selling:
  - Borrow asset through dealer/broker
  - Sell asset and deposit proceeds plus margin
  - Close position by buying asset and returning it
  - Profit = initial price - (ending price + dividends)
  - Must pay dividends or coupons to lender during borrowing period
- Margin requirements for short positions
  - Margin ratio: equity divided by value of assets owed
  - Initial margin typically 50%
  - Maintenance margin typically 30%
  - Margin calls when price rises too much
  - Balance sheet view: assets (cash), liabilities (borrowed shares), equity
- Risks of short-selling:
  - Potentially unlimited losses (no upper bound on price)
  - Short-squeeze risk (forced to close at unfavorable time)
  - Borrowing costs can be substantial
  - Regulatory restrictions during market distress
  - Controversial practice blamed for market manipulation
- Alternative short positions through derivatives
  - Using futures or options to create synthetic short
  - May be less expensive or restricted than direct shorting

# Example of A Short Trade

Initial scenario:

- We are bearish on DotBomb stock (currently \$100)
- Short-sell 1000 shares, generating \$100,000 proceeds
- Initial margin requirement: 50% (\$50,000 in cash/T-bills)
- Total margin account: \$150,000

**Balance sheet representation:**

| **Assets** | |
|---|---|
| Cash + T-bills | \$150,000 |

| **Liabilities** | |
|---|---|
| Short position in stock | \$100,000 |
| **Equity** | \$50,000 |

**If price falls to \$70:**

- Close position with \$30,000 profit

**If price rises to \$110:**

| **Assets** | |
|---|---|
| Cash + T-bills | \$150,000 |

| **Liabilities** | |
|---|---|
| Short position in stock | \$110,000 |
| **Equity** | \$40,000 |

Margin ratio: $\frac{\$40,000}{\$110,000} = 36\%$

# Example of A Short Trade (cont.)

**Limit price calculation:**

- Margin ratio: $\frac{\$150{,}000 - 1000P}{1000P}$
- Setting equal to maintenance margin (30%):

$$\begin{aligned}
\frac{\$150{,}000 - 1000P}{1000P} &= 30\% \\
\$150{,}000 - 1000P &= 0.3 \times 1000P \\
\$150{,}000 &= 1000P + 300P \\
\$150{,}000 &= 1300P \\
P &= \$115.38
\end{aligned}$$

**Key considerations:**

- Short-selling can be expensive (borrowing costs)
- Risk of short-squeeze (forced to close at unfavorable prices)
- Theoretically unlimited loss potential (no upper bound on prices)
- Alternatives: futures or options can create synthetic short positions

# Market Indexes

- Types of market indexes:
  - Equity indexes (DJIA, S&P500, NASDAQ)
  - Bond market indexes
  - Interest rate indexes (EURIBOR, LIBOR)
  - Volatility indexes (VIX)
  - Geographic scope: national, regional, global (e.g., MSCI)
  - Sector-specific indexes for industries
- Index construction methods:
  - Price-based (e.g., DJIA): equal weighting of price
    - Simple summation of prices divided by divisor
    - Higher-priced stocks have more influence
  - Market-value-weighted (e.g., S&P500): weighted by capitalization
    - Reflects actual economic importance
    - Large companies have more influence
- Mathematical formula: $I = \frac{1}{D} \sum_{k=1}^{m} w_k S_k$
  - $w_k$ are weights (1 for price-based, shares outstanding for market-value)
  - $D$ is divisor (adjusted for continuity)
  - Initially chosen to give "nice" value (e.g., 100 or 1000)
- Index adjustments for corporate actions
  - Stock splits and mergers require divisor changes
  - No adjustments for dividends (affects index value)
  - Different handling in price-based vs. market-value indexes

# Example of Price-based vs. Market-value-weighted Indexes

Scenario:

- Stock A: initial price \$25, increases 20% to \$30
  - 20 million shares outstanding (\$500M market cap)
- Stock B: initial price \$100, drops 10% to \$90
  - 1 million shares outstanding (\$100M market cap)

**Price-based index (e.g., DJIA):**

$$\text{Initial value} = \frac{25+100}{2} = 62.5$$
$$\text{Final value} = \frac{30+90}{2} = 60$$
$$\text{Change} = -4\%$$

**Market-value-weighted index (e.g., S&P500):**

$$\text{Initial value} = \frac{25\times 20M+100\times 1M}{10^6} = 600$$
$$\text{Final value} = \frac{30\times 20M+90\times 1M}{10^6} = 690$$
$$\text{Change} = +15\%$$

# Example of Index Adjustments

Initial scenario:

- Company A: 50 shares outstanding, price \$2
- Company B: 10 shares outstanding, price \$10
- Price-based index value: 6
- Market-value-weighted index value: 100

**Price changes:**

- Company A price increases to \$4
- Company B stock splits 2-for-1, price becomes \$5

**Price-based index adjustment:**

$$\text{Current divisor } D = 2 \text{ (since } \frac{2+10}{2} = 6)$$

$$\text{After A's price change} = \frac{4+10}{2} = 7$$

$$\text{New divisor } D' \text{ needed : } \frac{4+5}{D'} = 7 \Rightarrow D' = \frac{9}{7}$$

**Market-value-weighted index:**

$$\text{Current divisor } D = 2 \text{ (since } \frac{2\times 50+10\times 10}{D} = 100)$$

$$\text{No adjustment needed for stock split}$$

$$\text{New value} = \frac{50\times 4+20\times 5}{2} = 150$$

# Example 1.3: The Balance Sheet and Financial Ratios

| **Assets** | |
|---|---|
| *Current assets* | |
| Cash | \$80M |
| Accounts receivable | \$120M |
| *Fixed assets* | |
| Equipment | \$2,500M |
| **Total assets** | \$2,700M |

| **Liabilities** | |
|---|---|
| *Current liabilities* | |
| Accounts payable | \$300M |
| Long-term debt | \$1,800M |
| **Total liabilities** | \$2,100M |
| **Total equity** | \$600M |

- Book value per share (10M shares): $\frac{\$600M}{10M} = \$60$
- Book-to-market ratio (market price \$40): $\frac{\$60}{\$40} = 1.5$
- Total debt ratio: $\frac{\$2{,}100M}{\$2{,}700M} \approx 0.78$

# Example 1.3: The Balance Sheet and Financial Ratios (cont.)

Assuming net income = \$200M:

- Return on assets (ROA): $\frac{\$200M}{\$2{,}700M} \approx 7.4\%$
- Return on equity (ROE): $\frac{\$200M}{\$600M} \approx 33\%$
- Earnings per share (EPS): $\frac{\$200M}{10M} = \$20$
- Price-to-earnings (PE): $\frac{\$40}{\$20} = 2$

Stock classifications:

- **Value stocks**: Undervalued, low PE and price-to-book ratios
- **Growth stocks**: Look overvalued but promise growth, higher volatility

# Example 1.4: The Liquidity Trap in Thin Markets

In a deep and liquid market, a trade has little impact on prices, but:

- Markets can become thin during stress periods
- Hedge funds often purchase illiquid assets for additional return
- During market stress, flight to quality occurs (selling risky assets)

**The vicious feedback cycle:**

1. Asset values drop, eroding equity of leveraged hedge funds
2. Margin requirements force funds to liquidate assets to raise cash
3. Selling illiquid assets further reduces market prices
4. Lower prices lead to further equity erosion and more margin calls
5. Potential buyers wait for further price drops

**Historical examples:**

- LTCM collapse in 1998 (triggered by Russian default)
- Subprime mortgage crisis: Illiquid MBS couldn't be liquidated, forcing investors to sell liquid securities (stocks)

# Example 1.5: Are You On-The-Run?

- Treasury bonds are issued at regular intervals to finance government debt
- Most recently issued bonds are called *on-the-run*
- Older issues are called *off-the-run*

**Liquidity differences:**

- On-the-run bonds are more actively traded and more liquid
- This liquidity premium affects their price
- Off-the-run bonds trade at slightly lower prices (higher yields)

**Trading opportunities:**

- Traders may try to profit from this price differential
- Strategy: Buy cheaper off-the-run bonds and short-sell more expensive on-the-run bonds
- Requires careful risk management of yield curve shifts

# Example 1.7: A Long Hedge

Scenario:

- In six months we will need 500 ounces of gold
- Current forward price for delivery in 0.5 years: $F(0, 0.5) = 1250$ \$/ounce

**Hedging strategy:**

- Enter into a long position for 500 ounces to lock in the price
- Note: Real contracts may have standardized sizes (e.g., 100 ounces)

**Cash flows at maturity:**

- With physical delivery: $-1250$ \$/ounce $\times$ 500 ounces $= -\$625,000$
- With cash settlement (if spot price is 1150 \$/ounce):

$$[(1150 - 1250) - 1150] \text{ \$/ounce} \times 500 \text{ ounces} = -\$625,000$$

The hedger buys at a cheaper spot price, but the savings are offset by a loss on the forward position

# Example 1.9: Mechanics of Futures Markets

Initial conditions:

- Day 1: Gold futures price is \$1350 per ounce
- Enter long position for two contracts (100 ounces each)
- Initial margin: \$8000 per contract (total \$16,000)
- Maintenance margin: \$5000 per contract

| Day | Settlement price | Daily gain | Cumulative gain | Account balance |
|---|---|---|---|---|
| 1 | \$1346 | −\$800 | −\$800 | \$15,200 |
| 2 | \$1330 | −\$3,200 | −\$4,000 | \$12,000 |
| 3 | \$1334 | \$800 | −\$3,200 | \$12,800 |
| 4 | \$1315 | −\$3,800 | −\$7,000 | \$9,000 |
| 5 | \$1304 | −\$2,200 | −\$9,200 | \$7,800 |

# Example 1.9: Mechanics of Futures Markets (cont.)

| Day | Settlement price | Daily gain | Cumulative gain | Account balance |
|---|---|---|---|---|
| 4 | \$1315 | −\$3,800 | −\$7,000 | \$9,000 |
| 5 | \$1304 | −\$2,200 | −\$9,200 | \$7,800 |
| 6 | \$1320 | \$3,200 | −\$6,000 | \$13,200 |
| 7 | \$1330 | \$2,000 | −\$4,000 | \$15,200 |
| 8 | \$1328 | −\$400 | −\$4,400 | \$14,800 |
| 9 | | \$2,000 | −\$2,400 | \$16,800 |

**Key observations:**

- Day 4: Margin call for \$1,000 (account below maintenance margin)
- Day 5: Another margin call for \$2,200
- Day 6-7: Prices recover, improving account balance
- Day 9: Position closed at \$1338, with total loss of \$2,400

Daily marking-to-market ensures losses are recognized immediately, reducing counterparty risk

# Example 1.10: A Protective Put

Scenario:

- We hold an asset with current value $S_0 = S(t_0)$
- Concerned about potential loss over holding period $[t_0, T]$

**Hedging strategy:**

- Buy a put option with strike price $K$
- Overall portfolio value at maturity:

$$S_T + \max\{K - S_T, 0\} = \max\{K, S_T\}$$

**Tradeoffs:**

- Protection is not free - put option costs money
- Higher strike price = more protection = more expensive option
- Unlike hedging with forwards/futures (zero initial cost), options preserve upside potential
- We give up some profit potential to pay for downside protection

# Example 1.11: A Bullish Speculation

Scenario:

- Current asset price $S_0 = \$100$
- Strong belief price will rise in near future

**Strategy 1: Buy the asset**

- If price rises to \$120, return $= \frac{120-100}{100} = 20\%$

**Strategy 2: Buy a call option (strike price $K = \$100$, premium \$5)**

- If price rises to \$120, return $= \frac{\max\{120-100,0\}-5}{5} = \frac{15}{5} = 300\%$
- If price falls 1% to \$99, return $= \frac{\max\{99-100,0\}-5}{5} = \frac{-5}{5} = -100\%$

**Key tradeoff:**

- Options provide leverage - multiplying both gains and losses
- Limited downside risk (can only lose premium paid)
- But lose entire investment if option expires out of the money

# Example 1.12: A Structured Bond

Real-life example of a structured bond:

- Bond maturity: four years
- Face value payment guaranteed at maturity
- Single coupon, paid at maturity (no periodic coupons)
- Coupon linked to monthly average value of basket of 10 telecom stocks
- If average return negative, coupon = 0 (principal protected)

**Additional features:**

- Option to request anticipated coupon payment every six months (from year 2)
- Option to request early repayment of face value (with reduction)

**Underlying structure:**

- Zero-coupon bond to ensure principal protection
- Complex option on basket of stocks:

$$\max\left(0, \frac{1}{48}\sum_{i=1}^{48}\sum_{j=1}^{10} S_j(t_i) - K\right)$$

where $K$ is initial basket value