Surprises & Paradoxes III

St. Petersburg Paradox

- Proposed by Nicolas Bernoulli in 1713, analyzed by Daniel Bernoulli in 1738
- The game:
  - Flip a fair coin until it shows heads
  - If heads appears on the first flip, win \$2
  - If heads appears on the second flip, win \$4
  - If heads appears on the third flip, win \$8
  - In general, if heads appears on the n-th flip, win $\$2^n$
- The expected value calculation:

$$\mathrm{E}\, X = \sum_{n=1}^{\infty} 2^n \cdot \mathrm{P}(\text{heads on flip } n) = \sum_{n=1}^{\infty} 2^n \cdot \frac{1}{2^n} = \sum_{n=1}^{\infty} 1 = \infty$$

- Note that $\sum_{k=1}^{n} 2^k = 2^{n+1} - 2$
- The paradox: The game has infinite expected value, but most people would only pay a small amount to play
- If people maximize expected value, they should be willing to pay any finite amount to play

Early Solutions to the Paradox

- Practical resolution: No casino has infinite resources
  - With a capped maximum payout of M, $\mathrm{E}\,X \approx \lfloor \log_2 M \rfloor$
  - For $M = 2^{20} \approx \$1$ million, $\mathrm{E}\,X \approx \$20$
- Daniel Bernoulli's insight (1738): People value money differently
  - "The value of an item must not be based on its price, but rather on the utility it yields"
  - Proposed logarithmic utility function: $U(w) = \log(w)$
  - Diminishing marginal utility: Each extra dollar adds less utility
- Gabriel Cramer (1728) suggested: $U(w) = \sqrt{w}$
- Note that if $\forall\,|x| < 1$, $f(x) = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, then $\sum_{n=1}^{\infty} n x^n = x f'(x) = \frac{x}{(1-x)^2}$.
- With logarithmic utility, expected utility is finite:

$$\mathrm{E}\,U(X) = \sum_{n=1}^{\infty} \log(2^n) \cdot \frac{1}{2^n} = \sum_{n=1}^{\infty} \frac{n \log(2)}{2^n} = 2\log(2) < \infty$$

- This amounts to $\mathrm{E}\,U(X) \approx \$1.39$, explaining why people would only pay a small amount

The Expected Utility Hypothesis

Mathematical Formulation

The agent prefers the r.v. X to r.v. Y if and only if $\operatorname{E} U(X) > \operatorname{E} U(Y)$, where $U : \mathbb{R} \mapsto \mathbb{R}$ is the agent's utility function.

- Expected utility theory attempts to explain how people make decisions under uncertainty
- It seeks to address paradoxes like St. Petersburg by incorporating risk preferences
- Key insight: people care about the utility of outcomes, not just monetary values
- Decision-making is based on:
  1. Probabilities of different outcomes
  2. Subjective valuation (utility) of those outcomes

Properties of Utility Functions

- Three common assumptions about utility functions:
  - **More is better**: $U'(w) > 0$ (monotonically increasing)
  - **Risk aversion**: $U''(w) < 0$ (concave function)
  - **Decreasing absolute risk aversion**: $-\frac{U''(w)}{U'(w)}$ decreases as w increases
- Commonly used utility functions:
  - Logarithmic: $U(w) = \log(w)$
  - Power utility: $U(w) = \frac{w^{1-\gamma}-1}{1-\gamma}$ for $\gamma > 0, \gamma \neq 1$
  - Exponential utility: $U(w) = -e^{-\gamma w}$ for $\gamma > 0$
  - Quadratic utility: $U(w) = w - \frac{\gamma}{2}w^2$ for $w < \frac{1}{\gamma}$
- The parameter γ reflects the degree of risk aversion
- For power utility, $\gamma = 1$ corresponds to logarithmic utility (by L'Hôpital's rule)

Risk Aversion and Risk Premium I

- Risk Aversion
  - The **risk aversion** is the preference for a certain amount over a gamble with the same expected value
  - Example: Preferring $50 with certainty over a 50% chance of $100 (and 50% chance of $0)
  - Mathematically represented by a concave utility function: $U''(w) < 0$
- Risk Premium
  - Let $\widetilde{X}$ be a random variable with $\mathrm{E}\,\widetilde{X} = 0$ (a fair gamble)
  - A risk-averse person would pay to avoid this gamble
  - The **risk premium** π is the maximum amount they would pay:

$$U(w - \pi) = \mathrm{E}\, U(w + \widetilde{X}) \tag{1}$$

Risk Aversion and Risk Premium II

- For small risks, we can use Taylor expansion

$$U(w+\widetilde{X}) \approx U(w) + U'(w)\widetilde{X} + \frac{1}{2}U''(w)\widetilde{X}^2$$

- By $\mathrm{E}\,\widetilde{X} = 0$ and $\operatorname{var}\widetilde{X} = \mathrm{E}\,\widetilde{X}^2 - (\mathrm{E}\,\widetilde{X})^2 = \mathrm{E}\,\widetilde{X}^2$,

$$\mathrm{E}\,U(w+\widetilde{X}) \approx U(w) + U'(w)\,\mathrm{E}\,\widetilde{X} + \frac{1}{2}U''(w)\,\mathrm{E}\,\widetilde{X}^2 = U(w) + \frac{1}{2}U''(w)\operatorname{var}\widetilde{X}$$

- Similarly, for small premium, one can use Taylor expansion

$$U(w-\pi) \approx U(w) - \pi U'(w)$$

- Substitute into the risk premium formula (1) $U(w-\pi) = \mathrm{E}\,U(w+\widetilde{X})$,

$$U(w) - \pi U'(w) = U(w) + \frac{1}{2}U''(w)\operatorname{var}\widetilde{X} \implies \pi = \frac{1}{2}\left(-\frac{U''(w)}{U'(w)}\right)\operatorname{var}\widetilde{X}$$

Risk Aversion and Risk Premium - III

- The term $-\frac{U''(w)}{U'(w)}$ is the coefficient of absolute risk aversion (ARA)
  - Measures risk aversion in absolute dollar terms
  - Logarithmic utility: ARA $= \frac{1}{w}$ (decreasing with wealth)
  - Power utility: ARA $= \frac{\gamma}{w}$ (decreasing with wealth)
  - Exponential utility: ARA $= \gamma$ (constant regardless of wealth)
- The term $-\frac{U''(w)}{U'(w)} \cdot w$ is the coefficient of relative risk aversion (RRA)
  - Measures risk aversion relative to wealth level
  - Logarithmic utility: RRA $= 1$ (constant)
  - Power utility: RRA $= \gamma$ (constant)
  - Exponential utility: RRA $= \gamma w$ (increasing with wealth)

Axiomatic Foundation of Expected Utility

- John von Neumann and Oskar Morgenstern (1947) provided axioms for expected utility
- Four key axioms:
  - **Completeness**: For any lotteries L_1 and L_2, either $L_1 \succeq L_2$ or $L_2 \succeq L_1$ or both
  - **Transitivity**: If $L_1 \succeq L_2$ and $L_2 \succeq L_3$, then $L_1 \succeq L_3$
  - **Continuity**: If $L_1 \succeq L_2 \succeq L_3$, then there exists a probability $p \in [0, 1]$ such that $L_2 \sim pL_1 + (1-p)L_3$
  - **Independence**: For any lotteries L_1, L_2, L_3 and any probability $p \in (0, 1]$, $L_1 \succeq L_2$ if and only if $pL_1 + (1-p)L_3 \succeq pL_2 + (1-p)L_3$
- These axioms lead to the expected utility representation:

$$L_1 \succeq L_2 \iff \mathrm{E}_{L_1}[U(x)] \geqslant \mathrm{E}_{L_2}[U(x)]$$

- The independence axiom is particularly important and controversial
  - It states that preferences between lotteries should not be affected by mixing them with a third lottery
  - This axiom is violated in several famous paradoxes

Allais Paradox

- Game A

$$X = \begin{cases} 101 & \text{prob. } 0.33 \\ 100 & \text{prob. } 0.66 \\ 0 & \text{prob. } 0.01 \end{cases} \qquad Y = 100 \ \text{ with prob. } 1$$

Mostly prefer Y to X: from the Expected Utility Hypothesis

$$\begin{aligned} U(100) &> 0.33 \cdot U(101) + 0.66 \cdot U(100) + 0.01 \cdot U(0) \\ &\implies 0.34 \cdot U(100) > 0.33 \cdot U(101) + 0.01 \cdot U(0) \end{aligned} \tag{2}$$

- Game B

$$X = \begin{cases} 100 & \text{prob. } 0.34 \\ 0 & \text{prob. } 0.66 \end{cases} \qquad Y = \begin{cases} 101 & \text{prob. } 0.33 \\ 0 & \text{prob. } 0.67 \end{cases}$$

Mostly prefer Y to X: from the Expected Utility Hypothesis

$$\begin{aligned} 0.33 \cdot U(101) + 0.67 \cdot U(0) &> 0.34 \cdot U(100) + 0.66 \cdot U(0) \\ &\implies 0.33 \cdot U(101) + 0.01 \cdot U(0) > 0.34 \cdot U(100) \end{aligned} \tag{3}$$

Ellsberg Paradox

- Given an urn with 30 balls of colors red, yellow, and black
- There are 10 red balls; total 20 yellow / black balls, but the number of each type unknown
- The agent estimates the probability of drawing yellow as p where $0 < p < \frac{2}{3}$
- A single ball is drawn from the urn

Ellsberg Paradox (Cont'd)

- Game A

$$X = \begin{cases} 100 & \text{if red} \\ 0 & \text{if yellow or black} \end{cases} \qquad Y = \begin{cases} 100 & \text{if yellow} \\ 0 & \text{if red or black} \end{cases}$$

Mostly prefer X to Y: from the Expected Utility Hypothesis

$$\frac{1}{3} \cdot U(100) + \frac{2}{3} \cdot U(0) > p \cdot U(100) + (1-p) \cdot U(0)$$

$$\implies (\frac{1}{3} - p) \cdot U(100) > (\frac{1}{3} - p) \cdot U(0) \quad (4)$$

- Game B

$$X = \begin{cases} 100 & \text{if red or black} \\ 0 & \text{if yellow} \end{cases} \qquad Y = \begin{cases} 100 & \text{if yellow or black} \\ 0 & \text{if red} \end{cases}$$

Mostly prefer Y to X: from the Expected Utility Hypothesis

$$\frac{2}{3} \cdot U(100) + \frac{1}{3} \cdot U(0) > (1-p) \cdot U(100) + p \cdot U(0)$$

$$\implies (\frac{1}{3} - p) \cdot U(0) > (\frac{1}{3} - p) \cdot U(100) \quad (5)$$

Limitations of Expected Utility Theory

- The Allais and Ellsberg paradoxes highlight systematic violations of expected utility theory
- Key limitations:
  - Violation of the independence axiom
  - Inability to account for ambiguity aversion
  - Failure to explain reference-dependent preferences
  - Inconsistent treatment of gains versus losses
- These limitations led Kahneman and Tversky to develop alternative models
- Their research culminated in Prospect Theory (1979) and Cumulative Prospect Theory (1992)
- Won the Nobel Prize in Economics in 2002 (Kahneman; Tversky had passed away)

Prospect Theory – Key Innovations

- Developed by Daniel Kahneman and Amos Tversky (1979)
- Four major innovations over expected utility theory:
  - **Reference dependence**: Outcomes evaluated as gains or losses relative to a reference point
  - **Loss aversion**: Losses hurt more than equivalent gains feel good
  - **Diminishing sensitivity**: Marginal value decreases with distance from reference point
  - **Probability weighting**: People overweight small probabilities and underweight large ones
- Mathematical representation:

$$V(\text{prospect}) = \sum_i \pi(p_i) \cdot v(x_i)$$

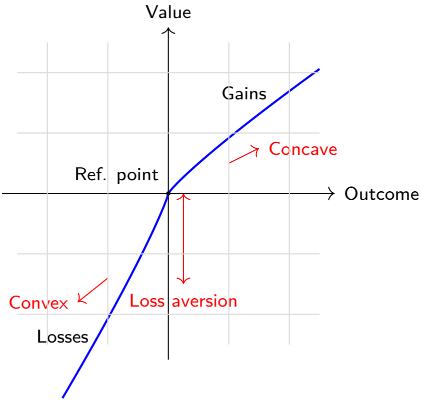
where $v(x)$ is the value function and $\pi(p)$ is the probability weighting function

The Value Function

- The value function $v(x)$ is:
  - Defined on gains and losses (deviations from reference point)
  - Steeper for losses than for gains (loss aversion)
  - Concave for gains (risk aversion)
  - Convex for losses (risk seeking)
- Typical parametrization:

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$

- Parameter values:
  - $\alpha, \beta \approx 0.88$ (diminishing sensitivity)
  - $\lambda \approx 2.25$ (loss aversion coefficient)

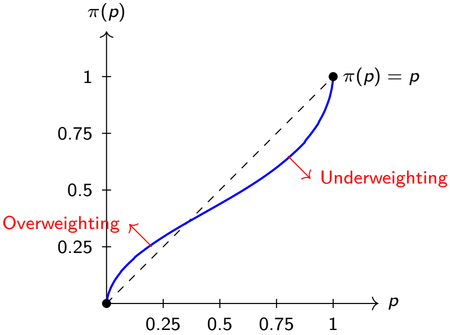


Probability Weighting Function

- People do not process probabilities linearly
- Probability weighting function $\pi(p)$:
  - Overweights small probabilities: $\pi(p) > p$ for small p
  - Underweights large probabilities: $\pi(p) < p$ for large p
  - Fixed points at $p = 0$ and $p = 1$
  - Inverse S-shaped curve
- Tversky and Kahneman's (1992) formulation:

$$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

- Typical parameter value: $\gamma \approx 0.65$



Key Findings: Asian Disease Problem

- Asian Disease Problem (Tversky & Kahneman, 1981)
  - One of the most famous demonstrations of framing effects
  - Scenario: "Imagine the U.S. is preparing for an outbreak of an unusual Asian disease expected to kill 600 people. Two alternative programs to combat the disease have been proposed."
  - **Gain frame** (first group of participants):
    - Program A: "200 people will be saved" (certainty)
    - Program B: "1/3 probability 600 people will be saved, and 2/3 probability no people will be saved" (risk)
    - Result: 72% chose Program A (risk-averse preference)
  - **Loss frame** (second group of participants):
    - Program C: "400 people will die" (certainty)
    - Program D: "1/3 probability nobody will die, and 2/3 probability 600 people will die" (risk)
    - Result: 78% chose Program D (risk-seeking preference)
  - Programs A and C are identical in outcomes, as are B and D
  - The shift in preferences demonstrates that people are risk-averse for gains (saving lives) but risk-seeking for losses (avoiding deaths)

Key Findings: The Reflection Effect

- The Reflection Effect (Kahneman & Tversky, 1979)
  - Core finding: Risk preferences "reflect" (reverse) when prospects are transformed from gains to losses
  - Positive prospects (gains): People generally risk-averse
  - Negative prospects (losses): People generally risk-seeking
- Experimental evidence:
  - Study participants chose between pairs of prospects
  - **Problem 3** (N=95):

$$\text{A: } (4,000, 0.80) \text{ vs. B: } (3,000, 1.00)$$

$$\text{Result: 20\% chose A, 80\% chose B (risk aversion)}$$

  - **Problem 3'** (N=95):

$$\text{C: } (-4,000, 0.80) \text{ vs. D: } (-3,000, 1.00)$$

$$\text{Result: 92\% chose C, 8\% chose D (risk seeking)}$$

  - Exact same probabilities and magnitudes, just with opposite signs
  - Theoretical implication: If $v(x) = -v(-x)$ (symmetry), then preferences would be consistent
  - Observed preference reversal supports S-shaped value function

Key Findings: The Certainty Effect

- The Certainty Effect (Kahneman & Tversky, 1979)
  - People overweight outcomes considered certain compared to merely probable outcomes
  - Reduction from certainty (100%) to uncertainty has more psychological impact than proportionally equivalent reductions (e.g., 80% to 20%)
  - Violates the independence axiom of expected utility theory
- Experimental evidence:
  - **Problem 1** (N=72):

    $$\text{A: } (2,500, 0.33; 2,400, 0.66; 0, 0.01) \text{ vs. B: } (2,400, 1.00)$$

    Result: 18% chose A, 82% chose B
  - **Problem 2** (N=72):

    $$\text{C: } (2,500, 0.33; 0, 0.67) \text{ vs. D: } (2,400, 0.34; 0, 0.66)$$

    Result: 83% chose C, 17% chose D
  - Inconsistency: $B \succ A$, but $C \succ D$
  - In Problem 1, certainty of B is highly attractive despite lower expected value
  - Problem 2 removes certainty, and now people maximize expected value
  - Mathematical violation of independence: If $B \succ A$, then $B' \succ A'$ for any mixture

Key Findings: The Common Ratio Effect

- Common Ratio Effect (Allais, 1953; tested by Kahneman & Tversky, 1979)
  - A manifestation of the certainty effect when probabilities are scaled by common factor
  - Demonstrates nonlinear probability weighting
  - Systematically violates expected utility theory
- Experimental evidence:
  - **Problem 7** (N=66):

    $$\text{A: } (6,000, 0.45) \text{ vs. B: } (3,000, 0.90)$$

    $$\text{Result: 14\% chose A, 86\% chose B}$$

  - **Problem 8** (N=66):

    $$\text{C: } (6,000, 0.001) \text{ vs. D: } (3,000, 0.002)$$

    $$\text{Result: 73\% chose C, 27\% chose D}$$

  - Both problems have identical ratios of probabilities ($0.45/0.90 = 0.001/0.002 = 1/2$)
  - Both have identical outcome magnitudes (6,000 vs. 3,000)
  - Preference reversal occurs because very small probabilities are overweighted
  - When probabilities are low, people prefer the option with the larger payoff
- This violation supports the inverse S-shaped probability weighting function

Key Findings: Probabilistic Insurance

- Probabilistic Insurance experiment (Kahneman & Tversky, 1979)
  - Regular insurance: Pay premium to fully eliminate risk
  - Probabilistic insurance: Pay half premium, but 1% chance insurance doesn't cover the loss
  - Expected utility theory: Probabilistic insurance should be attractive
  - Reality: Most people strongly dislike probabilistic insurance
- Experimental evidence:
  - Scenario: Regular insurance costs $200 to protect against 0.1% risk of $100,000 loss
  - Probabilistic insurance option: Pay $100 premium, but 1% chance insurance is void
  - Would need 95% discount (not 50%) to make people indifferent
  - Mathematical demonstration of nonlinear probability weighting:

$$\begin{aligned}\pi(0.001) &> \pi(0.000) + \pi(0.001) - \pi(0.000) \cdot \pi(0.01) \\ &\approx \pi(0.000) + \pi(0.001) \cdot (1 - \pi(0.01))\end{aligned}$$

- Real-world applications:
  - Explains demand for low-deductible insurance policies
  - Insurance with exclusion clauses perceived as much less valuable
  - People pay significant premium for "peace of mind" (certainty)

Experimental Evidence: The Endowment Effect I

- The Endowment Effect (Thaler, 1980; tested by Kahneman, Knetsch & Thaler, 1990)
  - Definition: People assign higher value to objects they own compared to identical objects they don't own
  - Direct implication of loss aversion in Prospect Theory
  - Formal hypothesis: The disutility of giving up an object is greater than the utility of acquiring it
- Classic Mug Experiment (Kahneman, Knetsch & Thaler, 1990):
  - Participants randomly assigned to three groups:
    - Sellers: Given coffee mugs and asked minimum selling price
    - Buyers: Asked maximum buying price for same mugs
    - Choosers: Asked to choose between mug or cash at different amounts
  - Results:
    - Median selling price: $7.12
    - Median buying price: $2.87
    - Median "choose mug" price: $3.12
  - Selling prices approximately double buying prices
  - Choosers behaved like buyers, not sellers
  - Rules out alternative explanations (income effects, transaction costs)

Experimental Evidence: The Endowment Effect II

- Endowment Effect - Exchange Asymmetries (Knetsch, 1989)
  - Experiment design:
    - Group 1: Given coffee mugs, then offered opportunity to exchange for chocolate bars
    - Group 2: Given chocolate bars, then offered opportunity to exchange for mugs
    - Group 3 (control): Offered direct choice between mugs and chocolate bars
  - Results:
    - Group 1: Only 11% traded mug for chocolate
    - Group 2: Only 10% traded chocolate for mug
    - Group 3: 56% chose mugs, 44% chose chocolate
  - Implications: Strong tendency to stick with status quo regardless of which item received
  - Preference reversals cannot be explained by conventional consumer theory
- Subsequent investigations of the Endowment Effect:
  - Market experience can reduce but not eliminate the effect (List, 2003)
  - Effect stronger for goods valued for use than for exchange (Kahneman et al., 1990)
  - Time of ownership increases the effect (Strahilevitz & Loewenstein, 1998)
  - Not found in cultures with limited private ownership (Apicella et al., 2014)

Experimental Evidence: Framing Effects

- Framing Effects: Same information presented differently leads to different decisions
  - Framing systematically affects preferences across numerous domains
  - Violates description invariance principle of rational choice
  - Demonstrates reference-dependence of value function
- Medical Treatment Frames (McNeil et al., 1982):
  - Physicians and patients chose between surgery and radiation therapy
  - Survival frame: "Surgery has 90% survival rate"
  - Mortality frame: "Surgery has 10% mortality rate"
  - Results:
    - Survival frame: 84% chose surgery
    - Mortality frame: Only 50% chose surgery
  - Same statistical information led to dramatically different choices
  - Professional physicians showed similar biases as patients
- Attribute Framing (Levin & Gaeth, 1988):
  - Product rated more favorably when described in positive terms
  - Ground beef labeled "75% lean" rated higher than "25% fat"
  - Product evaluations:
    - "75% lean" rated 5.8/7 on quality scale
    - "25% fat" rated 3.2/7 on quality scale
  - Effect persisted even after consumers tasted the identical product

Experimental Evidence: Mental Accounting I

- Mental Accounting (Thaler, 1985, 1999): A cognitive process where people categorize and evaluate financial activities
  - People keep separate mental accounts for different types of expenditures
  - Money is not treated as fungible across these accounts
  - Strongly violates standard economic assumptions
- Theater Ticket Experiment (Kahneman & Tversky, 1984):
  - Scenario 1: "You've bought a $10 theater ticket but lost it. Would you buy another?"
    - Result: 46% would buy another ticket
  - Scenario 2: "You plan to buy a $10 theater ticket but realize you've lost $10 cash. Would you still buy the ticket?"
    - Result: 88% would still buy the ticket
  - Economic equivalence: Both scenarios involve losing $10 and deciding whether to spend another $10
  - Psychological difference: Loss is assigned to different mental accounts
  - In Scenario 1, the "theater ticket" account now shows $20 expense (over budget)
  - In Scenario 2, the lost cash is assigned to a general "cash" account, not the "entertainment" account

Experimental Evidence: Mental Accounting II

- Calculator vs. Jacket Experiment (Tversky & Kahneman, 1981):
  - Scenario 1: "You're buying a $15 calculator and learn it's $5 cheaper at another store 10 minutes away. Would you make the trip?"
    - Result: 68% would travel to save $5
  - Scenario 2: "You're buying a $125 jacket and learn it's $5 cheaper at another store 10 minutes away. Would you make the trip?"
    - Result: Only 29% would travel to save $5
  - Same absolute saving ($5) and same effort (10 minute trip)
  - Difference explained by relative thinking: $5 is 33% of $15 but only 4% of $125
  - People respond to relative savings, not absolute amounts
- Sunk Cost Effect (Arkes & Blumer, 1985):
  - Theater season ticket experiment:
    - Random customers given discounts on season tickets
    - Those paying full price attended more shows initially
    - Effect disappeared in second half of season
  - Explanation: People create mental accounts for investments and are reluctant to close them at a loss
  - Standard economics: Sunk costs should be ignored in forward-looking decisions
  - Mental accounting: Past expenditures influence current decisions

Experimental Evidence: The House Money Effect

- House Money Effect (Thaler & Johnson, 1990): Prior gains increase risk-taking
  - Experiment design:
    - Participants either given $15 initially or won it in a prior gamble
    - Then offered various risky bets with positive expected value
  - Results:
    - Those who "won" the initial $15 took significantly more risk
    - 77% accepted a 50/50 bet to win $9 or lose $4.50 after prior gain
    - Only 41% accepted the same bet without prior gain
  - "Playing with house money": Recent gains not fully integrated into wealth
  - Losses that follow gains are coded as "reduced gains" rather than "losses"
  - Diminishes the pain of potential losses
- Break-Even Effect (Thaler & Johnson, 1990): Prior losses can increase risk-taking
  - After experiencing losses, people become risk-seeking if gambles offer chance to break even
  - Betting more after losing ("doubling down")
  - Explanation: People reluctant to close mental accounts at a loss
  - Only applies when gamble offers possibility of complete recovery
  - Otherwise, prior losses typically increase risk aversion ("snake-bit" effect)

Experimental Evidence: The Money Illusion

- Money Illusion (Shafir, Diamond & Tversky, 1997): People think in nominal rather than real terms
  - Traditional economics: People should care only about real purchasing power
  - Reality: Nominal values strongly influence perceptions and decisions
- Economic Satisfaction Experiment:
  - Scenario: Two individuals, Ann and Barbara, graduate same year, same company; both starting salary $30,000
    - Ann: No raise first year; second year 2% increase
    - Barbara: No raise first year, economy experienced 4% annual inflation; second year 5% increase
  - Questions: Who is doing better economically? Who is happier?
    - Economically better: 71% chose Ann (correctly, real terms)
    - Happier: 64% chose Barbara (nominal illusion)
  - Real economic outcomes: Ann (2% real)
  - Real economic outcomes: Barbara (5% nominal - 4% inflation = 1% real)
  - People recognize real vs. nominal in analytical contexts but default to nominal in emotional evaluations
- Implications:
  - Explains resistance to wage cuts even during deflation
  - Contributes to "sticky prices" in macroeconomics
  - Influences housing markets and long-term financial decisions

Beyond Prospect Theory: Heuristics and Biases I

- Tversky & Kahneman explored cognitive shortcuts (heuristics) beyond Prospect Theory
- These heuristics serve as mental shortcuts that reduce cognitive load
- But they also lead to systematic errors and predictable biases
- Three main heuristics identified in their 1974 Science paper:
  1. Availability
  2. Representativeness
  3. Anchoring and adjustment

Beyond Prospect Theory: The Availability Heuristic

- Availability Heuristic: Judging frequency by ease of recall
  - Definition: Events more easily recalled are judged more probable
  - Leads to systematic biases in probability and frequency judgments
  - More memorable = more likely (in people's minds)
- Letter Frequency Experiment (Tversky & Kahneman, 1973):
  - Question: Are there more words starting with 'K' or with 'K' as the third letter?
  - Results: 70% believed more words start with 'K'
  - Reality: English has about $3\times$ more words with 'K' in third position
  - Explanation: Words beginning with 'K' more easily retrieved from memory
- Causes of Death Estimates (Lichtenstein et al., 1978):
  - Participants estimated relative frequencies of causes of death
  - Results: Dramatic overestimation of rare but spectacular causes:
    - Tornadoes judged $20\times$ more common than asthma (actually 1/20 as common)
    - Homicide judged more frequent than diabetes (actually 1/4 as common)
  - Media coverage strongly correlated with overestimation
  - Vivid, dramatic events much more available in memory
- Financial implications: Investors overreact to recent, vivid market events

Beyond Prospect Theory: The Representativeness Heuristic

- Representativeness Heuristic: Judging probability by similarity to stereotypes
  - People assess probabilities based on resemblance to mental prototypes
  - Neglect relevant statistical information (base rates, sample sizes)
  - Focus on "matching" characteristics rather than probability principles
- The Linda Problem (Tversky & Kahneman, 1983):
  - Description: "Linda is 31, single, outspoken, very bright. Philosophy major. Concerned with discrimination, social justice. Participated in anti-nuclear demonstrations."
  - Question: Which is more probable?
    - A: Linda is a bank teller
    - B: Linda is a bank teller AND active in the feminist movement
  - Results: 85% of participants chose B as more probable
  - Conjunction fallacy: P(A B) cannot exceed P(A)
  - People choose the option that better matches (represents) the description
  - Called the "conjunction fallacy" - violates basic probability laws
- Base Rate Neglect Experiment (Kahneman & Tversky, 1973):
  - Participants given personality descriptions of individuals randomly selected from group of 30 engineers and 70 lawyers
  - Asked to estimate probability each person was an engineer
  - Results: Estimates primarily based on how stereotypically "engineer-like" the description was
  - Prior probabilities (base rates) of 30/70 largely ignored

Beyond Prospect Theory: Anchoring and Adjustment

- Anchoring and Adjustment: Initial values heavily influence final estimates
  - People start with an initial value (anchor) and adjust insufficiently
  - The anchor can be completely irrelevant yet still exert strong influence
  - This creates systematic biases in numerical estimation and valuation
- United Nations Experiment (Tversky & Kahneman, 1974):
  - Participants spun a rigged "wheel of fortune" landing on either 10 or 65
  - Then asked to estimate percentage of African nations in the UN
  - Results:
    - Group with anchor of 10: median estimate was 25%
    - Group with anchor of 65: median estimate was 45%
  - Completely arbitrary numbers significantly influenced estimates
- Real Estate Anchoring (Northcraft & Neale, 1987):
  - Professional real estate agents and students evaluated same property
  - Given different listing prices (anchors): $119,900, $129,900, $139,900, $149,900
  - Results:
    - Appraisals directly influenced by the listing price
    - Both professionals and amateurs affected similarly
    - Pros denied using listing price in their valuations
  - Demonstrates anchoring works even with domain experts
- Psychological mechanisms: Both conscious adjustments and unconscious priming effects

Beyond Prospect Theory: Heuristics and Biases II

- Overconfidence Bias: Systematic overestimation of knowledge and abilities
  - Calibration studies (Fischhoff, Slovic & Lichtenstein, 1977):
    - Participants answered general knowledge questions and stated confidence
    - Results: When "99% certain," correct only 80% of the time
    - When "80% certain," correct only 65% of the time
  - Better-than-average effect (Svenson, 1981):
    - 93% of American drivers rated themselves as above-average
    - Statistically impossible for 93% to be above the median
    - Similar results found for professors, entrepreneurs, and students
  - Financial implications: Excessive trading, inadequate diversification, underestimation of risk
- Hindsight Bias: "I knew it all along" effect
  - After learning an outcome, people believe they would have predicted it
  - Fischhoff's (1975) experiment:
    - Participants read historical scenarios with multiple possible outcomes
    - Group 1: Predicted likelihood of each outcome
    - Group 2: Told which outcome actually occurred, then asked what they would have predicted
    - Results: Group 2 assigned much higher probabilities to the outcome they were told happened
  - Creates illusion of predictability and understanding
  - Makes us poor judges of our past decision quality

Beyond Prospect Theory: Heuristics and Biases III

- Status Quo Bias: Tendency to prefer the current state of affairs (Samuelson & Zeckhauser, 1988)
  - Experiment: Participants inherited hypothetical portfolio
    - Group 1: Given portfolio and asked if they want to change investments
    - Group 2: Given cash and asked how to invest from scratch
  - Results: Group 1 far more likely to keep inherited investments
  - Real-world manifestation: Default options in retirement plans
    - When enrollment is opt-in, participation 40%
    - When enrollment is opt-out (automatic), participation 90%
    - Same options available, dramatically different outcomes
  - Related to loss aversion, endowment effect, and regret avoidance
- Present Bias: Overweighting immediate outcomes (O'Donoghue & Rabin, 1999)
  - Time inconsistency: Preferences between today and tomorrow differ from between next year and next year plus one day
  - Example: Prefer $100 today over $110 tomorrow, but prefer $110 in 31 days over $100 in 30 days
  - Hyperbolic discounting: Discount rates decline over time
  - Results in procrastination, undersaving, addiction patterns

Real-World Applications of Prospect Theory in Finance

- Equity Premium Puzzle:
  - Historical excess return of equities over bonds (6%) too large for standard risk models
  - Benartzi & Thaler (1995): Explained by "myopic loss aversion"
    - Investors evaluate portfolios frequently (myopia)
    - Loss aversion makes short-term volatility especially painful
    - Mathematical analysis: 1-year evaluation period + loss aversion coefficient of 2.25 generates 6% premium
    - The more frequently investors check portfolios, the higher risk premium they demand
- Disposition Effect:
  - Empirical finding: Investors sell winning stocks too early, hold losing stocks too long
  - Odean (1998): Analysis of 10,000 trading accounts shows:
    - Investors realize 15% of gains but only 10% of losses
    - Effect cannot be explained by tax considerations, rebalancing, or transaction costs
  - Prospect Theory explanation:
    - S-shaped value function makes investors risk-averse for gains (sell winners)
    - But risk-seeking for losses (hold losers hoping for recovery)
    - Mental accounting keeps track of gains/losses for individual stocks
  - Leads to predictable price patterns and market inefficiencies

Real-World Applications of Prospect Theory in Finance II

- IPO Underpricing Puzzle:
  - Average first-day returns for IPOs 15-20% (money ”left on table”)
  - Loughran & Ritter (2002): Issuers not upset about underpricing when total valuation exceeds expectations
  - Prospect Theory explanation:
    - Issuing firms integrate two outcomes: pre-IPO price revision (gain) and underpricing (loss)
    - Due to diminishing sensitivity, large gain partly offsets smaller loss
    - Reference point is initial expected value before banker's involvement
    - Mental accounting combines the two monetary outcomes
  - Empirical support: Underpricing higher following positive pre-market price revisions
- House Market Anomalies:
  - Genesove & Mayer (2001): Sellers in down markets set prices too high
    - Sellers facing nominal losses (relative to purchase price) set prices 25-35% higher
    - Leads to longer time on market and fewer successful sales
    - Explained by loss aversion relative to original purchase price reference point
  - Anenberg (2011): Volume drops dramatically in down markets
    - Loss aversion creates ”lock-in effect” when market prices fall below original purchase prices
    - Reduces market liquidity and mobility

Critiques and Limitations of Prospect Theory I

- Parameter Calibration Challenges:
  - Original Kahneman & Tversky parameters based on median responses
  - Substantial individual heterogeneity in:
    - Loss aversion coefficient (ranges from 1.5 to 4.5)
    - Risk aversion for gains (from 0.7 to 0.9)
    - Risk seeking for losses (from 0.7 to 0.9)
    - Probability weighting parameter (from 0.5 to 0.8)
  - Parameter stability questionable across domains and contexts
  - Harrison & Rutström (2009): Parameter estimates highly sensitive to experimental design
  - Birnbaum (2008): Different elicitation methods yield different parameters for same individuals
- Descriptive vs. Normative Limitations:
  - Prospect Theory deliberately descriptive, not normative
  - Unlike Expected Utility Theory, not axiomatically derived from "rational" principles
  - Creates tension for applications: Should we design policies that accommodate or correct biases?
  - No clear guidance on debiasing techniques
  - Raises philosophical questions about nature of rational choice
  - Difficult to distinguish "errors" from legitimate preferences

Critiques and Limitations of Prospect Theory II

- Reference Point Ambiguity:
  - Theory critically depends on reference point but doesn't specify how it's determined
  - Multiple potential reference points in real-world scenarios:
    - Current wealth/status (status quo)
    - Expected outcomes
    - Aspiration levels (goals)
    - Past peak experiences
    - Social comparison (peer outcomes)
  - Köszegi & Rabin (2006): Reference points are rational expectations
  - Heath, Larrick & Wu (1999): Goals serve as reference points
  - Multiple reference points may operate simultaneously
  - Reference points may be unstable and manipulable
- Dynamic Inconsistency Issues:
  - Original theory static, not addressing multi-period decisions
  - Reference points likely shift with experience, creating time-inconsistent preferences
  - Thaler & Johnson (1990): Prior outcomes affect risk attitudes
  - Barberis, Huang & Santos (2001): Reference points depend on investment history
  - Difficult to model path-dependent reference points mathematically
  - Creates challenges for dynamic optimization problems

Critiques and Limitations of Prospect Theory III

- Simplicity vs. Reality Tradeoffs:
  - Theory omits important factors in real decisions:
    - Emotions beyond simple gain/loss utility (regret, disappointment, anxiety)
    - Social preferences (fairness, reciprocity, social image)
    - Self-control problems
    - Skill vs. chance attributions
    - Ambiguity attitudes distinct from risk attitudes
  - Loewenstein (1999): "Visceral factors" often overwhelm cognitive evaluations
  - Multiple competing behavioral models for different contexts:
    - Regret Theory (Loomes & Sugden, 1982)
    - Disappointment Theory (Bell, 1985)
    - Rank-Dependent Utility (Quiggin, 1982)
    - Security-Potential/Aspiration Theory (Lopes, 1987)
- Empirical Challenges:
  - Some effects diminish with experience or market discipline (List, 2003)
  - Individual-level predictions often less accurate than aggregate patterns
  - Substantial heterogeneity in behavioral responses
  - Some studies fail to replicate classic findings (Plott & Zeiler, 2005 on endowment effect)
  - Cultural variations in behavioral patterns (Henrich et al., 2001)