

# Introduction to Financial Models

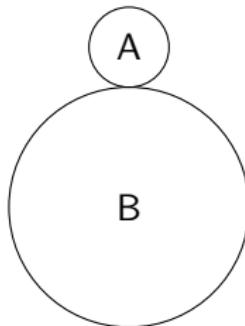
## Lecture 02: Surprises & Paradoxes II

1 Coin Rotation Paradox

2 Braess Paradox

## Coin Rotation Paradox

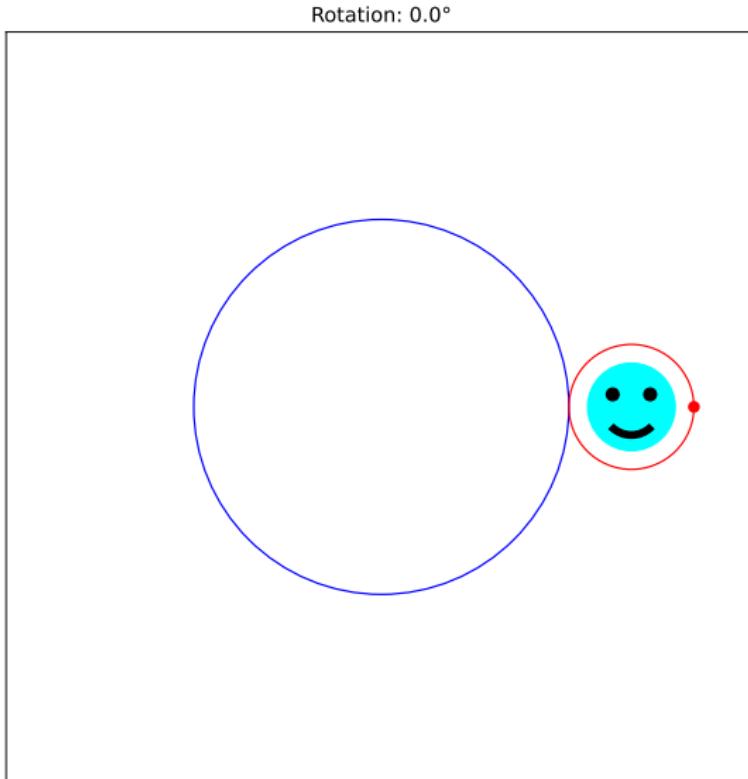
## The 1982 SAT Question Everyone Got Wrong



The radius of circle A is  $\frac{1}{3}$  of the radius of circle B. Circle A rolls around circle B one trip back to its starting point. How many times will circle A revolve in total?

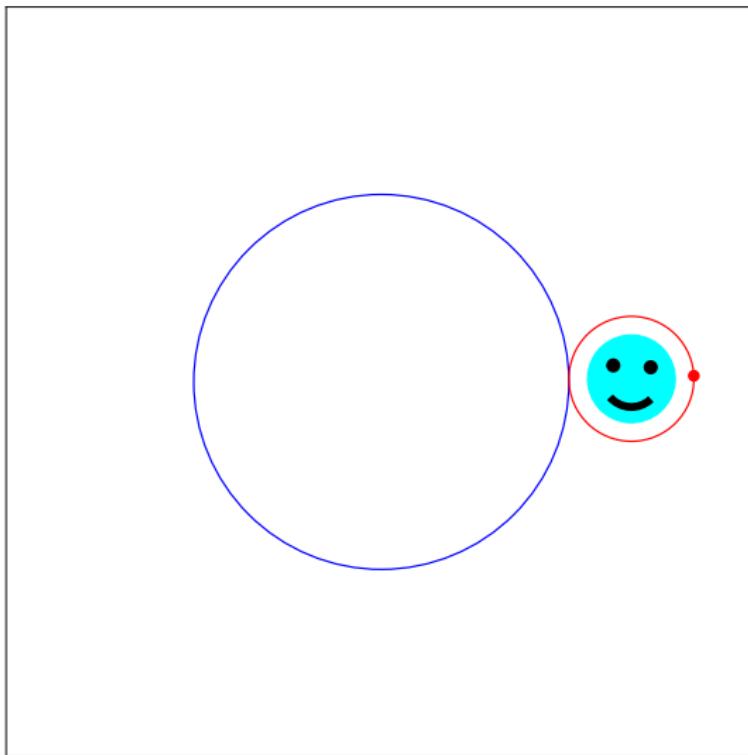
- (a)  $\frac{3}{2}$
- (b) 3
- (c) 6
- (d)  $\frac{9}{2}$
- (e) 9

# Animation of Coin Rotation Paradox

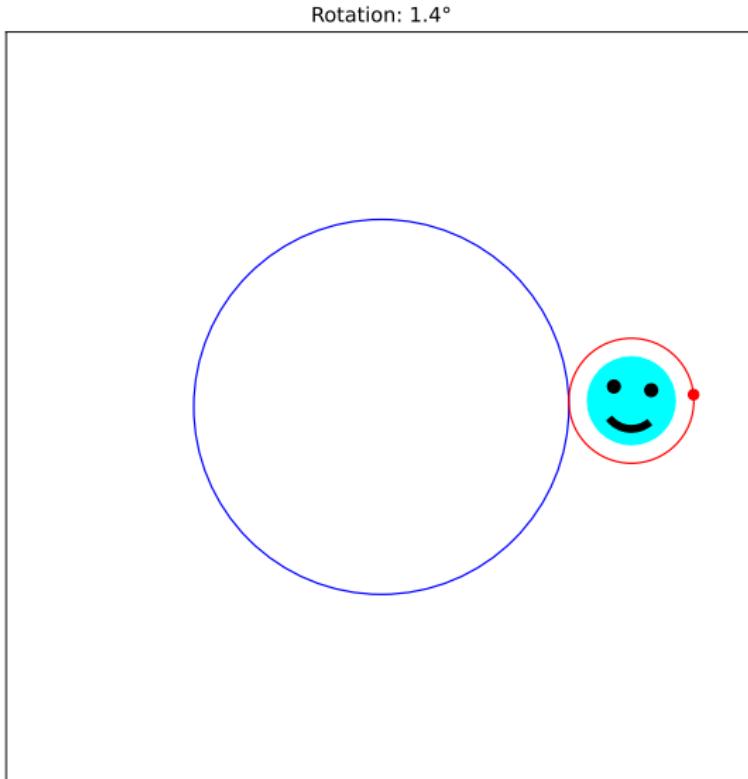


# Animation of Coin Rotation Paradox

Rotation: 0.7°

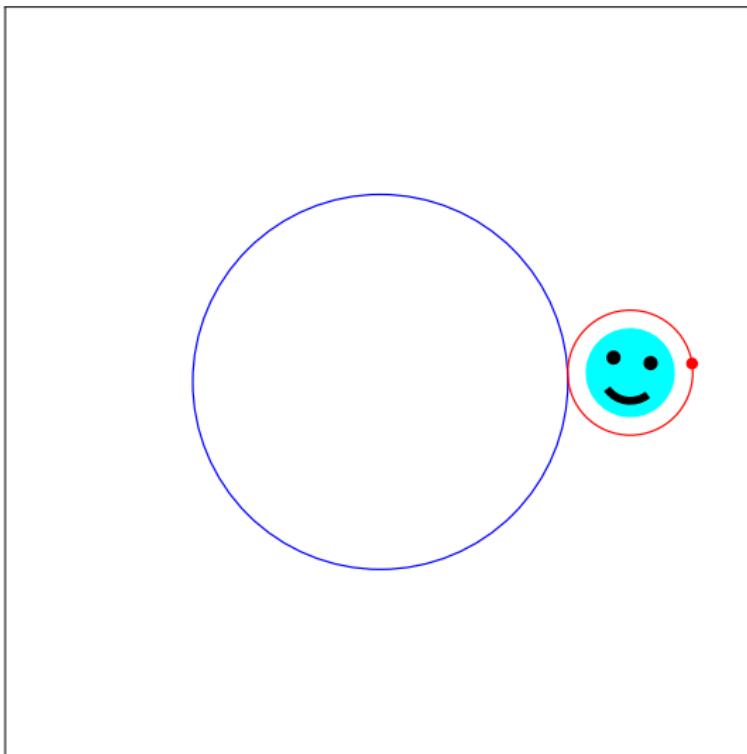


# Animation of Coin Rotation Paradox



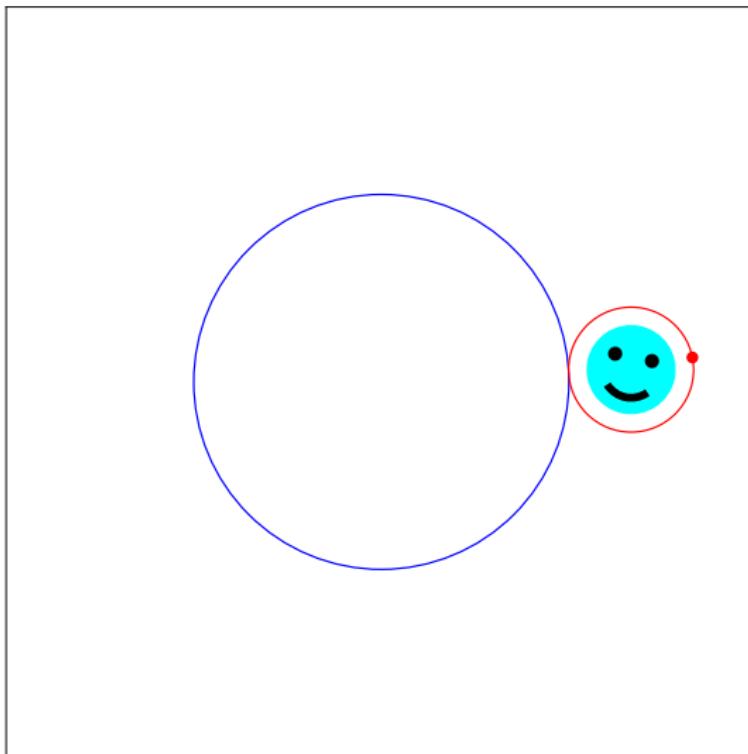
# Animation of Coin Rotation Paradox

Rotation: 2.1°



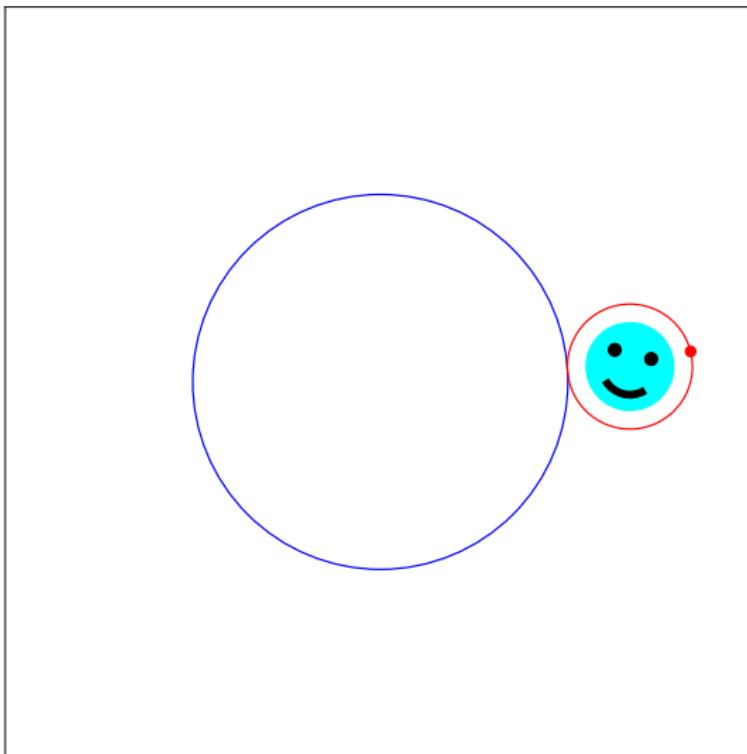
# Animation of Coin Rotation Paradox

Rotation: 2.8°



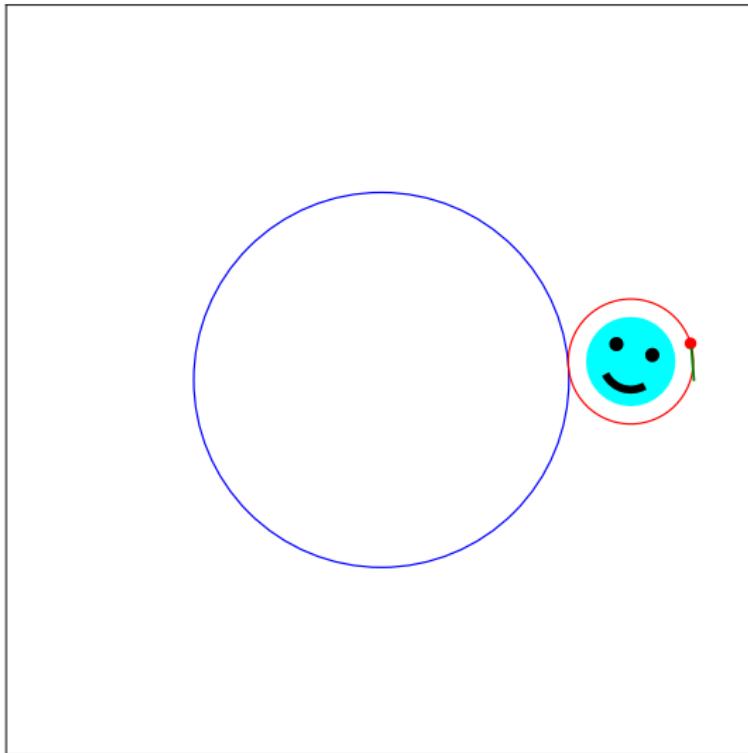
# Animation of Coin Rotation Paradox

Rotation: 3.5°



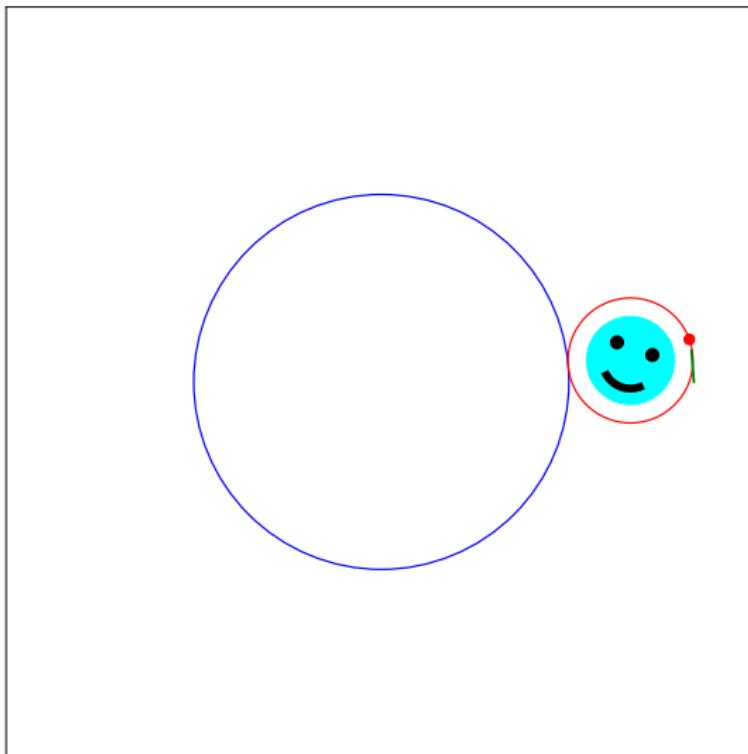
# Animation of Coin Rotation Paradox

Rotation: 4.2°



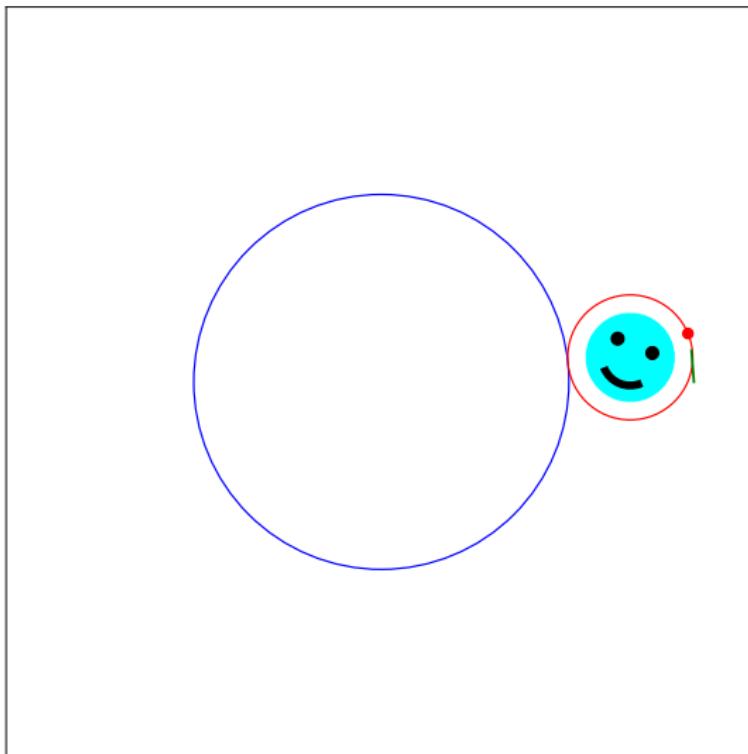
# Animation of Coin Rotation Paradox

Rotation: 4.9°



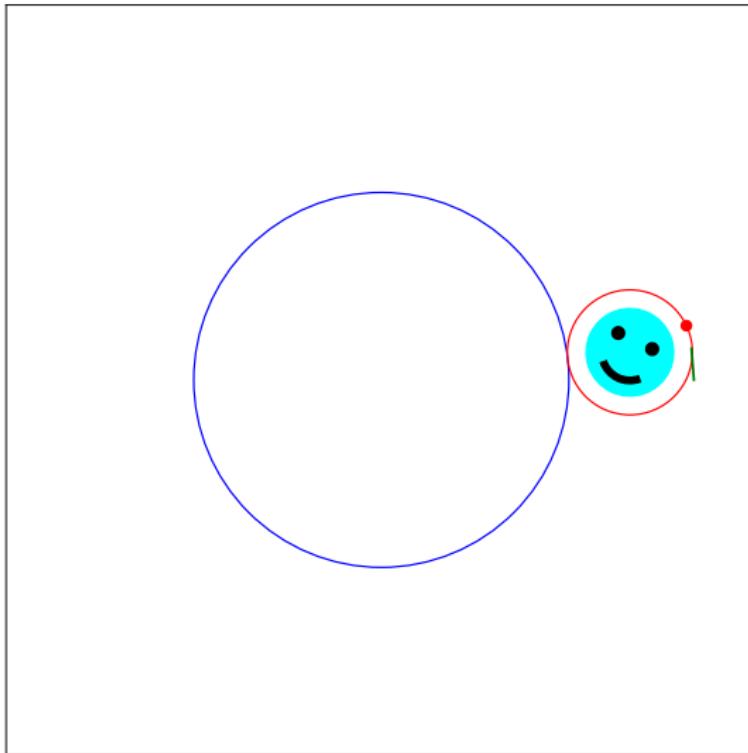
# Animation of Coin Rotation Paradox

Rotation: 5.6°



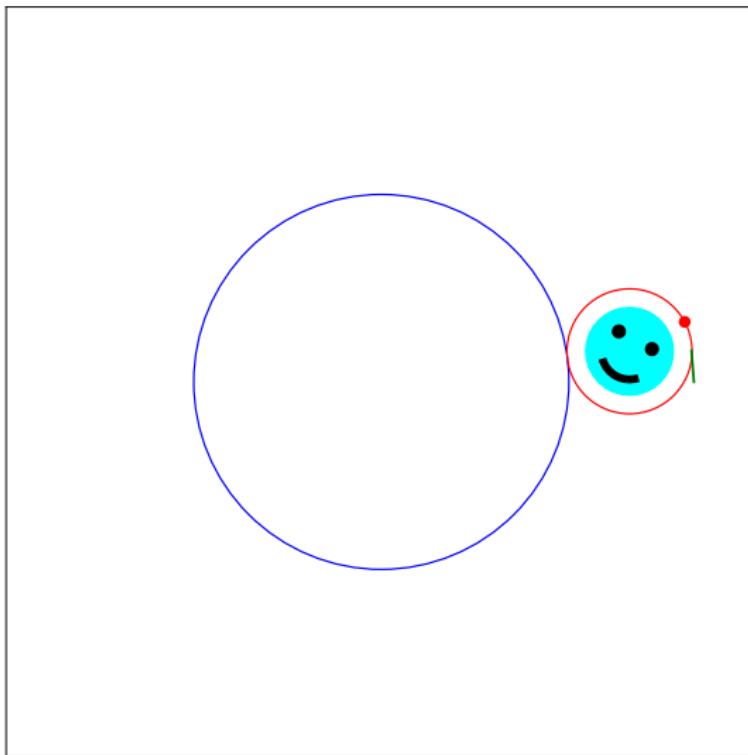
# Animation of Coin Rotation Paradox

Rotation: 6.3°



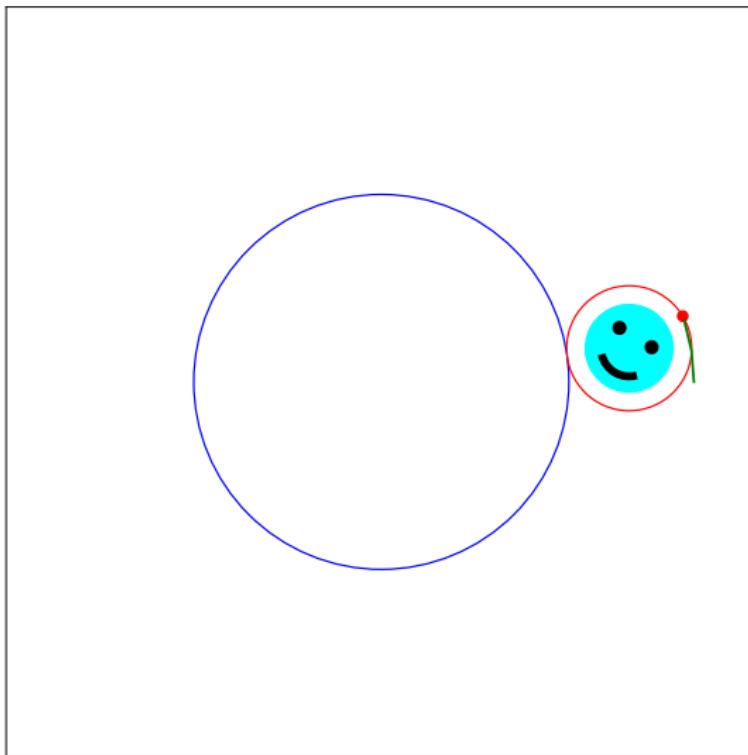
# Animation of Coin Rotation Paradox

Rotation: 7.0°



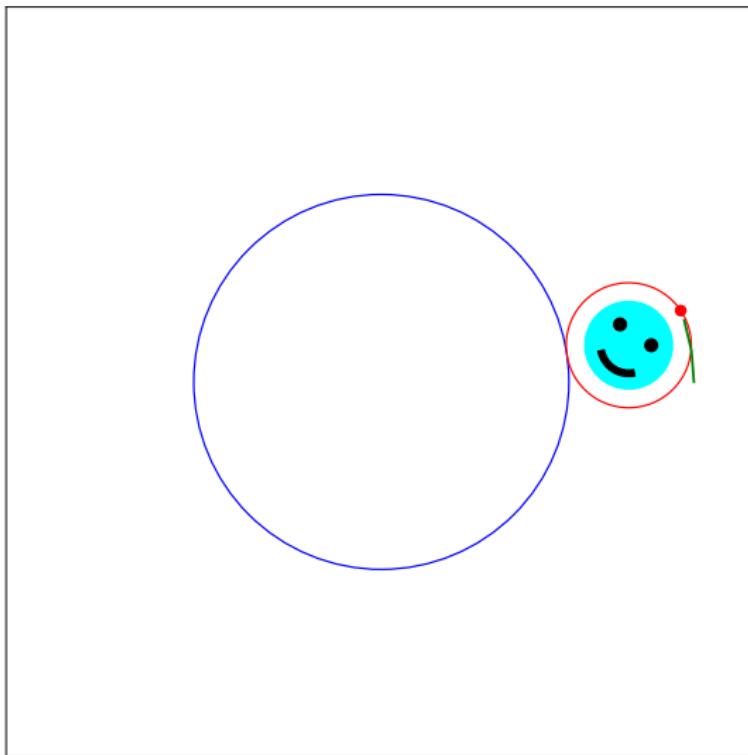
# Animation of Coin Rotation Paradox

Rotation: 7.7°



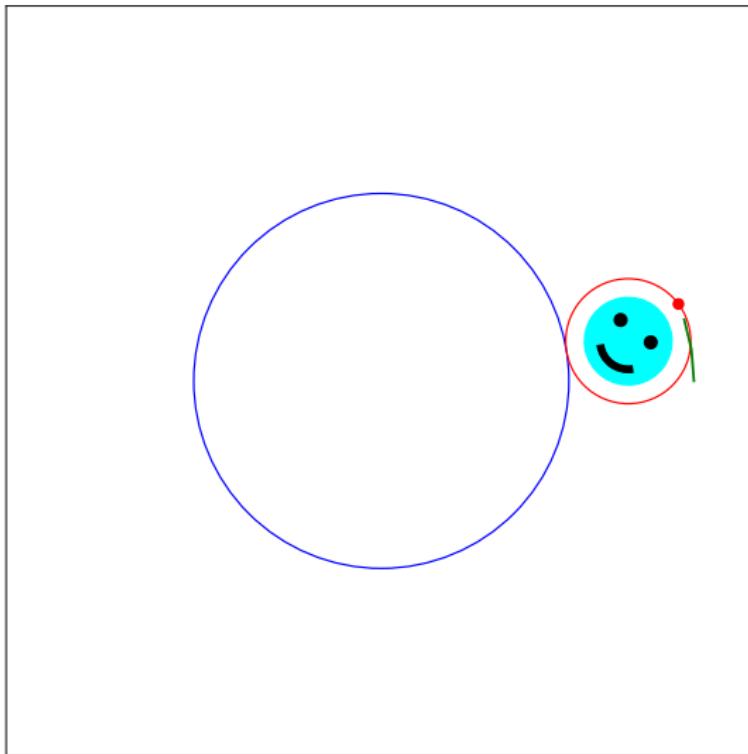
# Animation of Coin Rotation Paradox

Rotation:  $8.4^\circ$



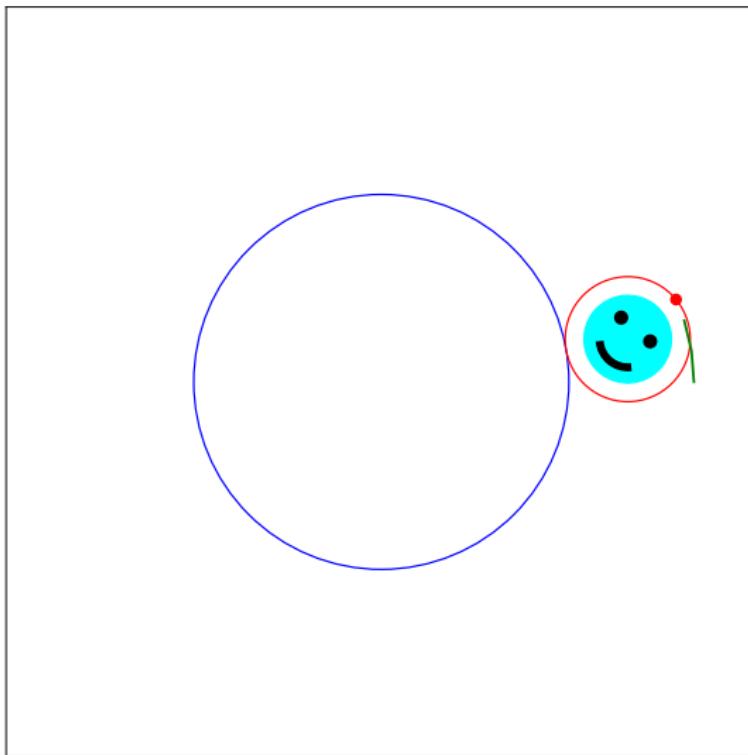
# Animation of Coin Rotation Paradox

Rotation: 9.1°



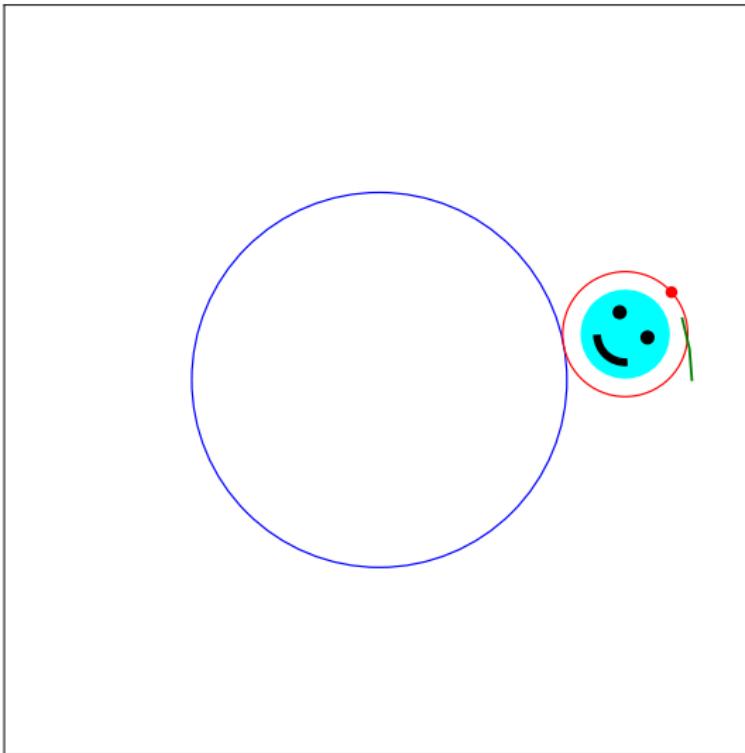
# Animation of Coin Rotation Paradox

Rotation: 9.8°

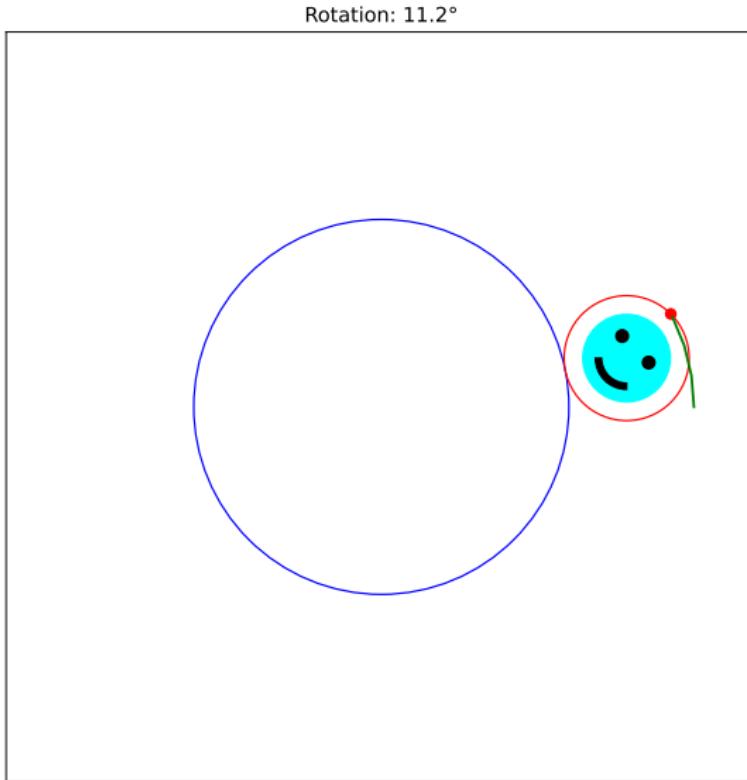


# Animation of Coin Rotation Paradox

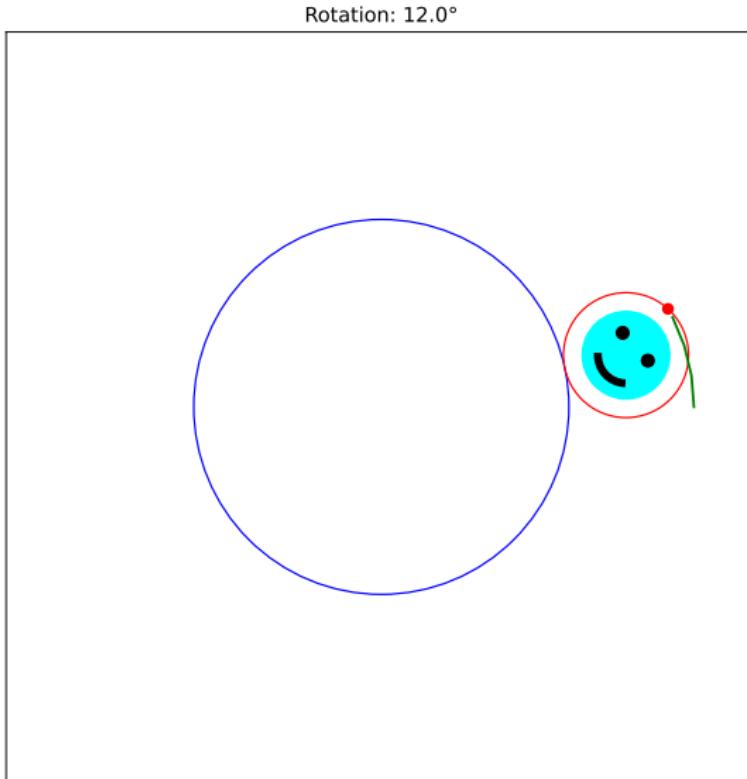
Rotation: 10.5°



# Animation of Coin Rotation Paradox

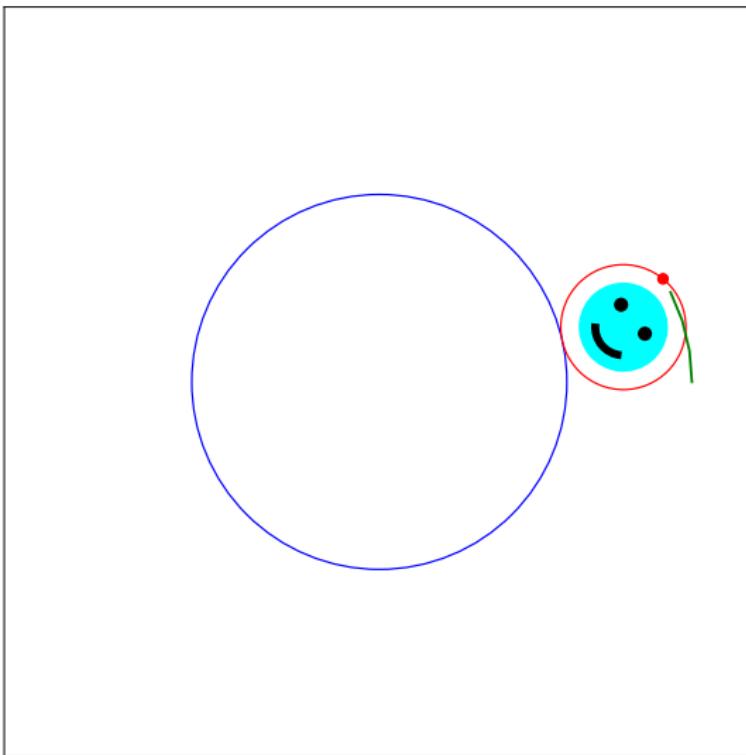


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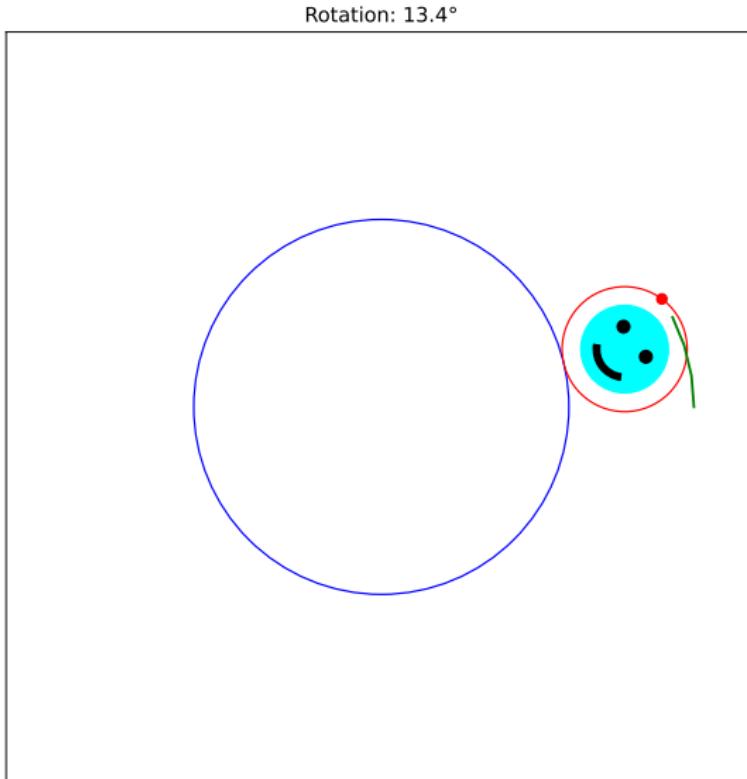


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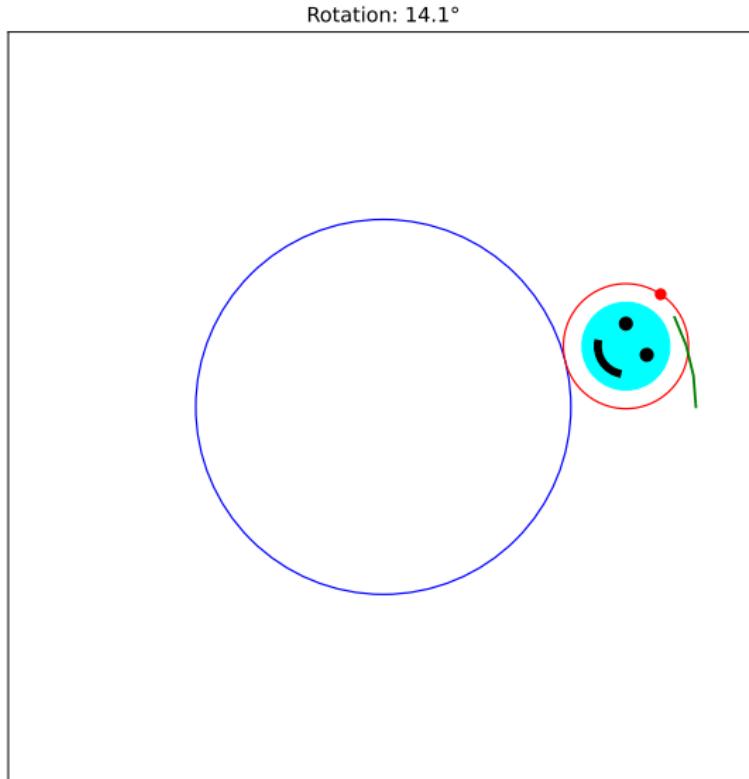
Rotation:  $12.7^\circ$



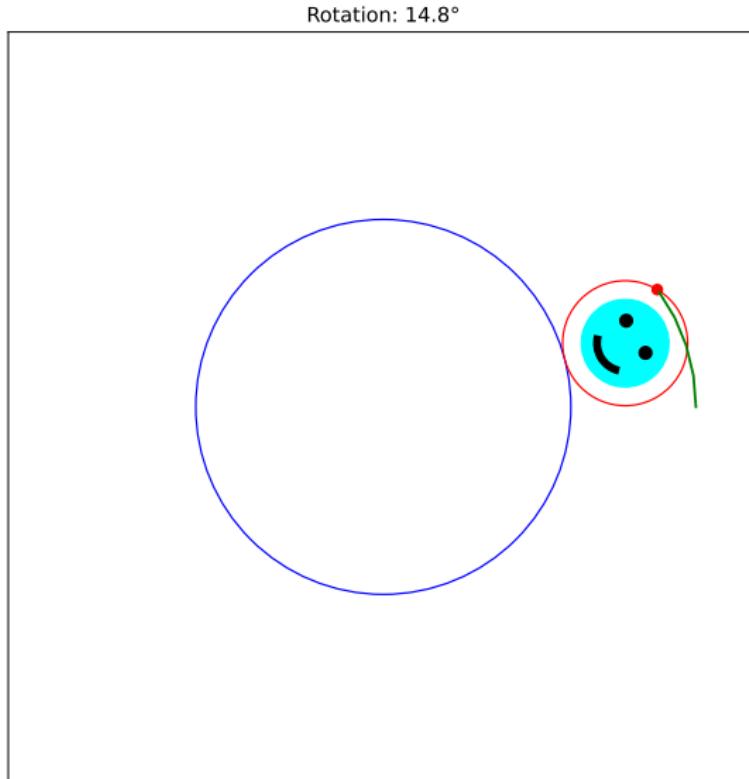
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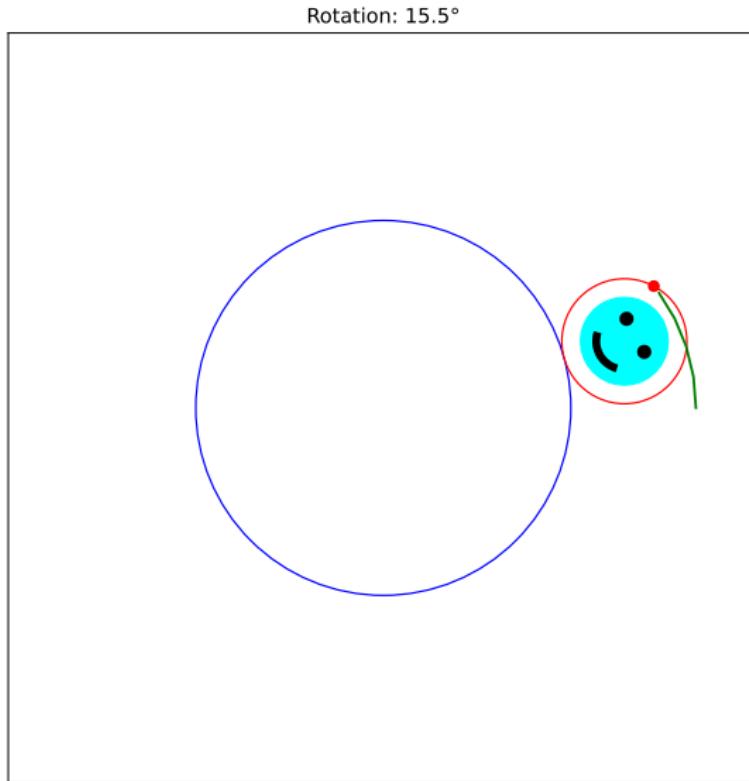
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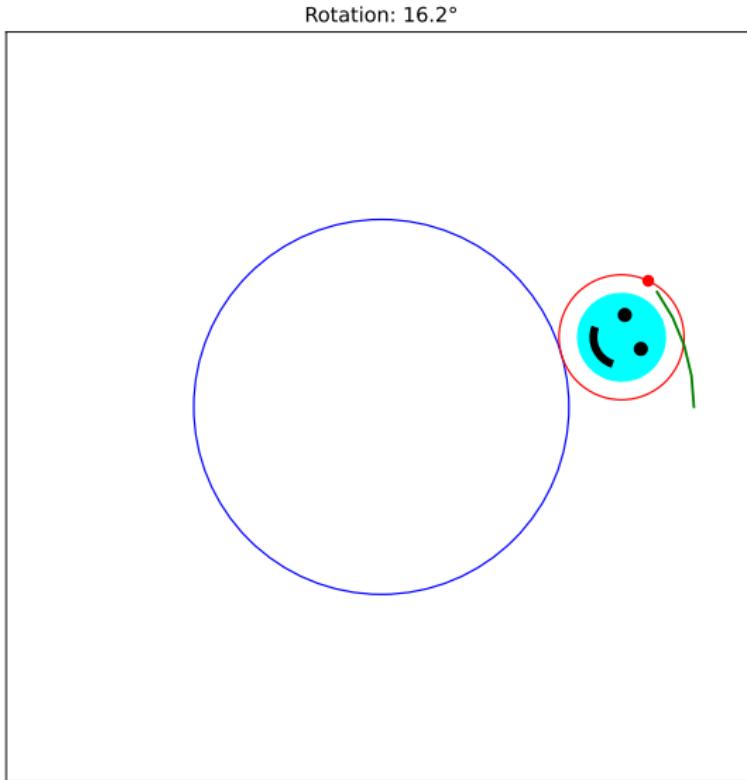
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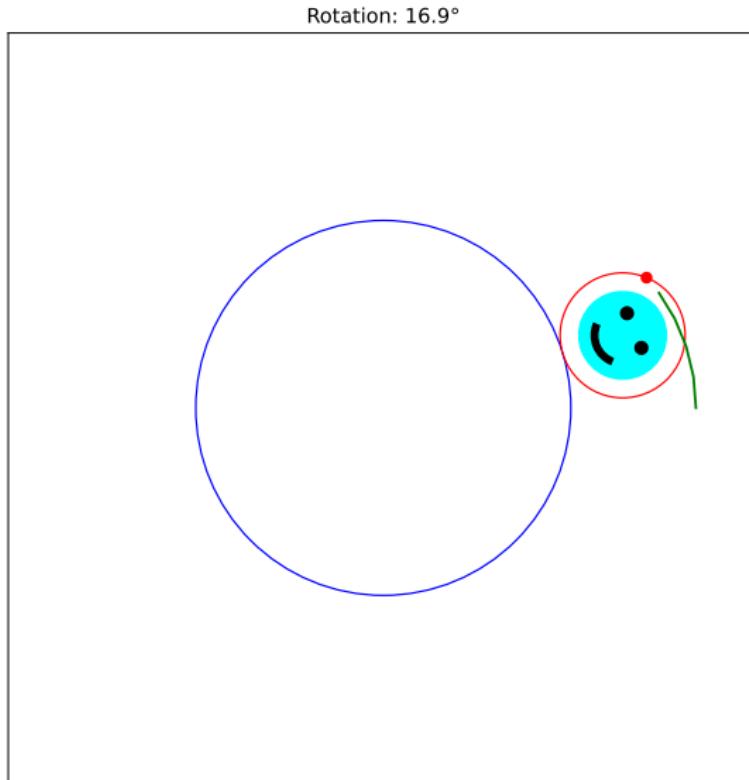
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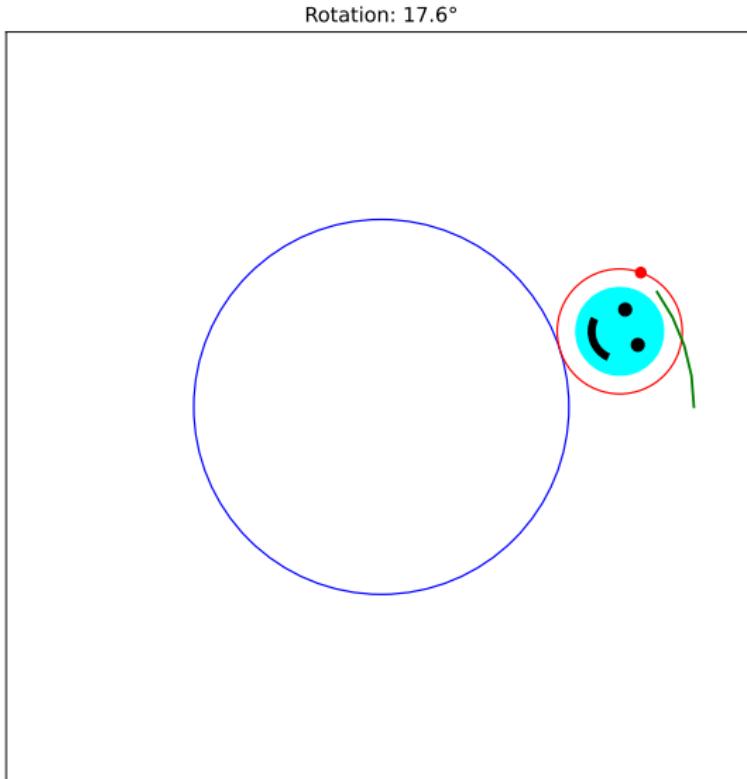
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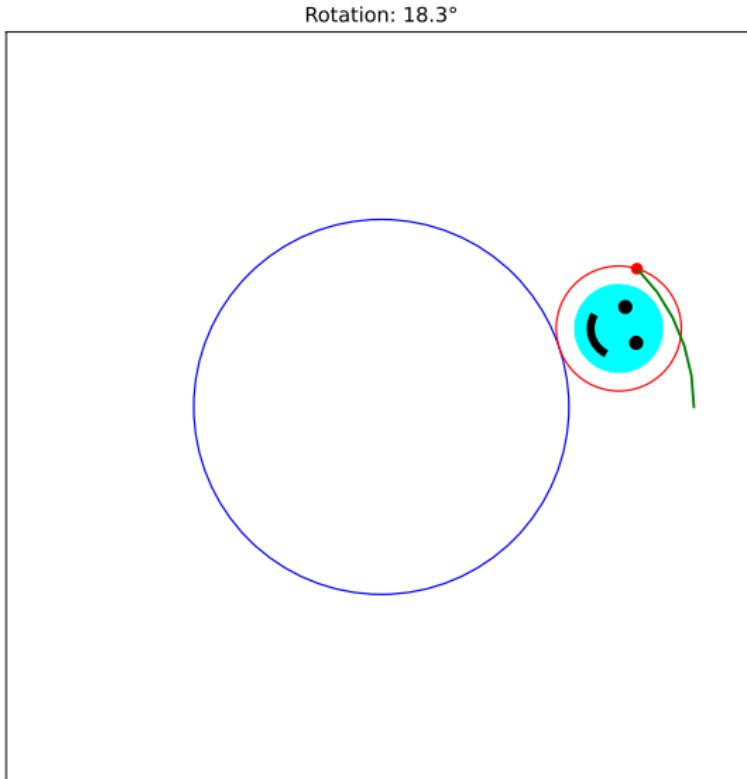
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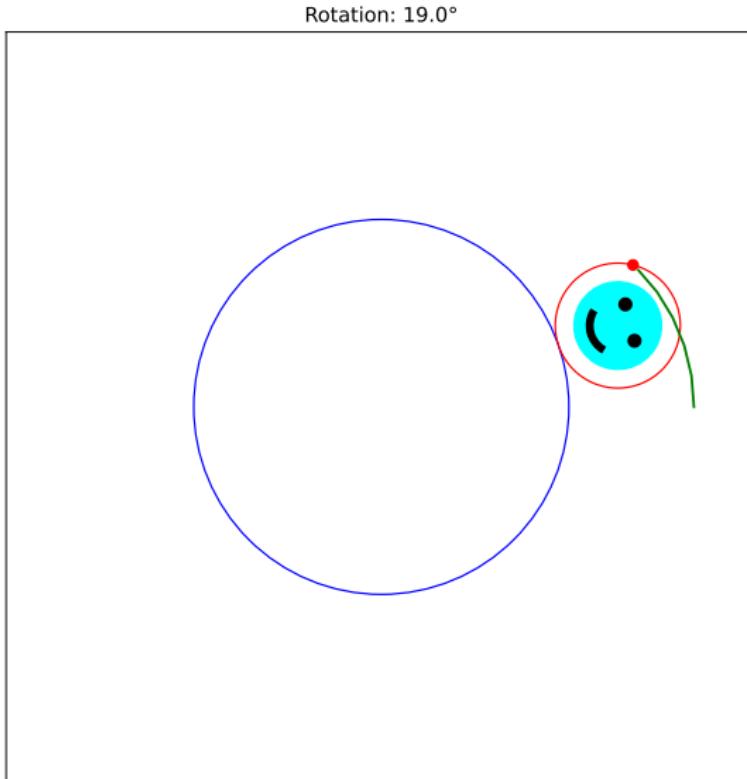
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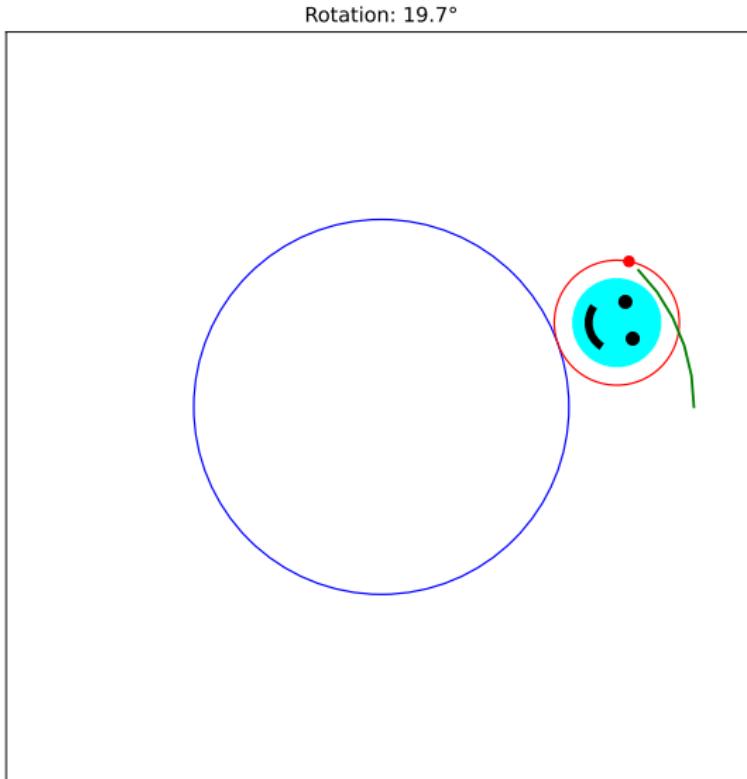
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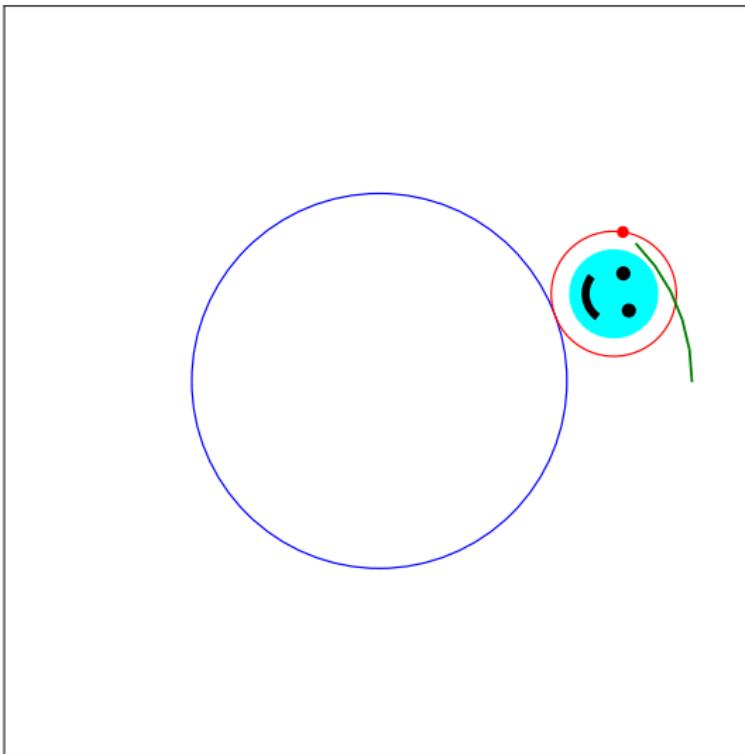


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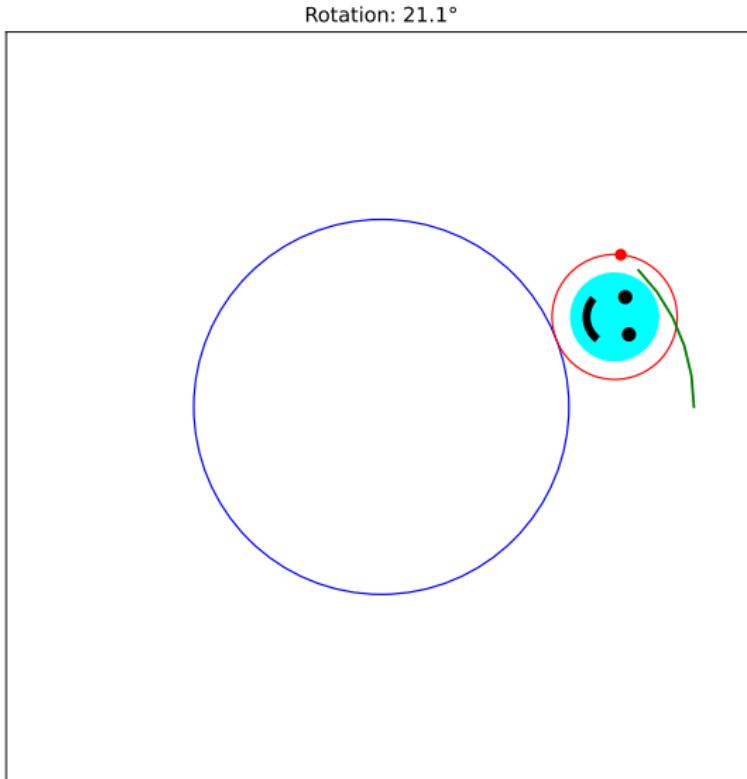


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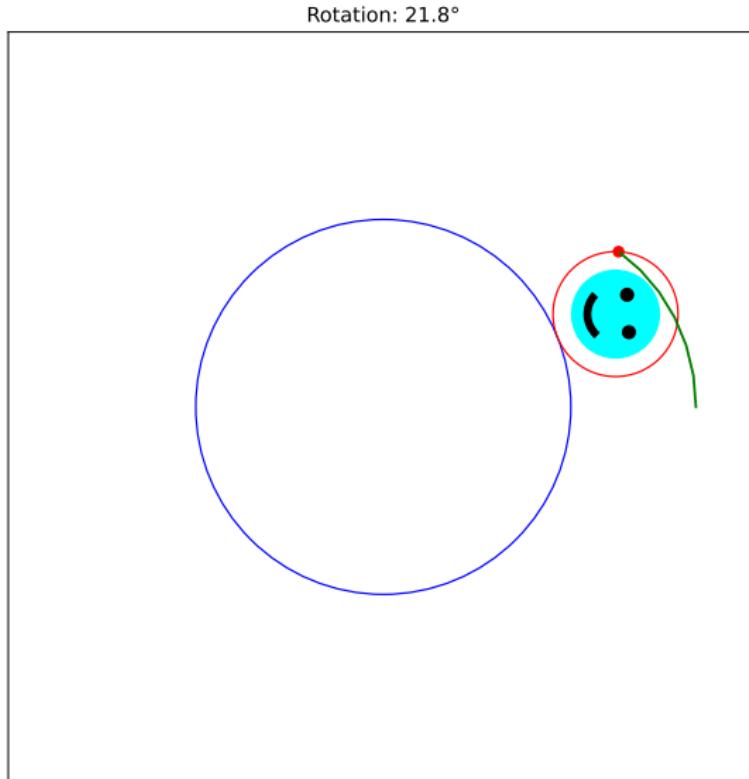
Rotation: 20.4°



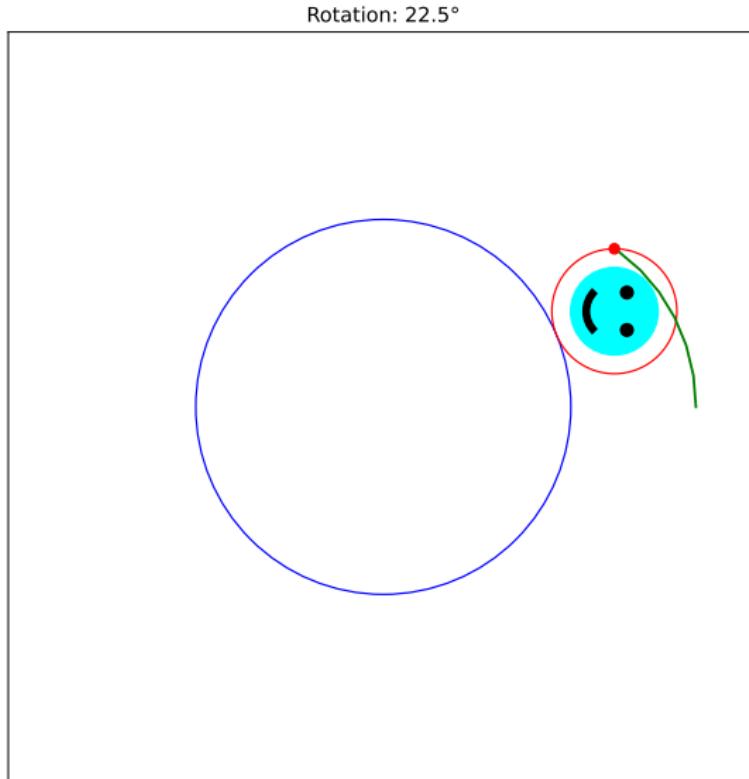
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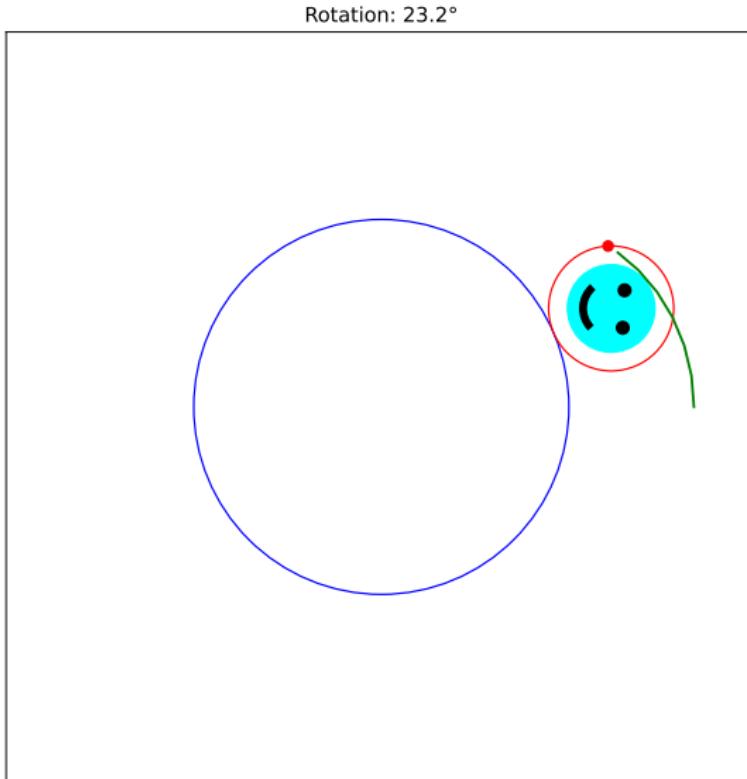
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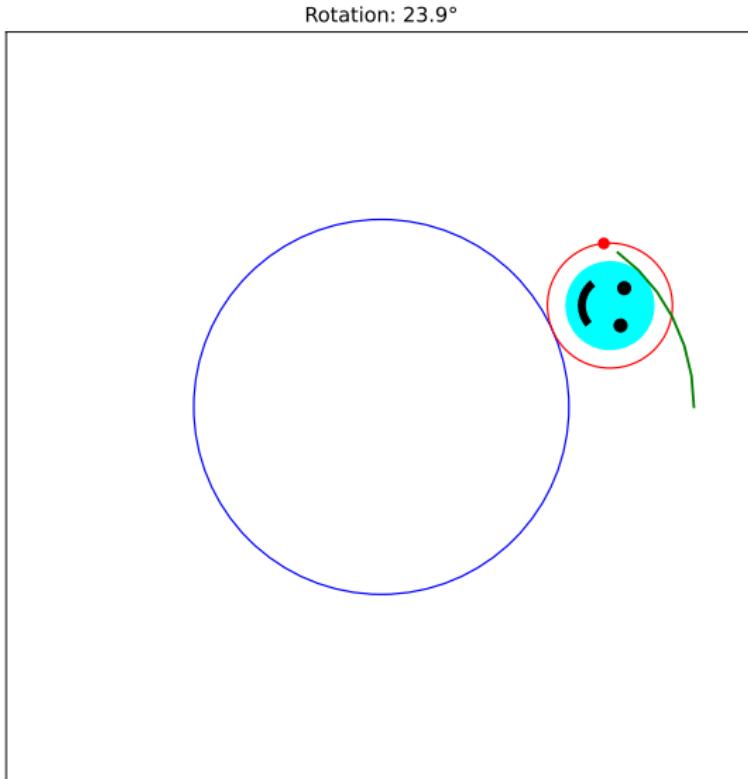
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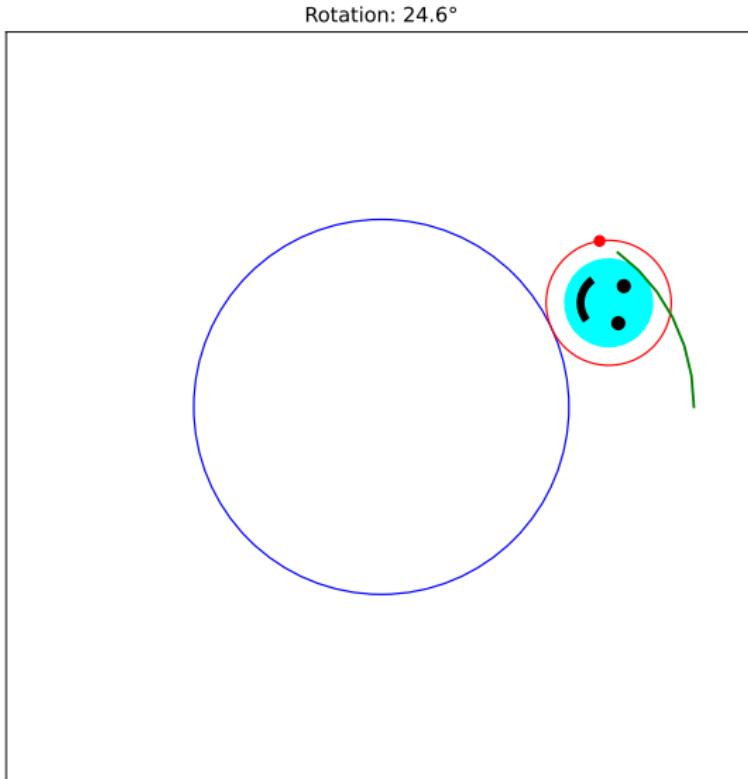
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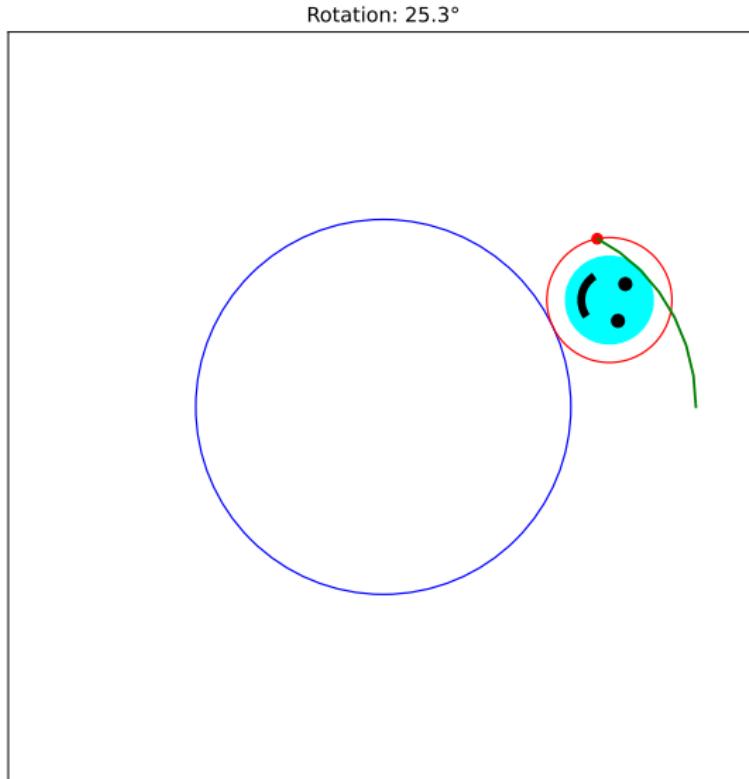
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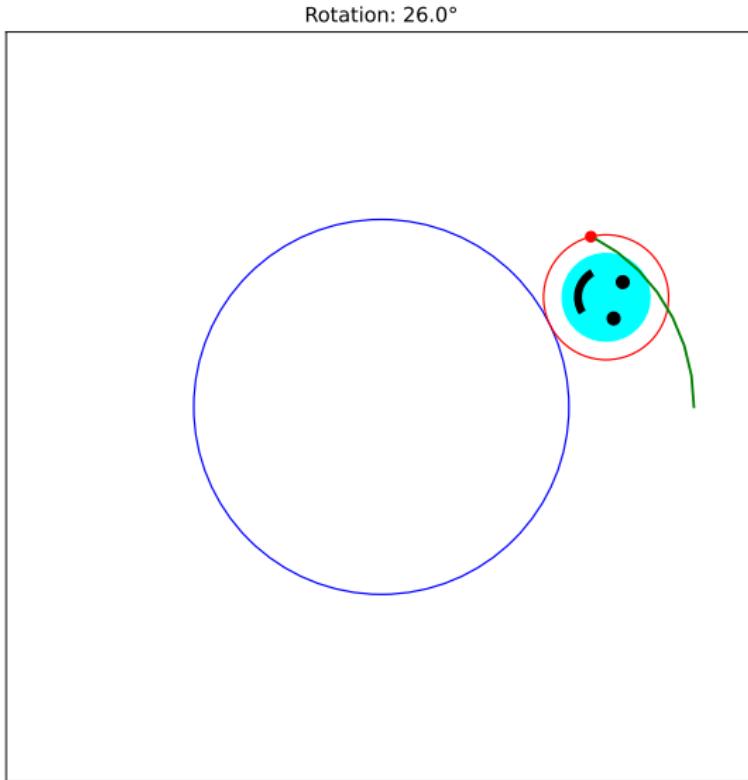
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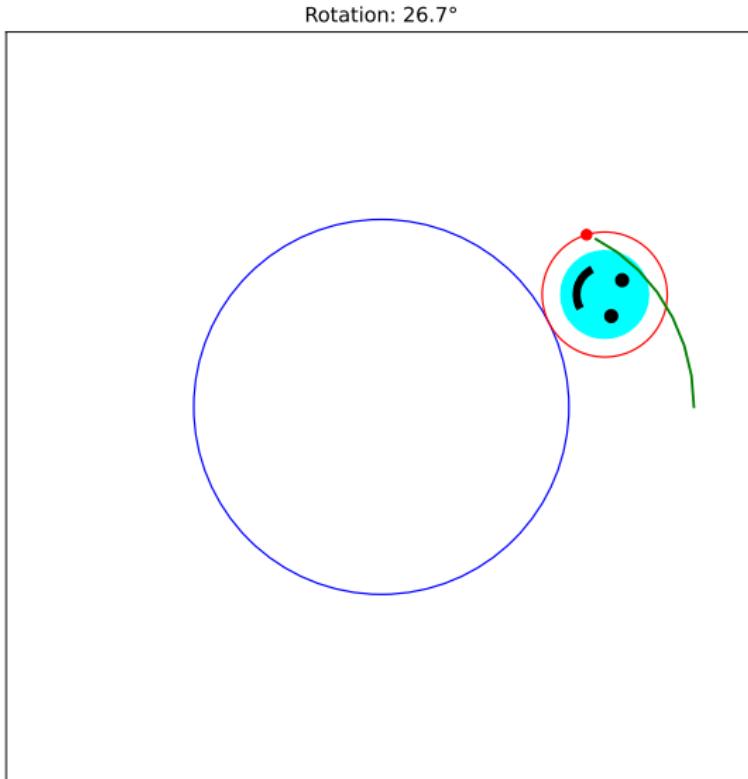
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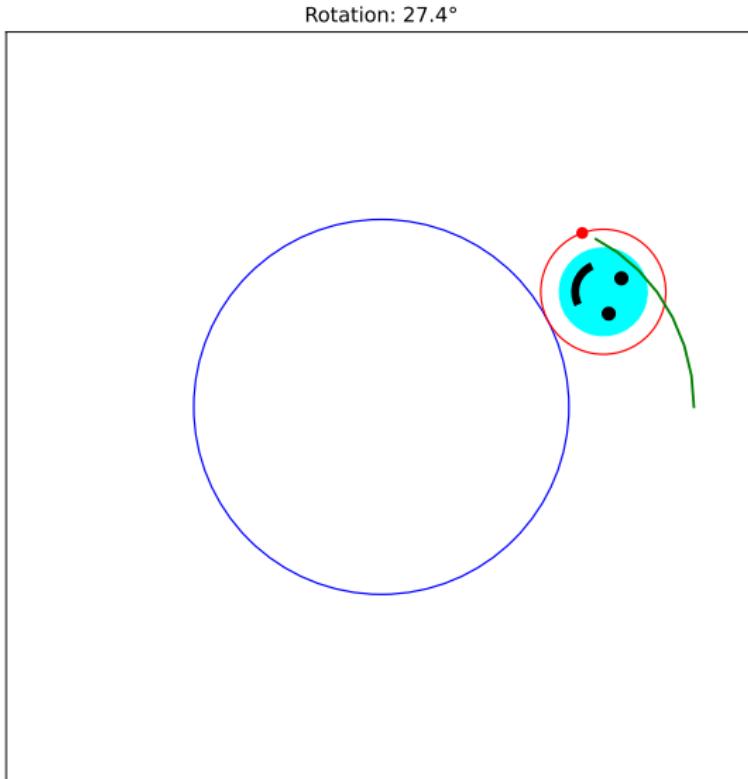
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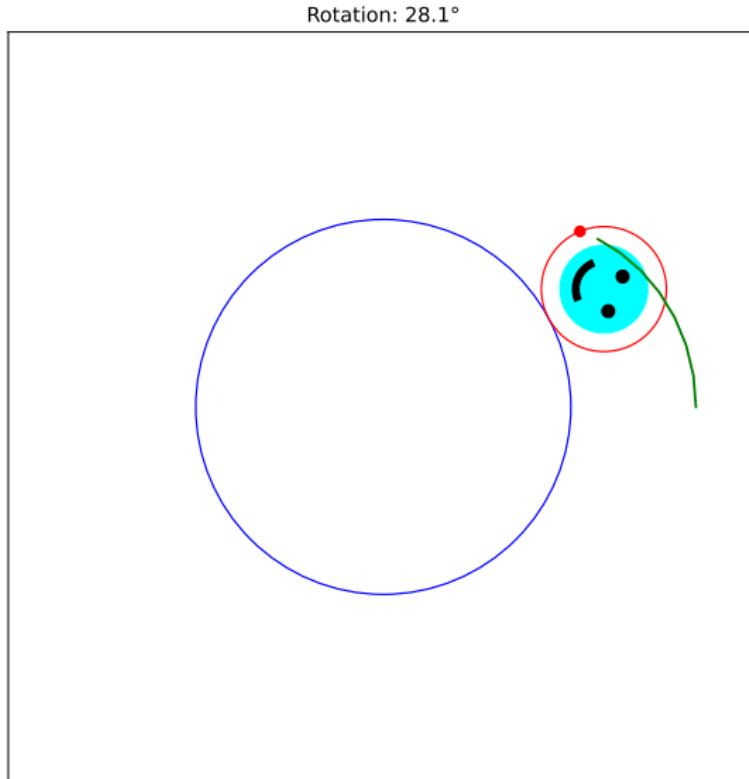
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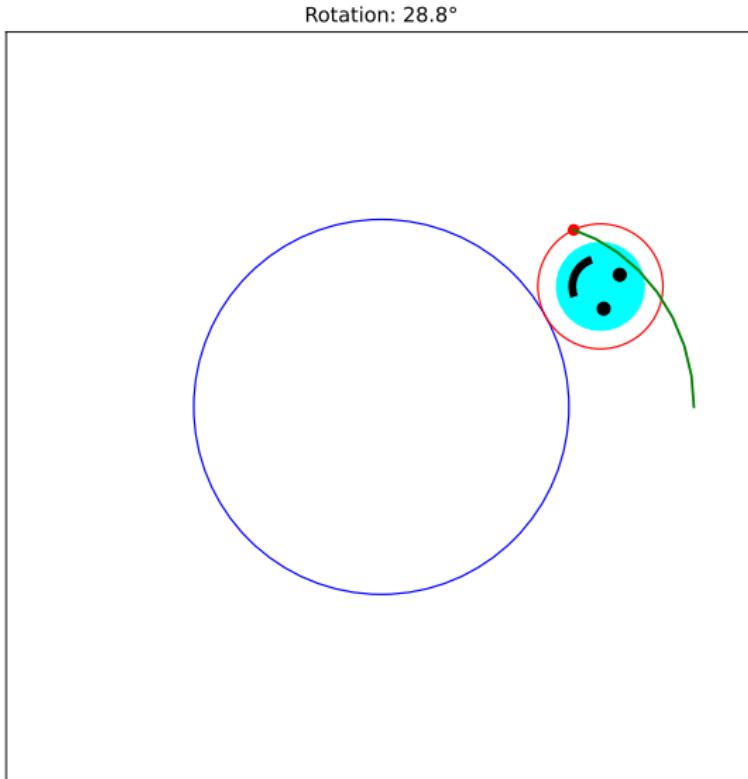
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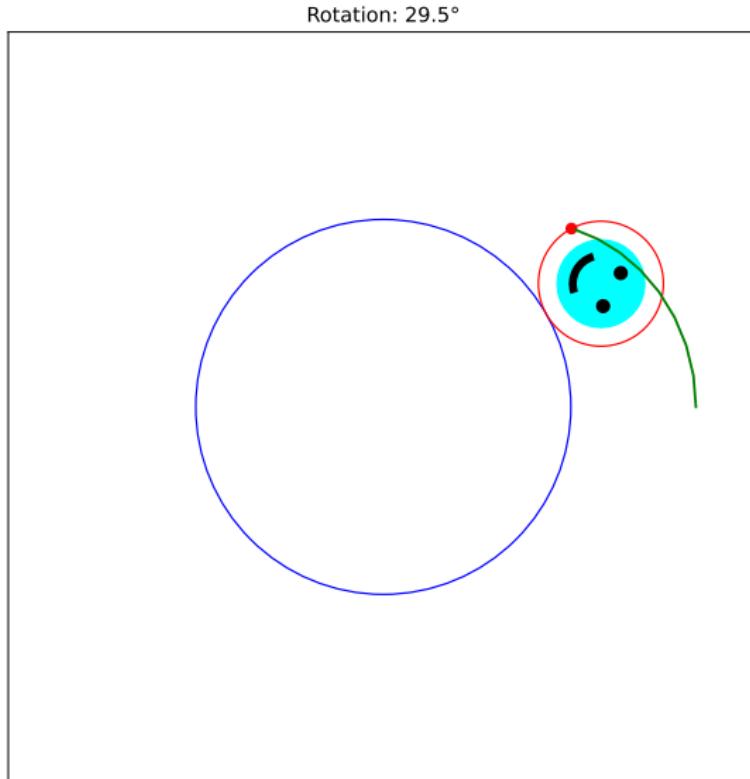
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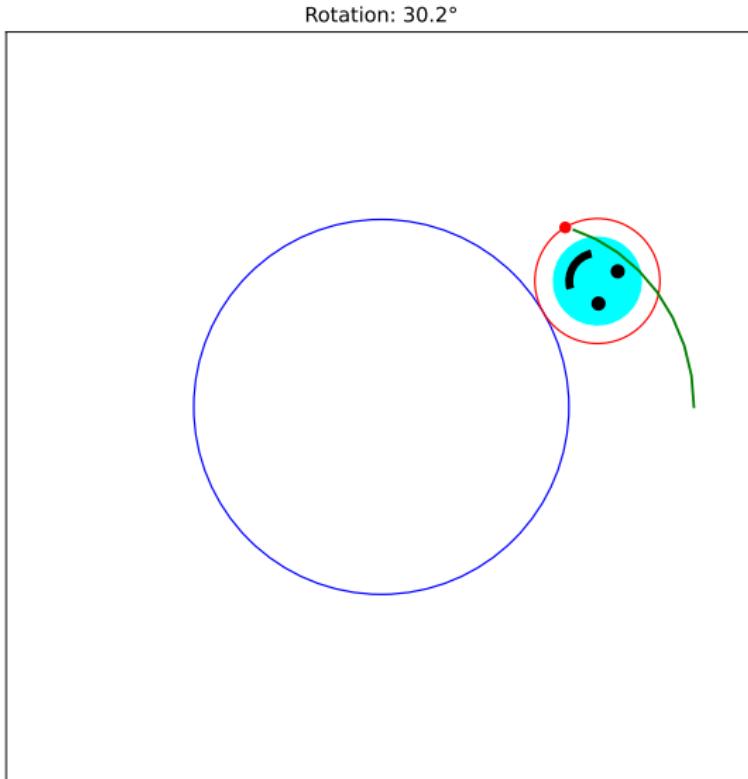
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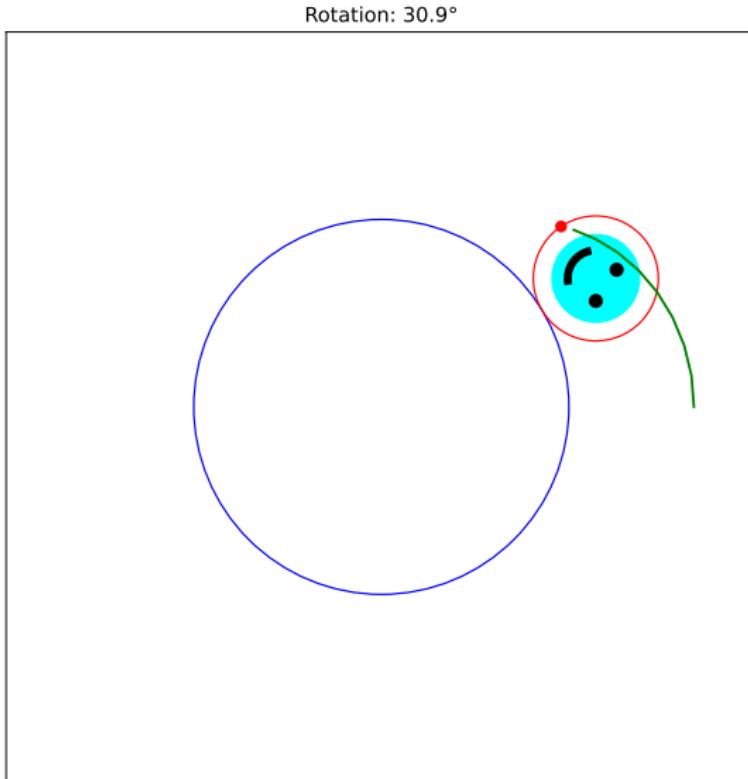
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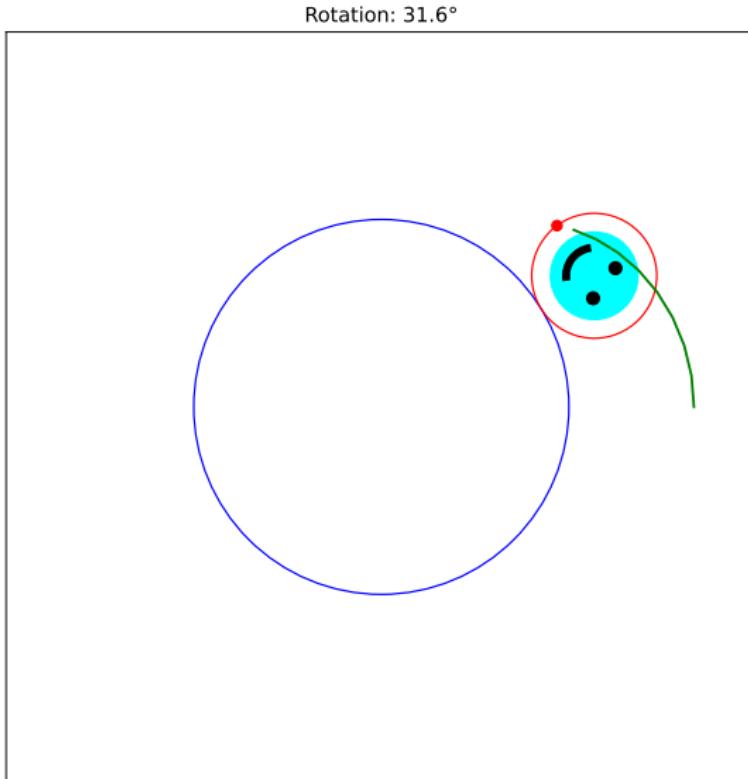
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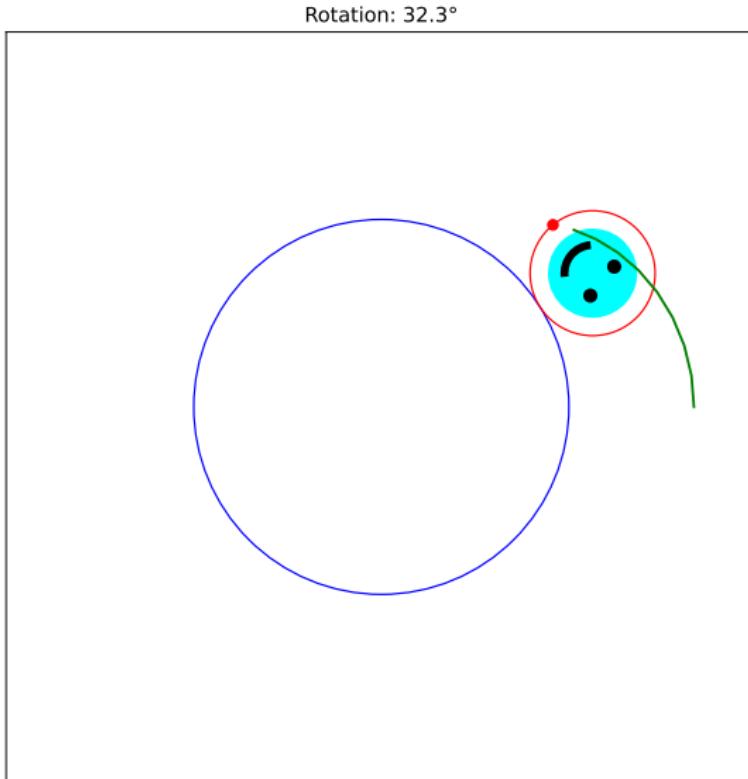
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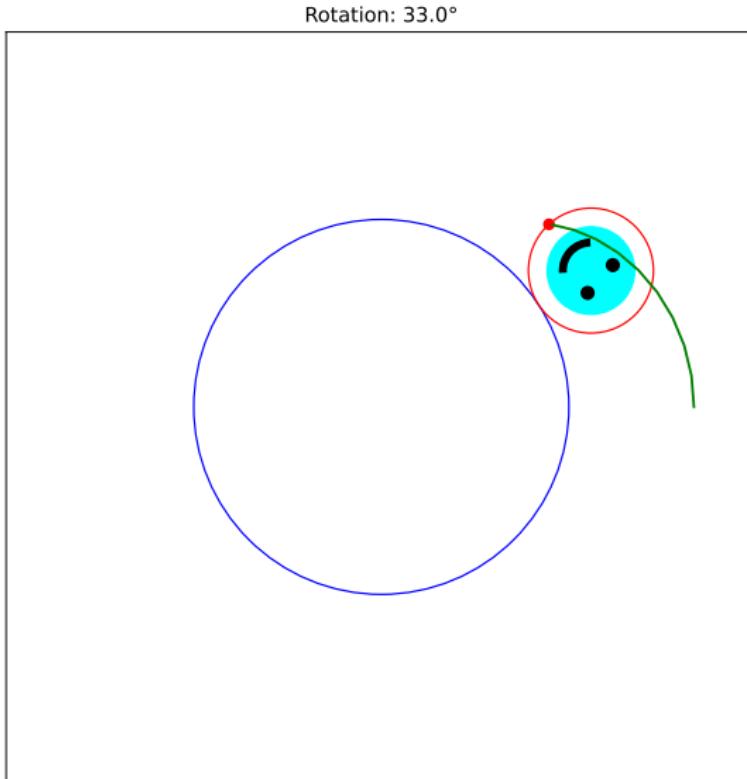
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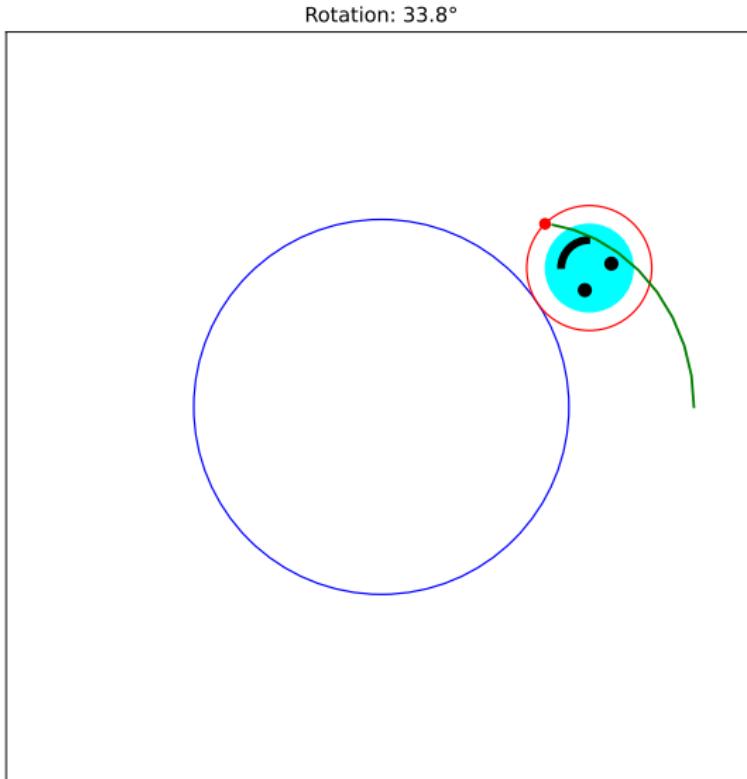
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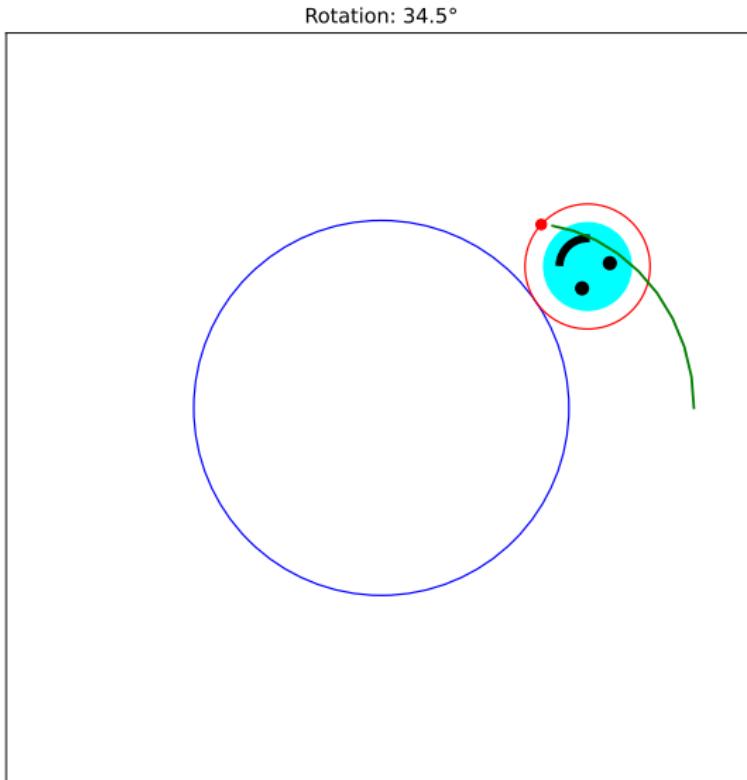
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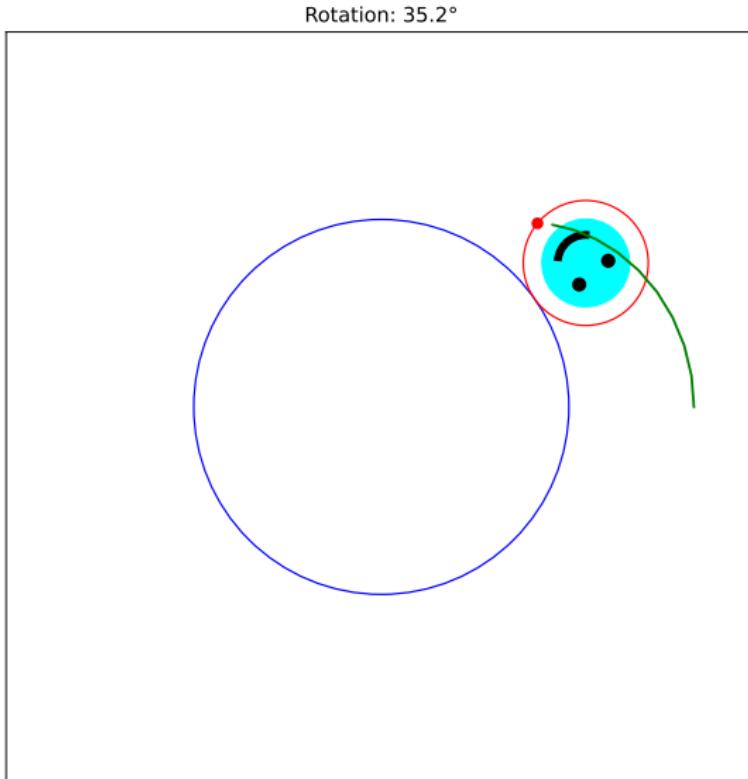
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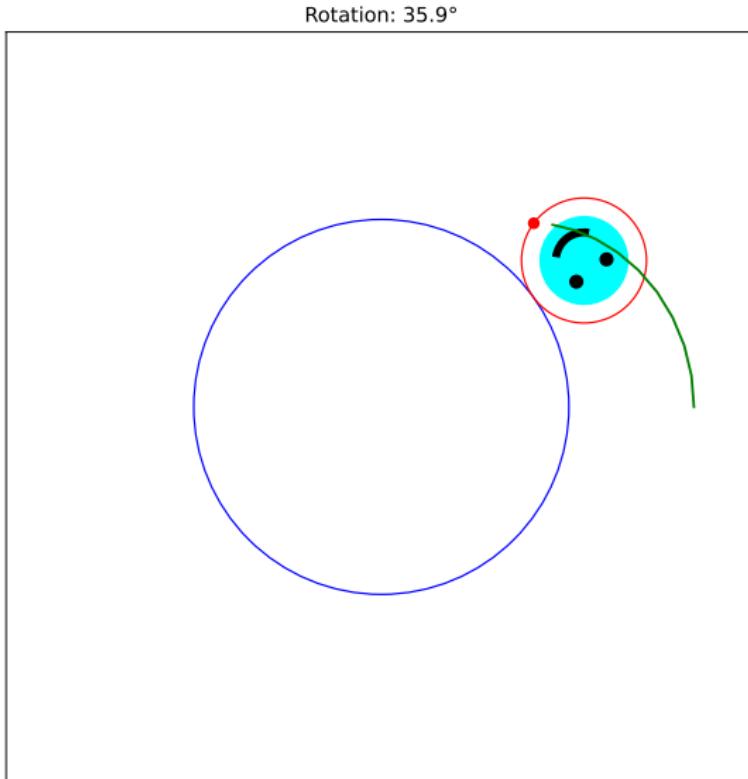
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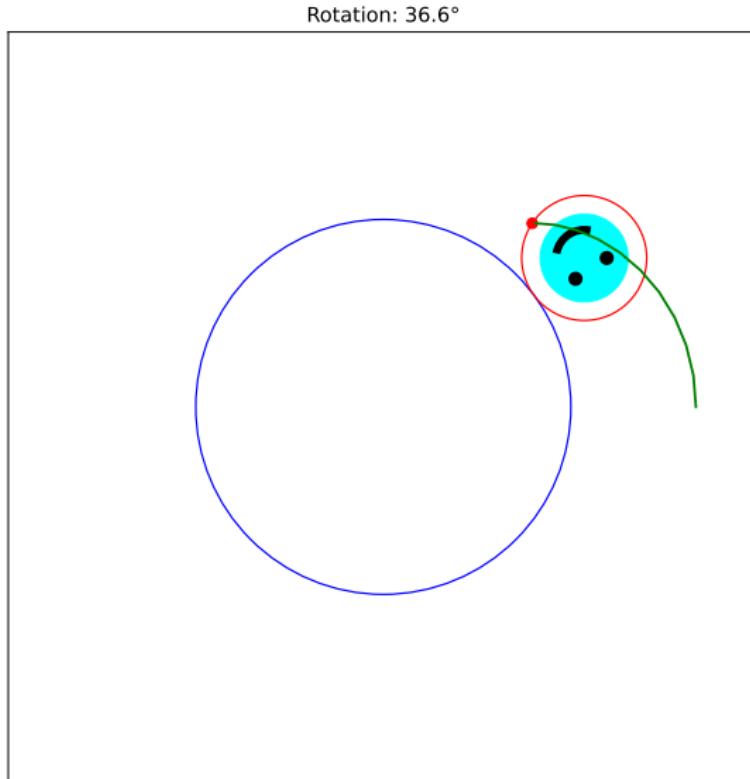
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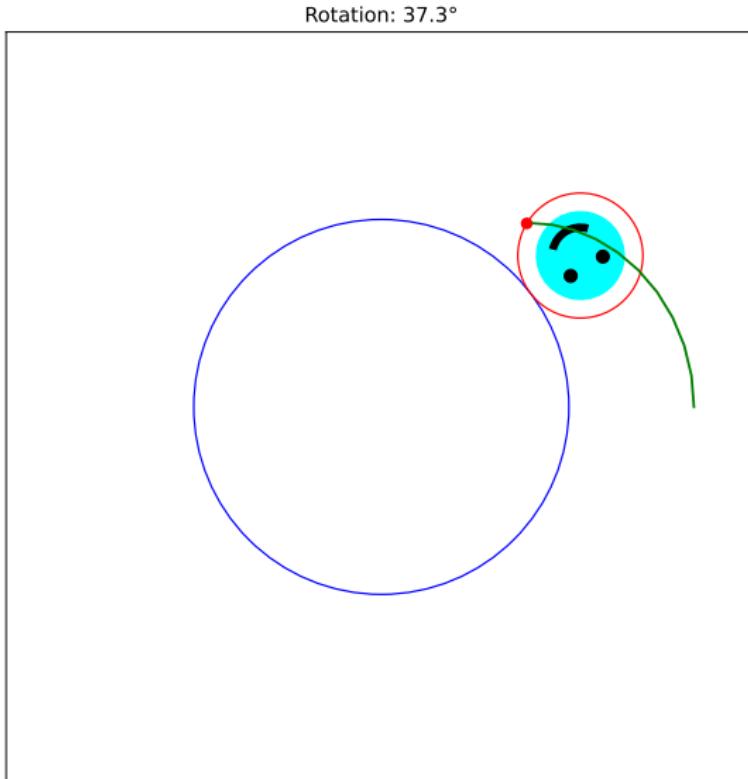
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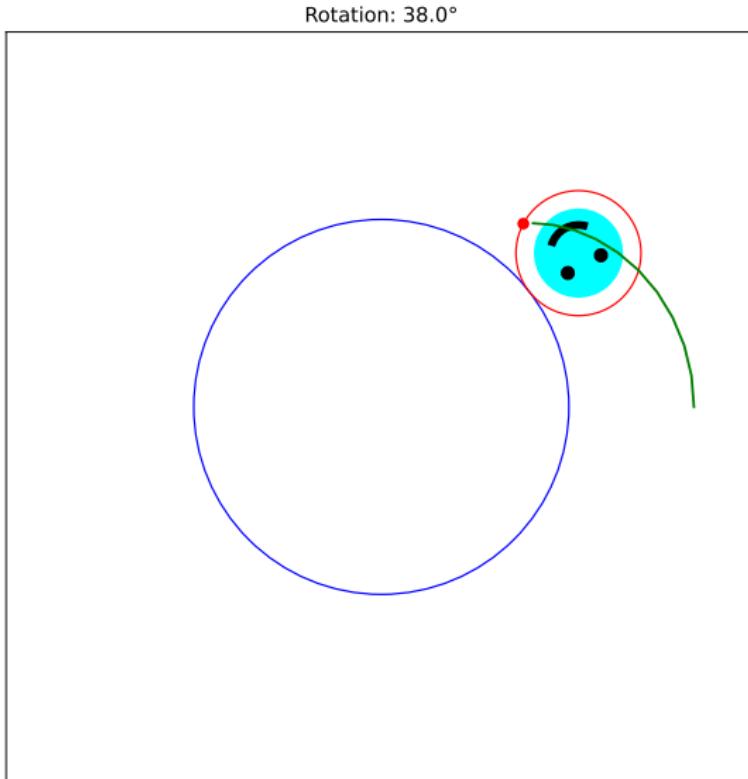
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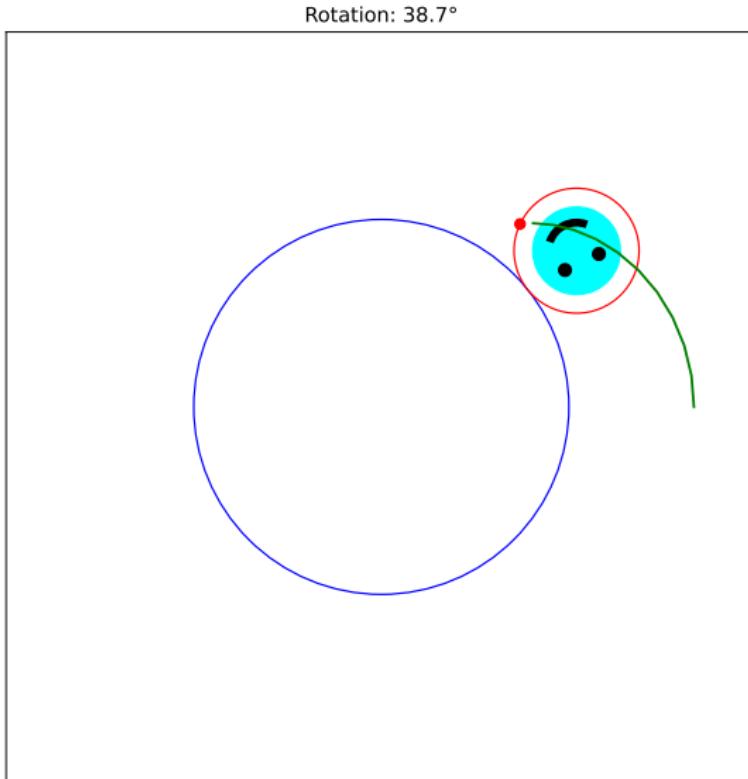
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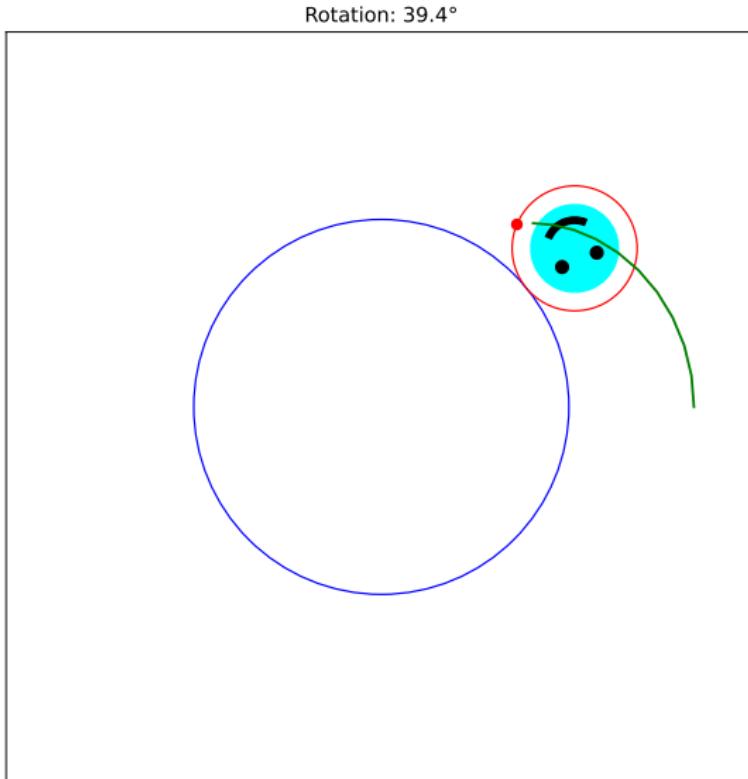
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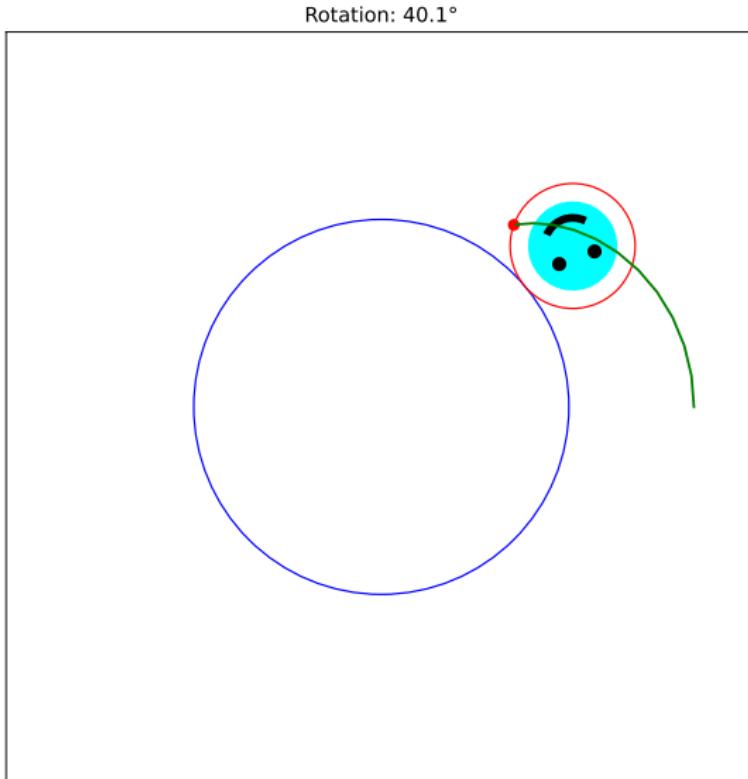
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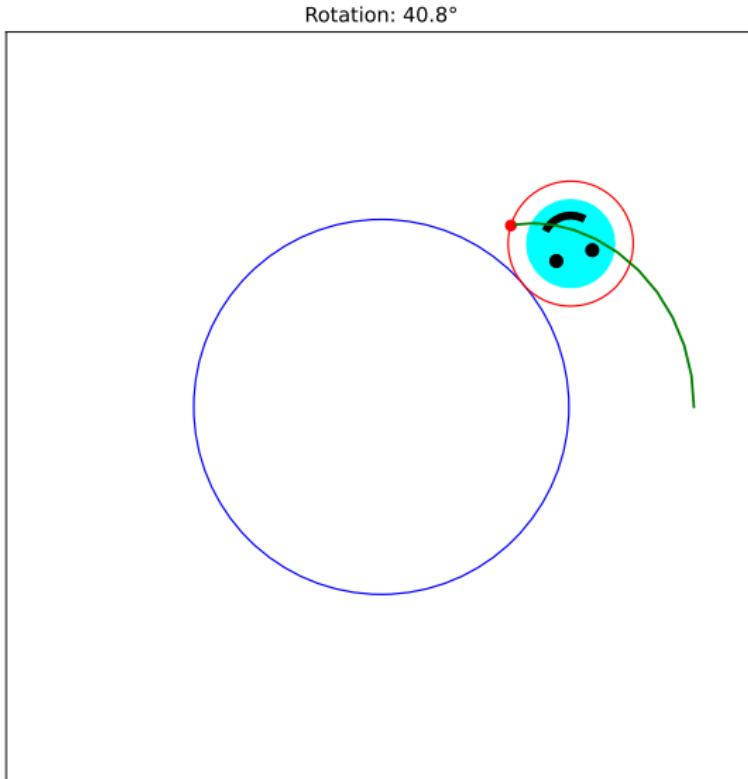
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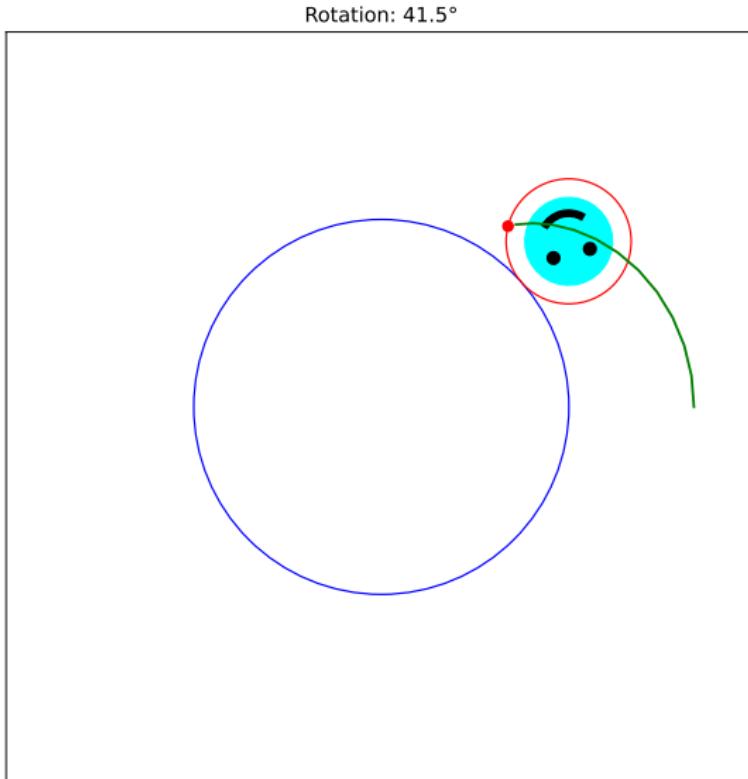
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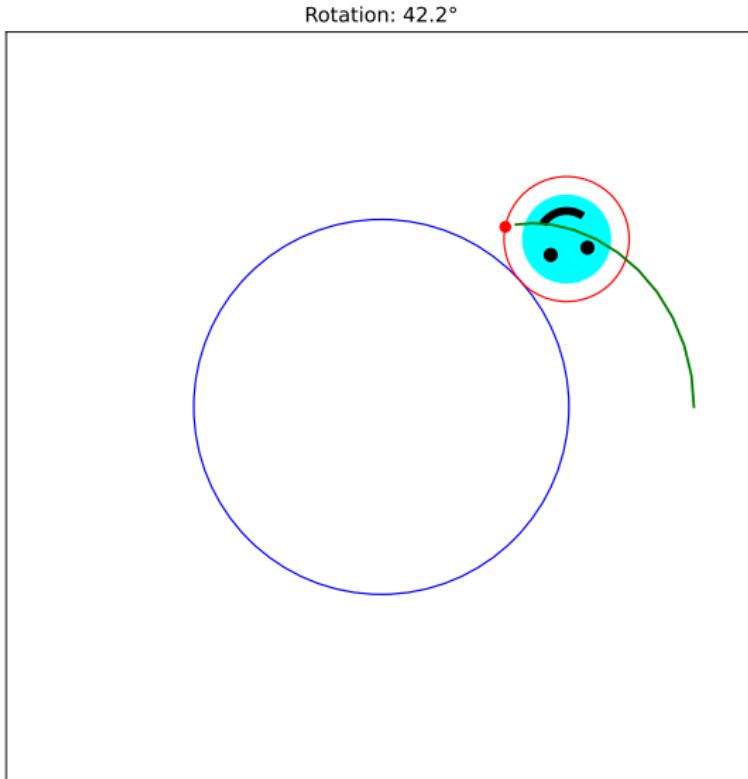
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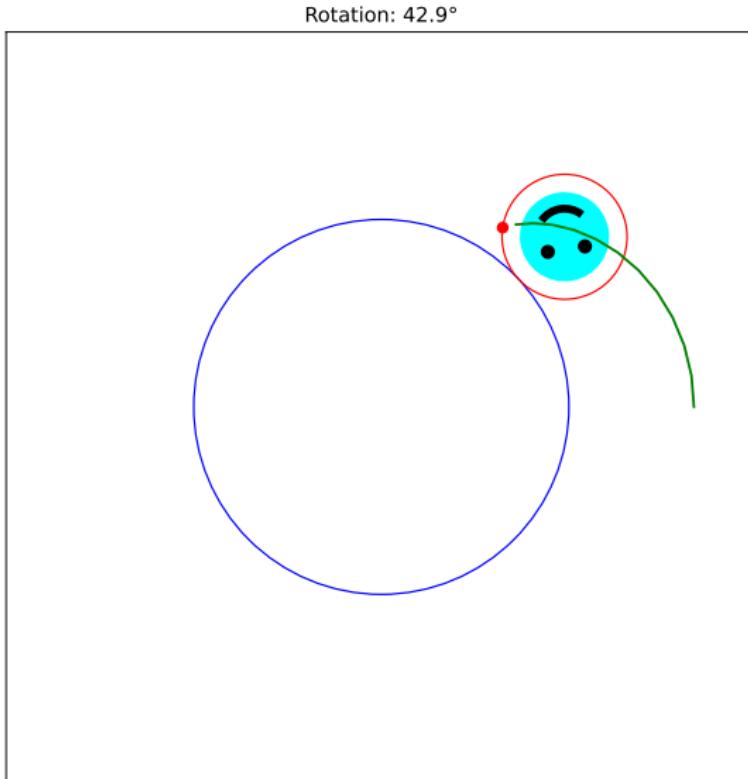
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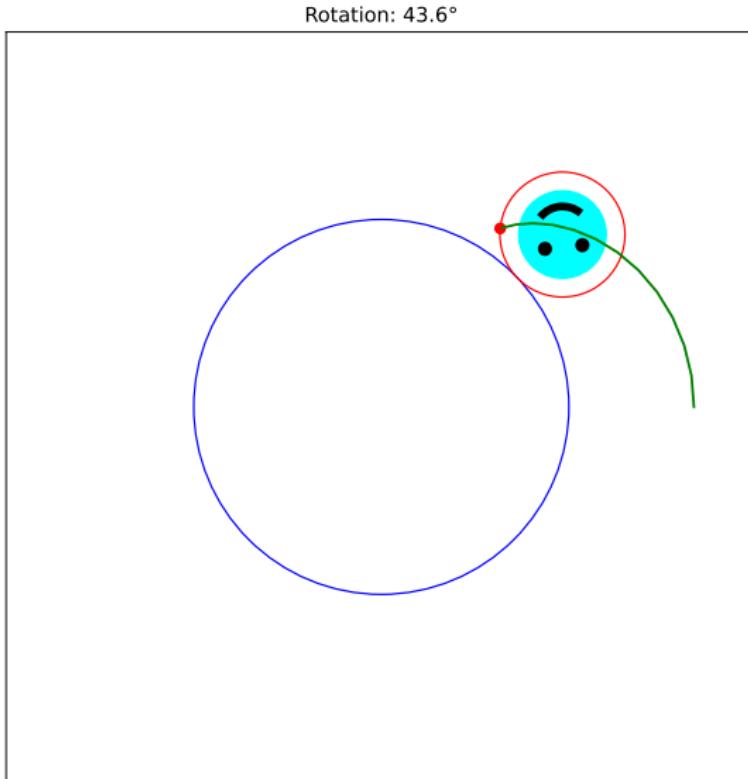
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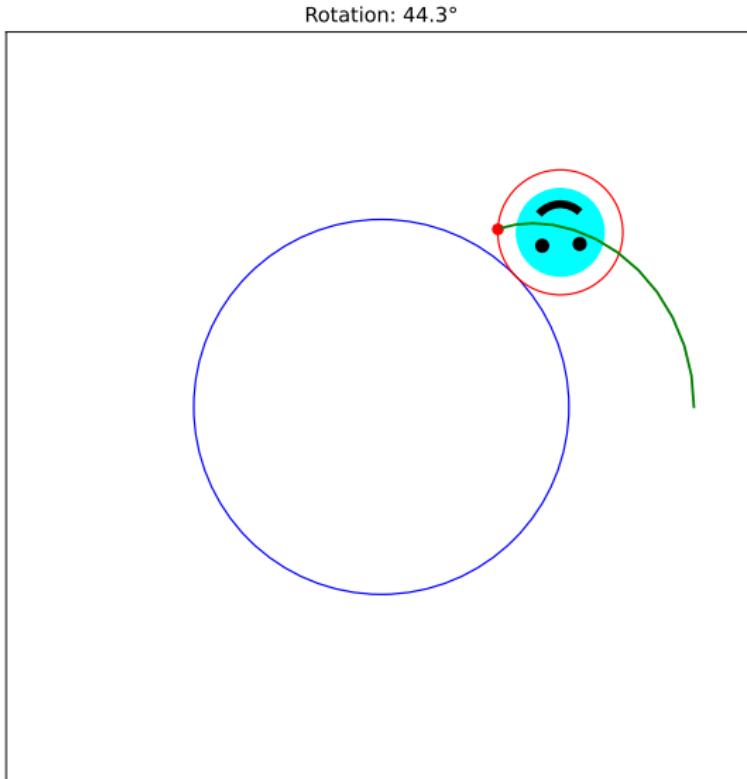
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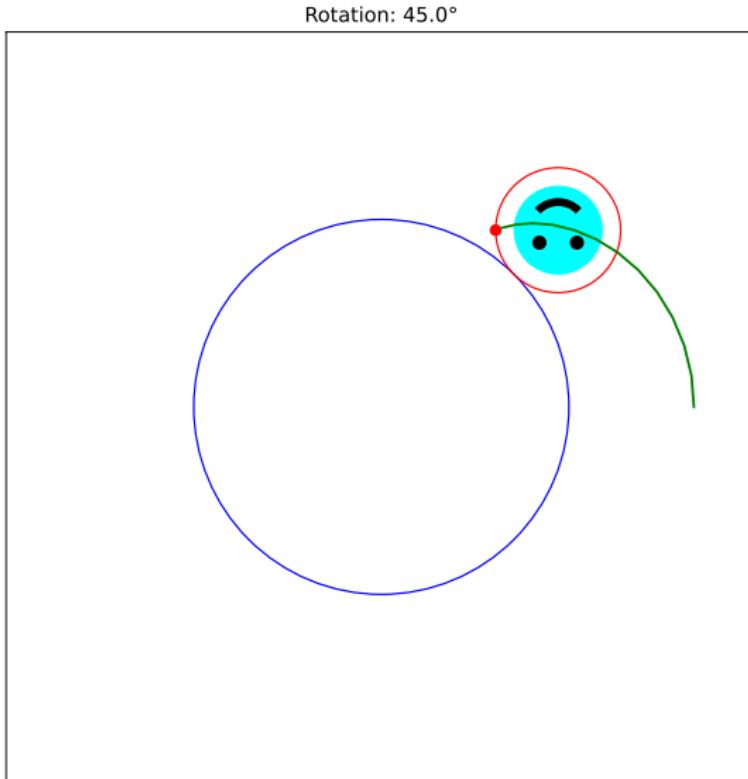
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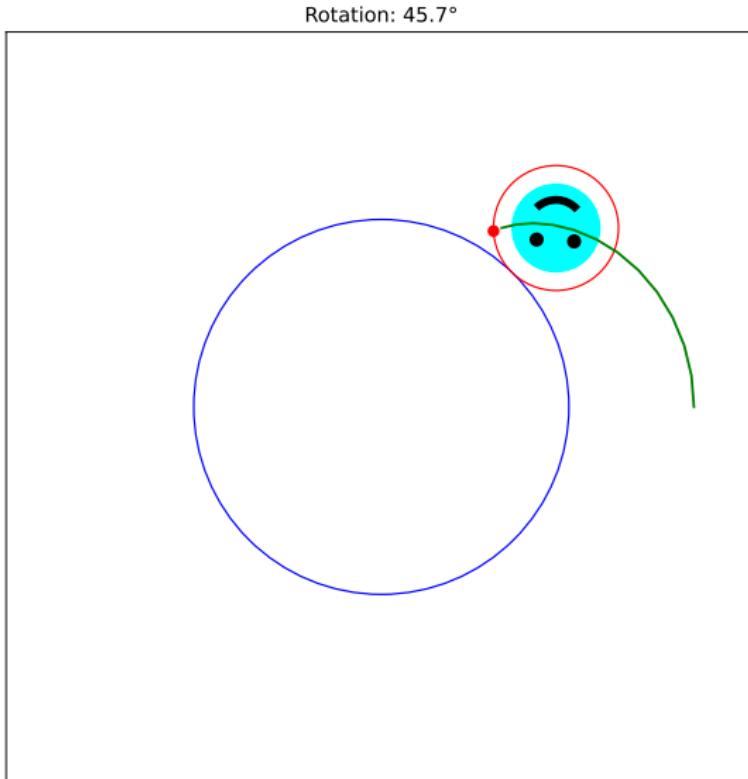
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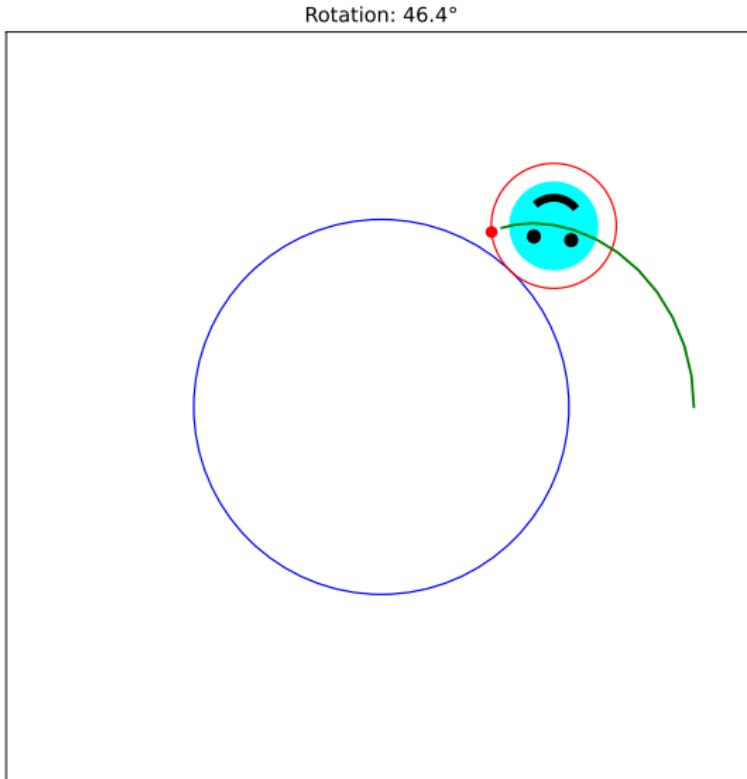
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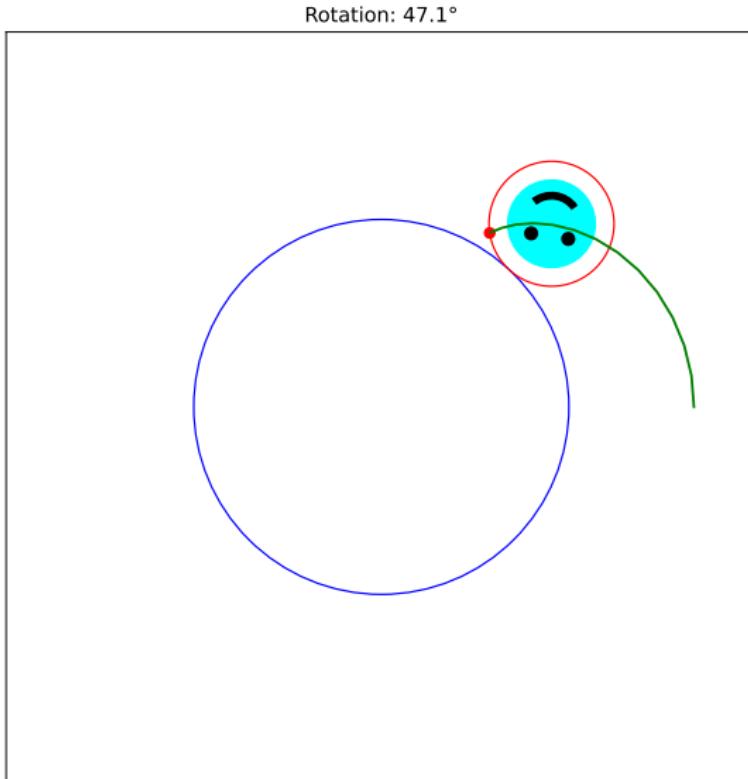
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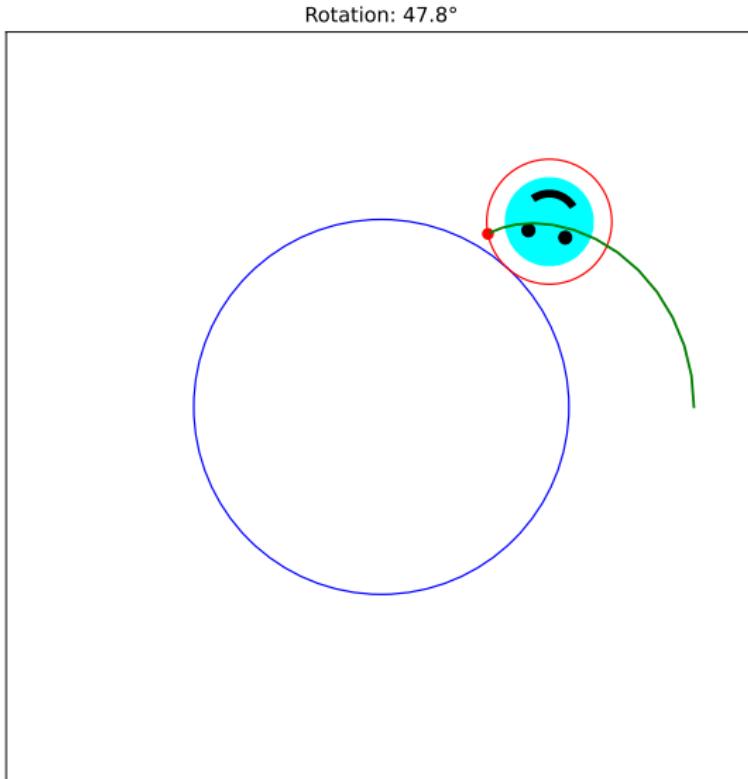
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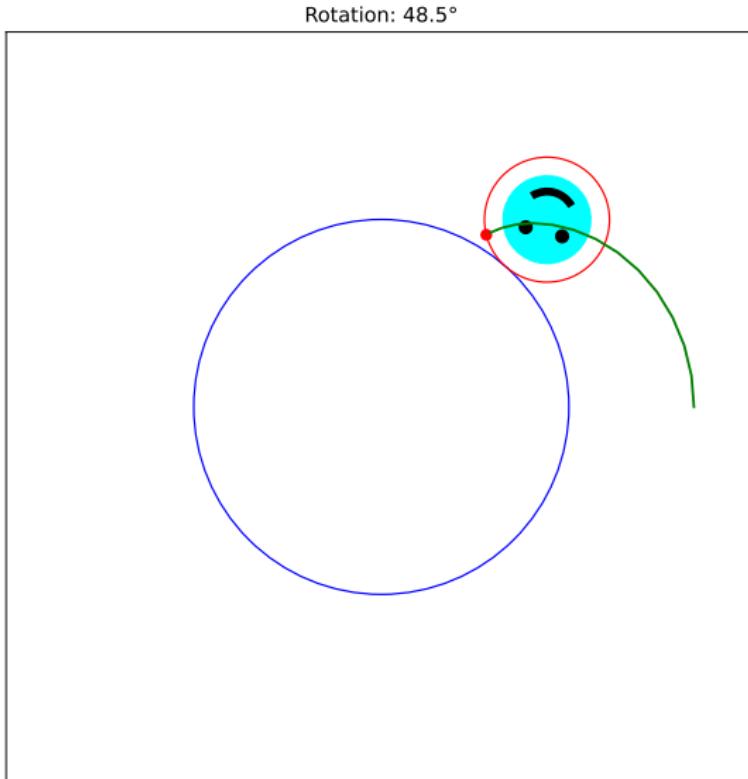
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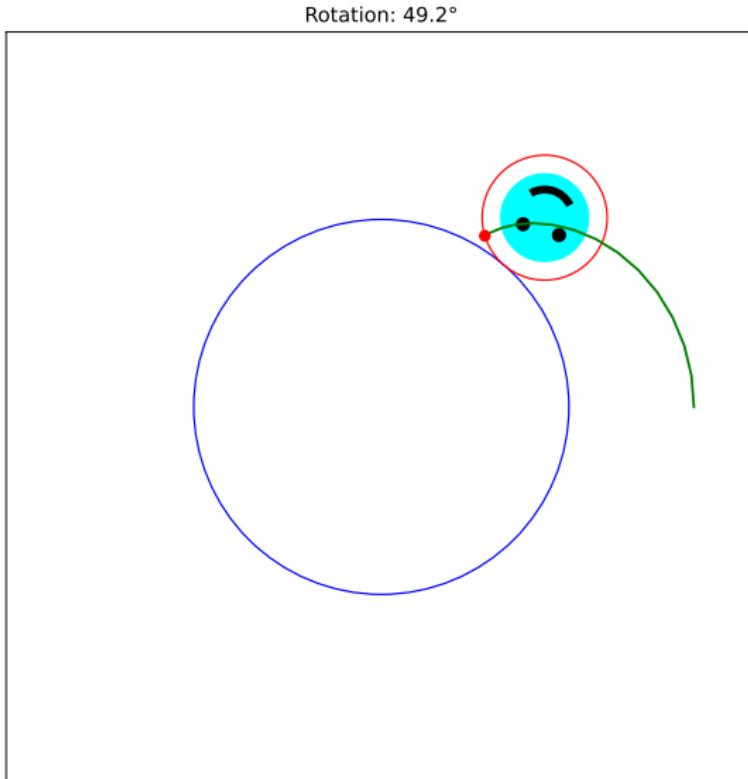
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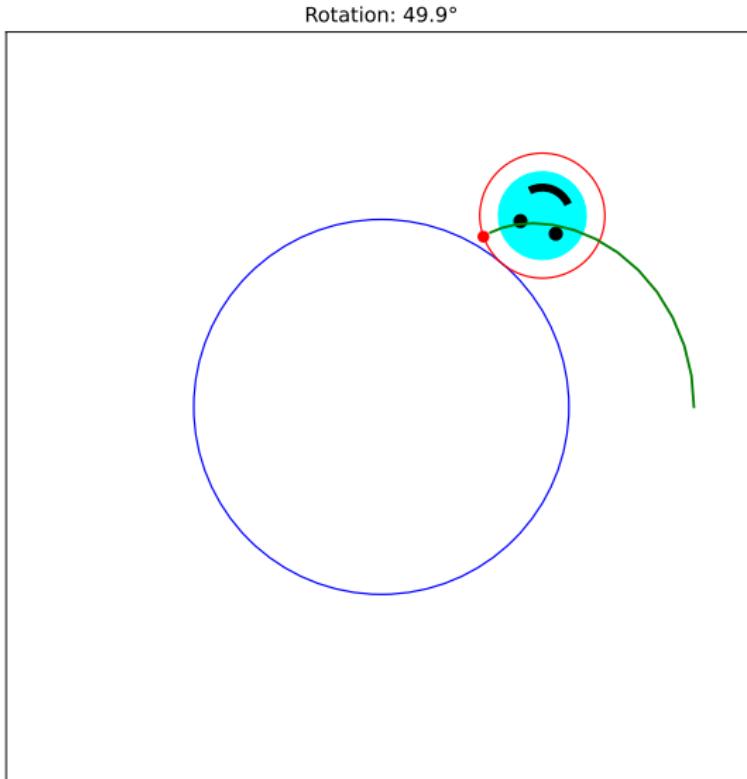
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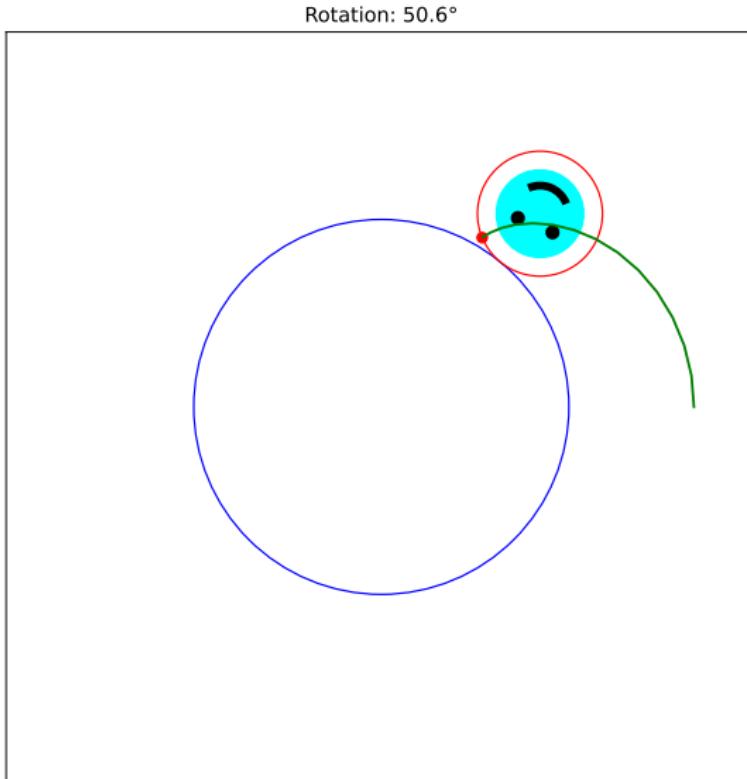
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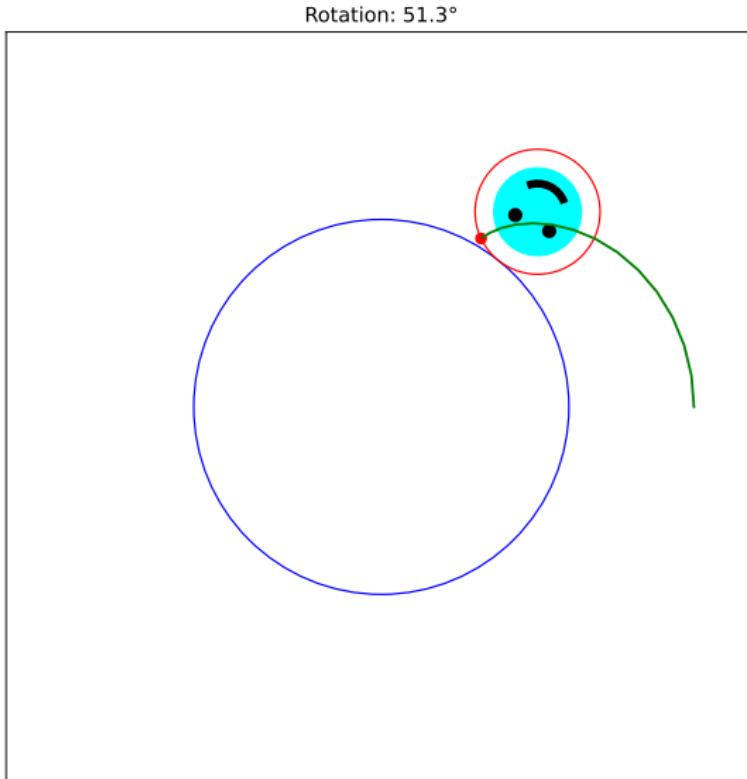
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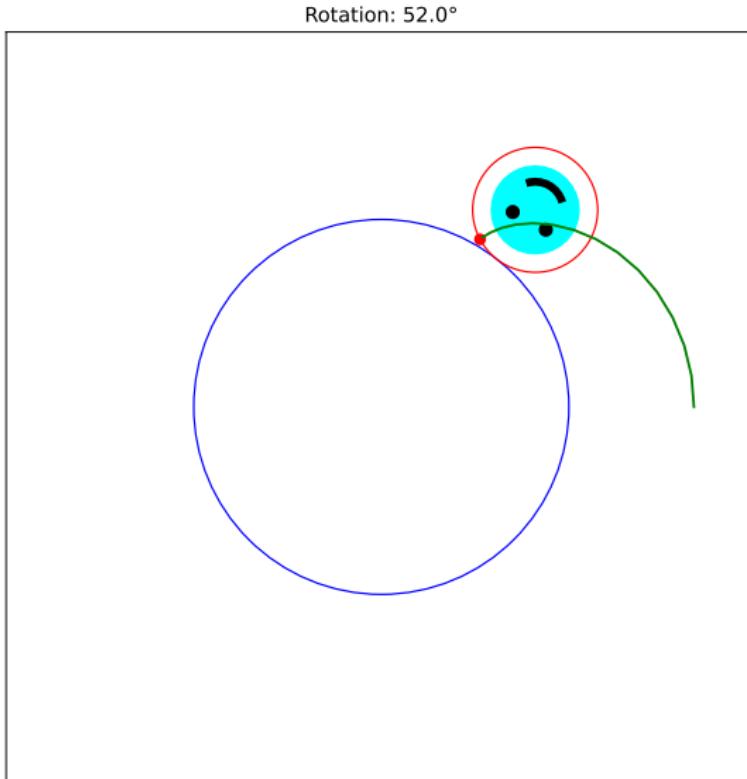
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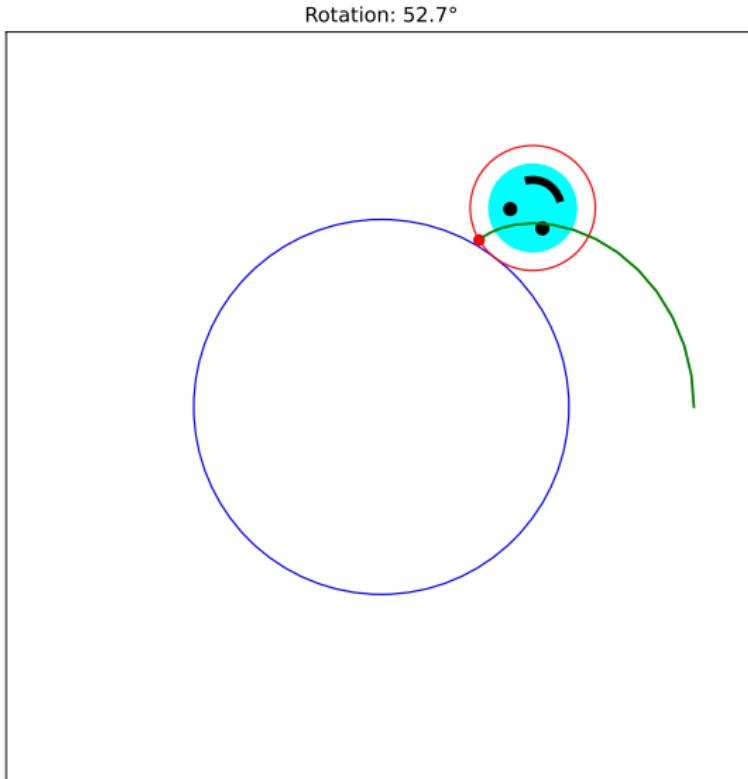
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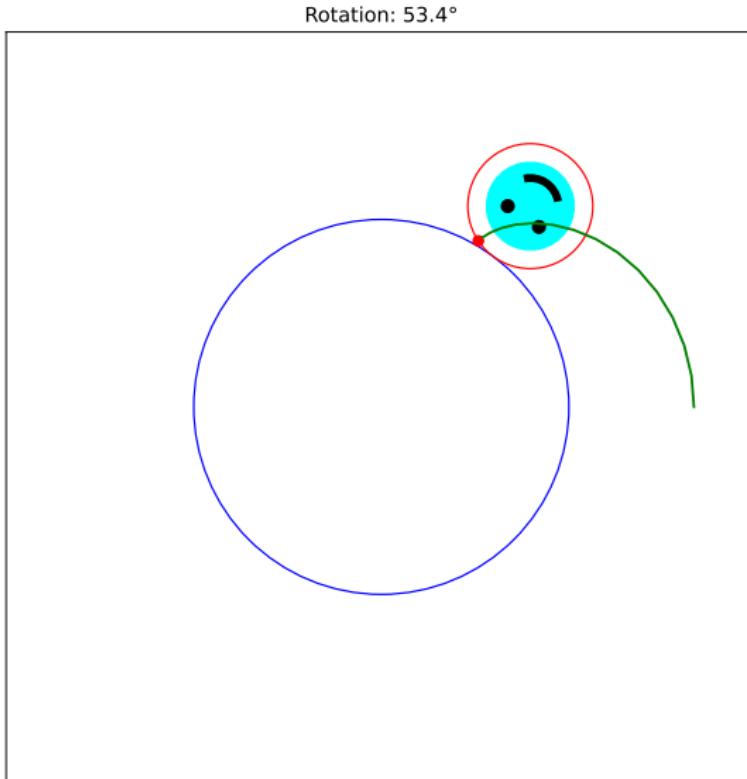
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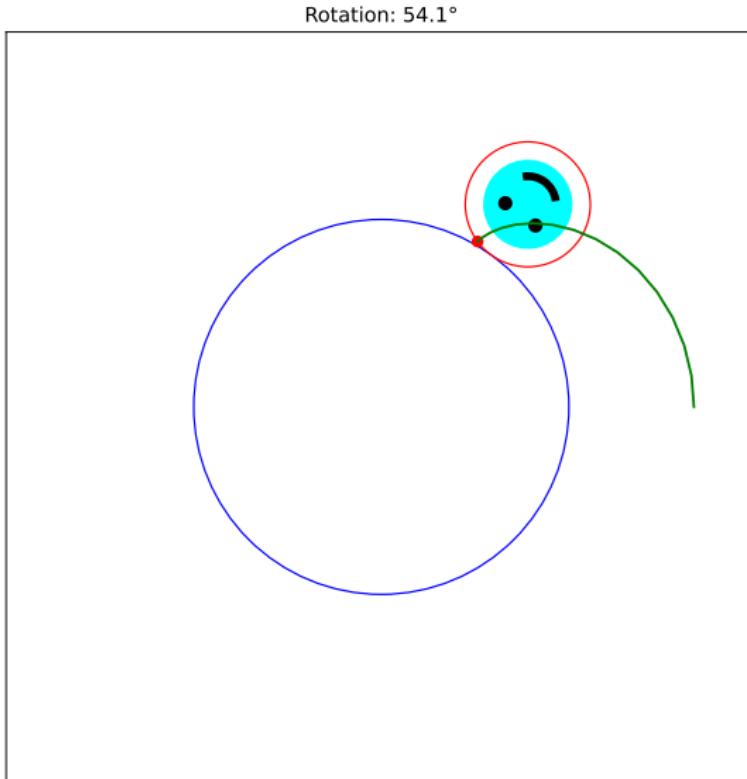
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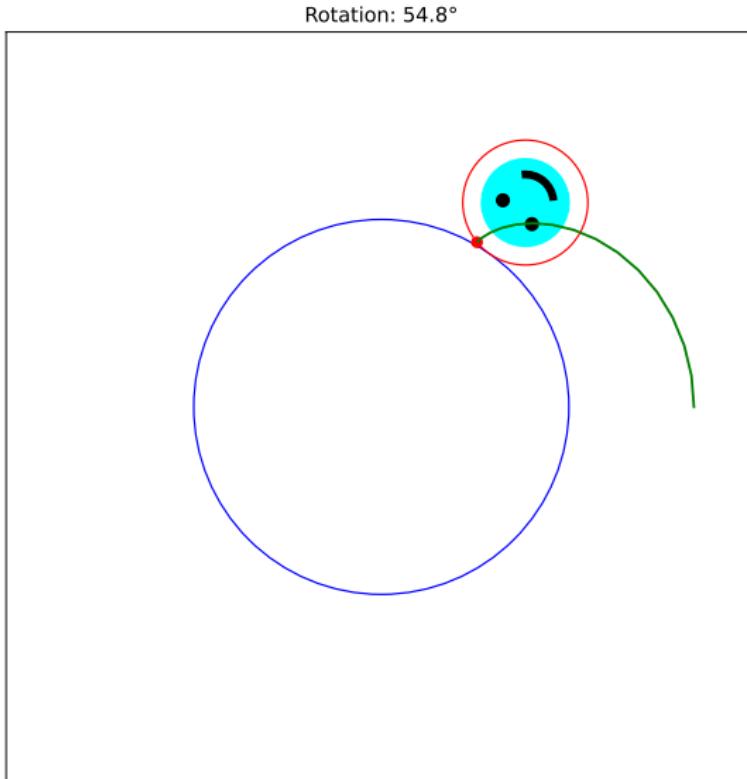
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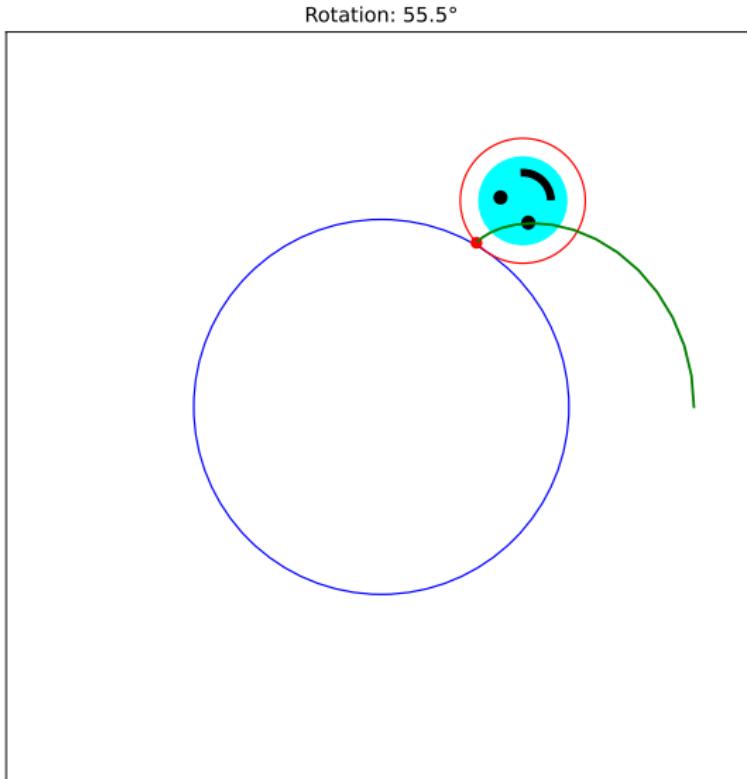
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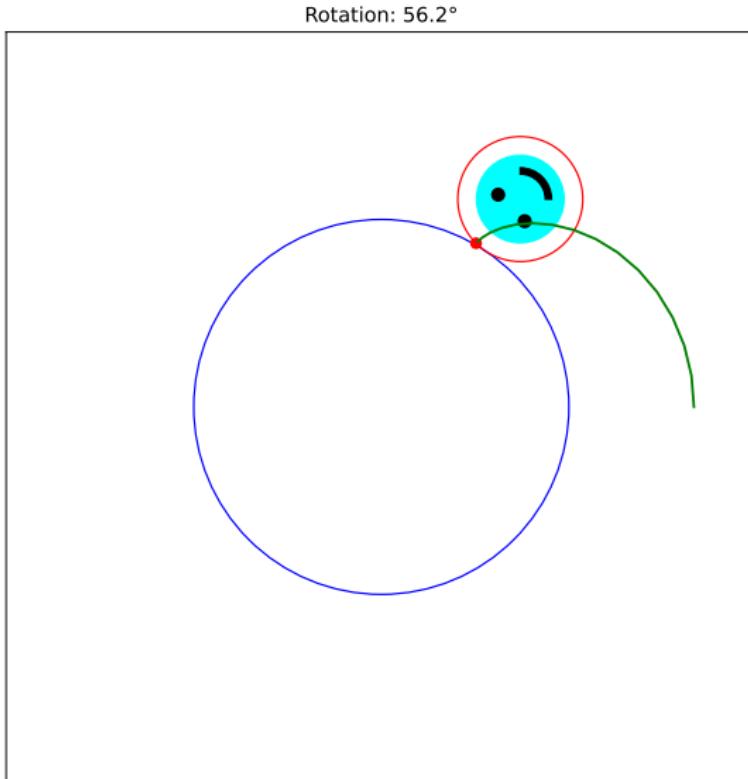
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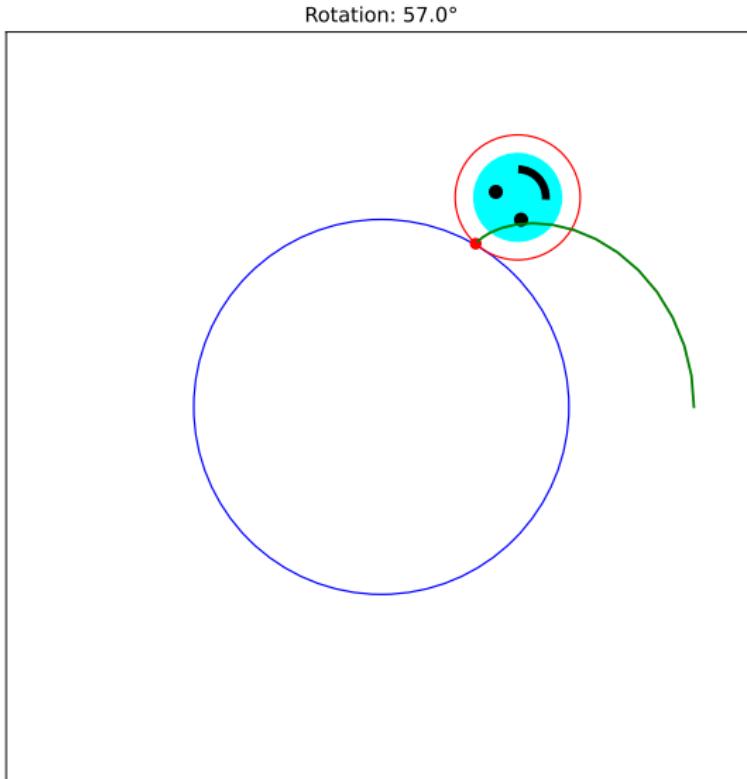
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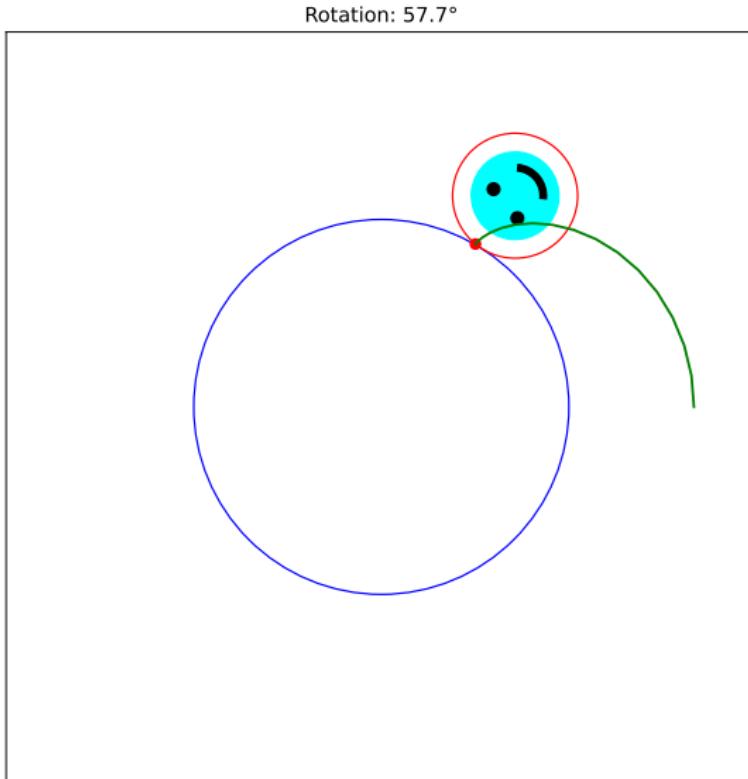
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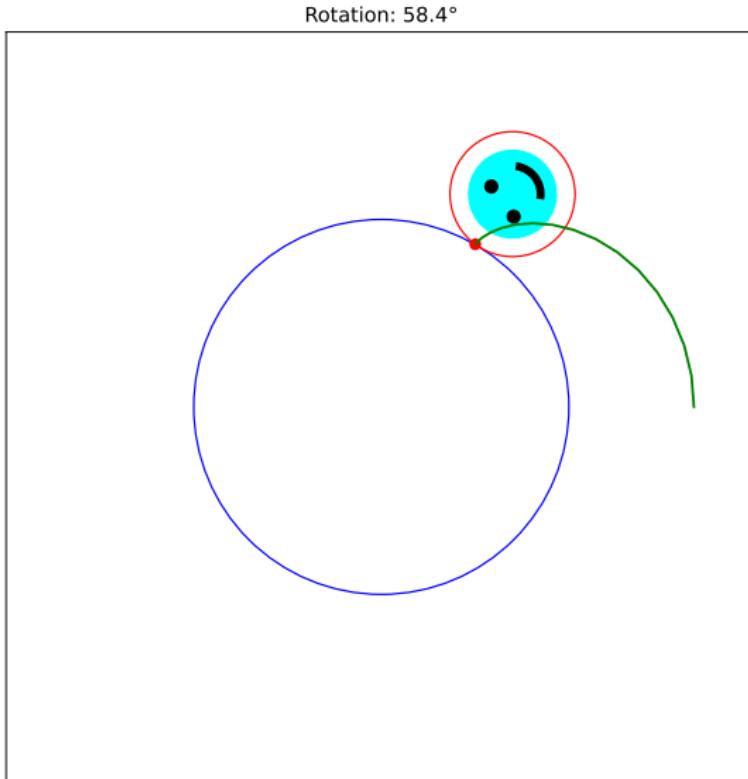
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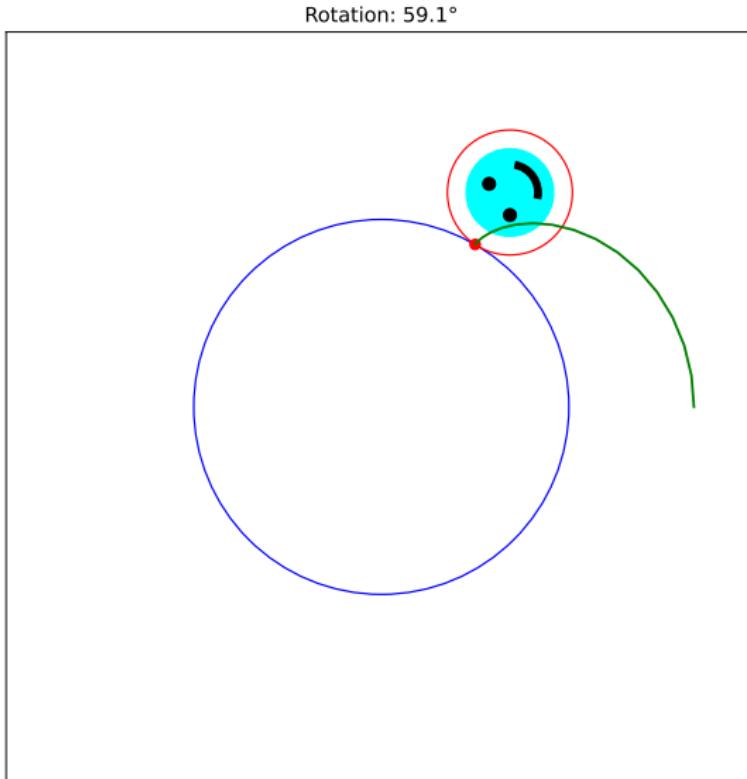
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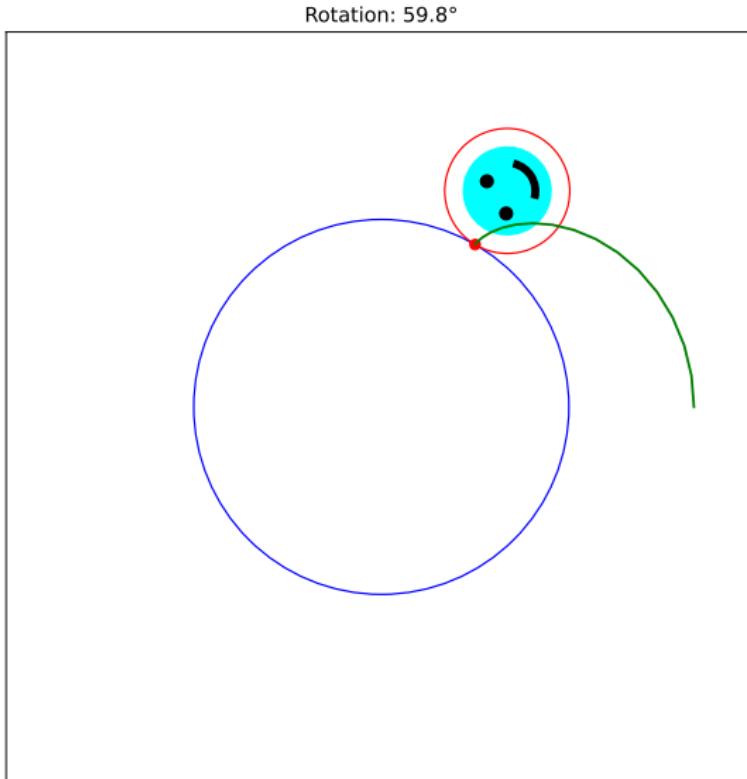
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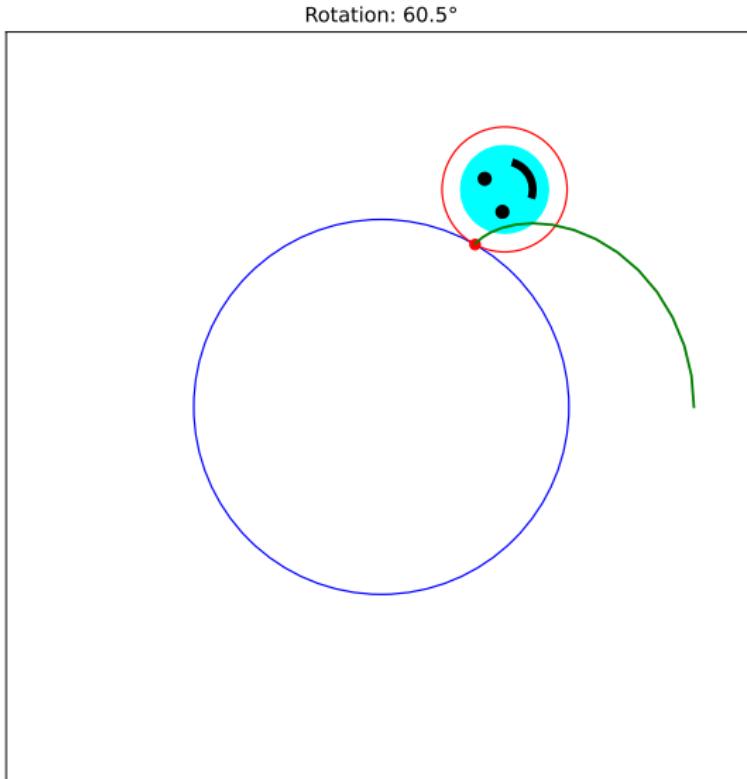
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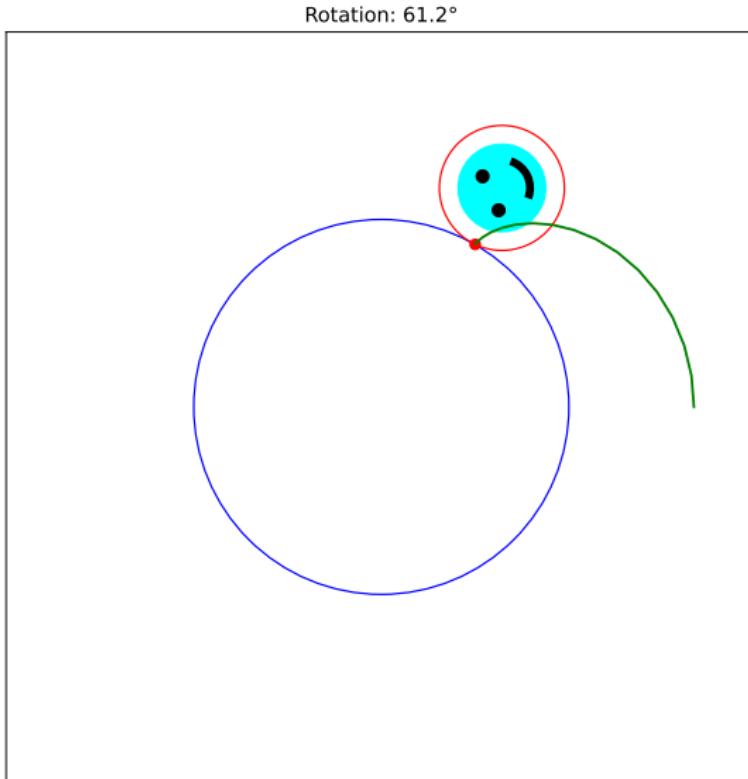
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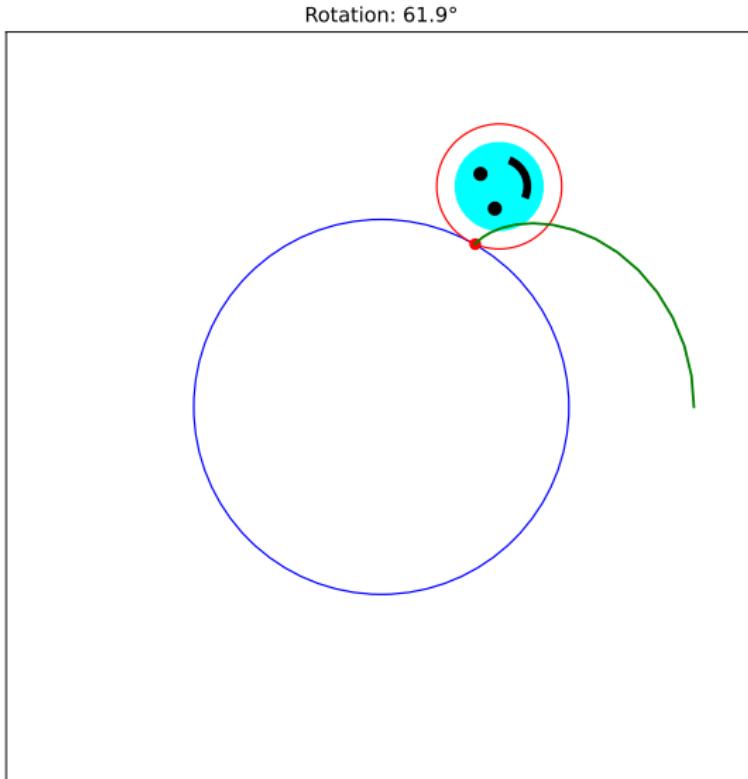
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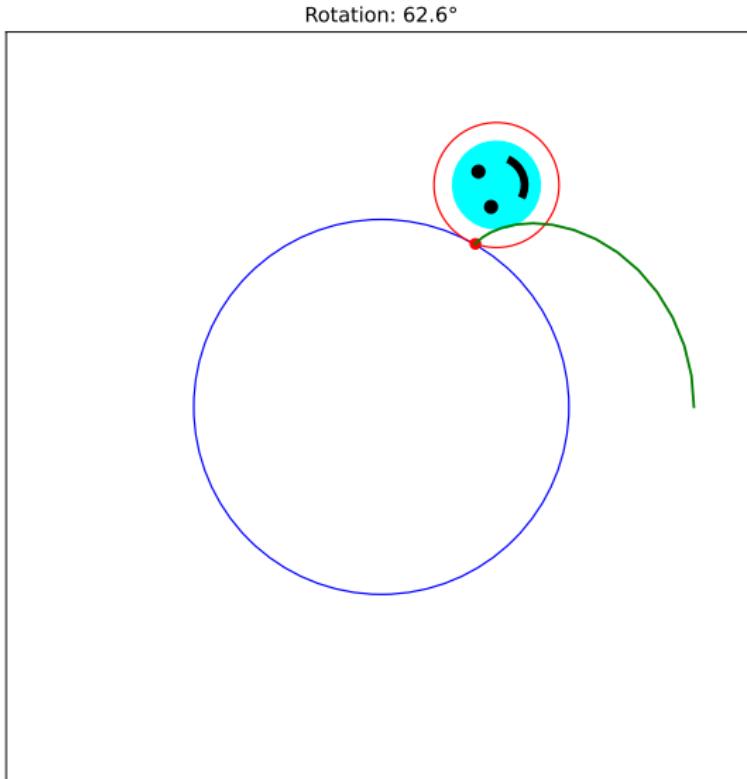
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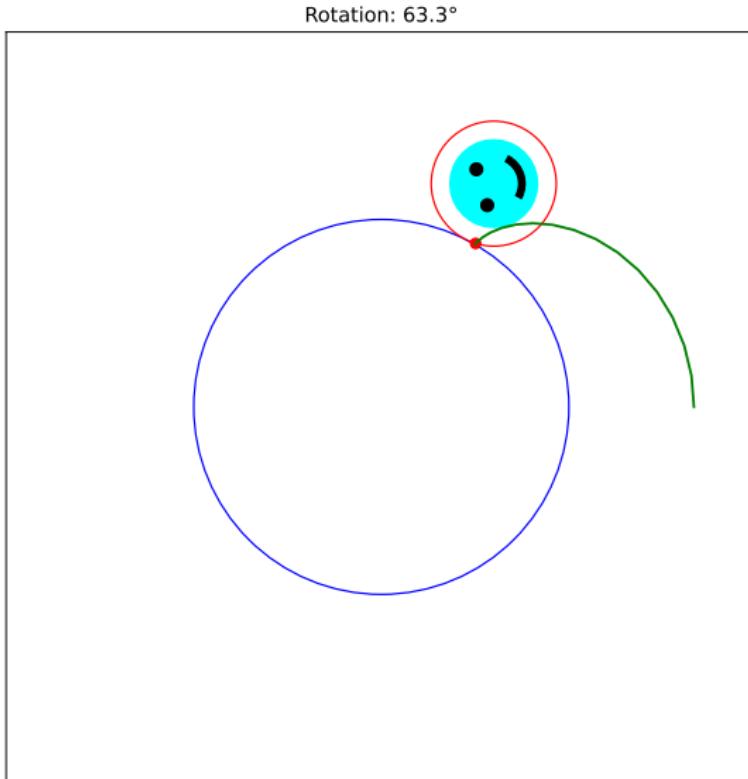
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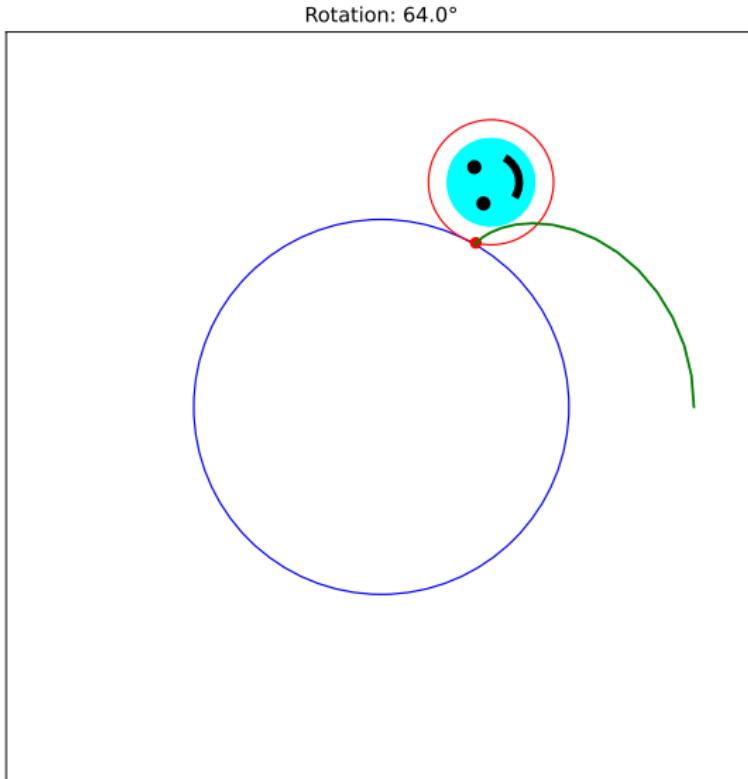
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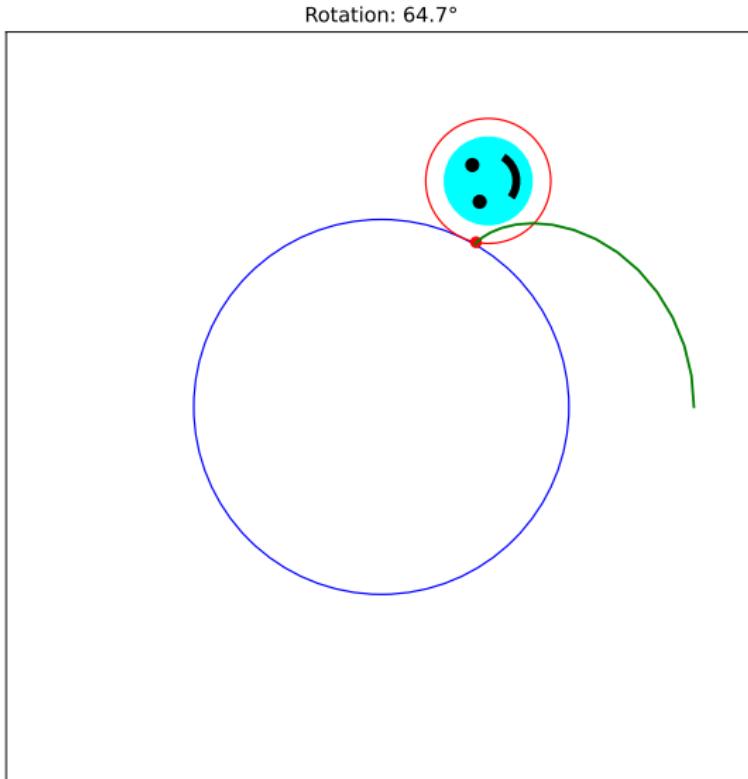
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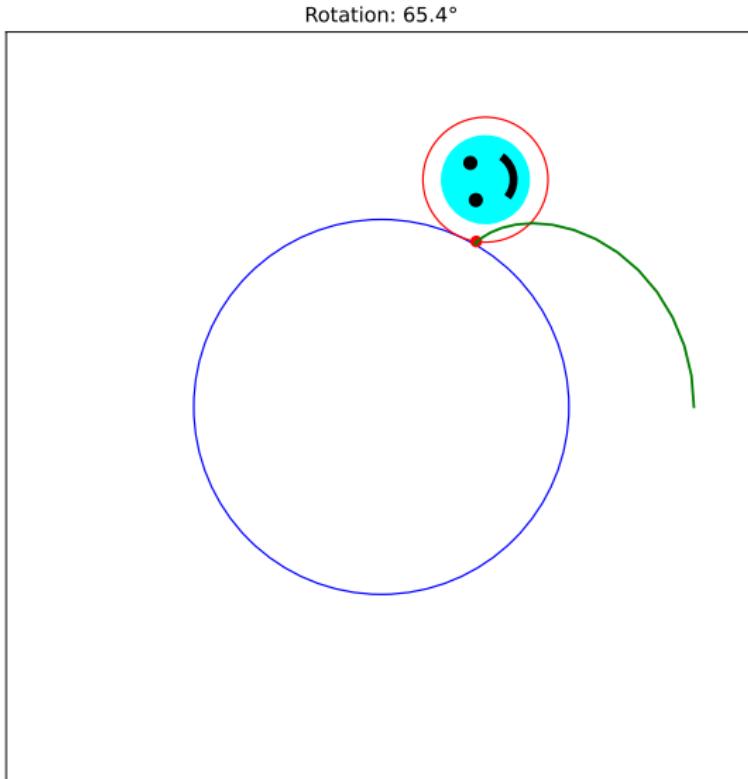
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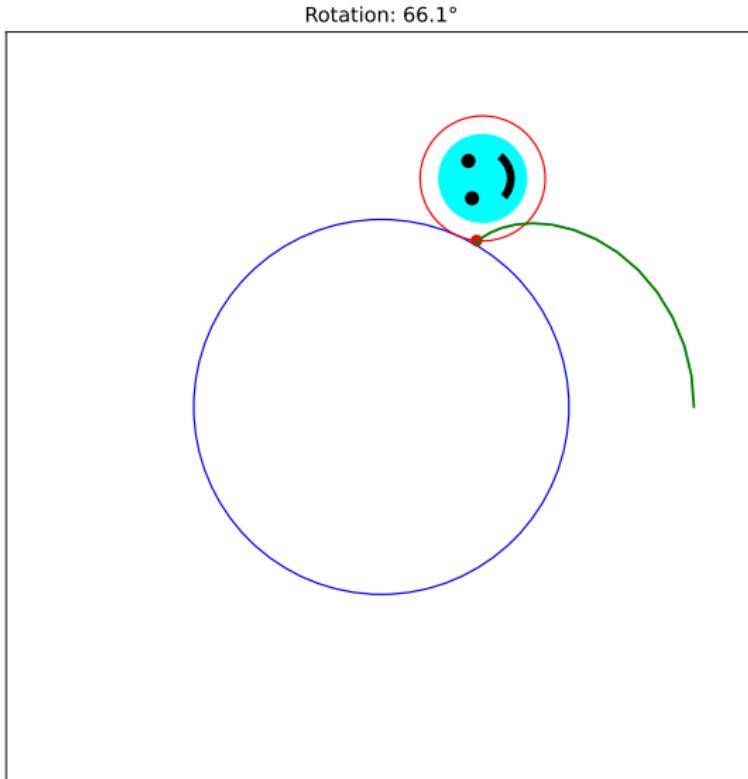
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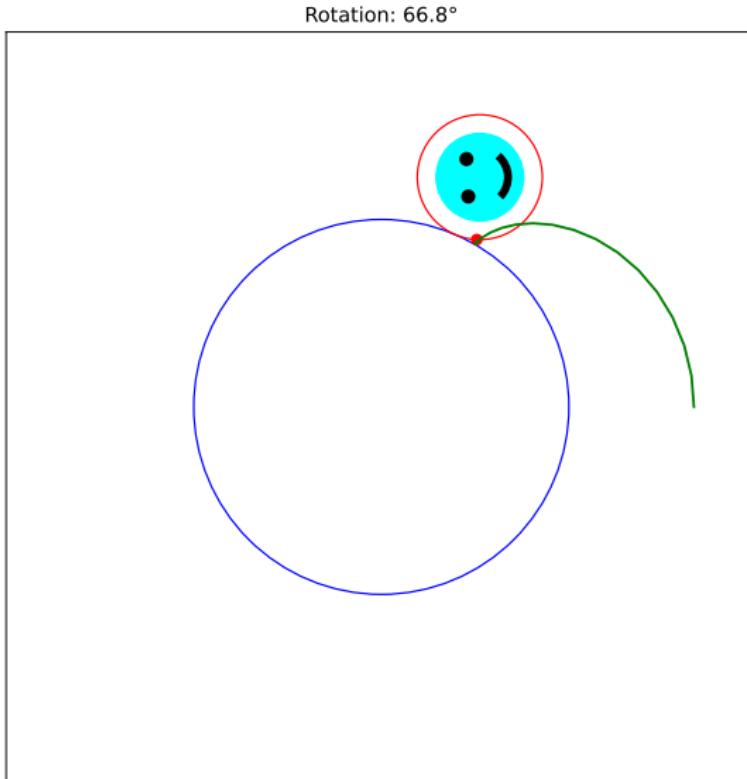
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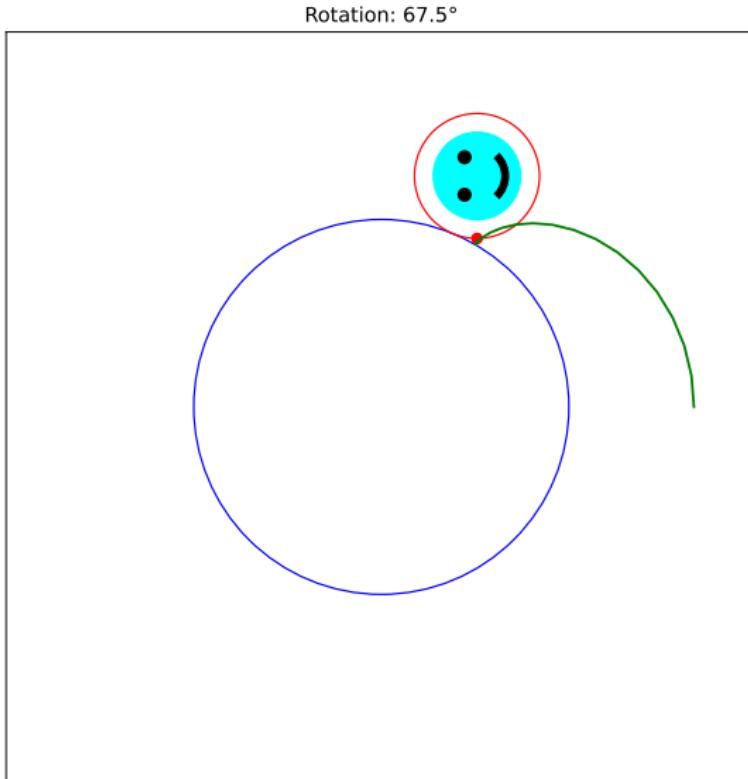
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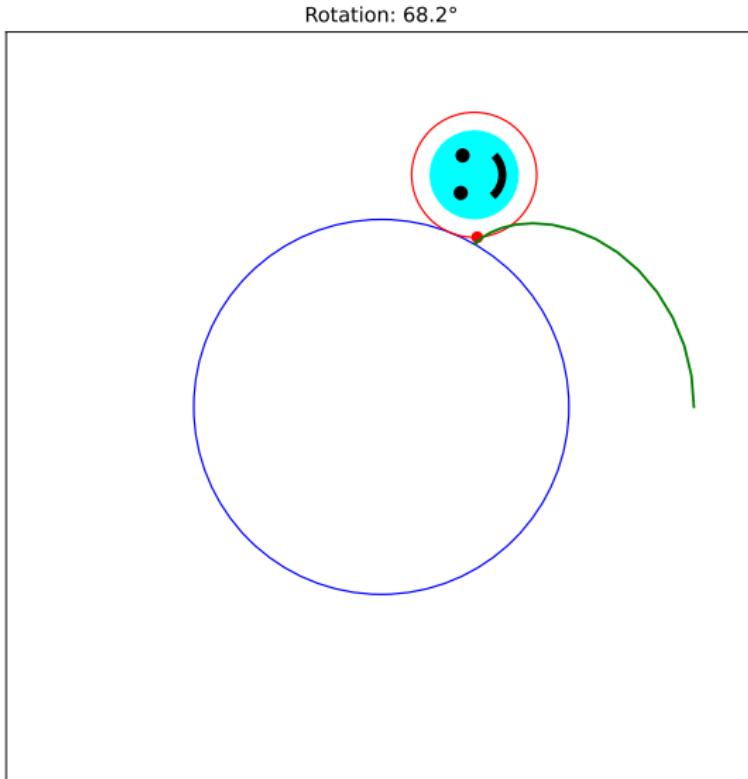
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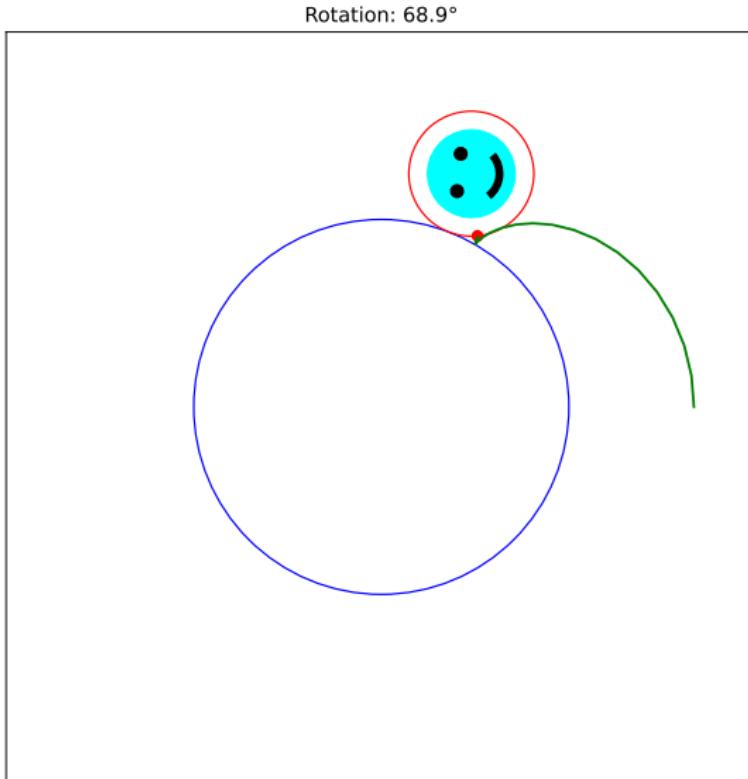
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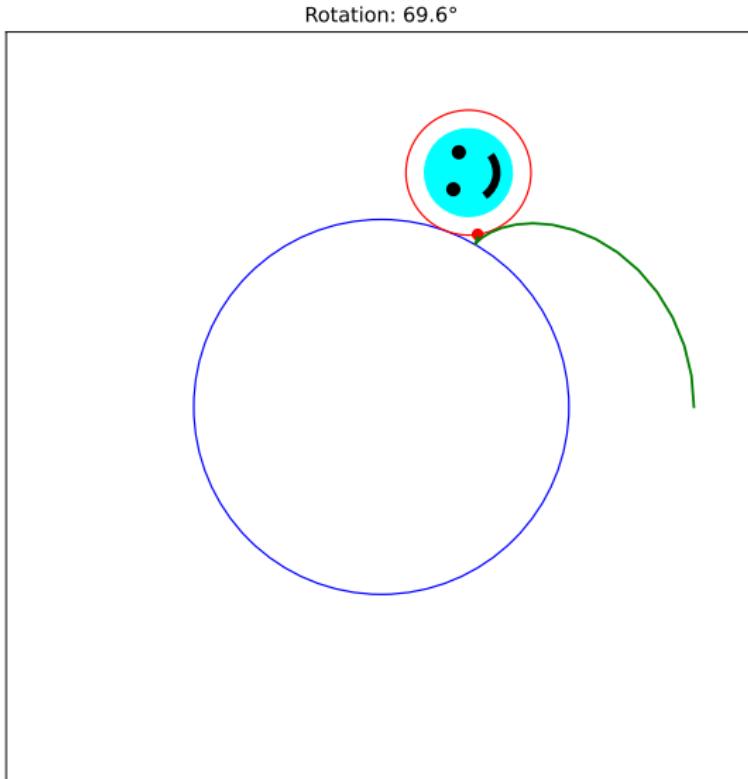
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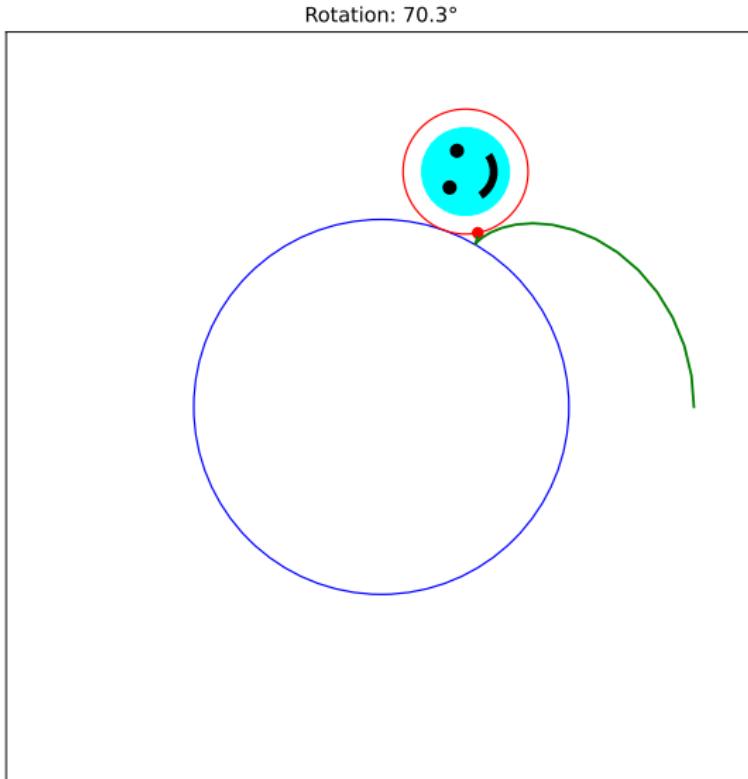
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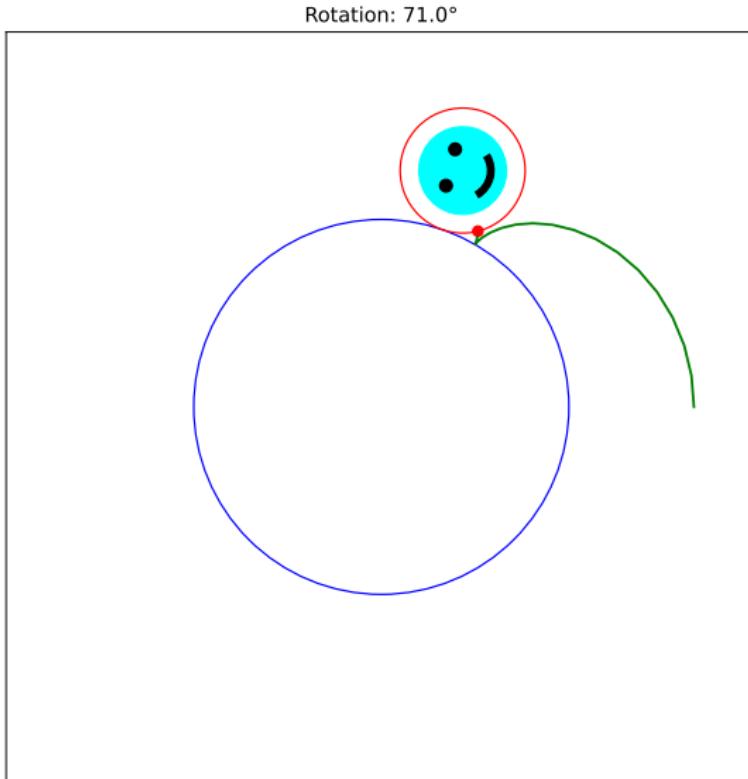
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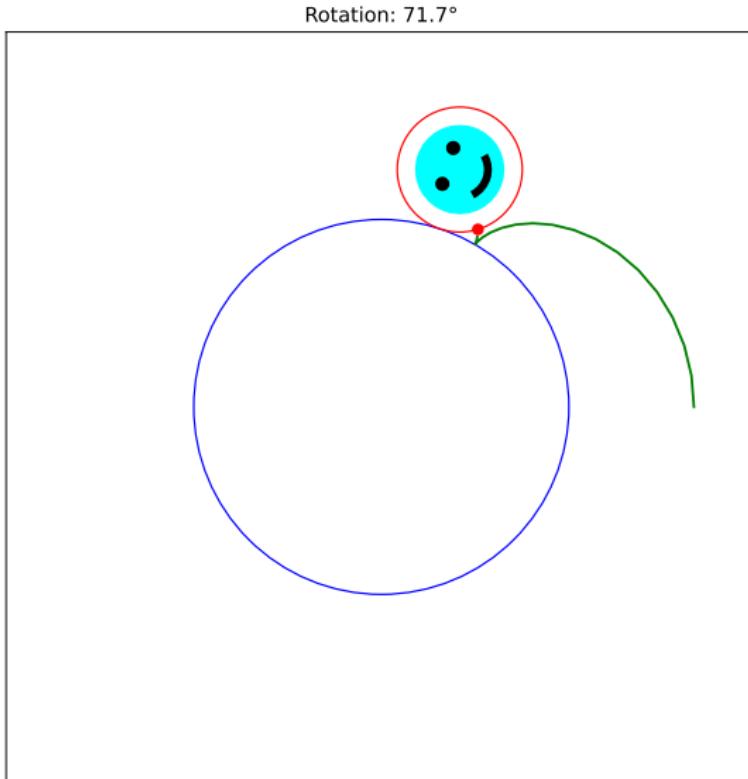
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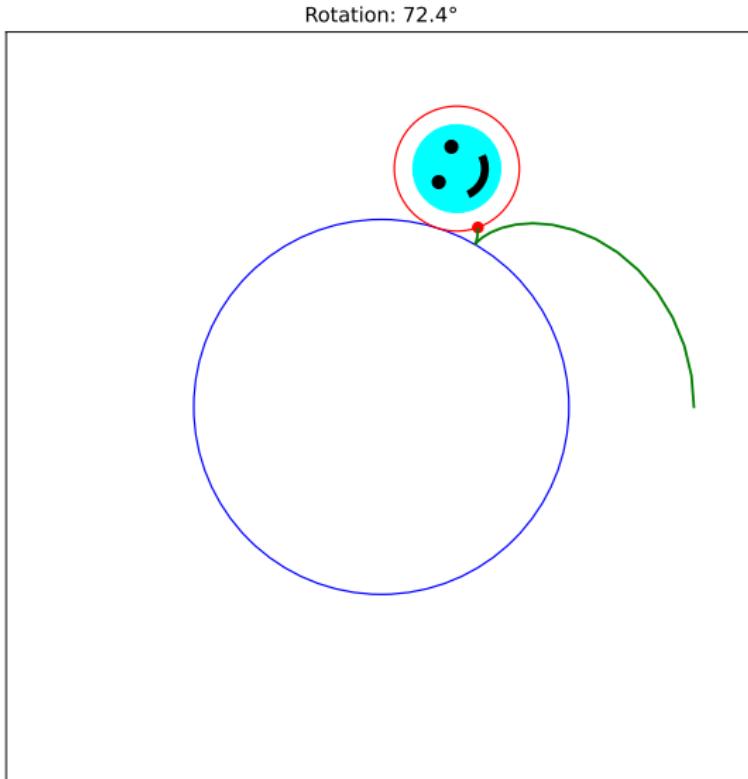
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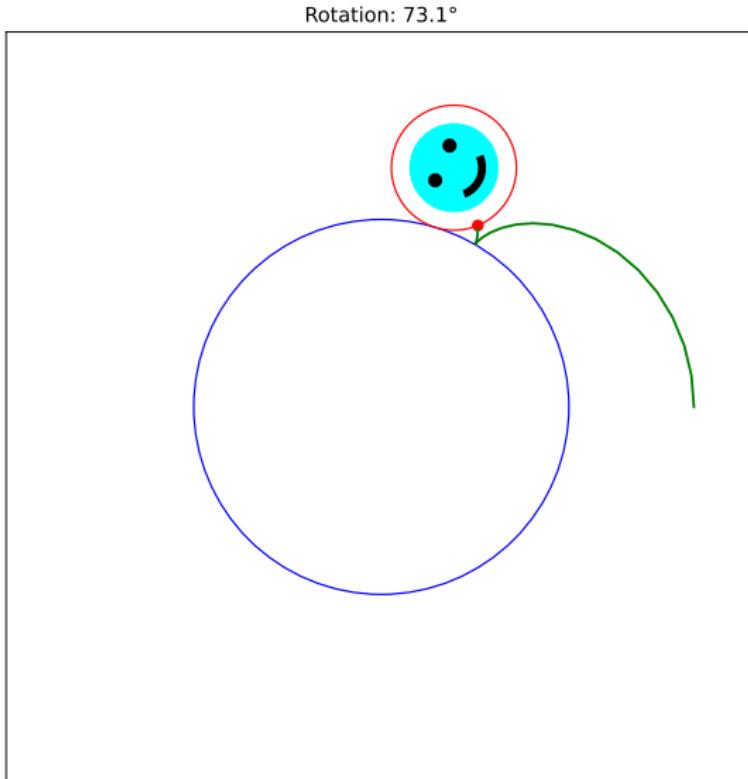
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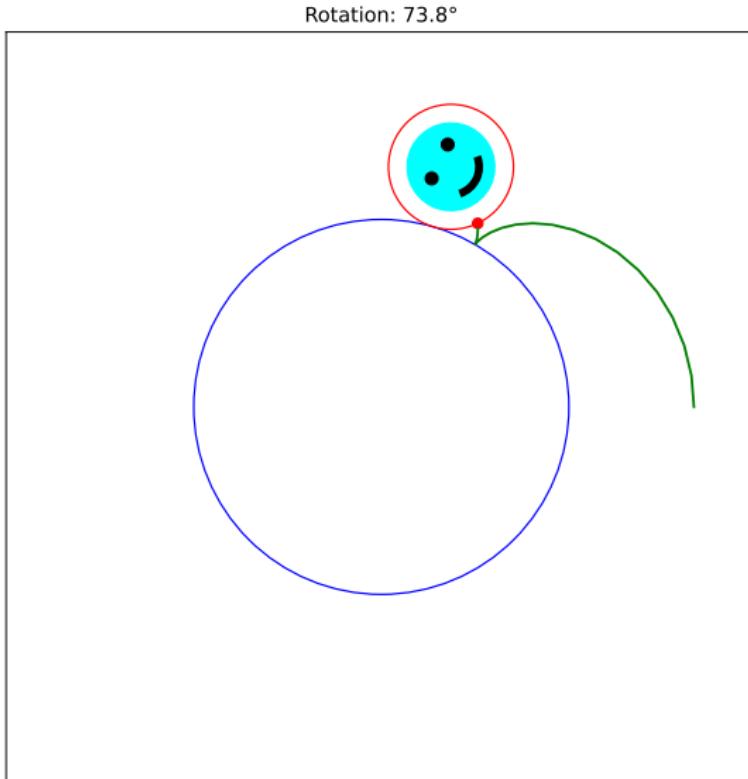
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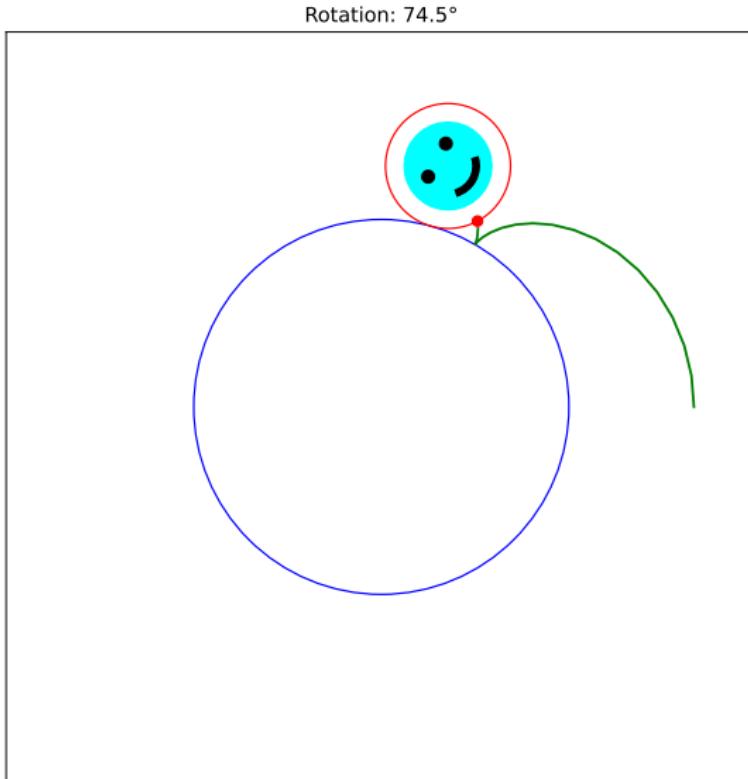
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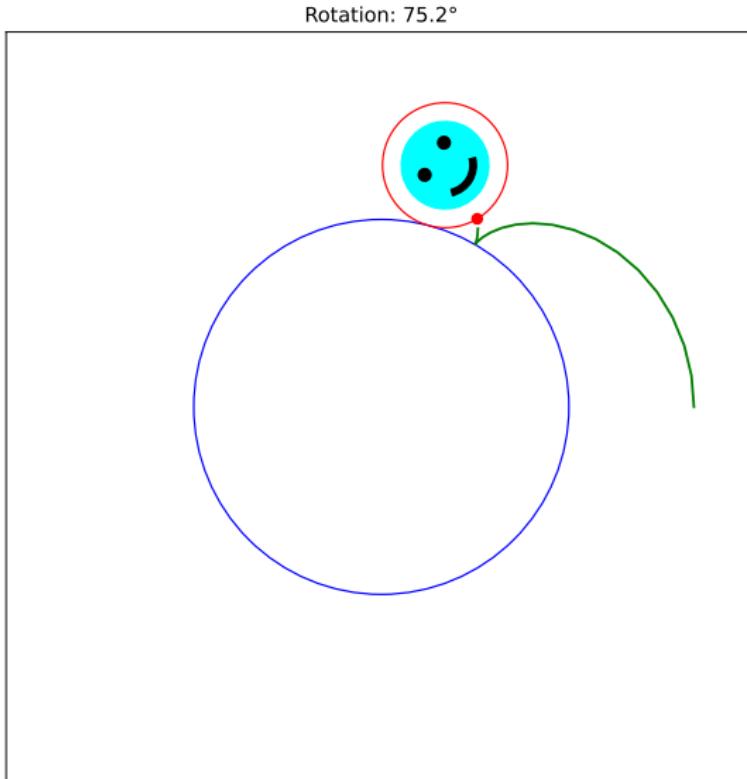
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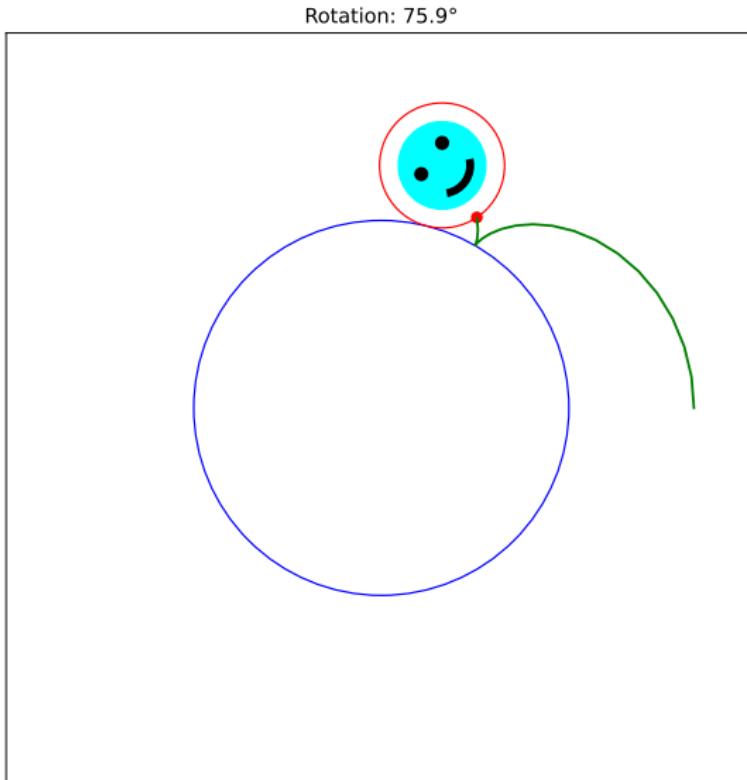
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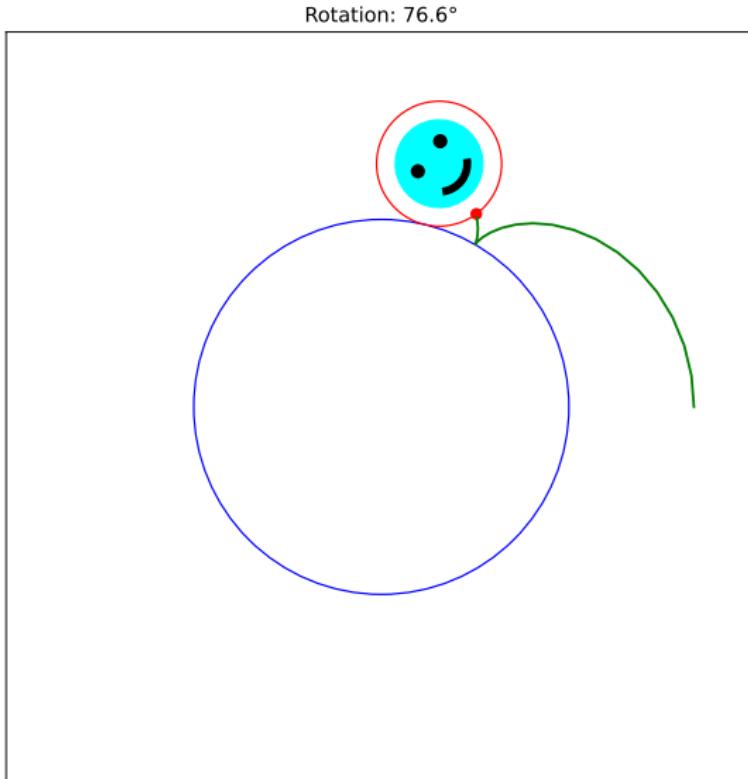
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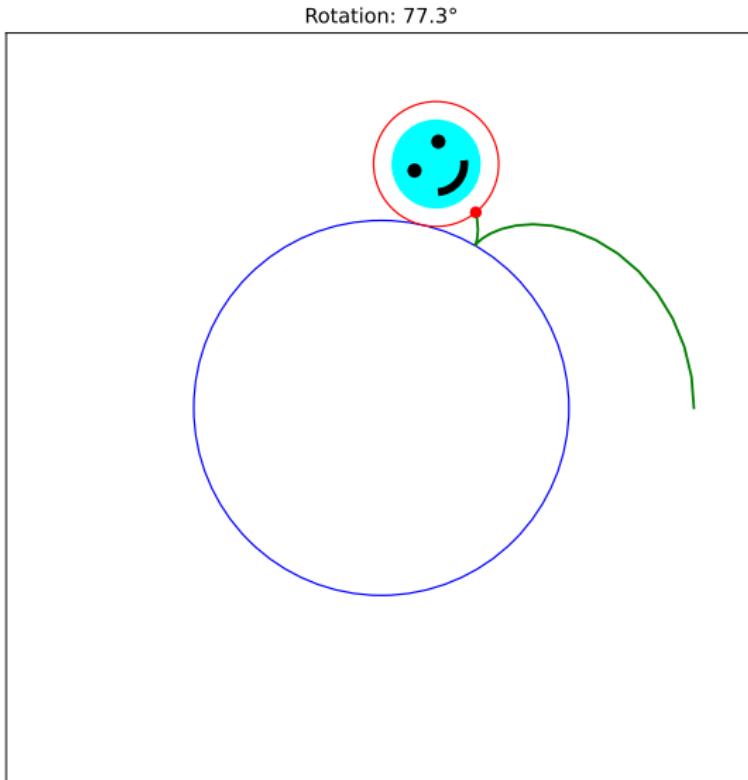
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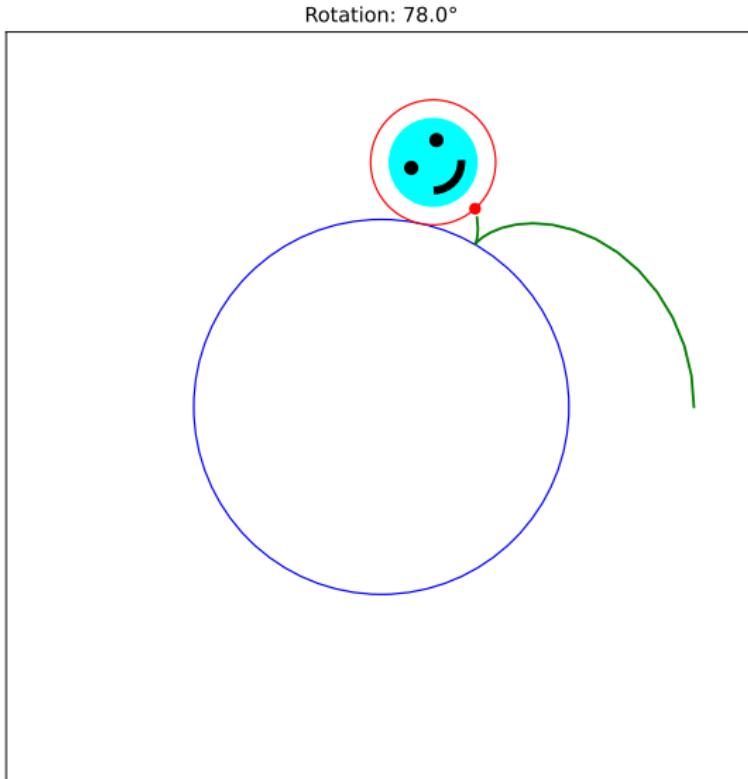
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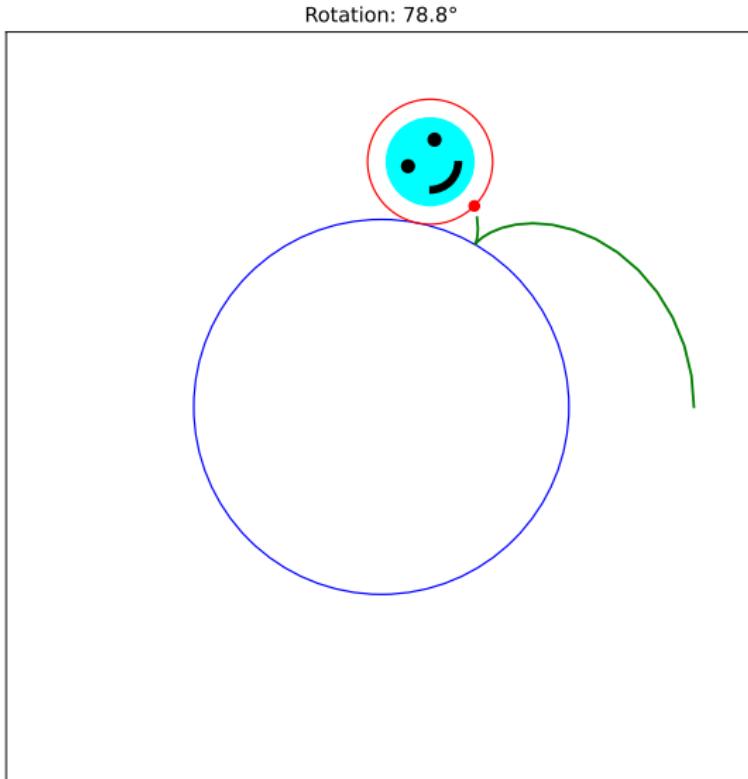
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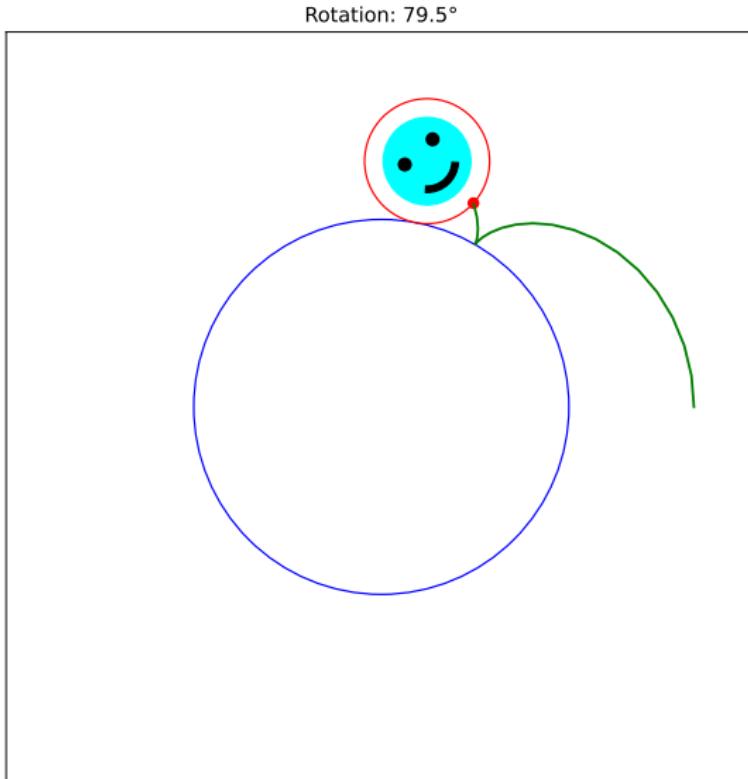
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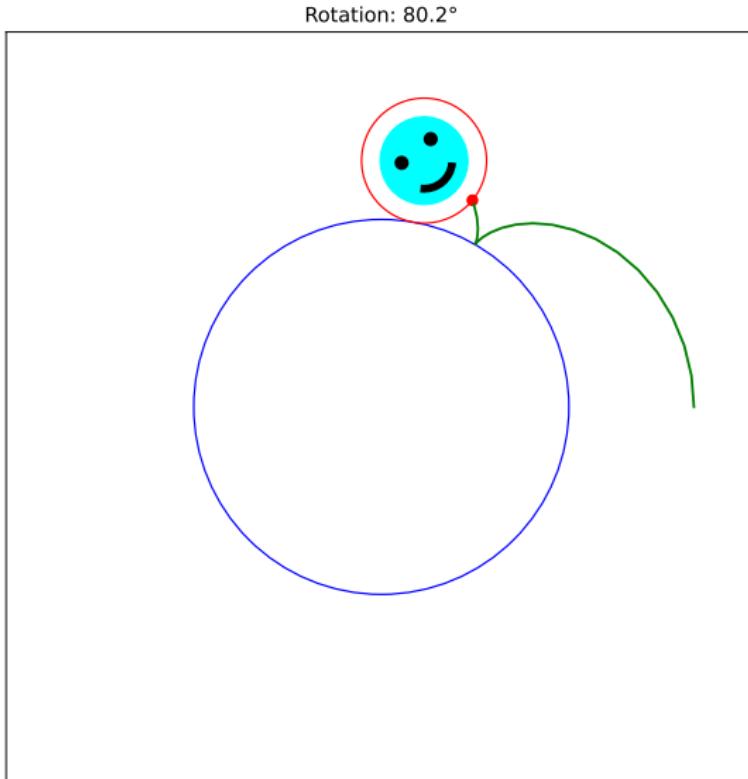
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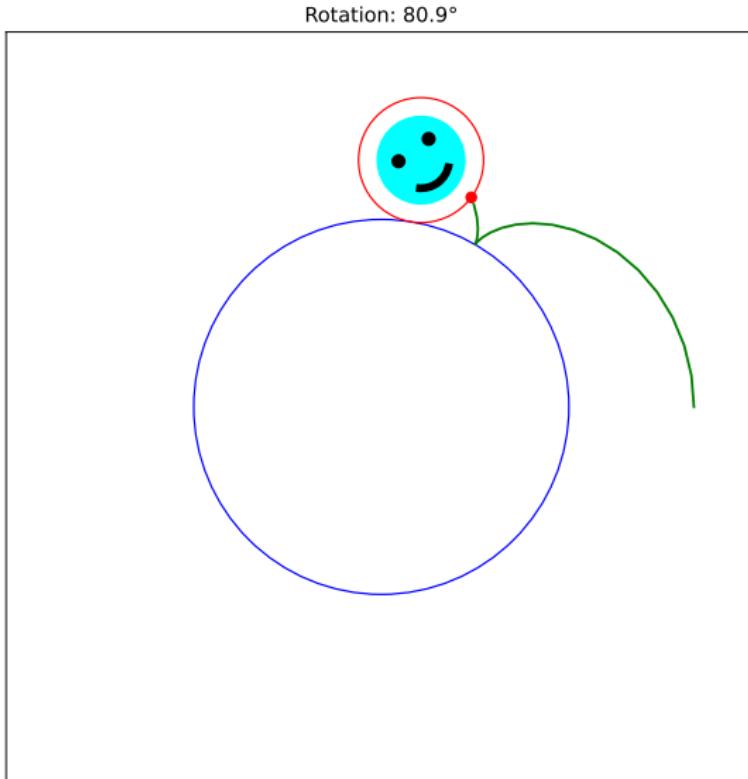
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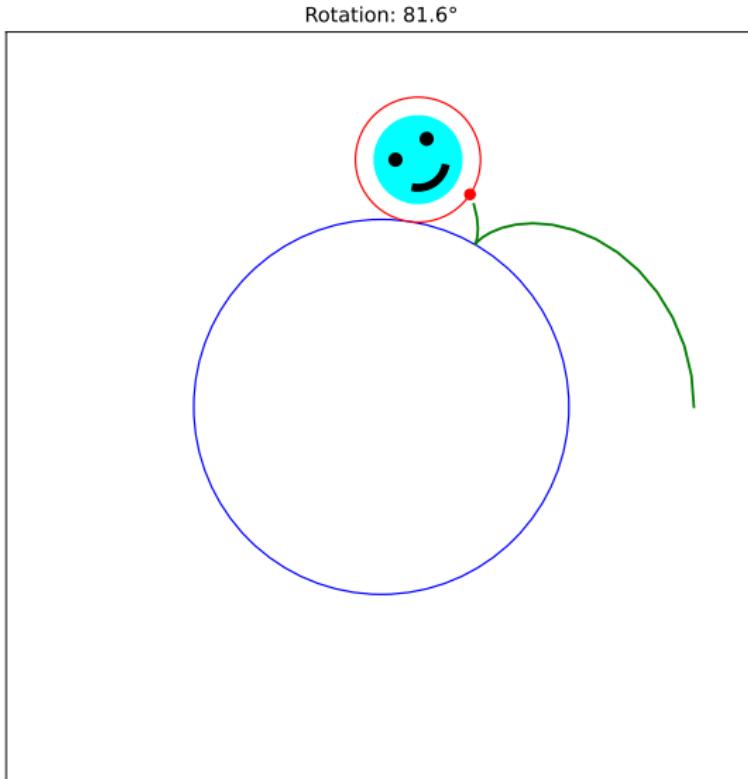
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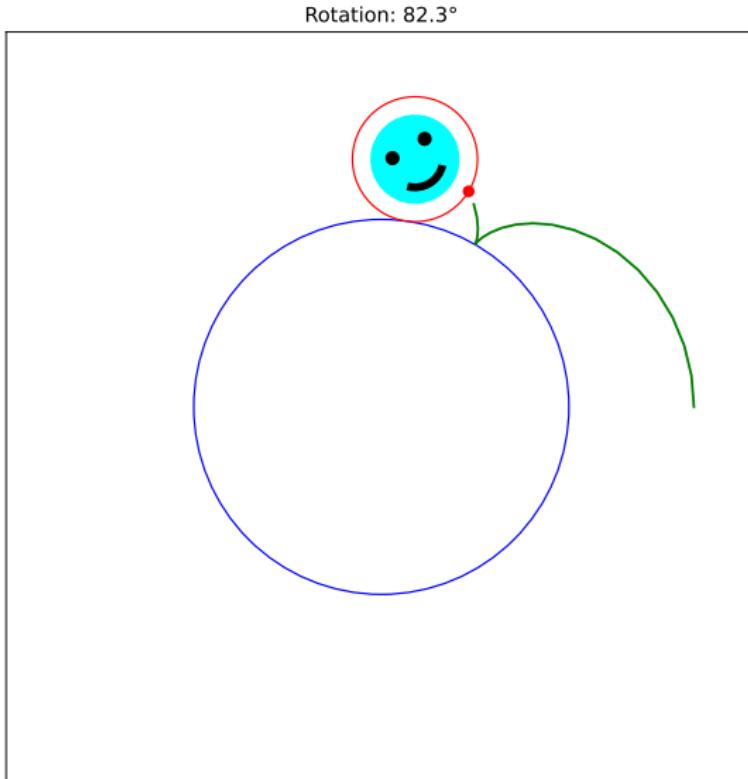
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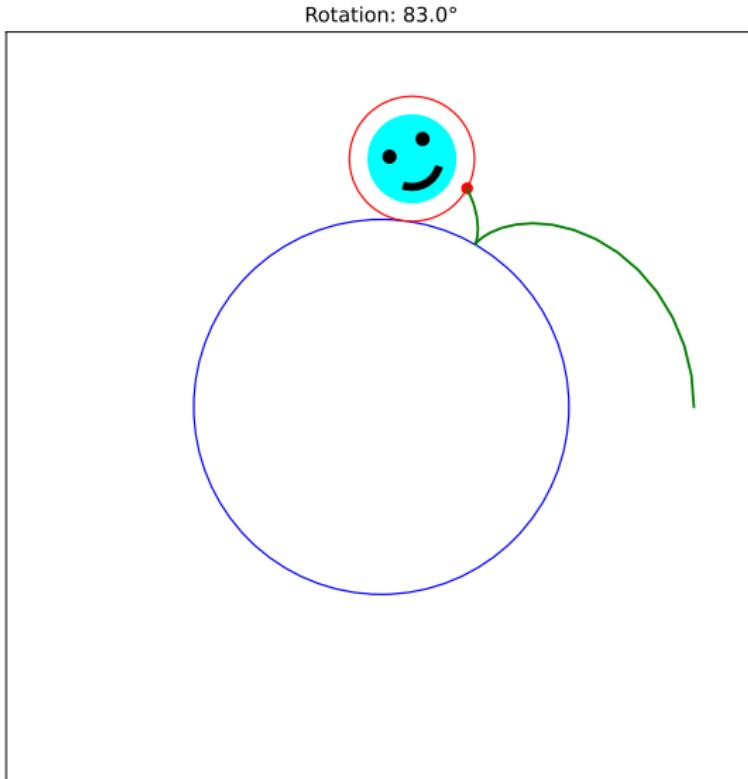
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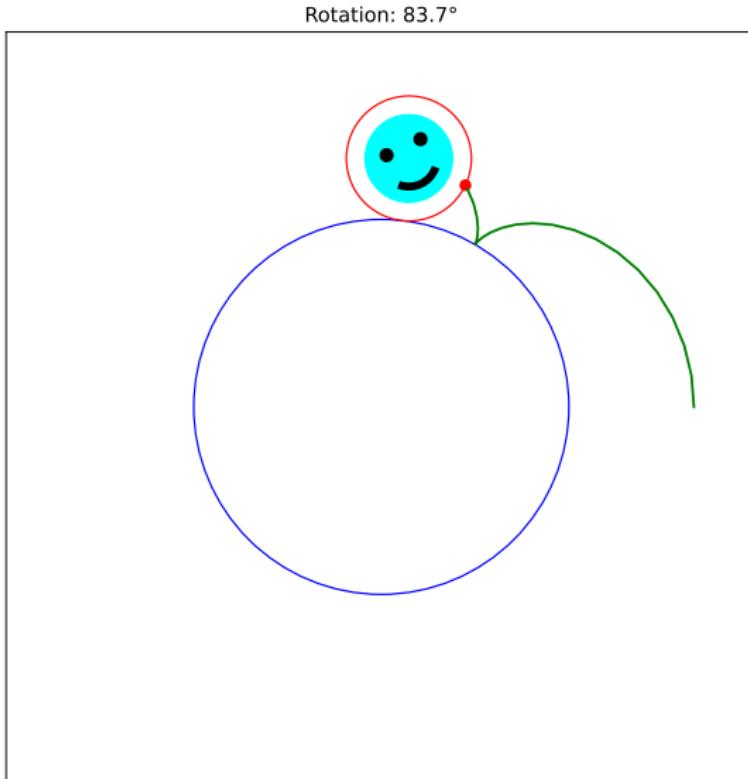
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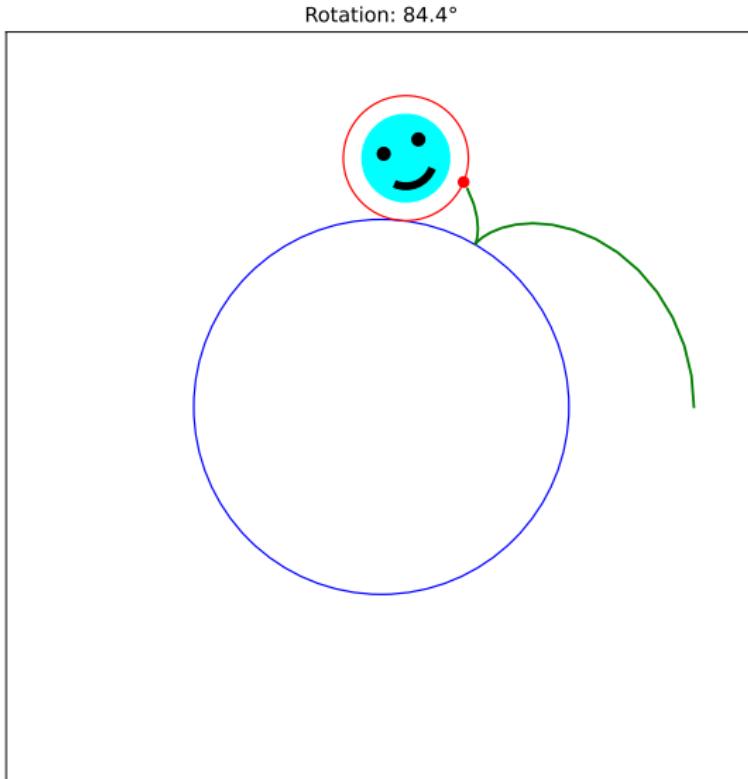
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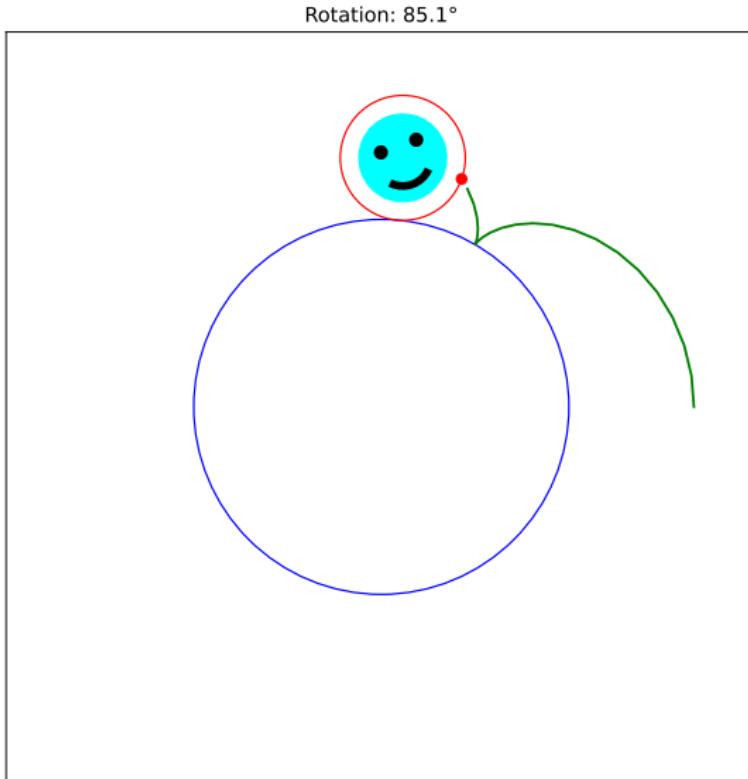
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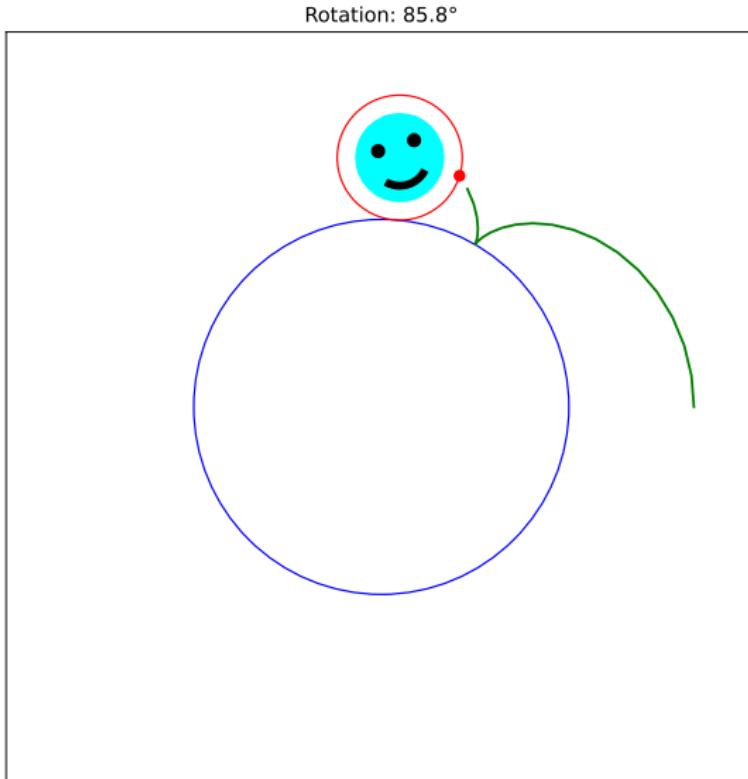
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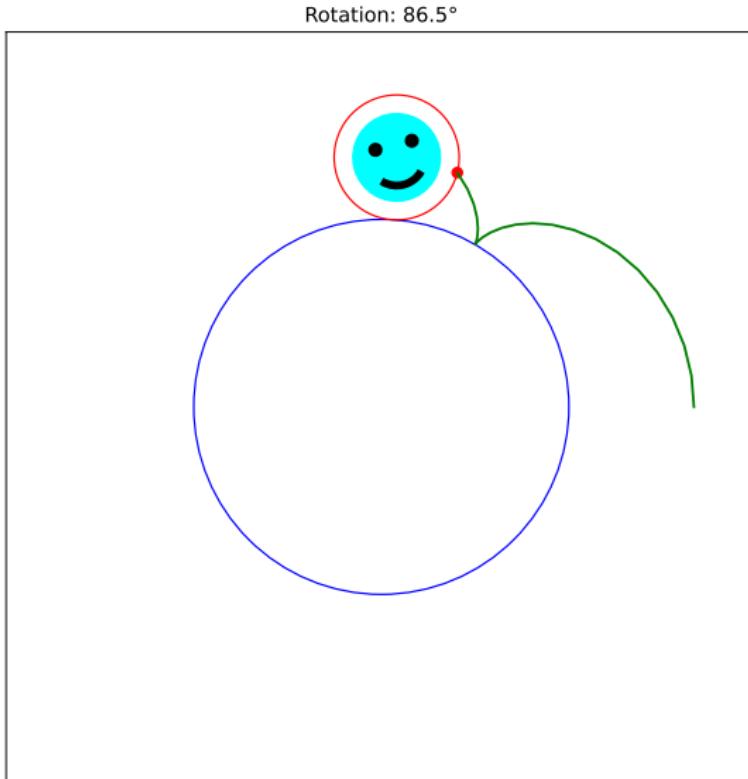
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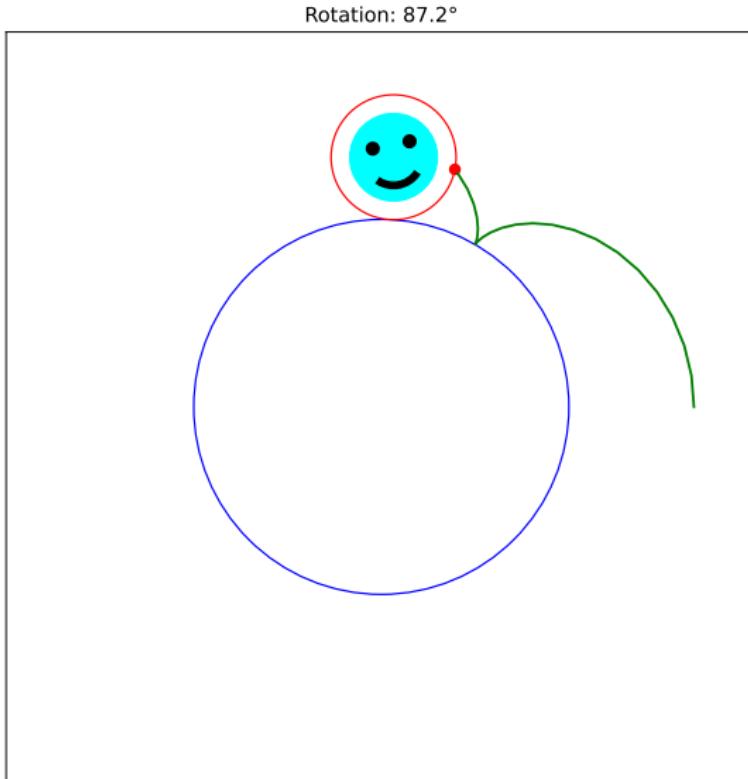
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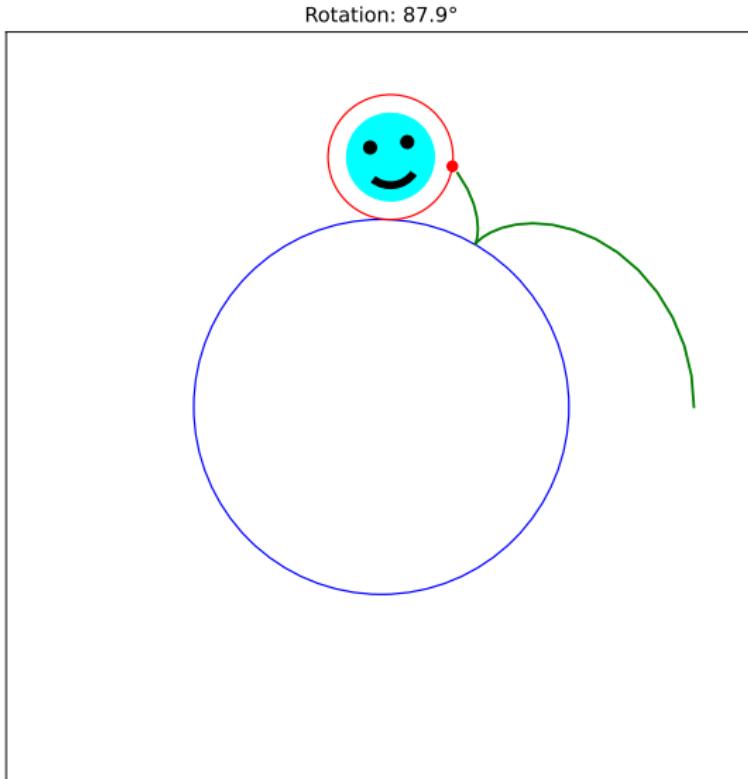
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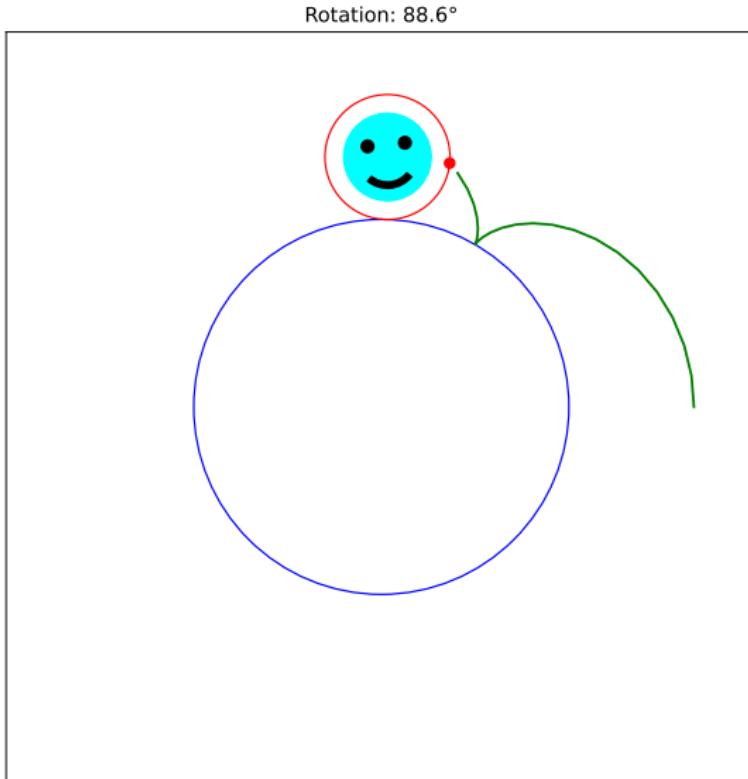
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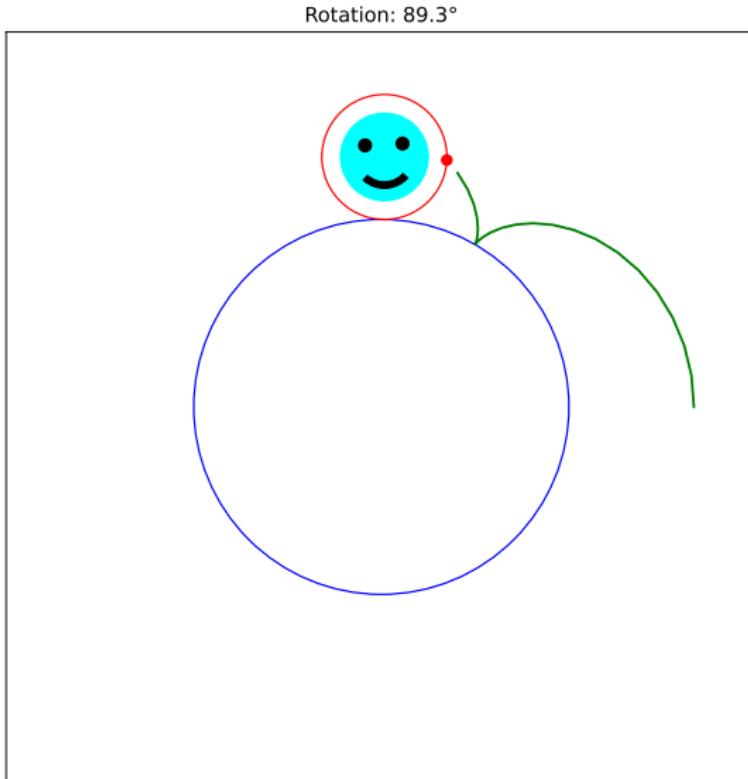
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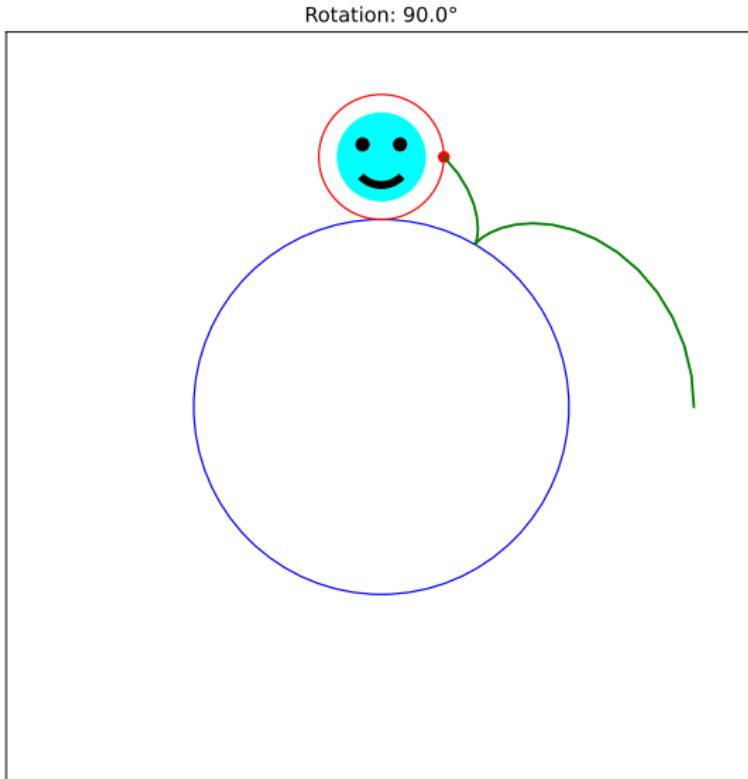
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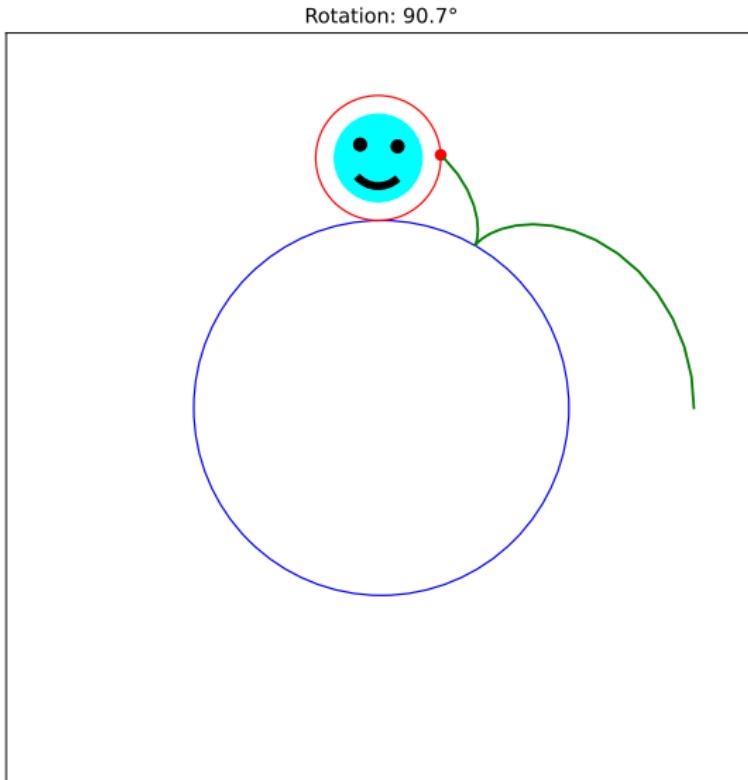
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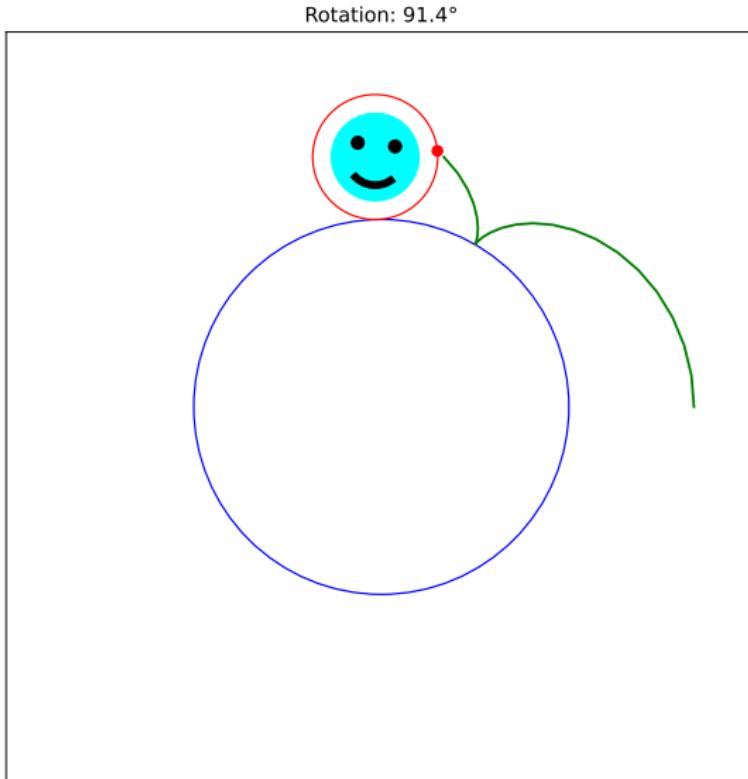
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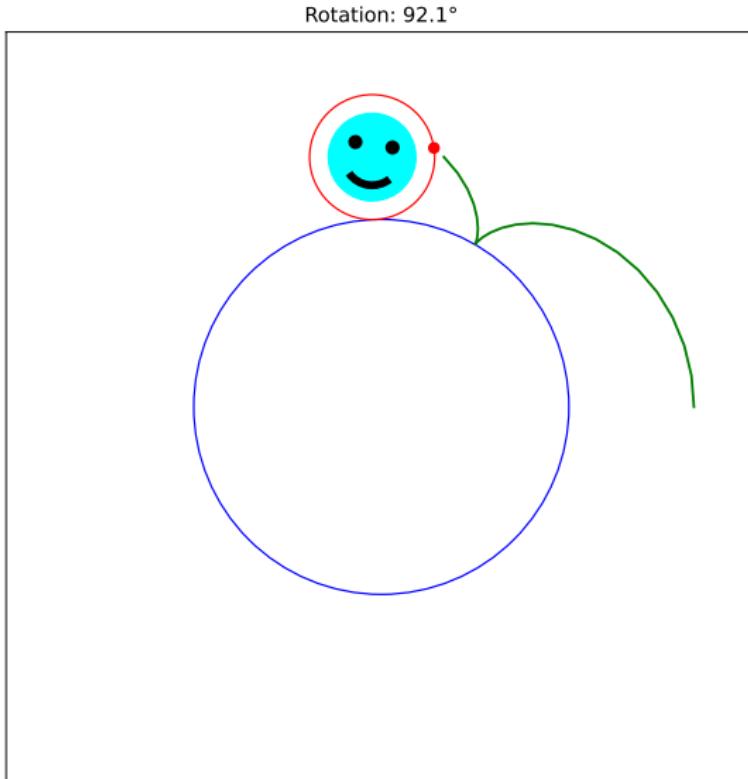
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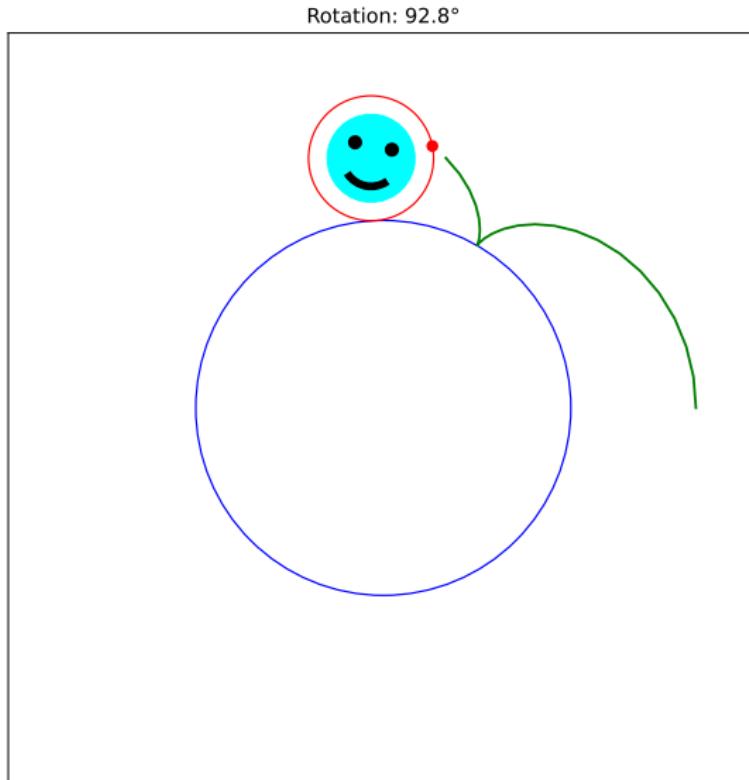
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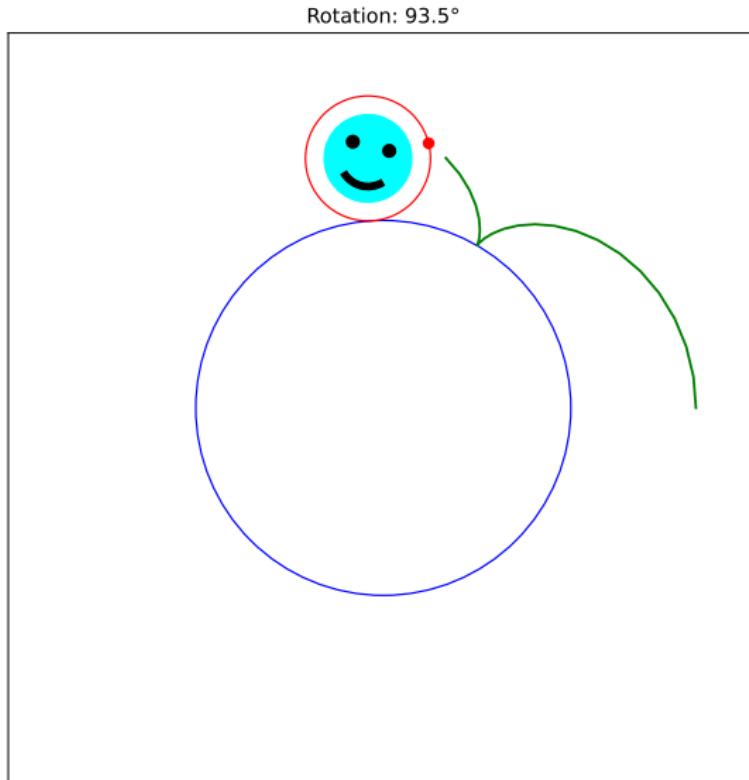
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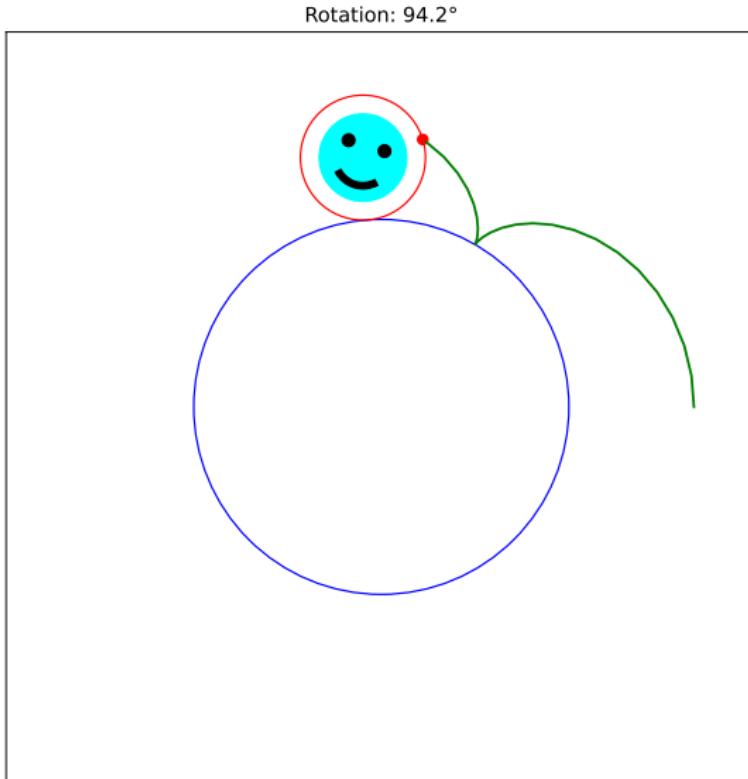
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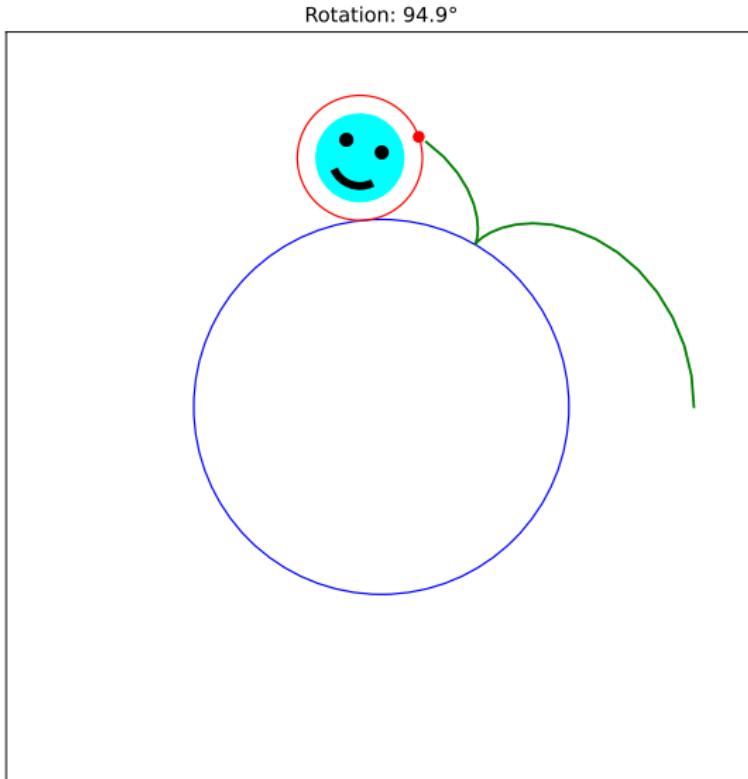
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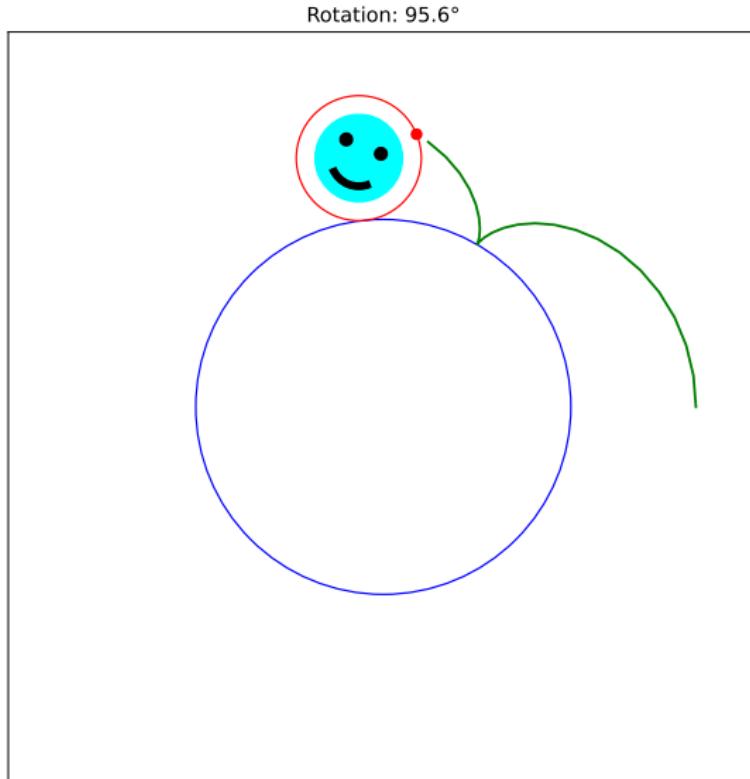
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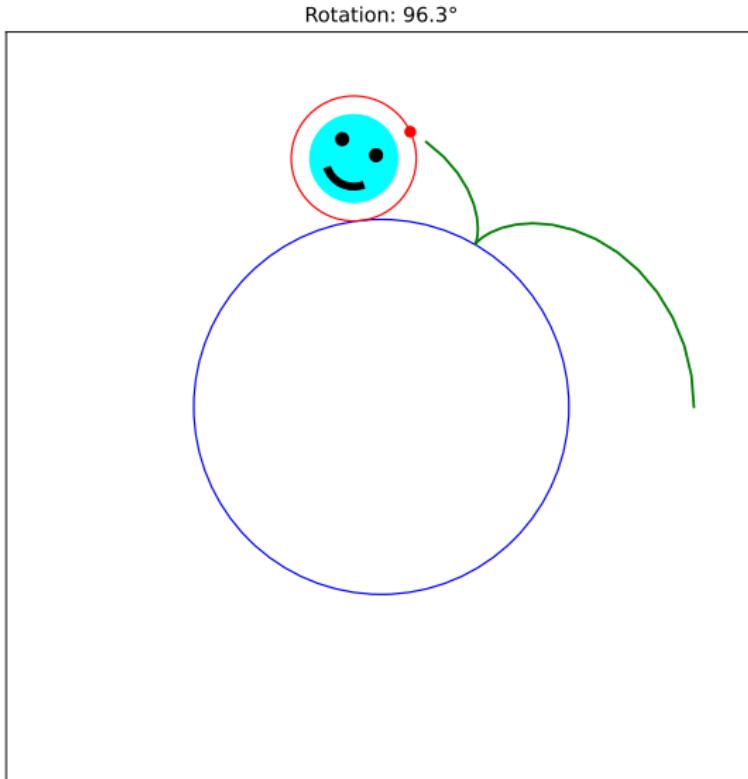
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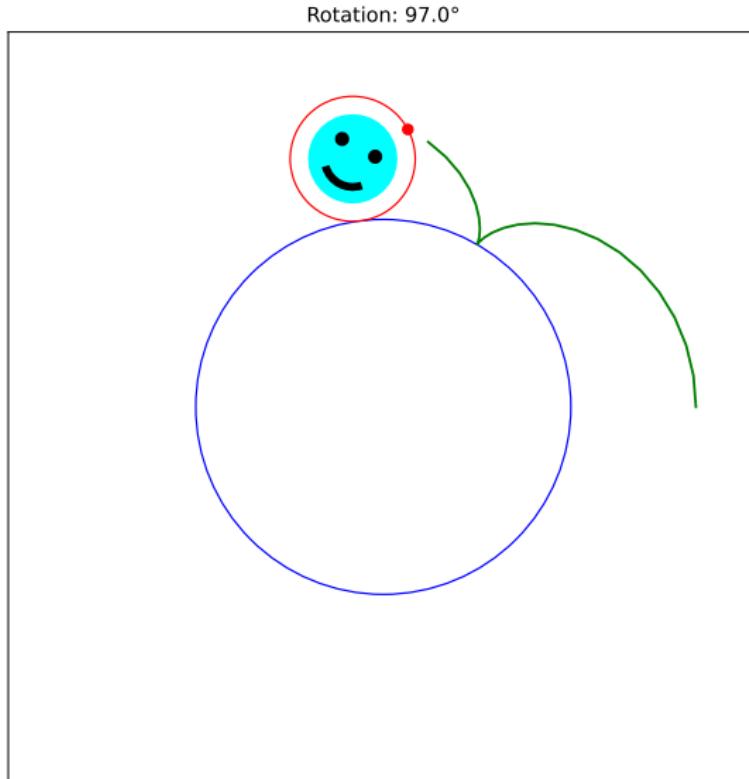
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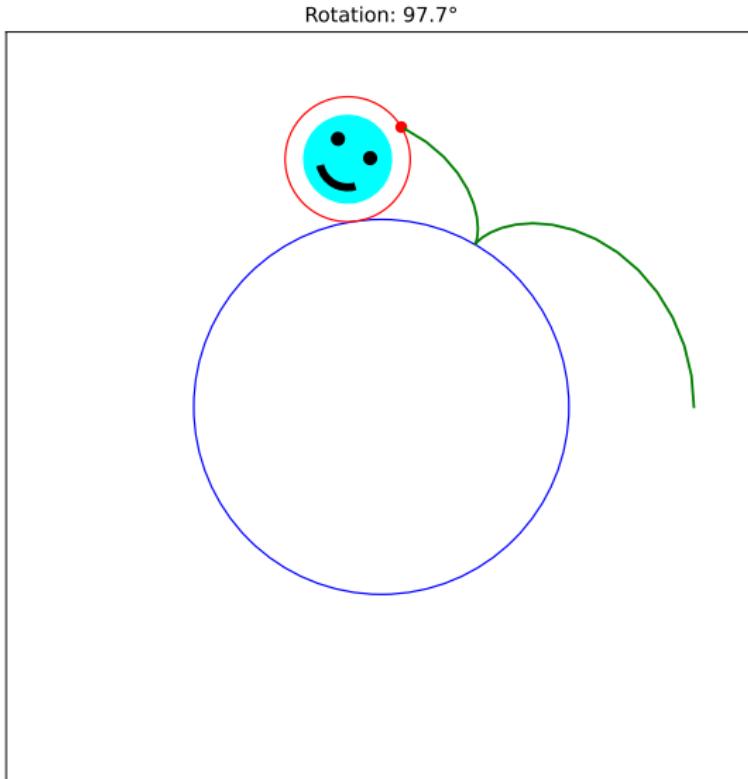
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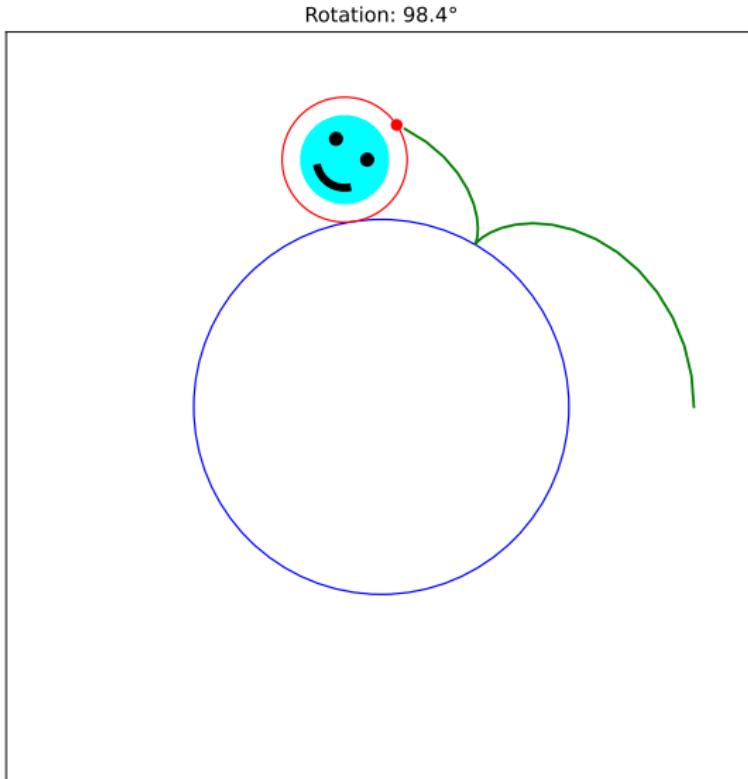
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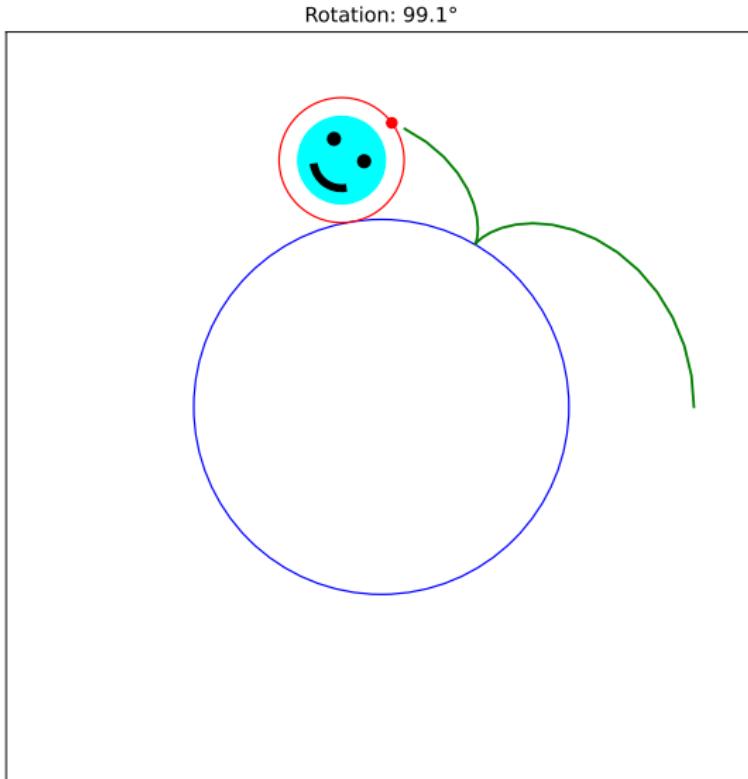
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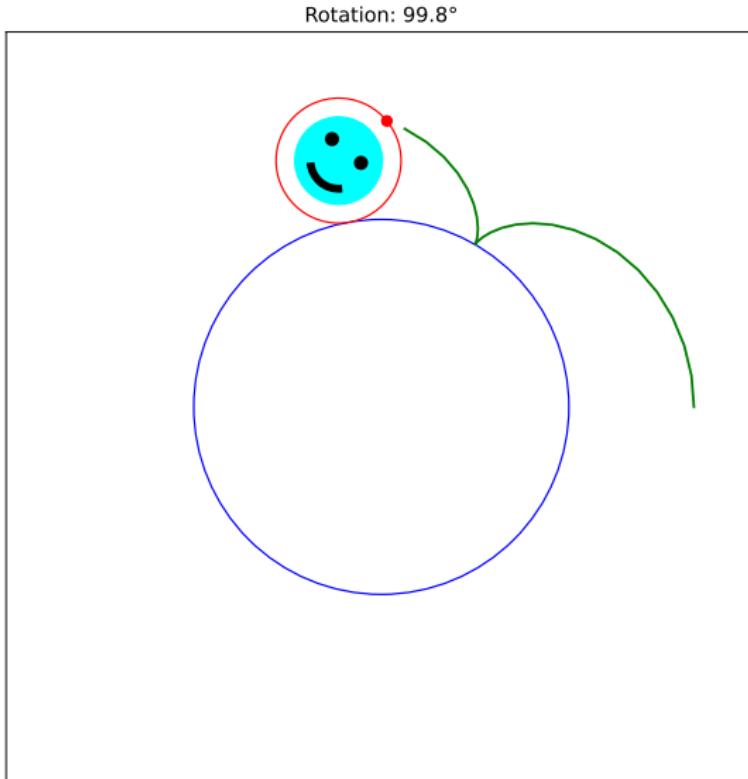
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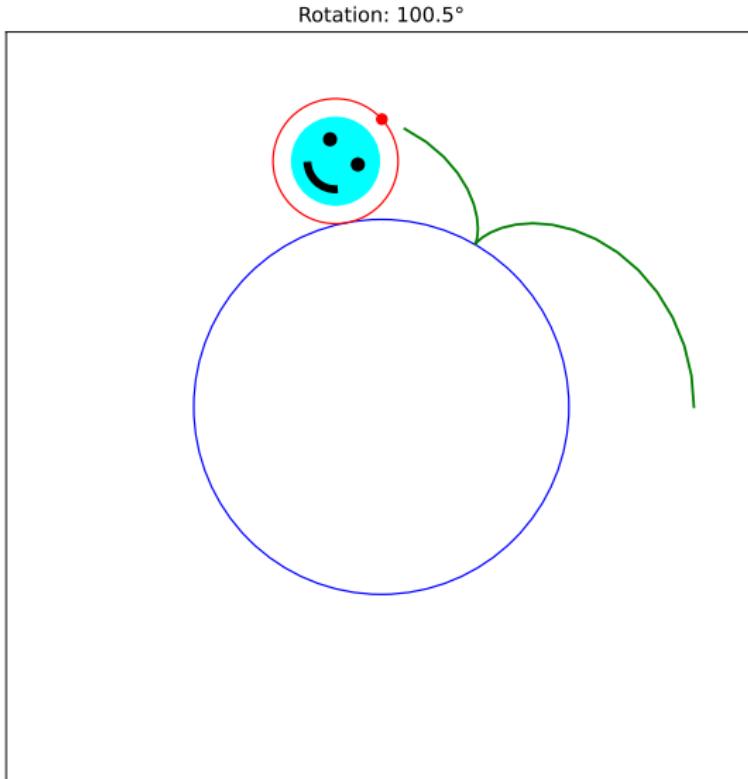
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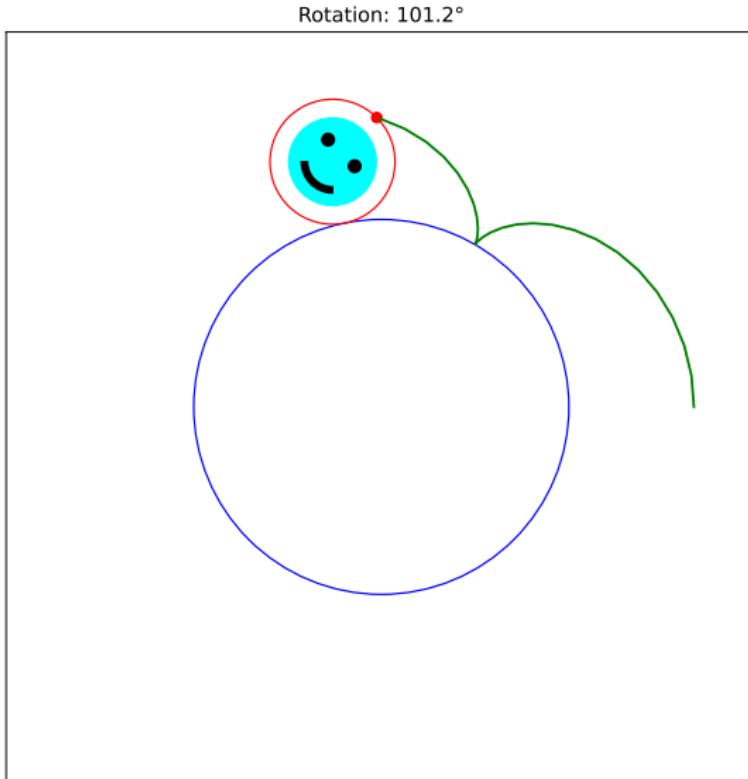
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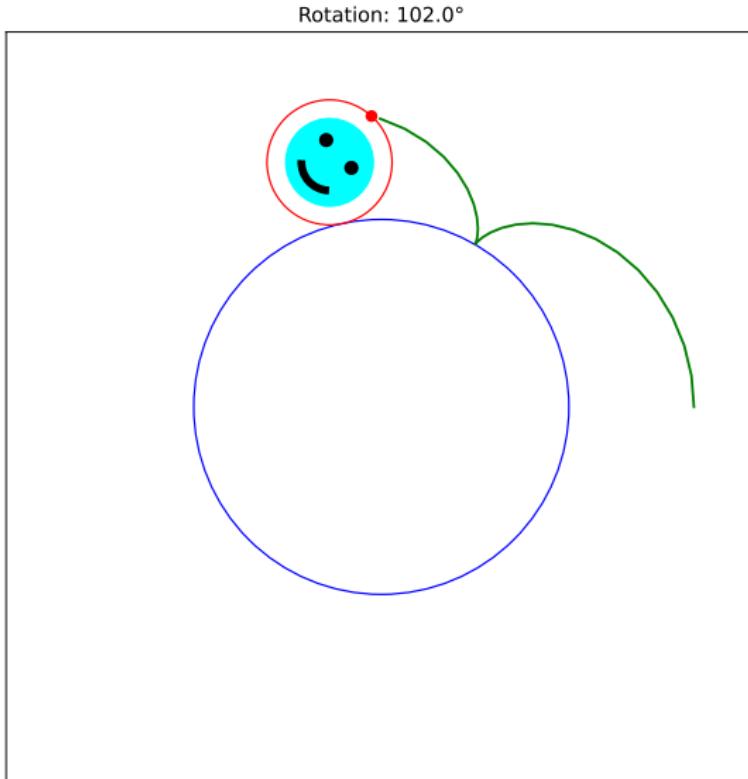
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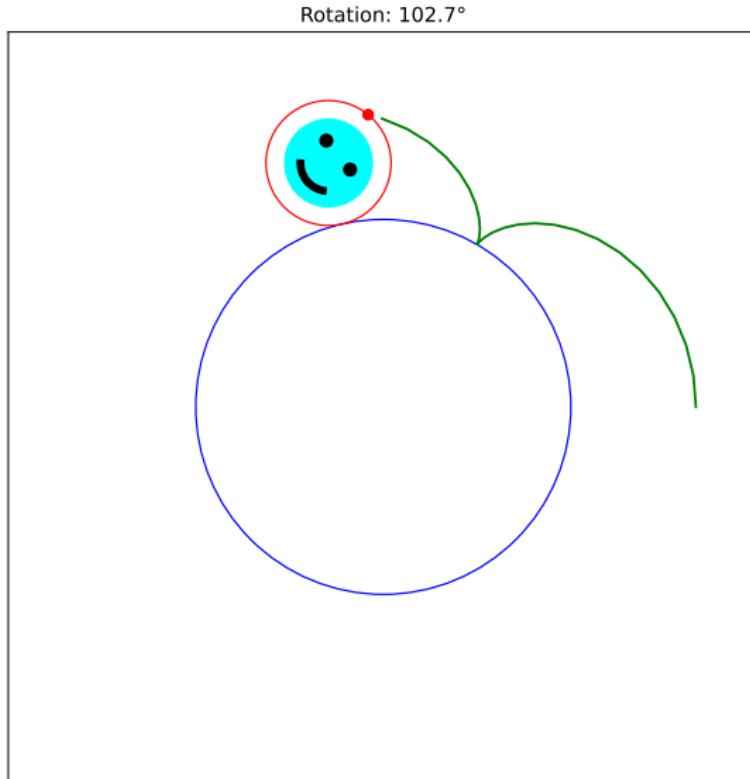
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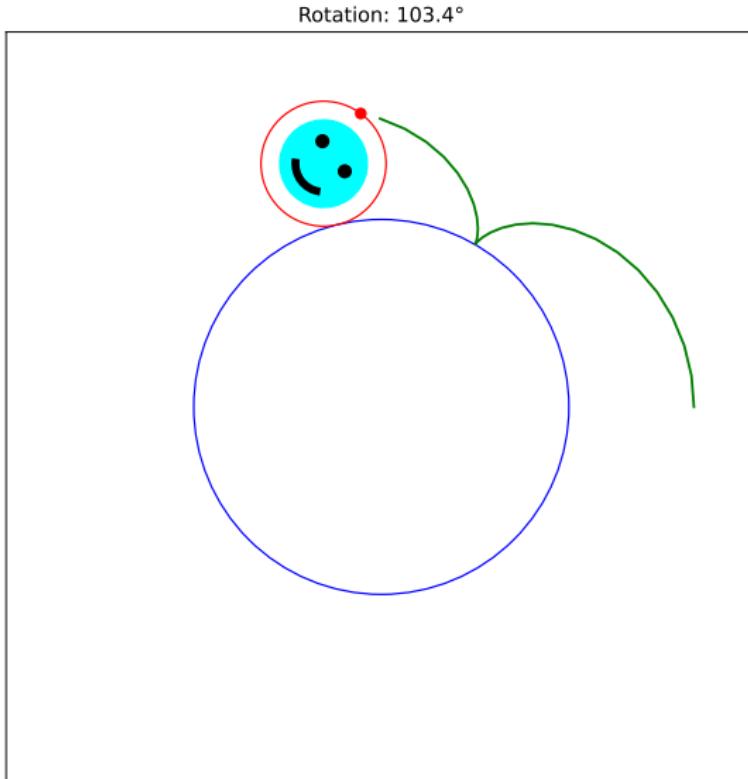
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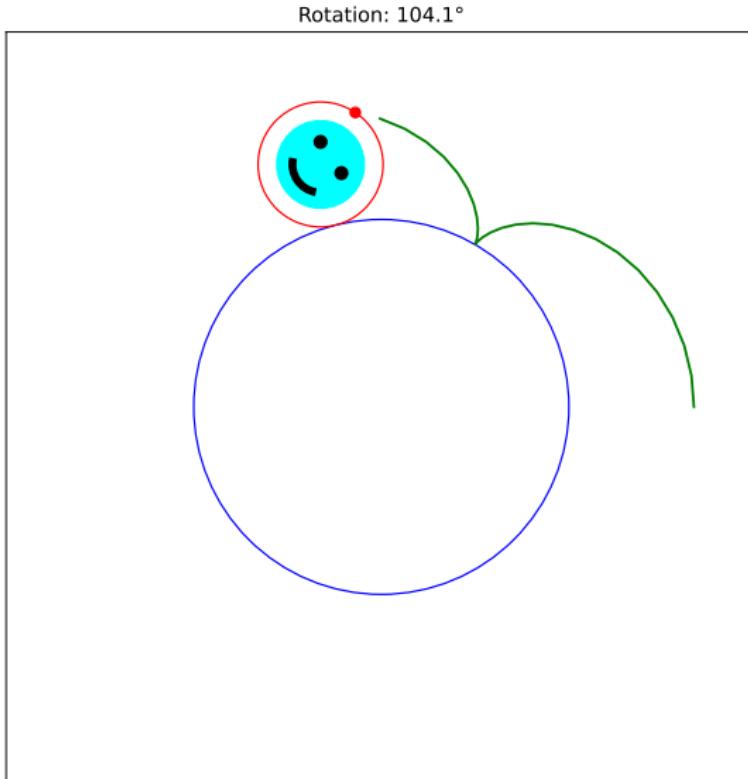
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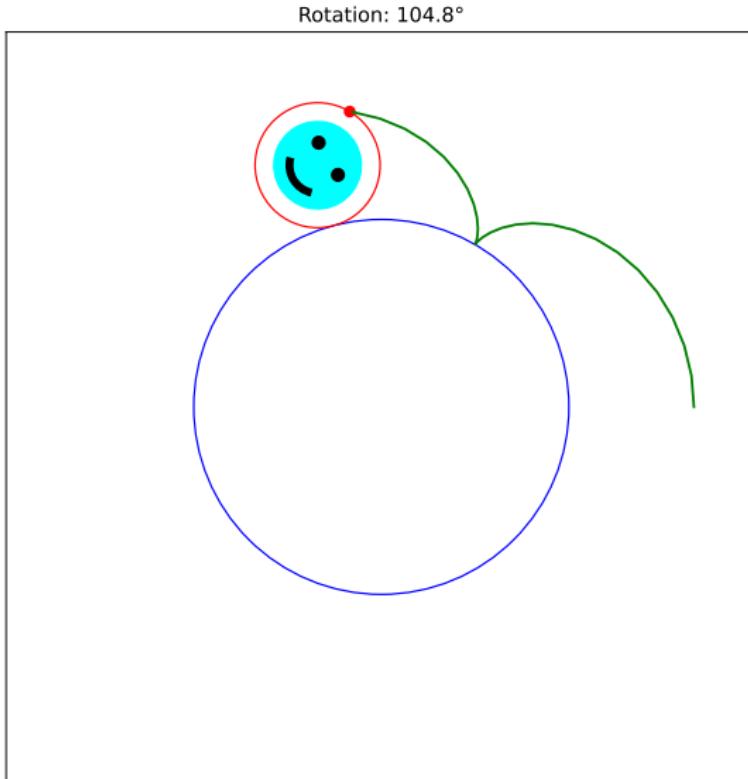
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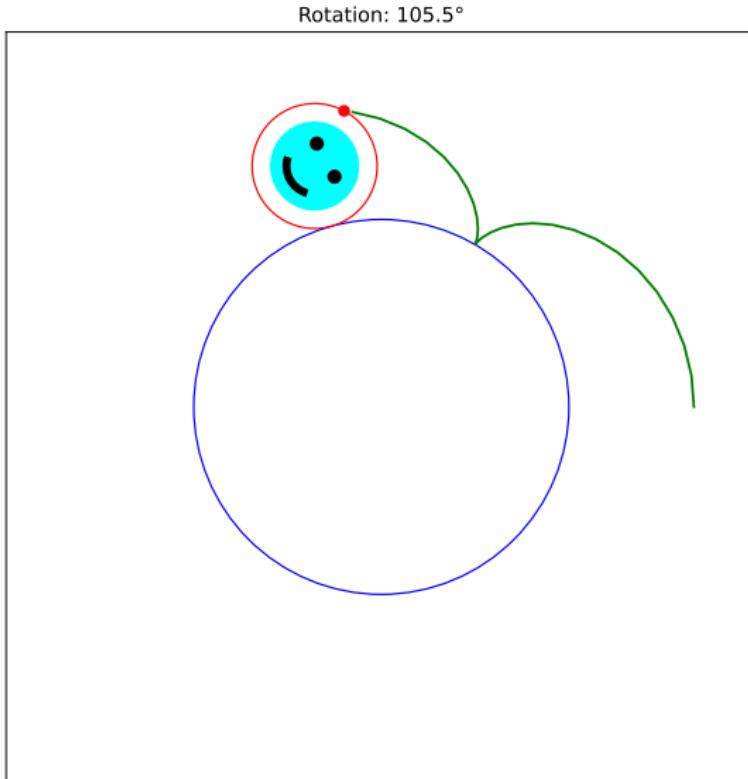
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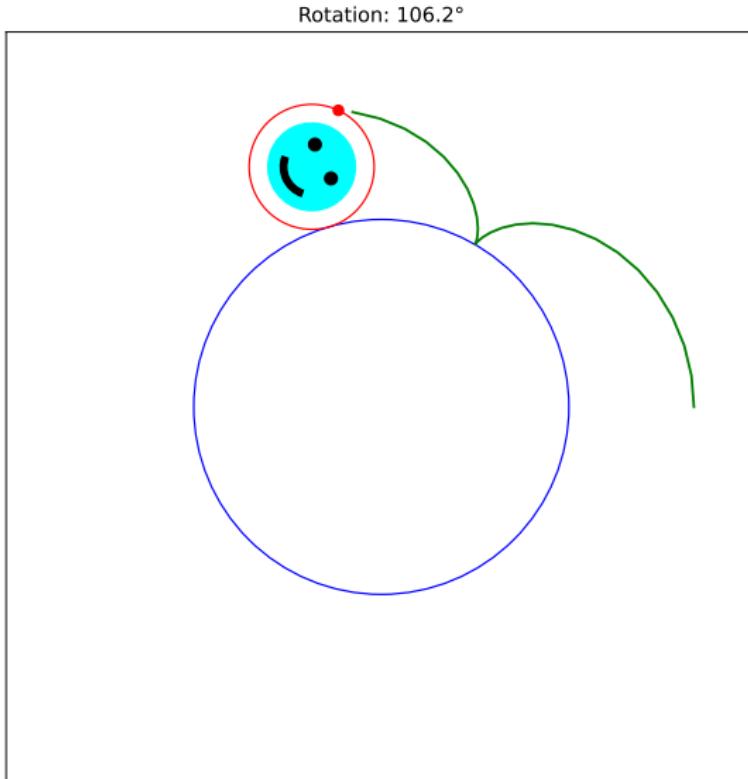
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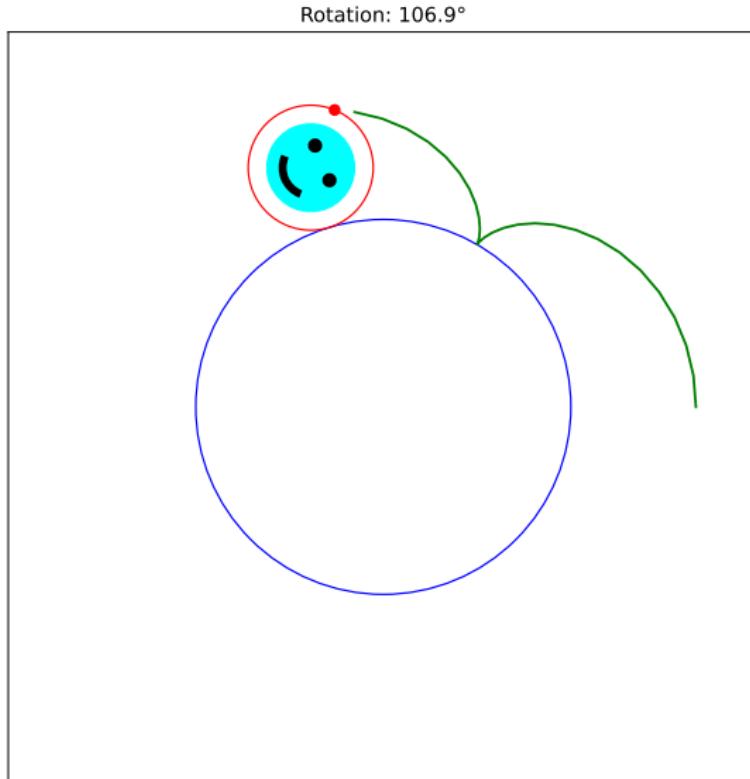
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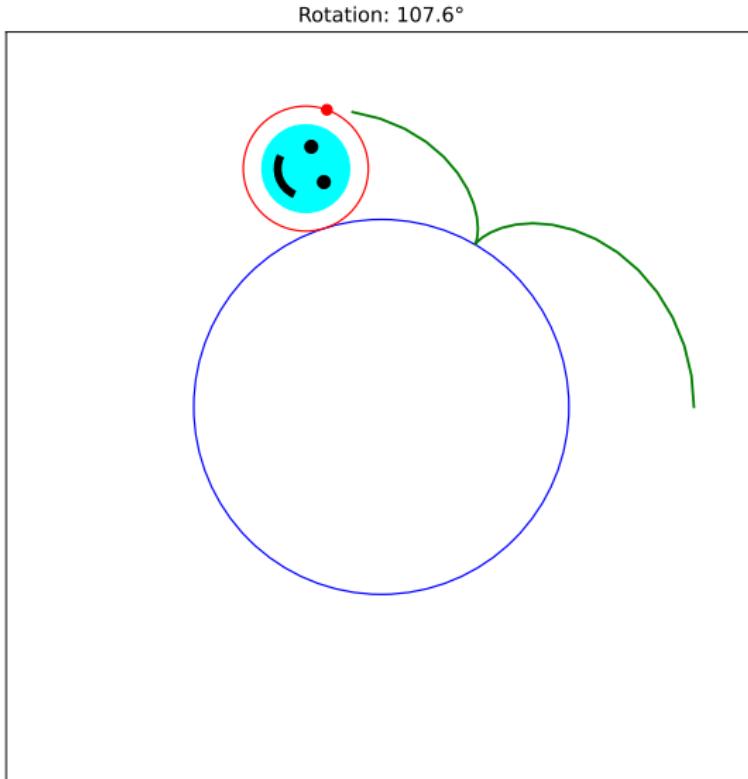
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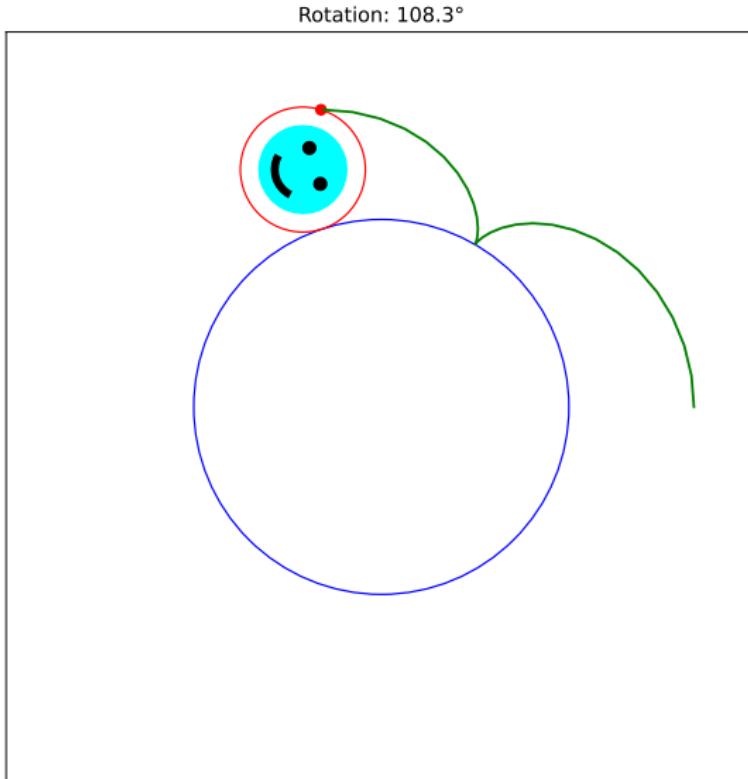
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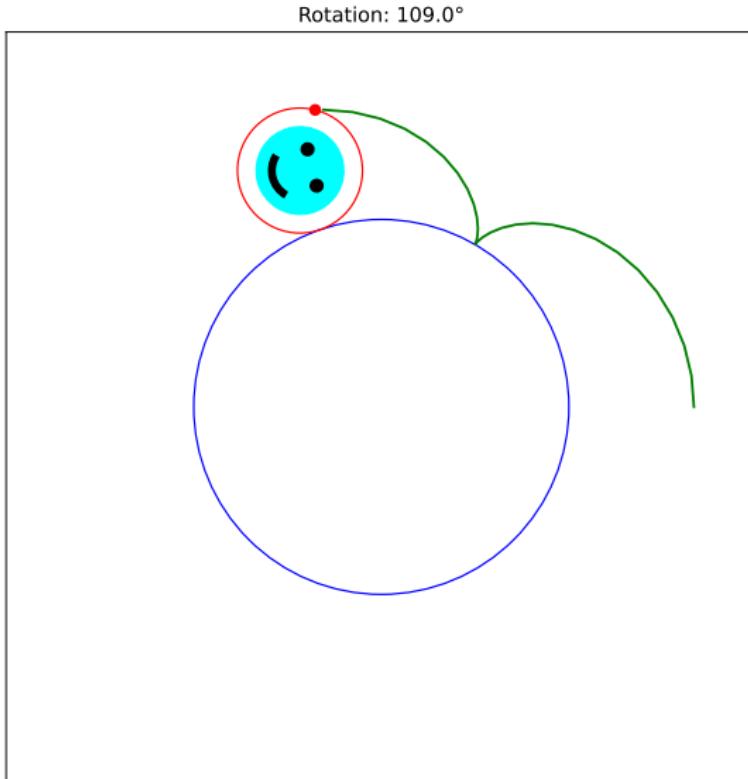
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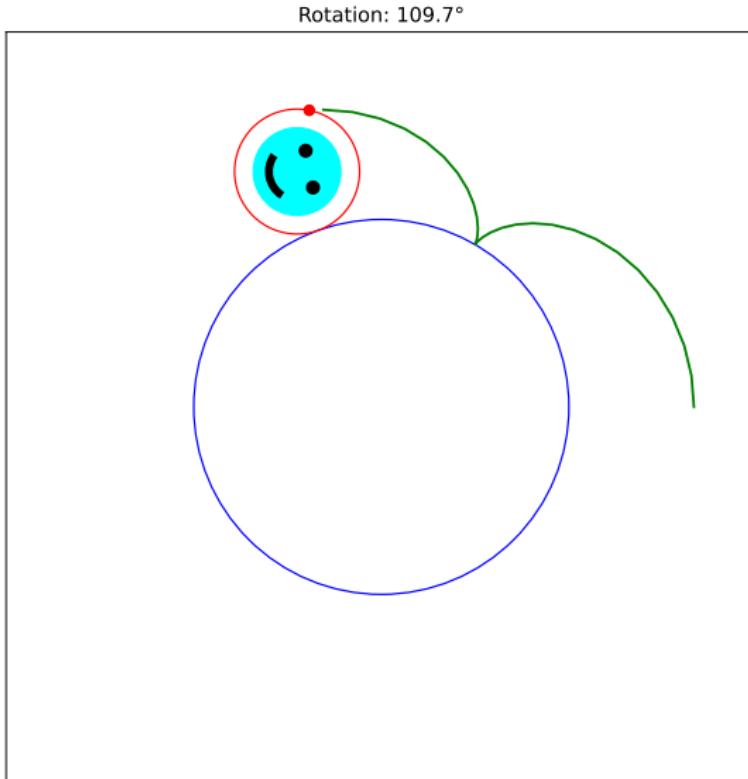
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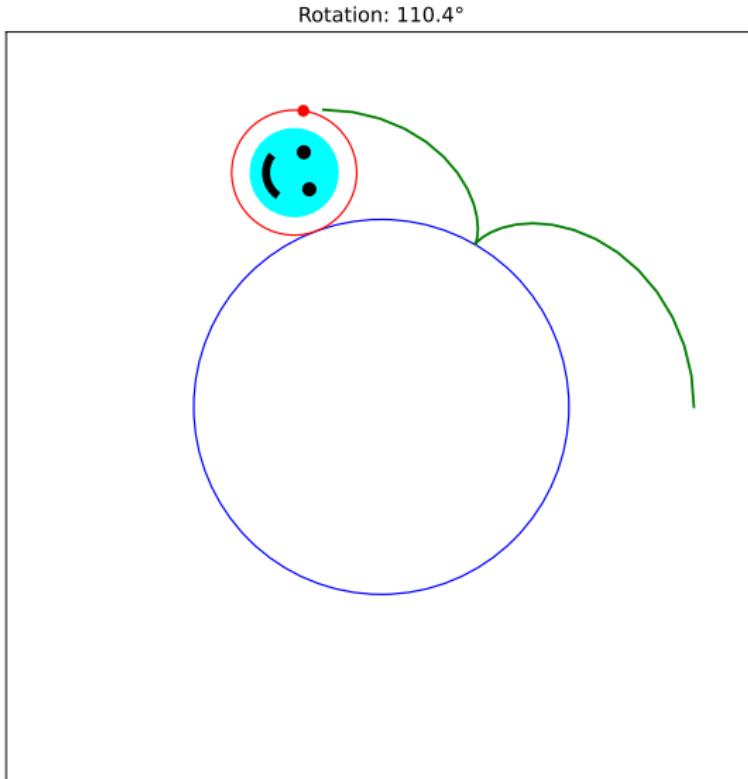
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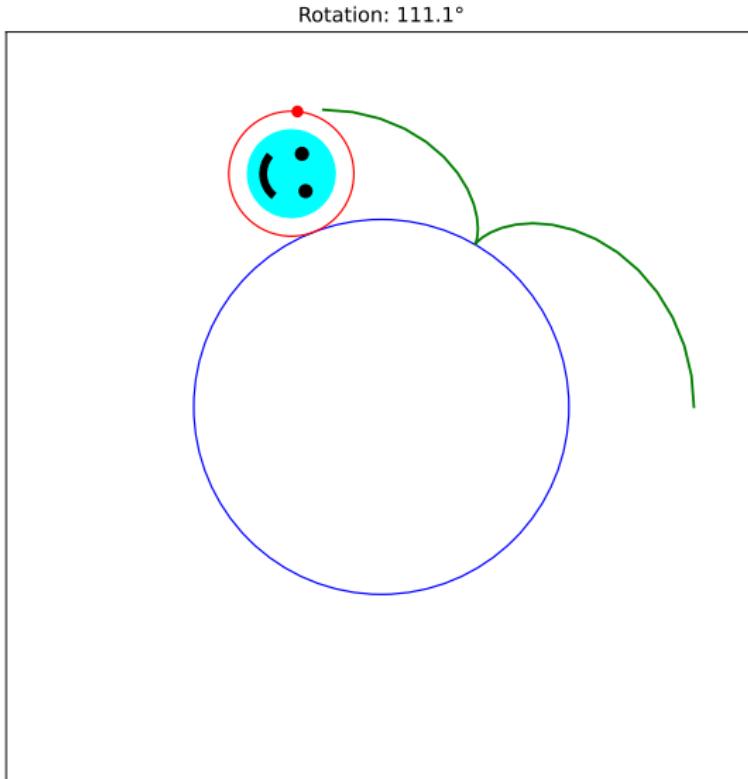
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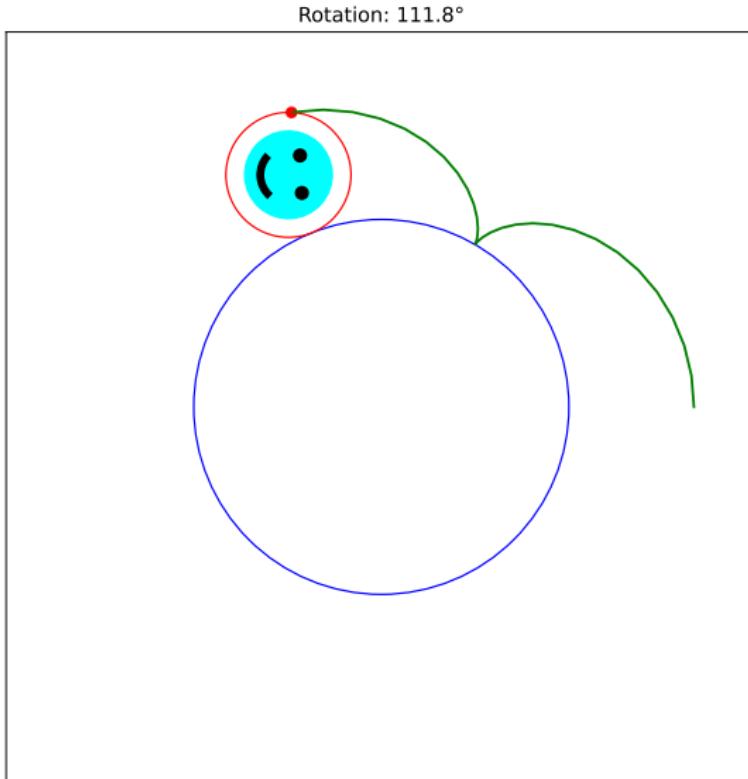
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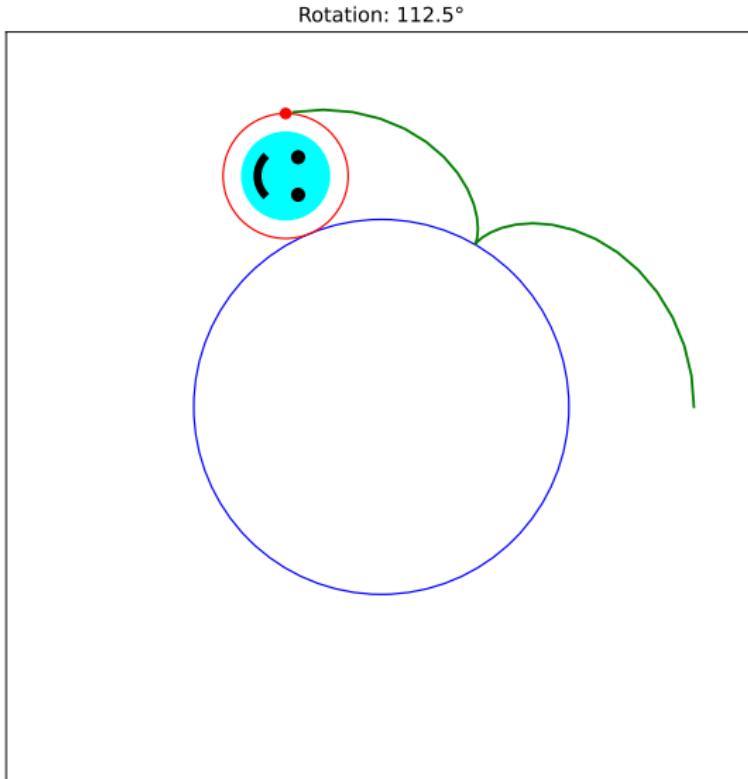
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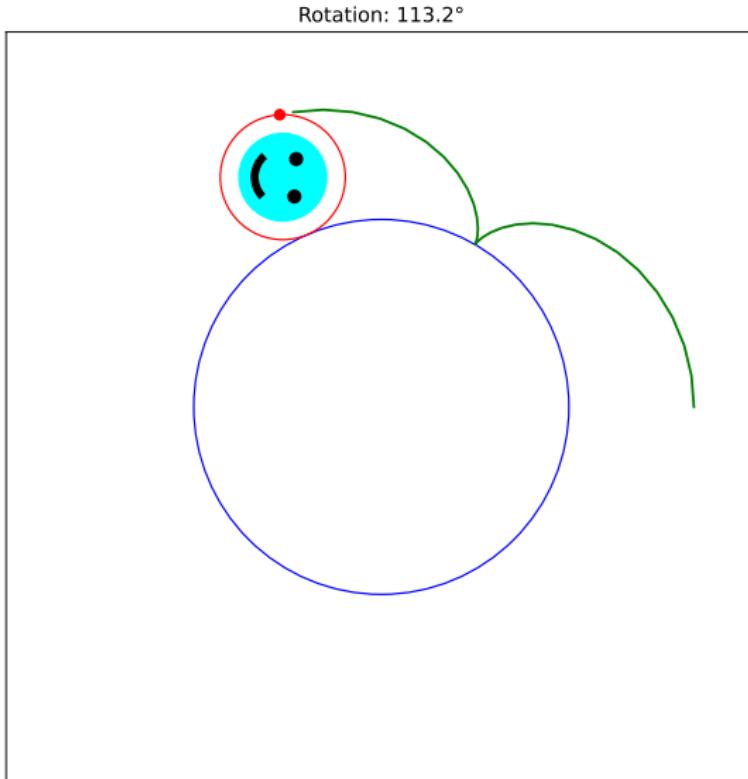
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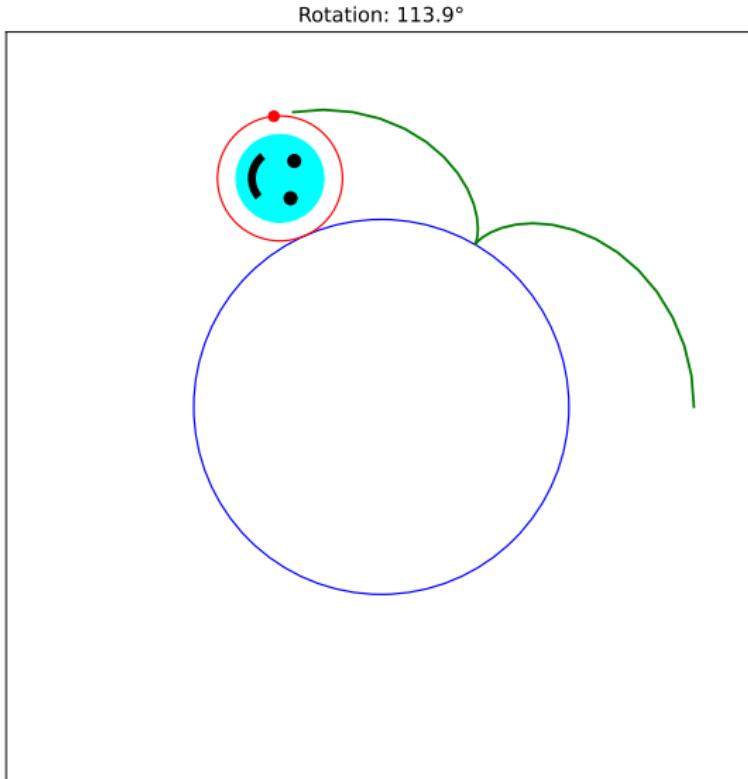
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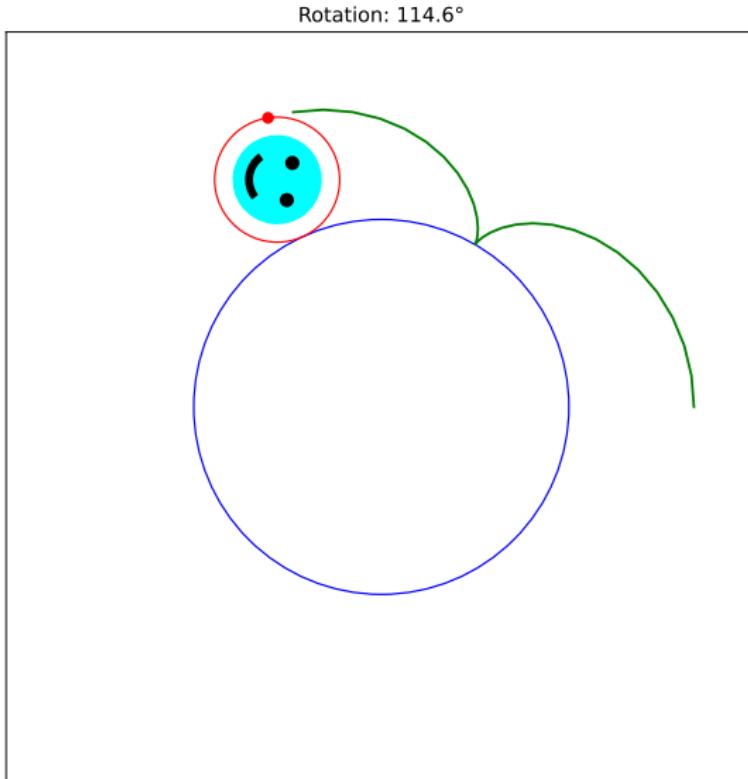
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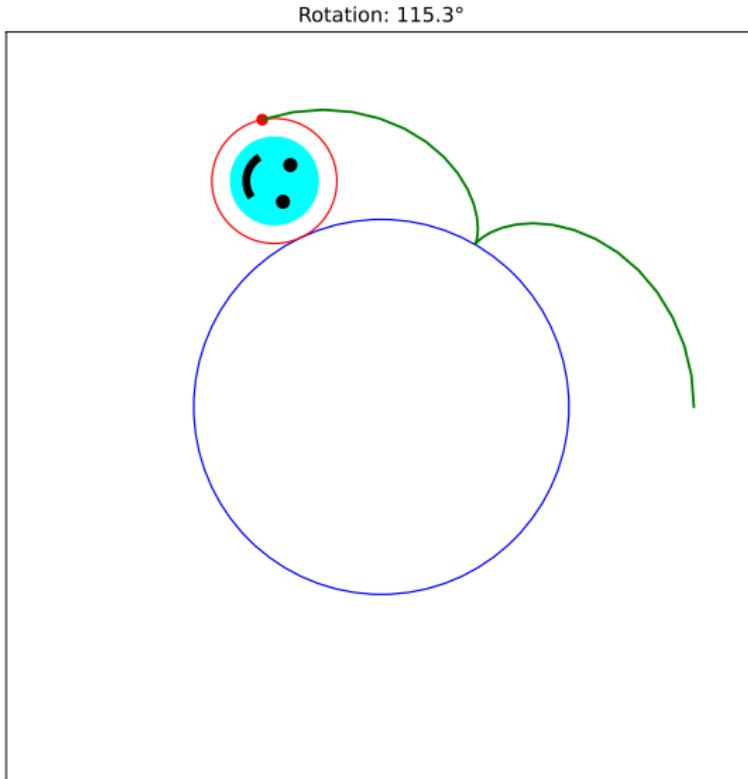
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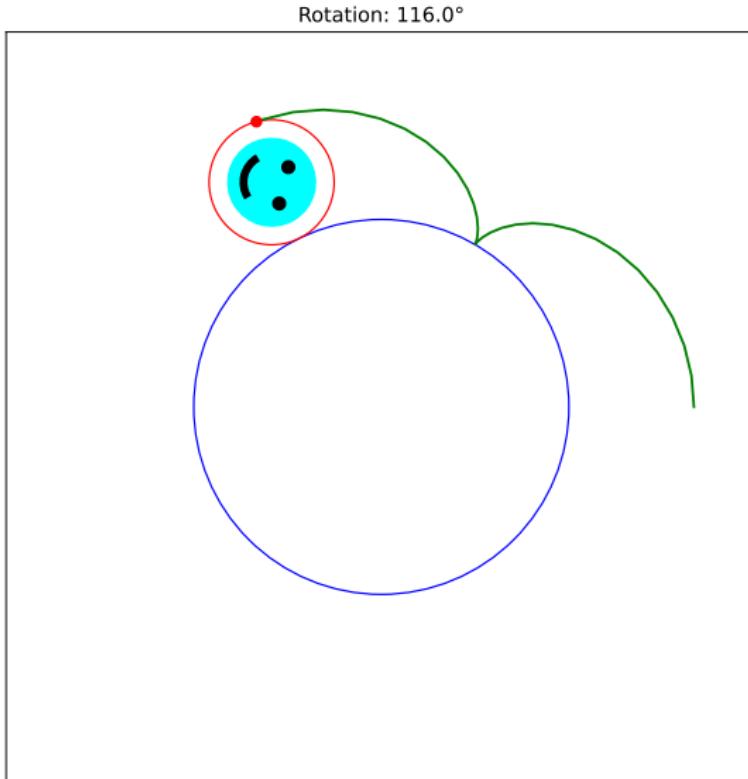
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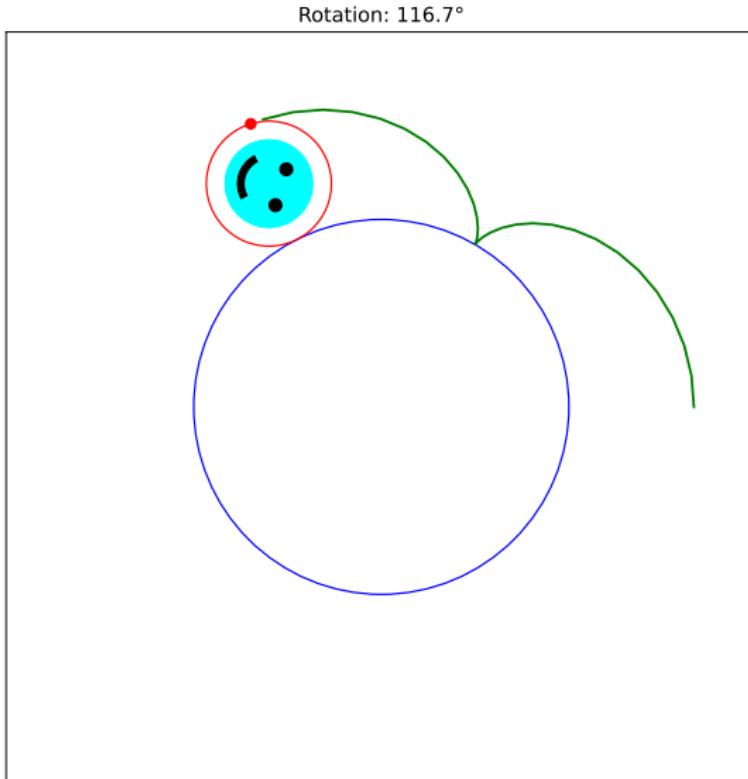
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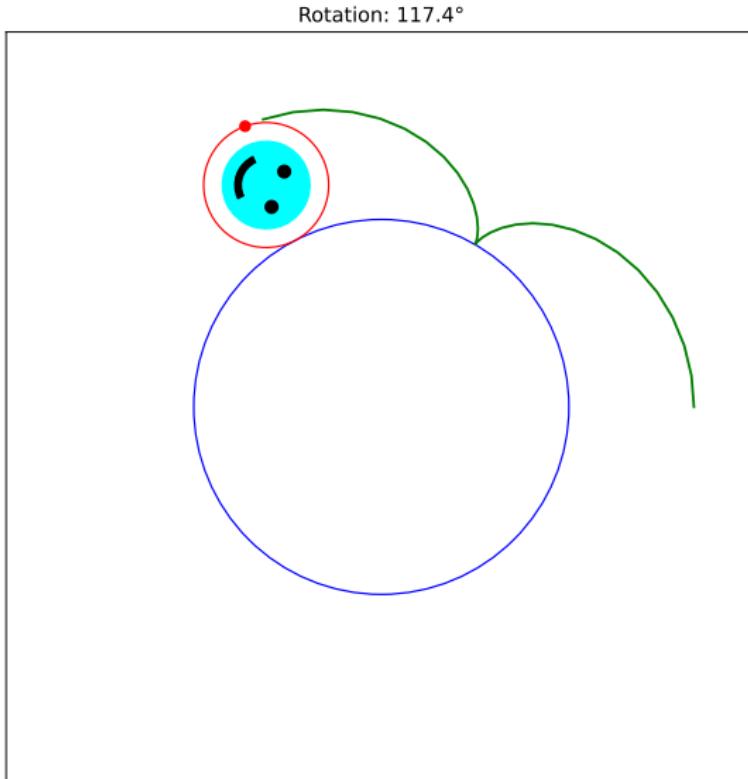
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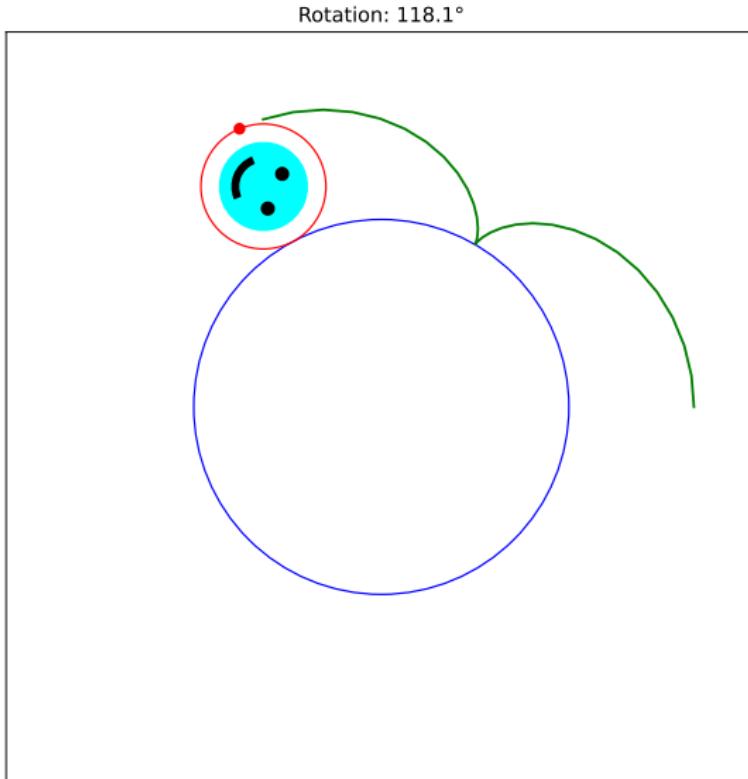
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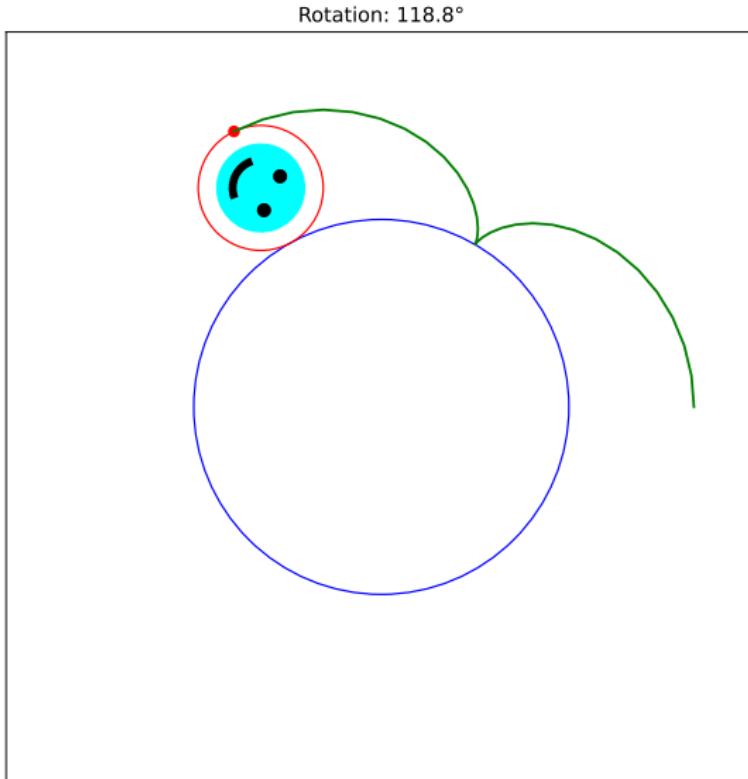
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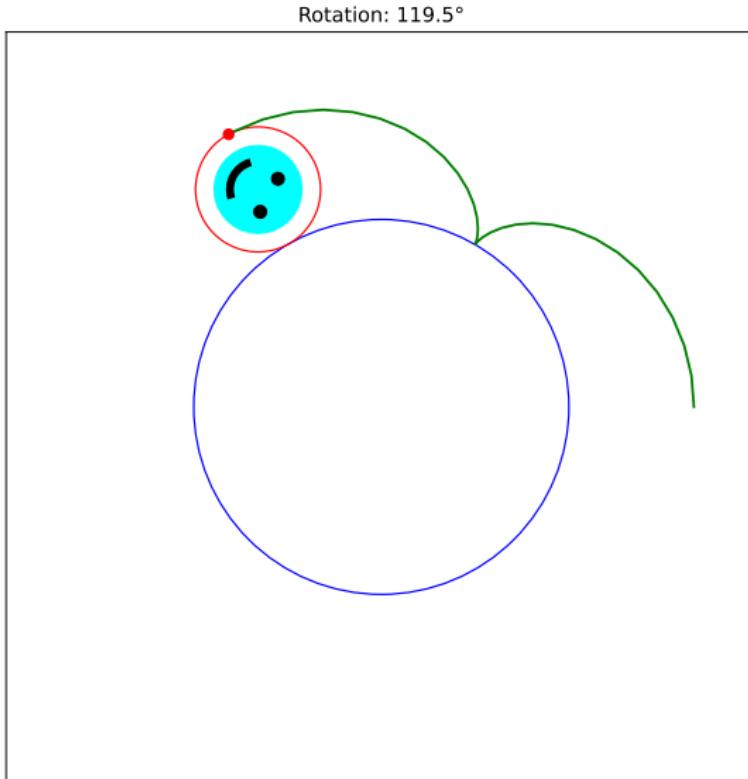
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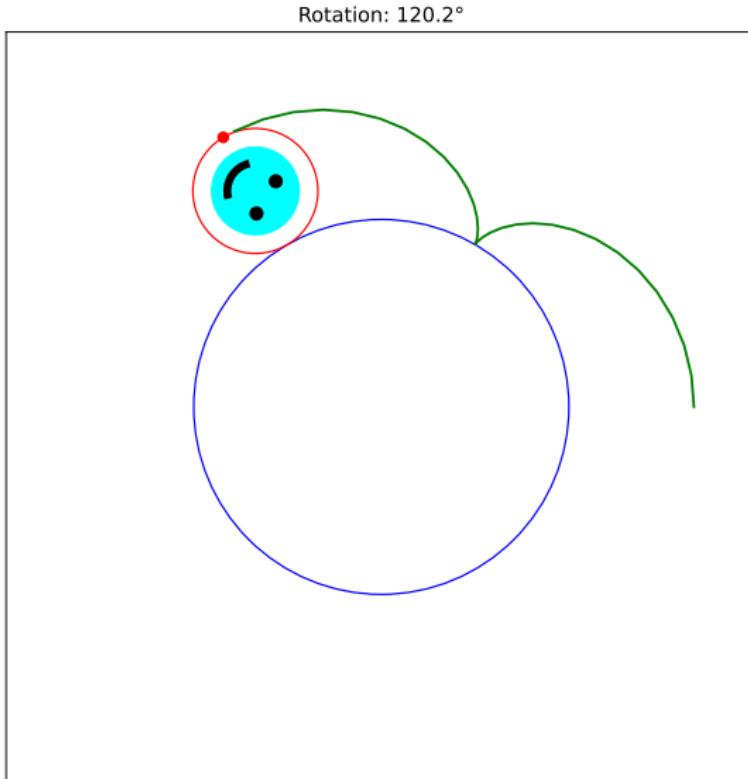
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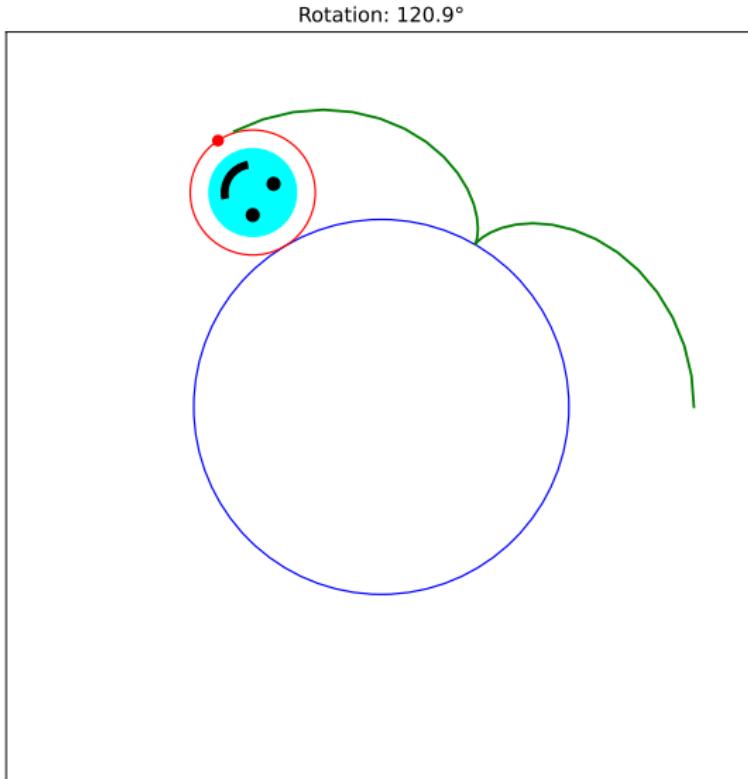
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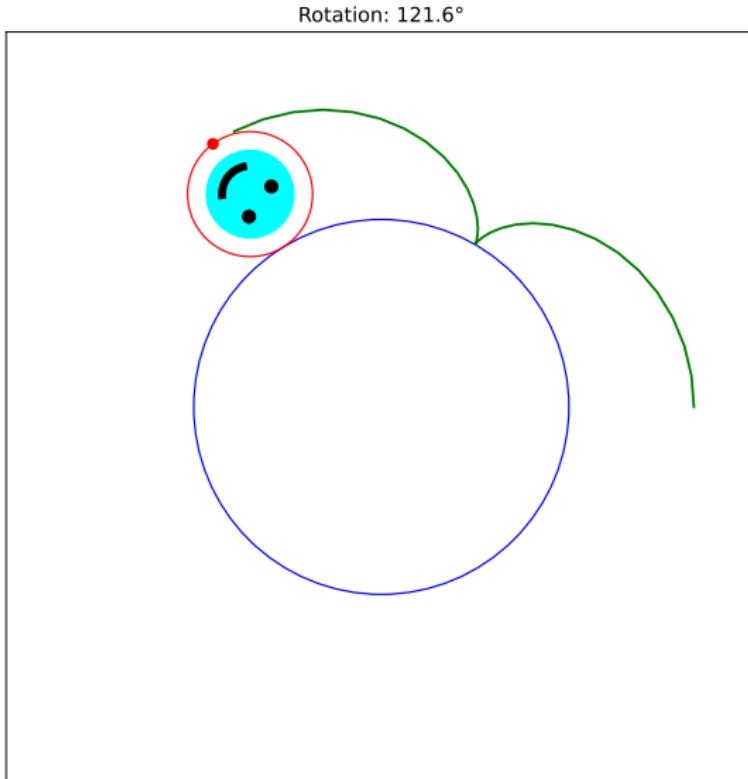
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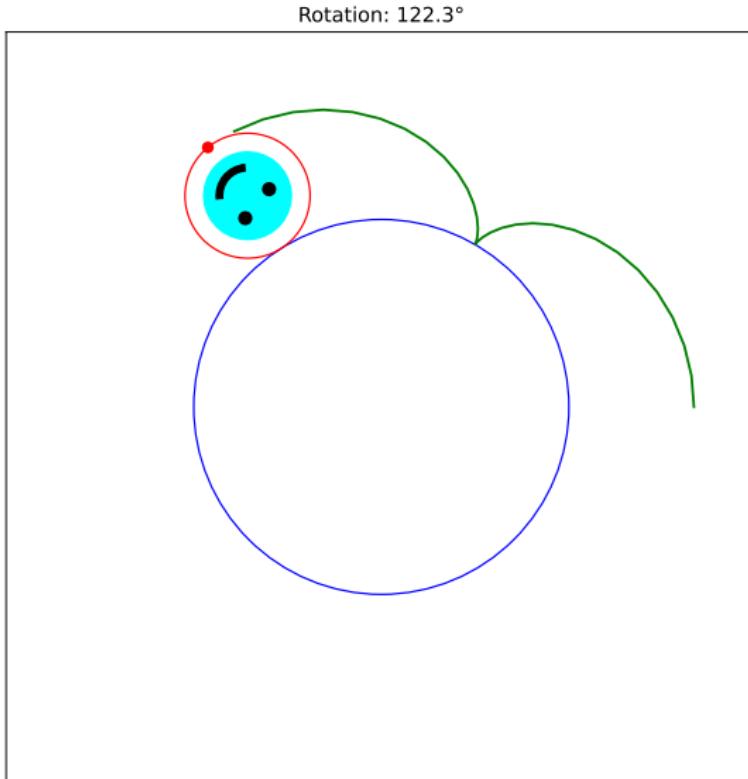
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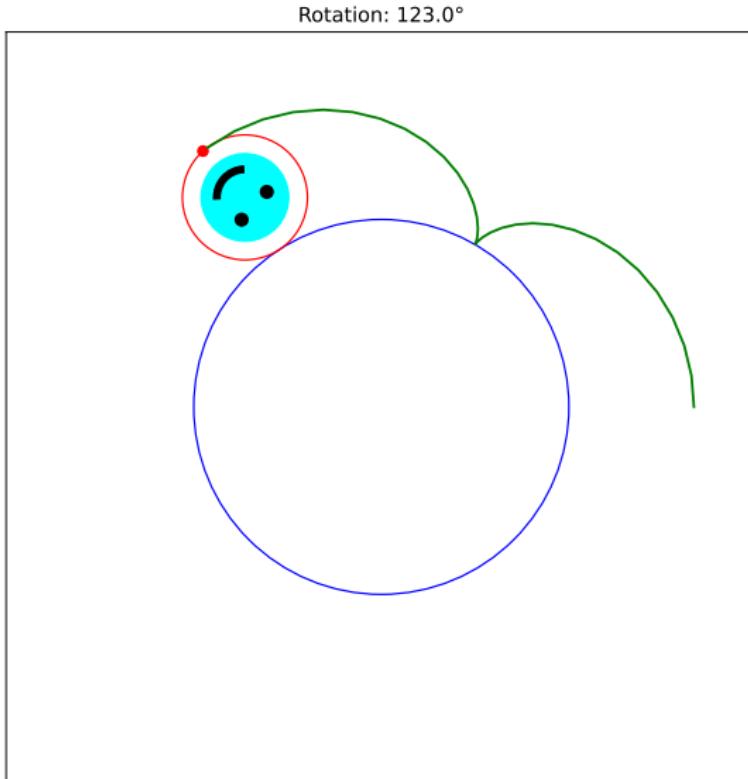
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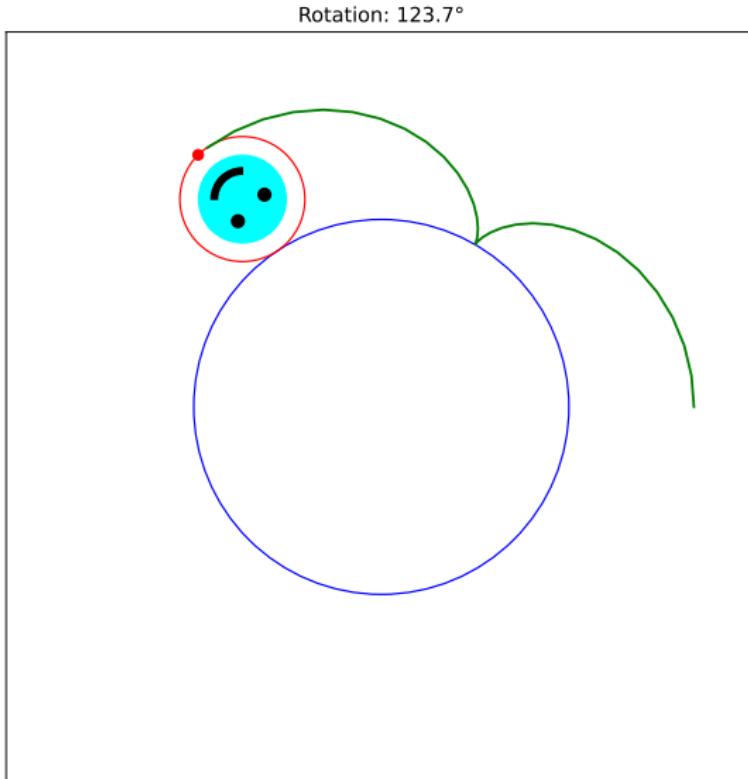
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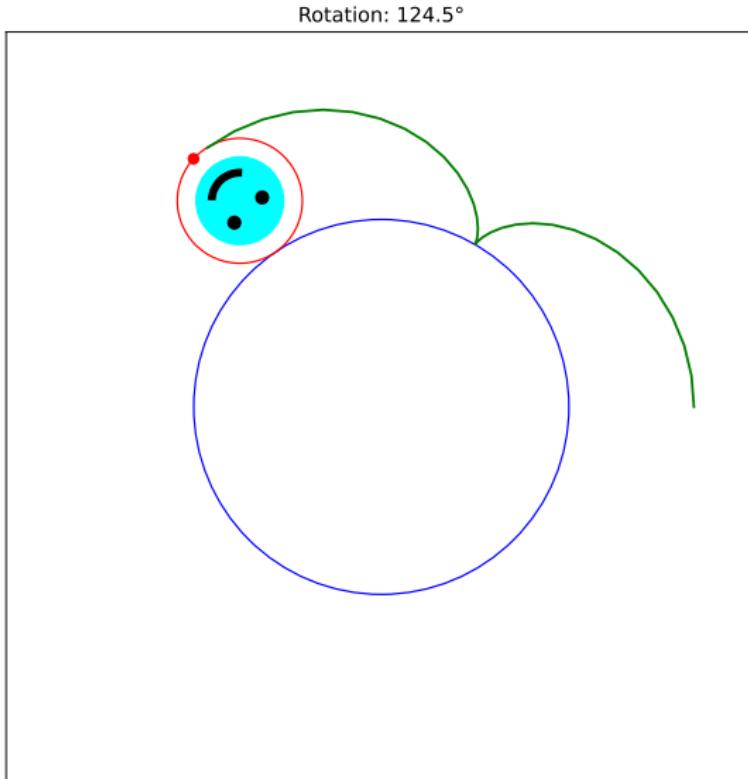
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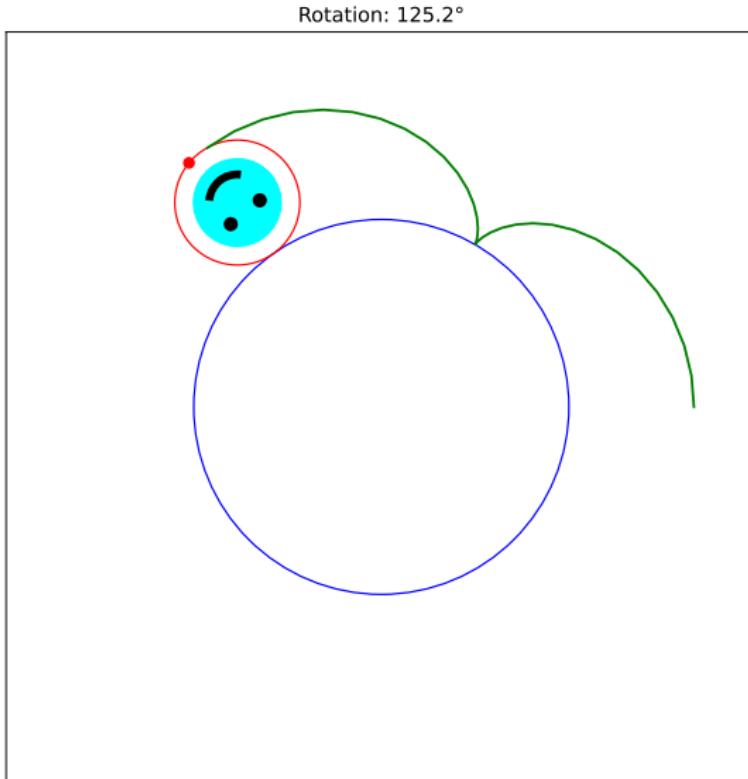
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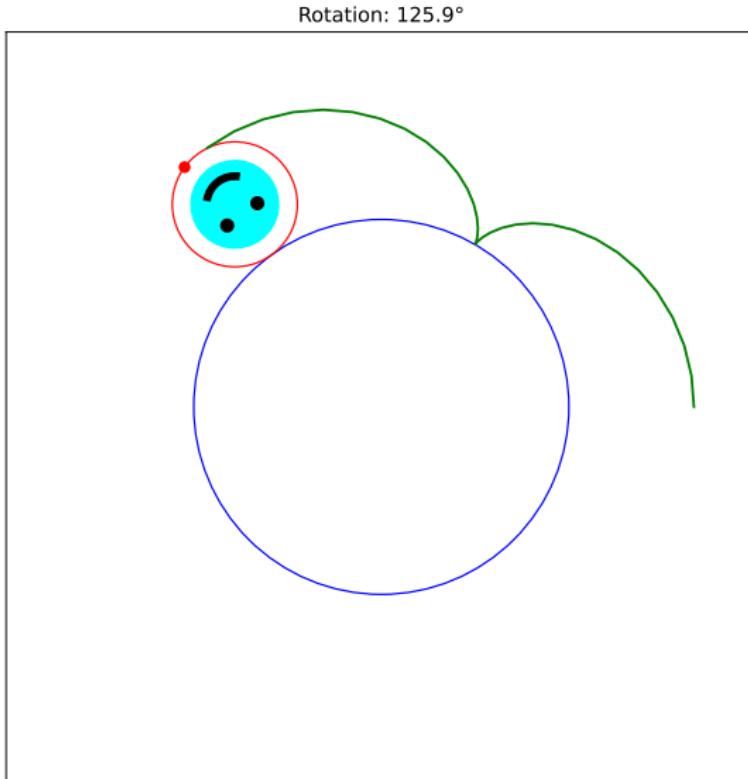
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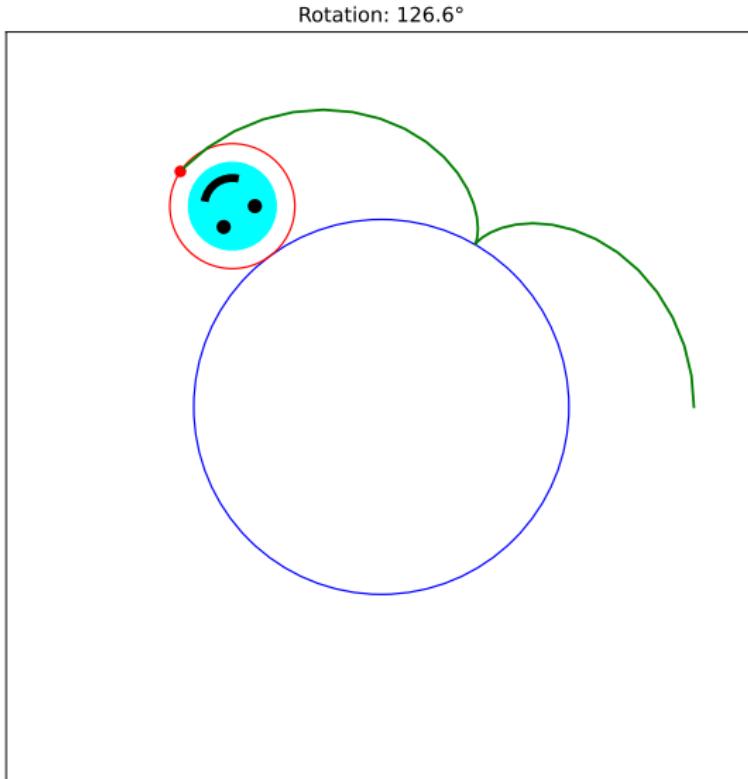
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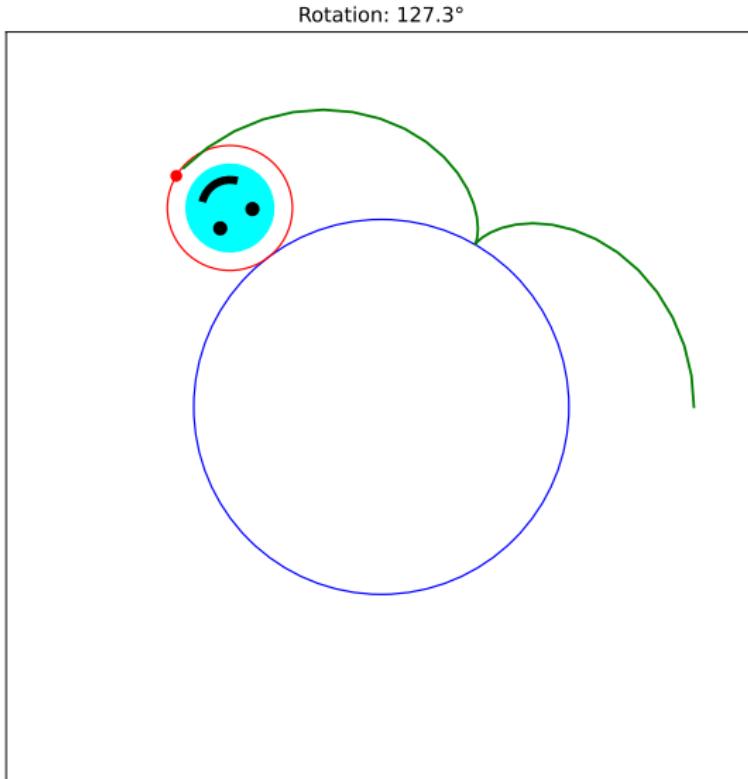
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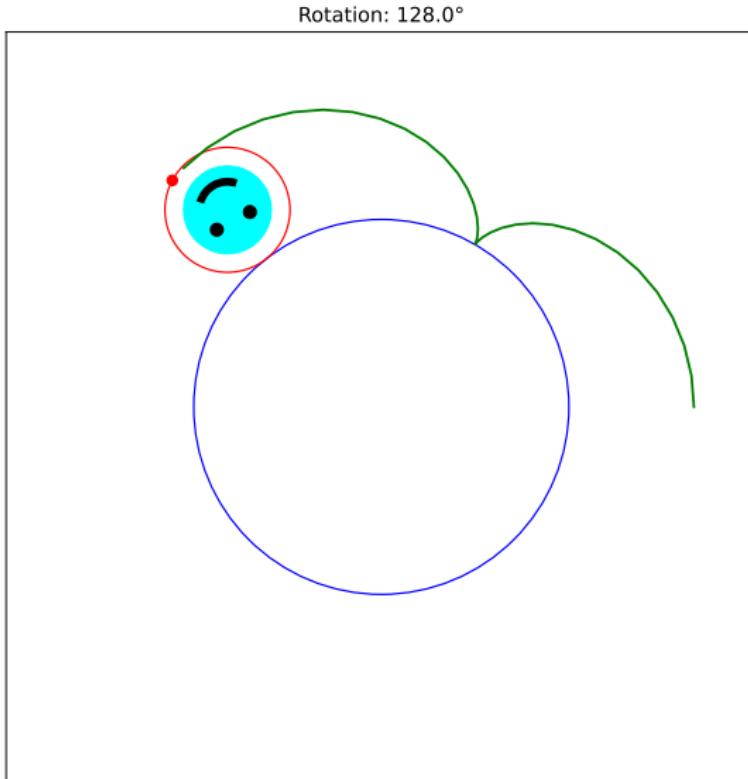
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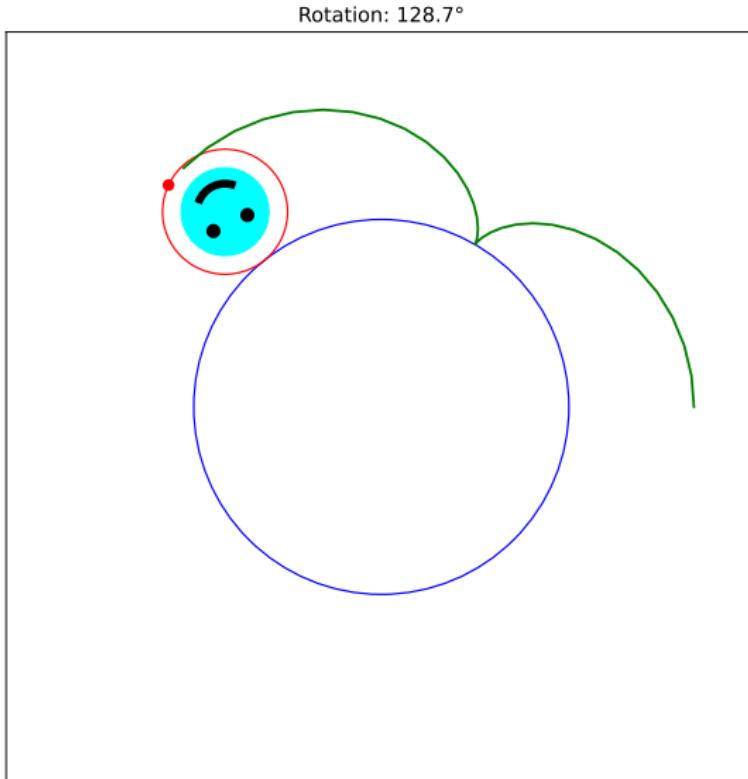
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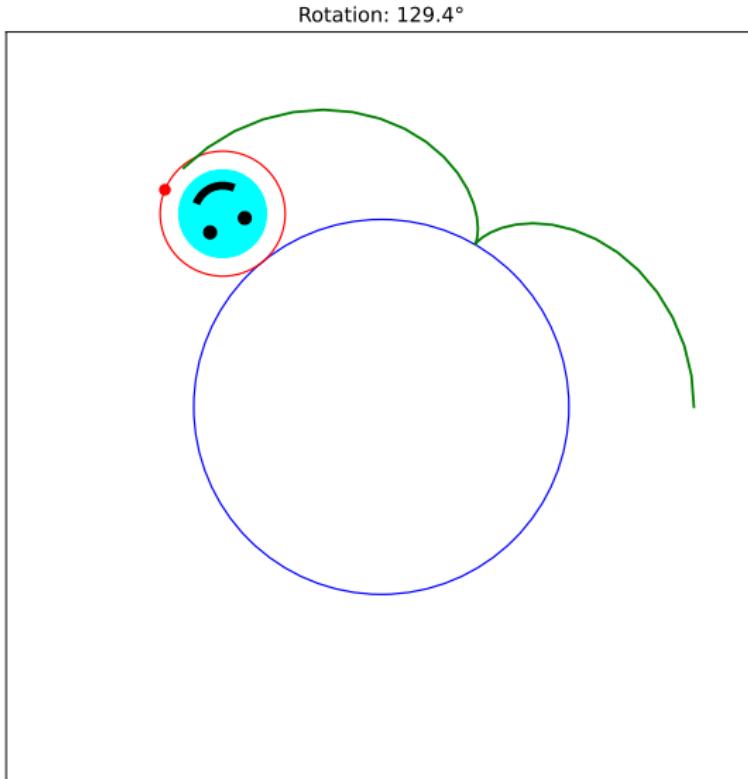
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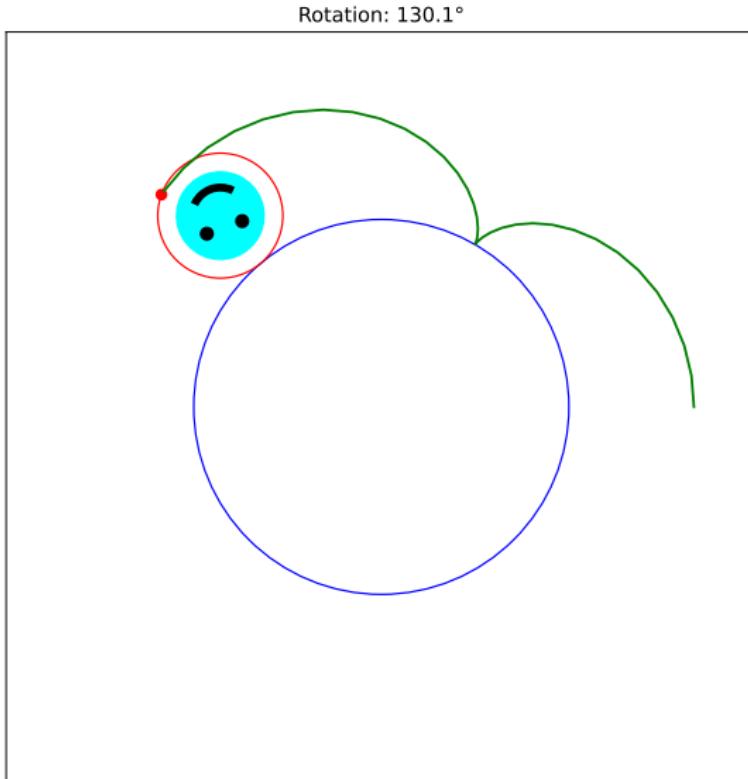
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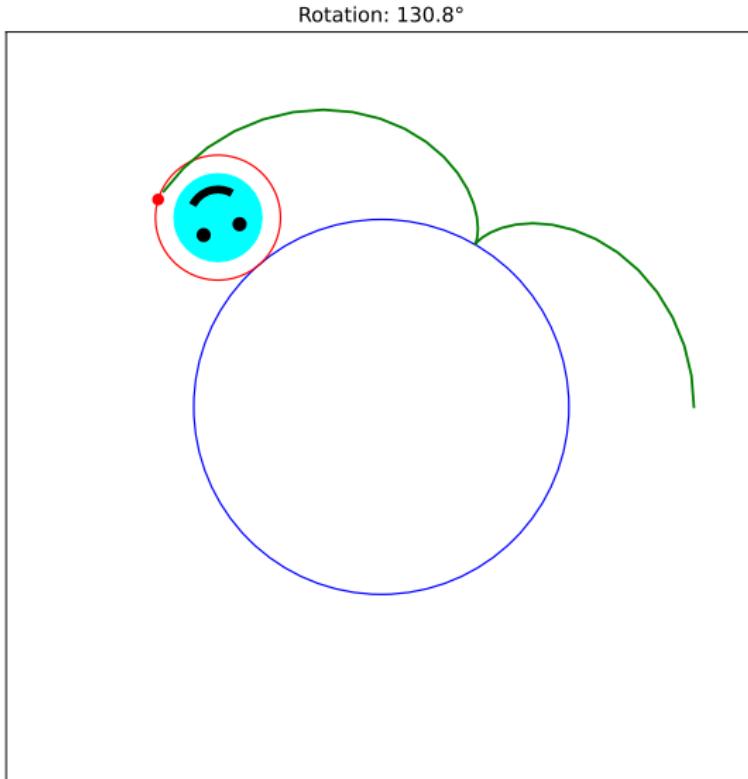
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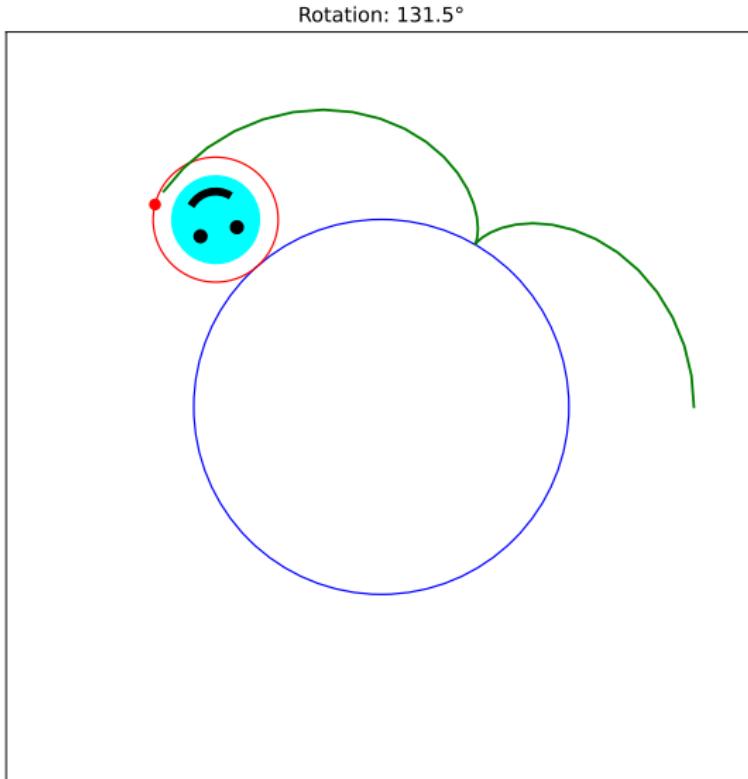
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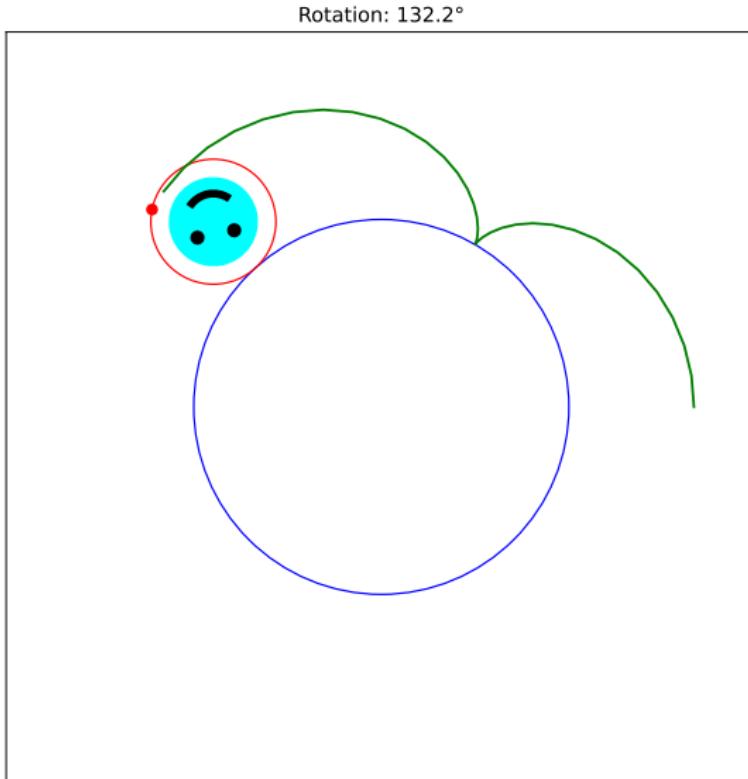
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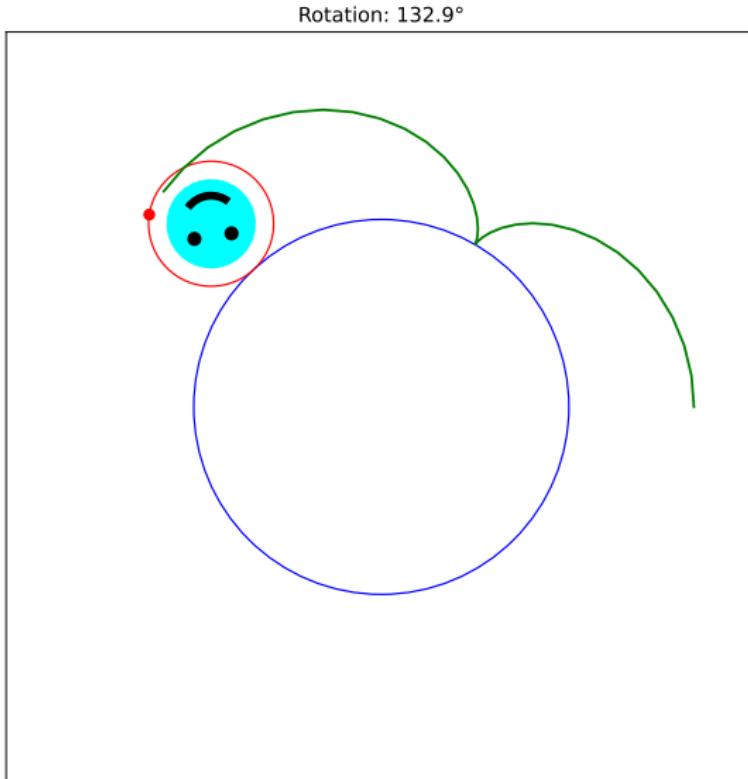
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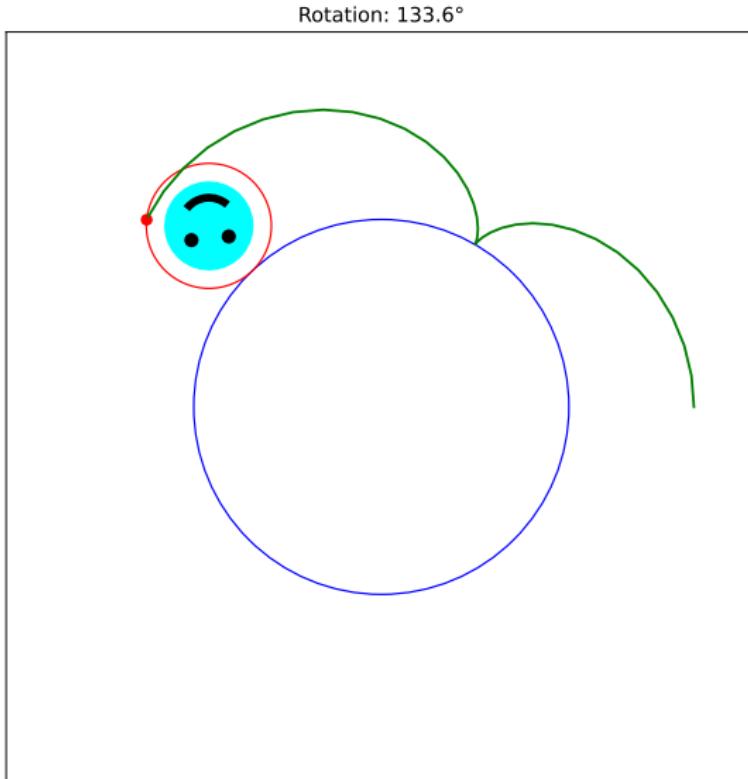
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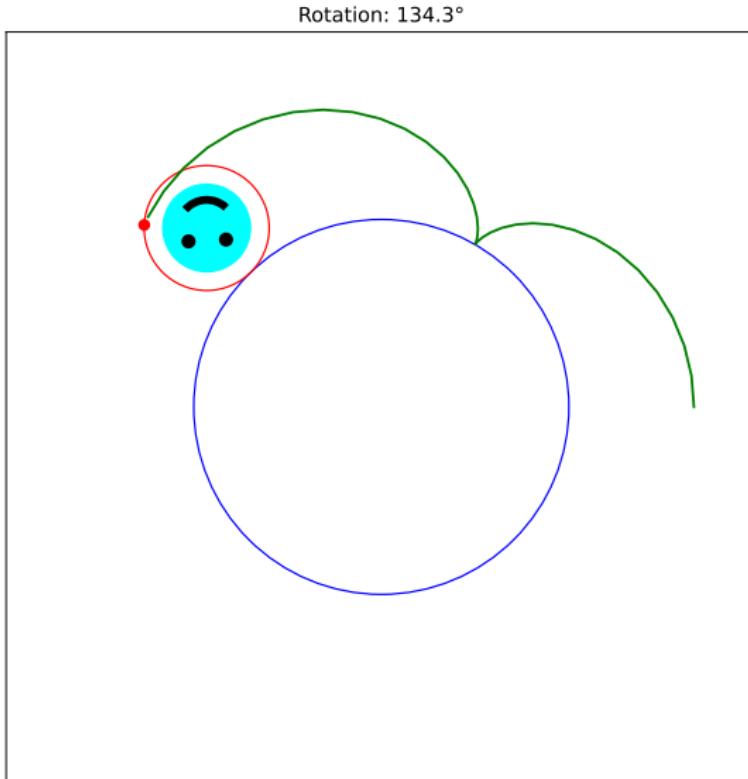
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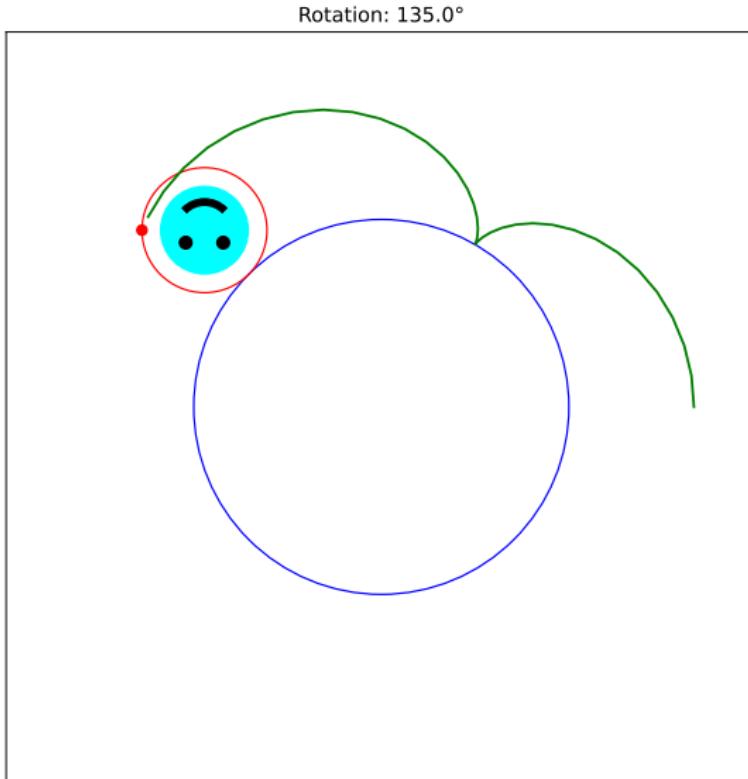
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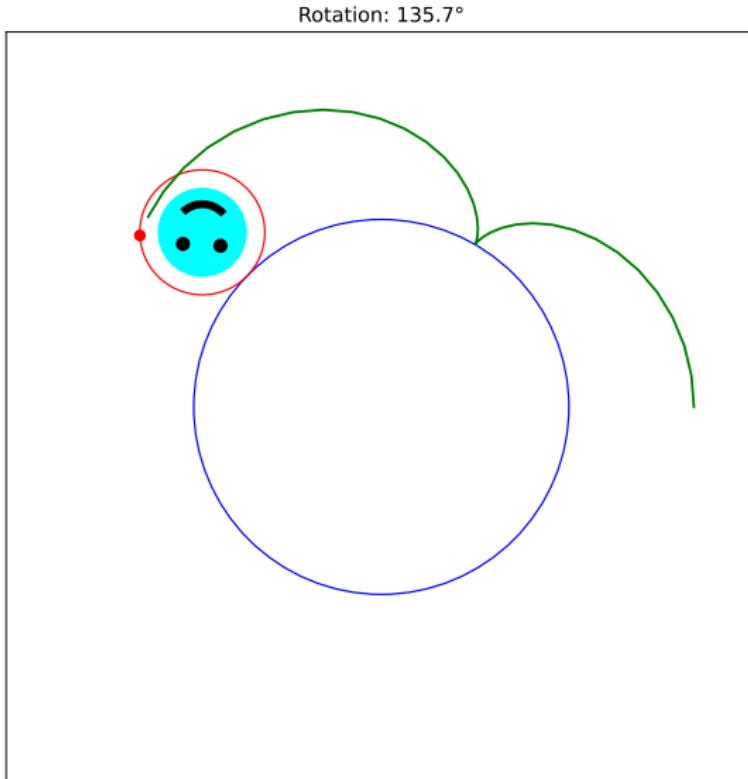
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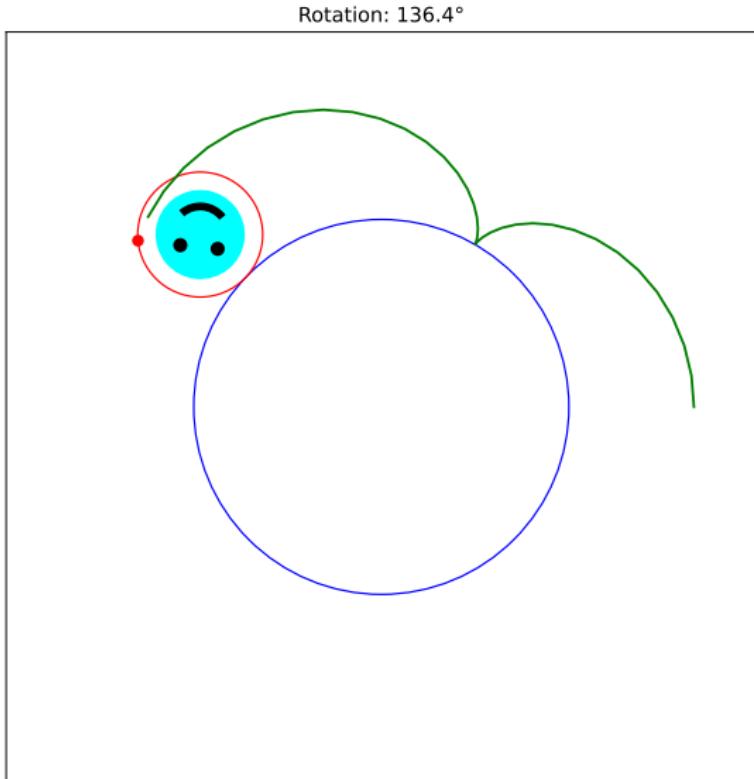
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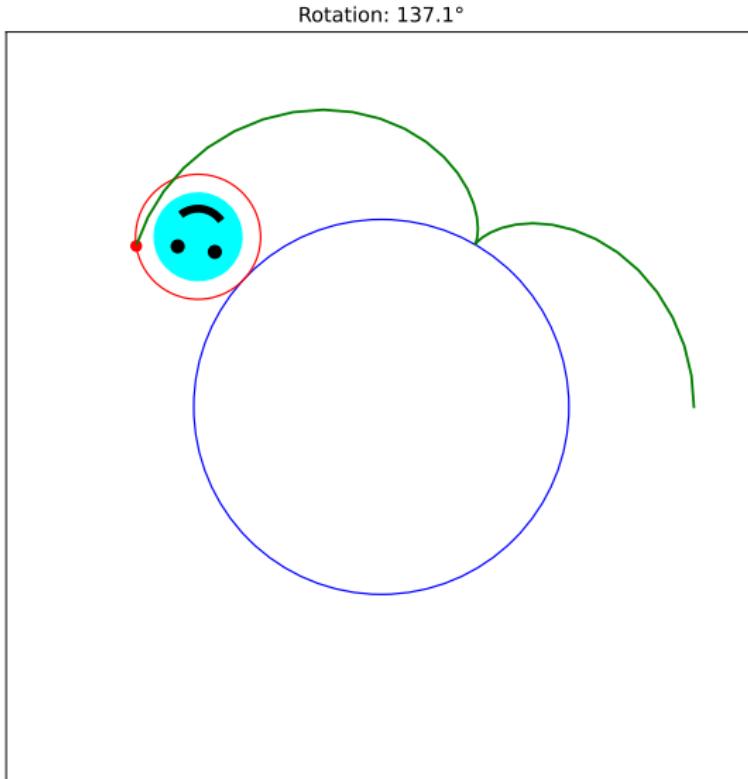
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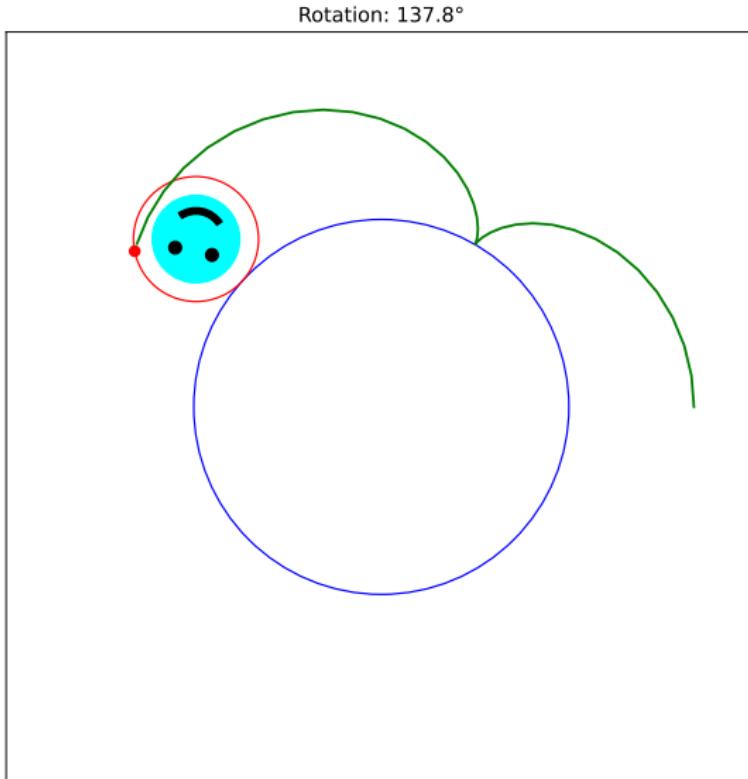
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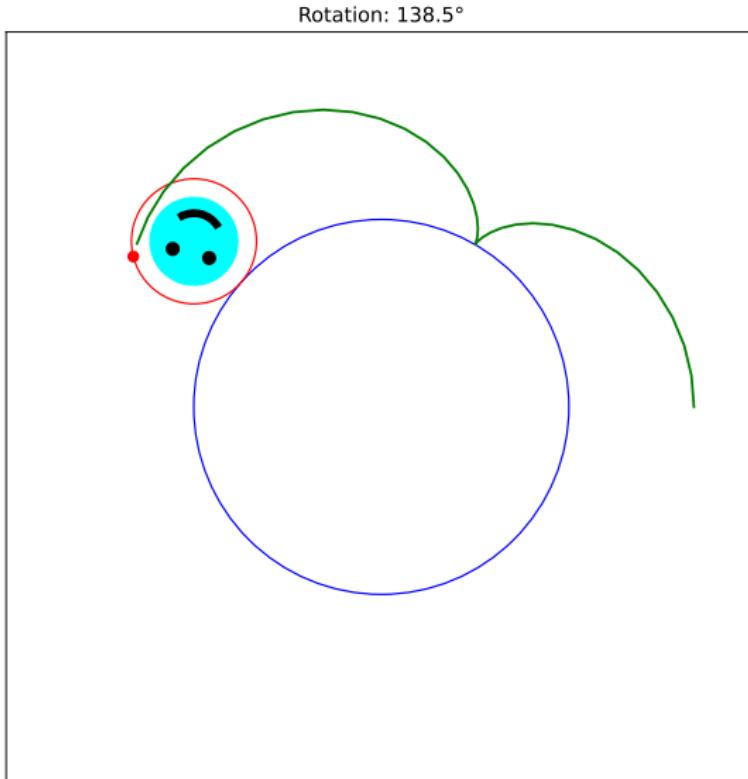
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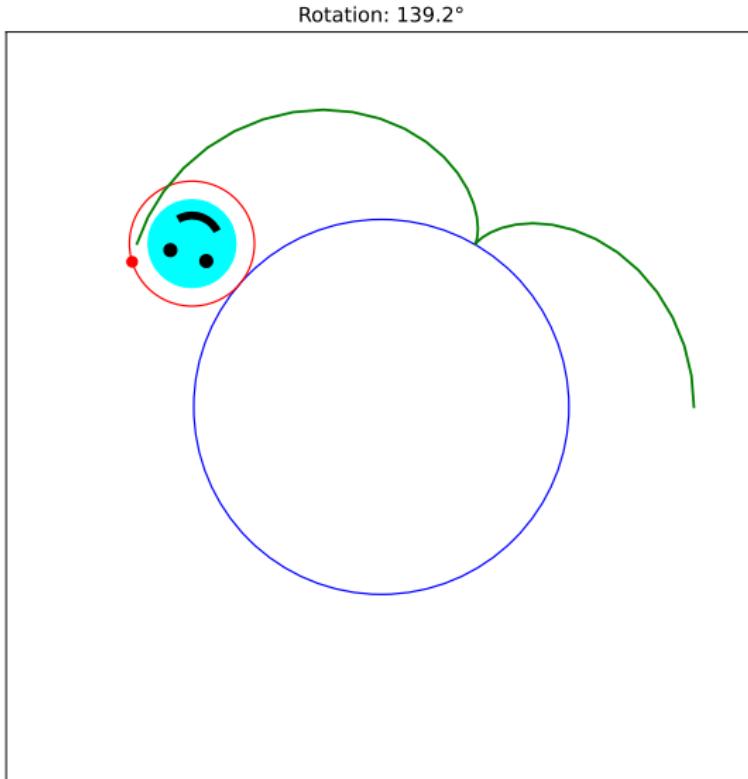
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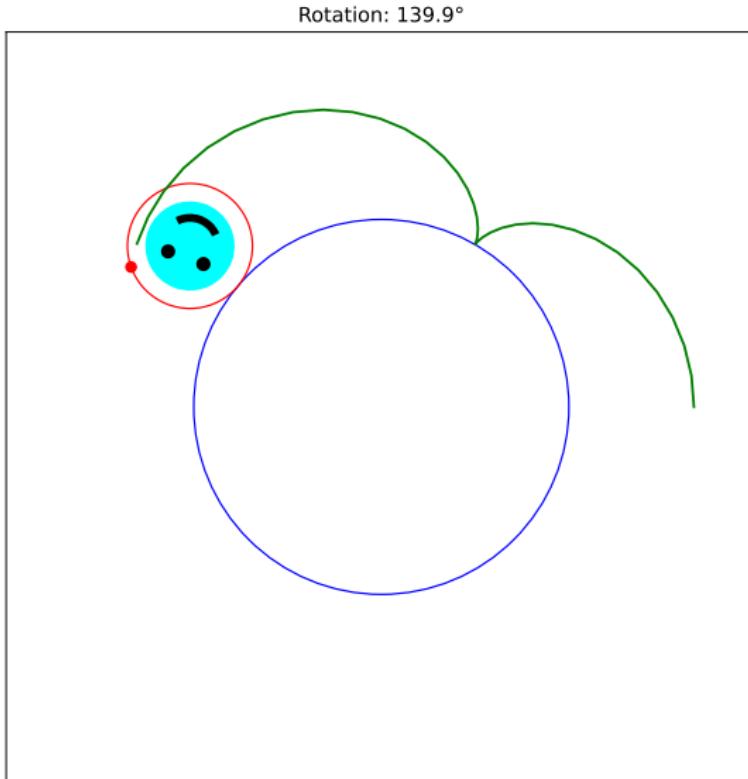
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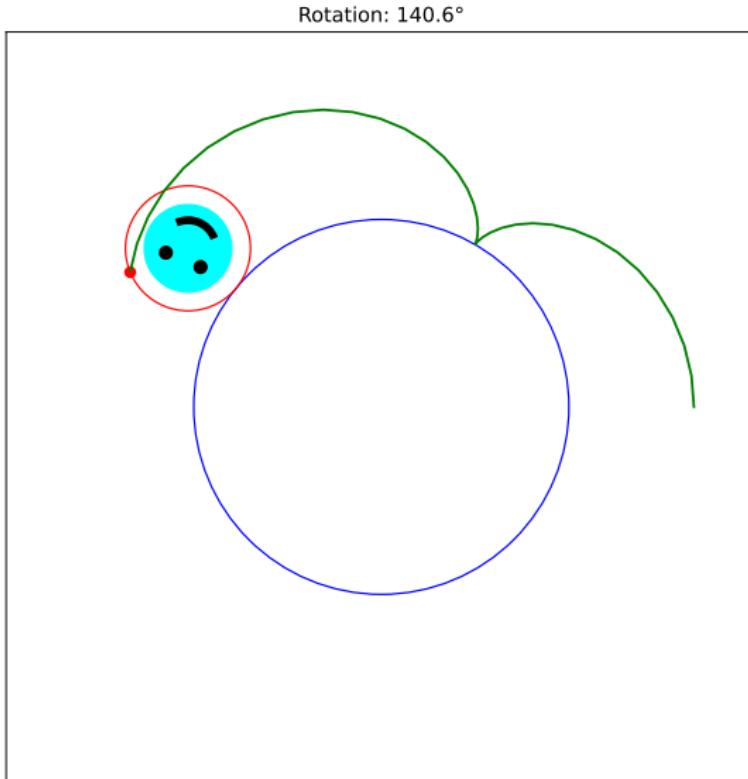
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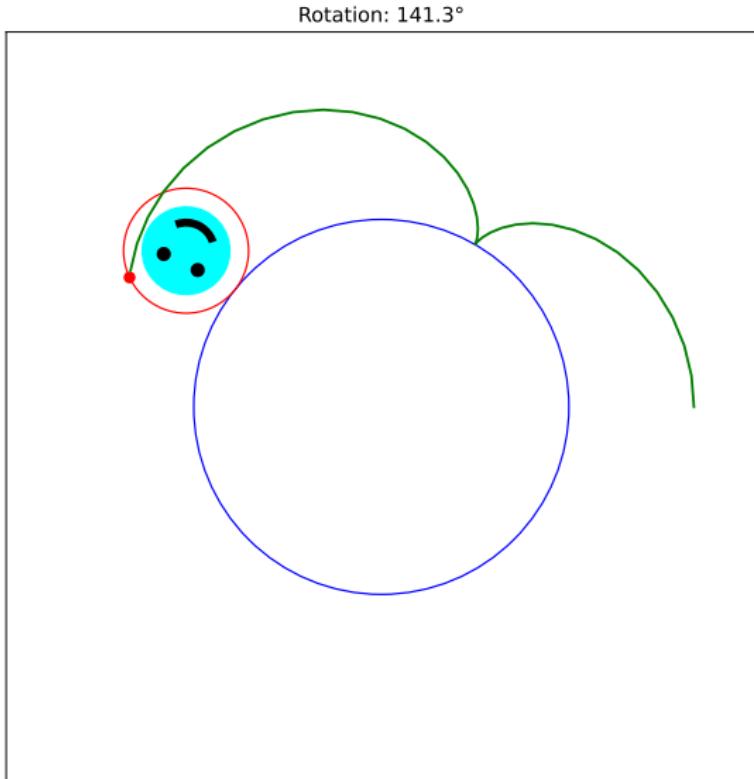
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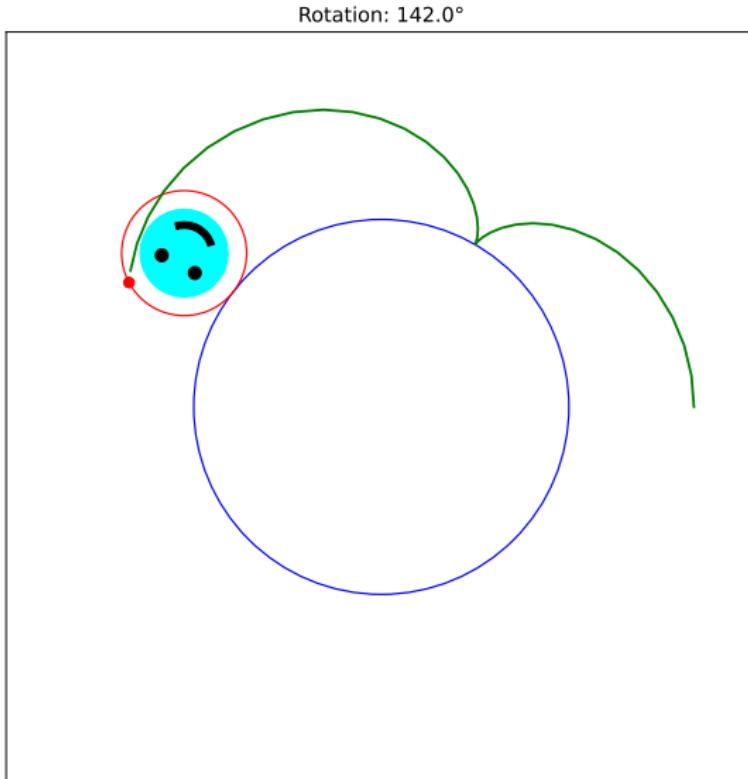
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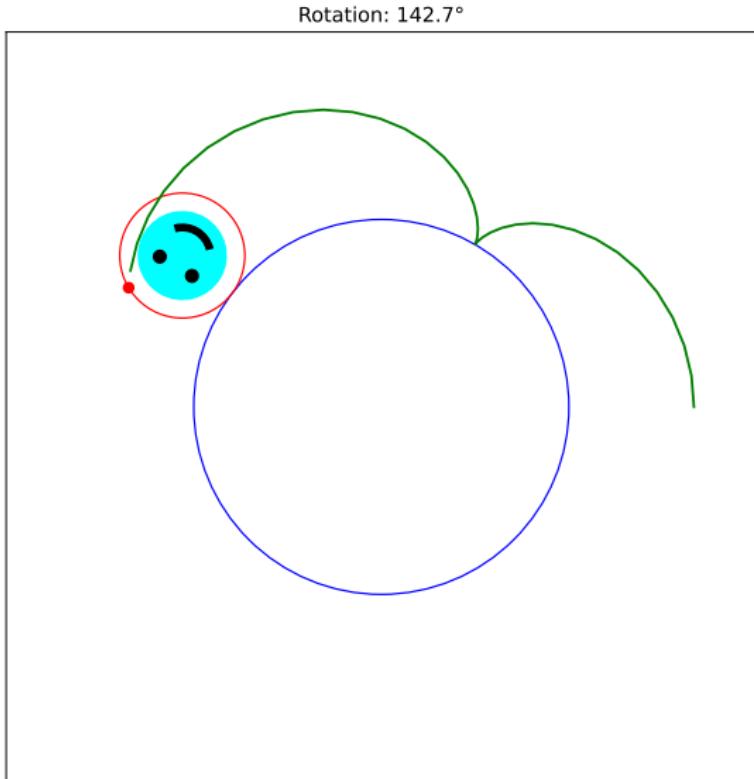
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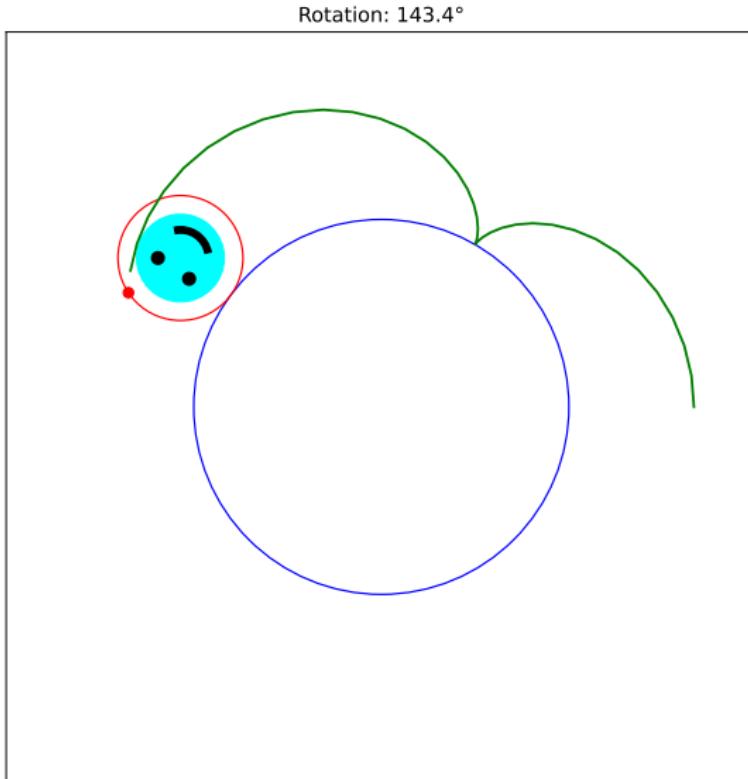
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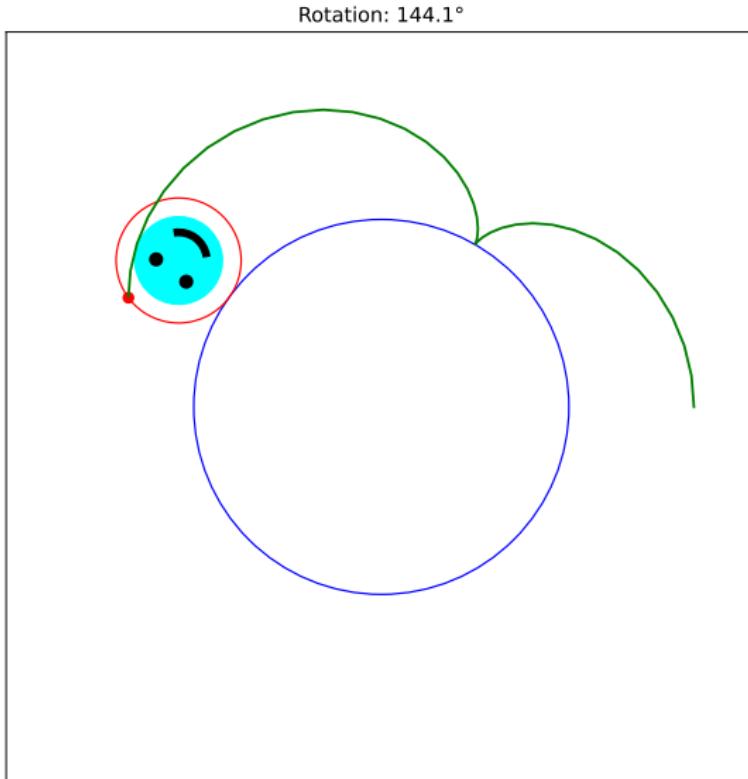
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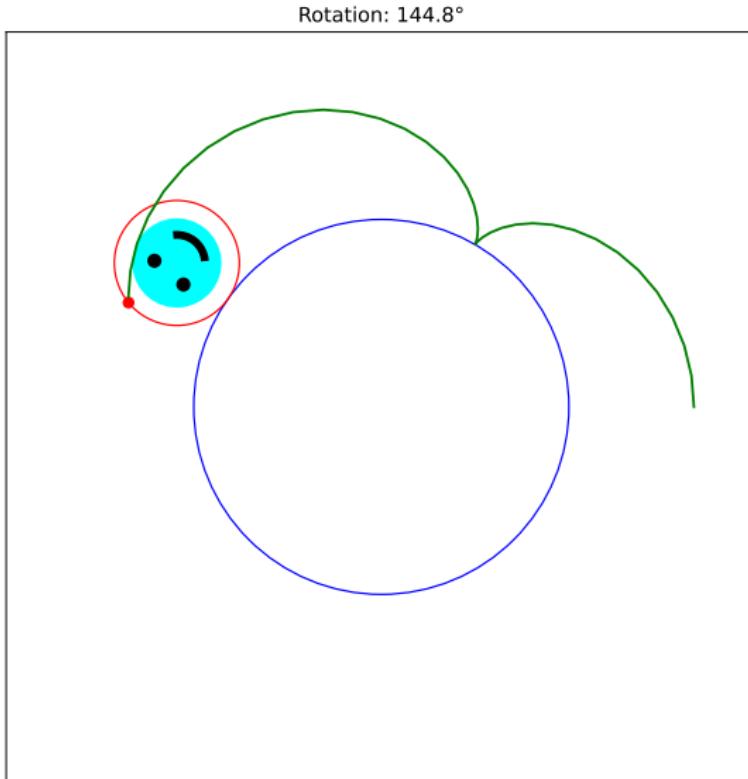
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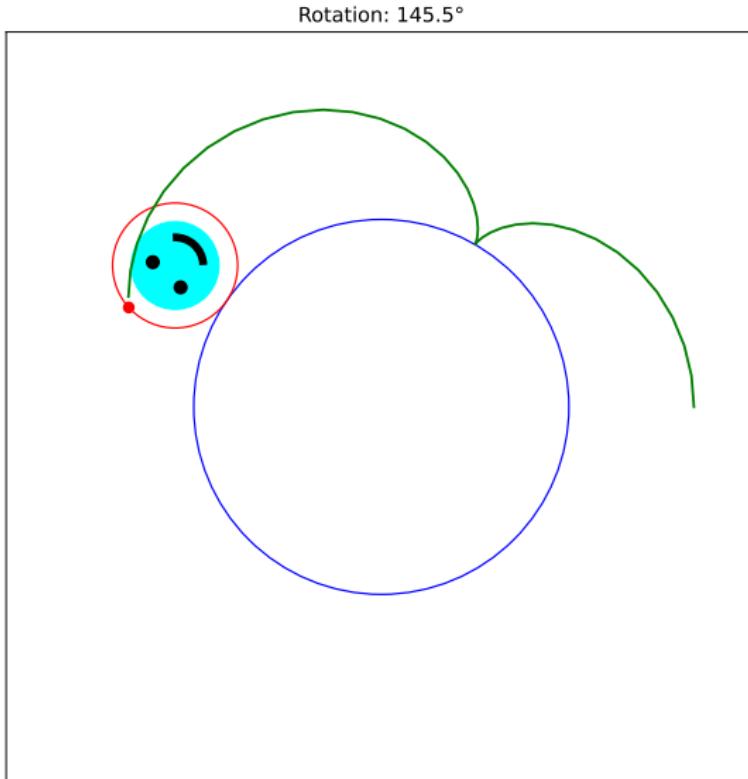
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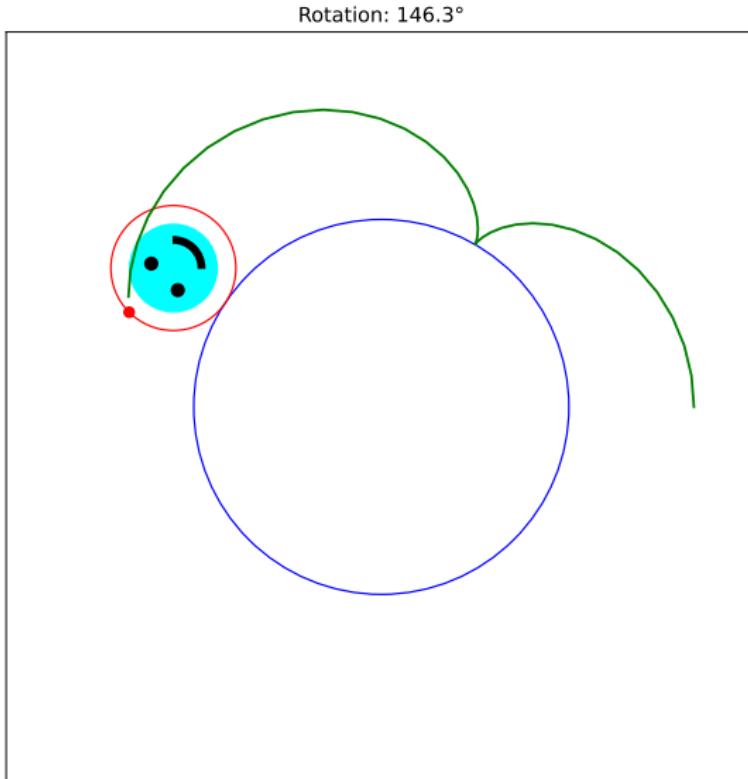
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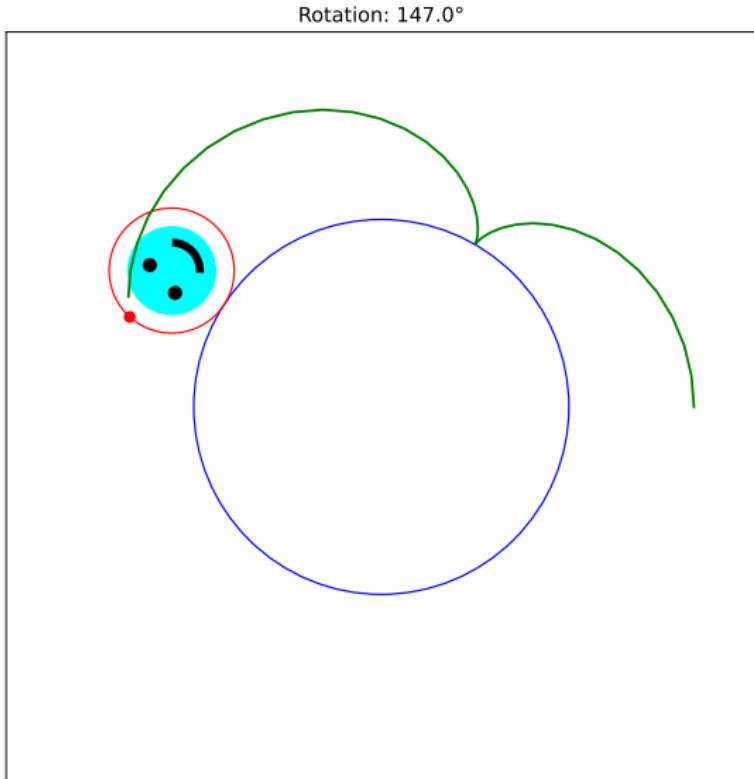
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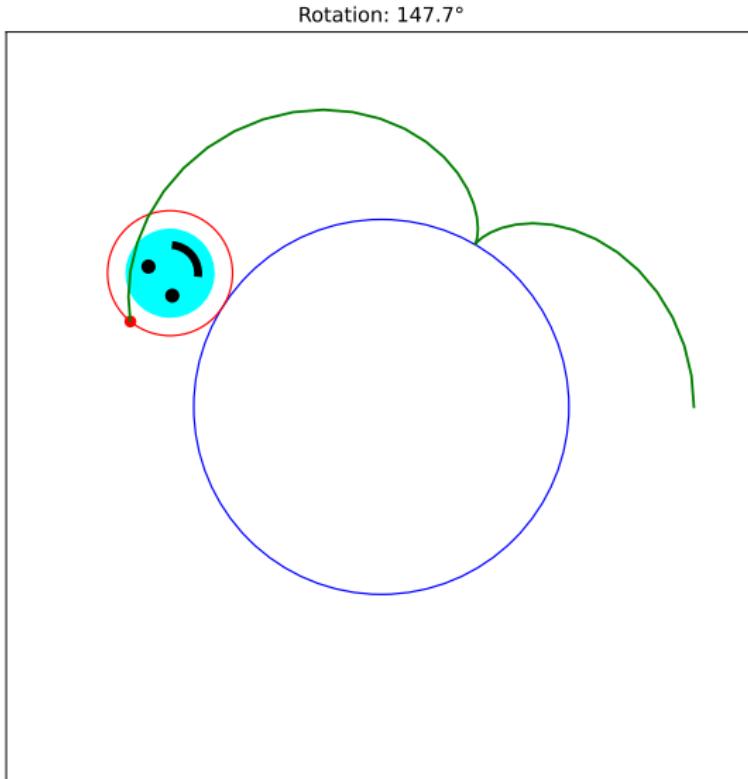
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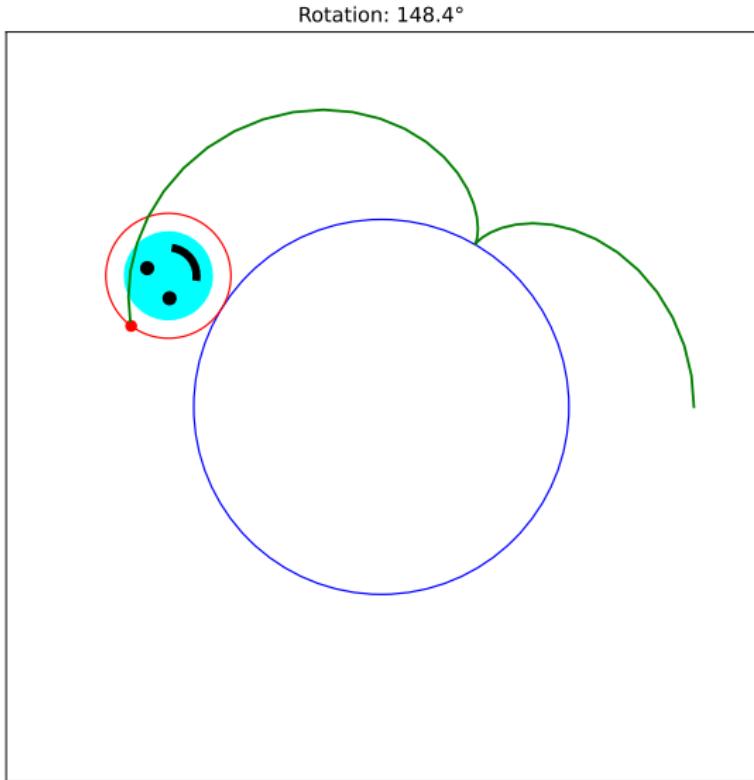
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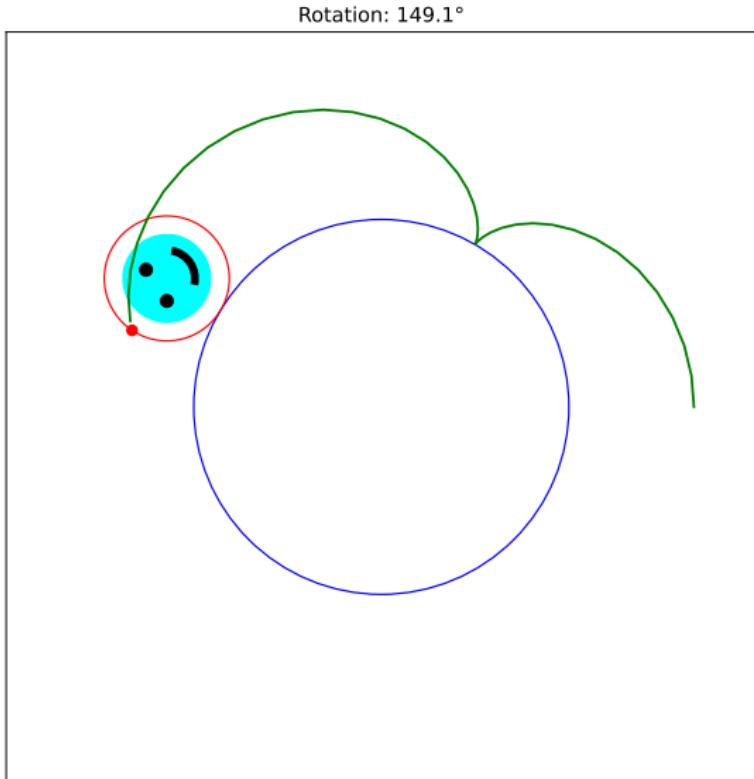
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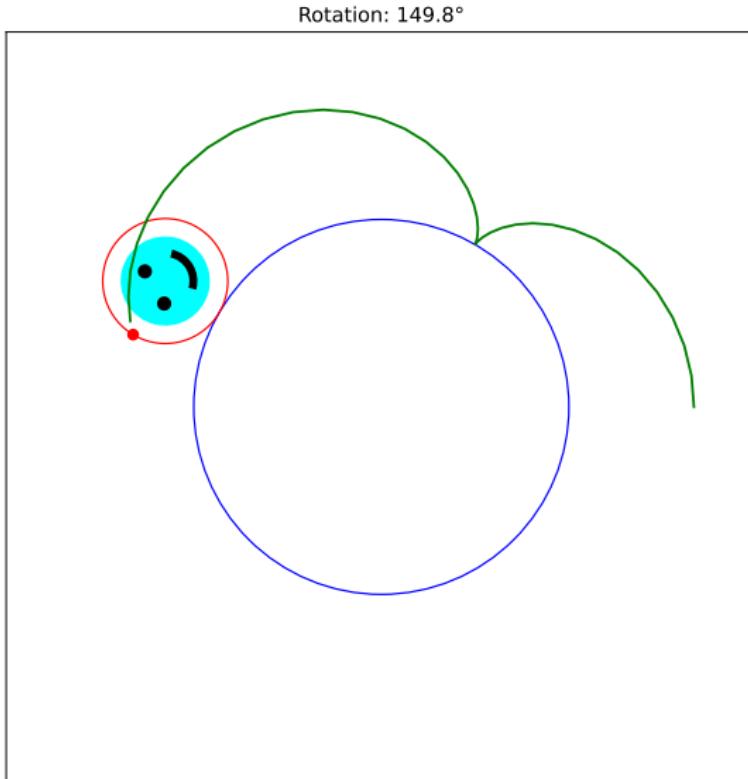
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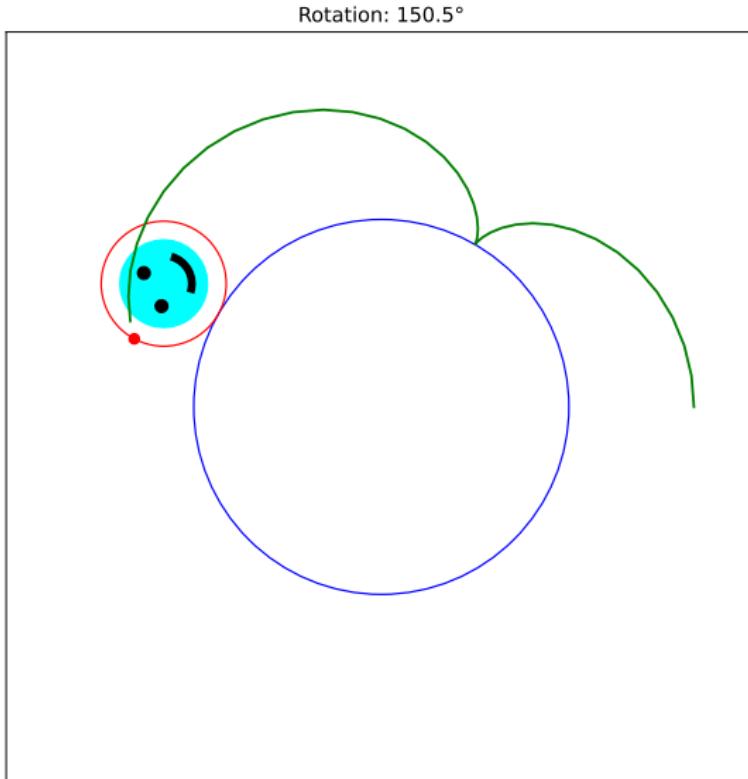
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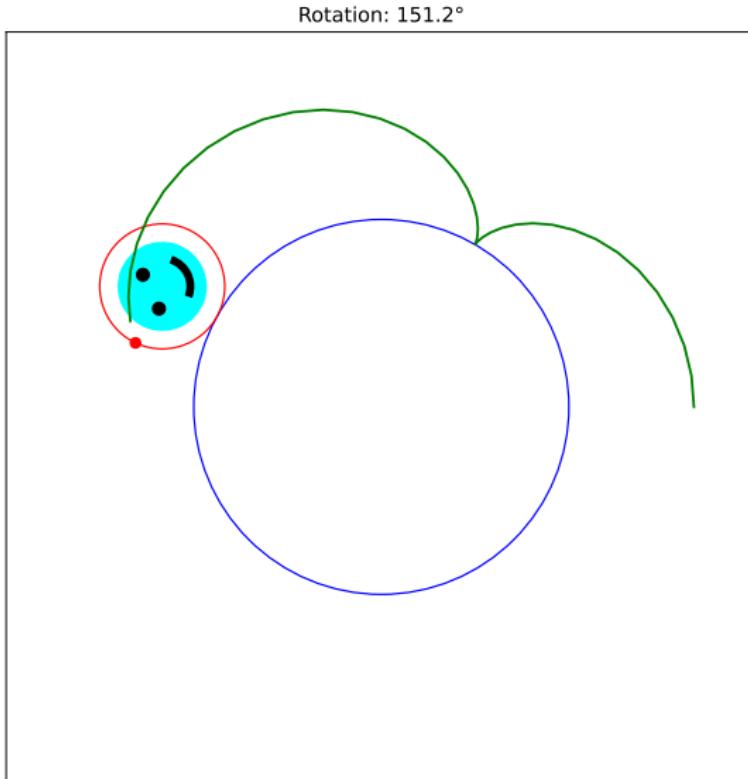
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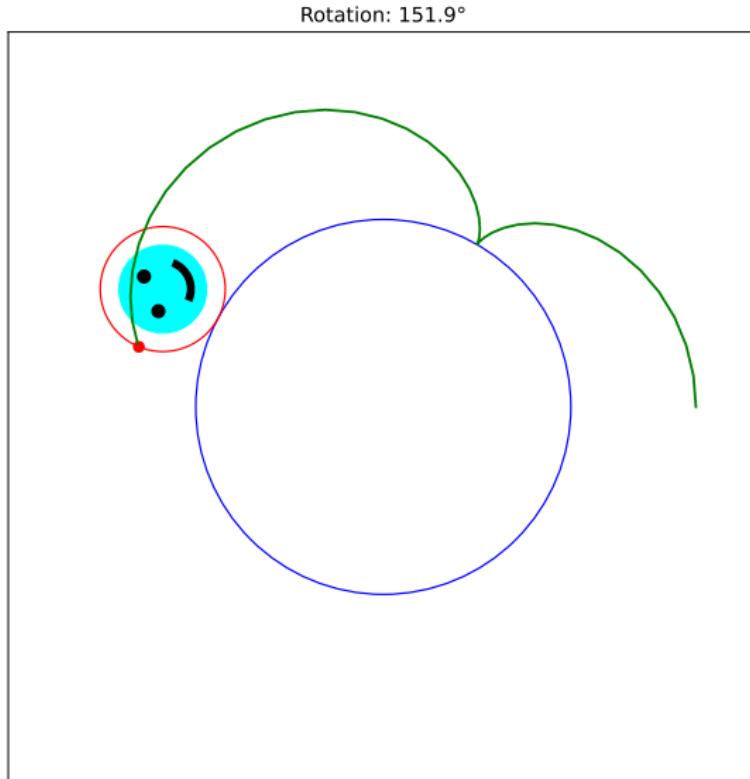
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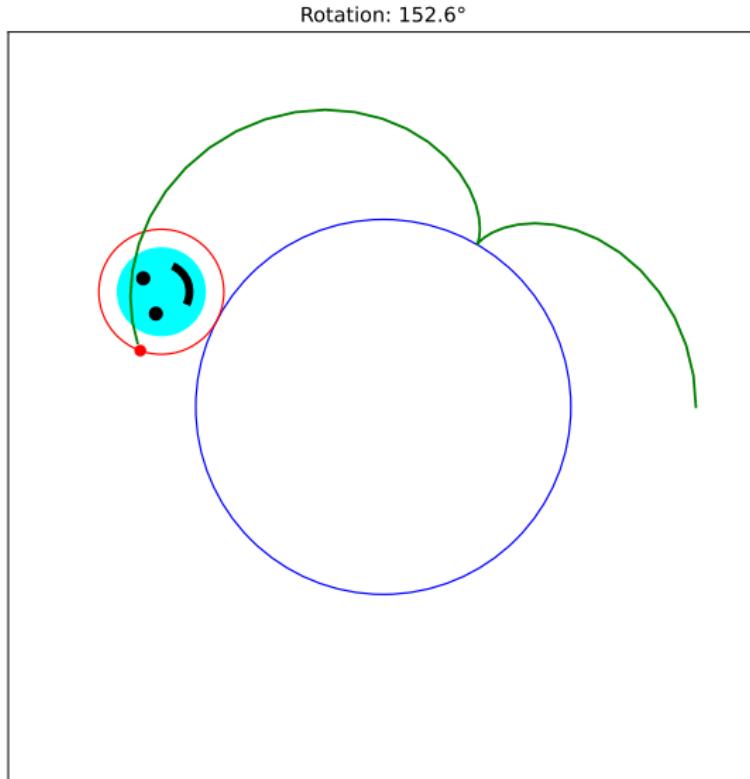
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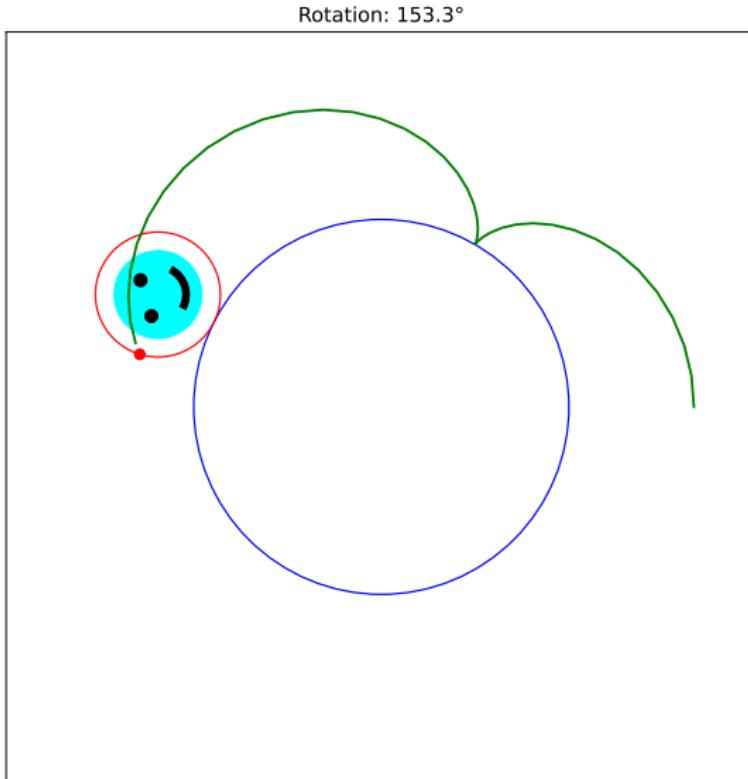
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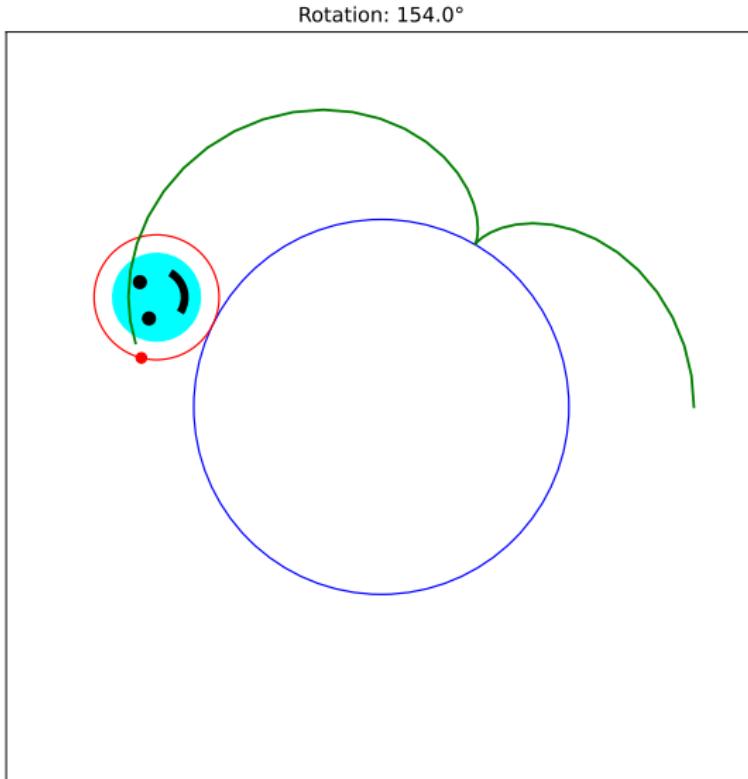
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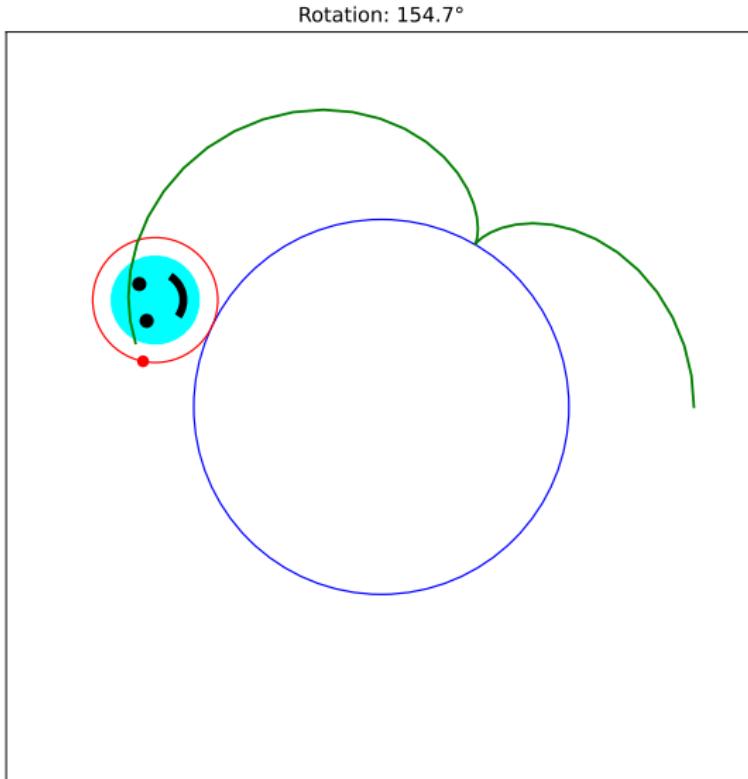
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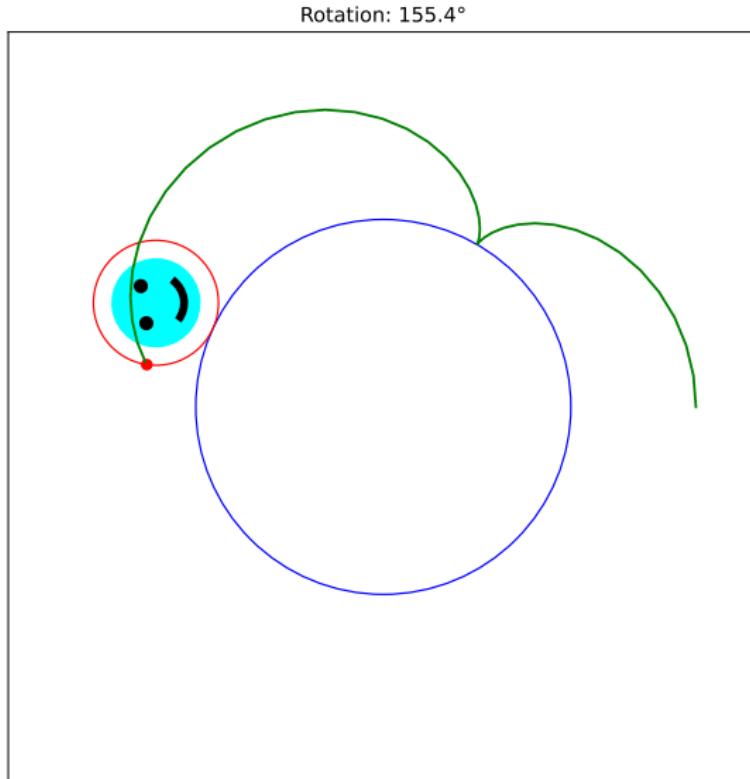
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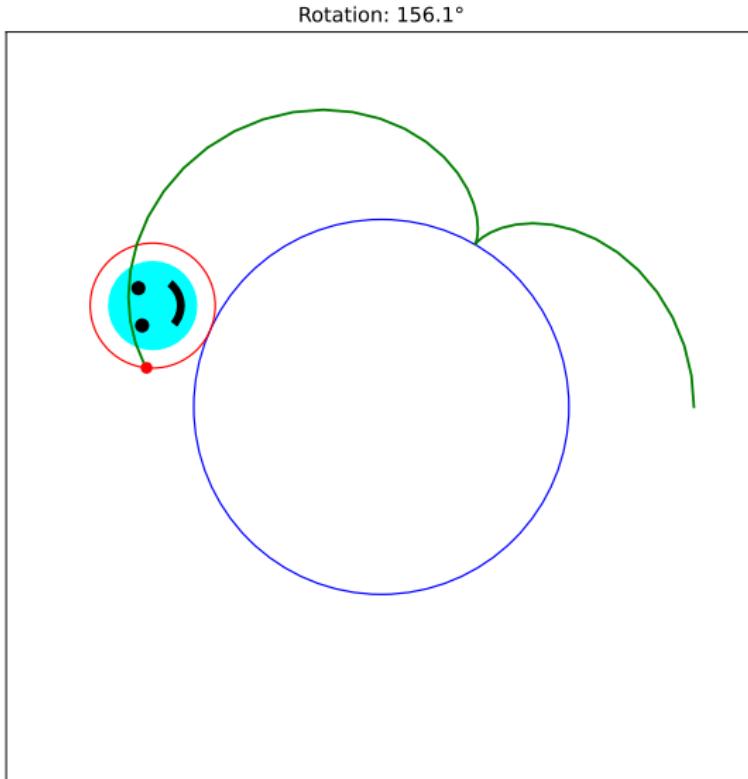
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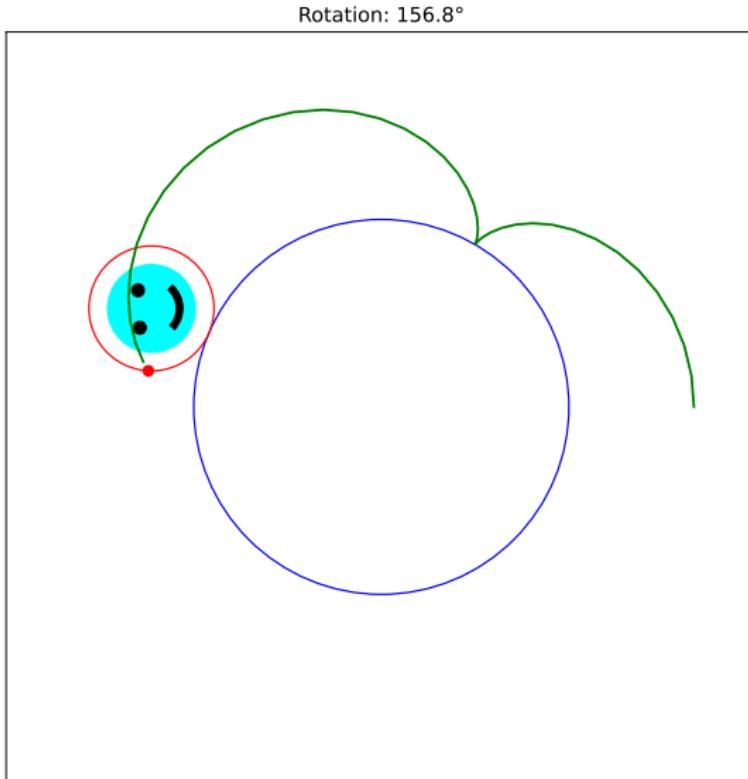
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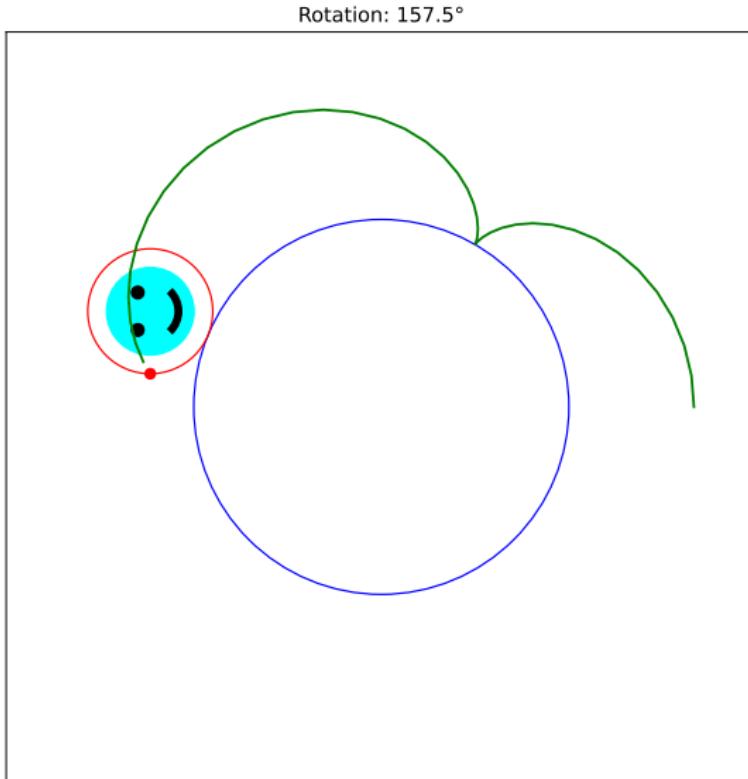
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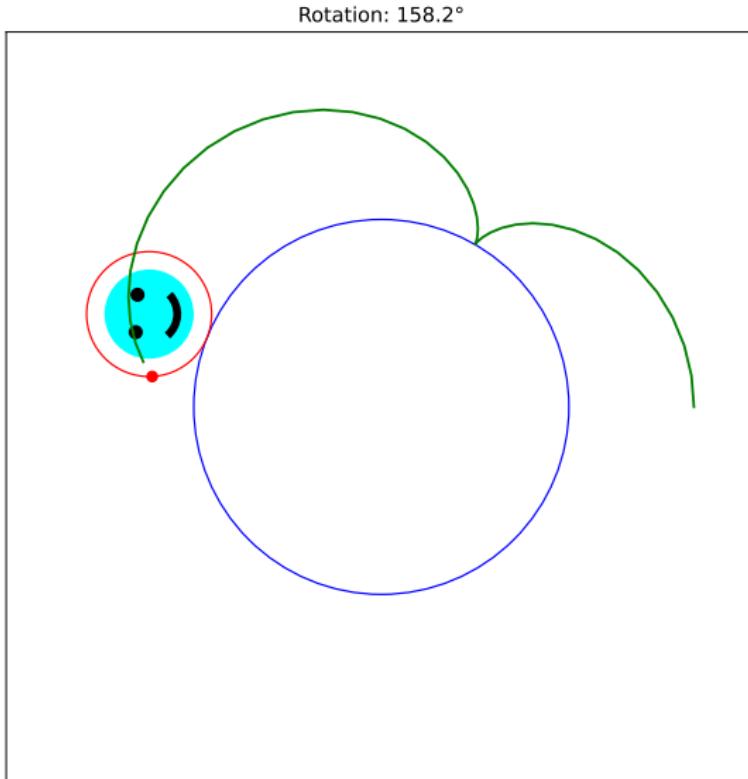
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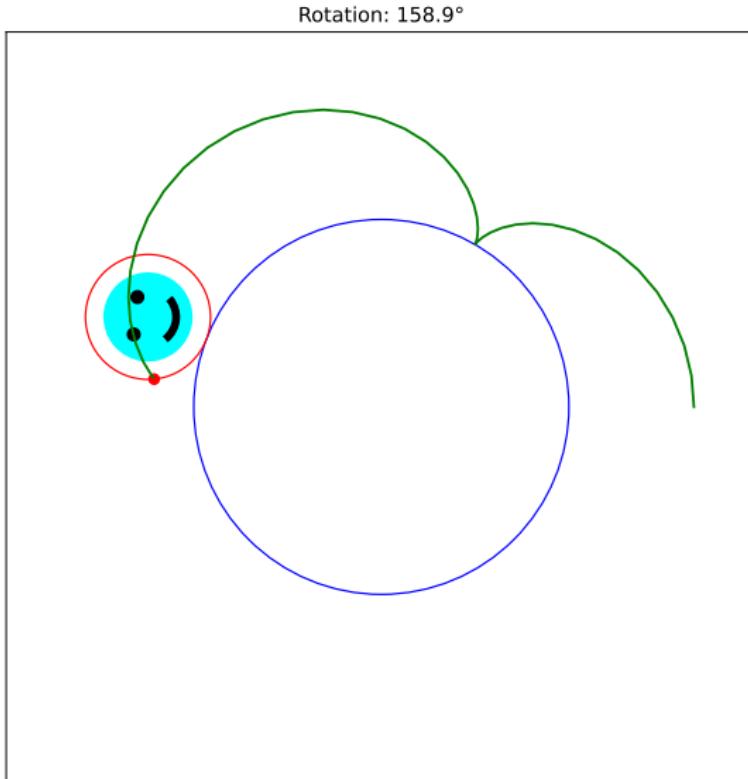
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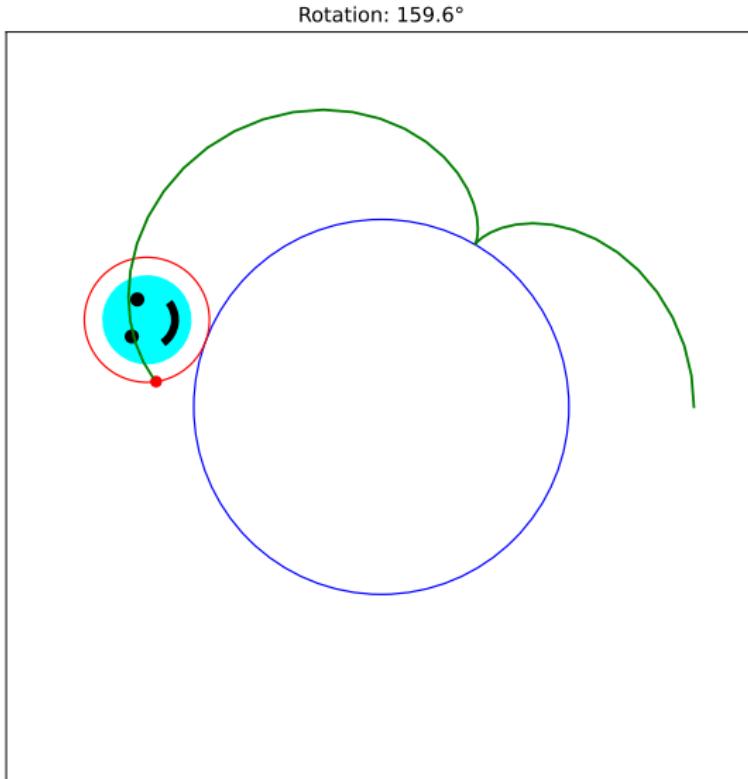
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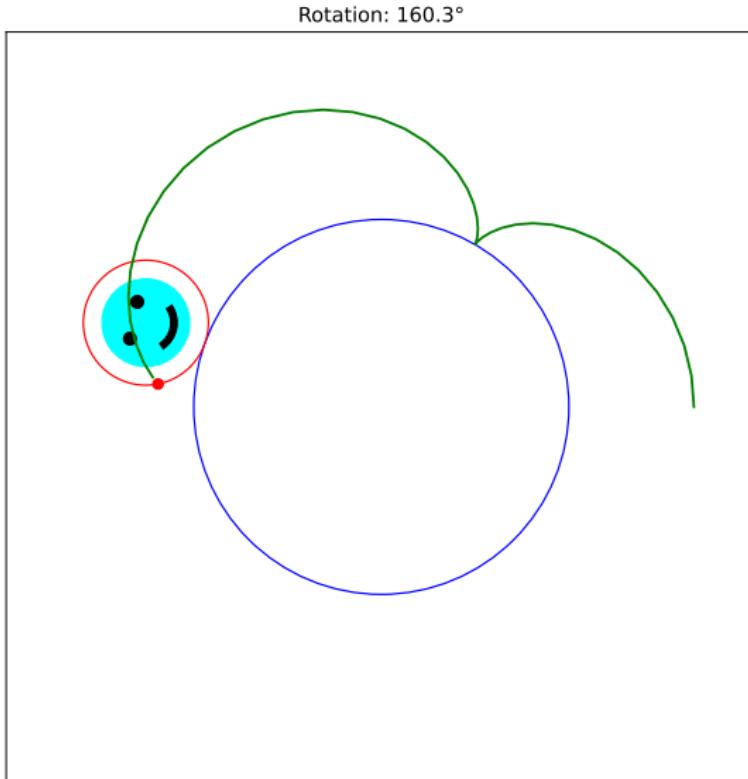
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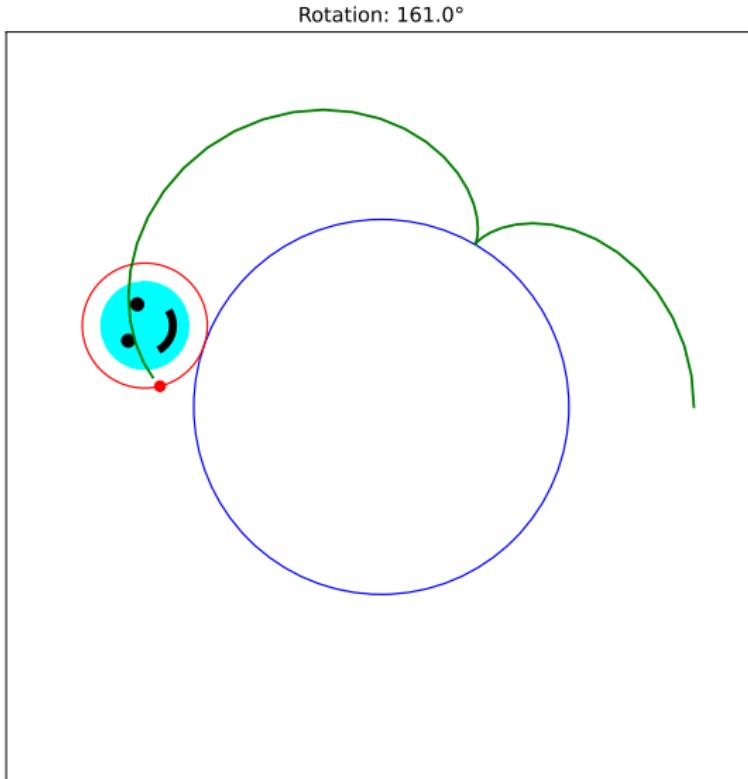
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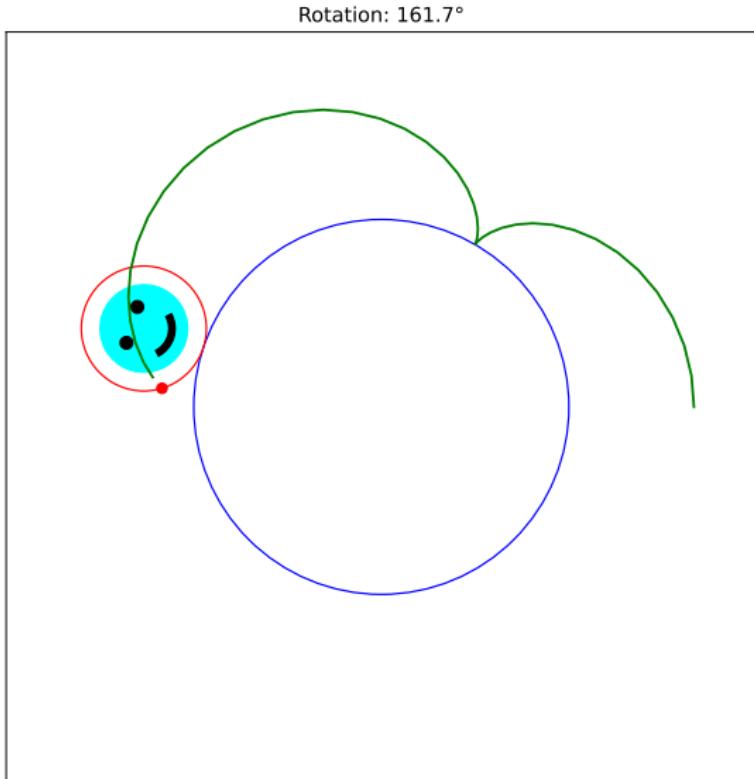
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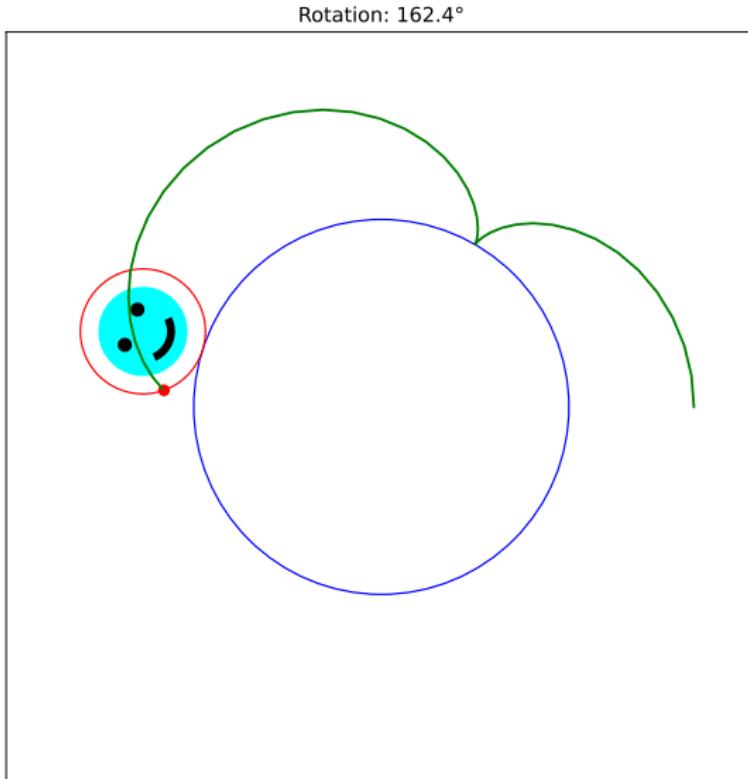
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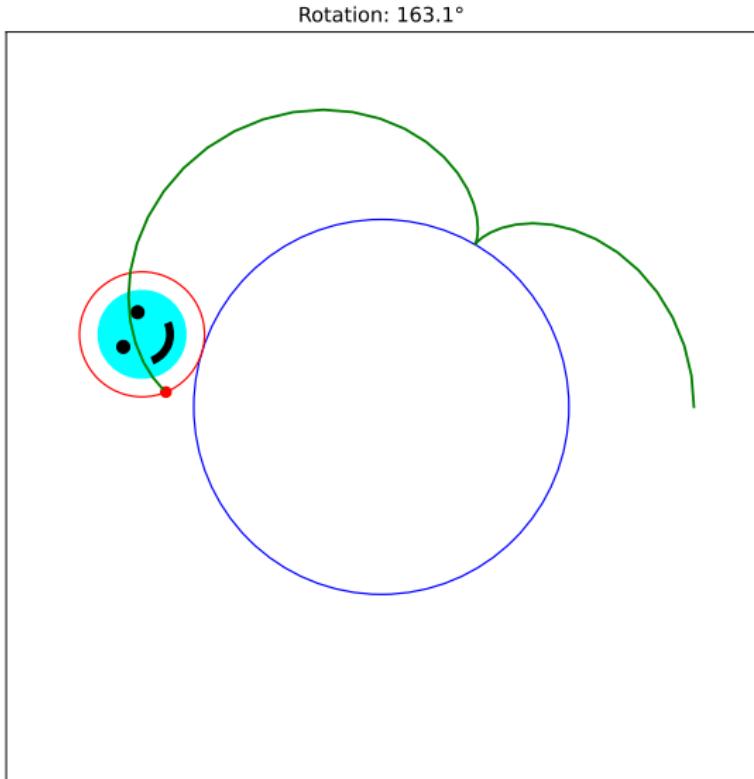
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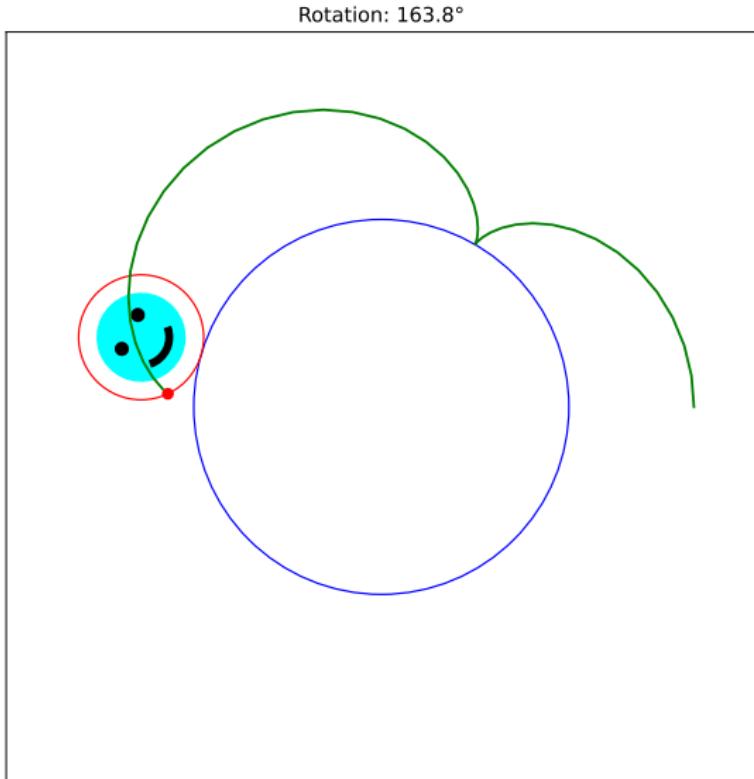
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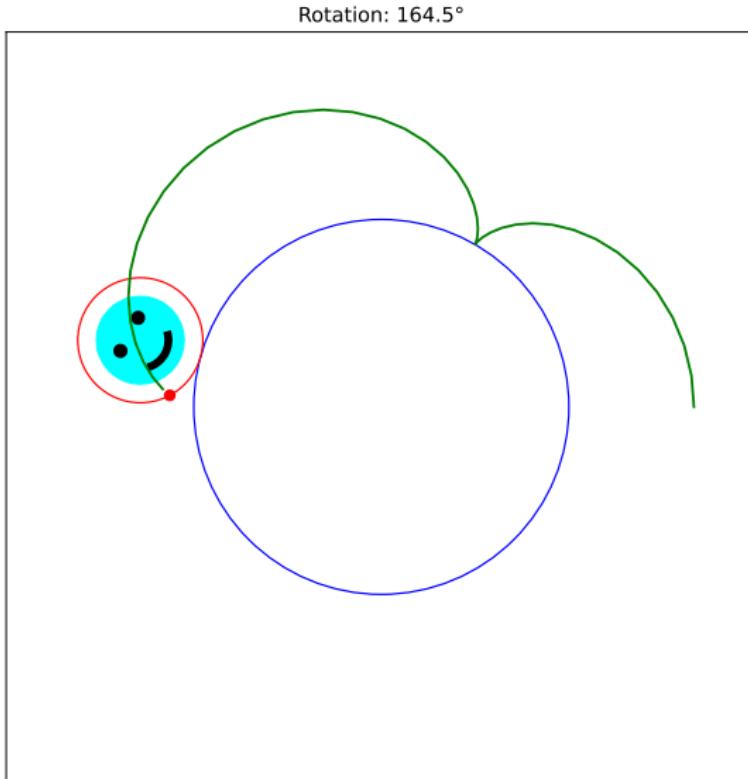
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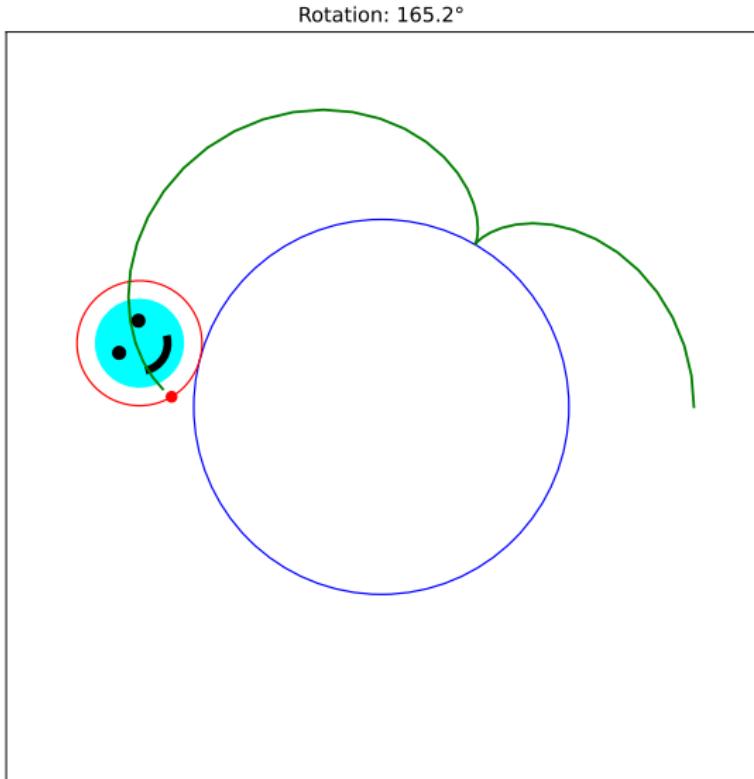
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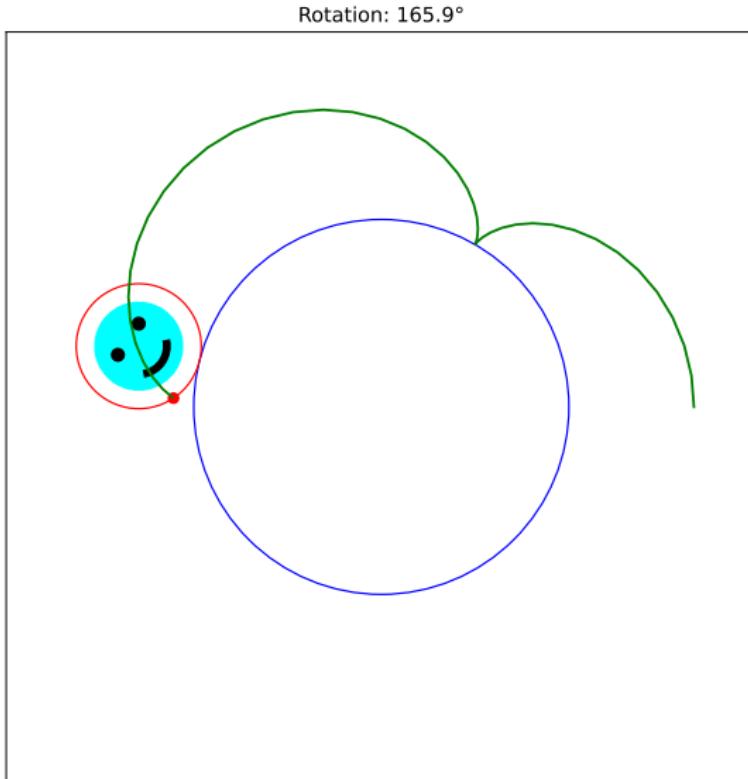
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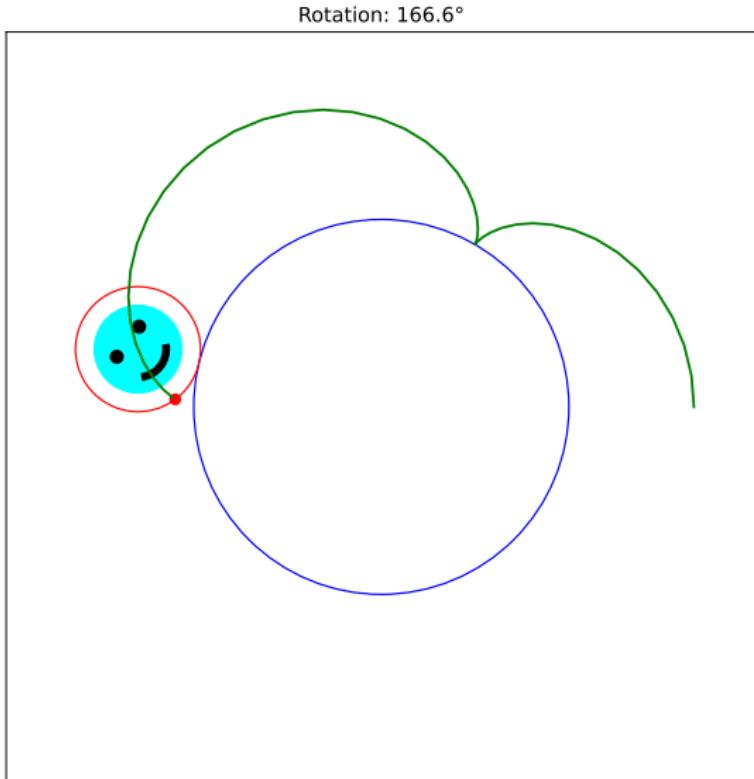
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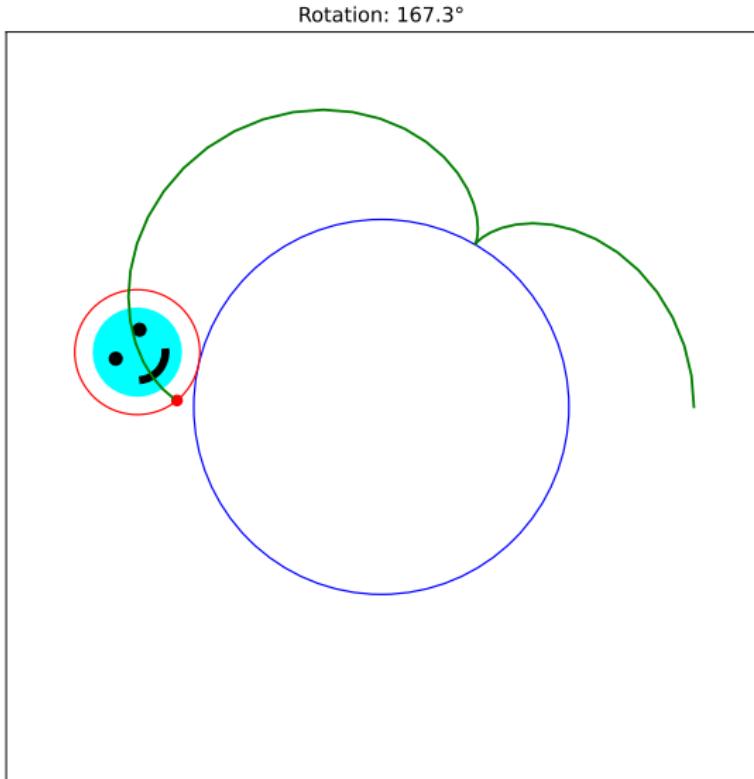
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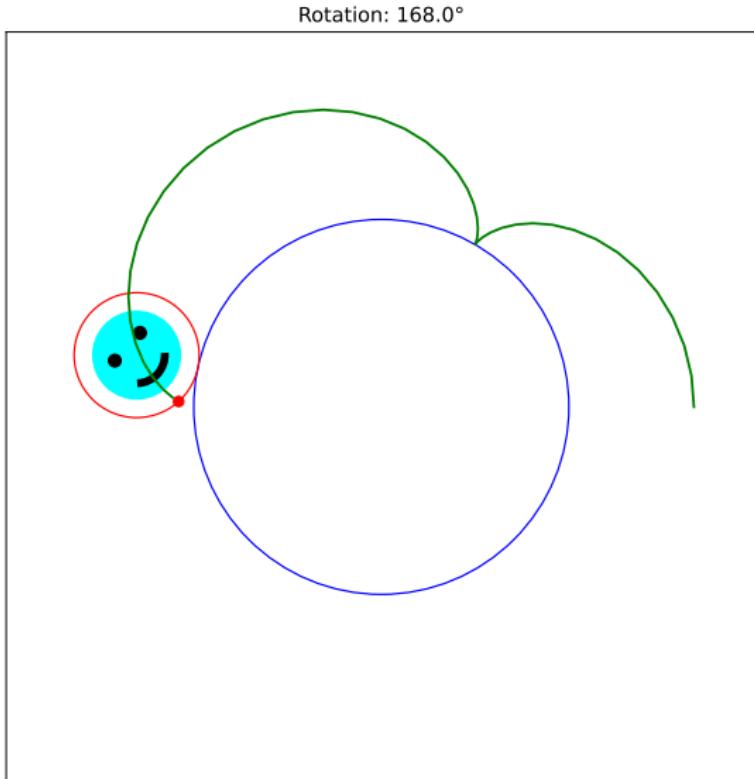
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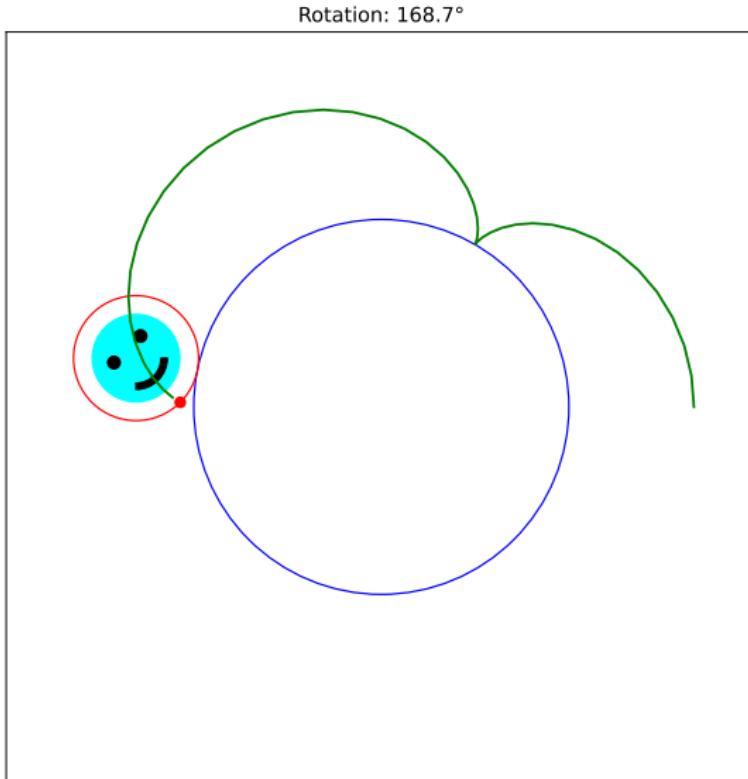
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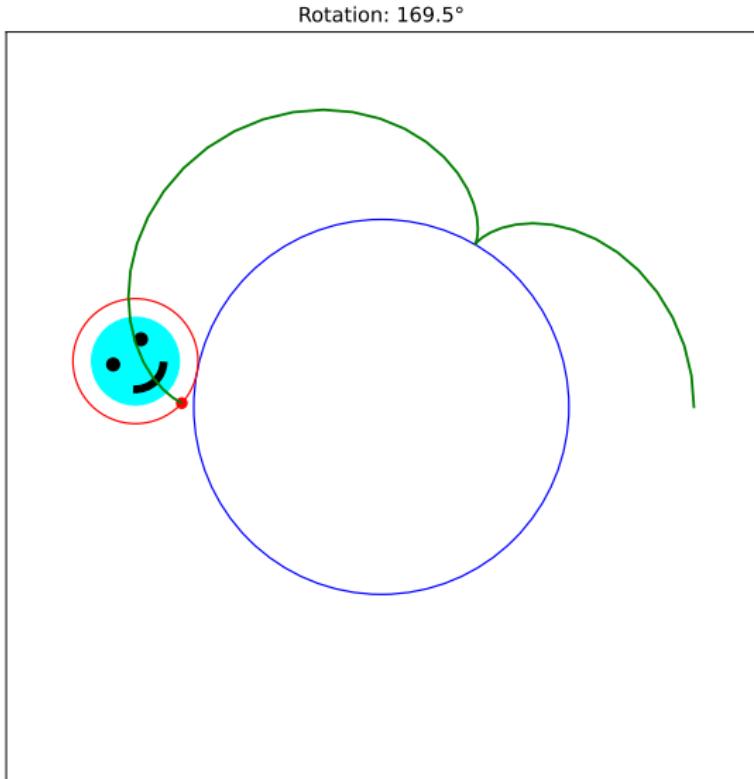
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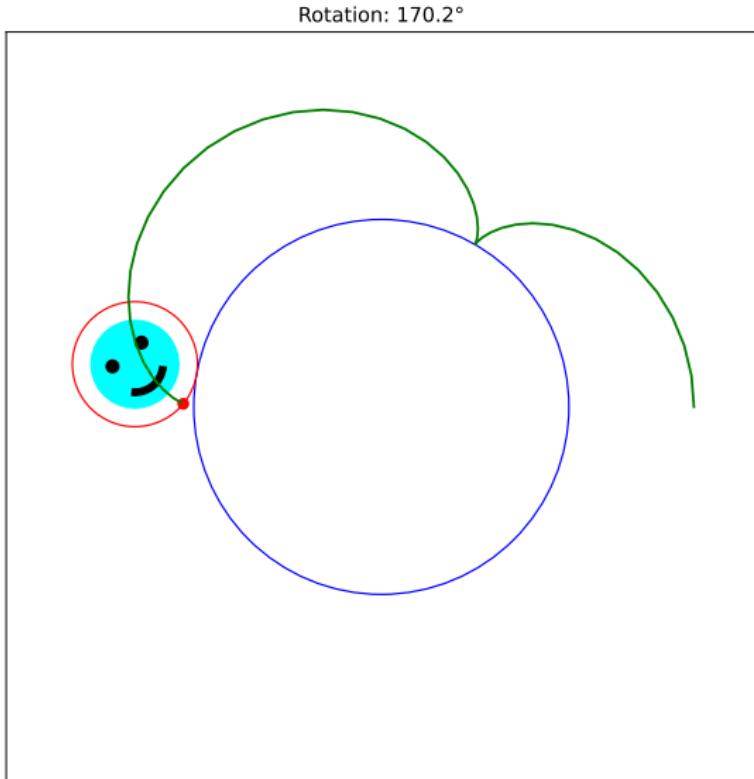
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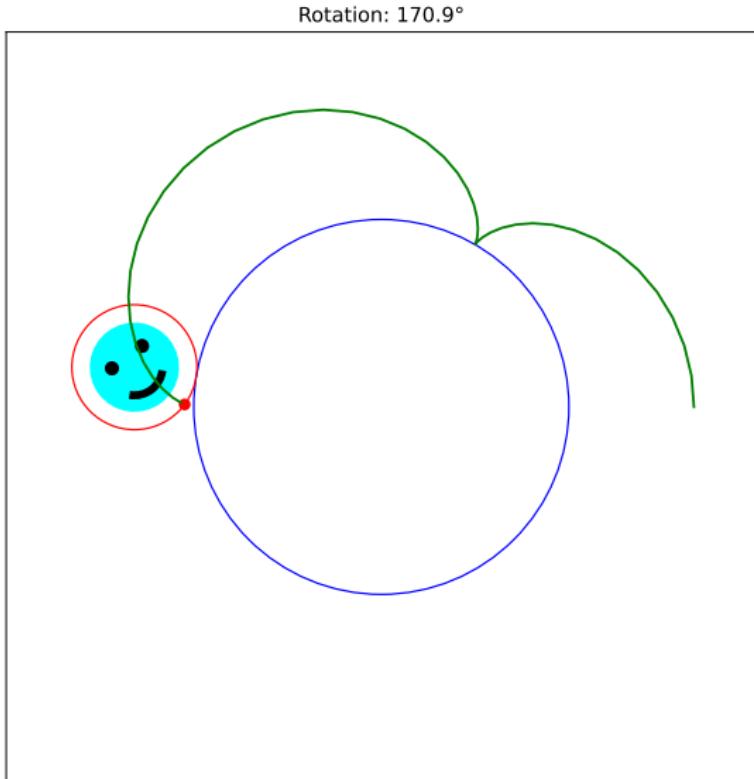
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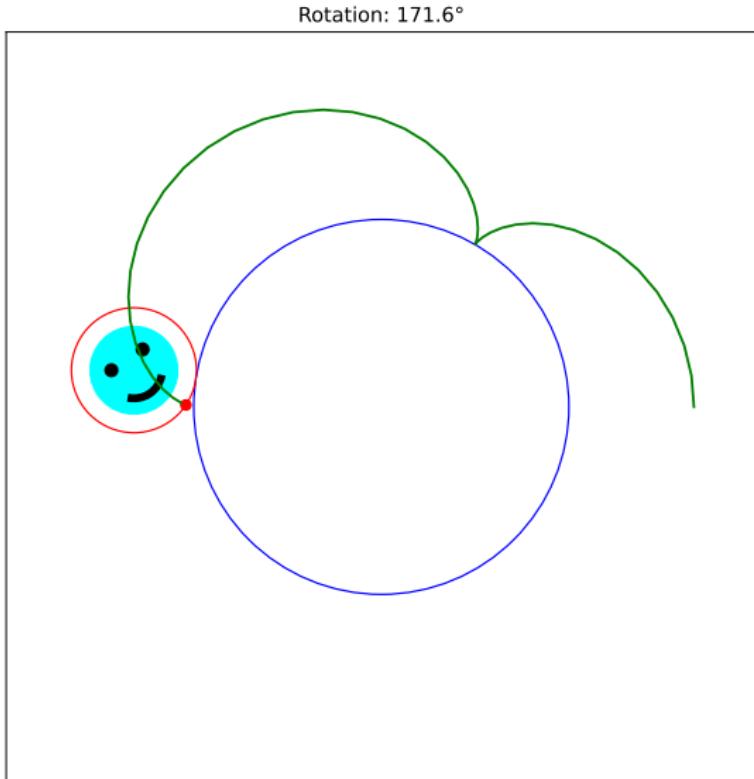
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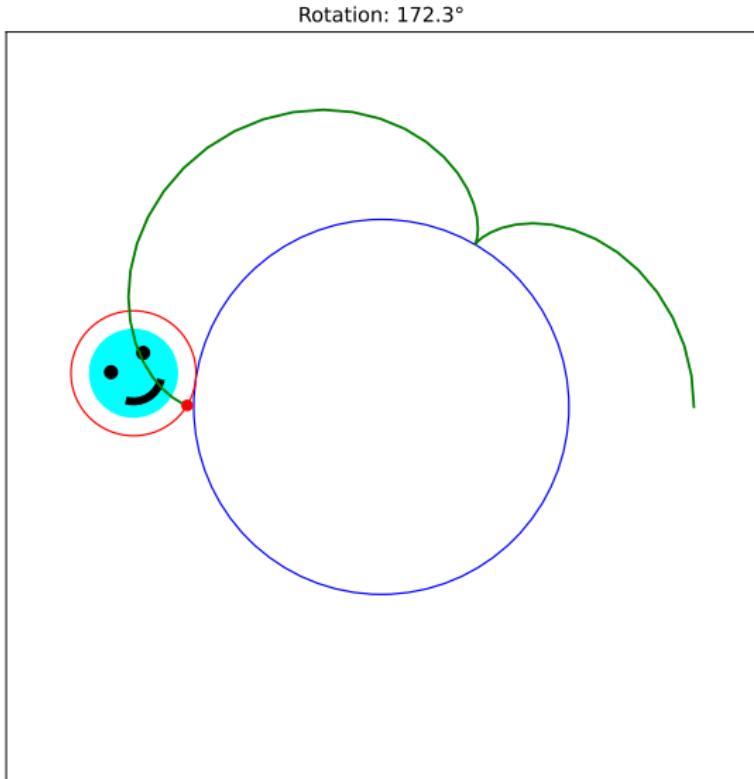
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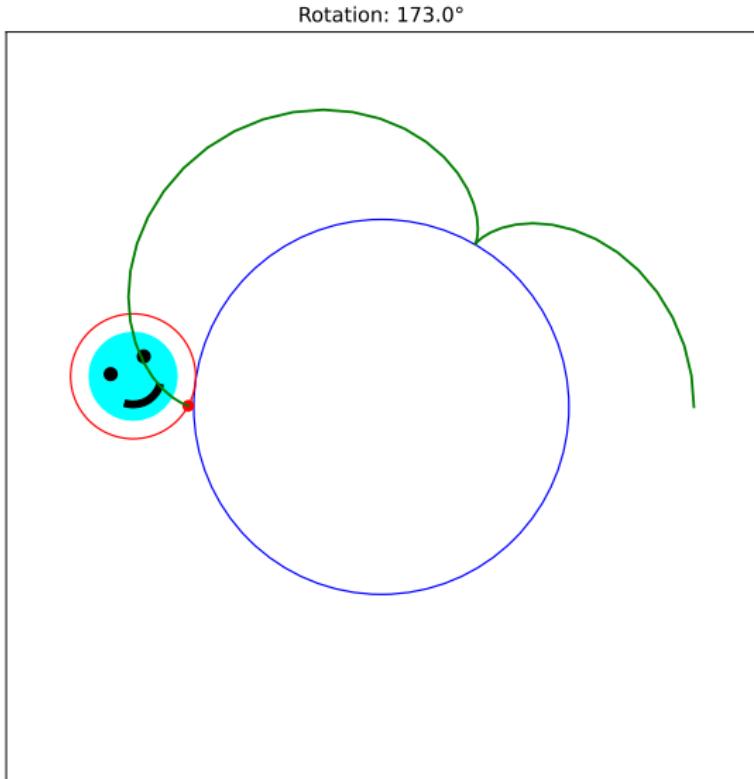
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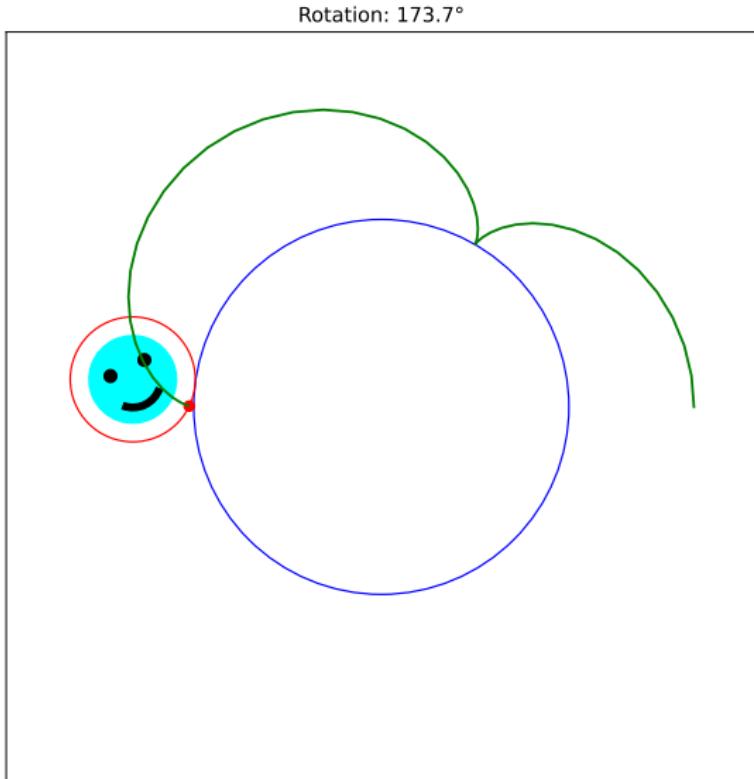
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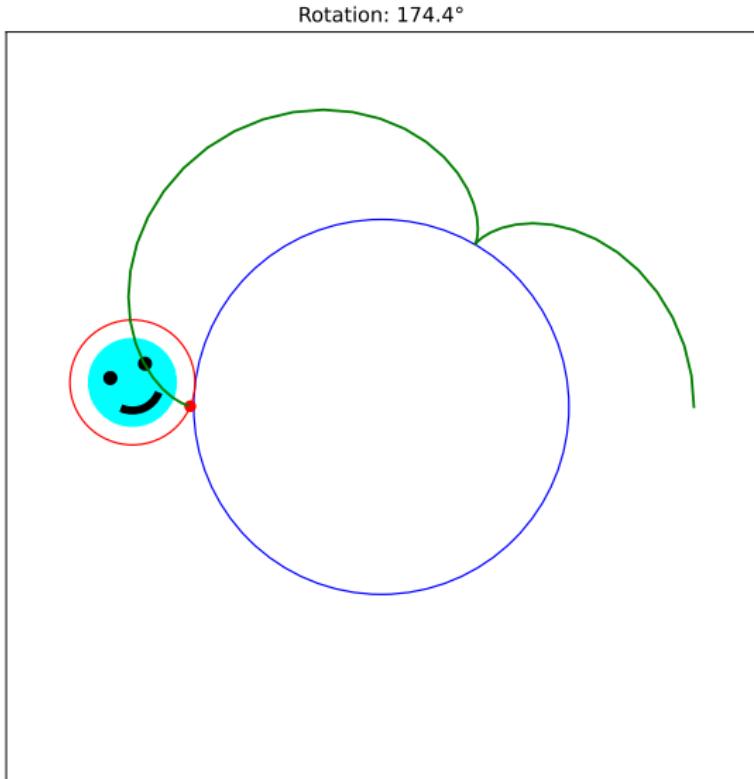
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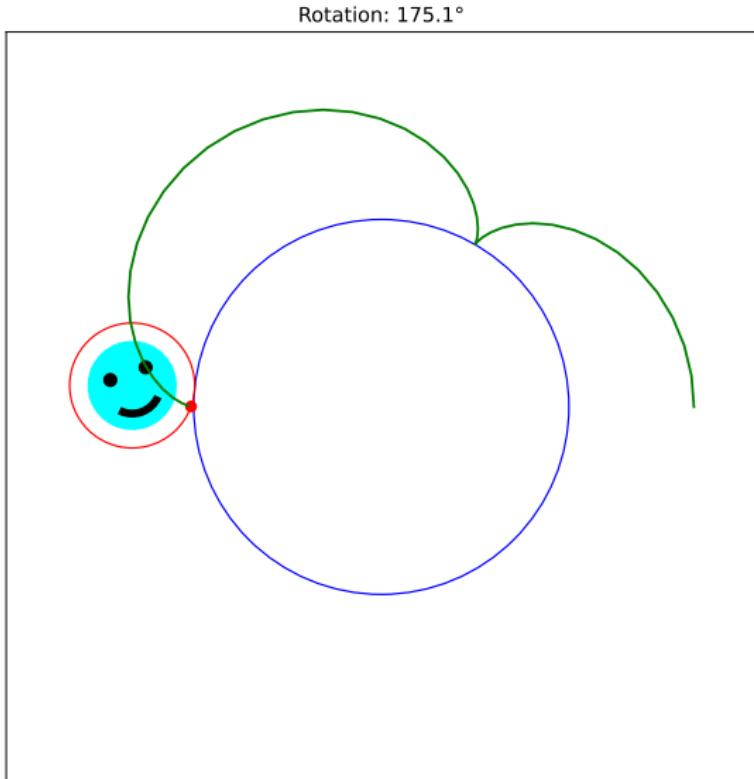
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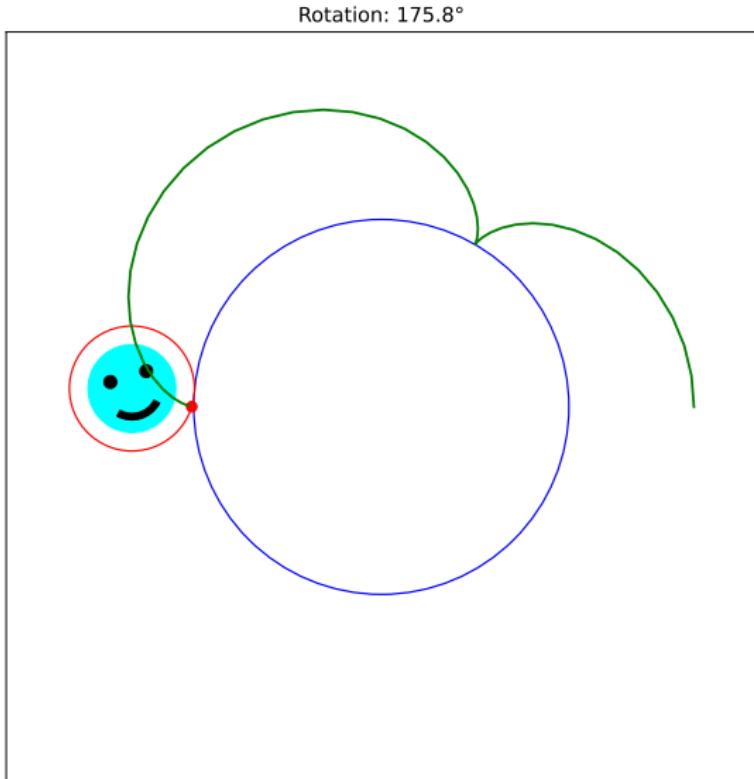
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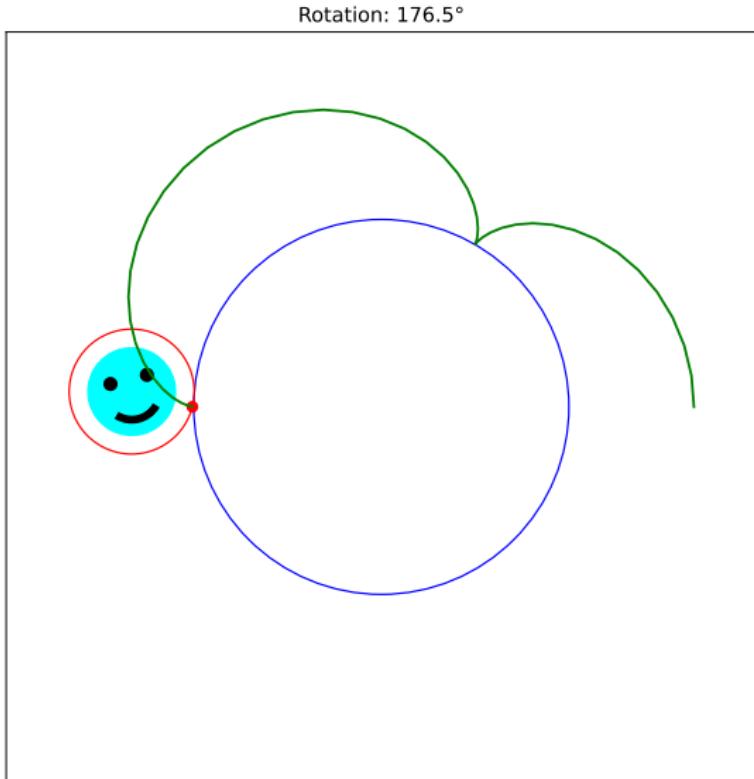
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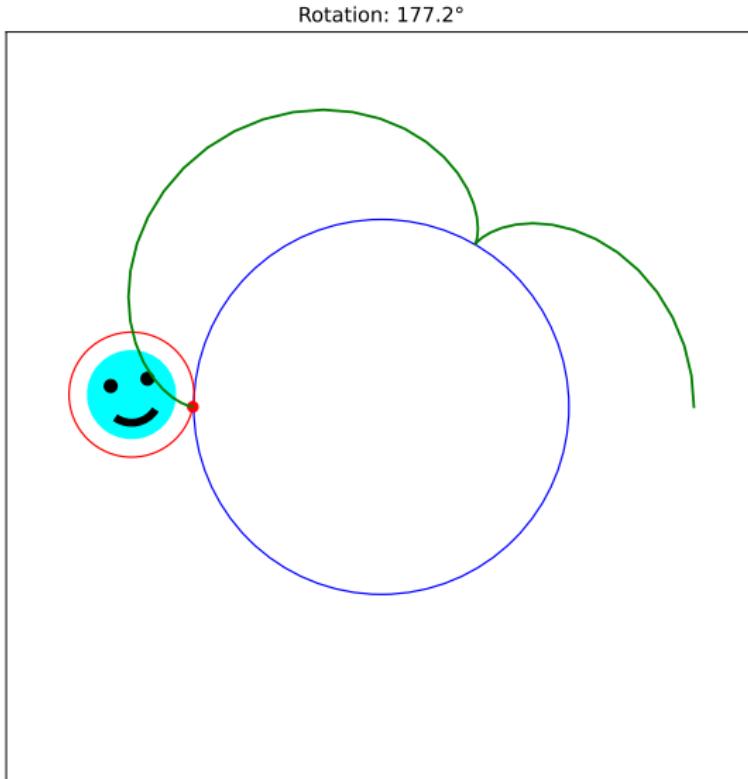
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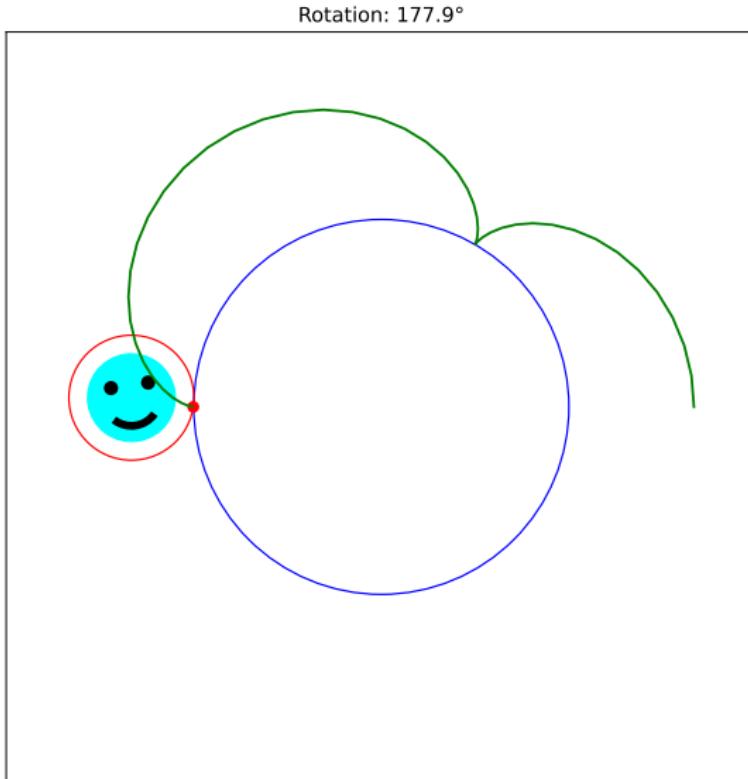
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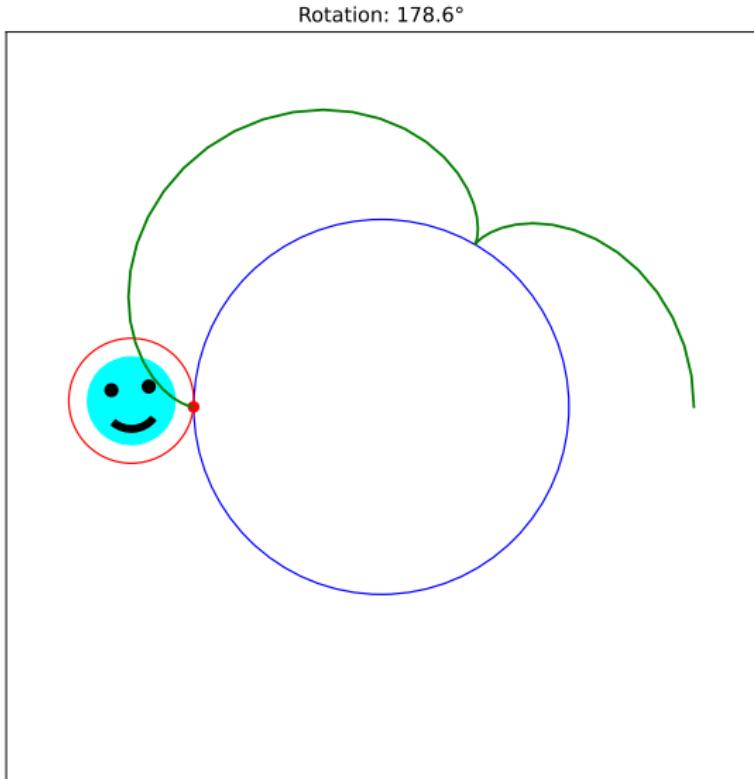
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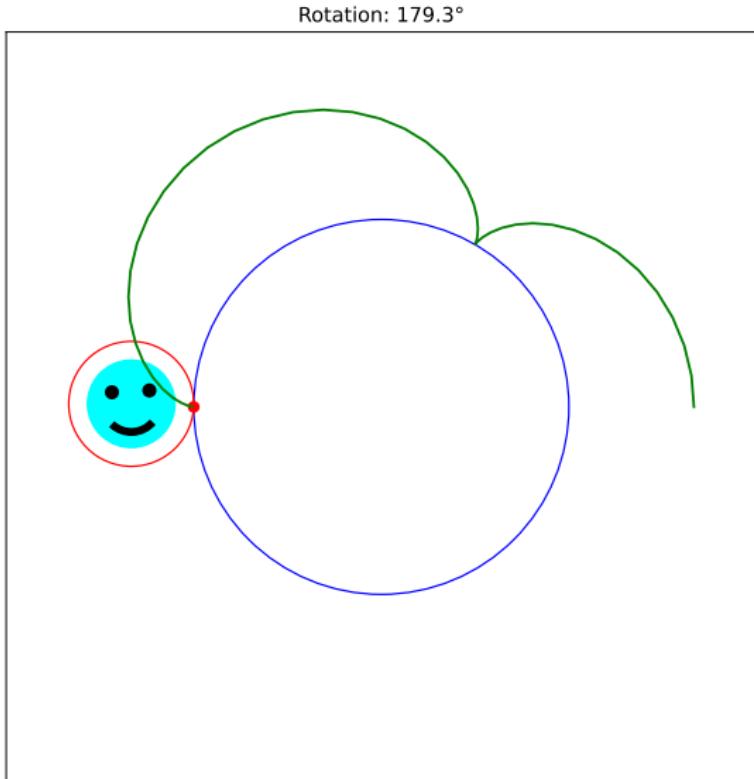
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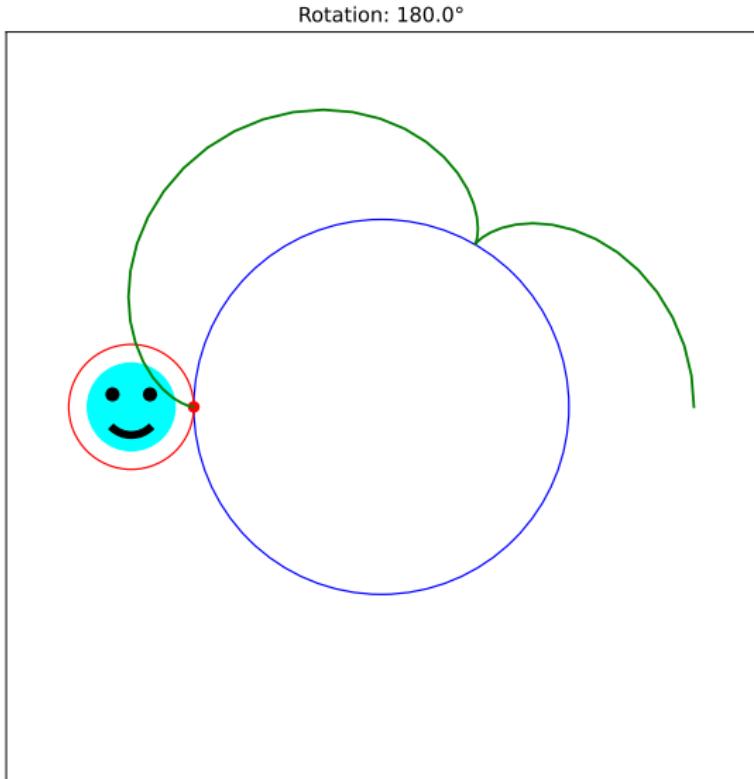
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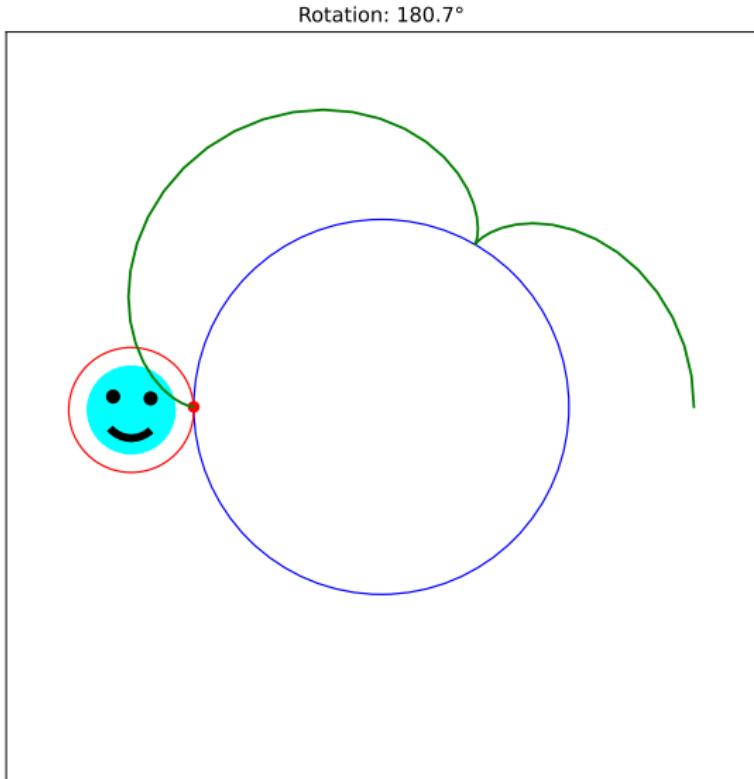
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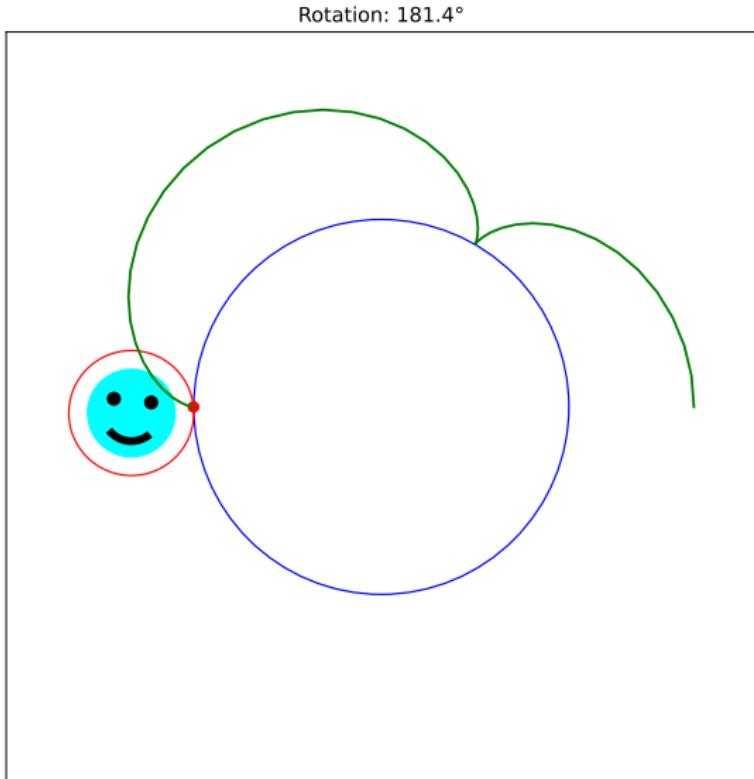
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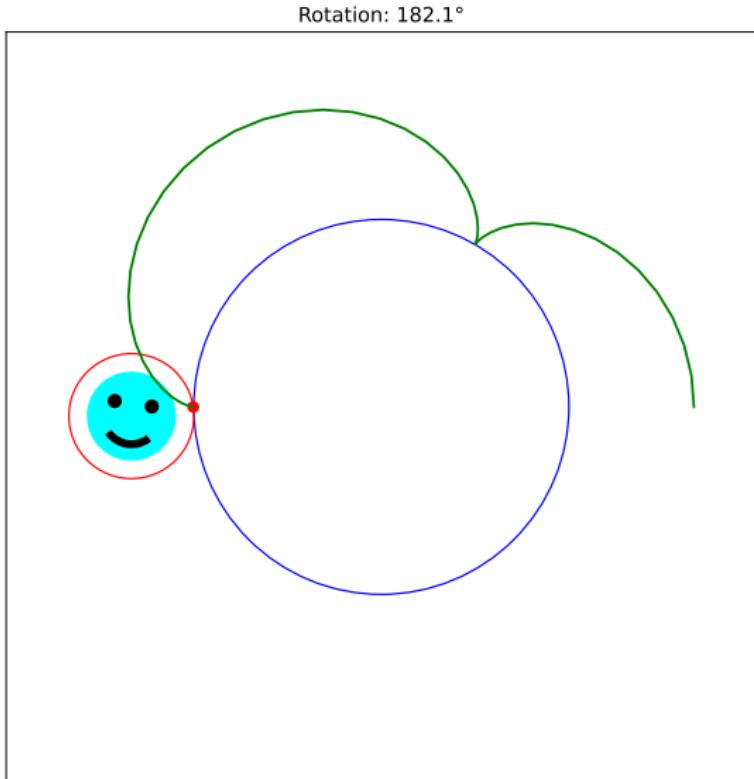
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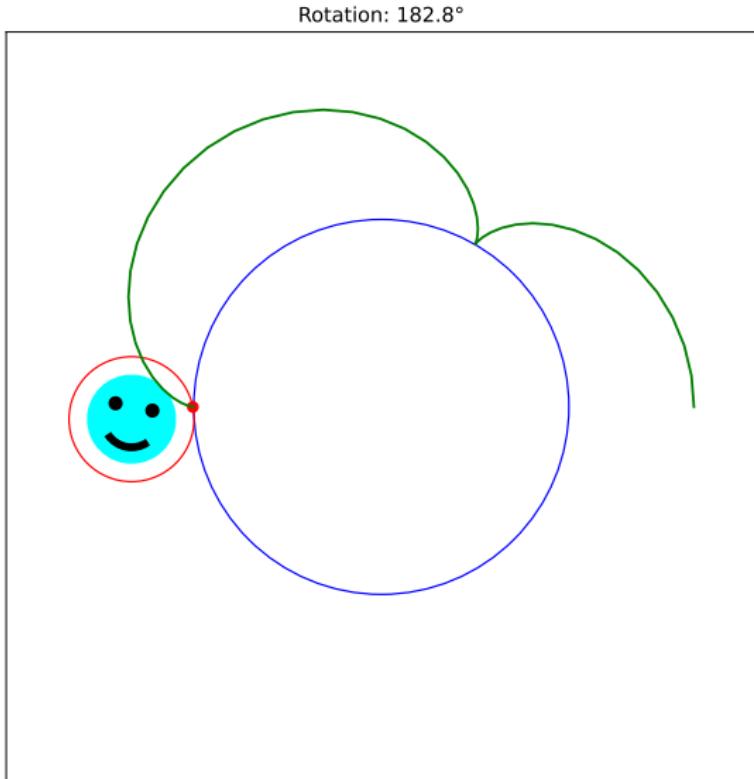
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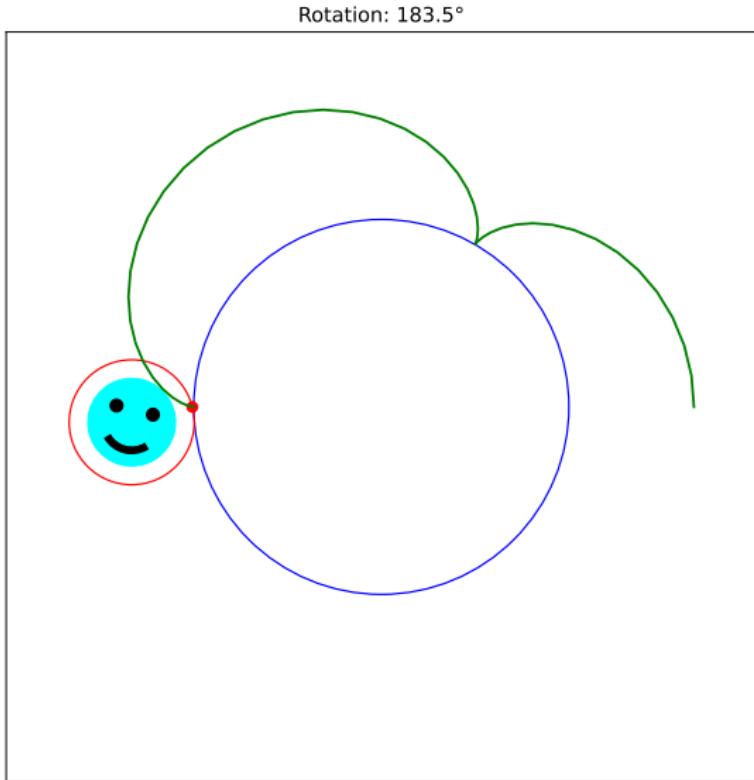
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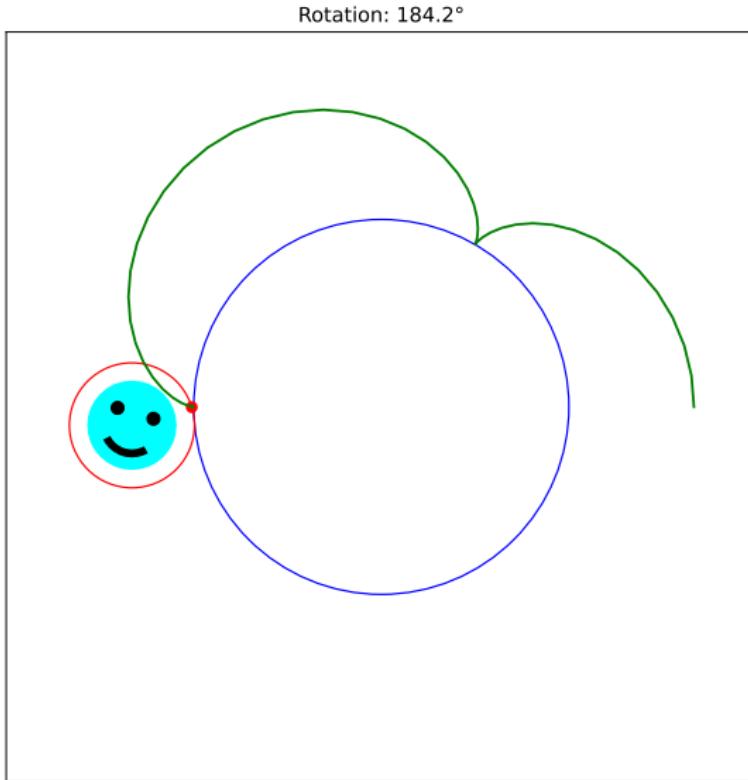
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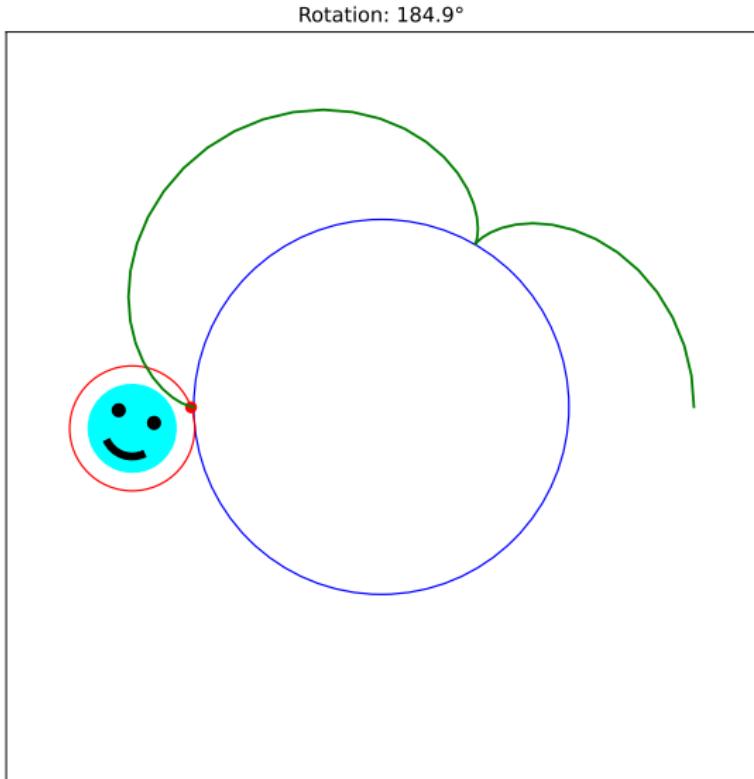
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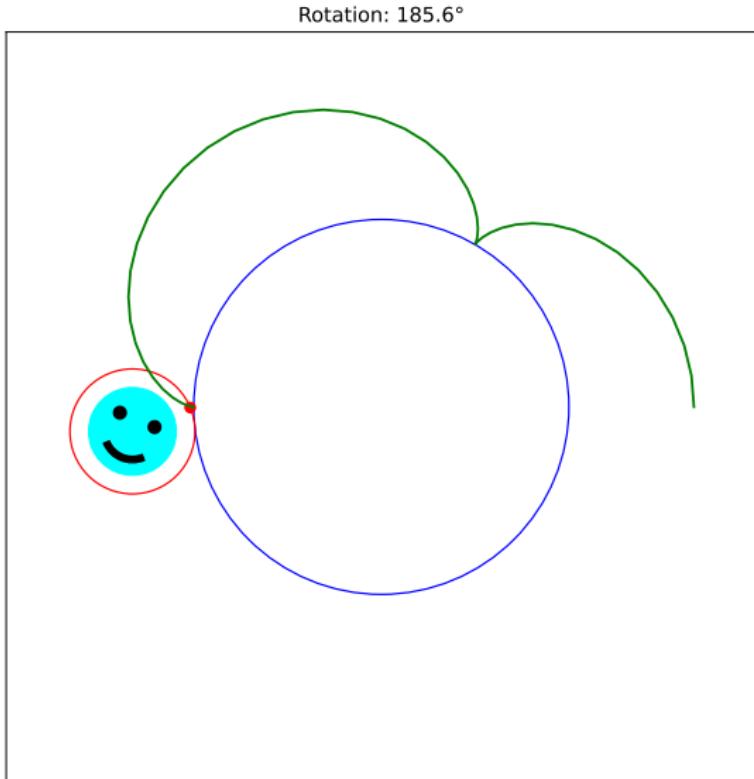
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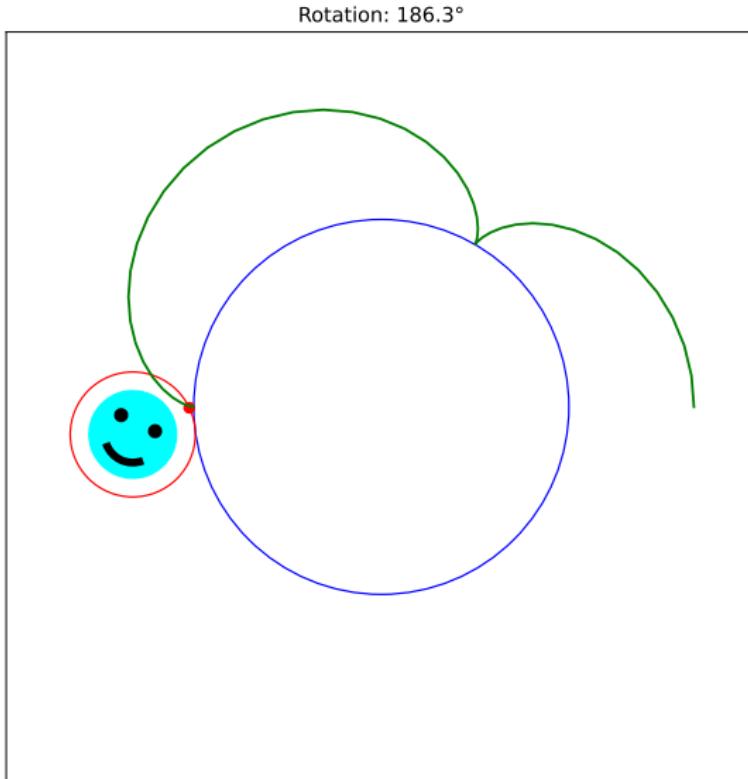
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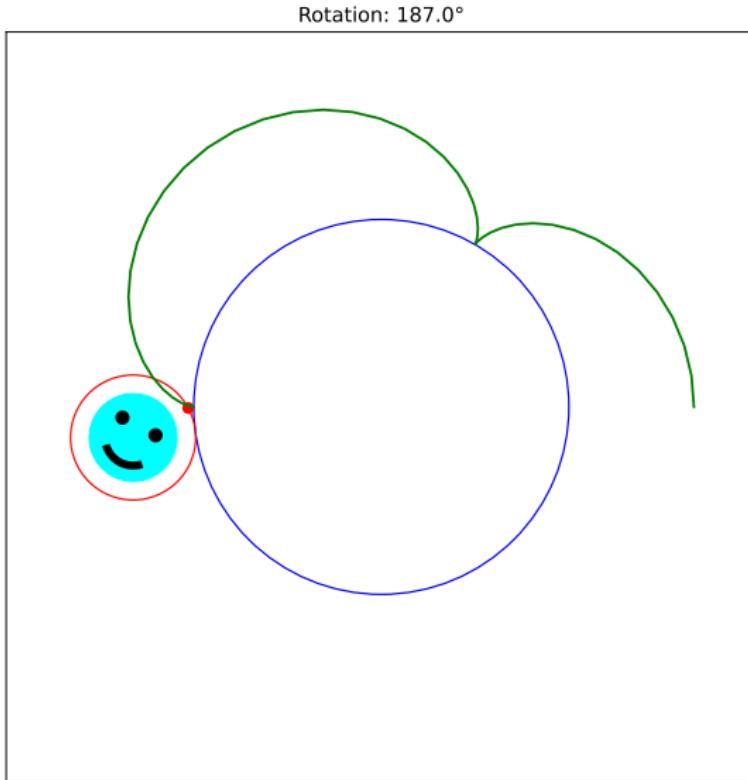
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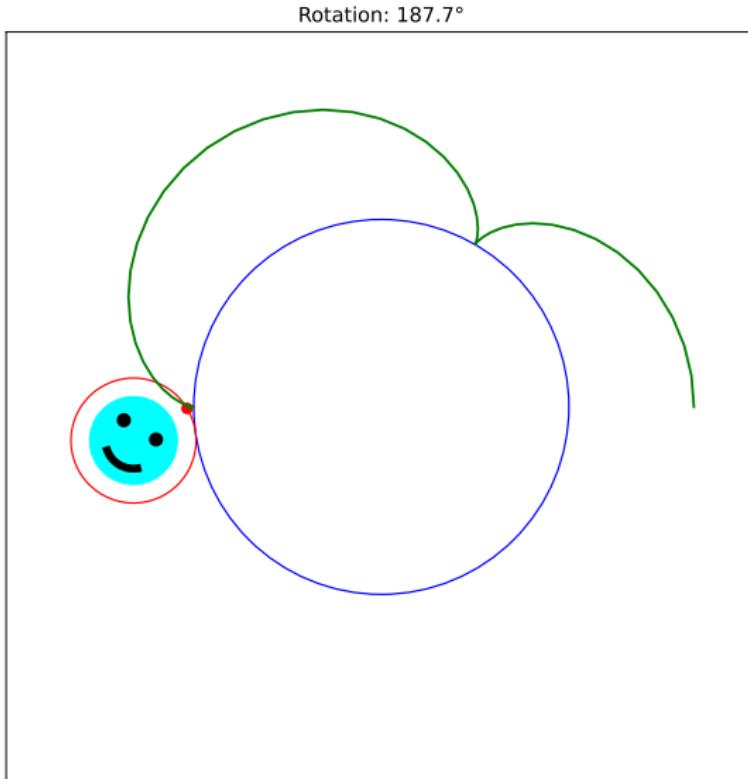
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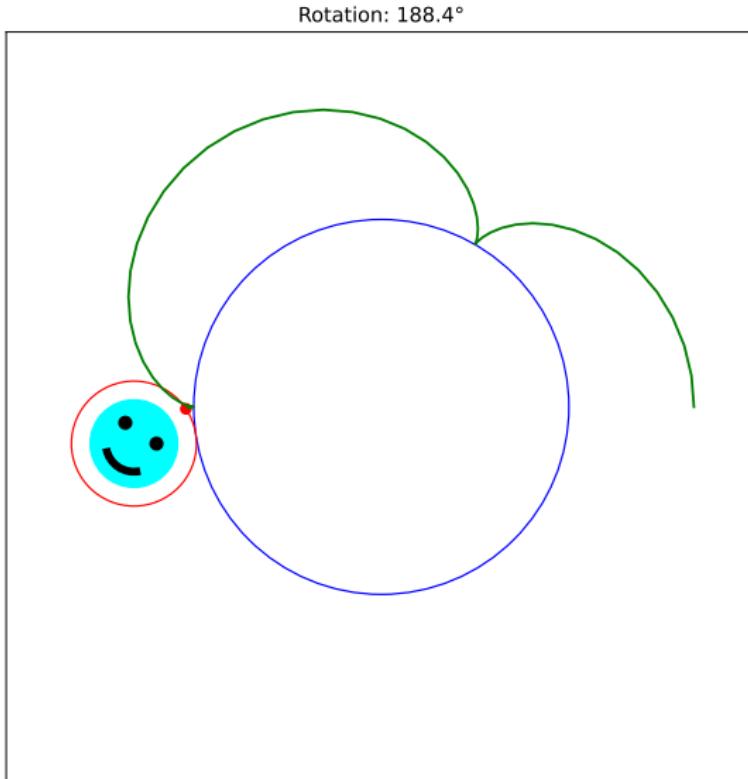
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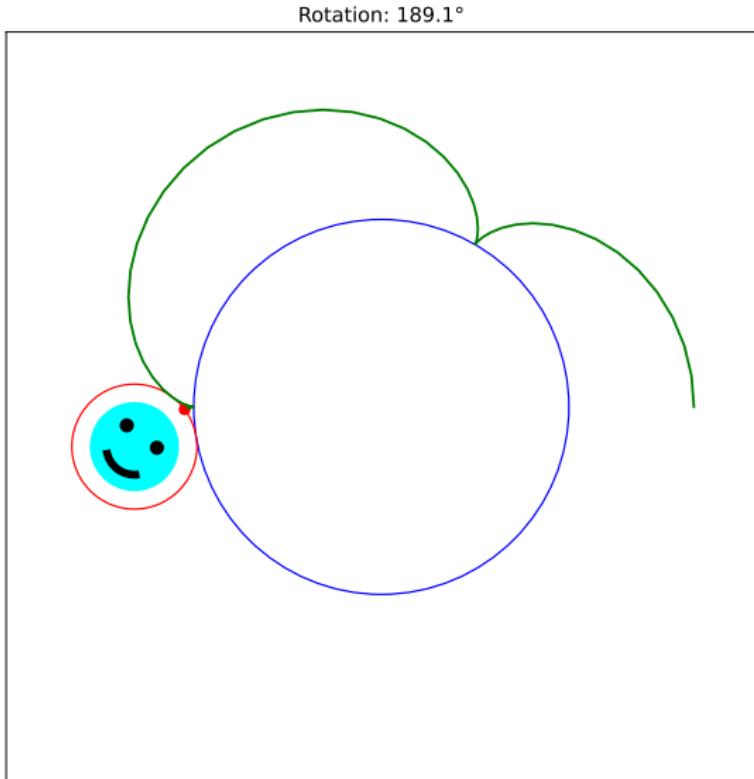
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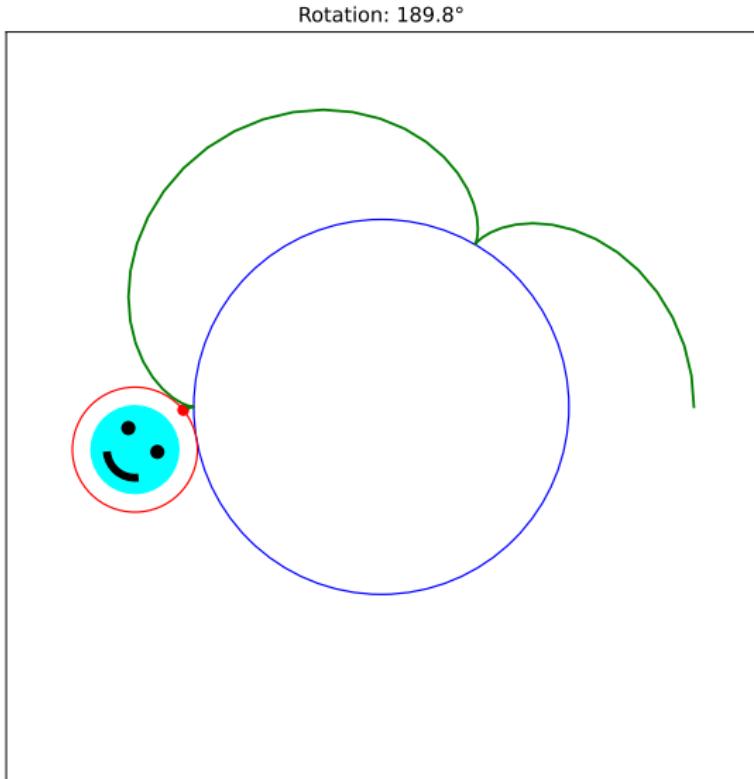
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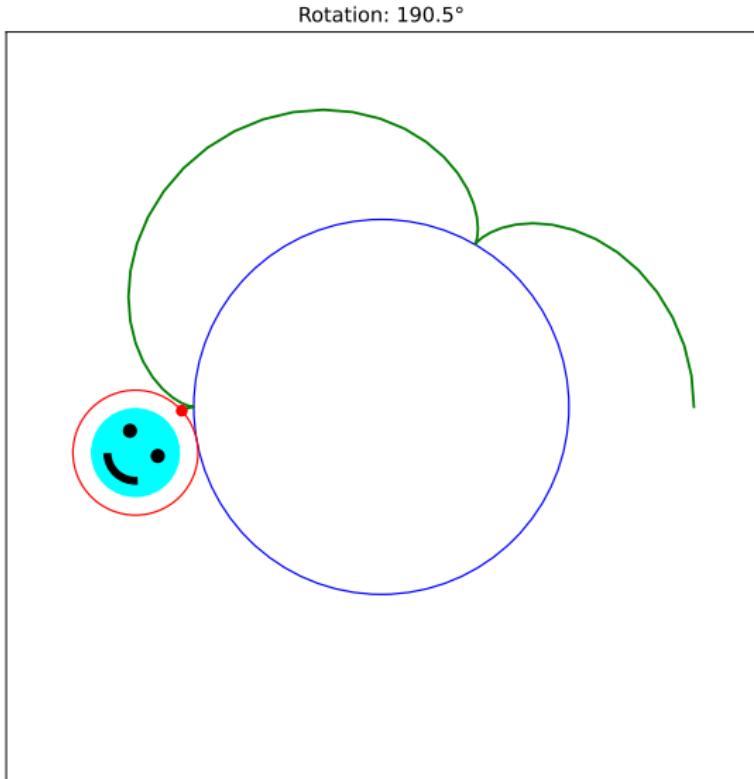
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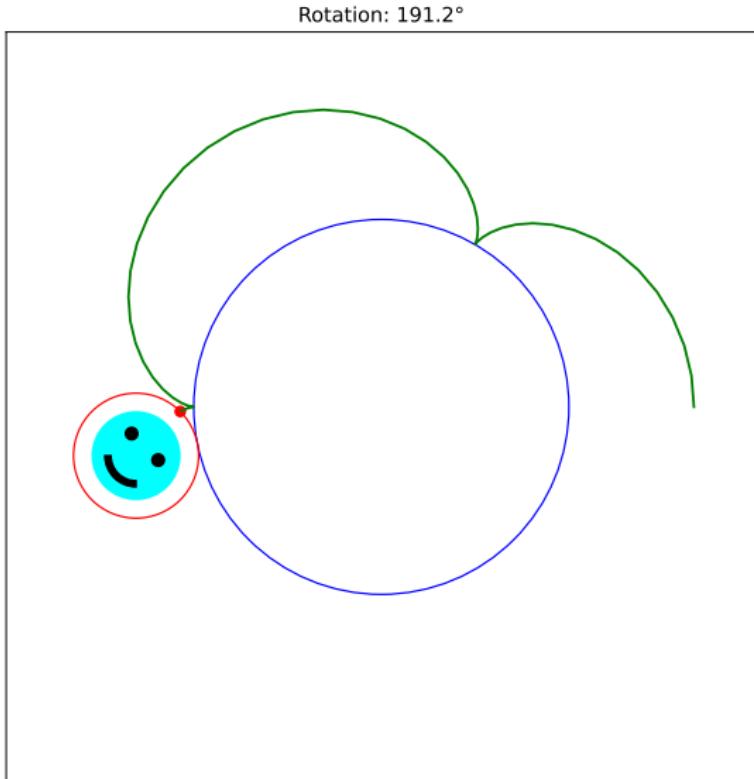
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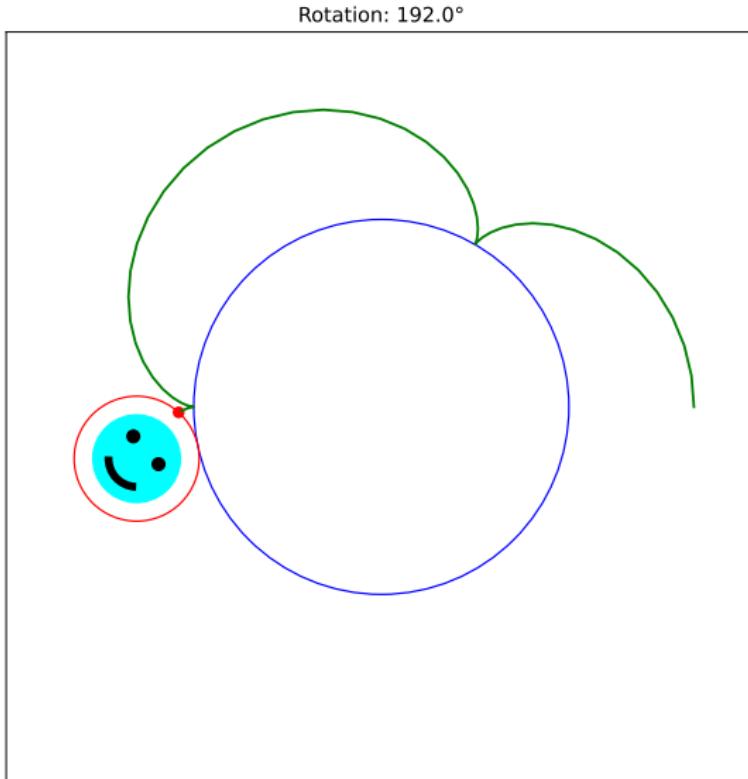
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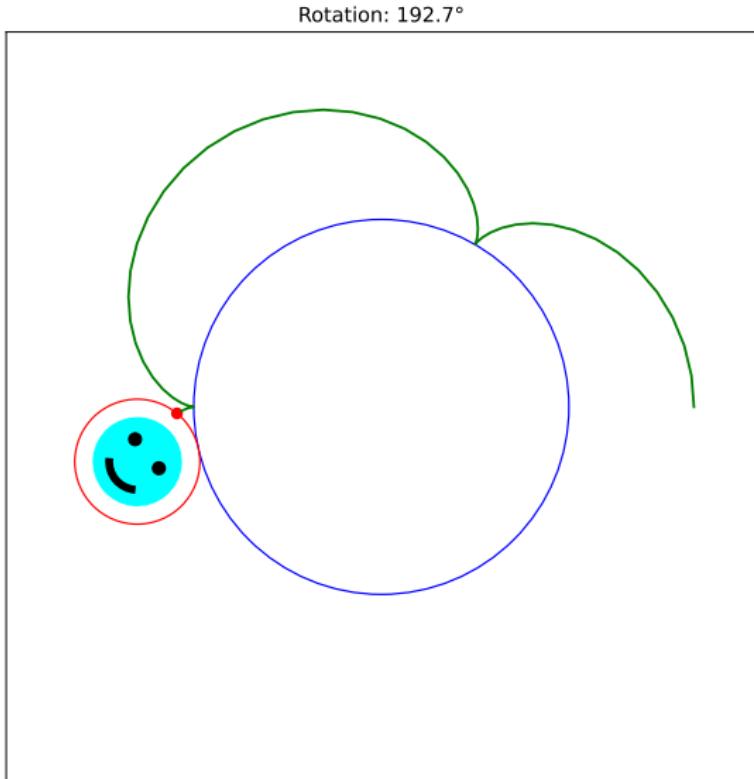
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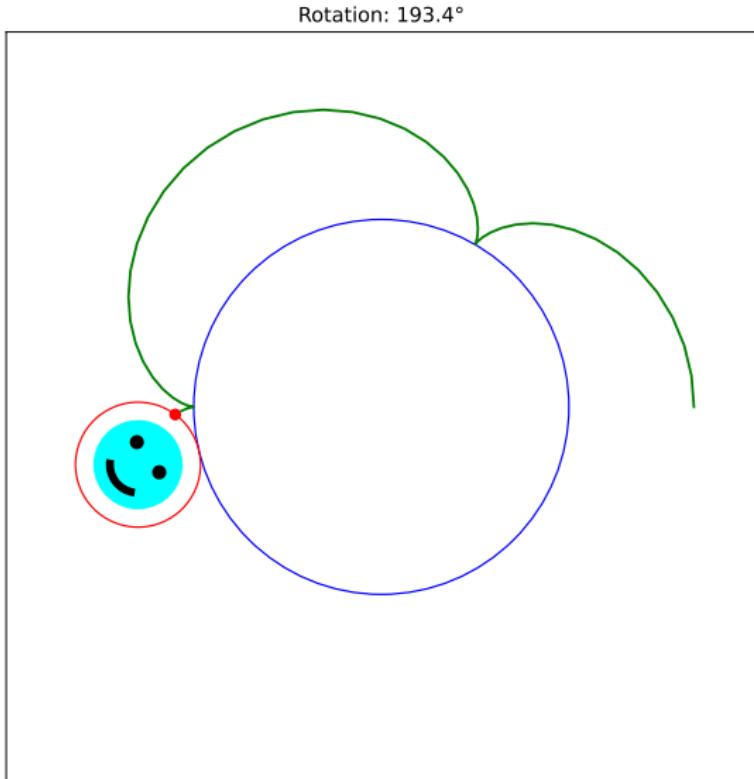
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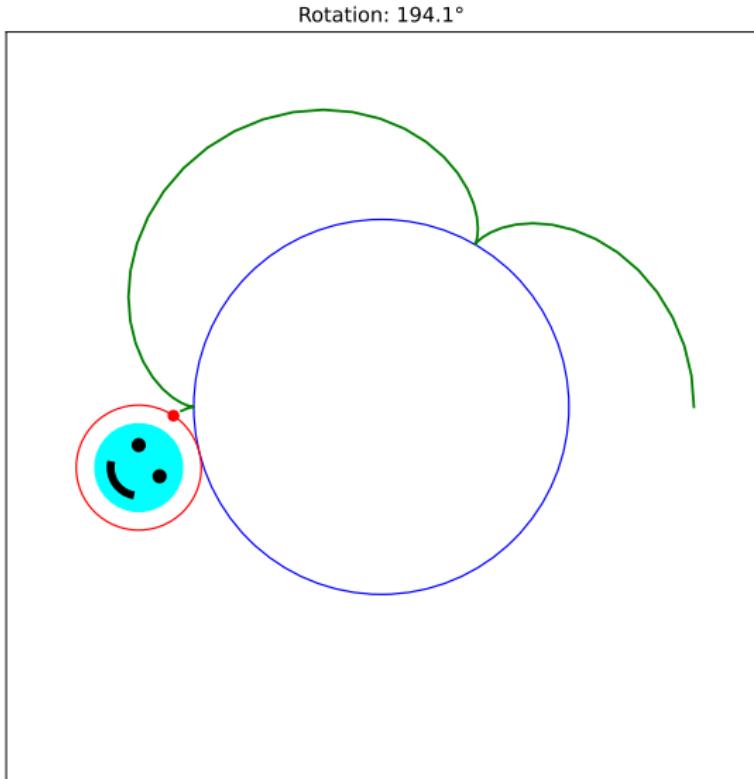
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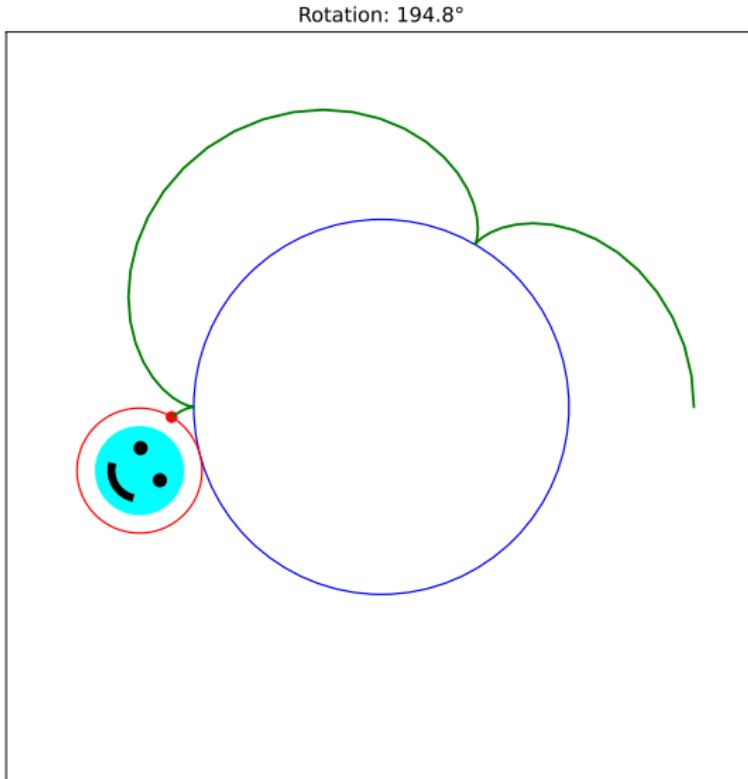
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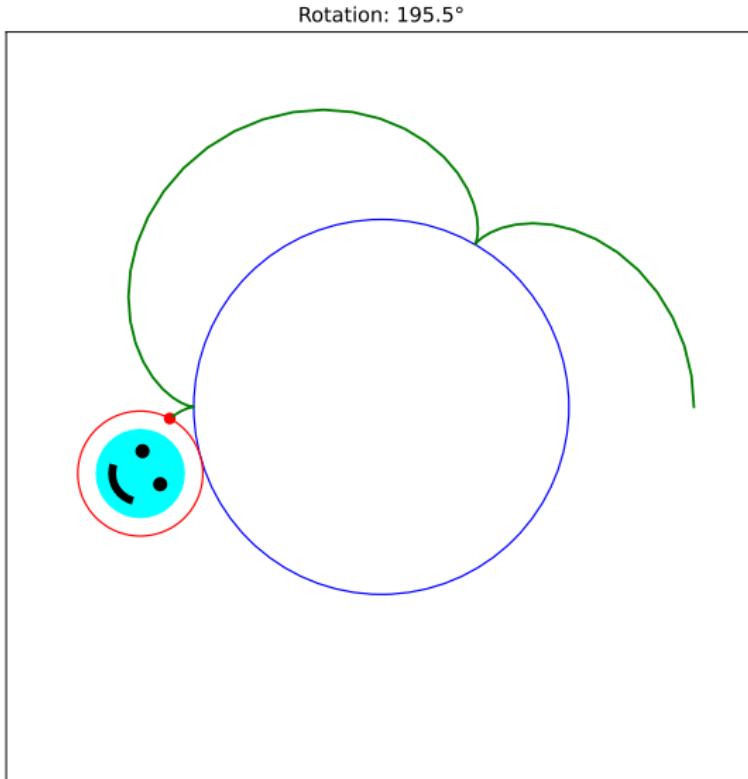
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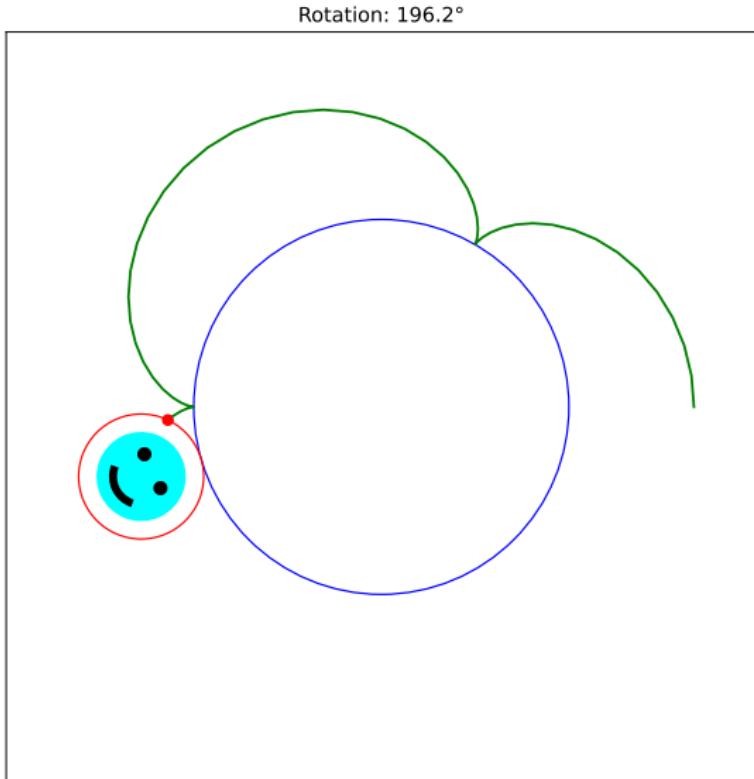
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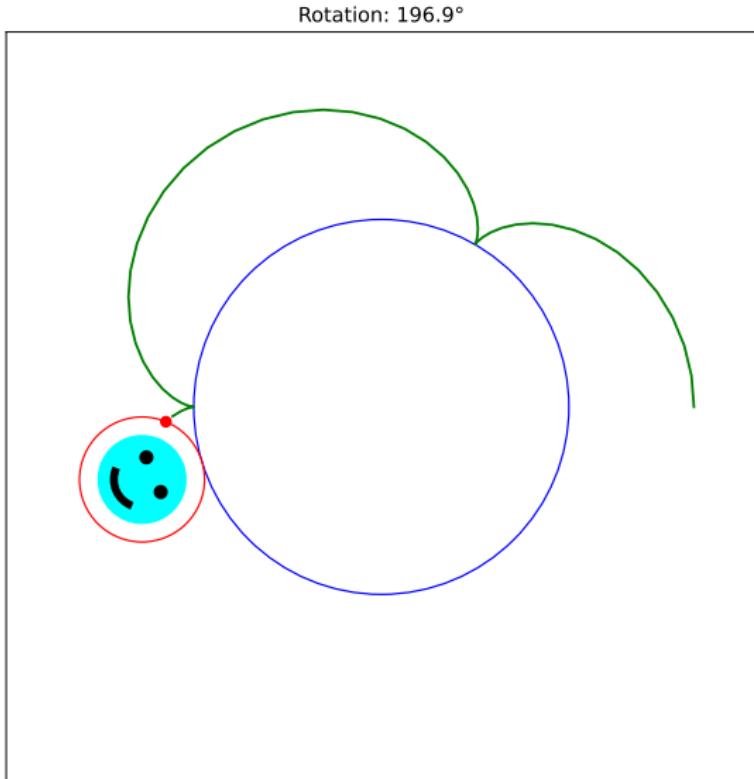
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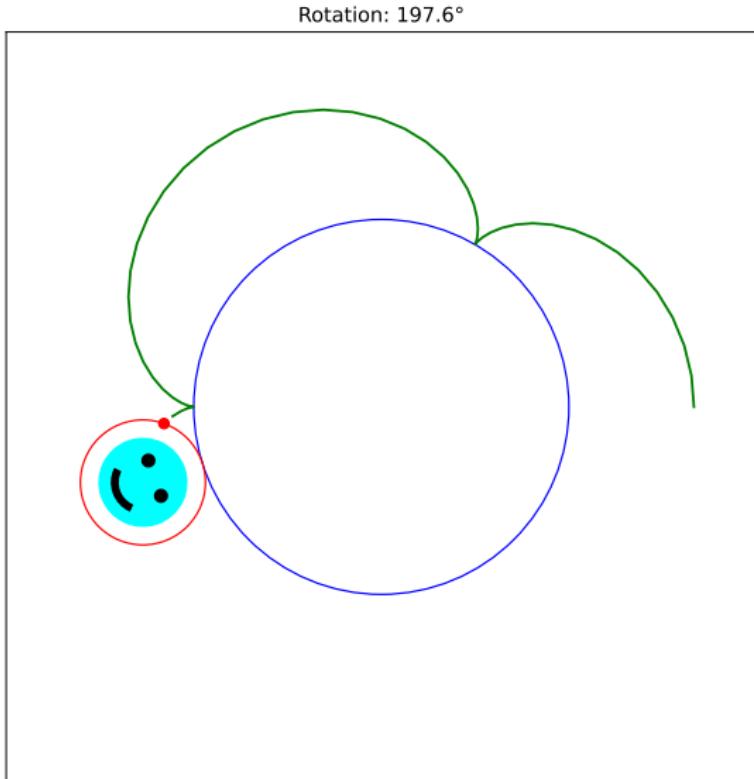
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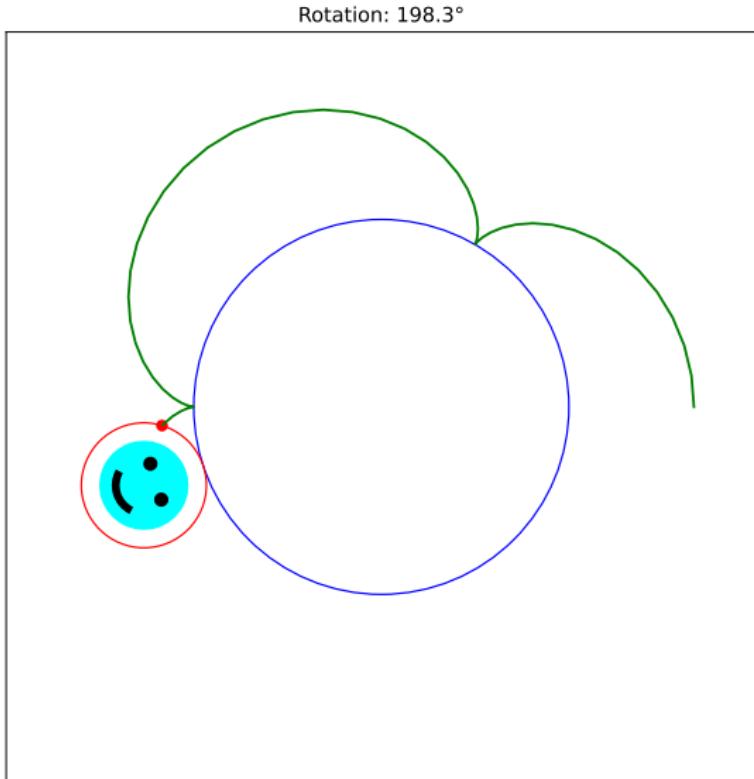
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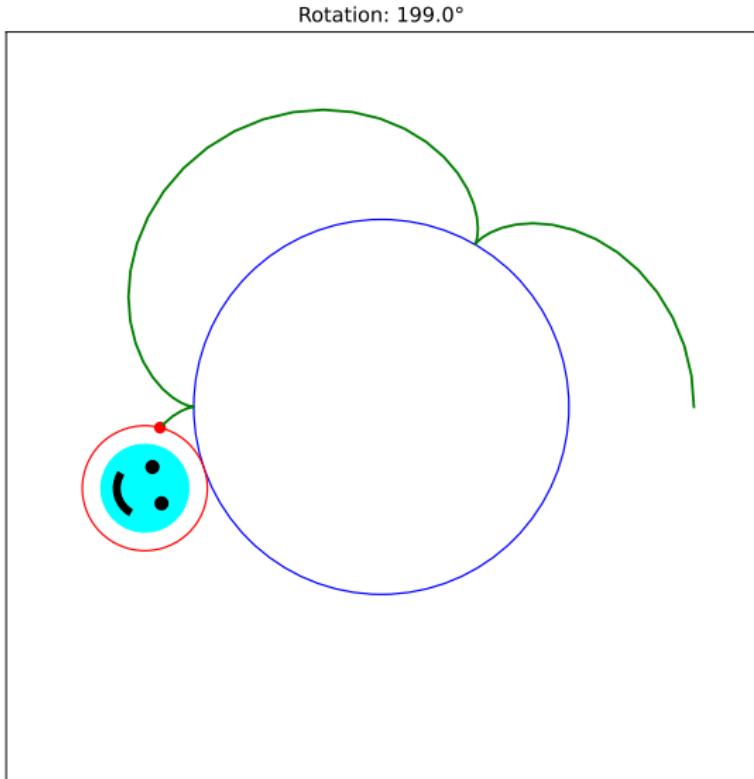
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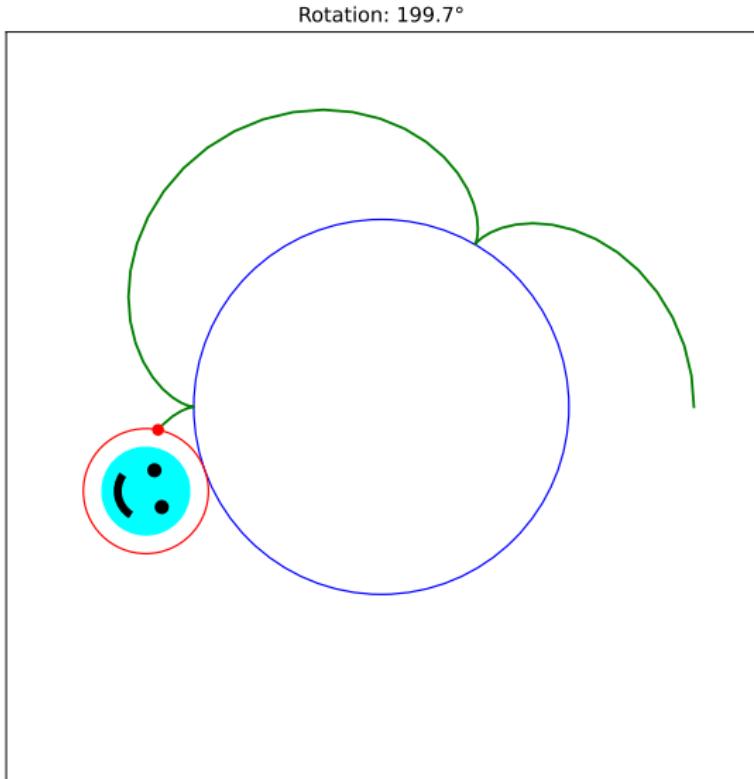
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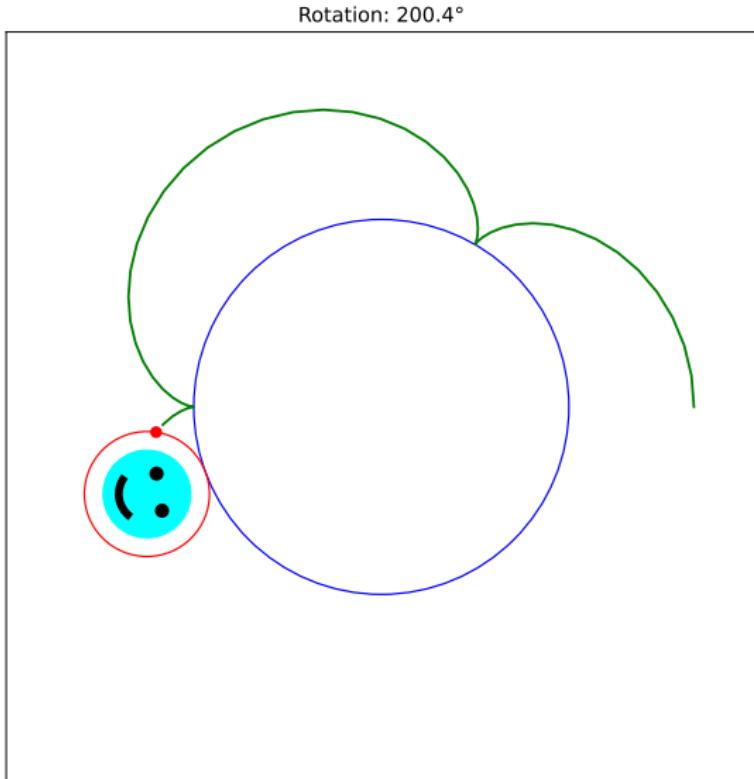
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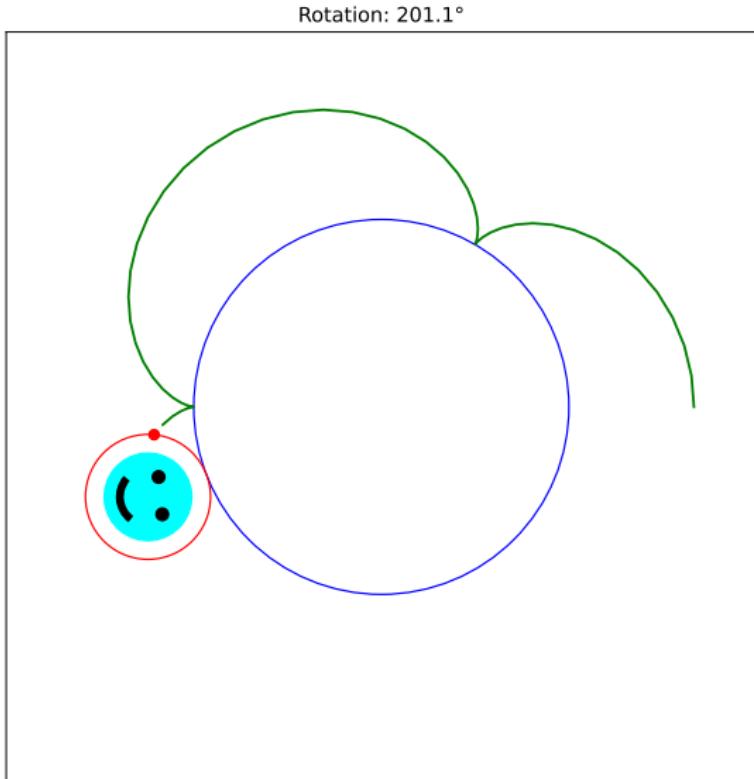
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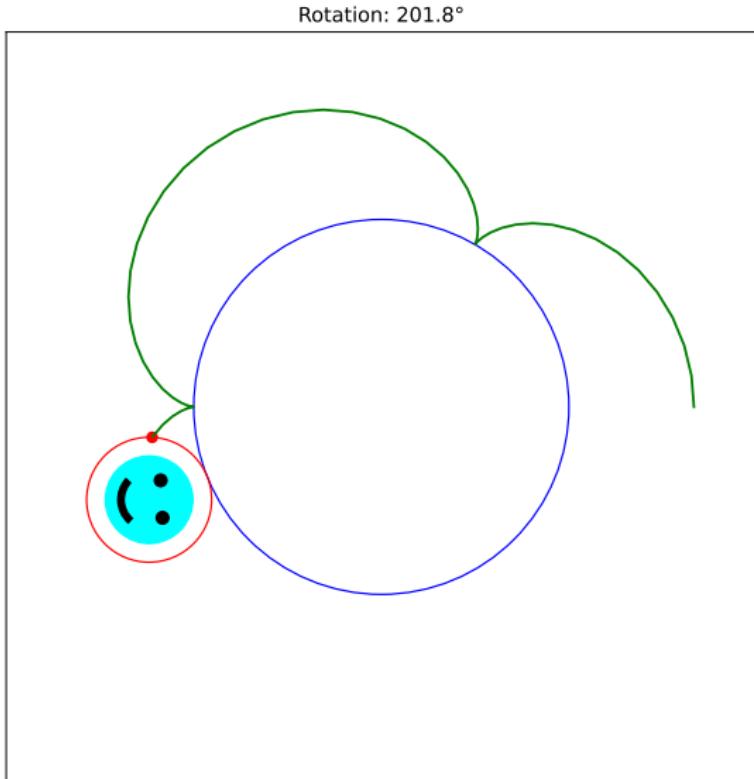
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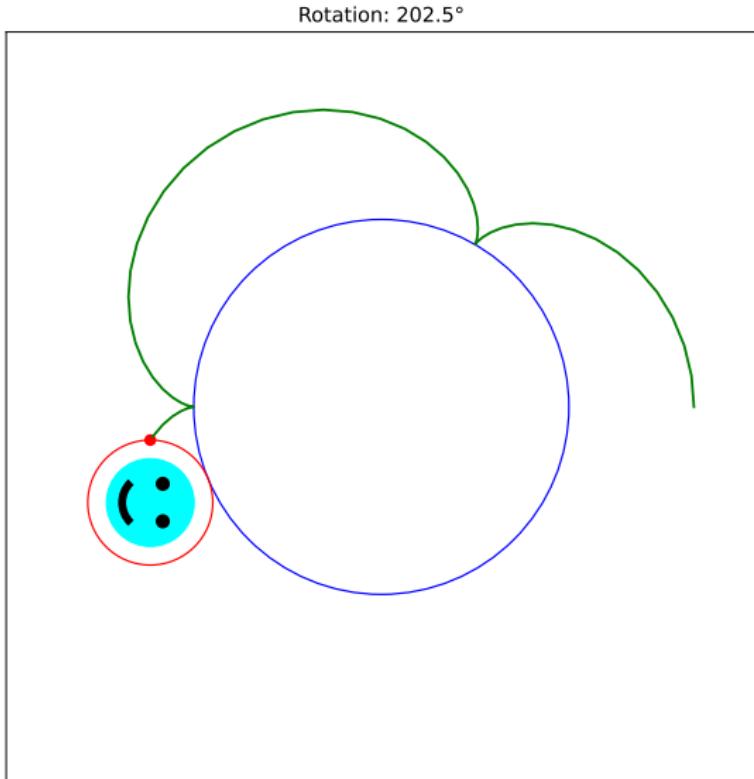
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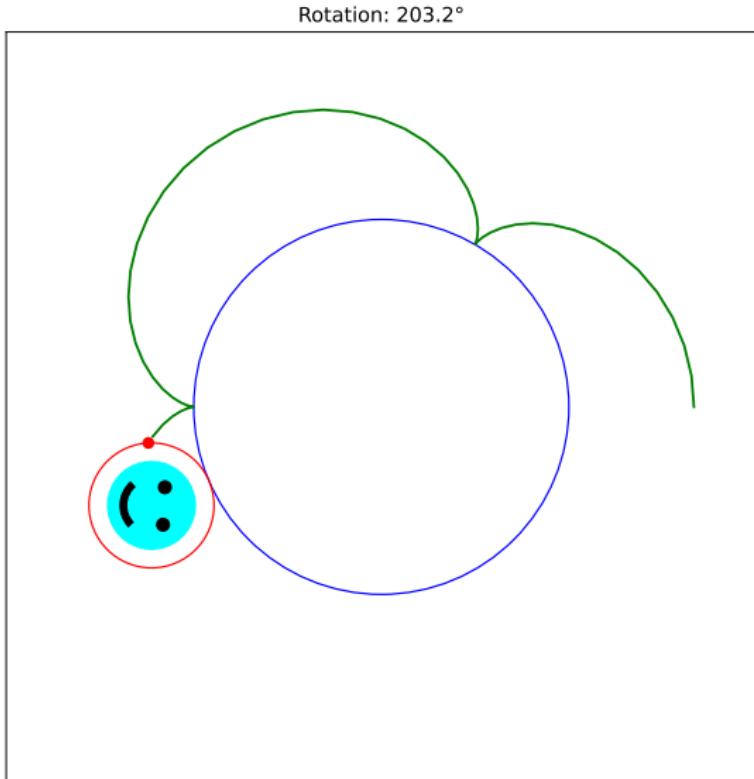
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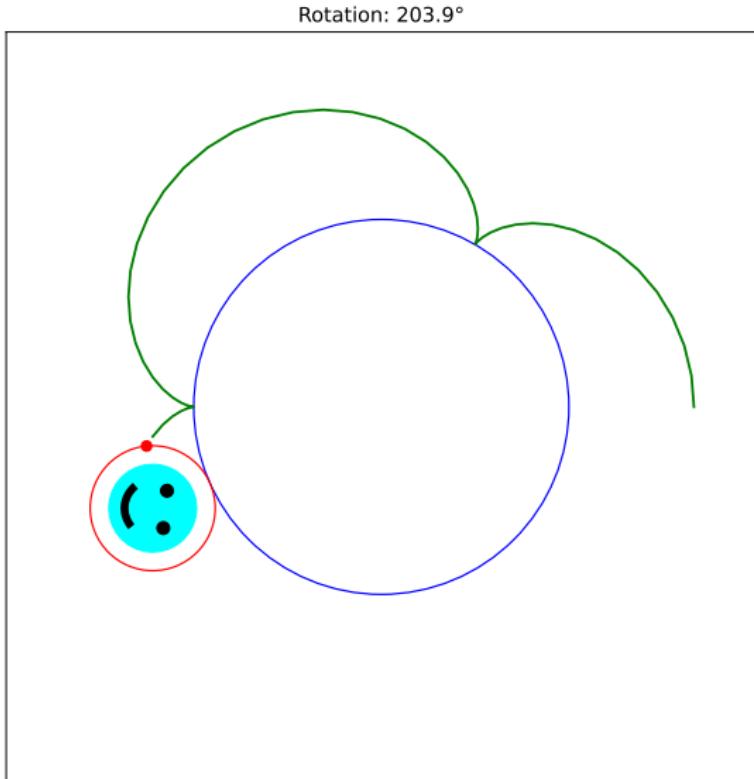
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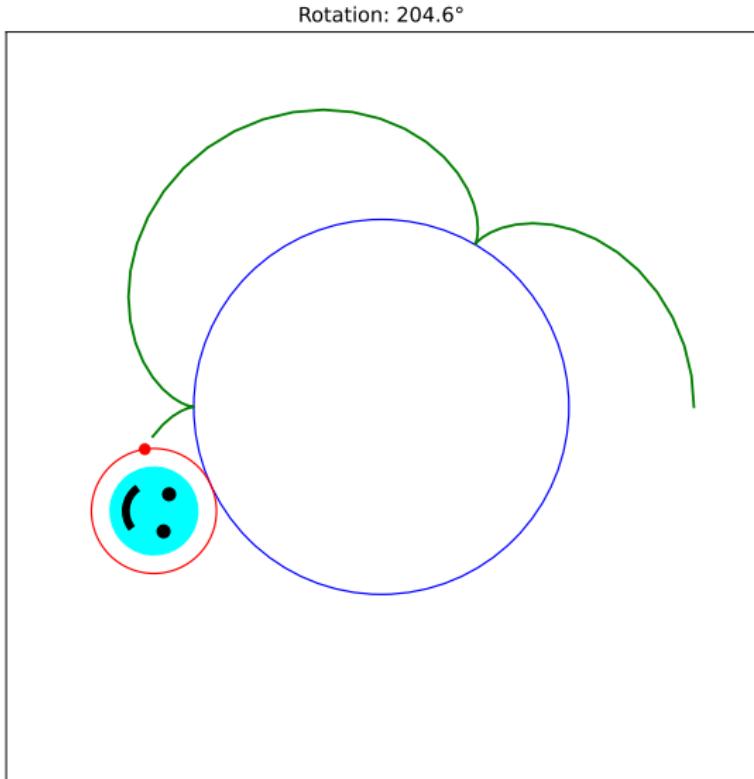
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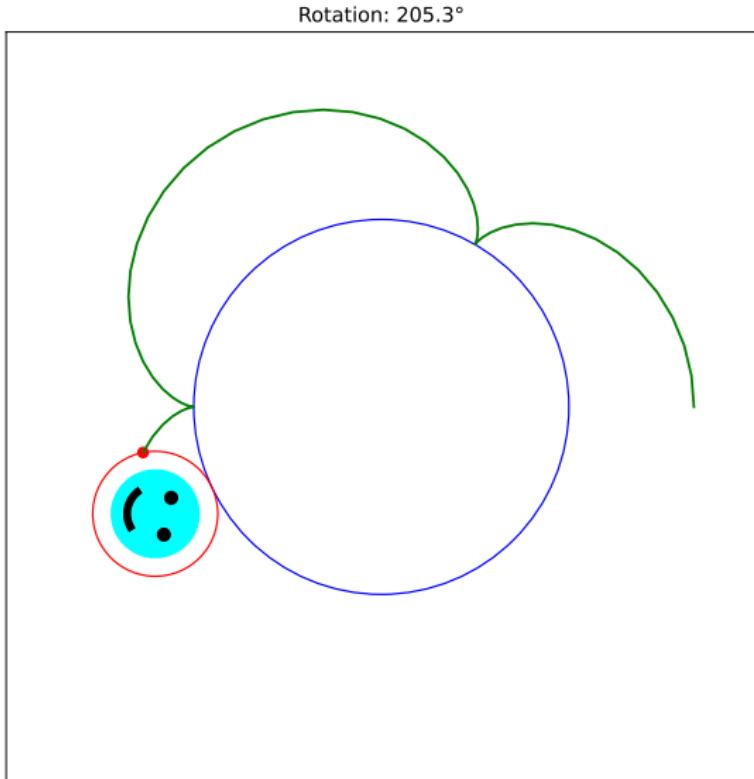
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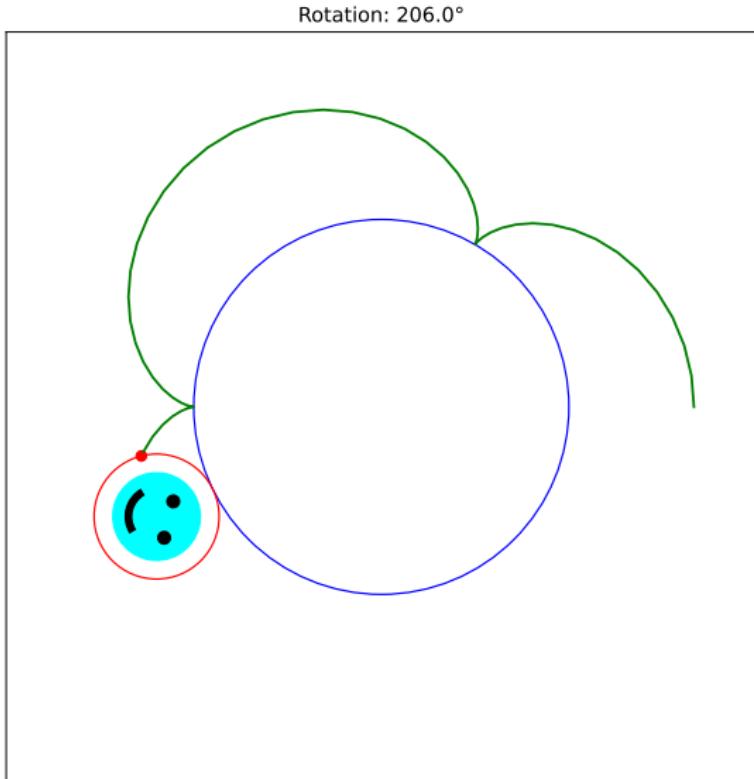
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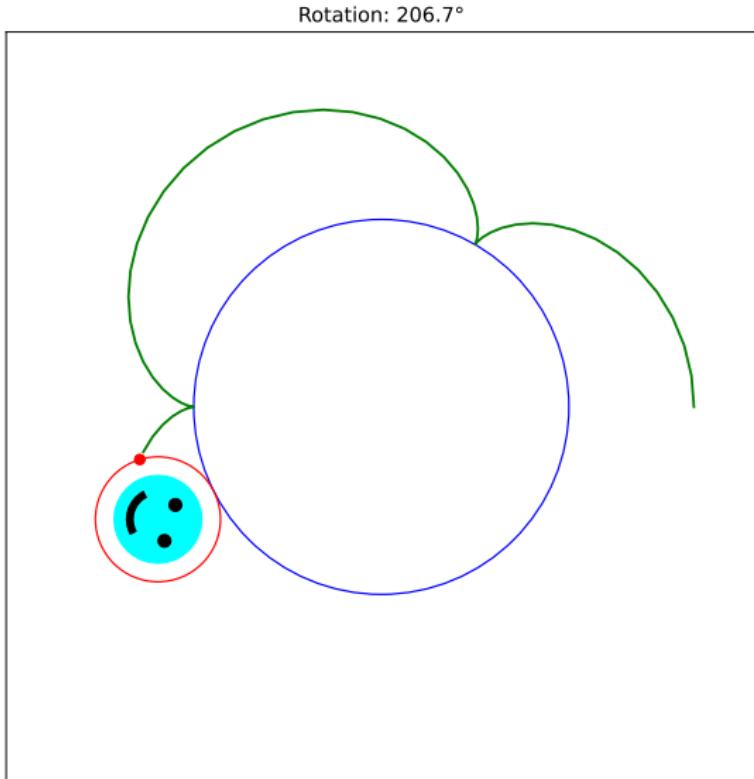
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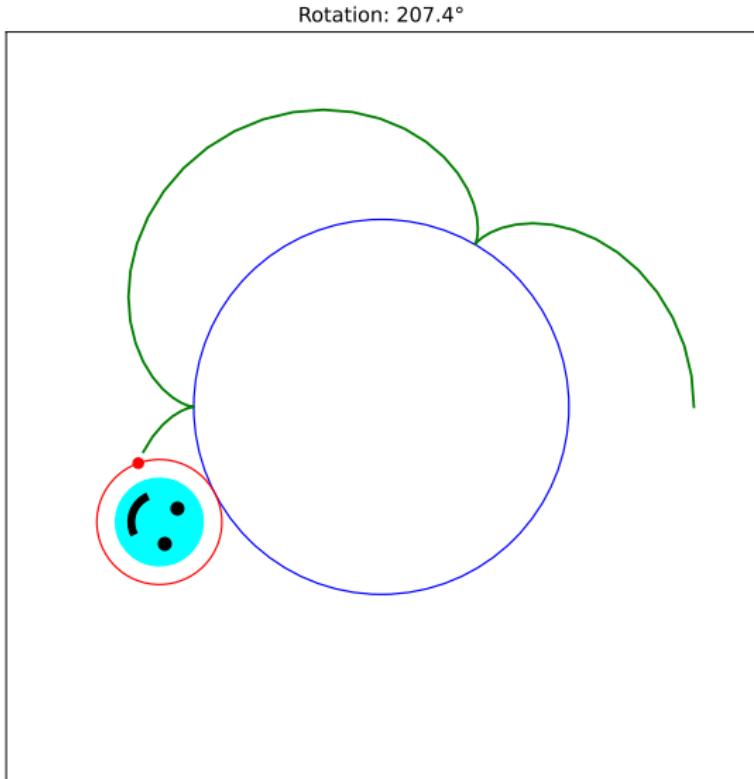
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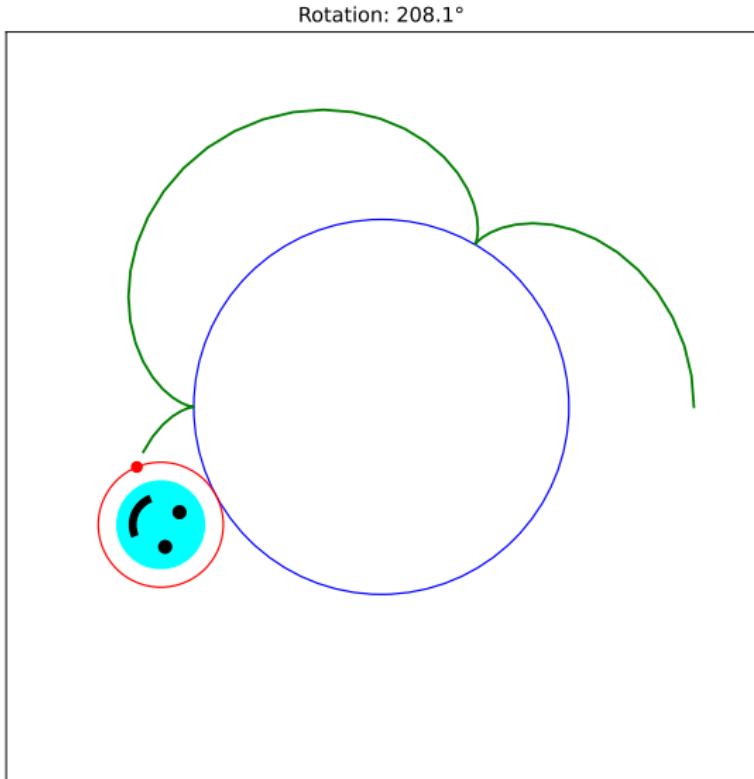
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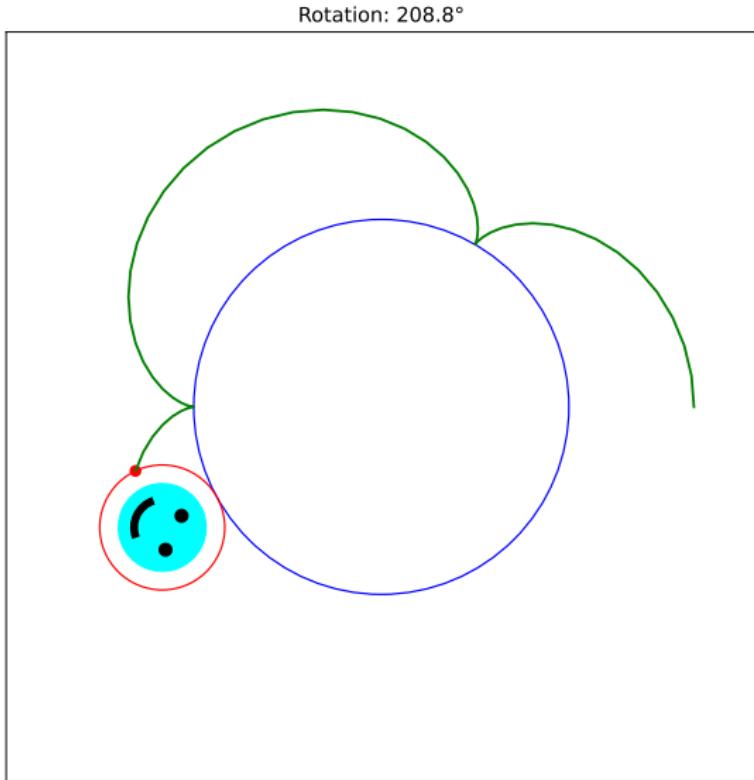
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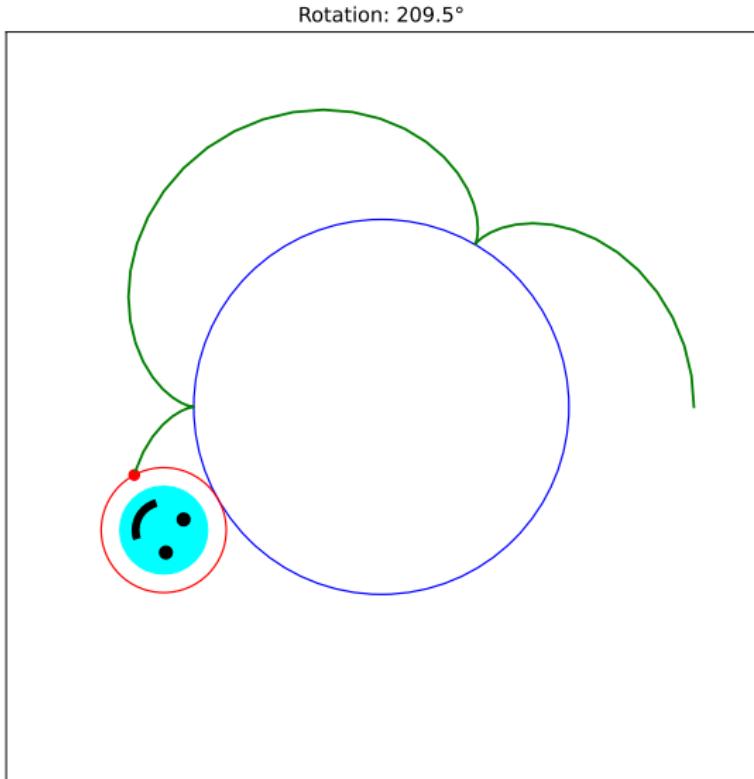
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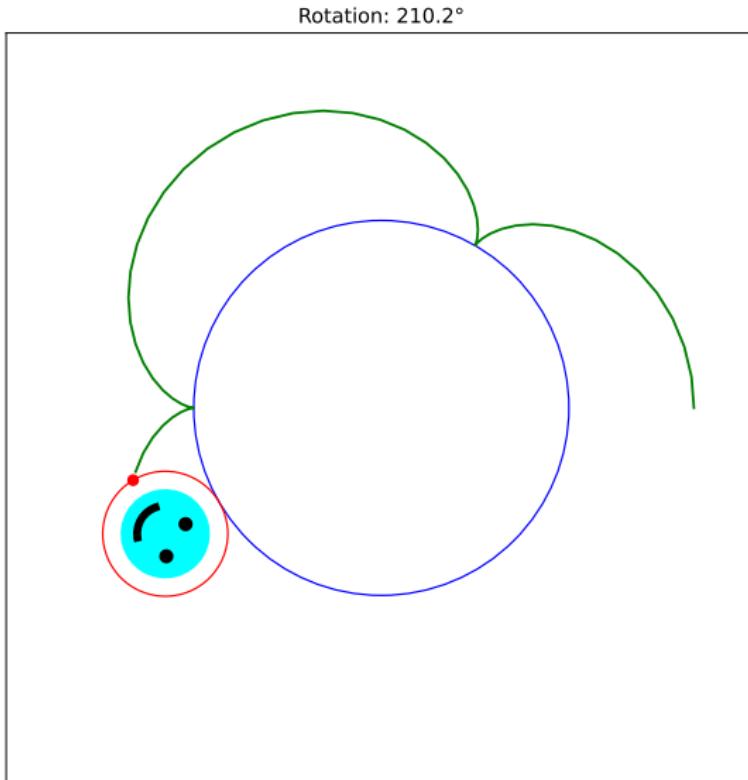
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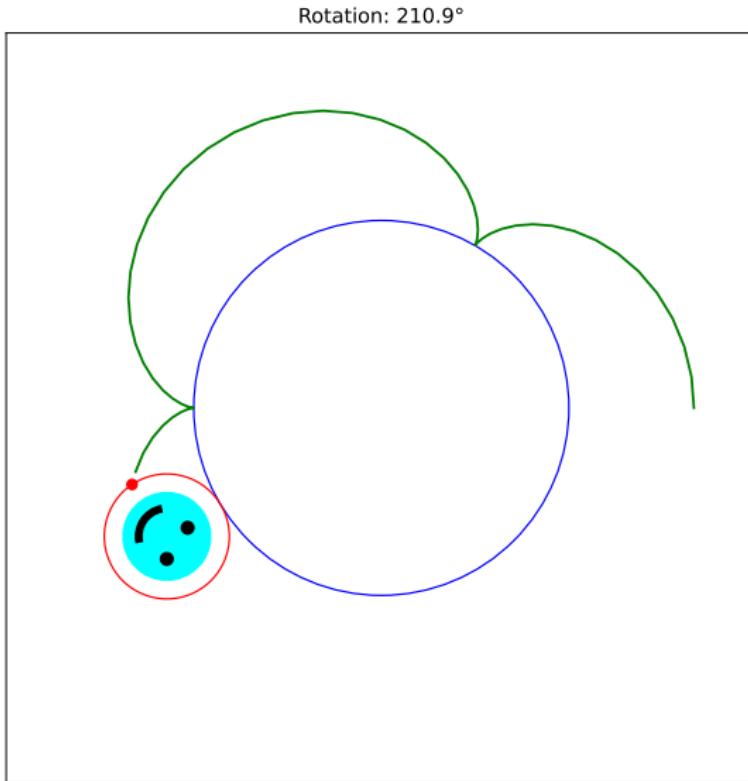
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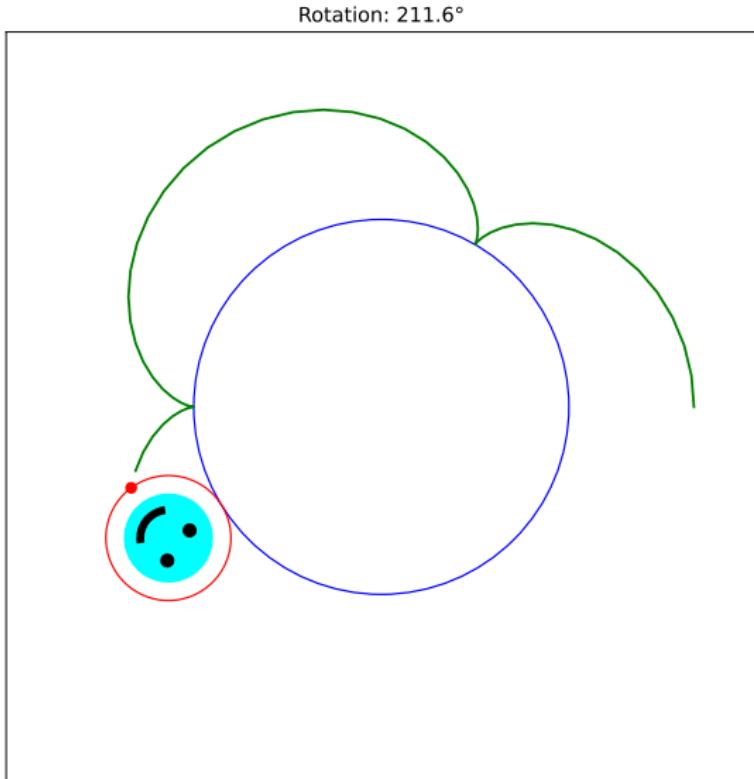
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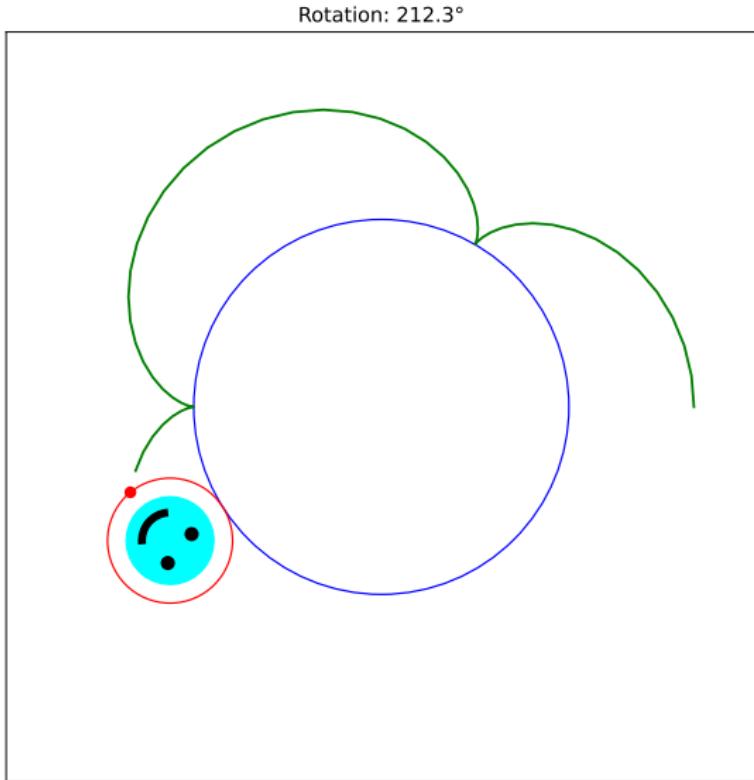
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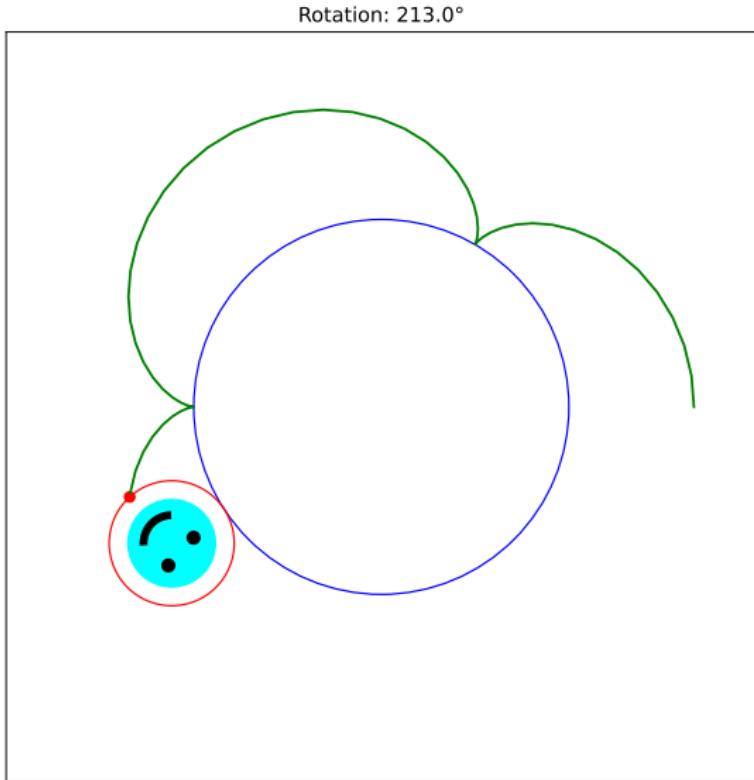
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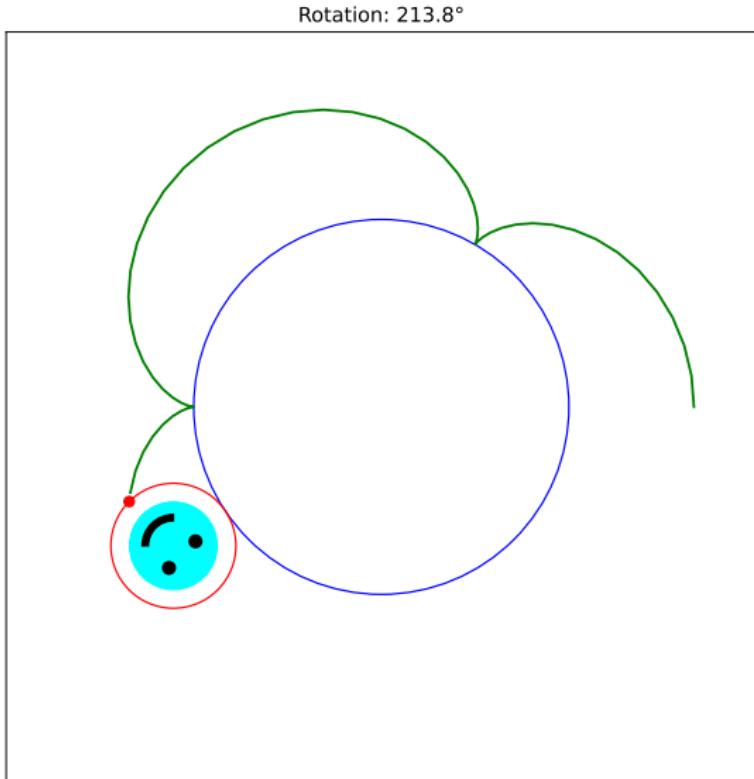
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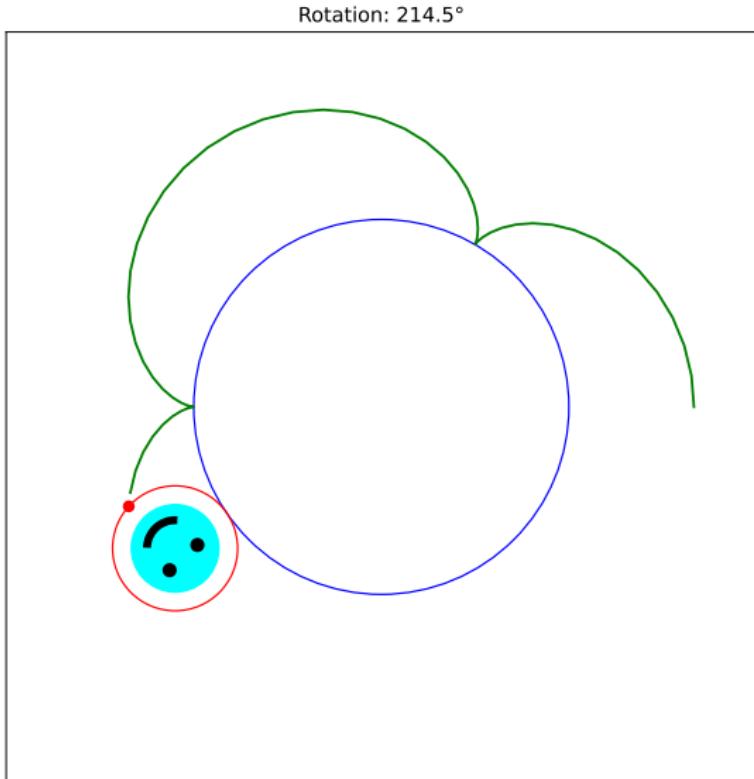
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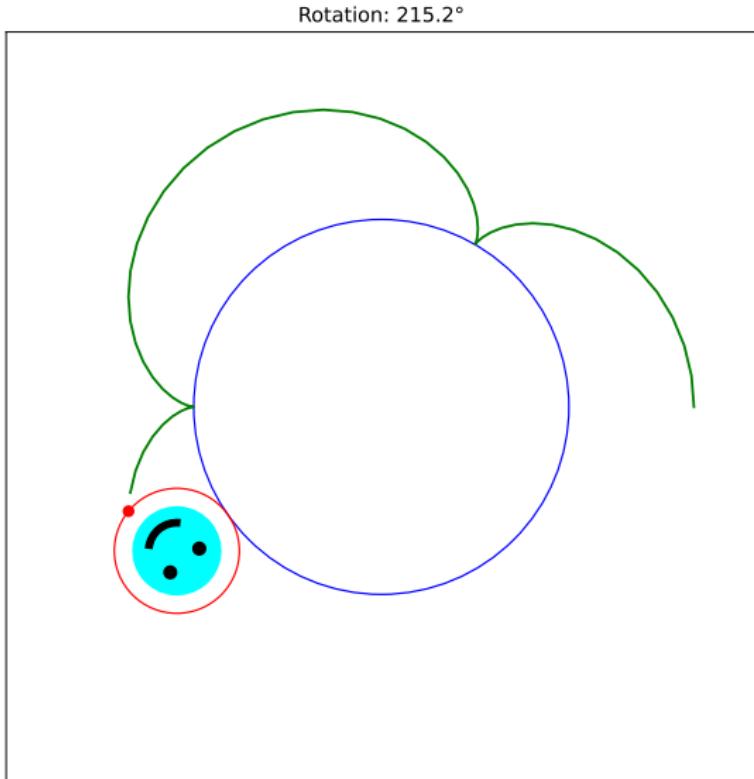
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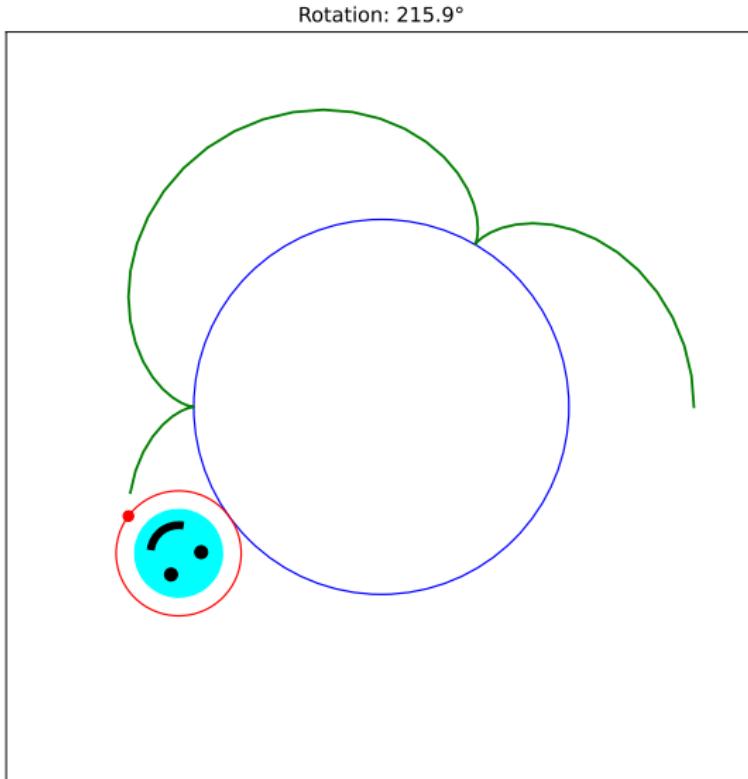
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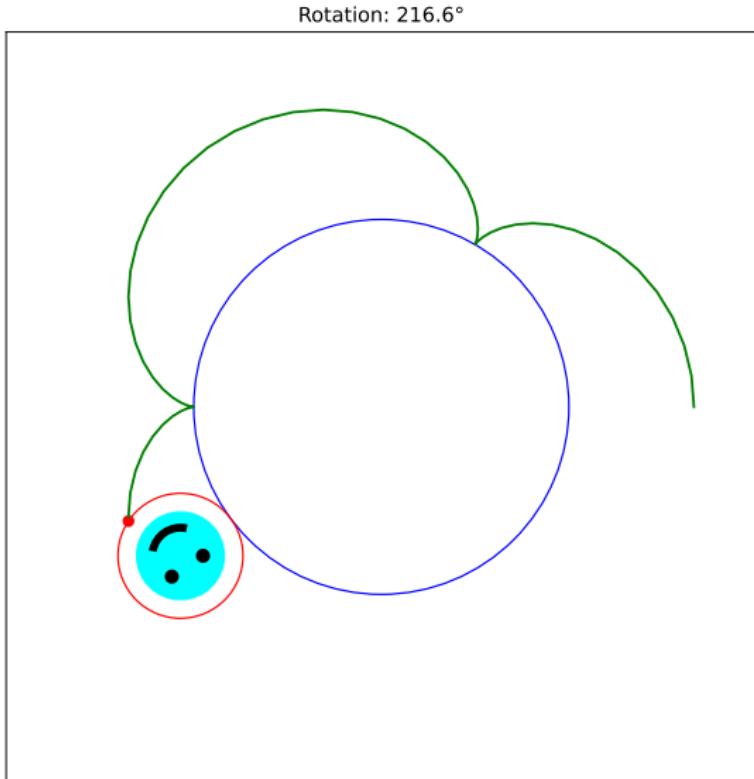
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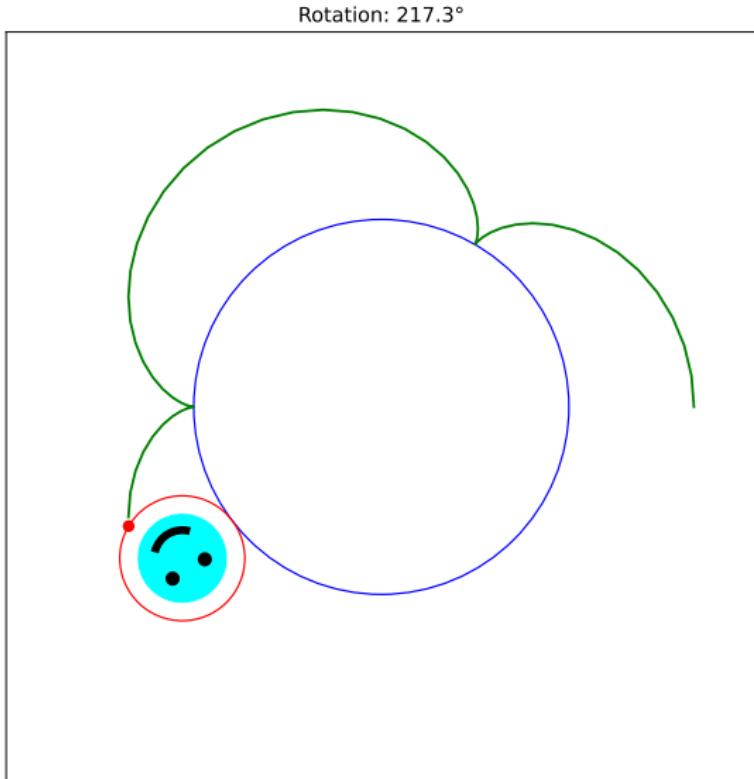
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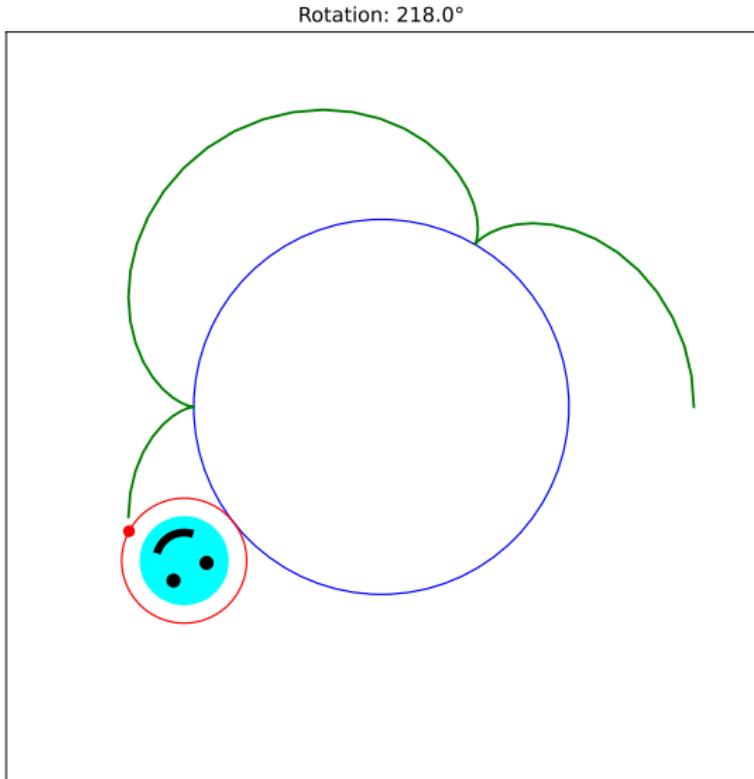
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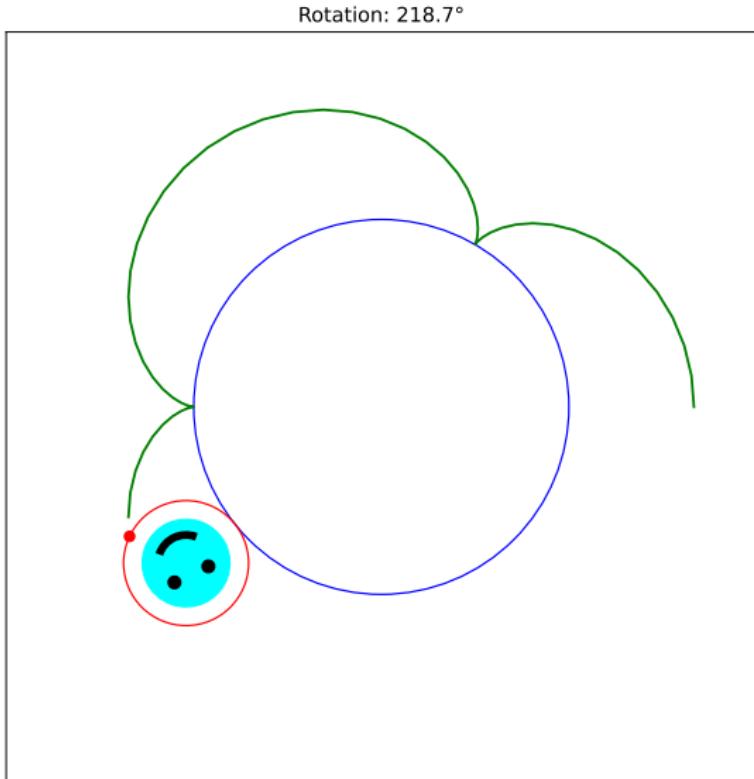
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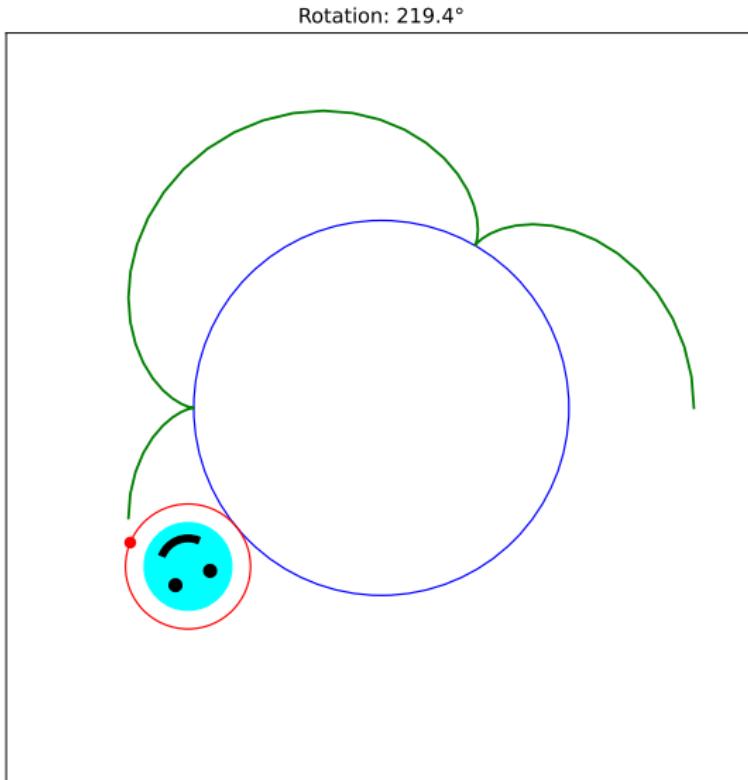
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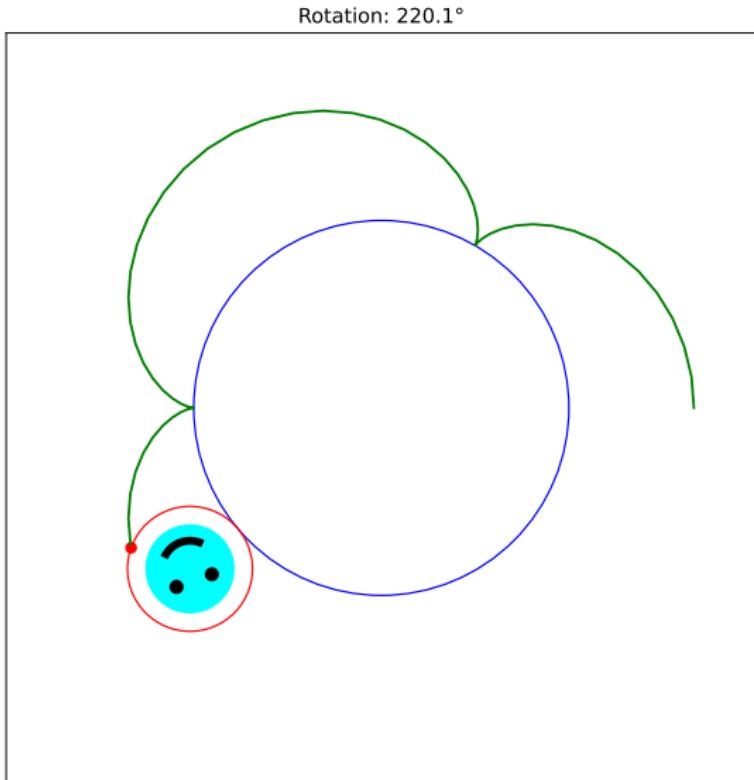
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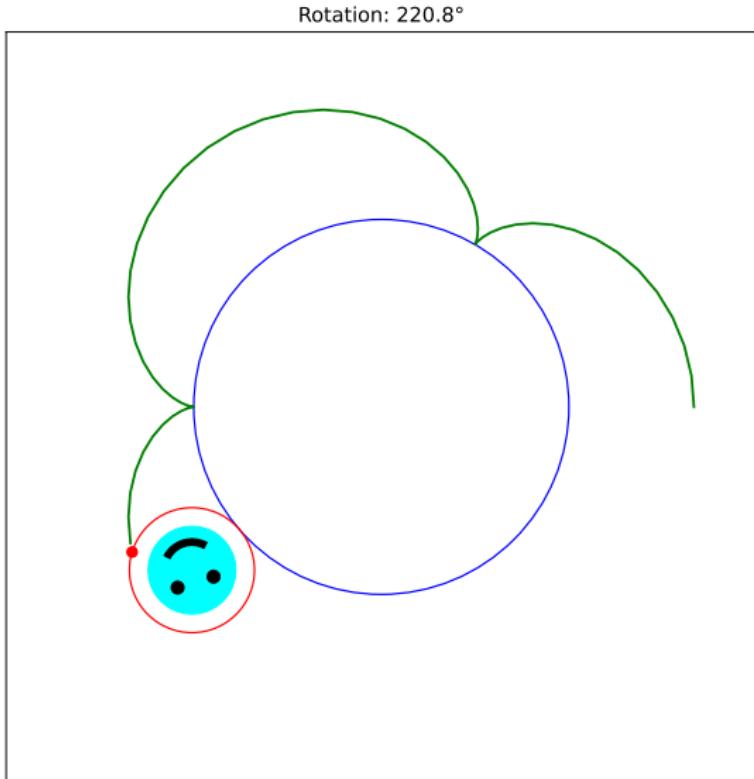
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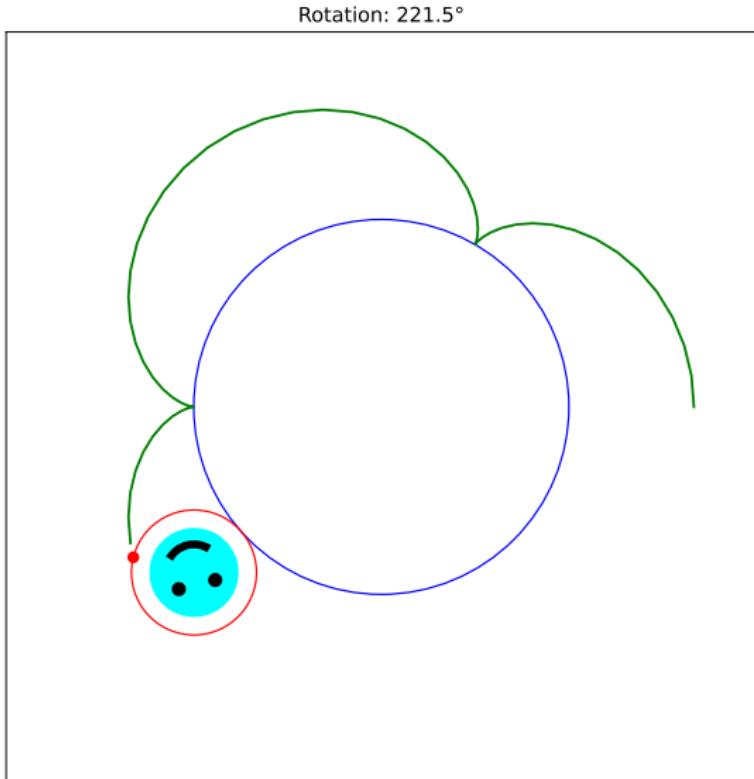
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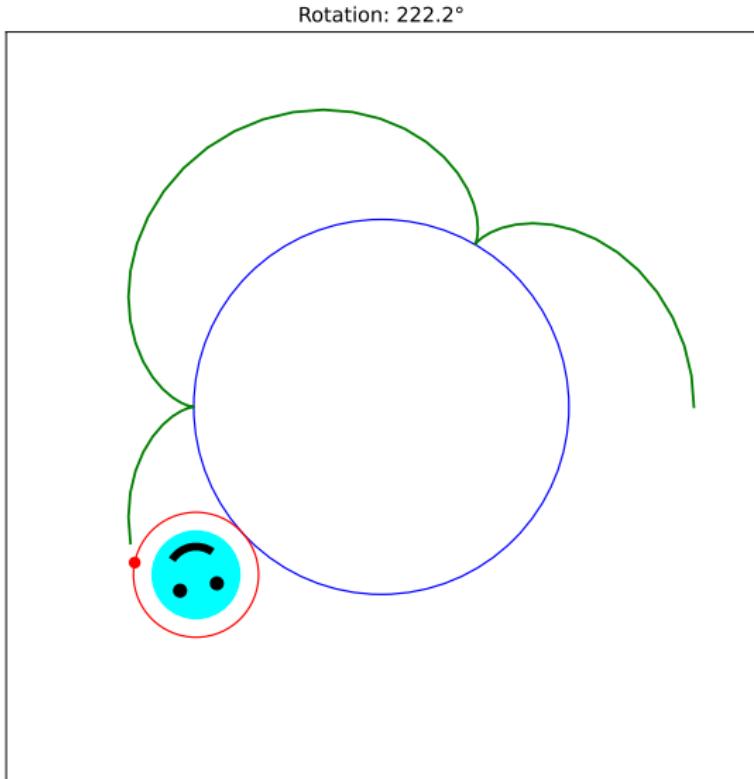
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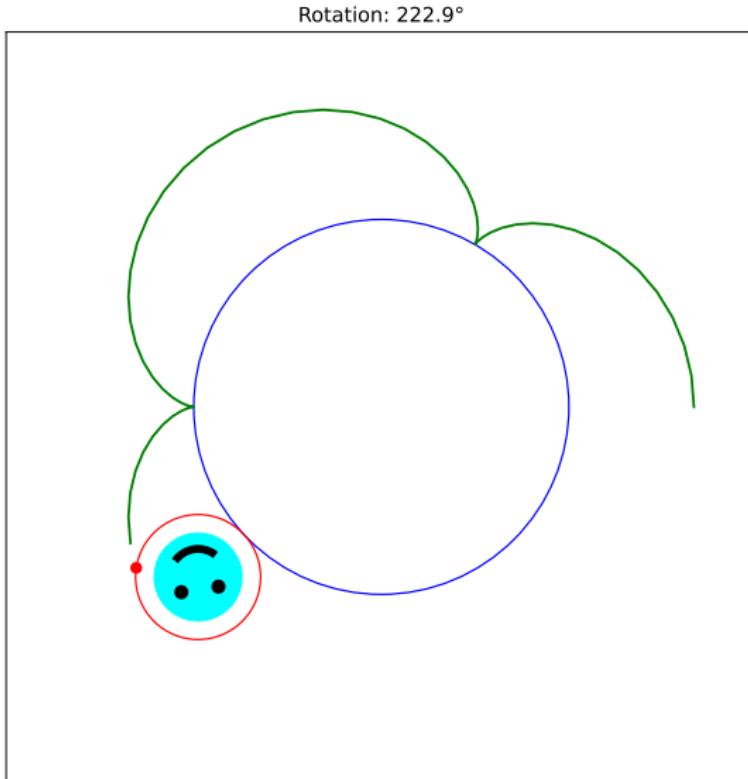
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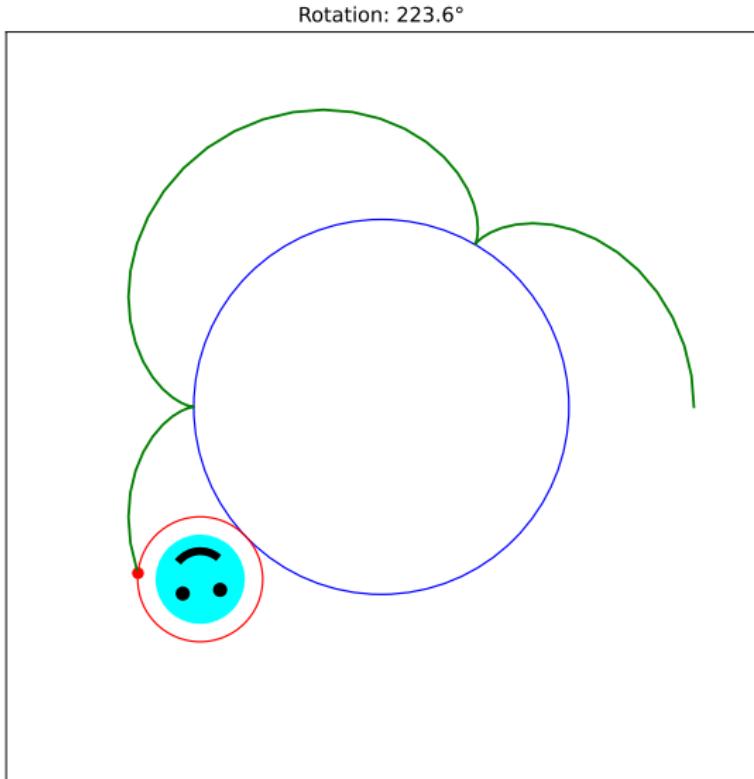
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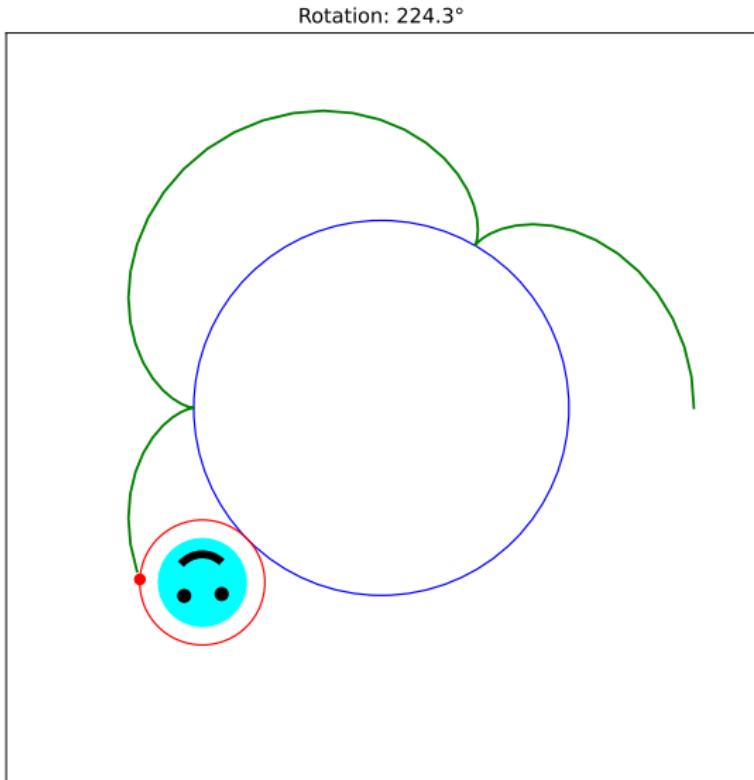
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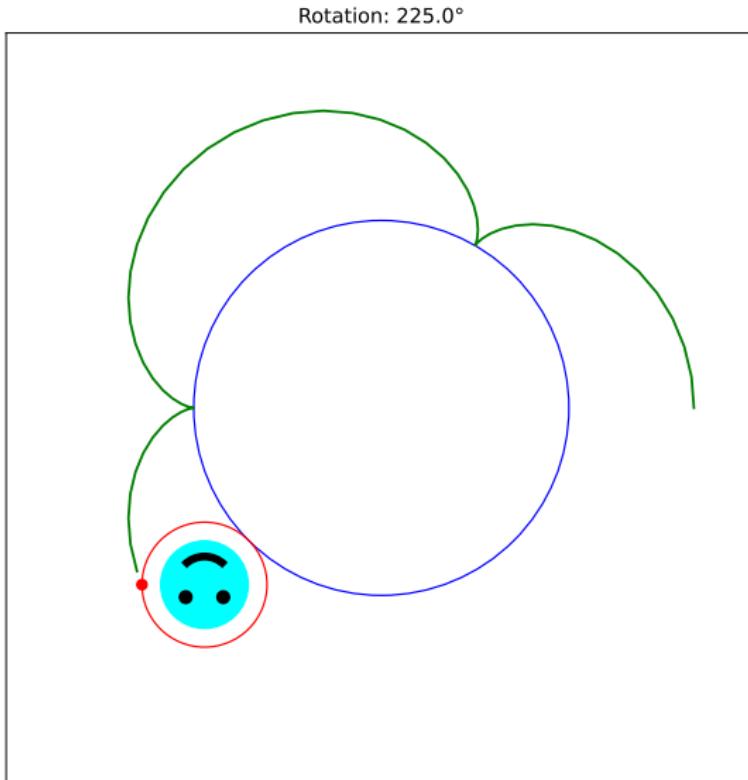
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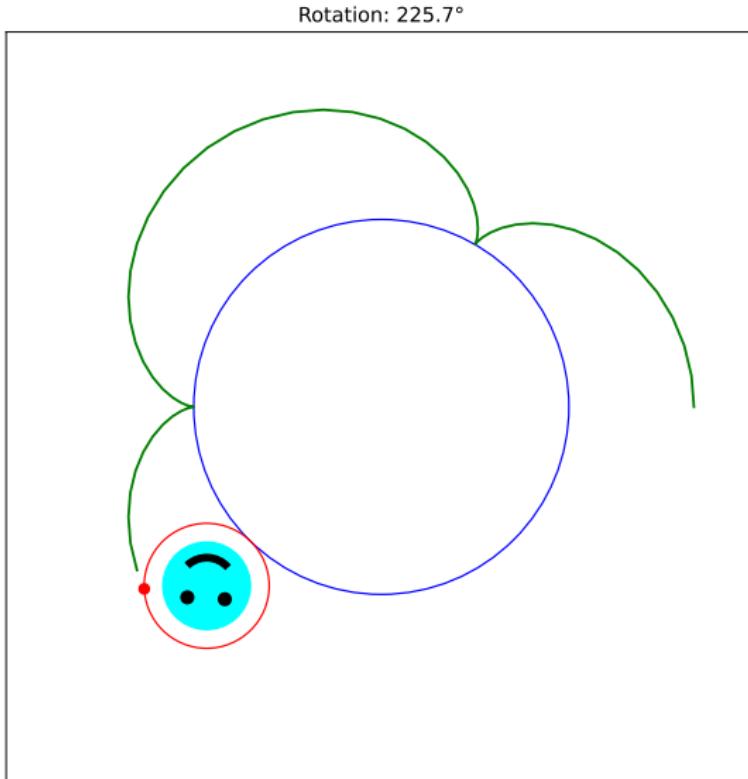
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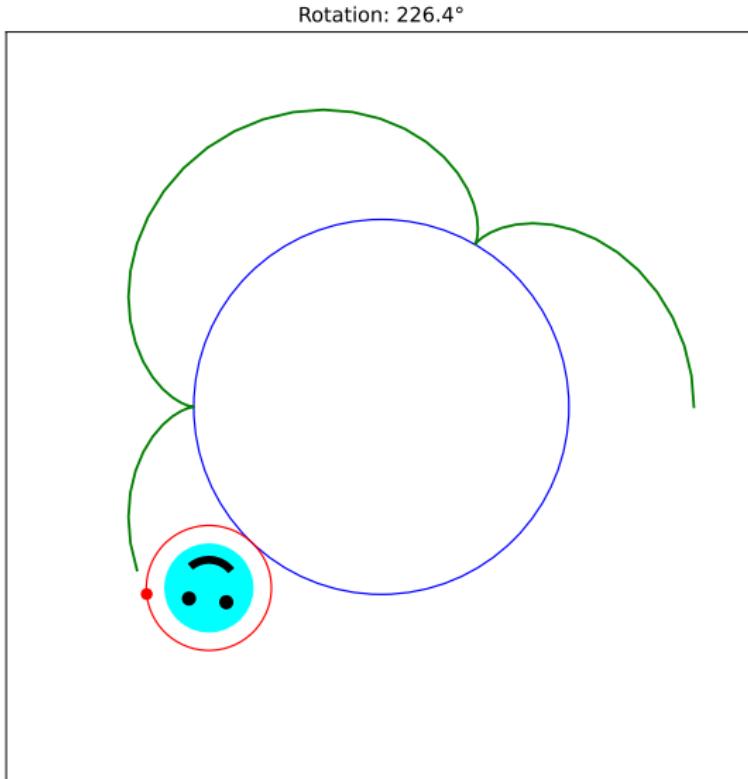
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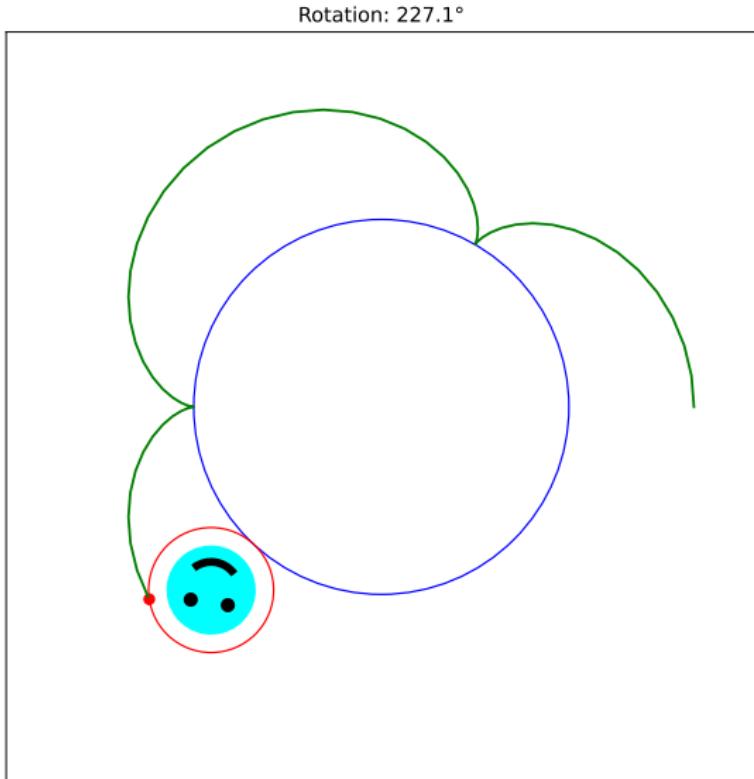
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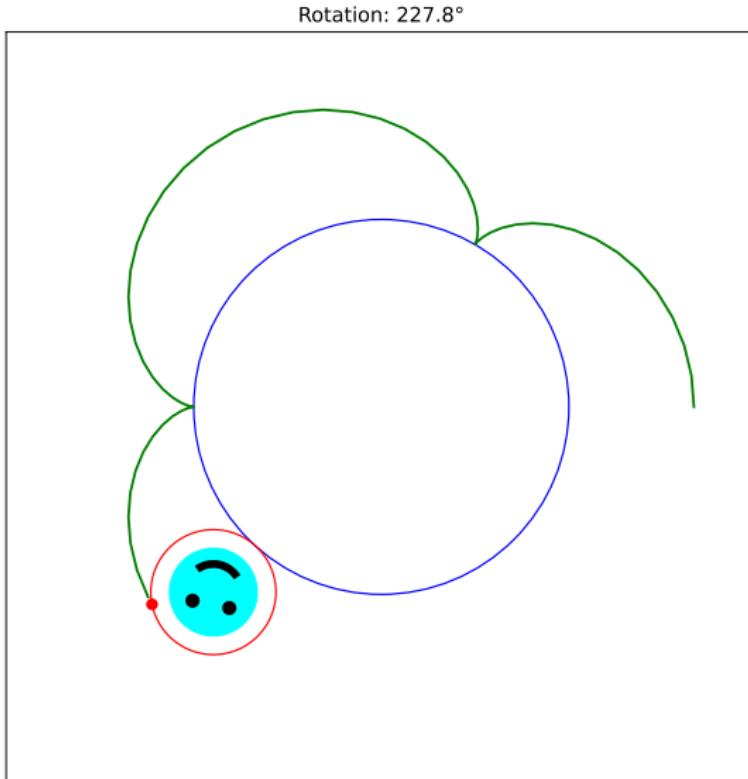
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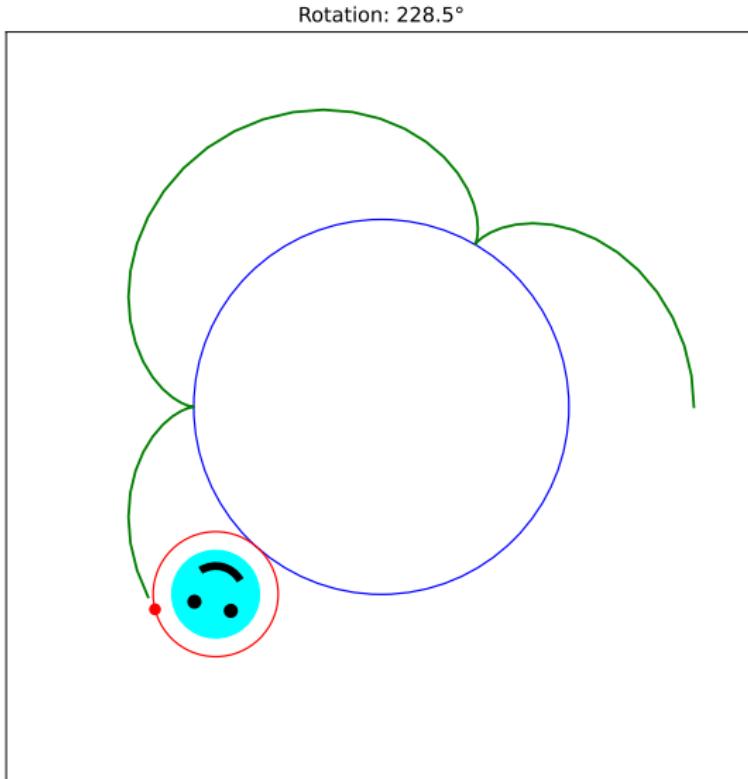
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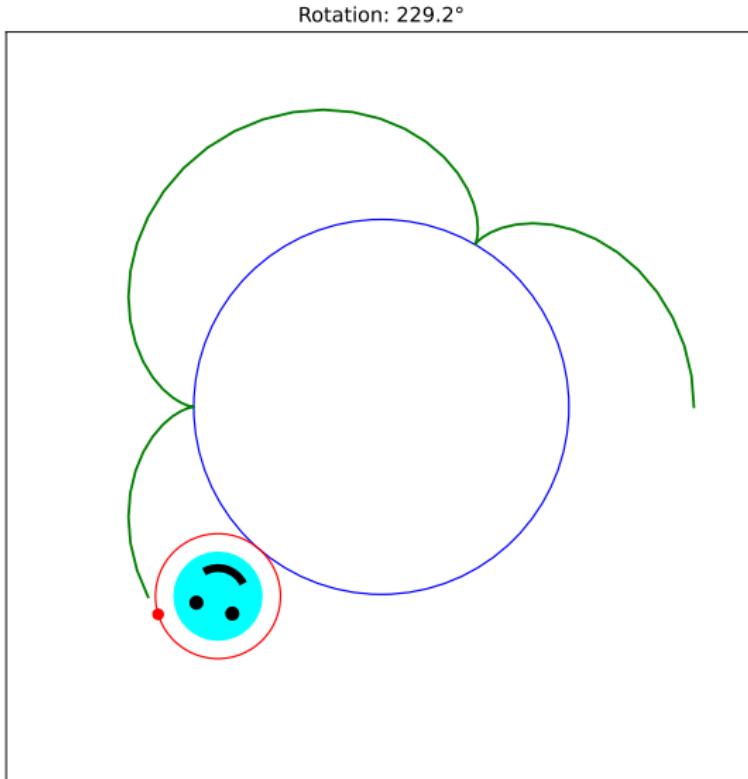
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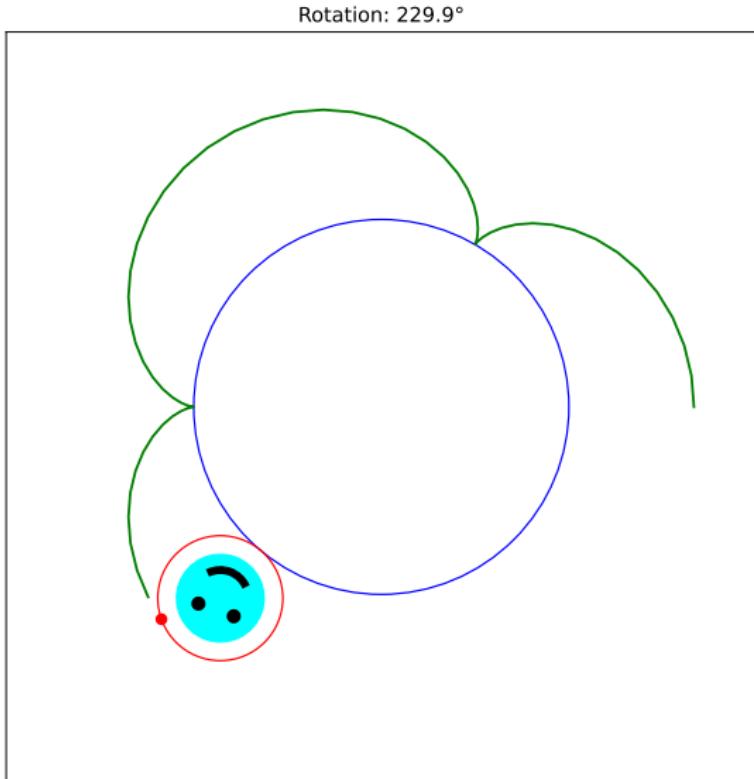
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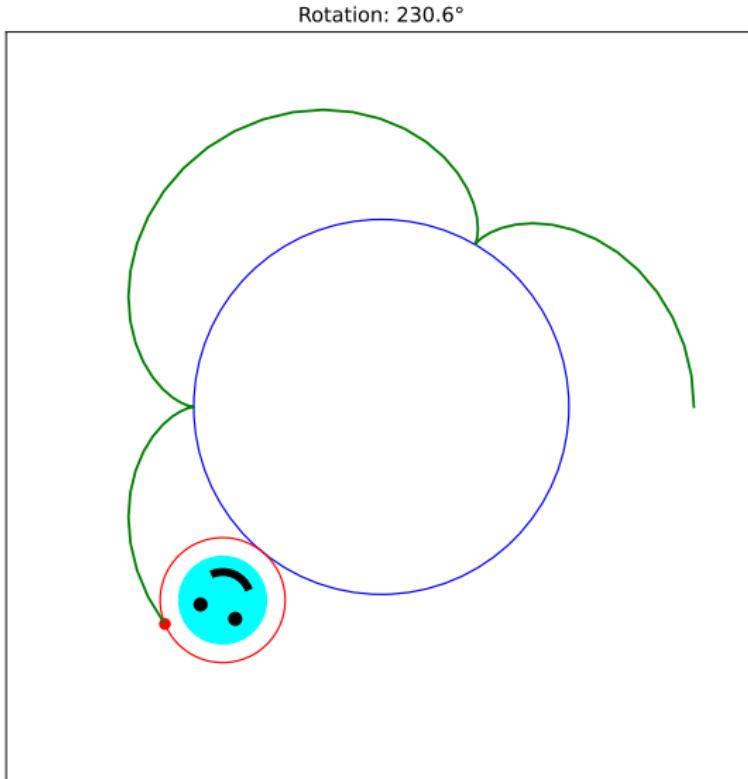
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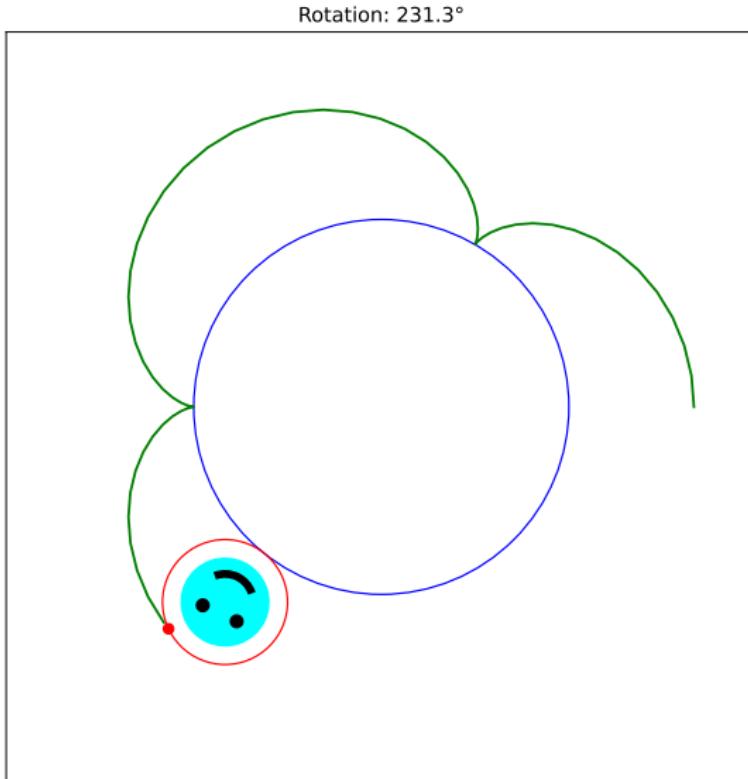
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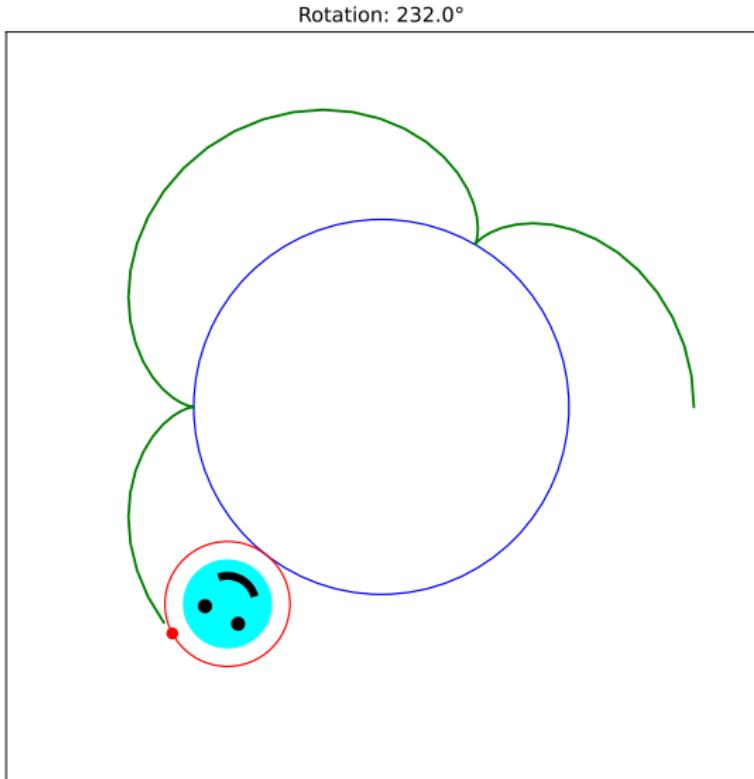
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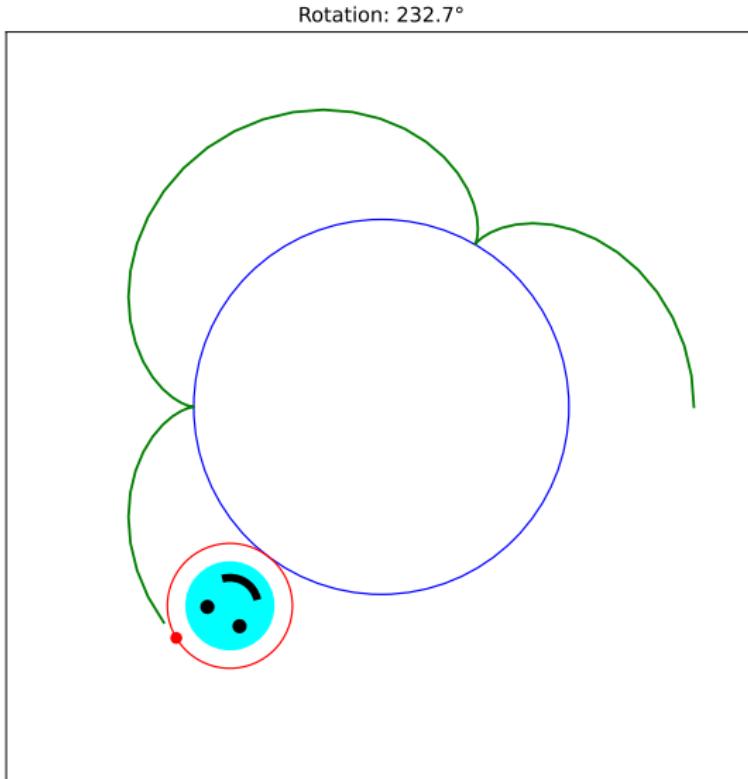
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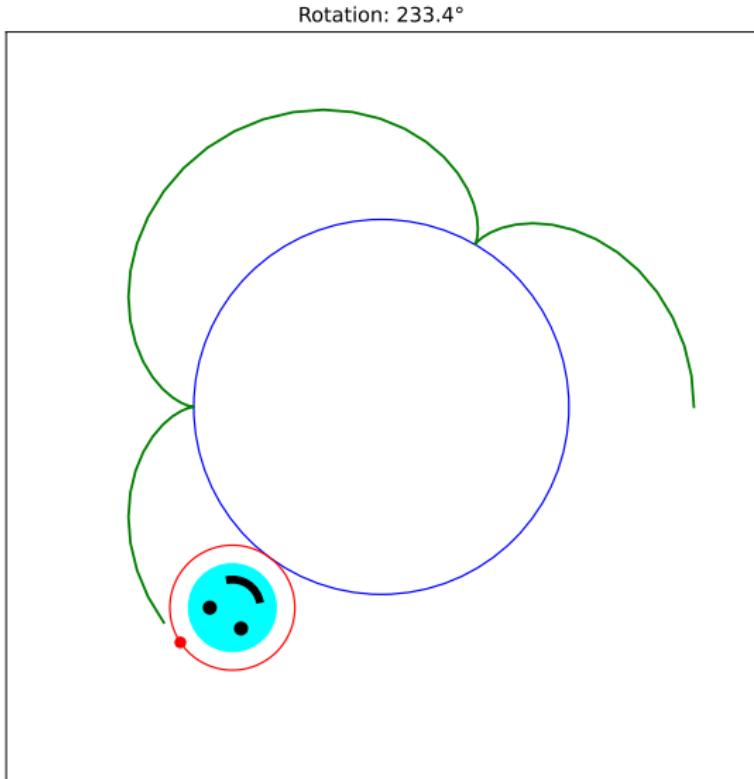
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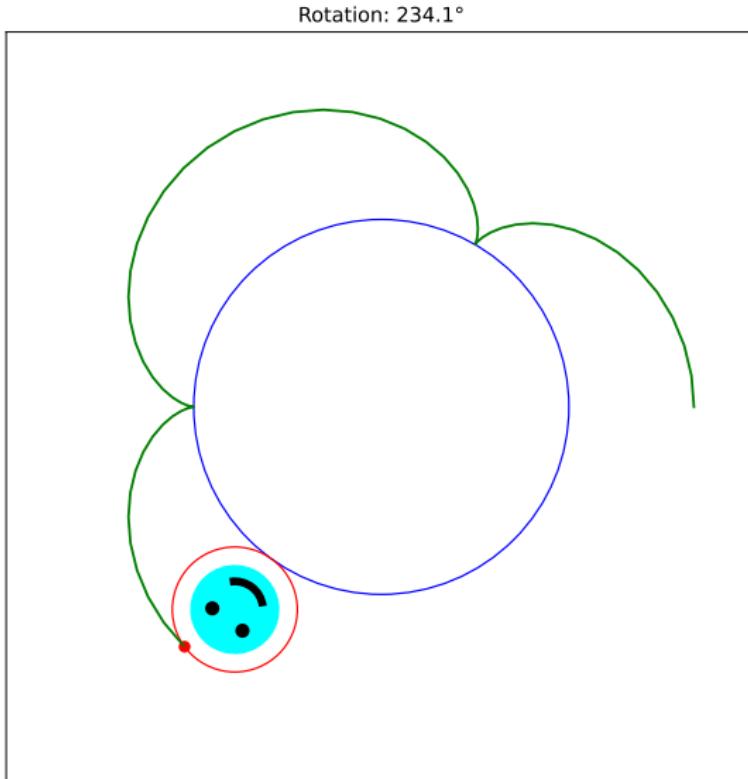
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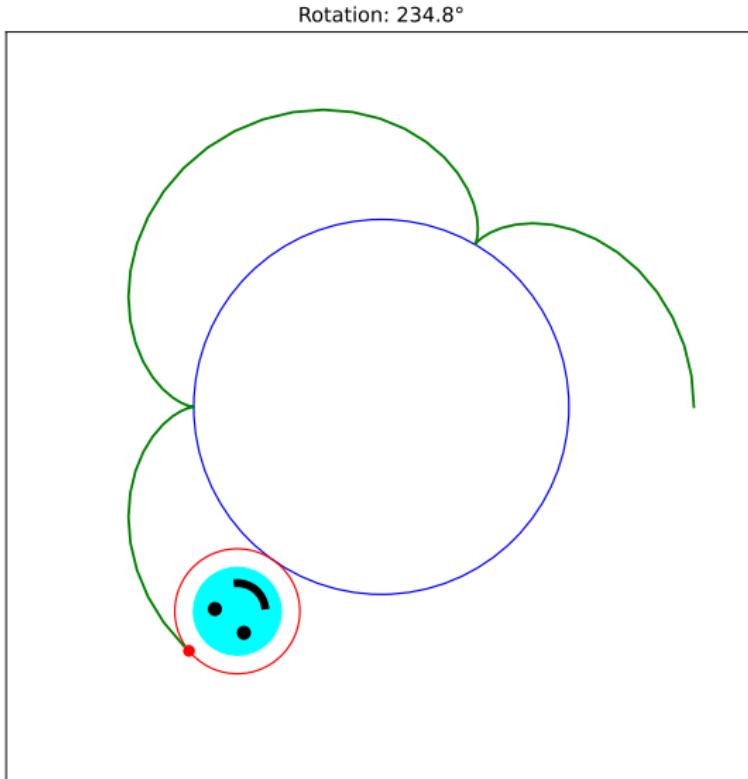
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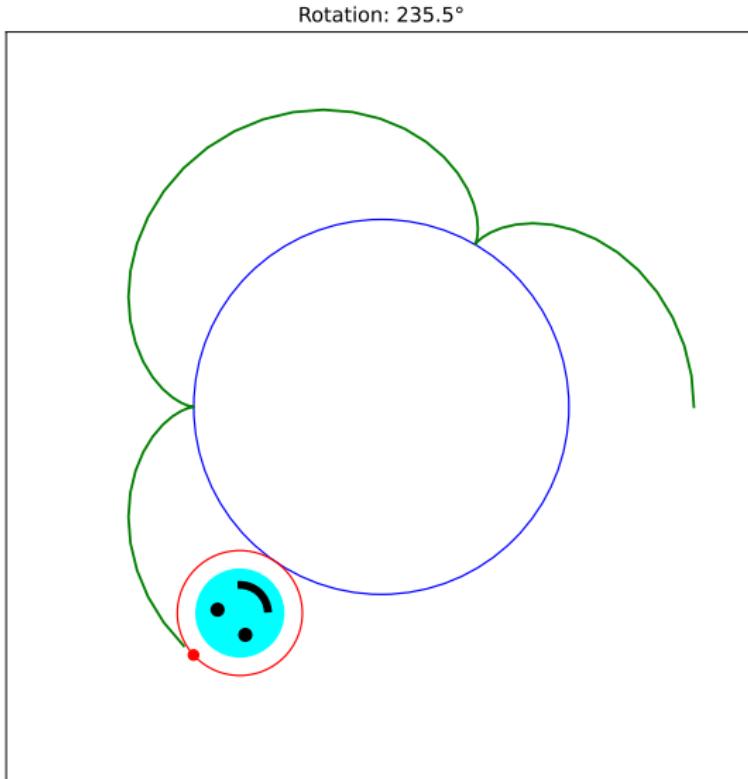
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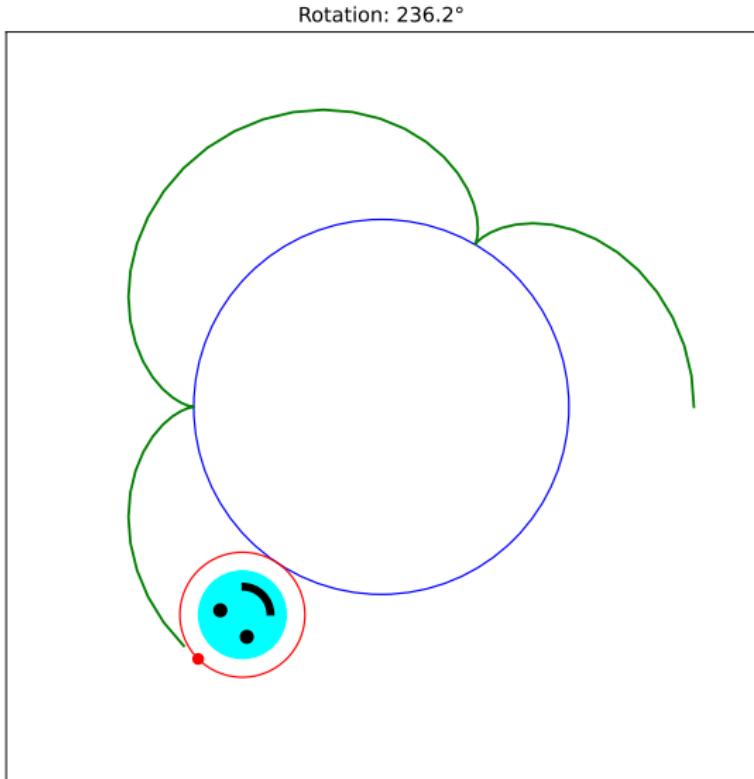
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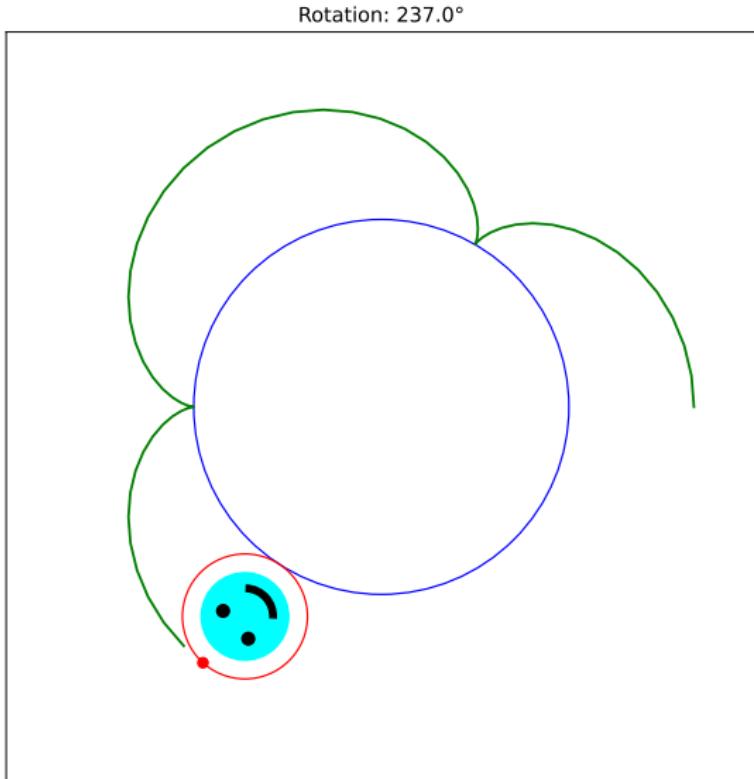
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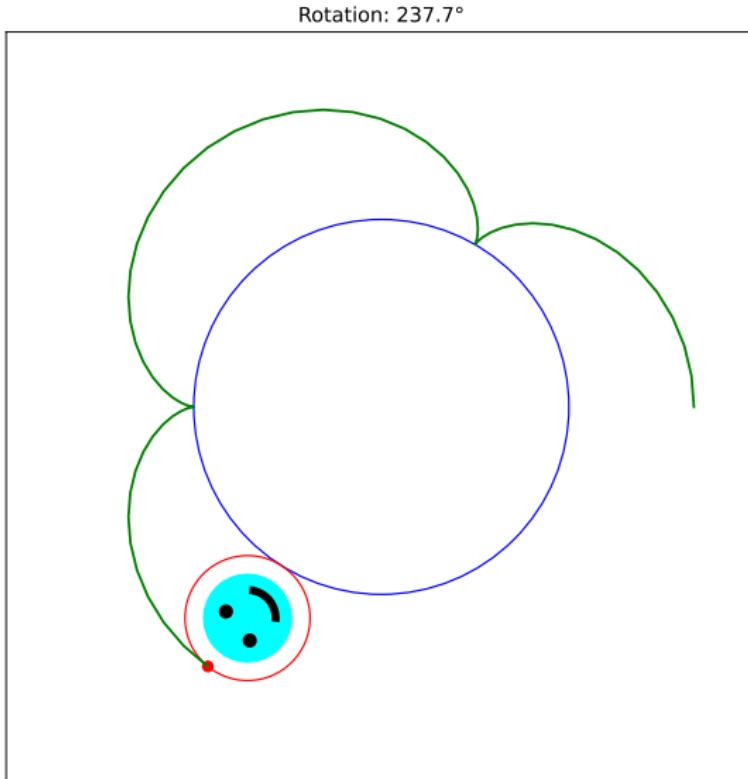
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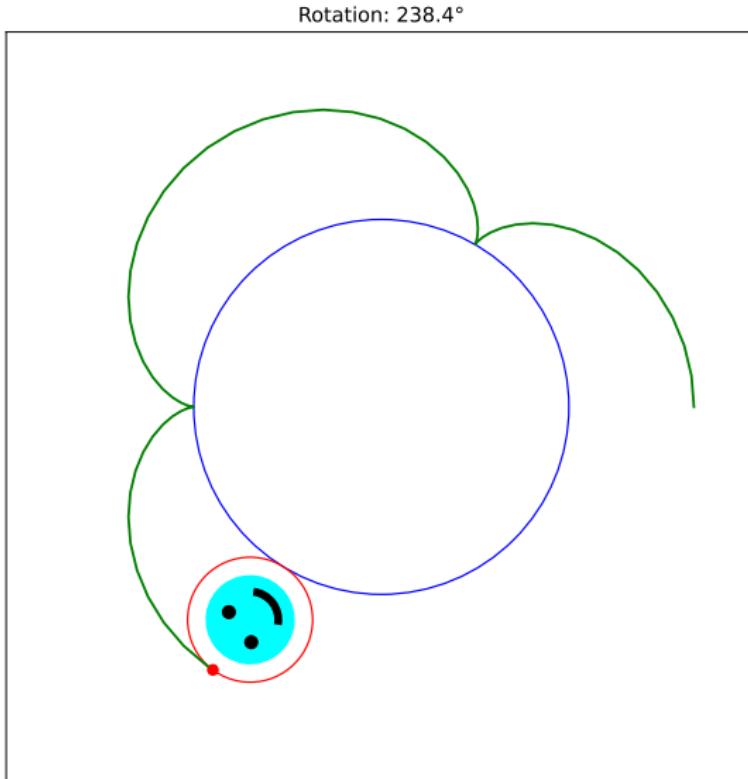
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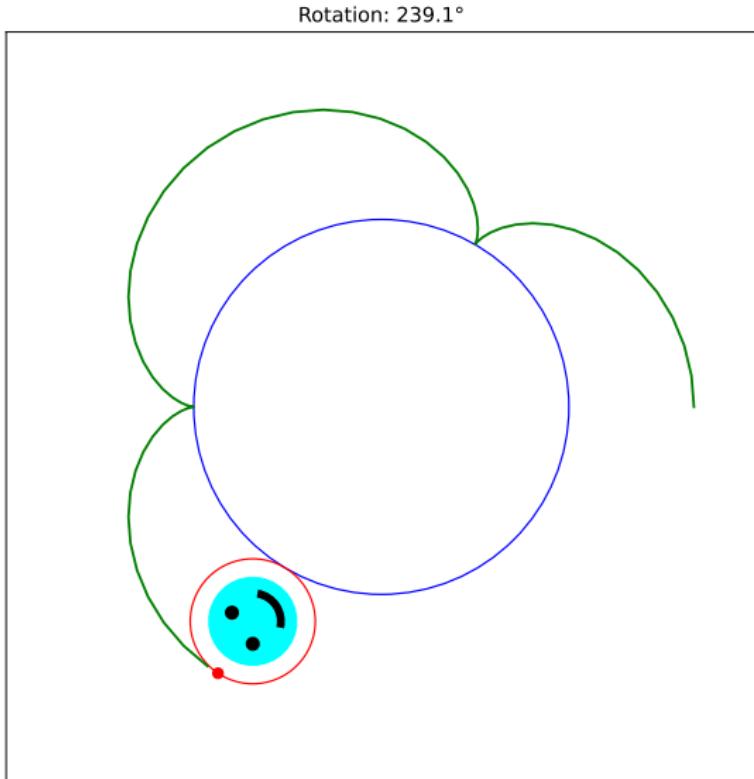
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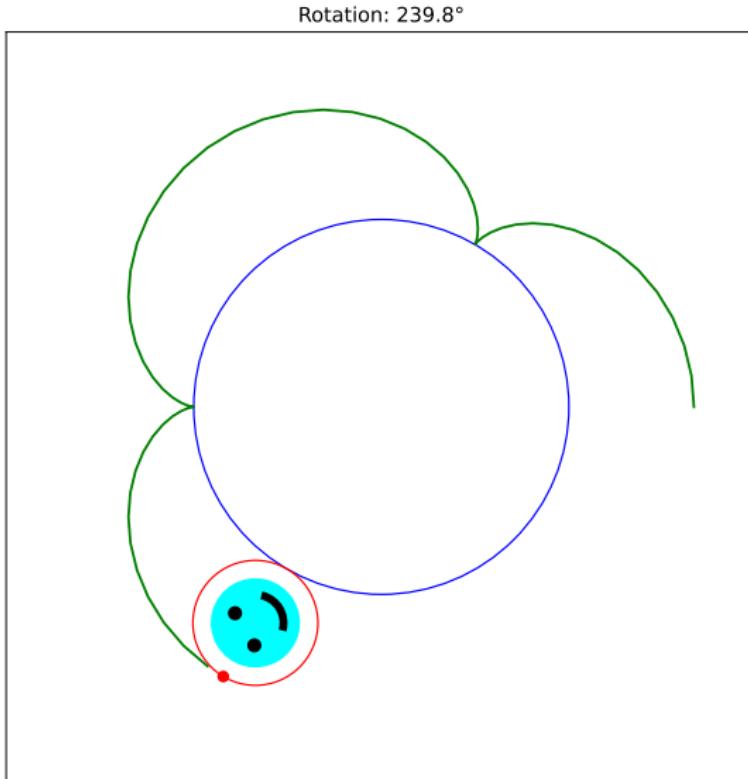
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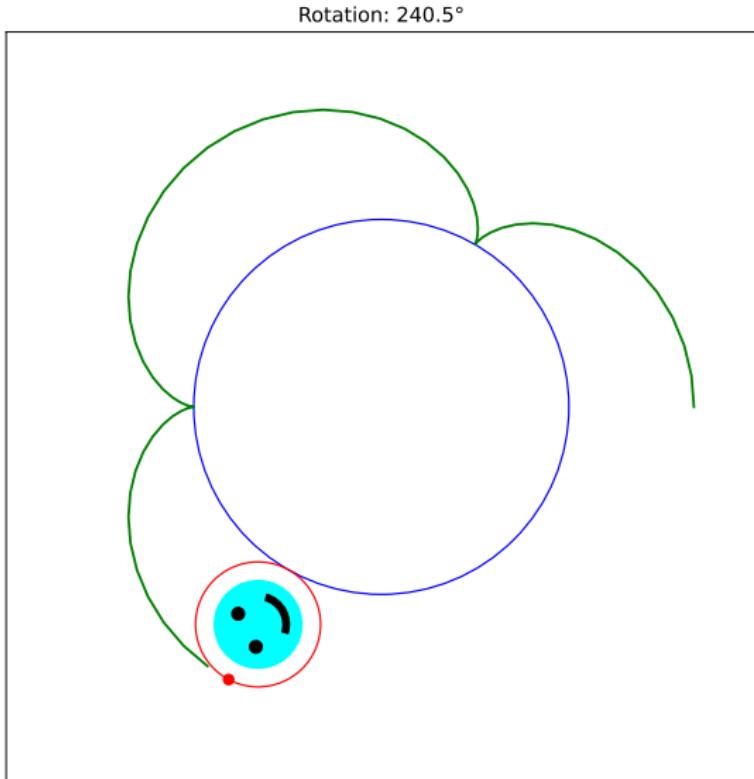
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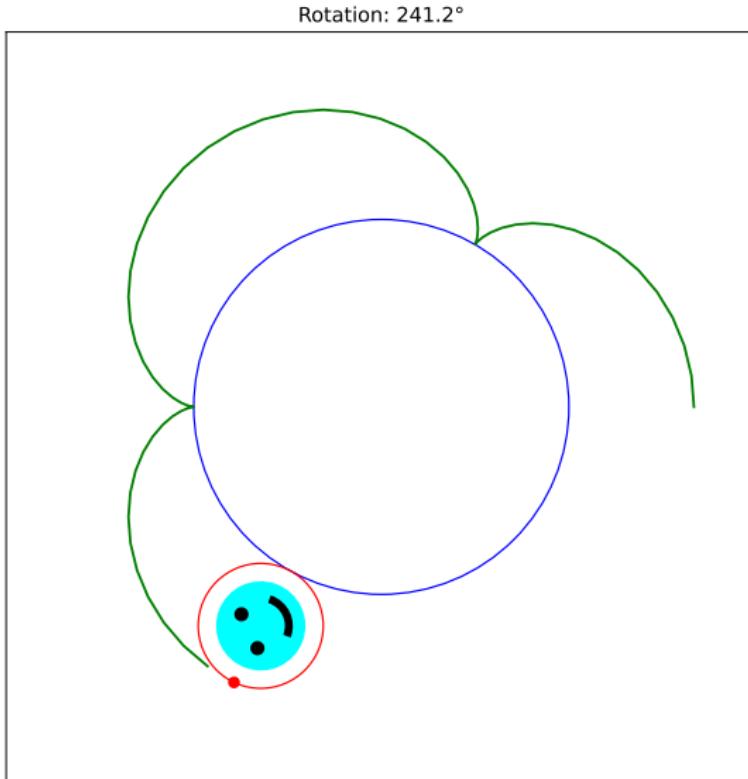
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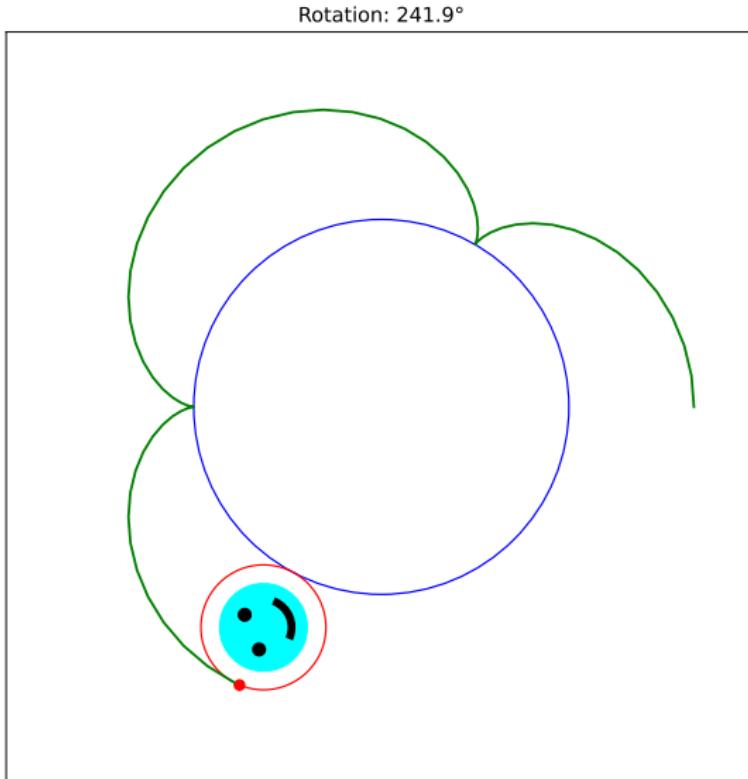
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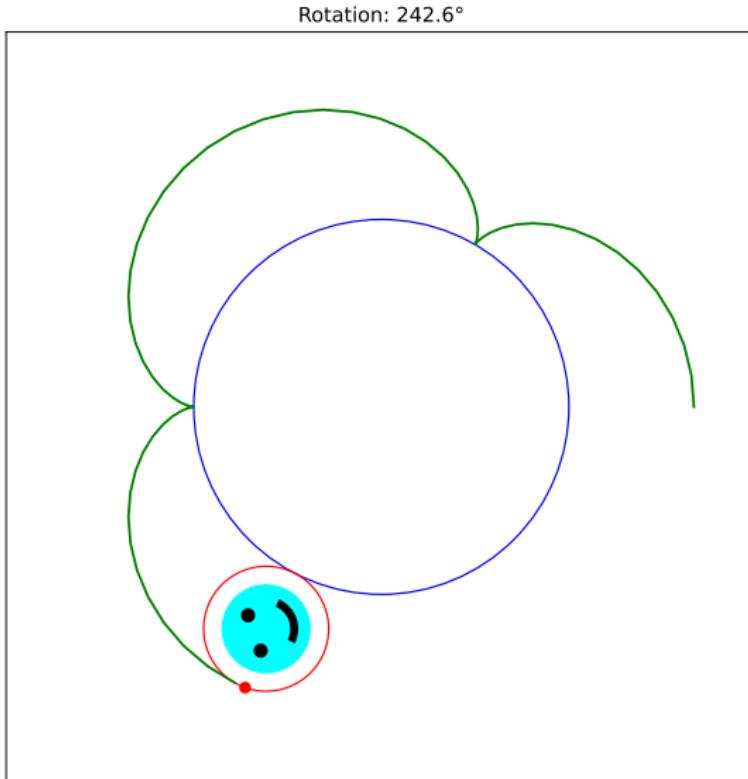
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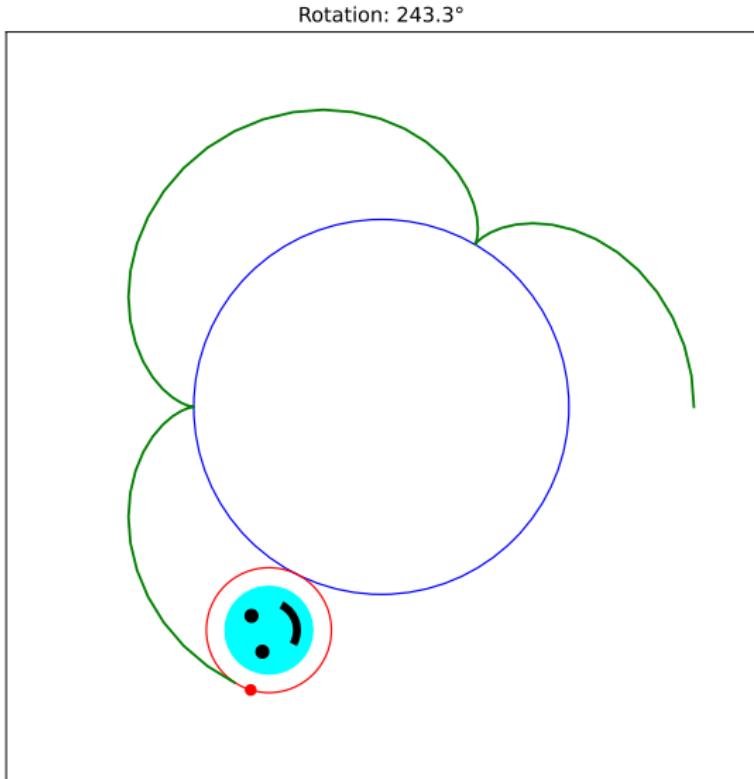
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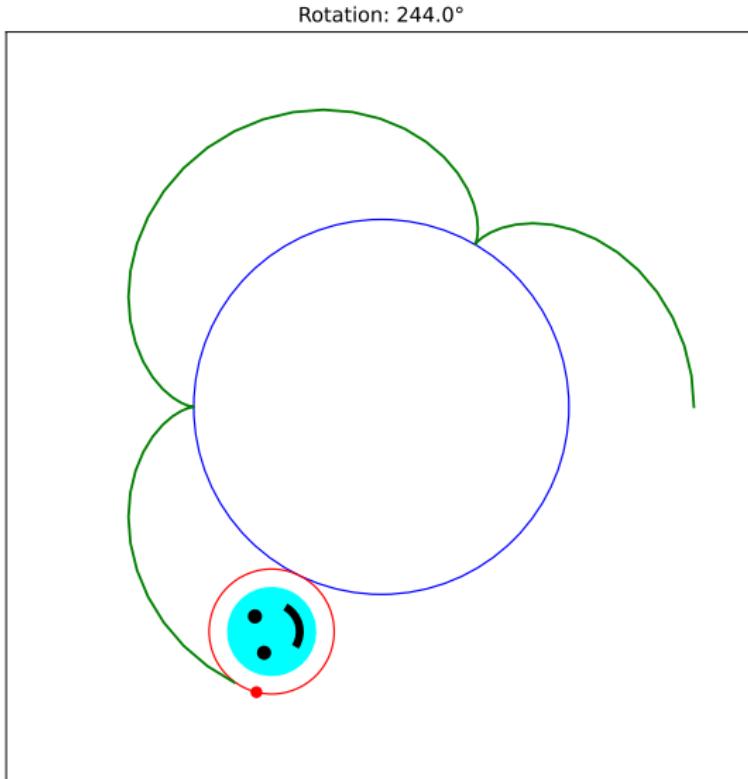
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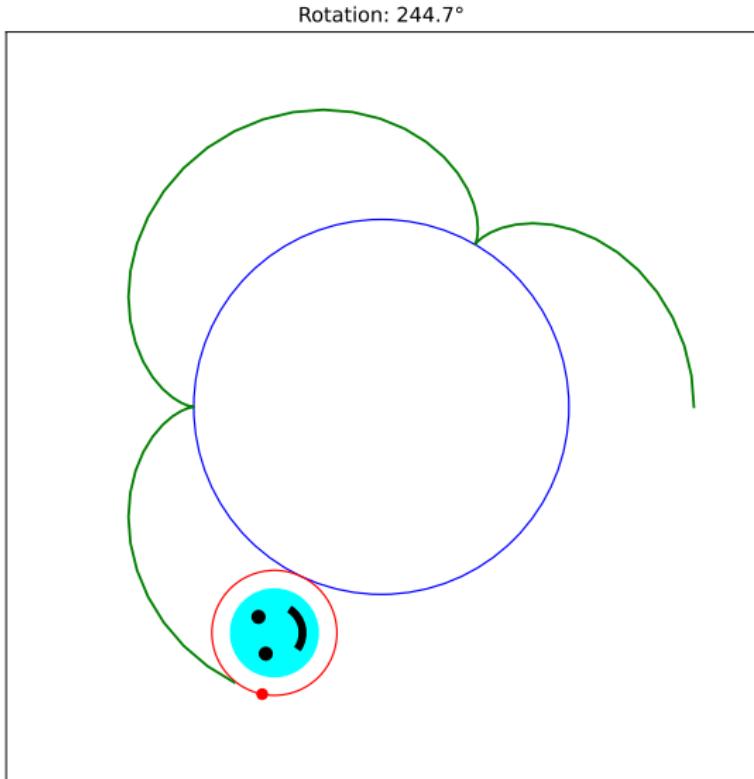
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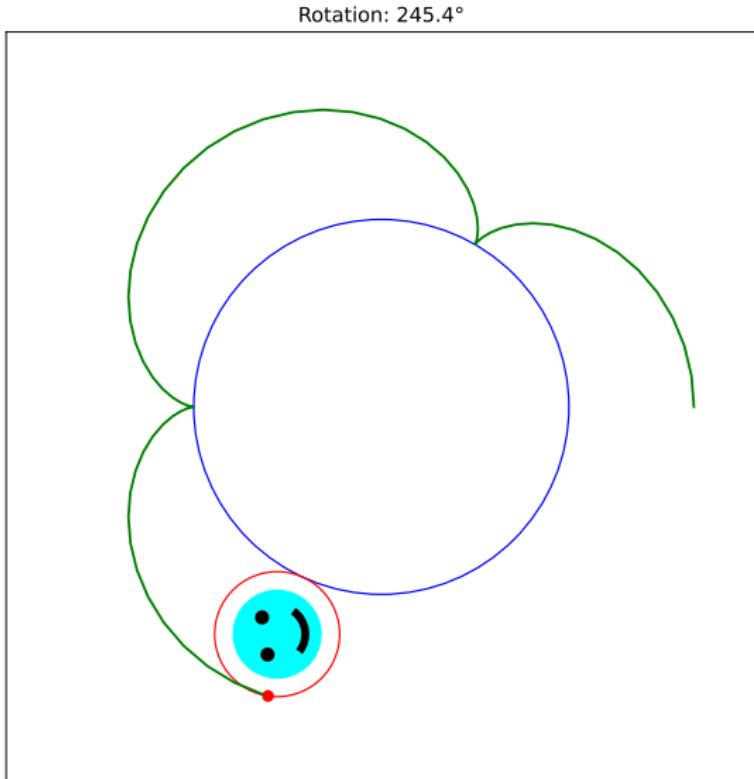
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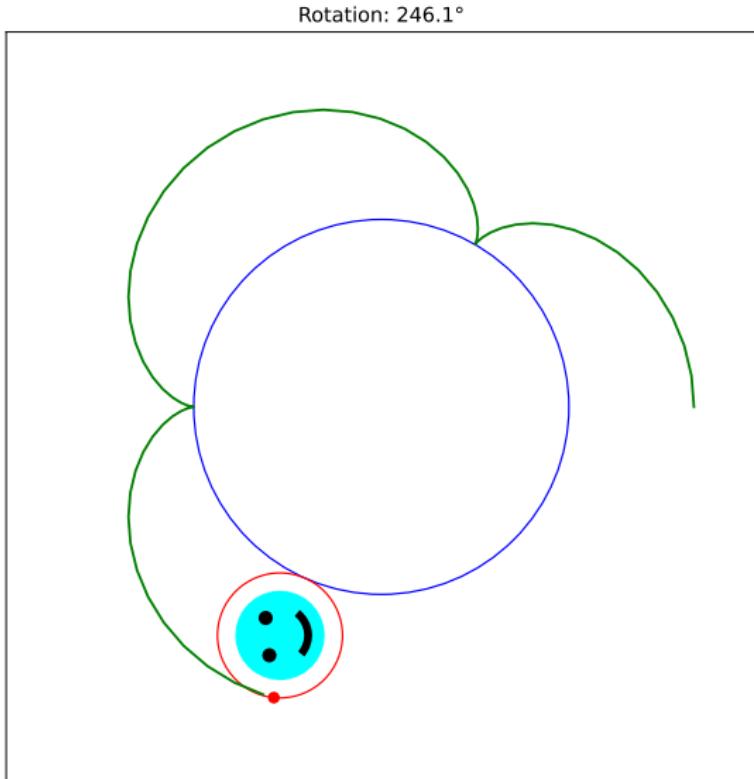
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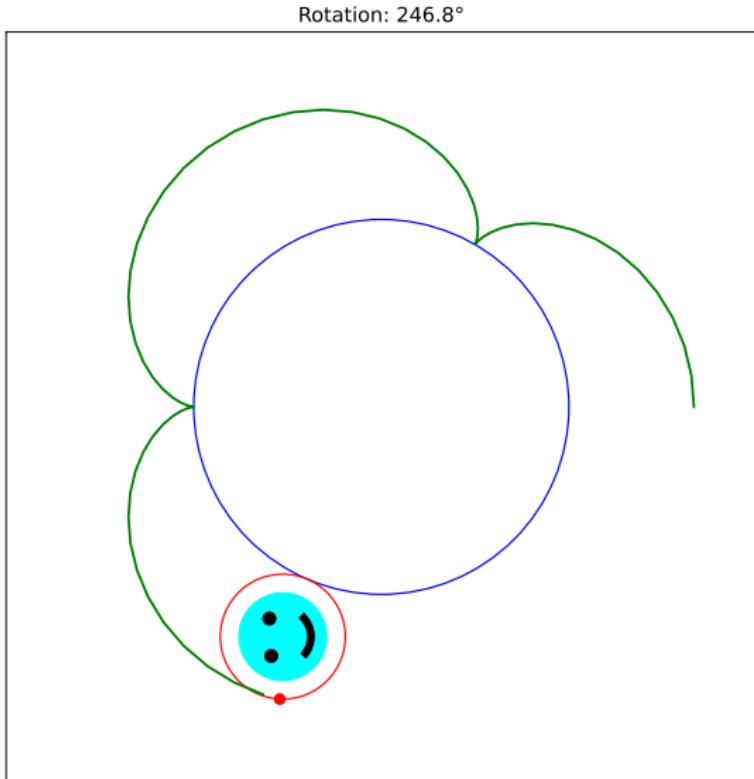
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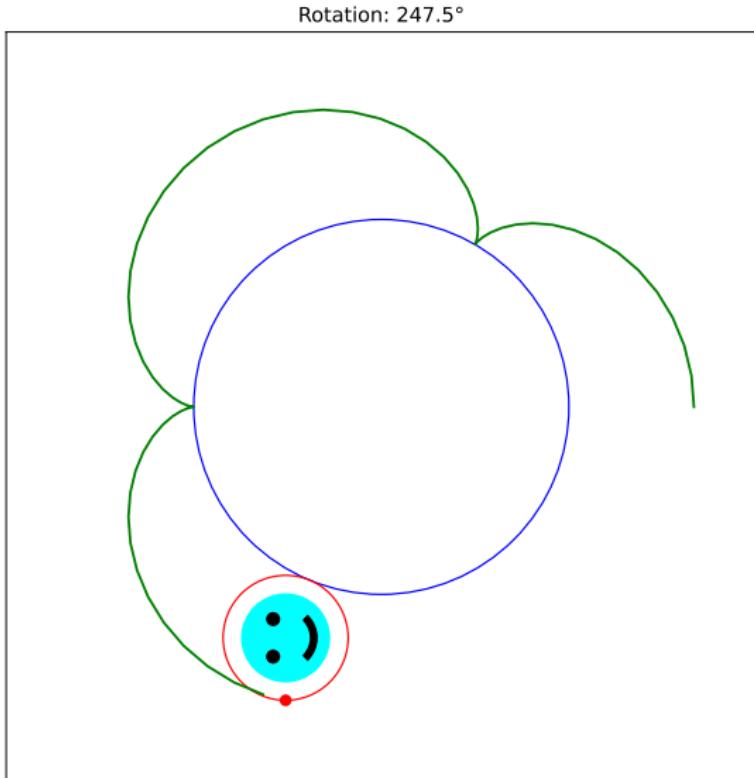
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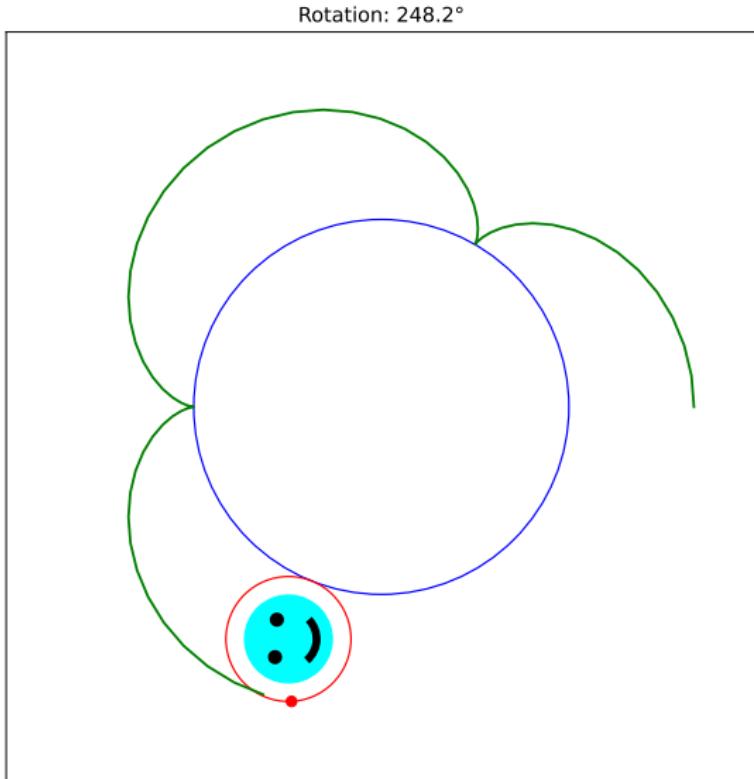
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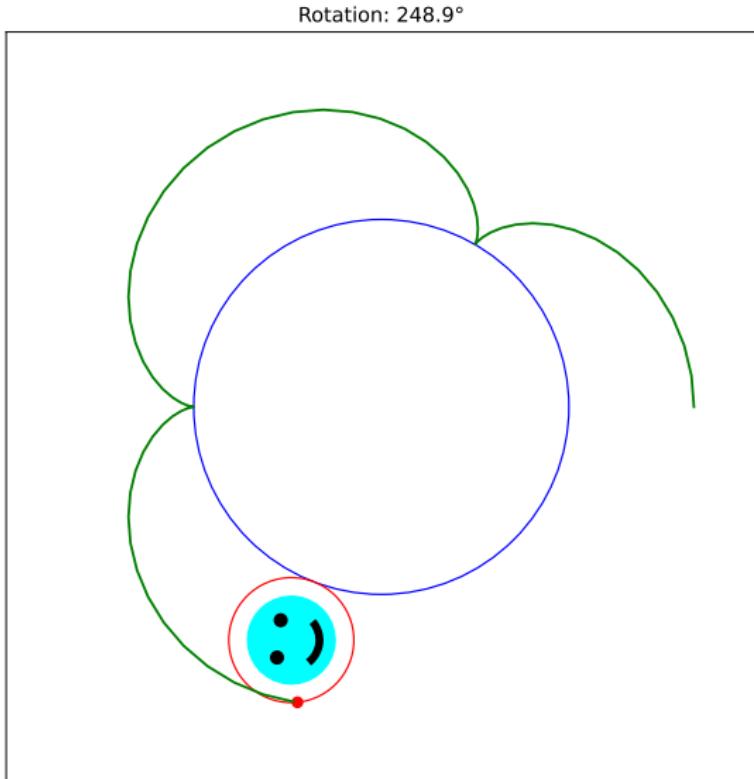
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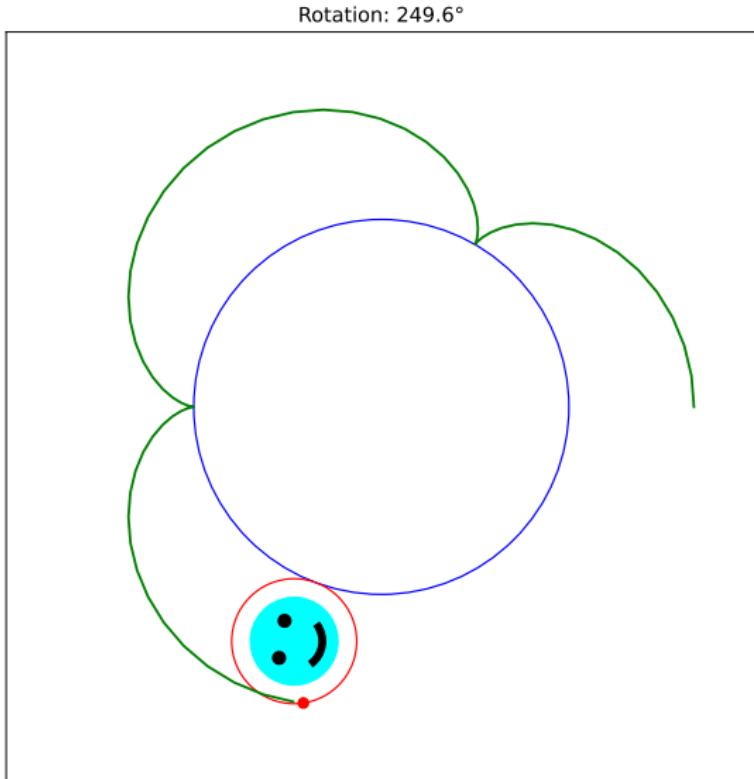
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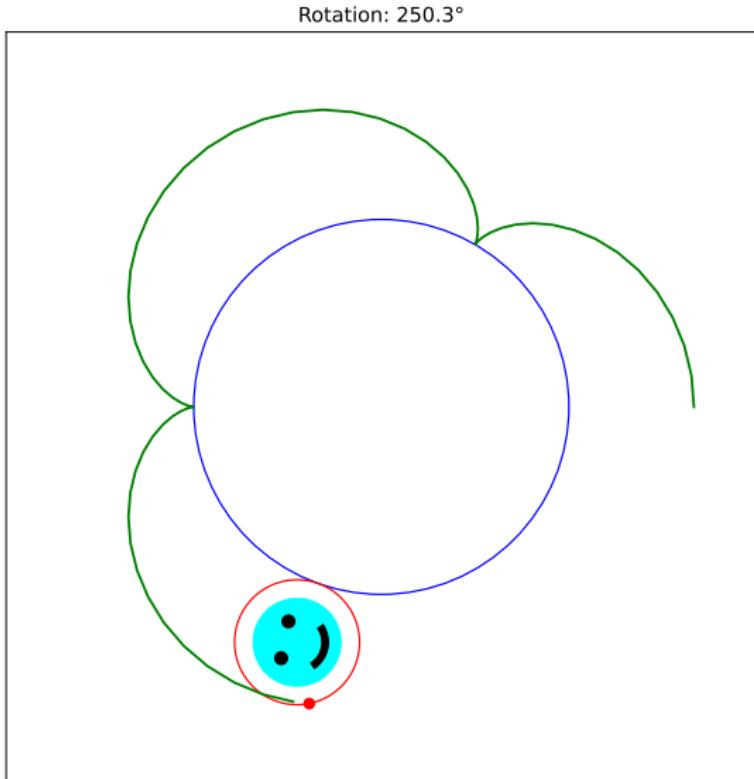
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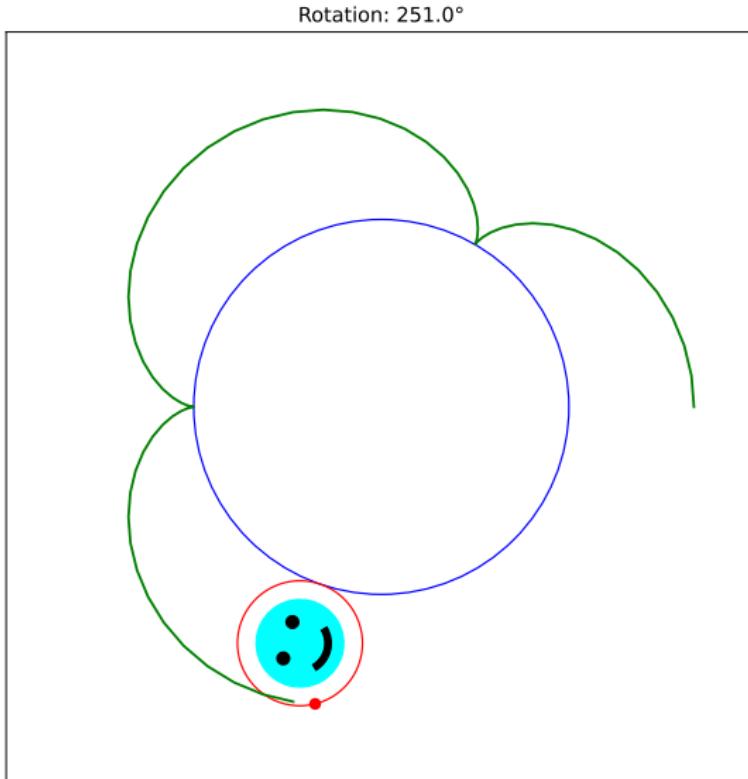
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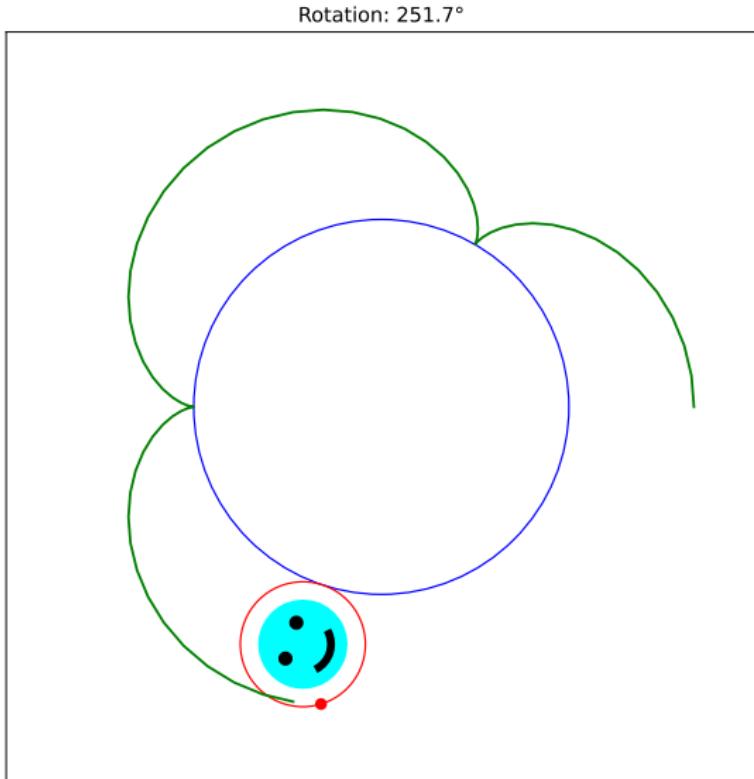
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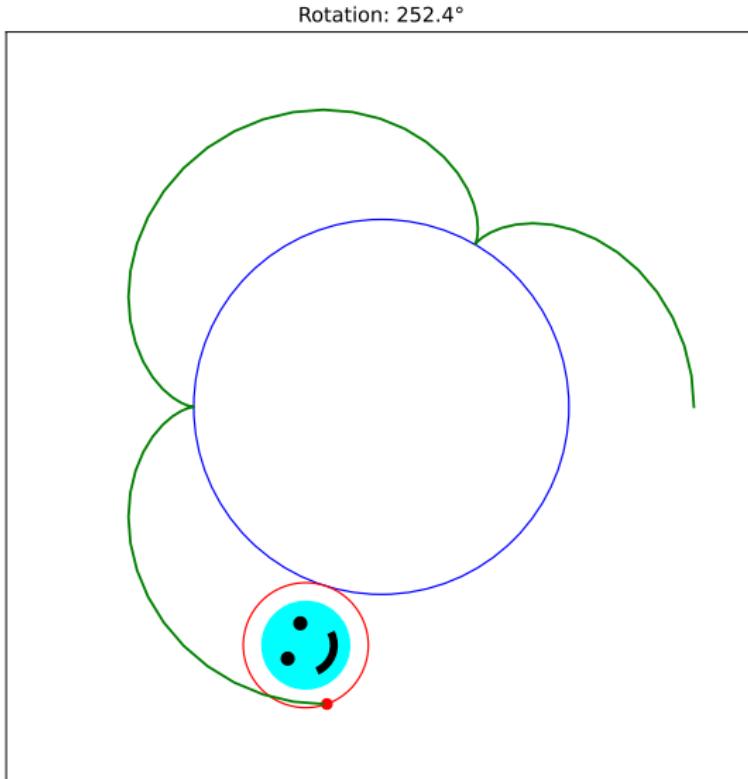
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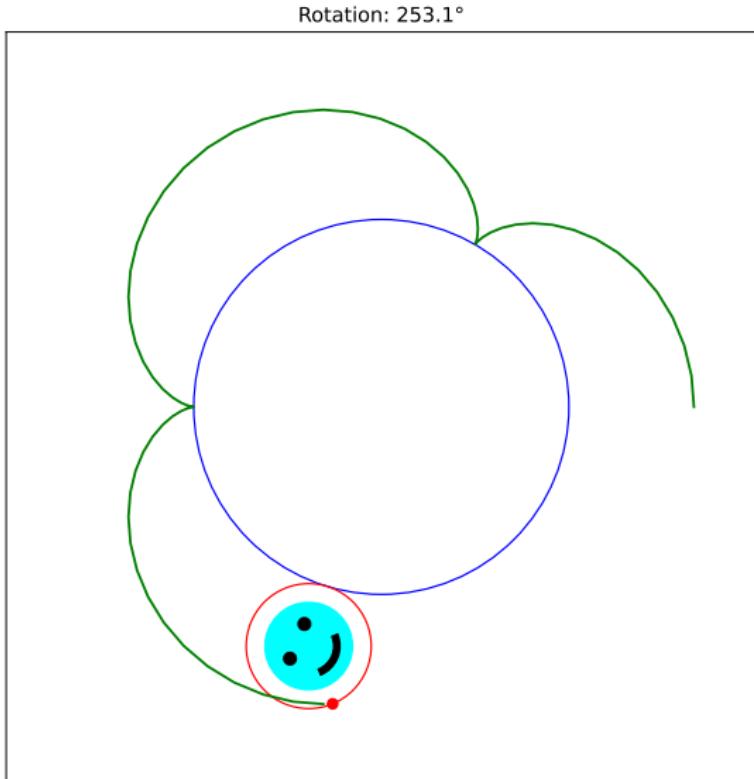
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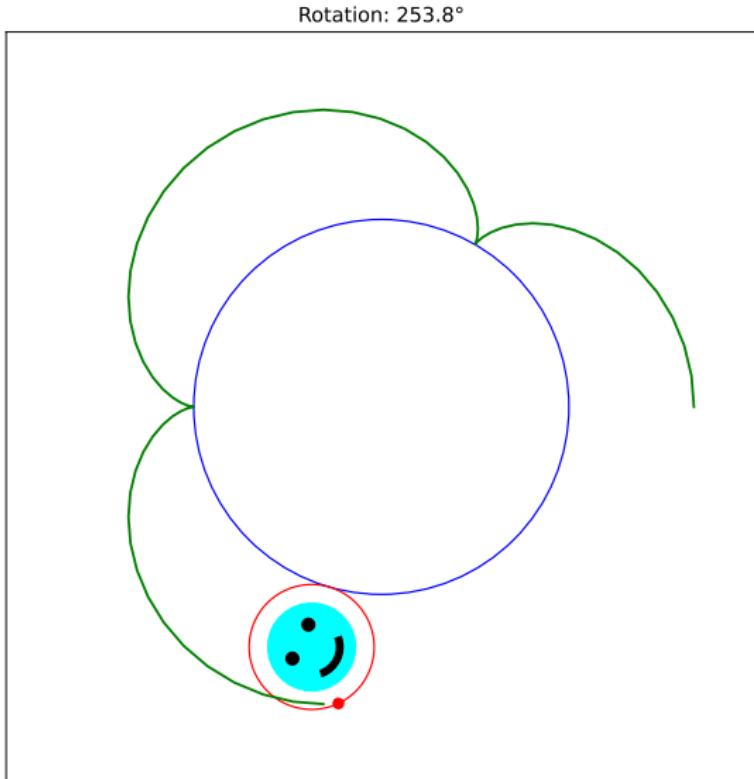
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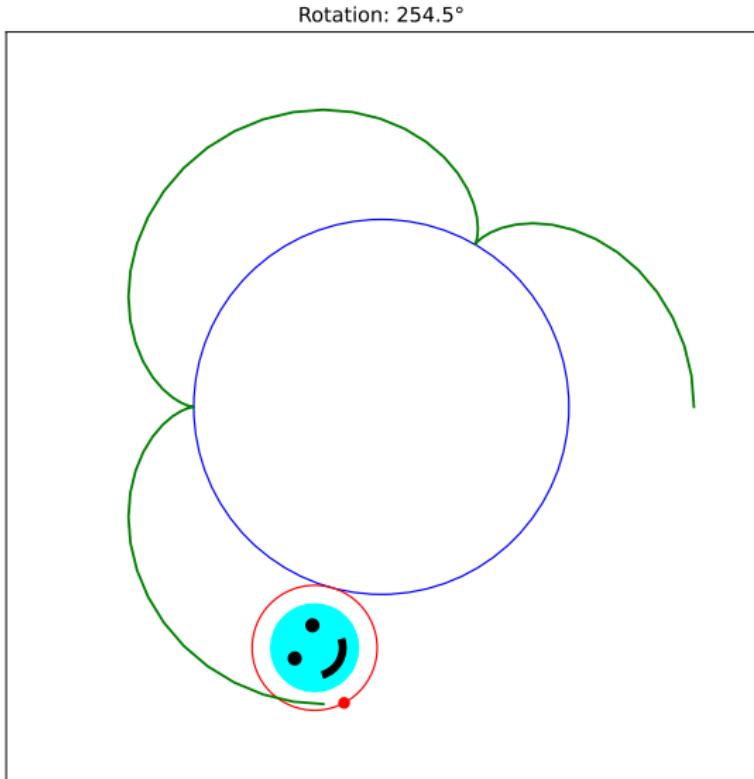
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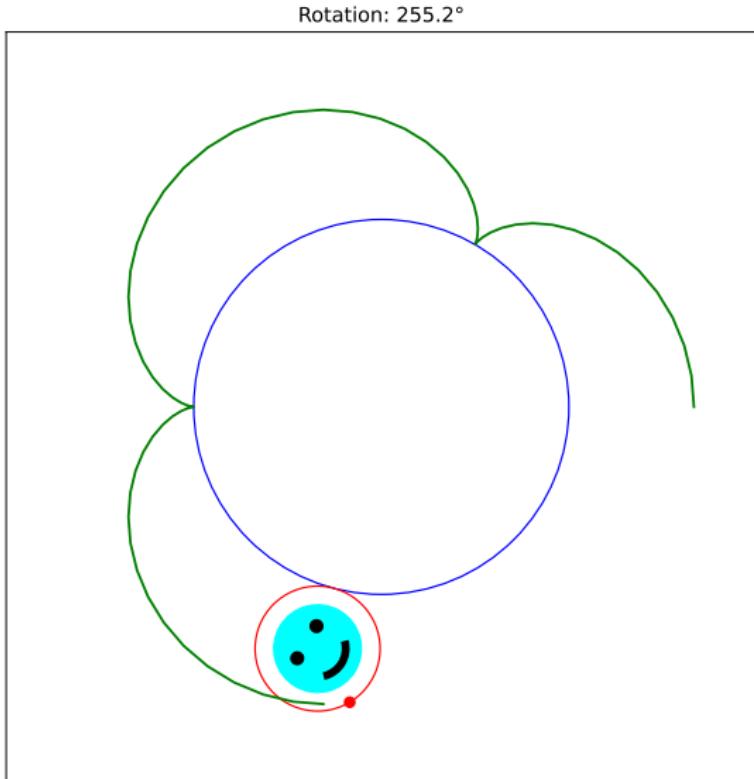
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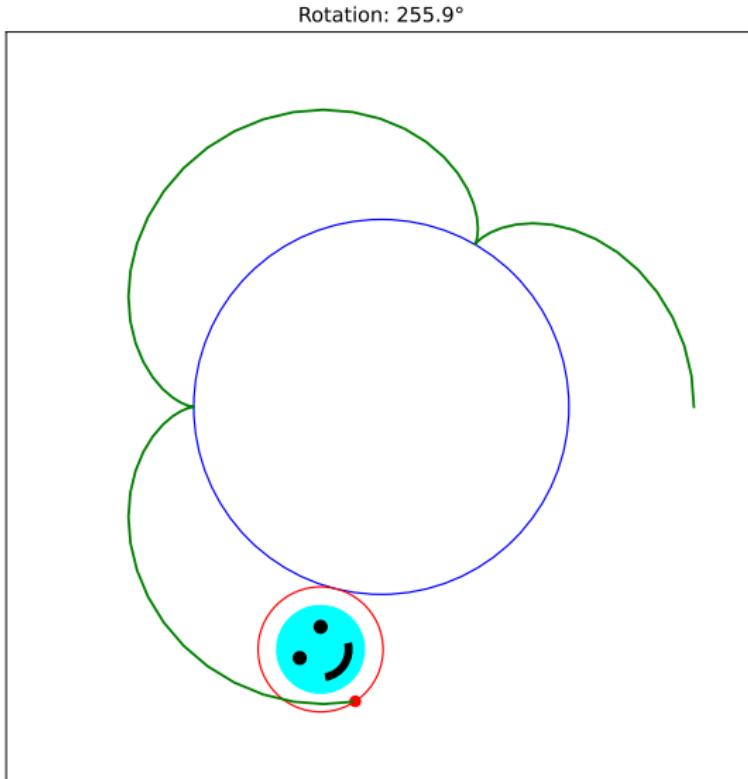
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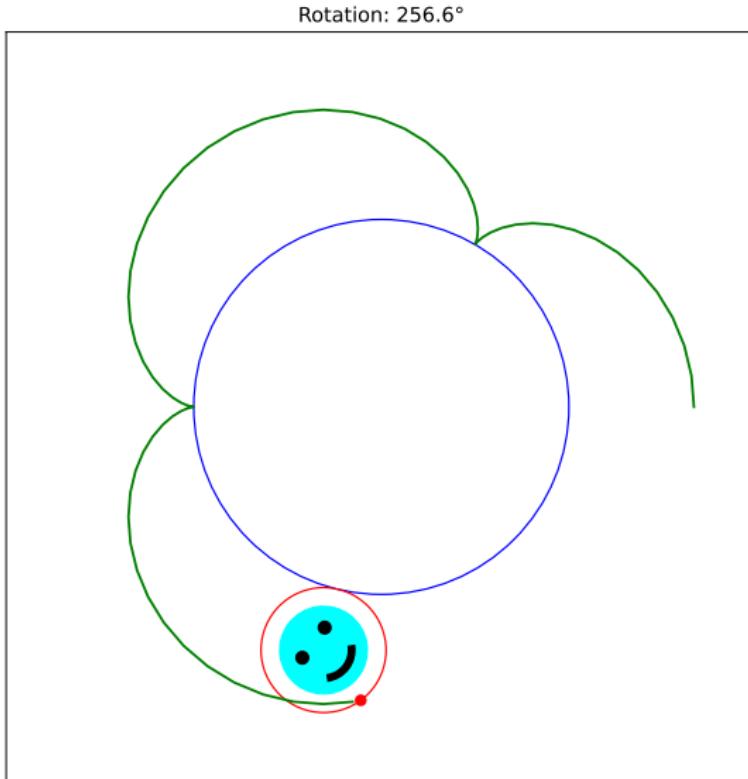
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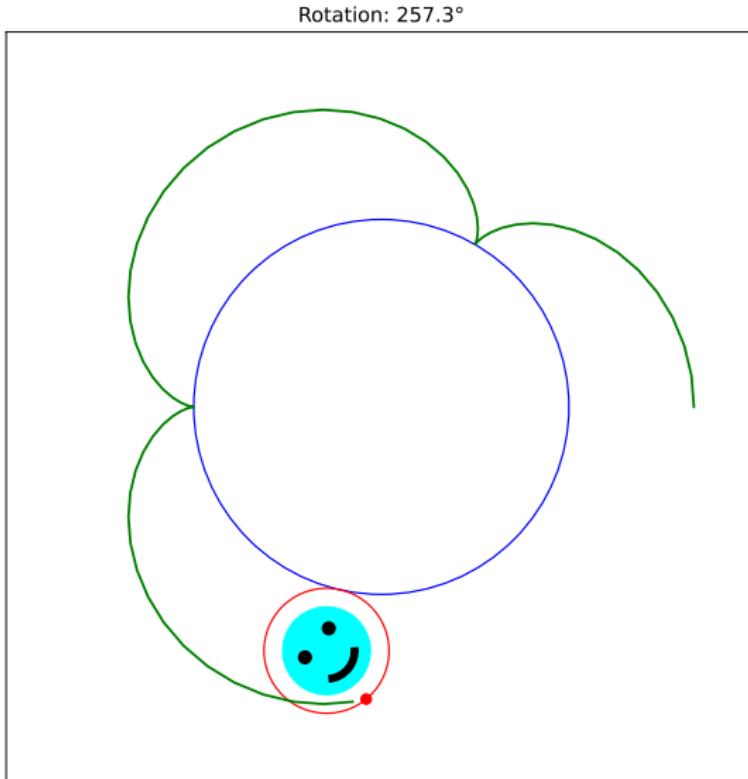
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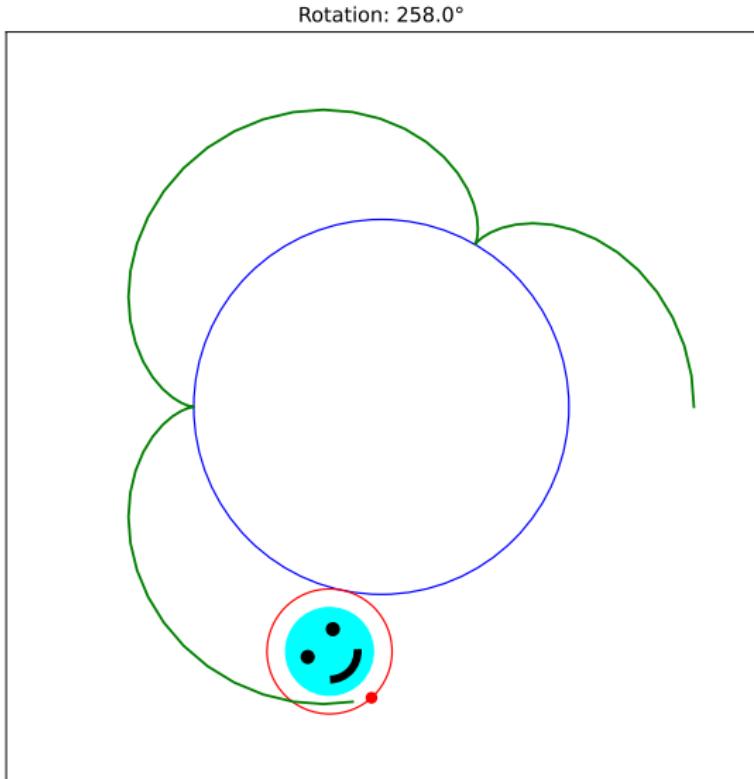
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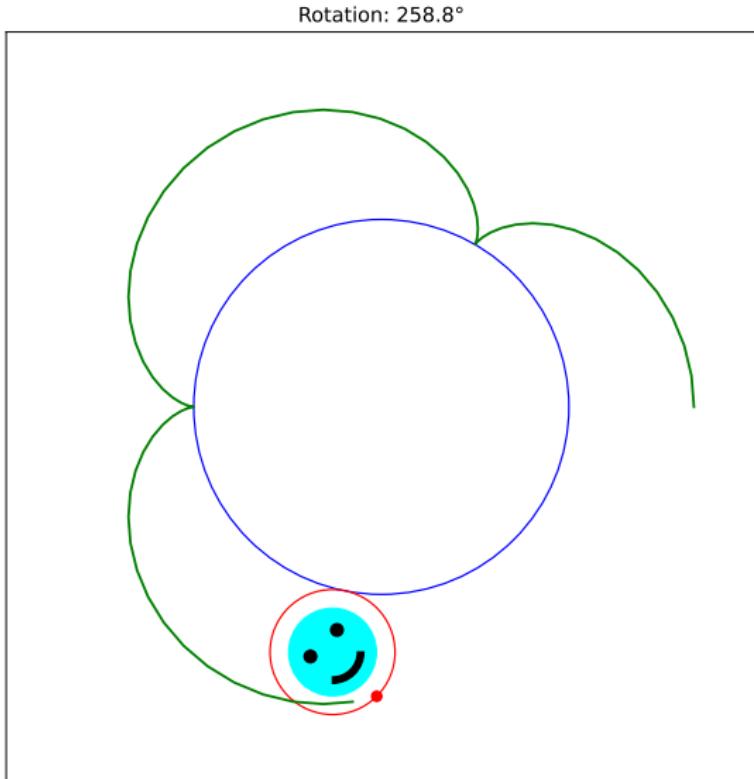
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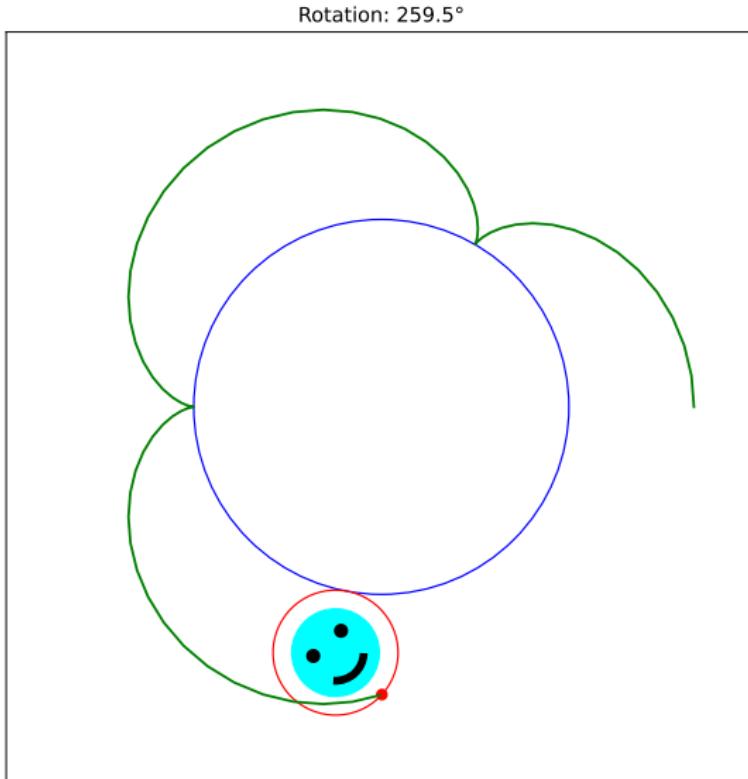
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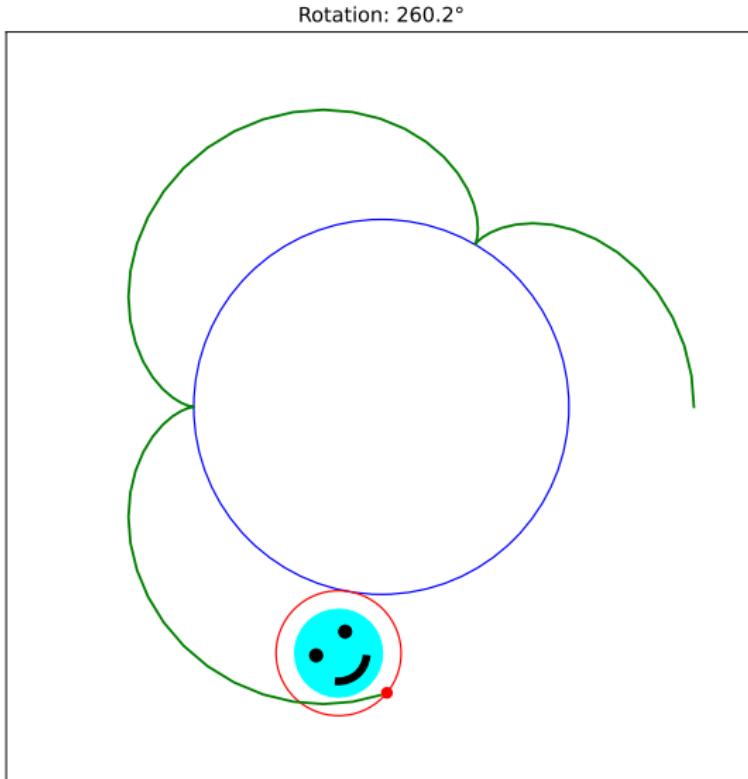
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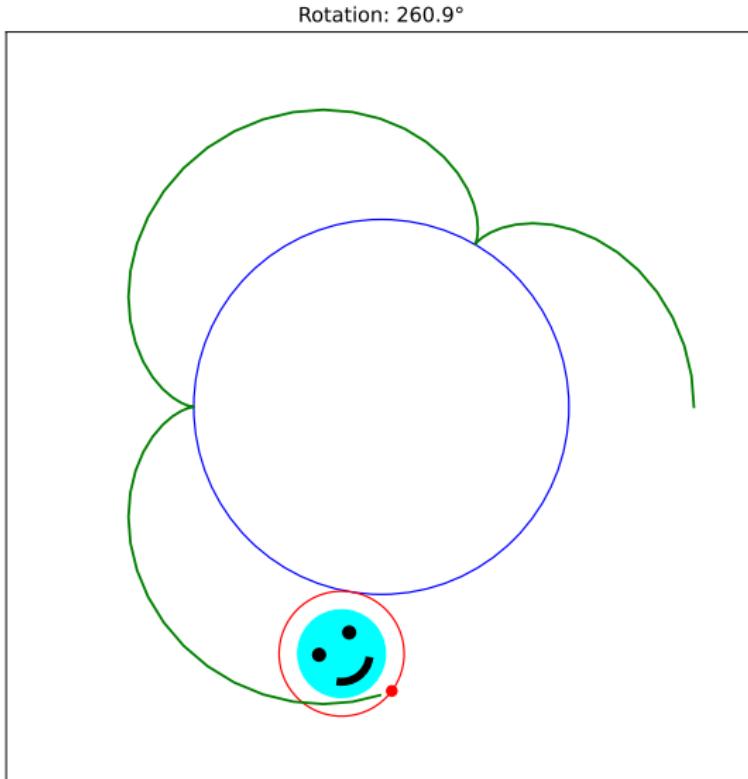
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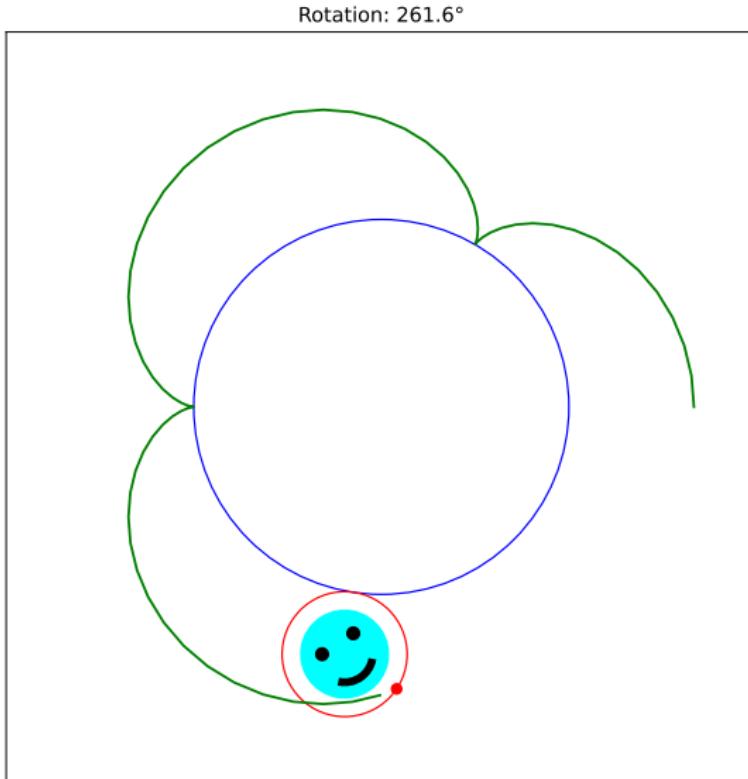
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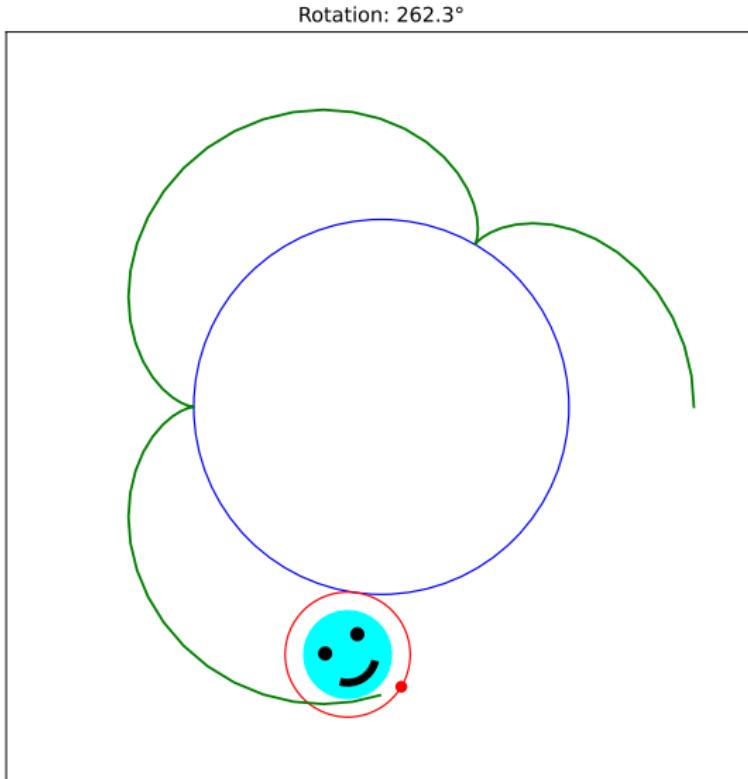
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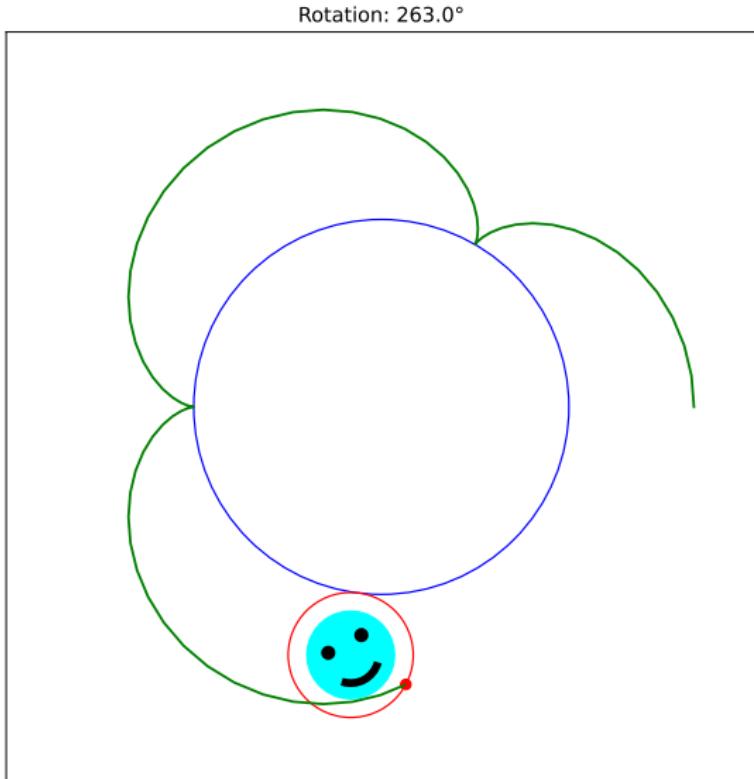
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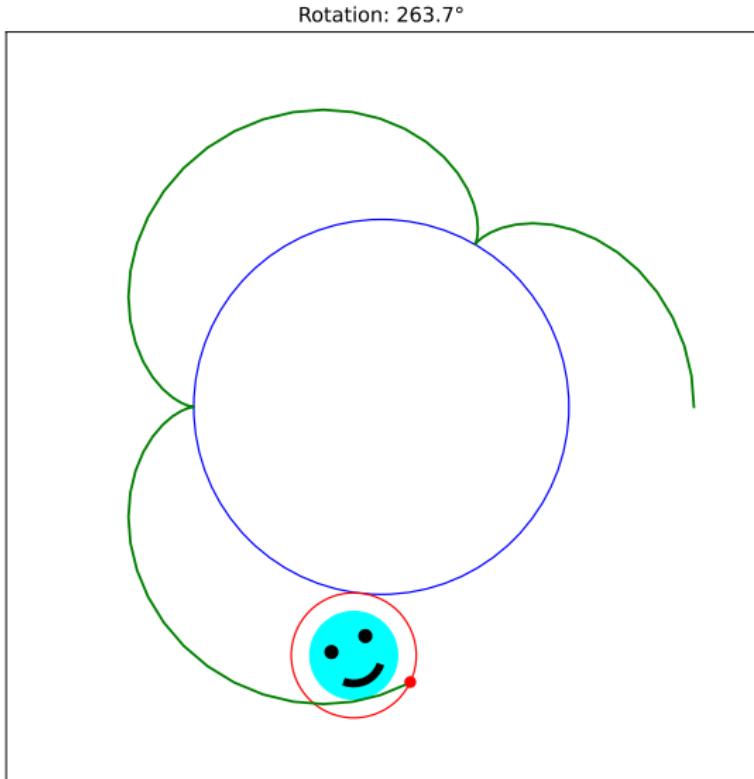
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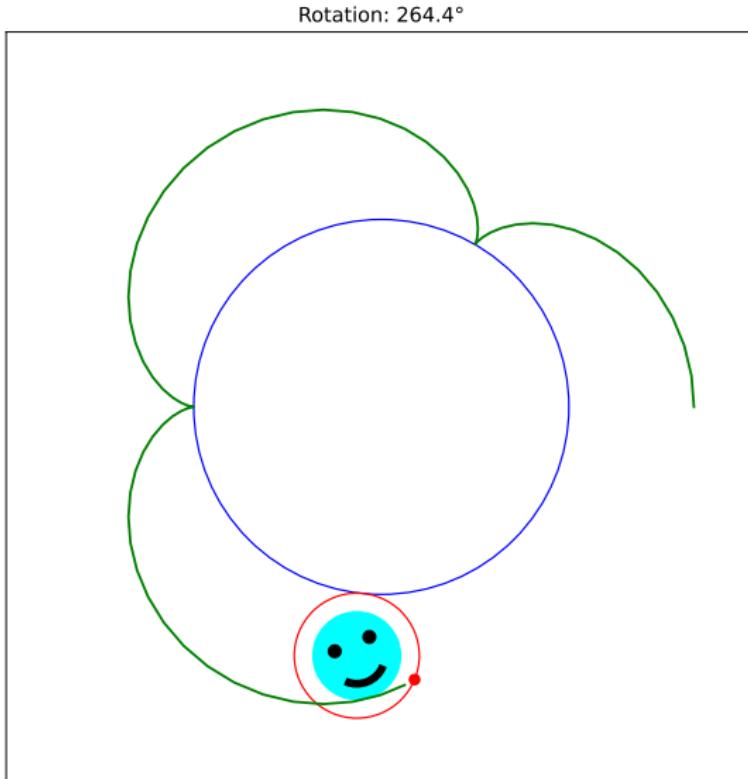
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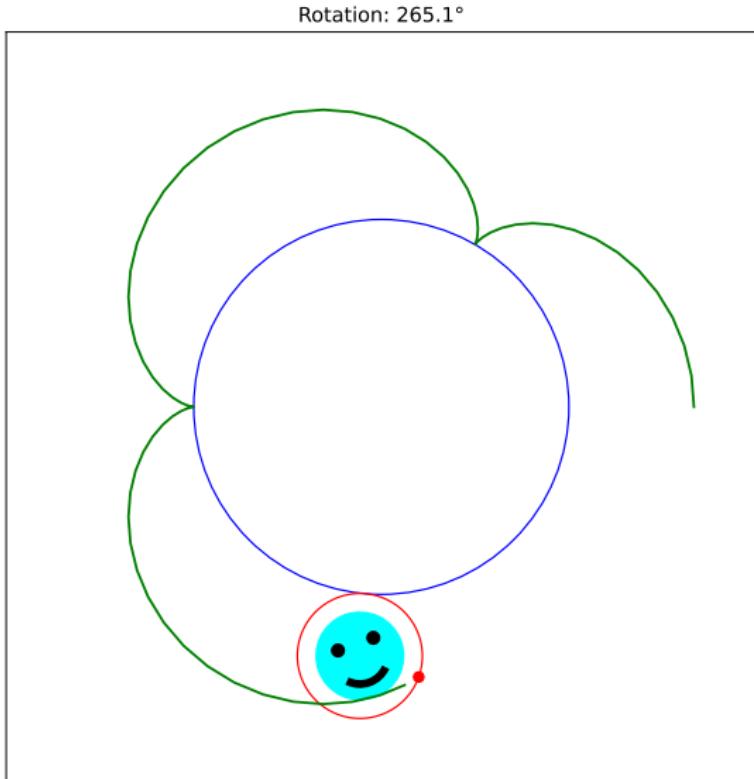
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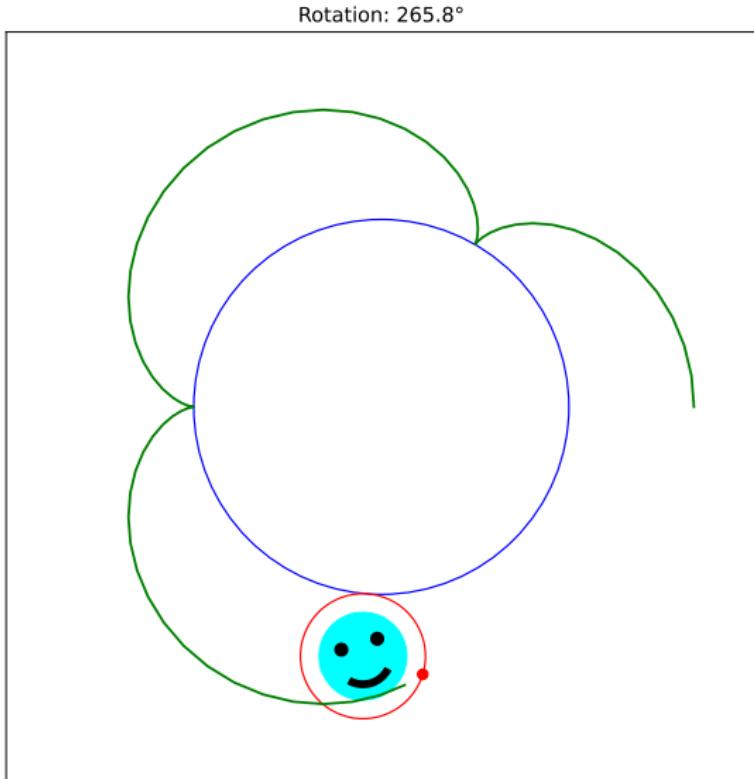
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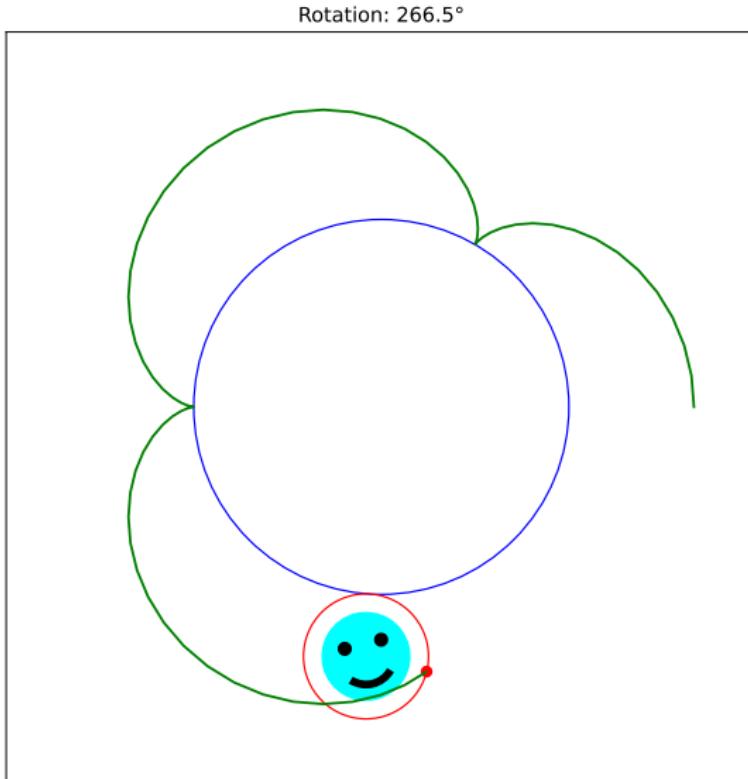
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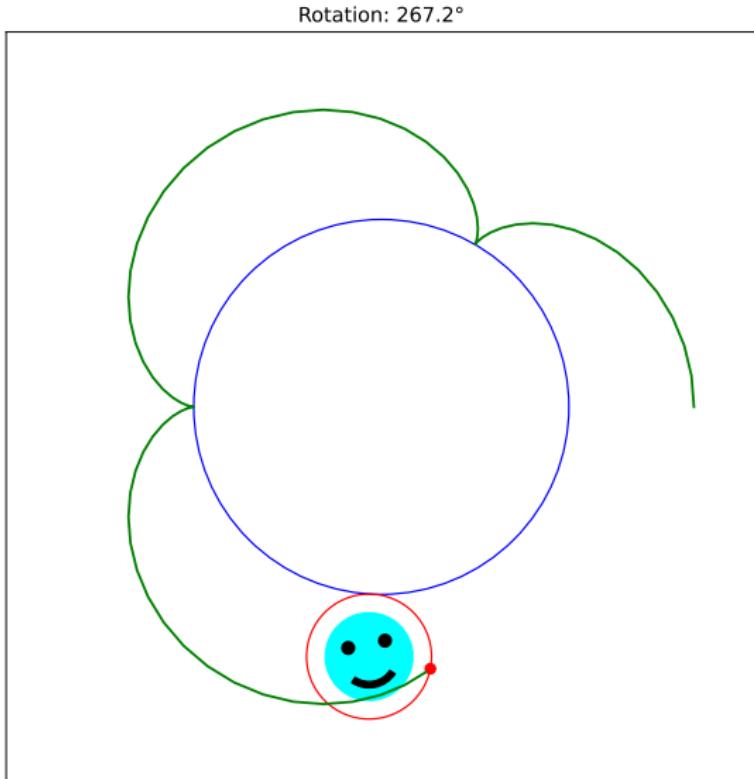
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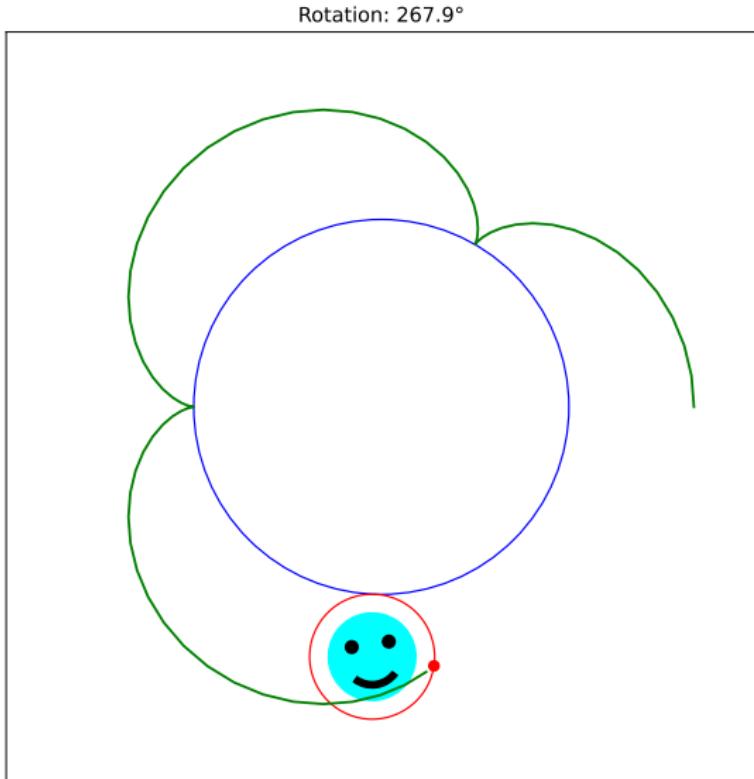
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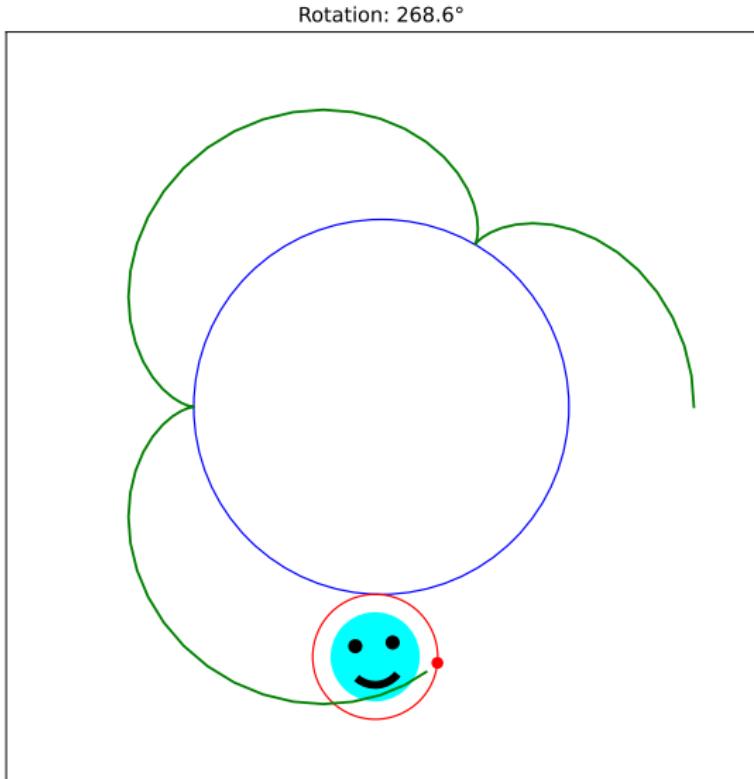
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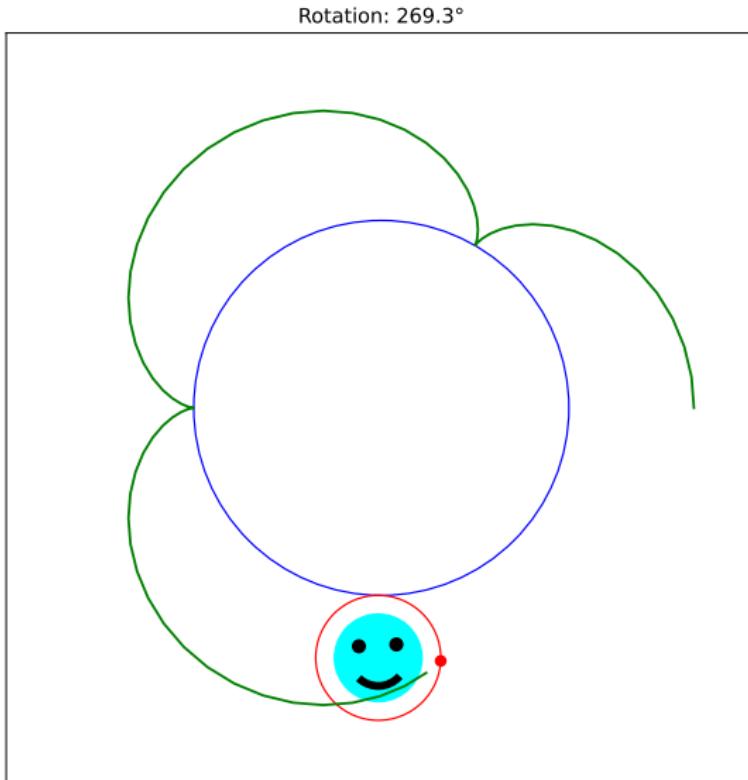
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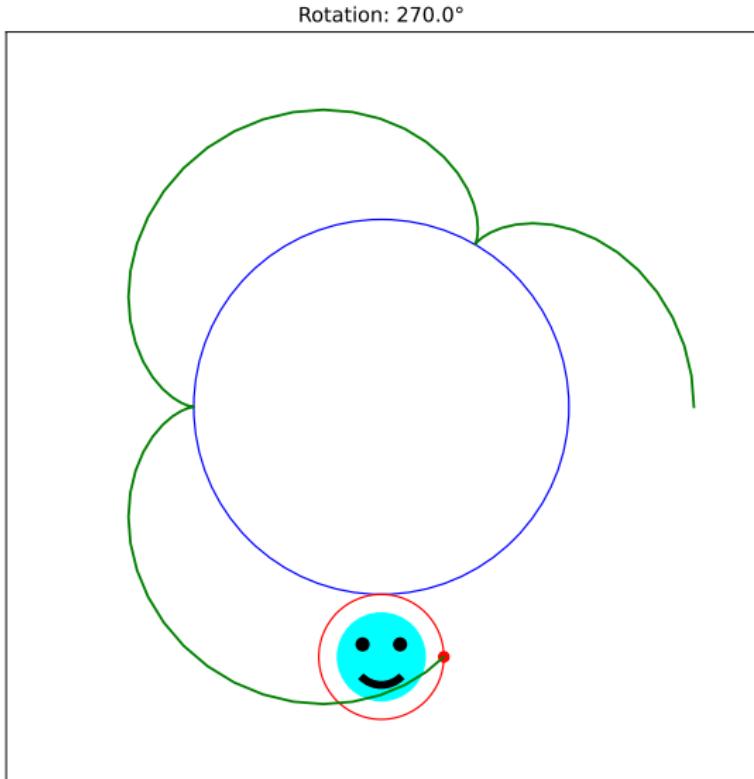
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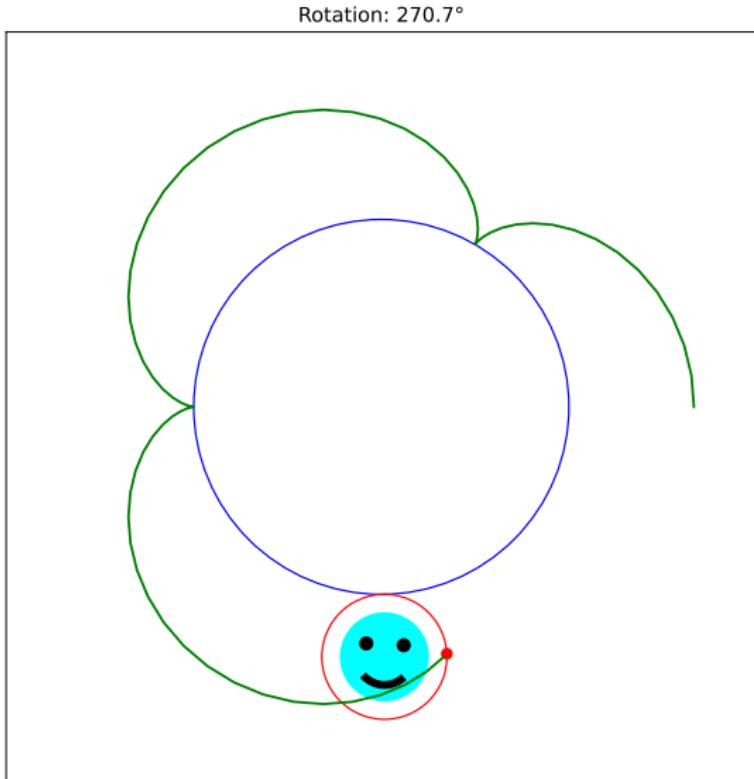
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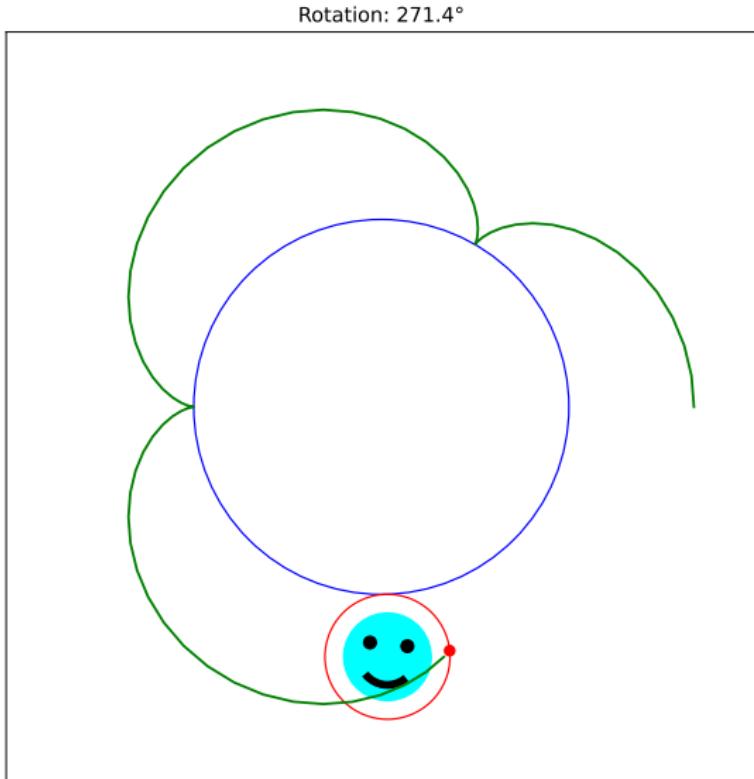
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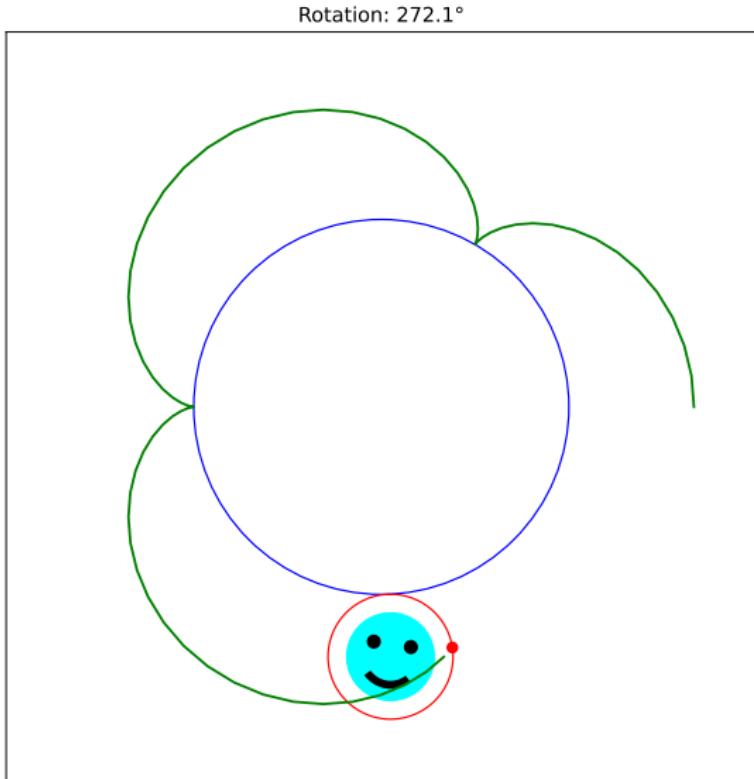
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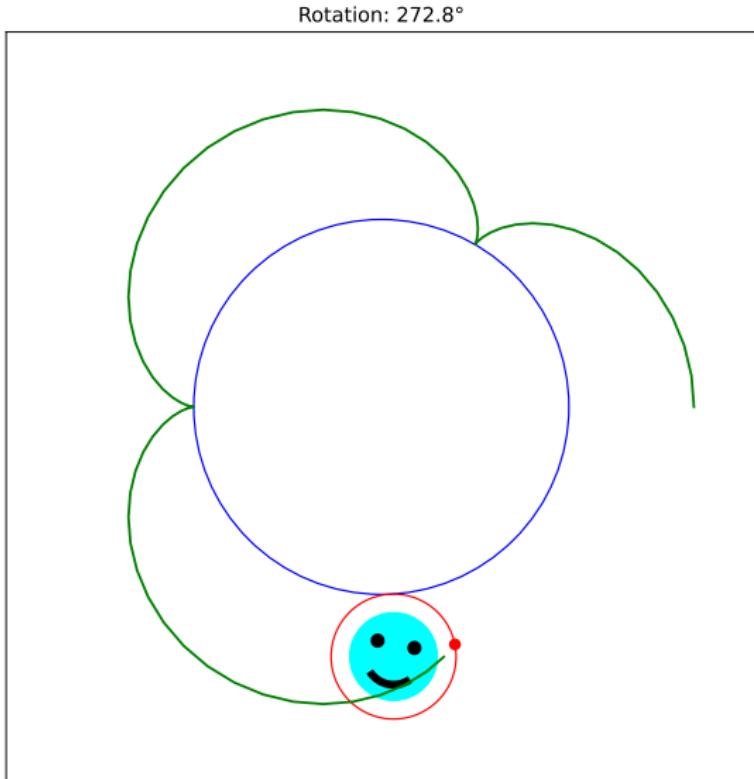
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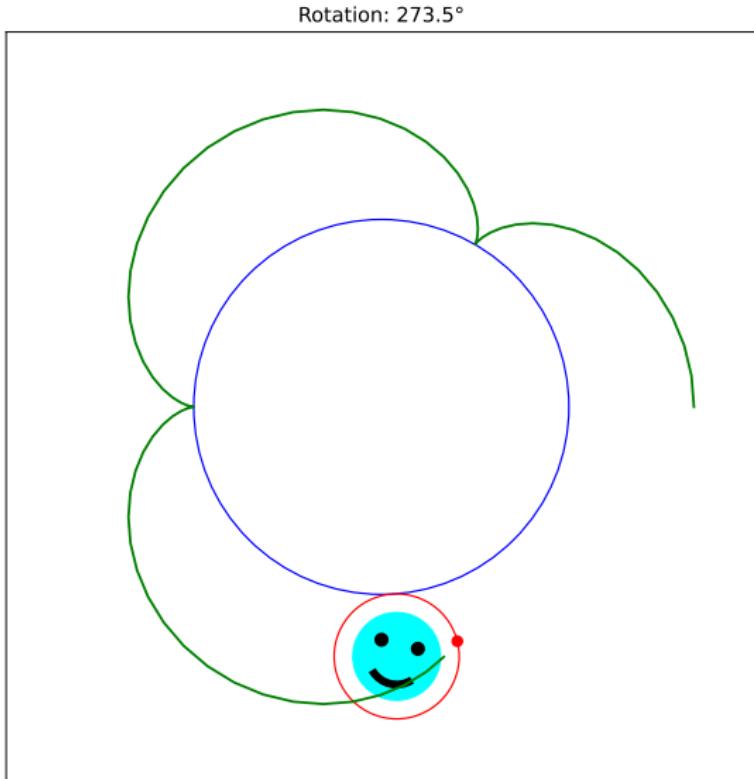
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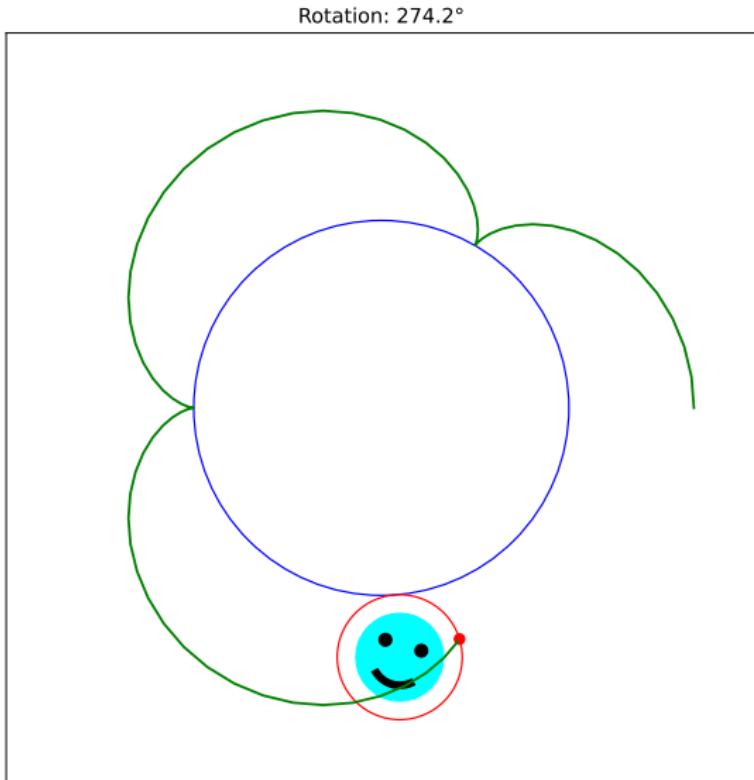
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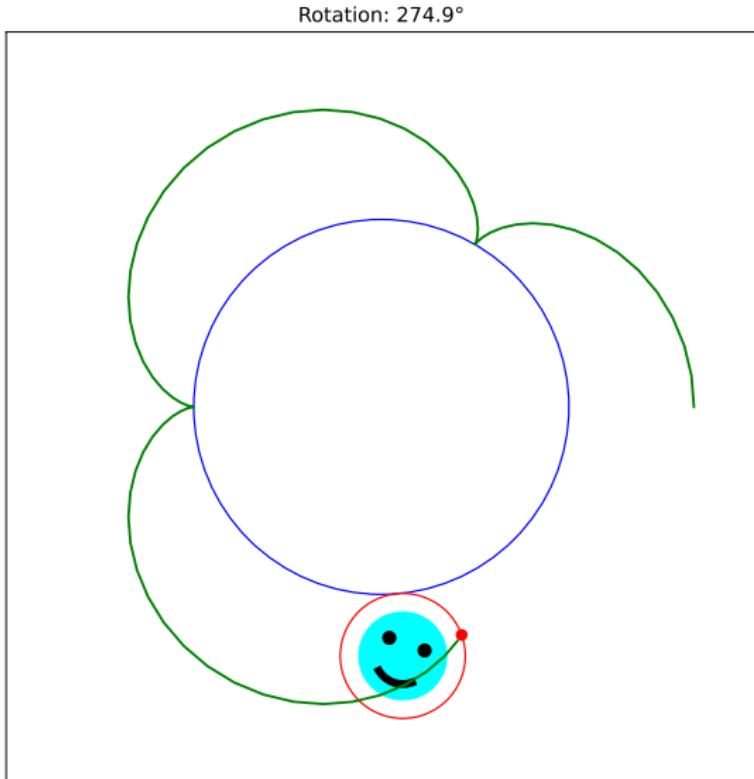
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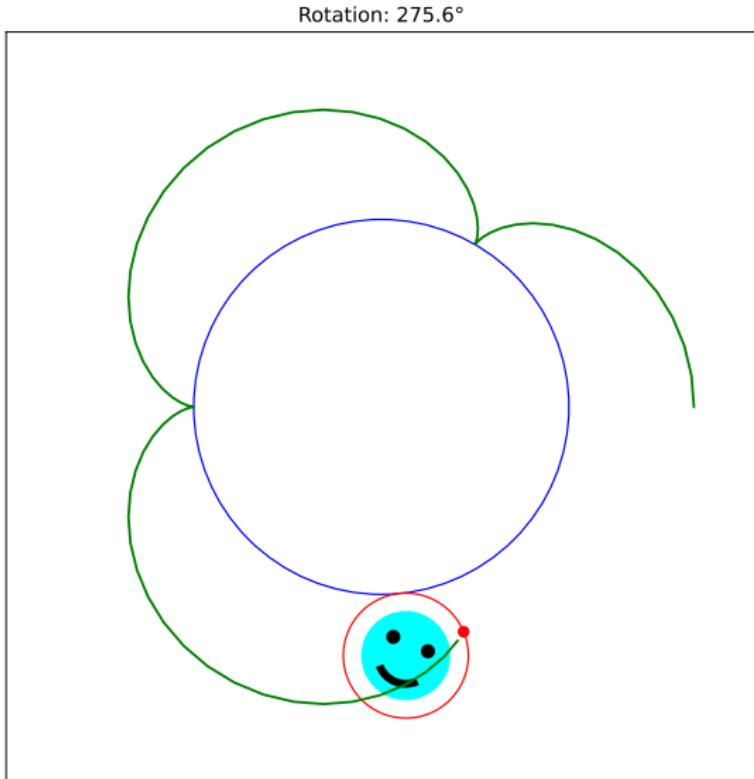
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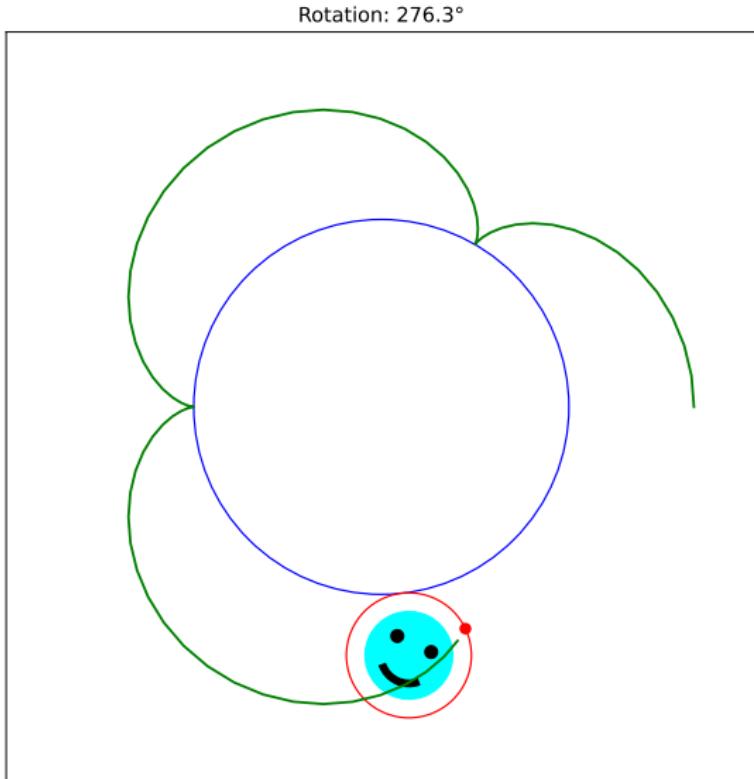
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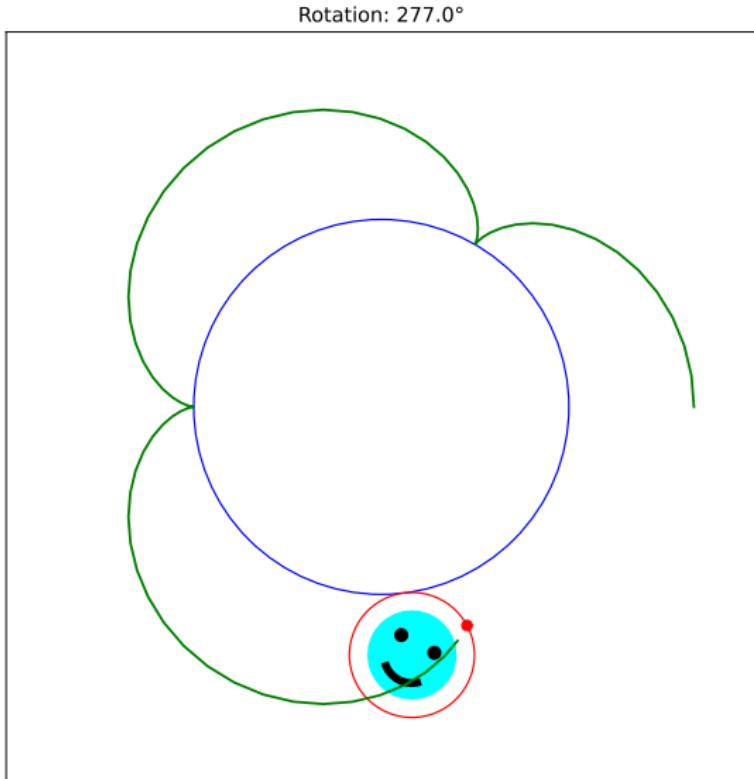
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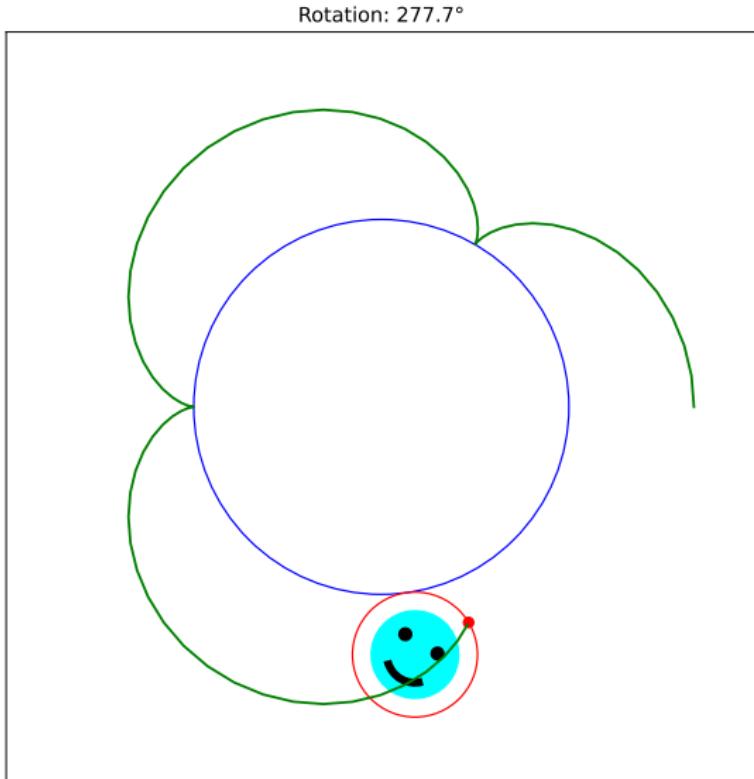
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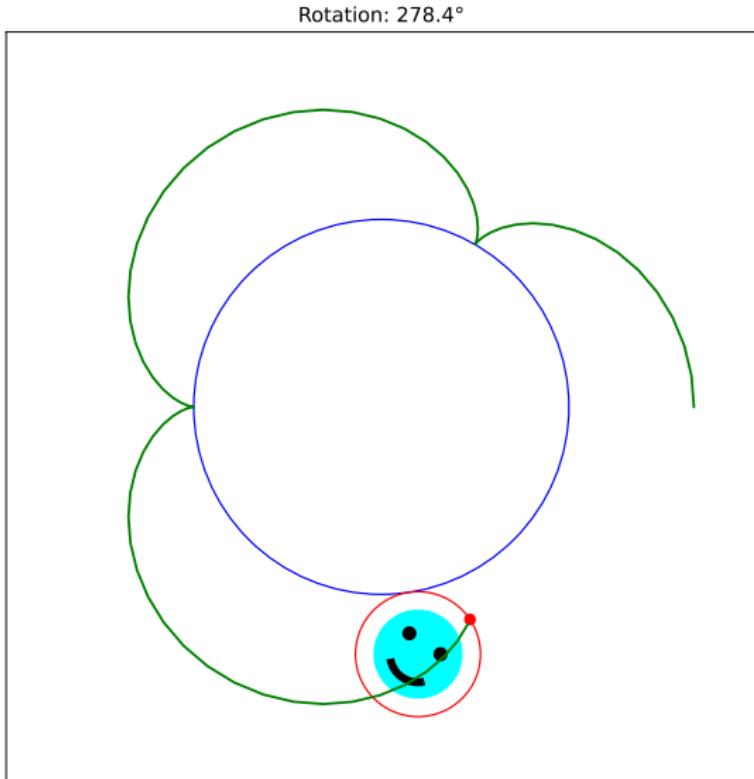
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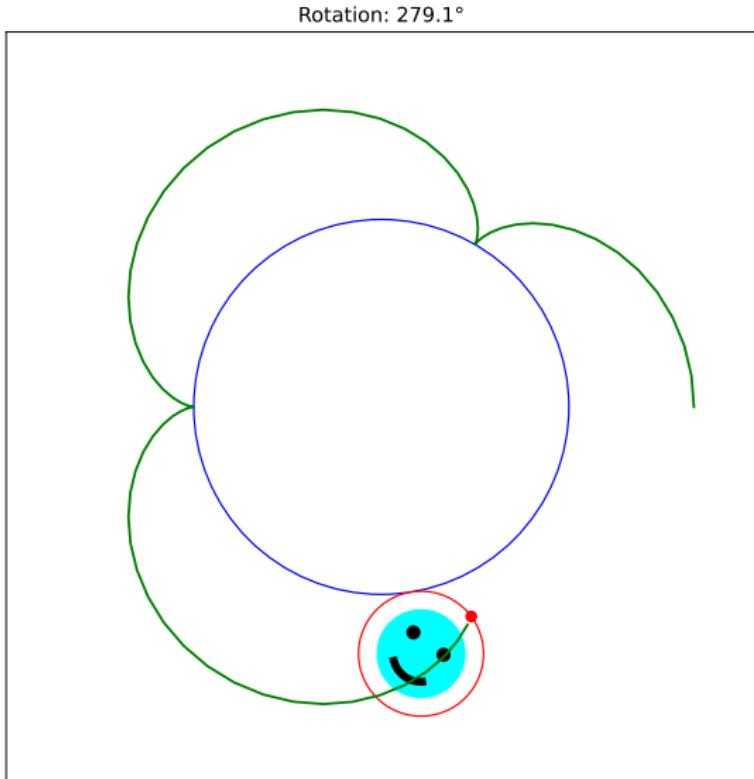
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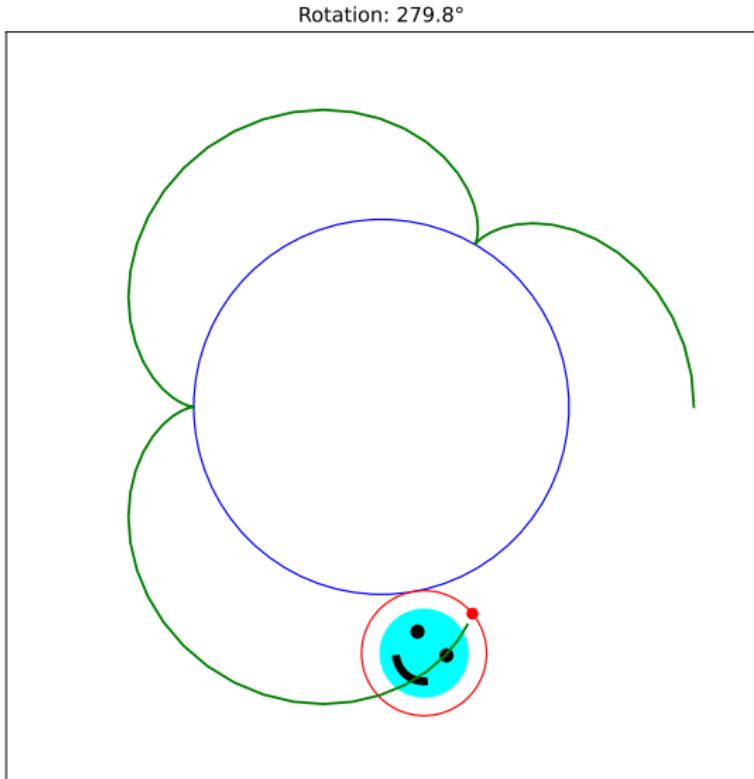
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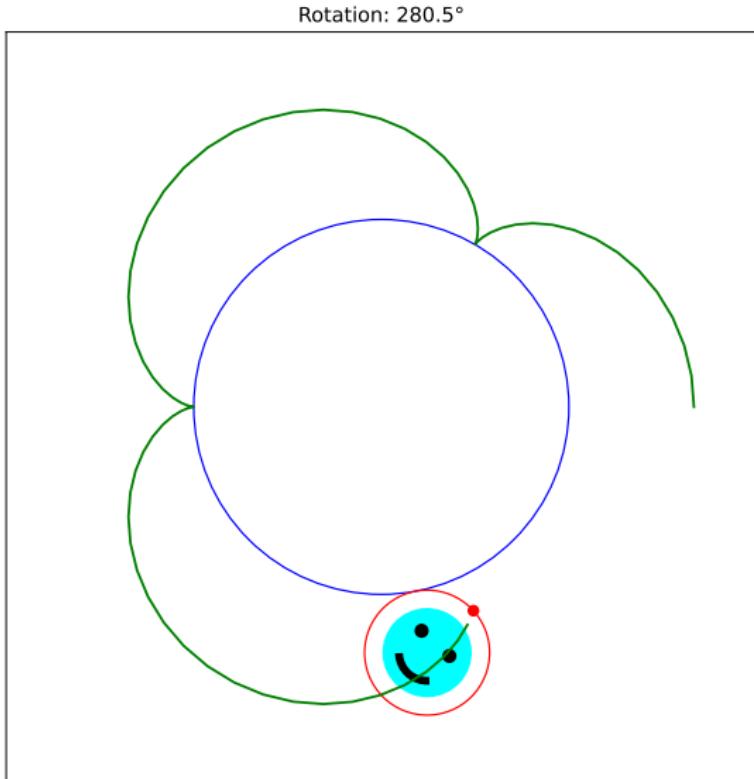
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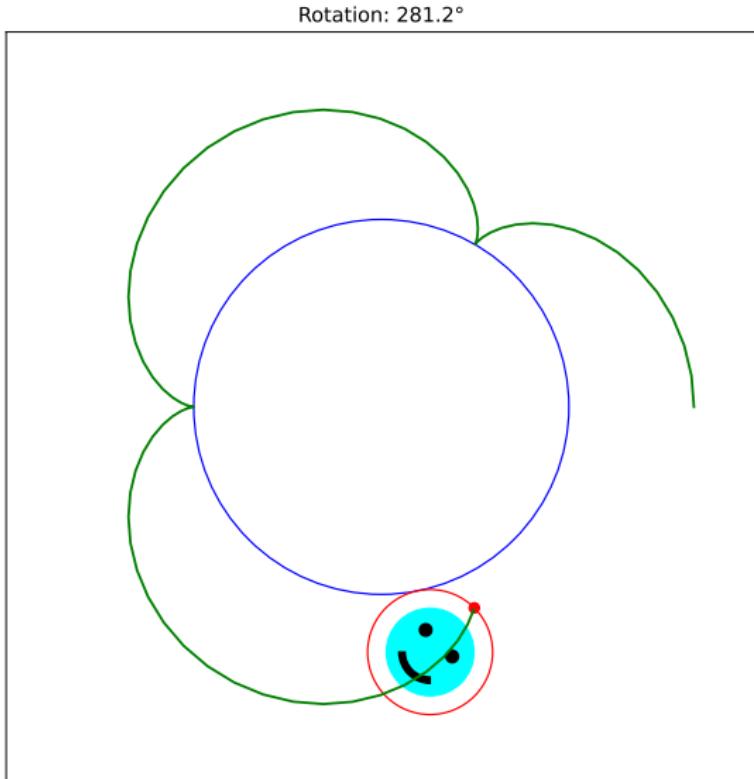
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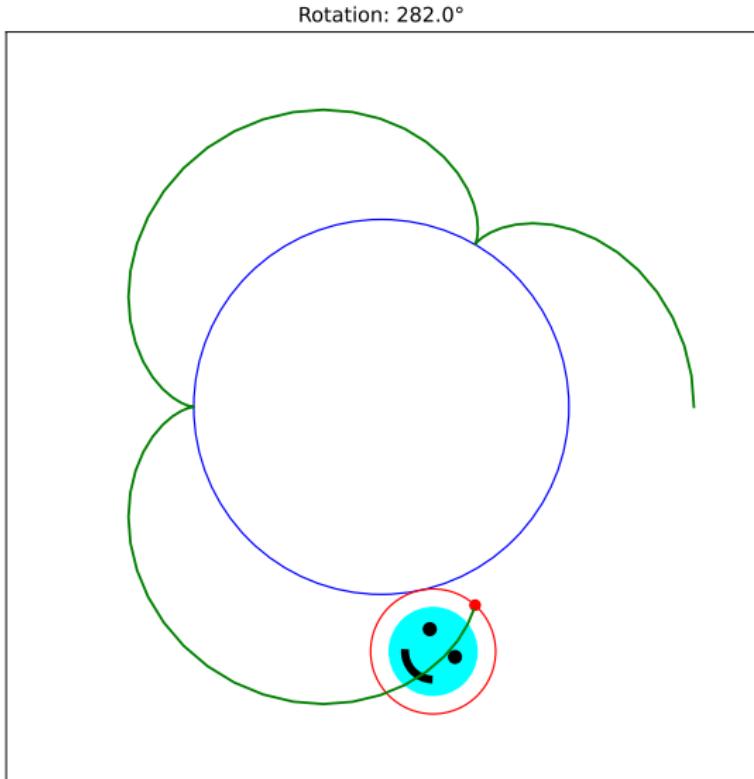
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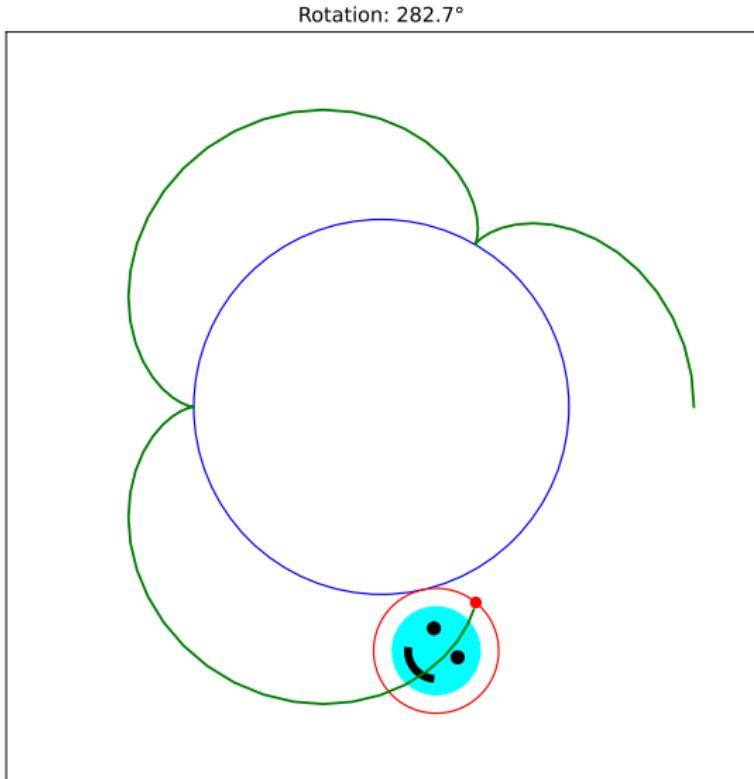
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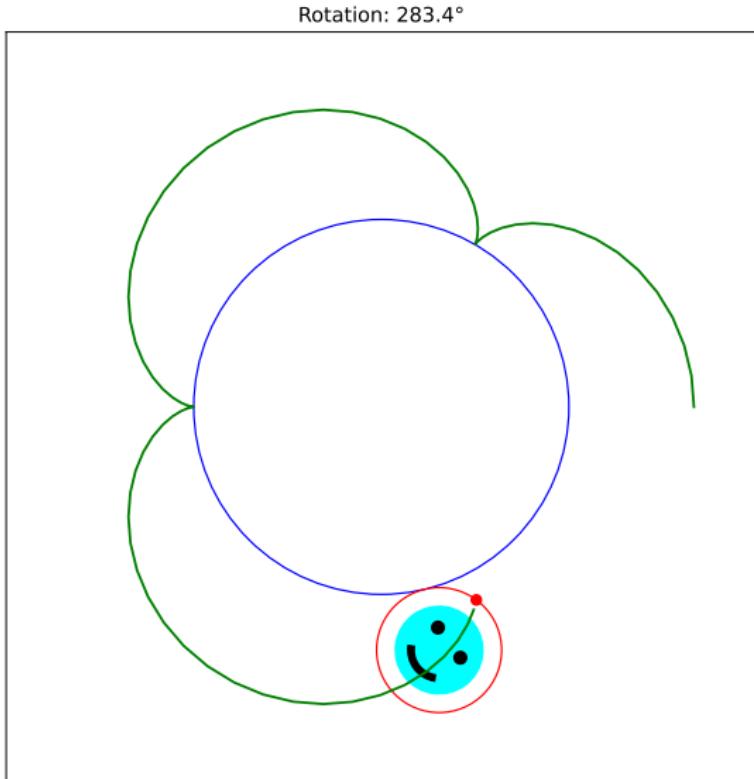
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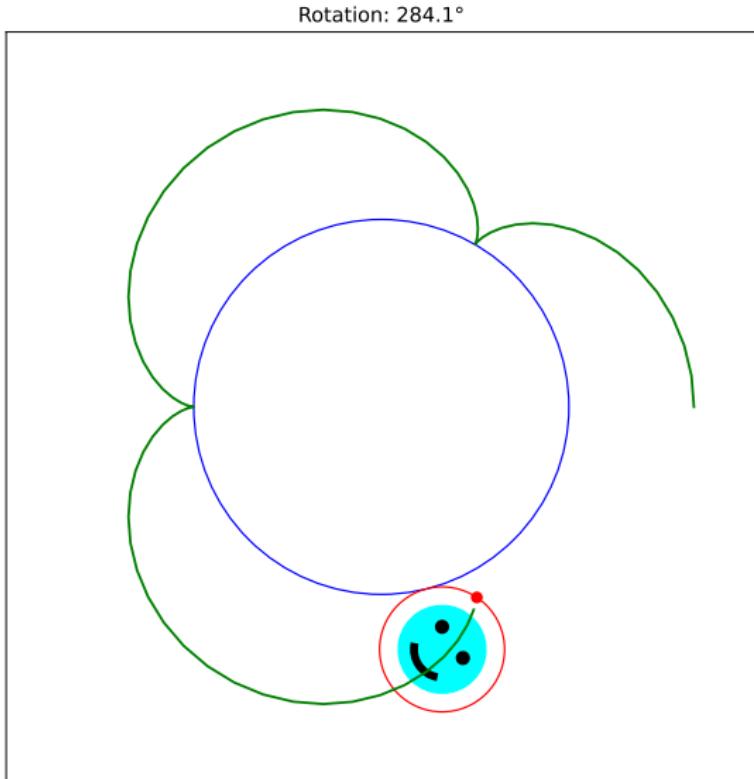
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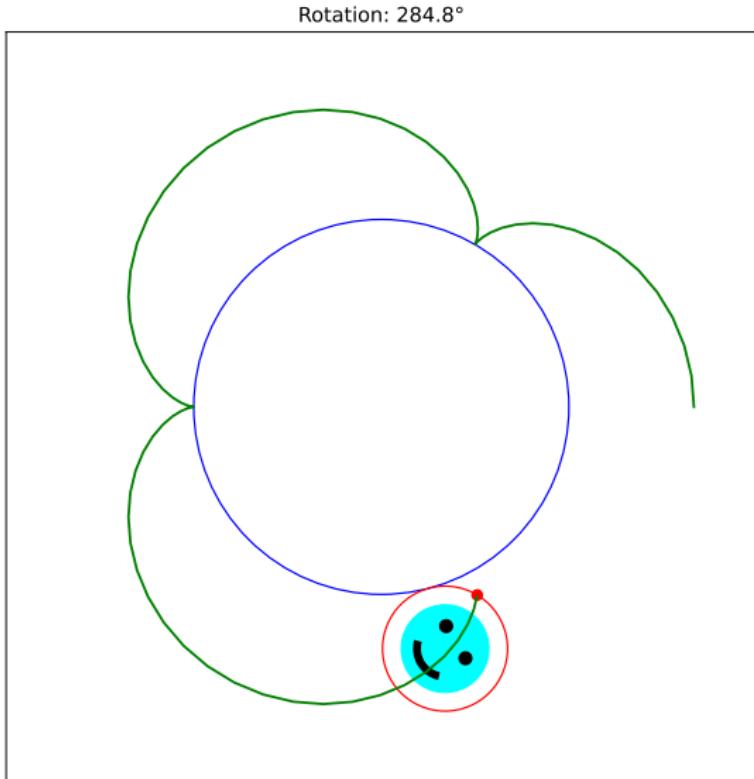
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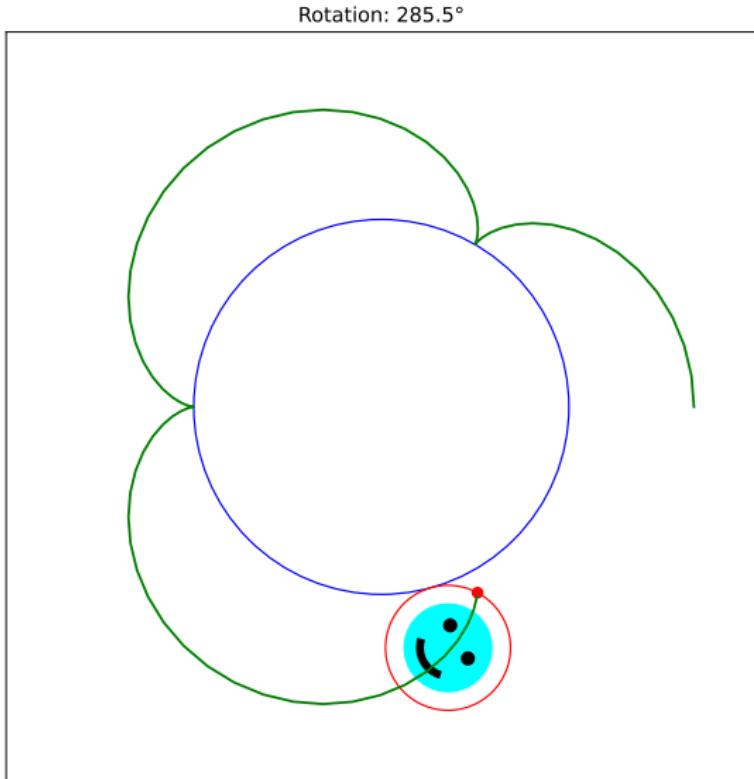
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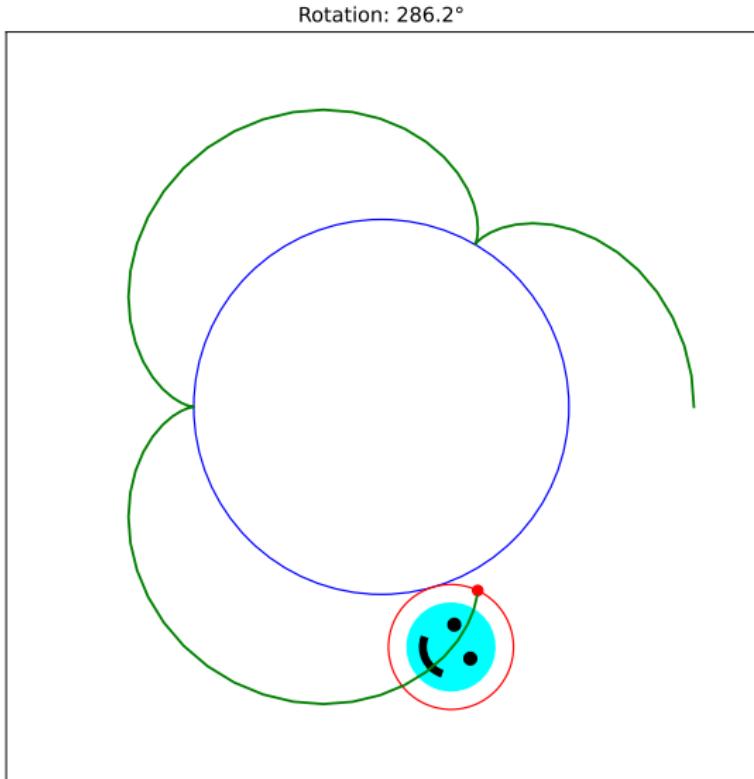
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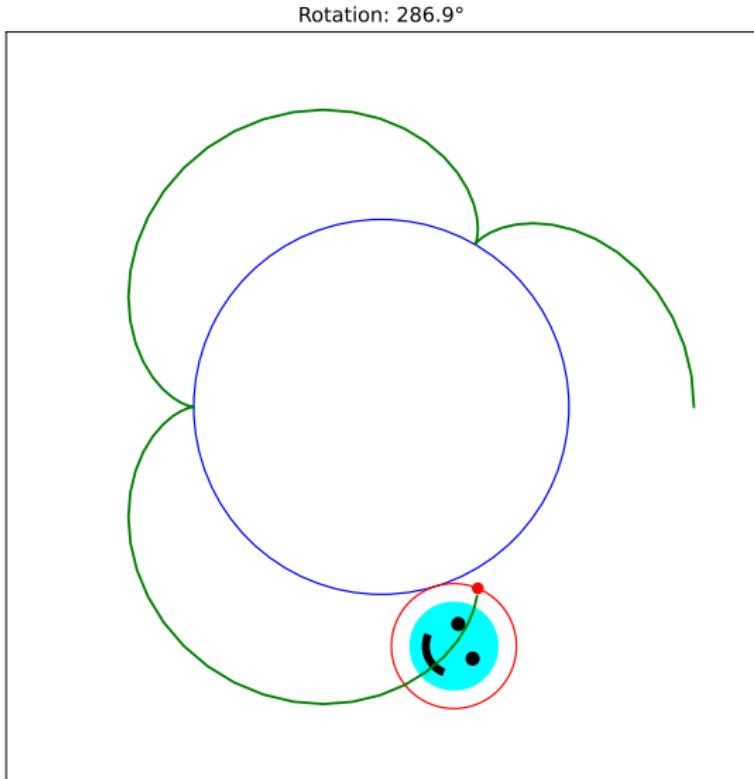
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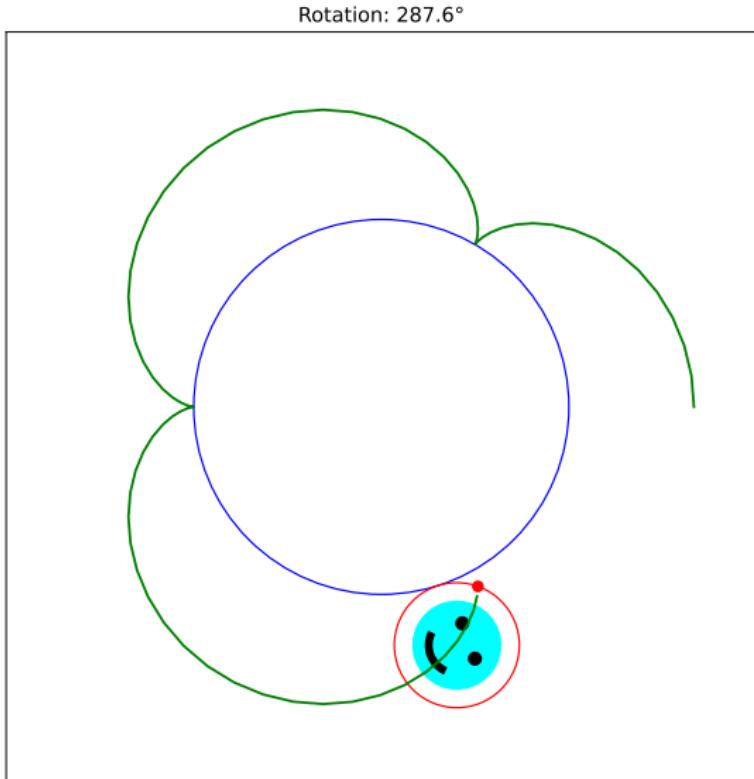
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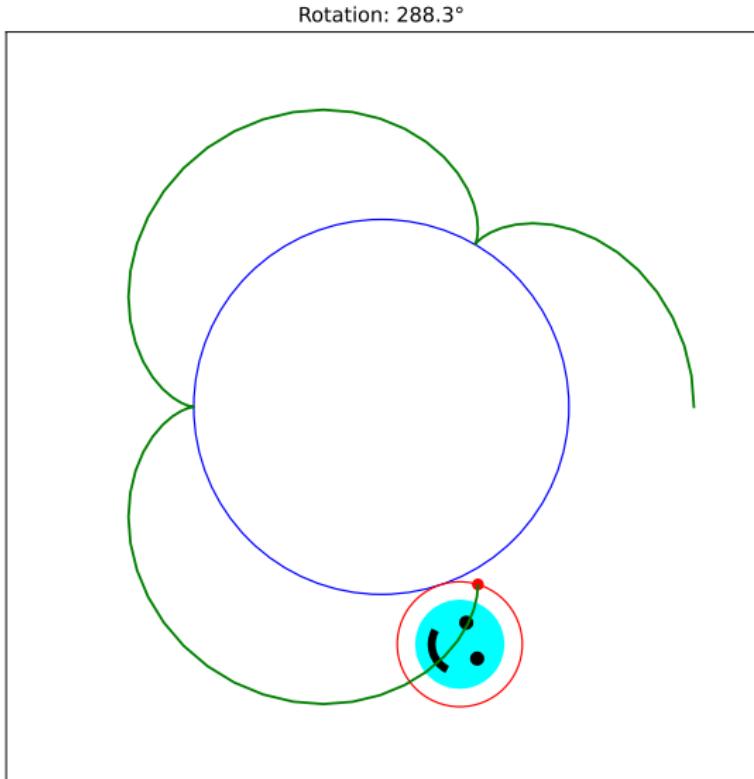
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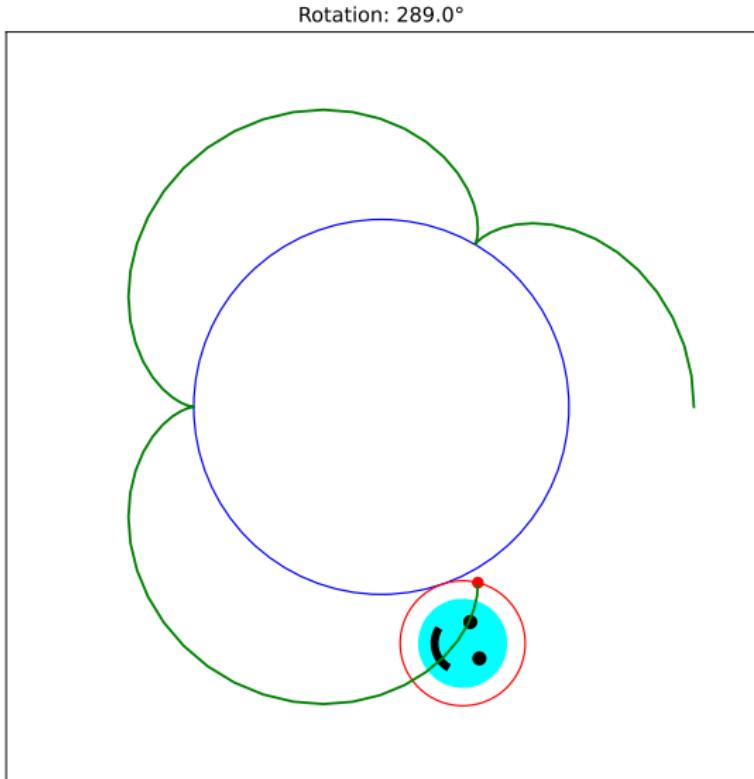
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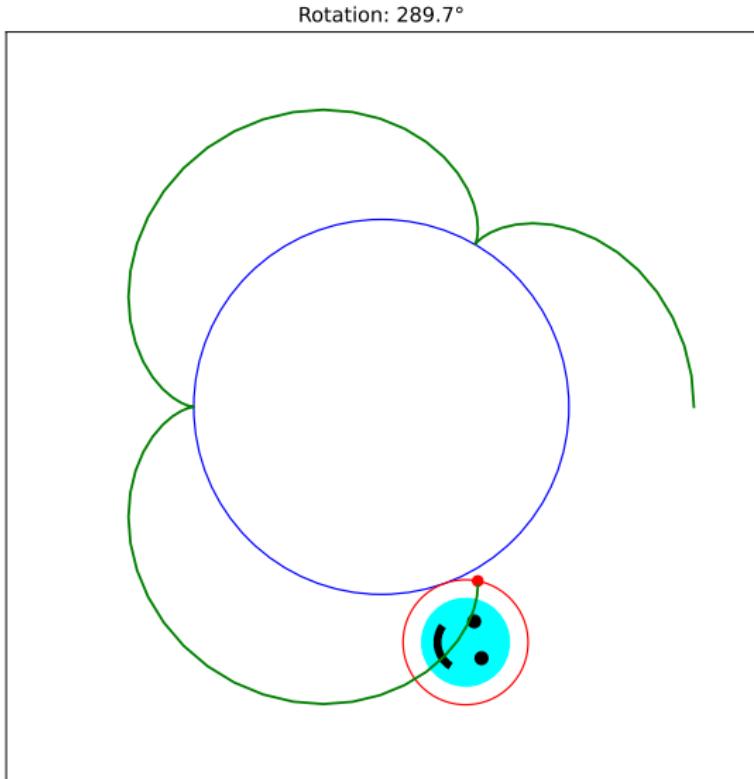
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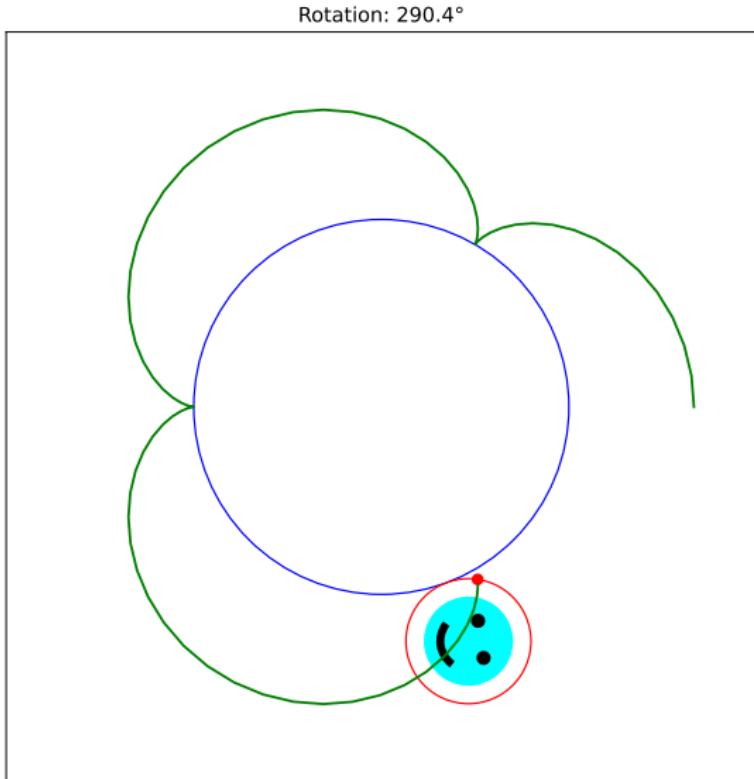
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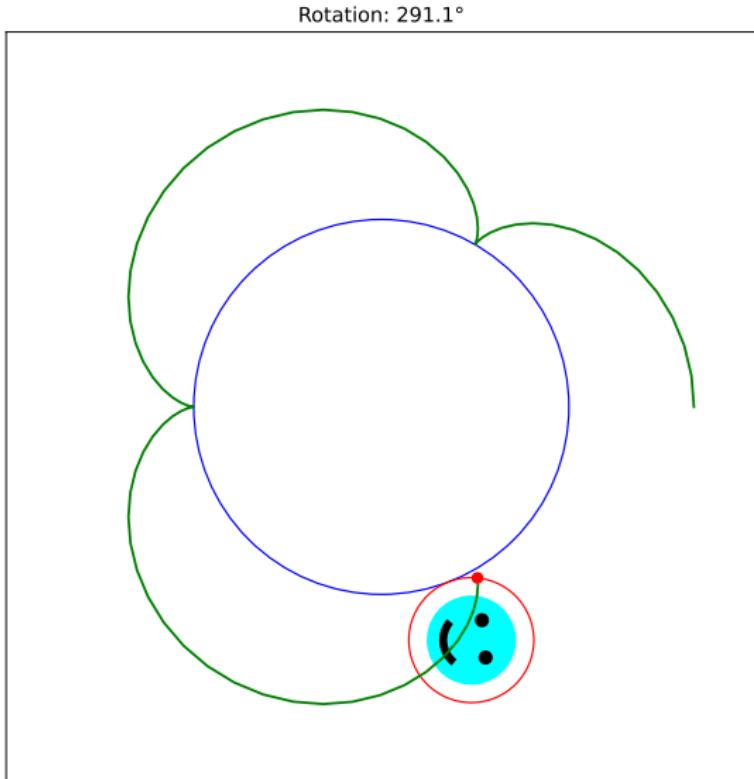
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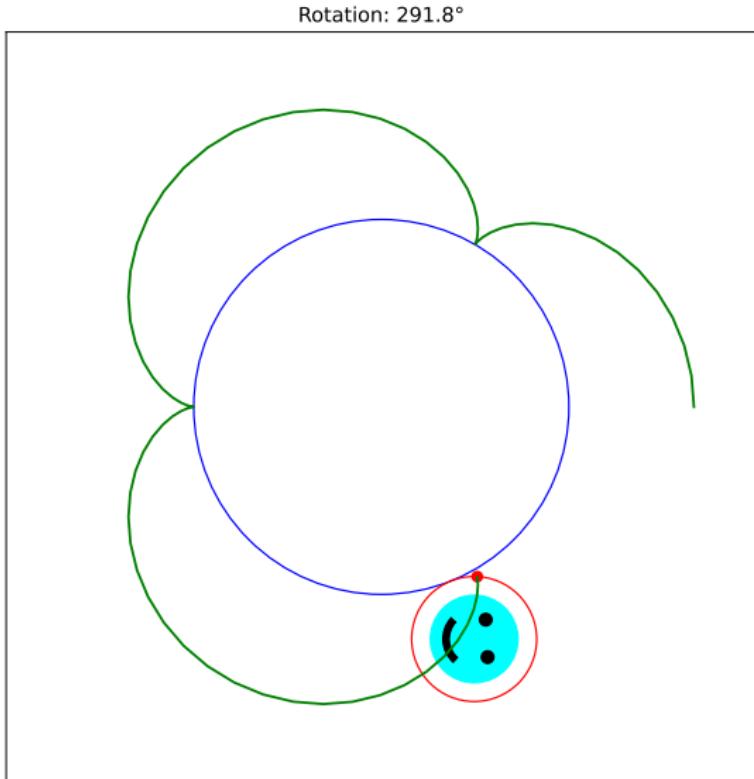
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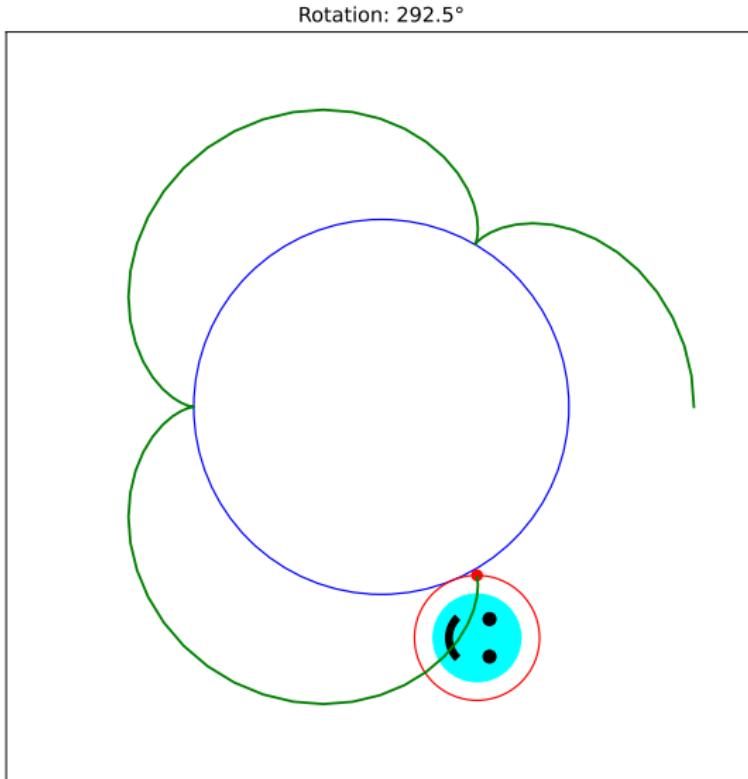
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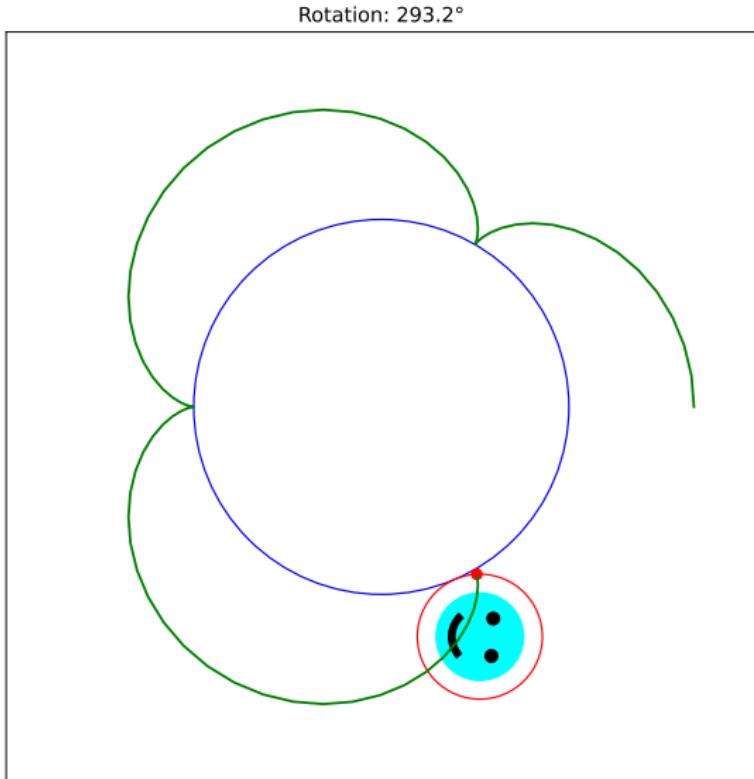
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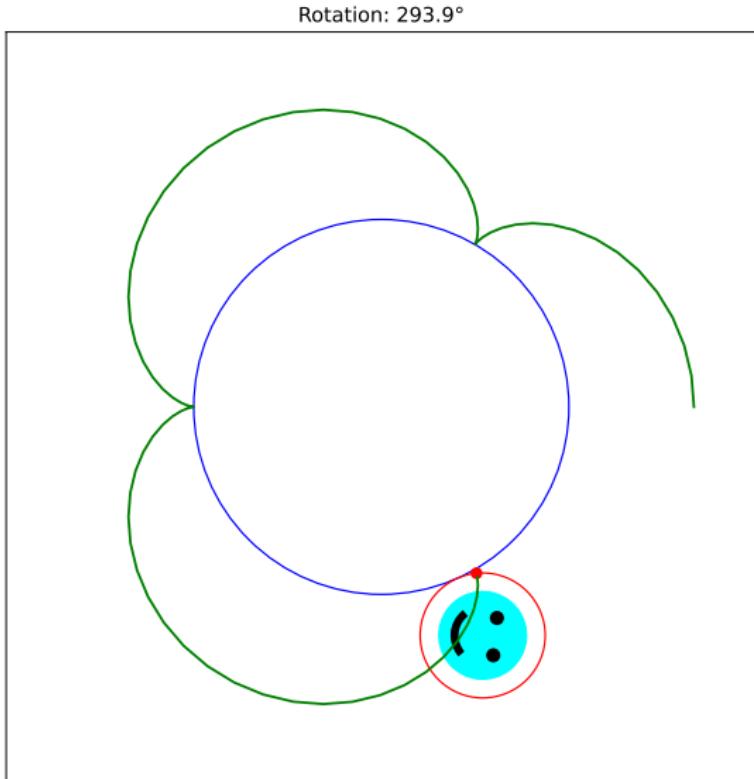
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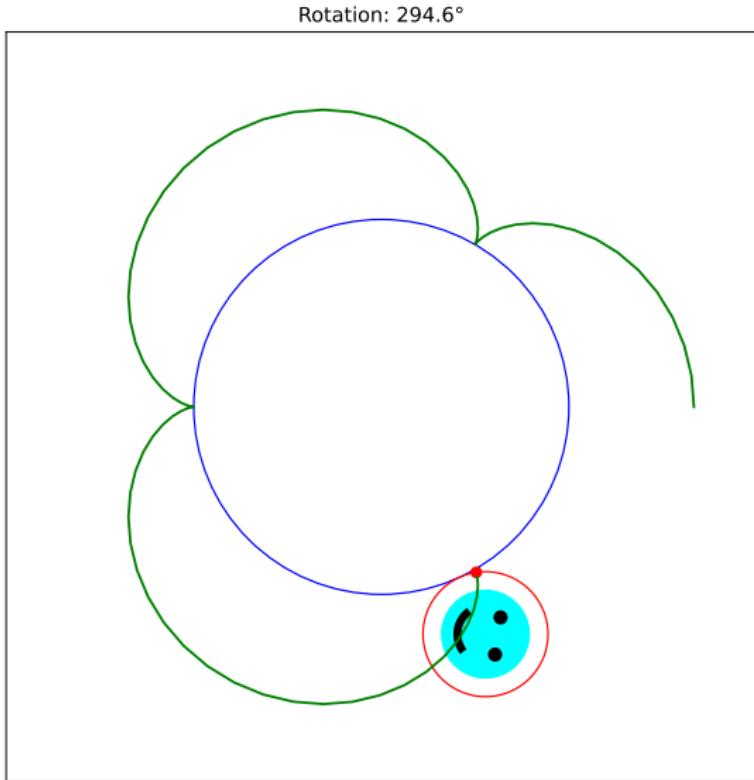
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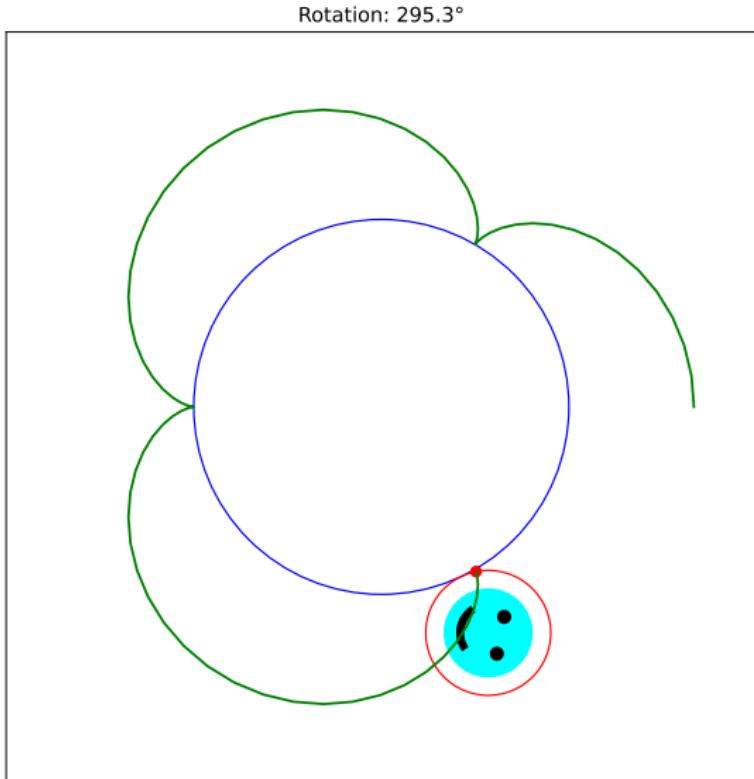
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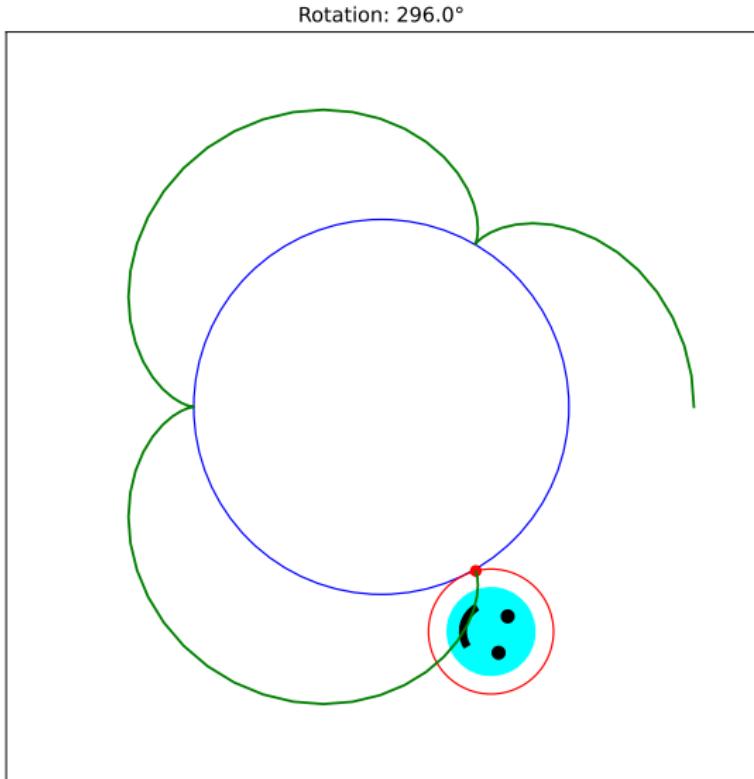
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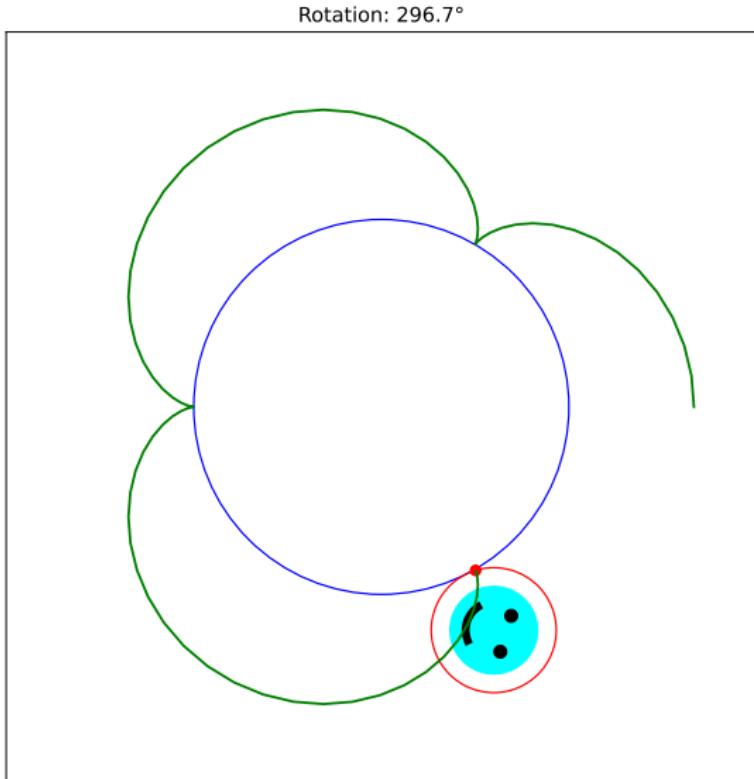
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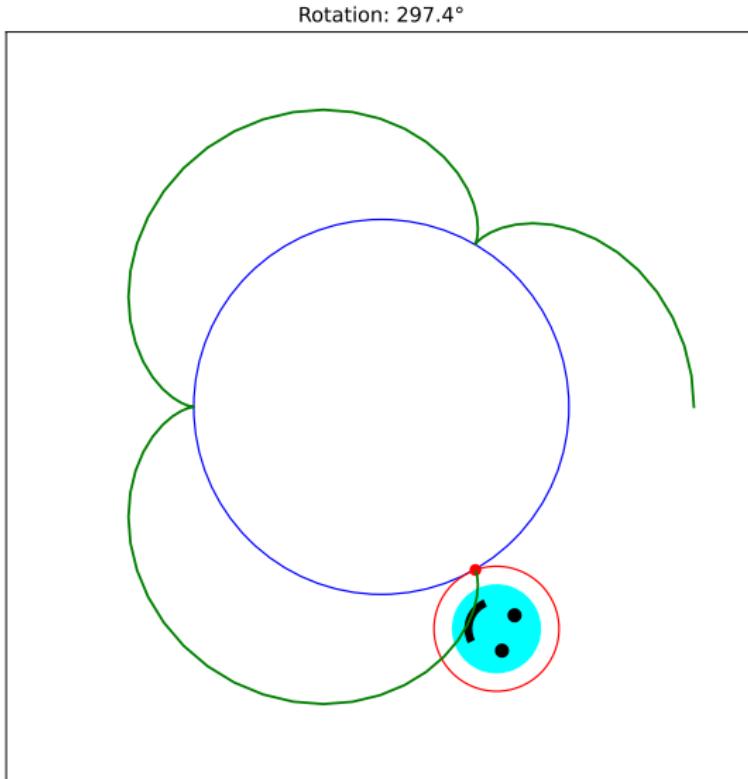
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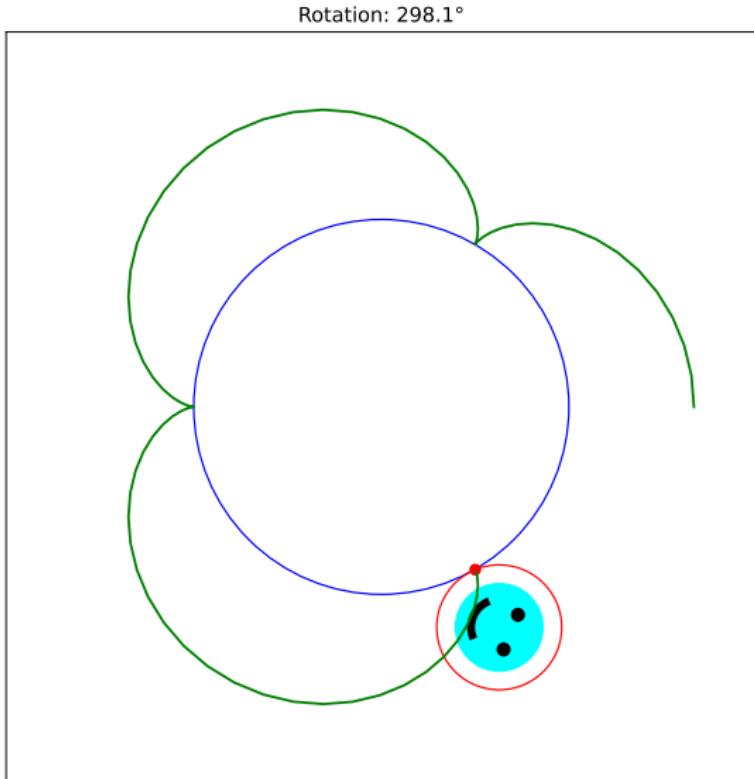
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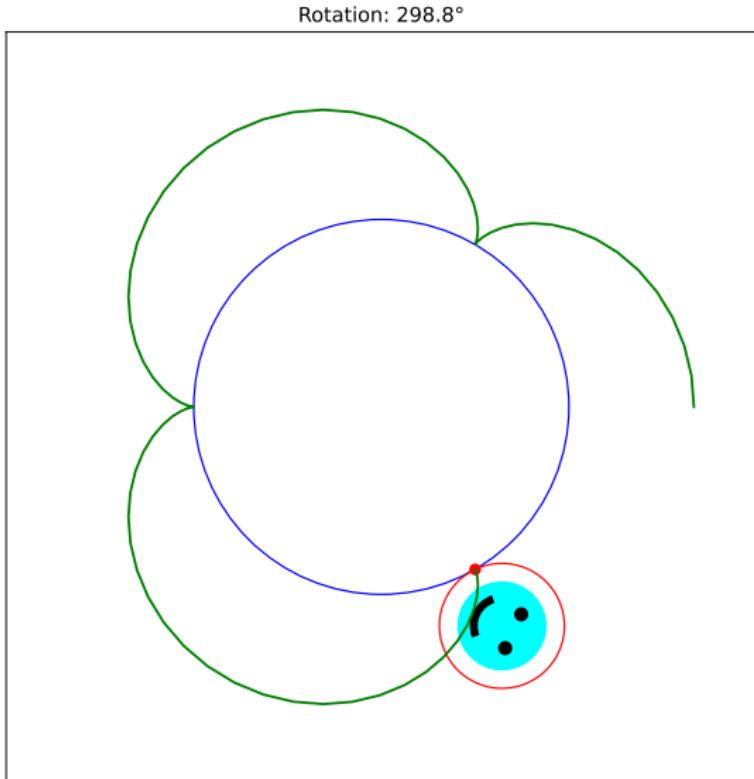
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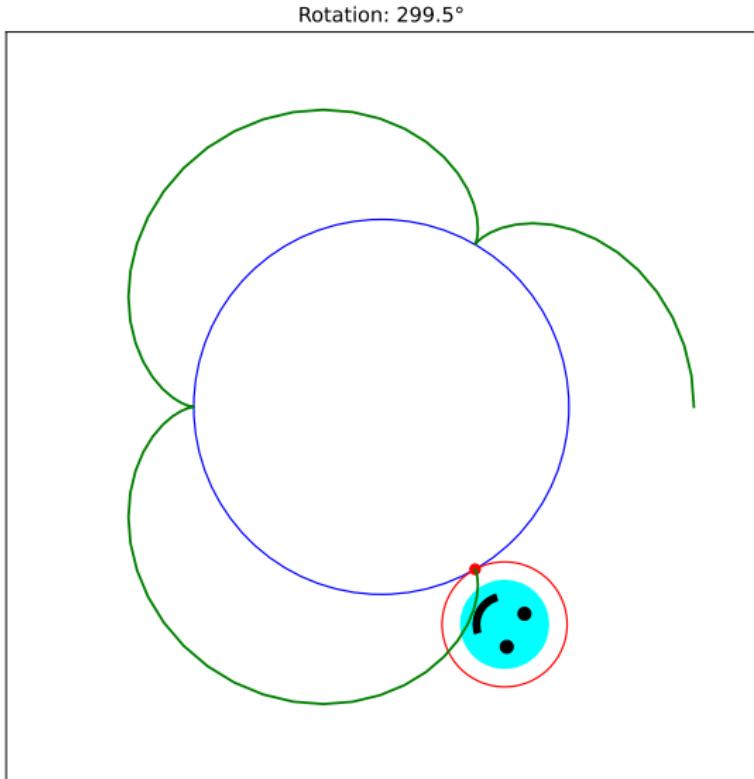
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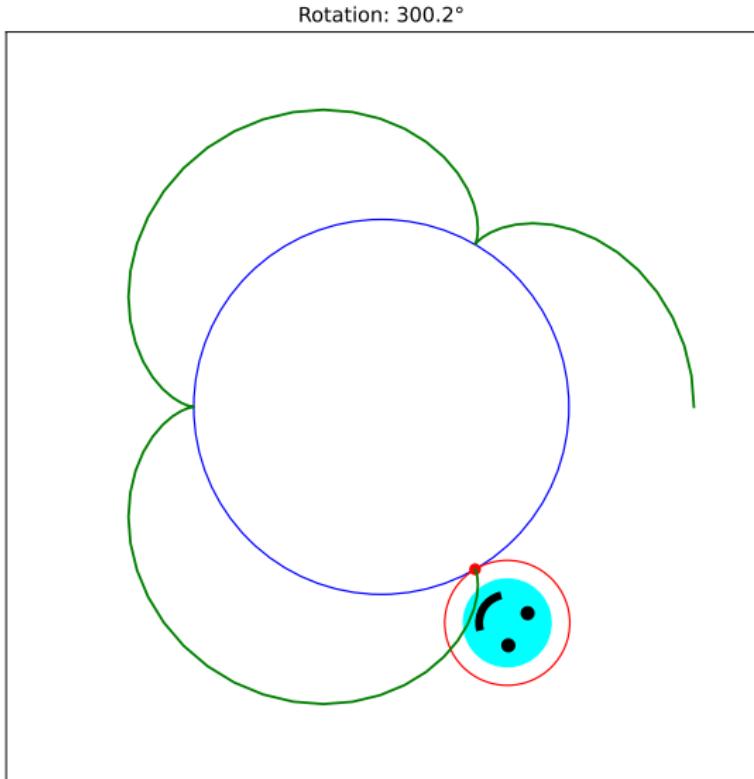
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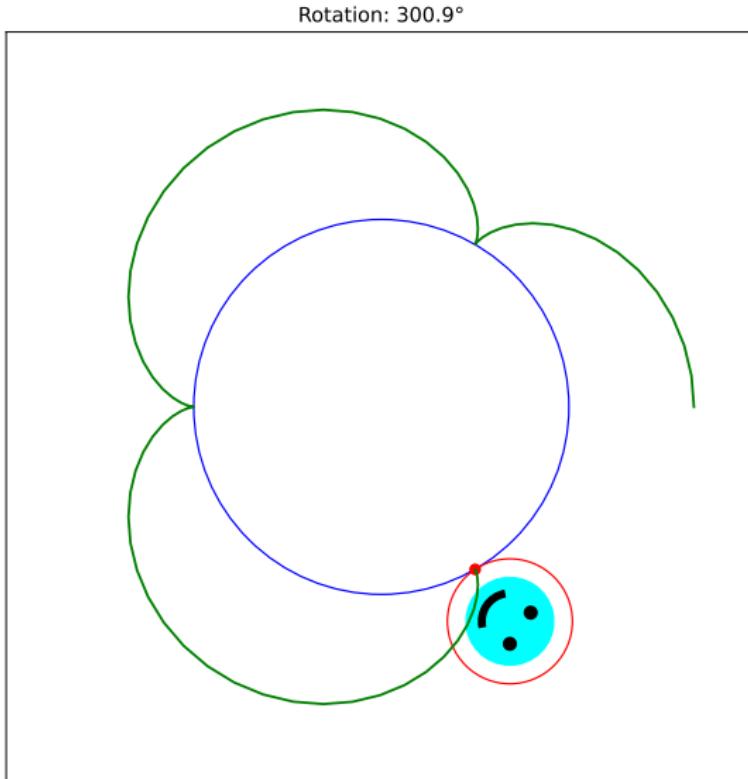
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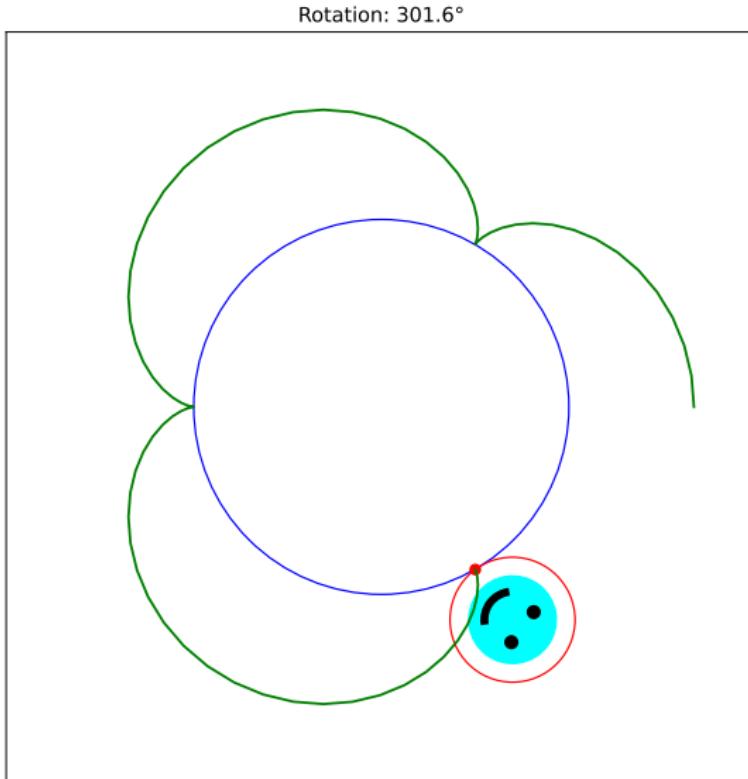
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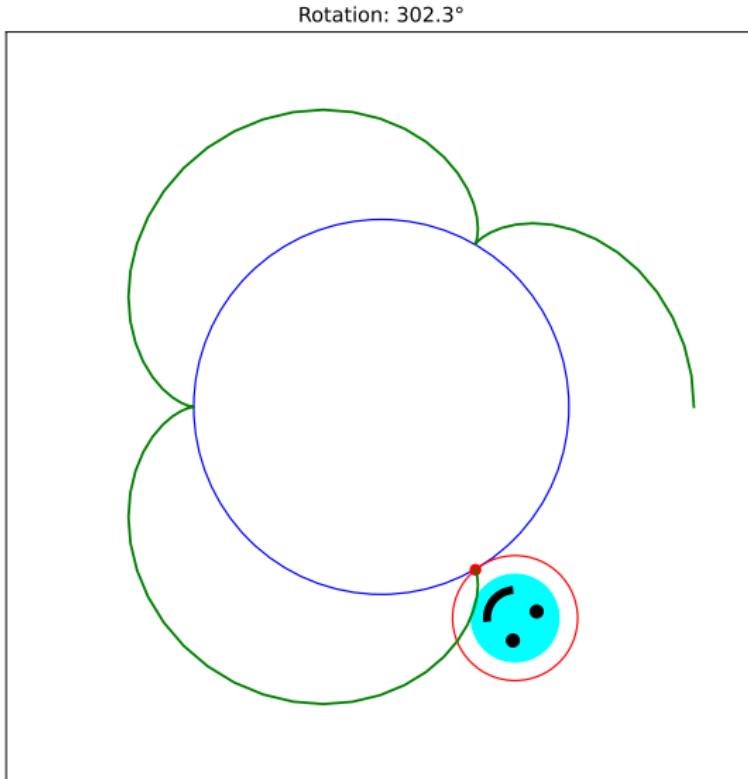
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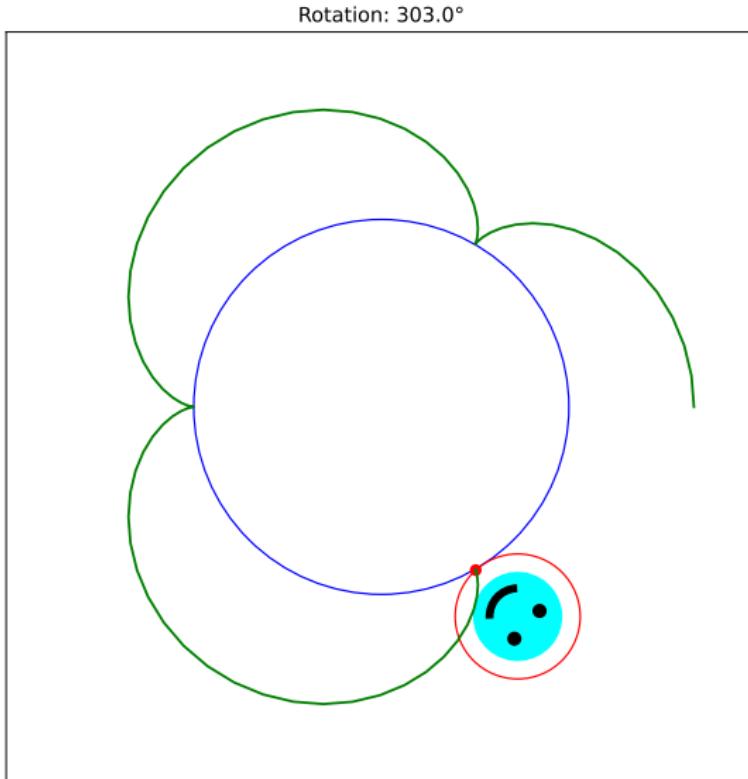
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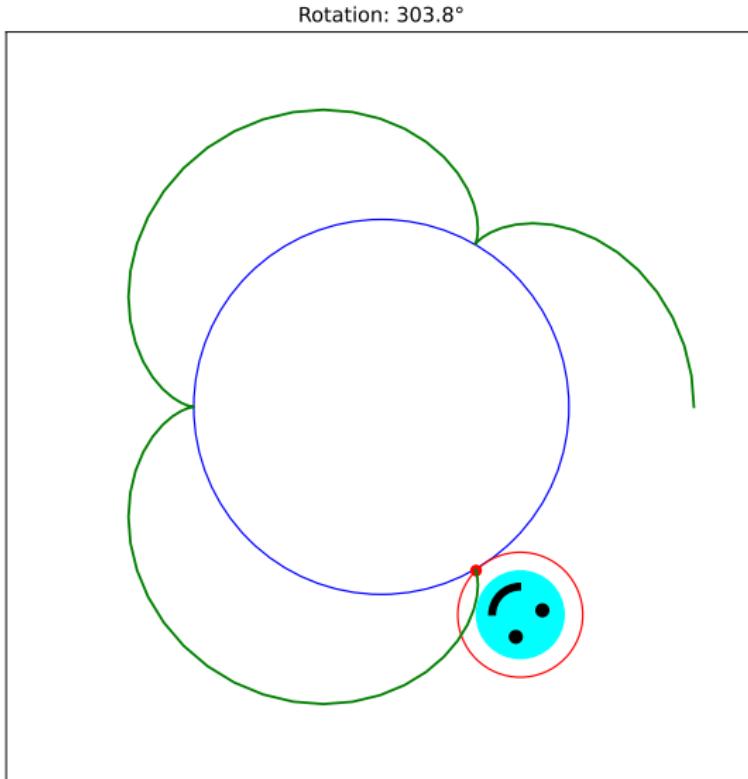
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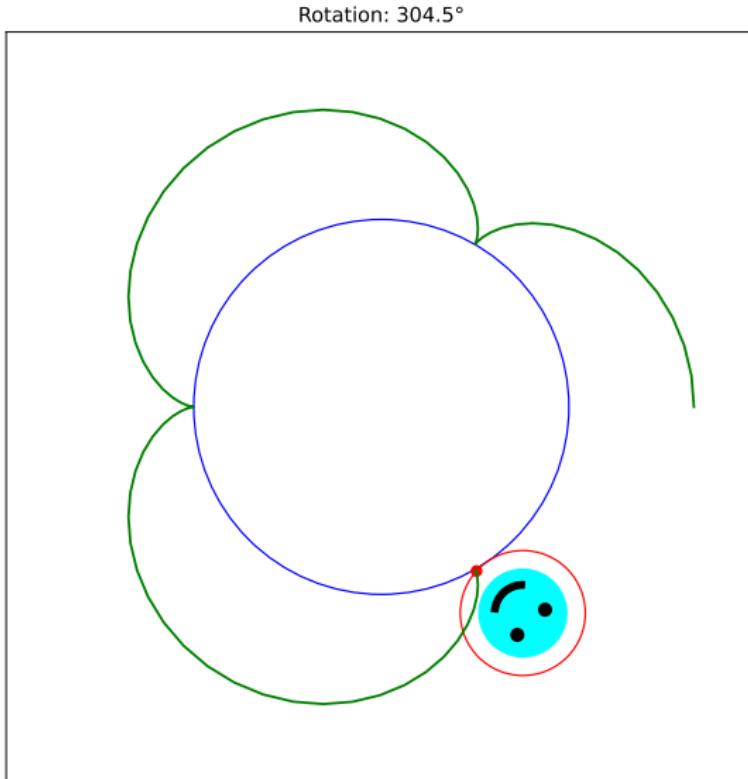
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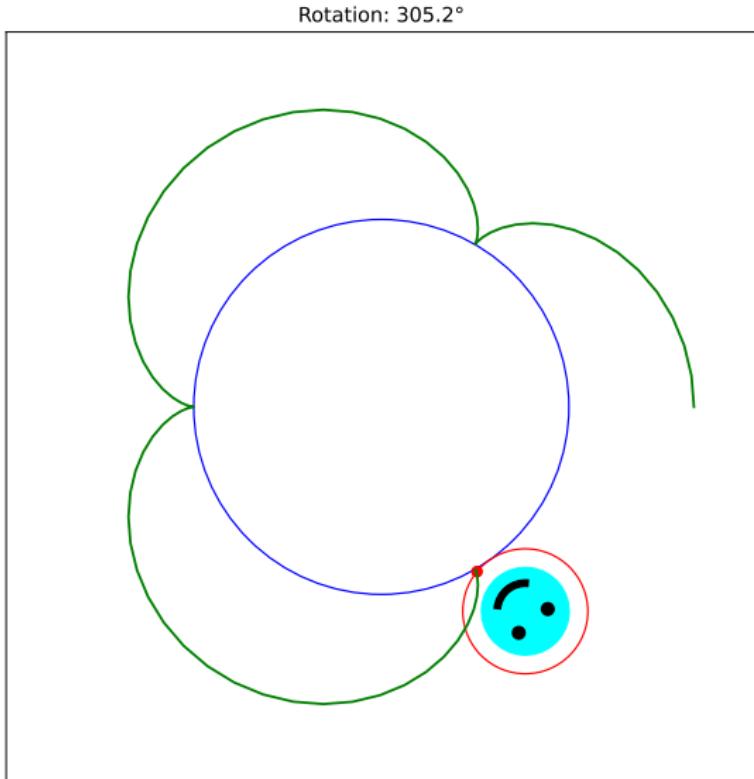
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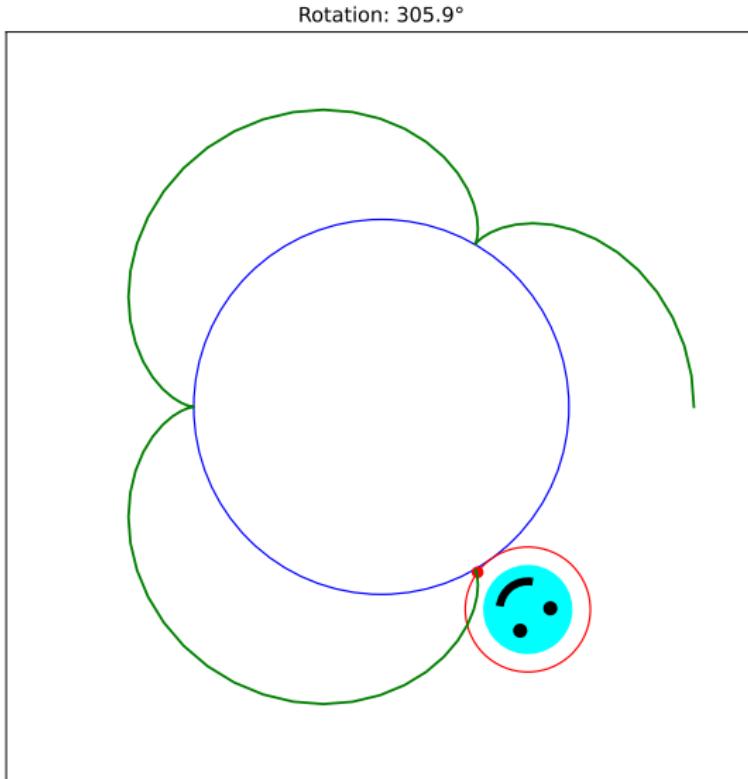
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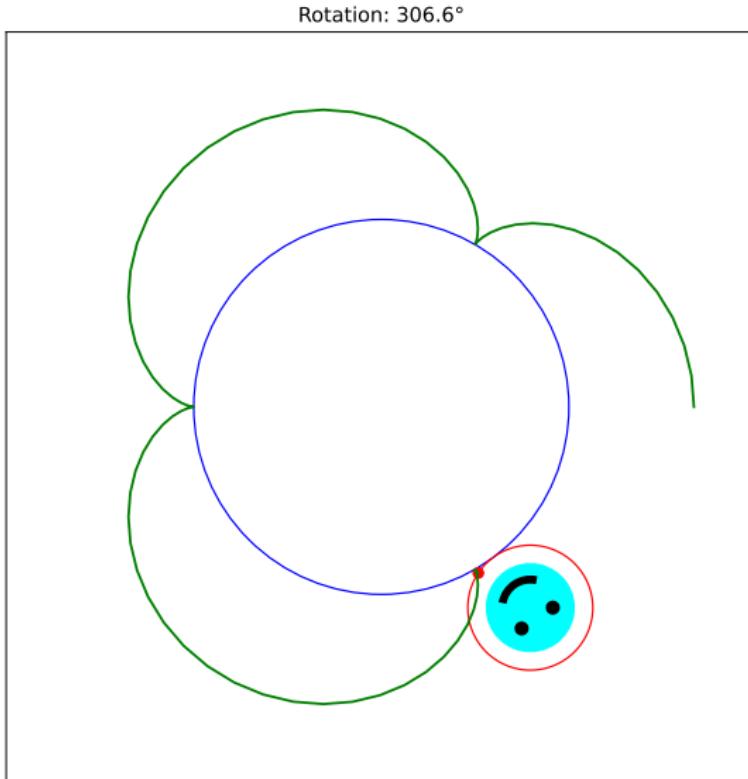
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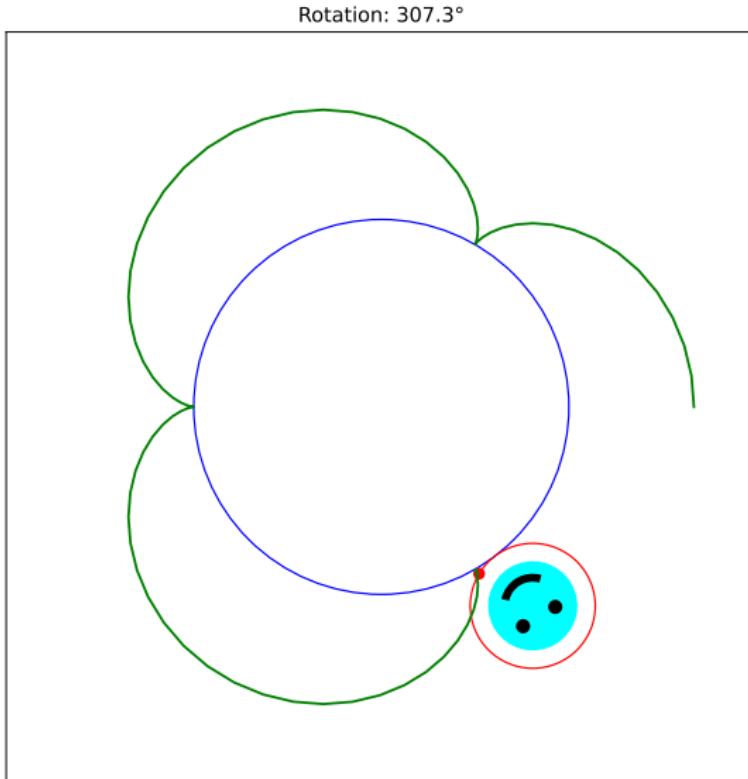
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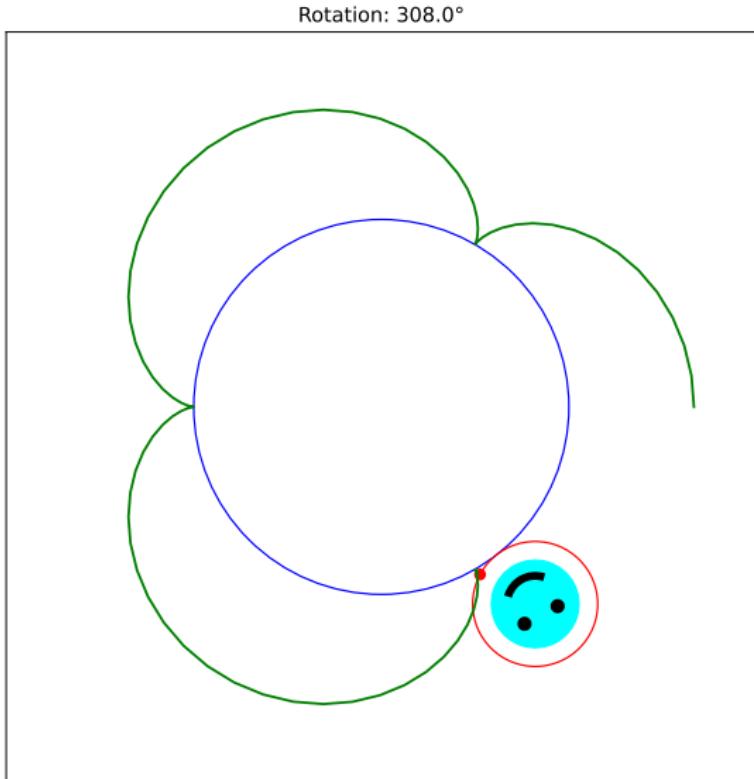
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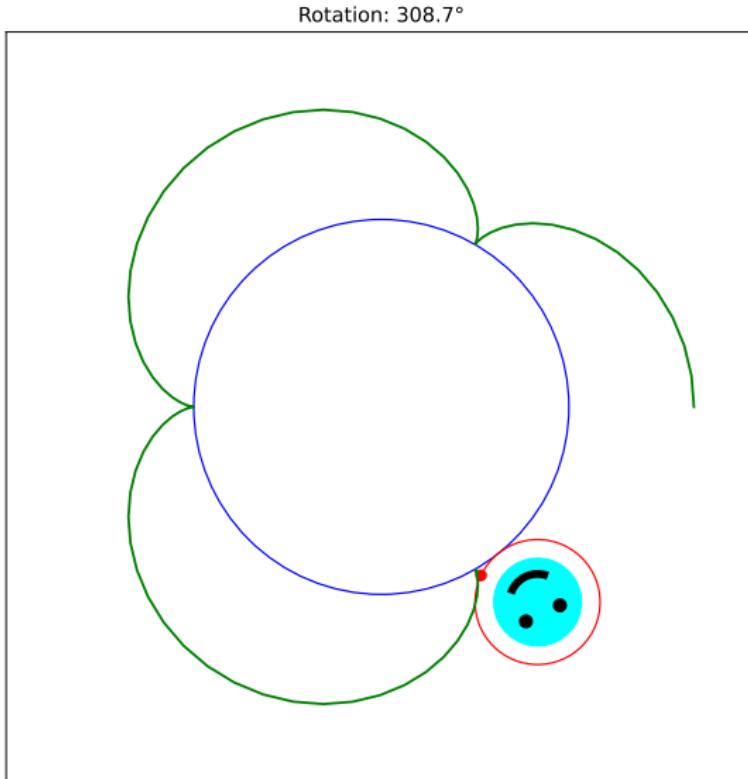
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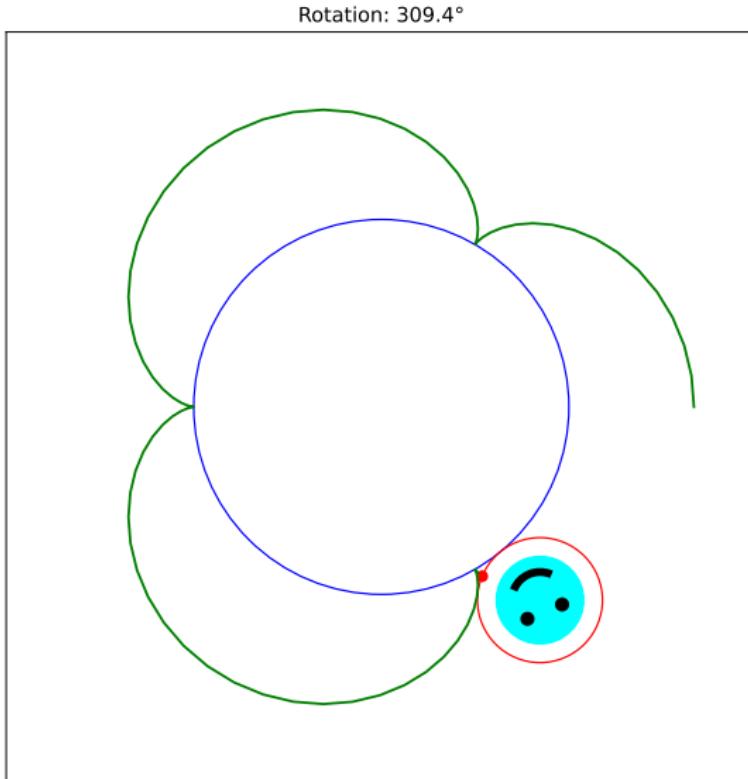
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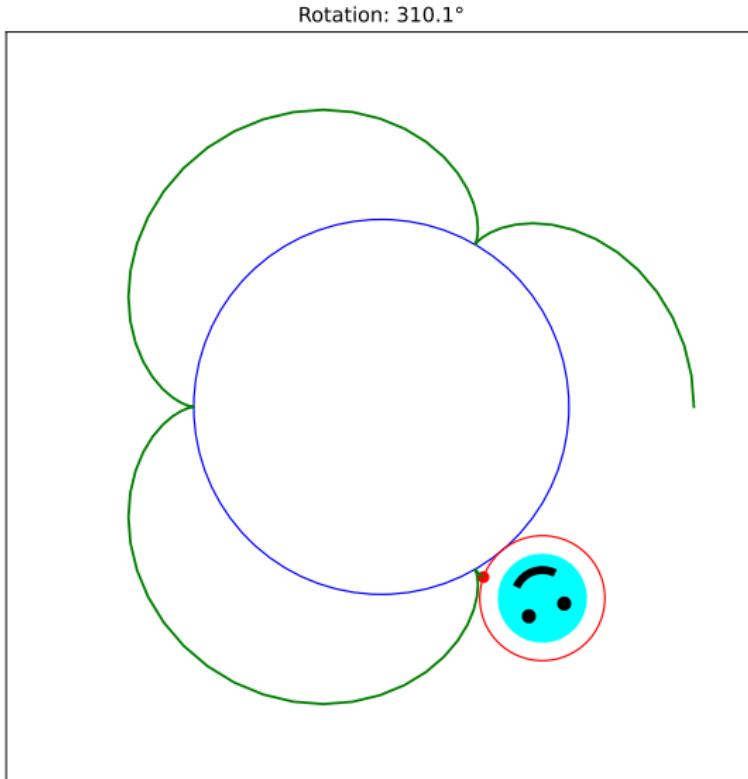
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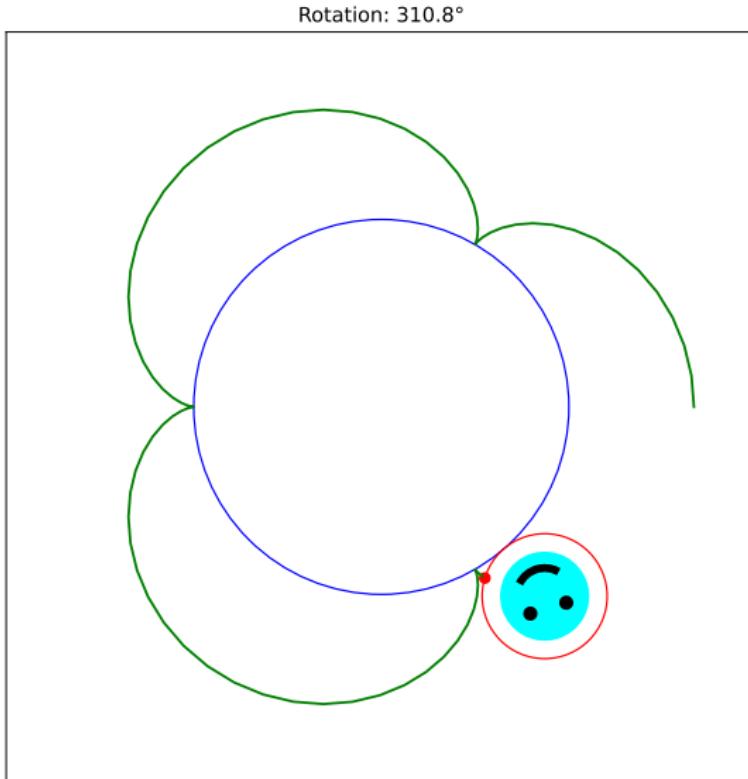
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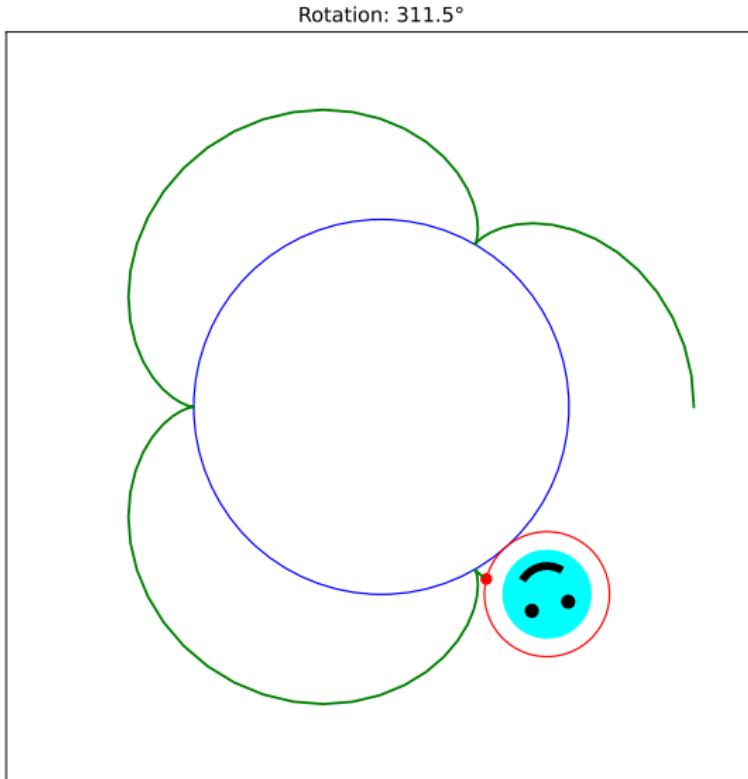
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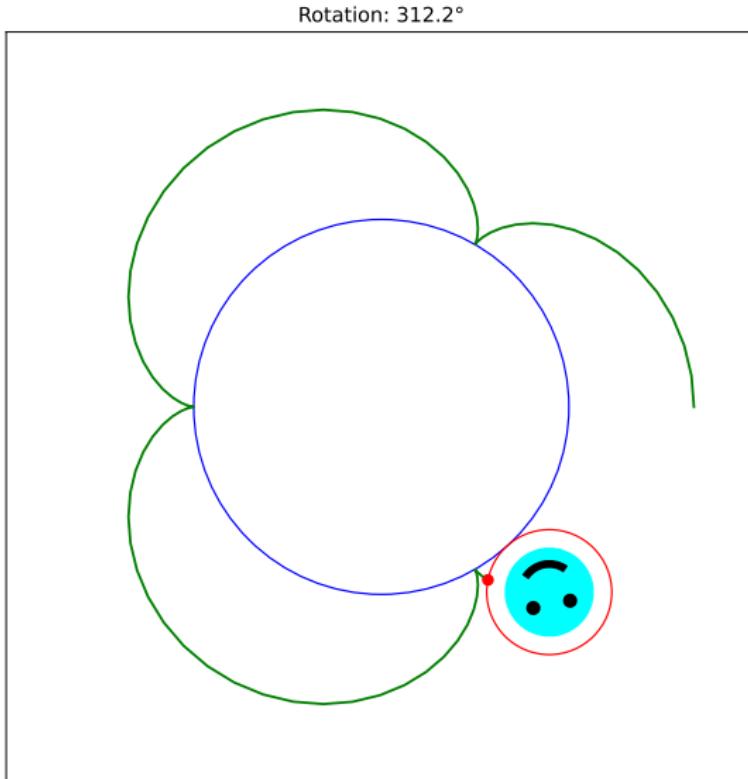
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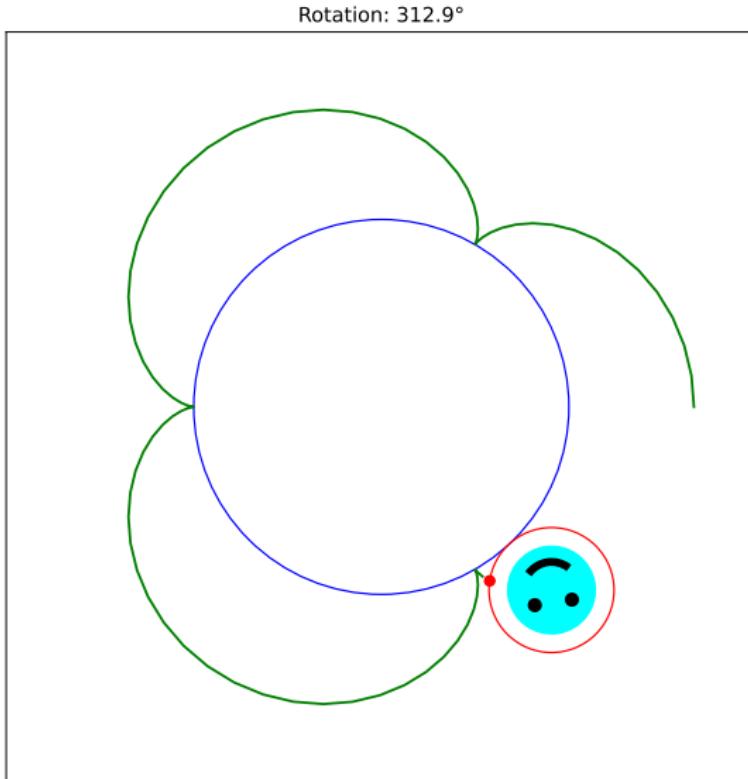
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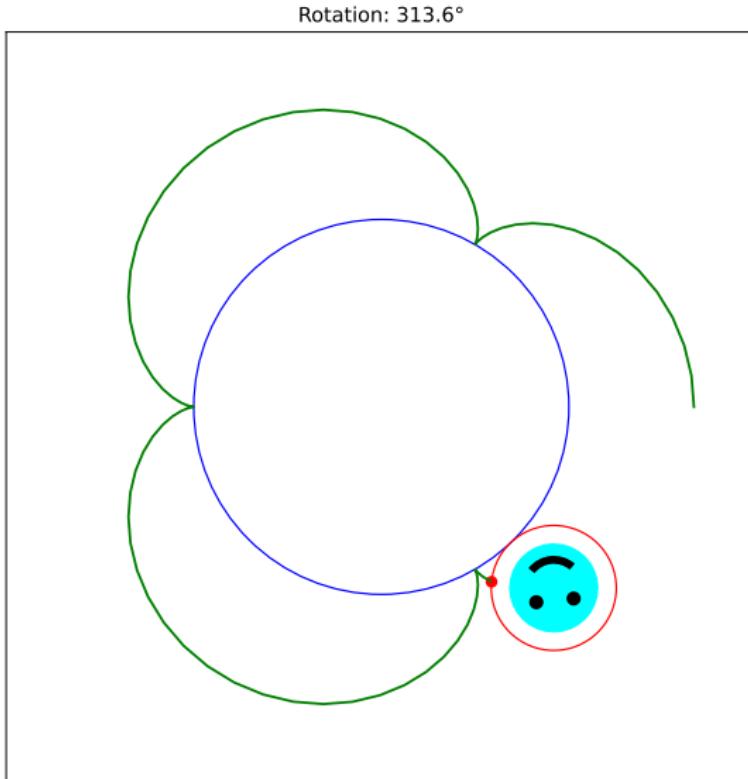
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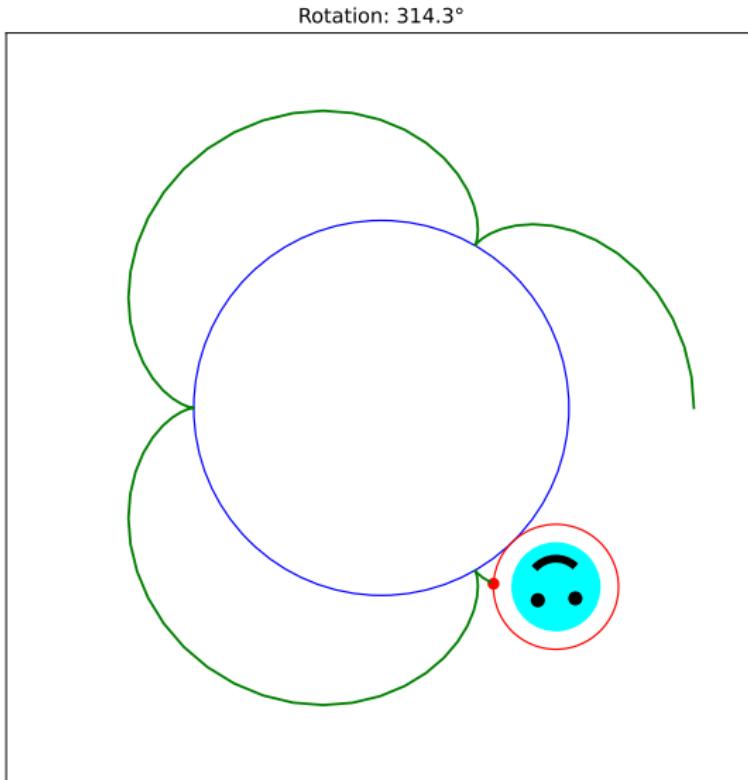
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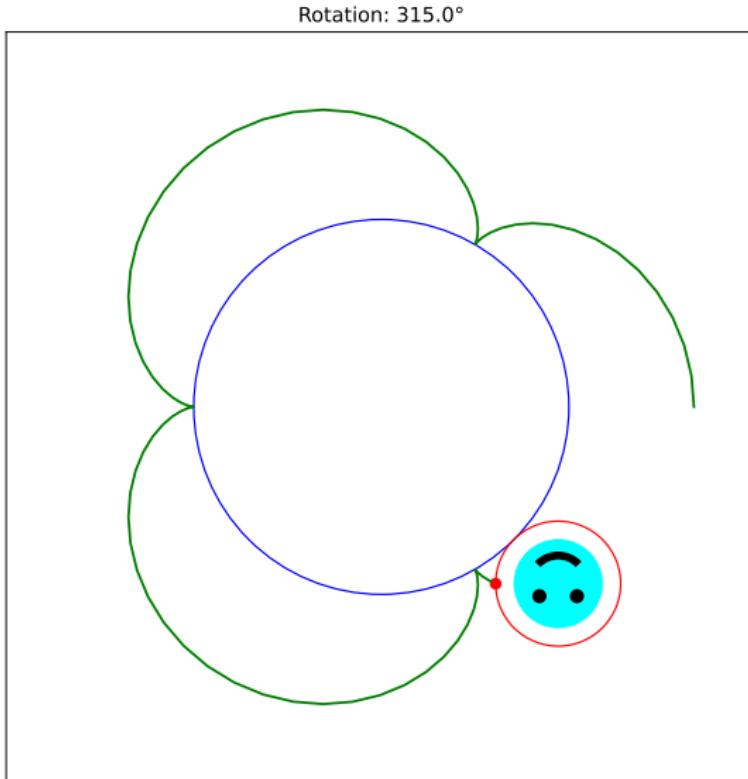
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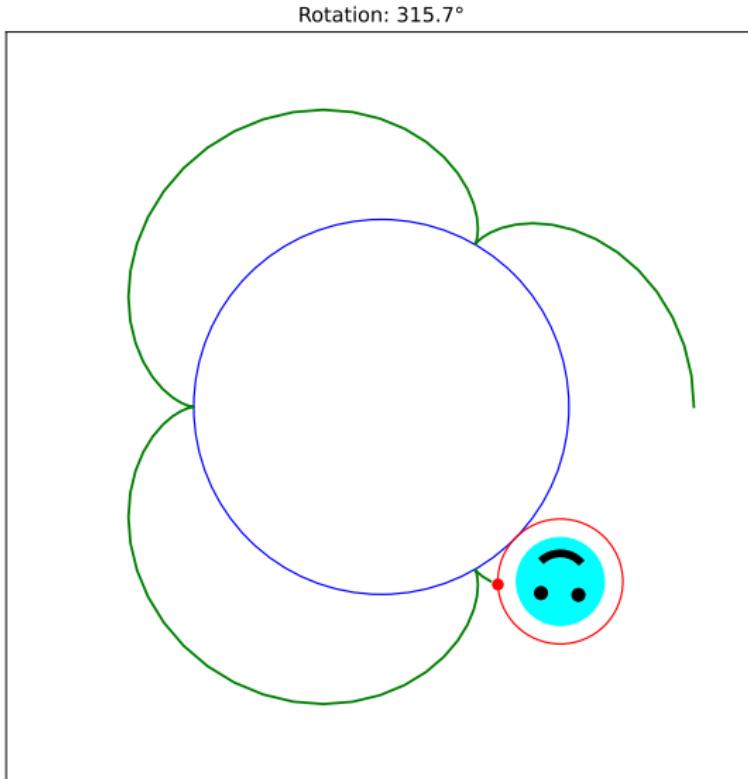
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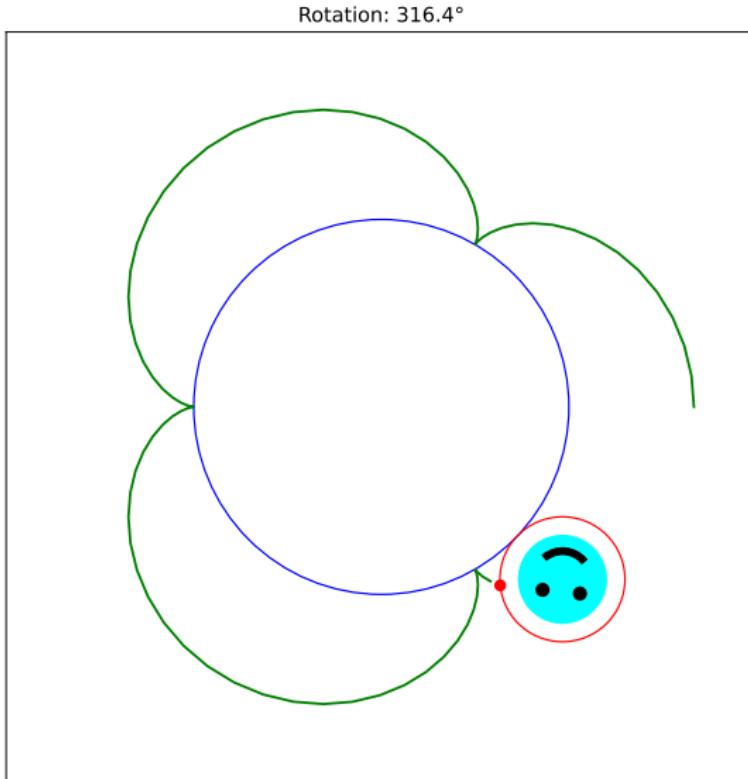
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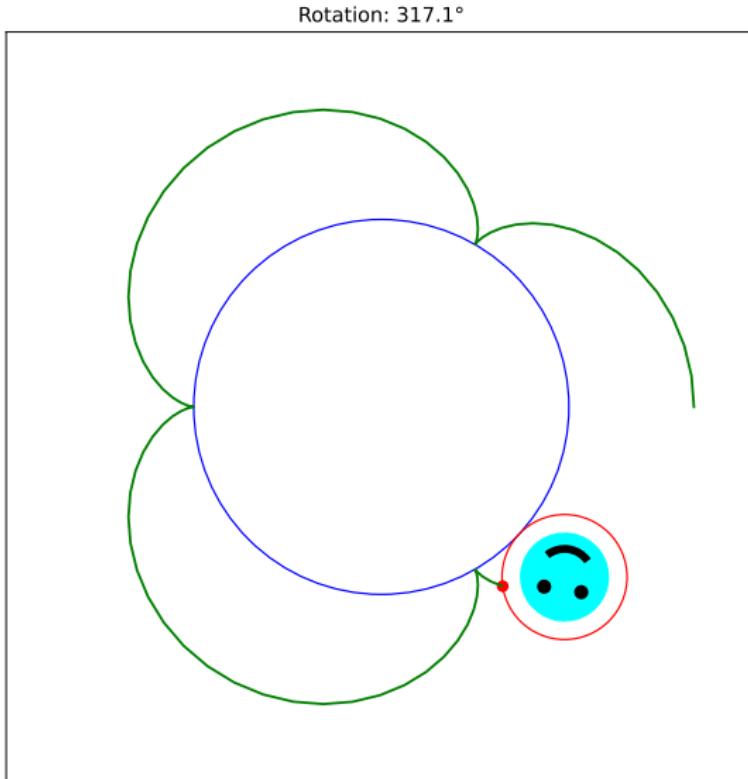
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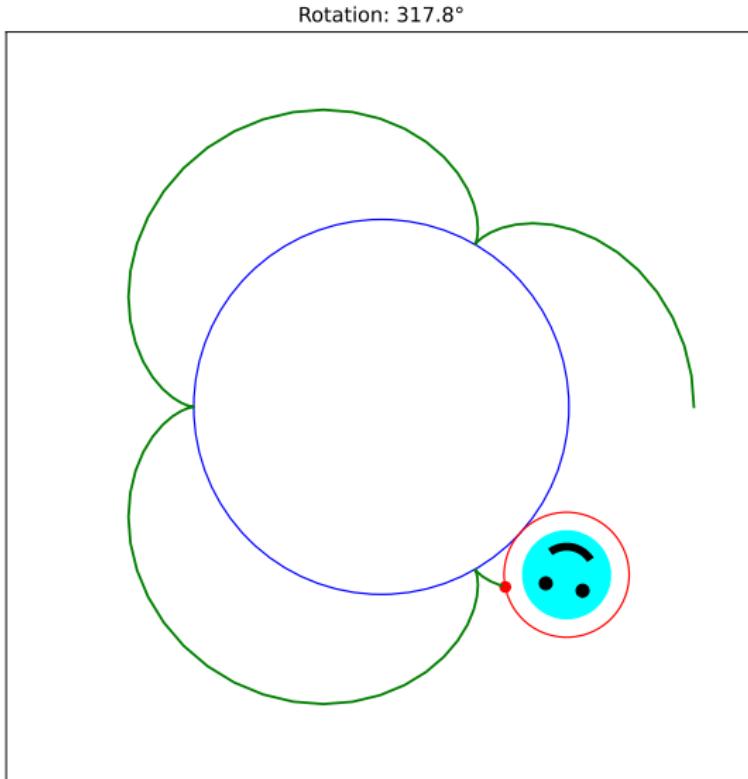
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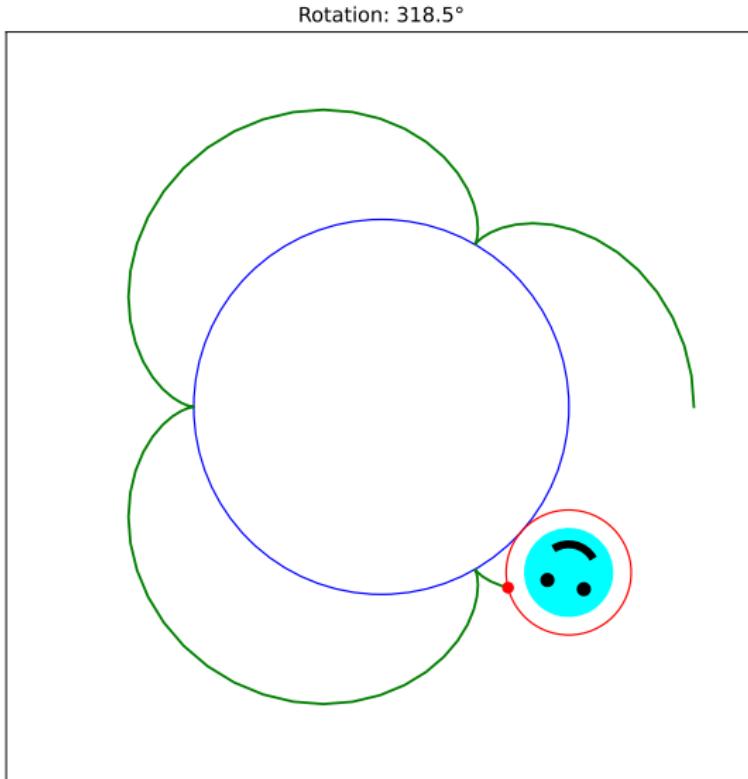
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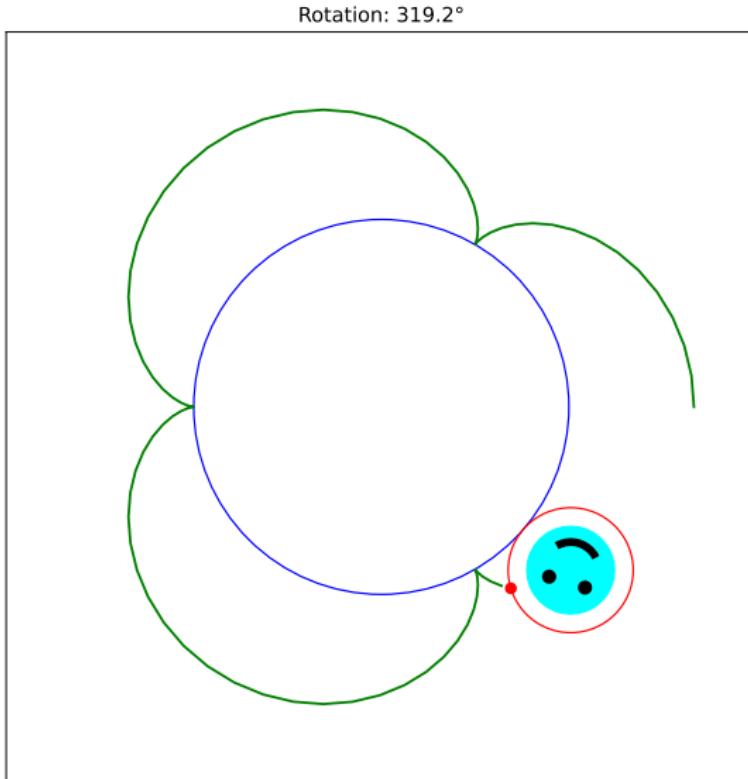
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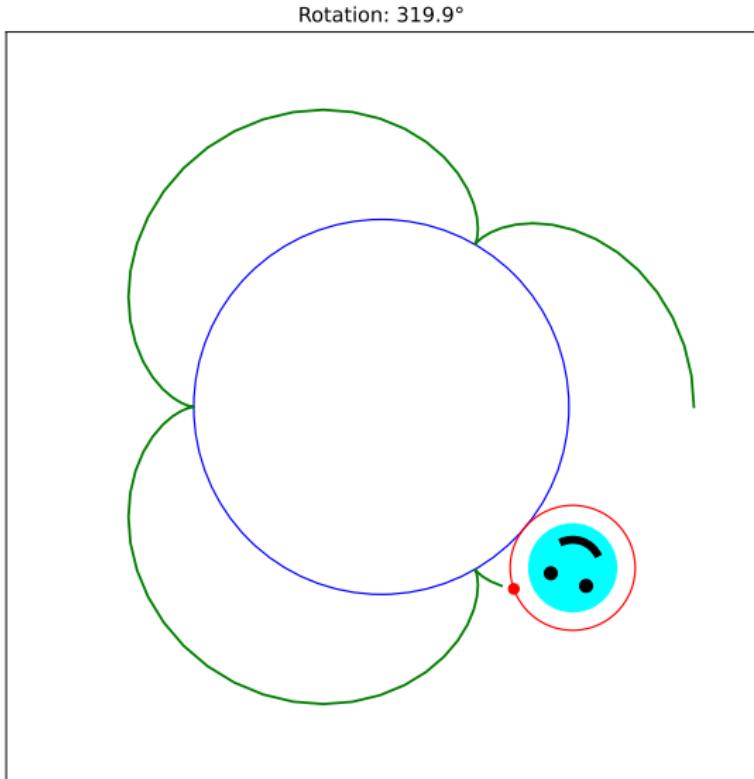
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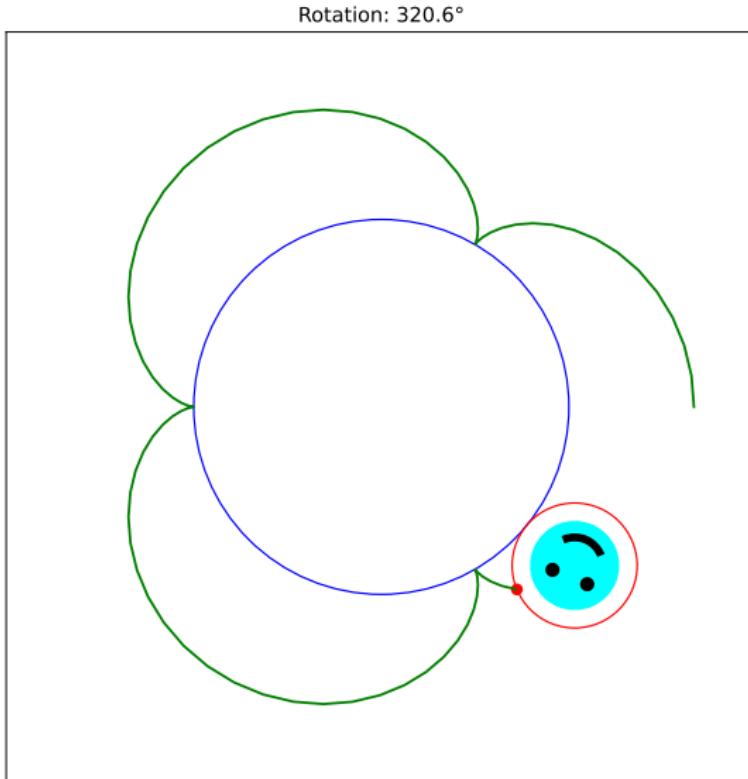
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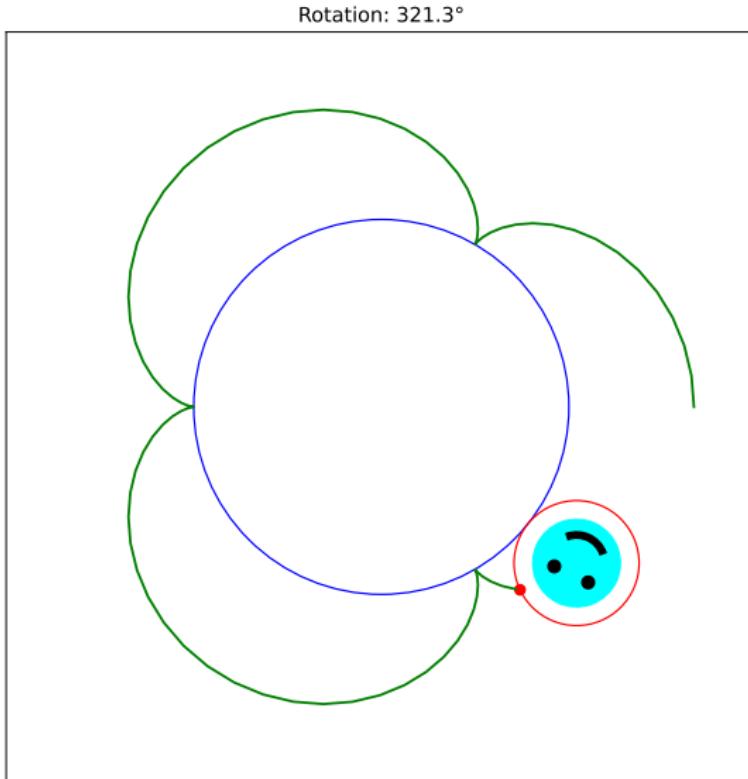
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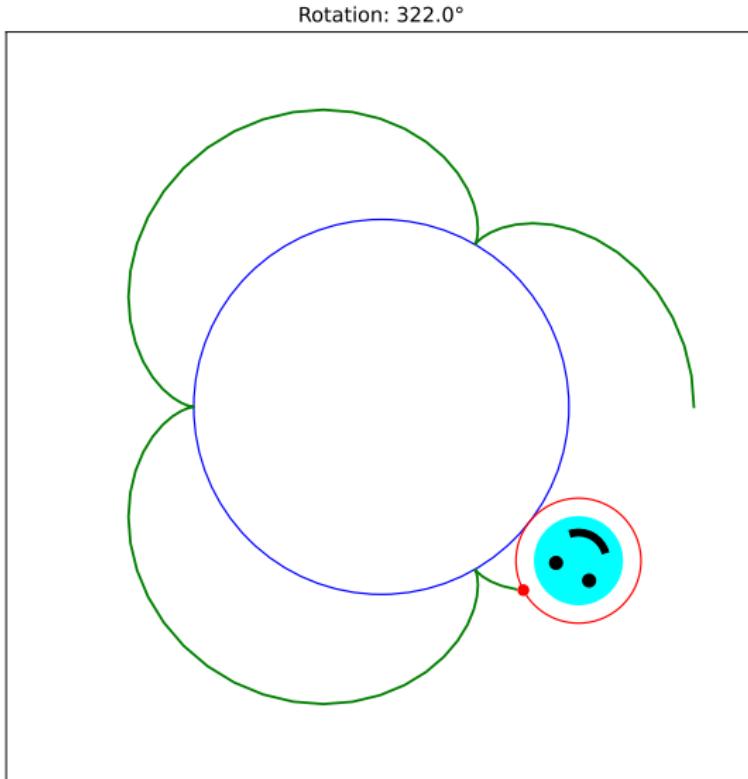
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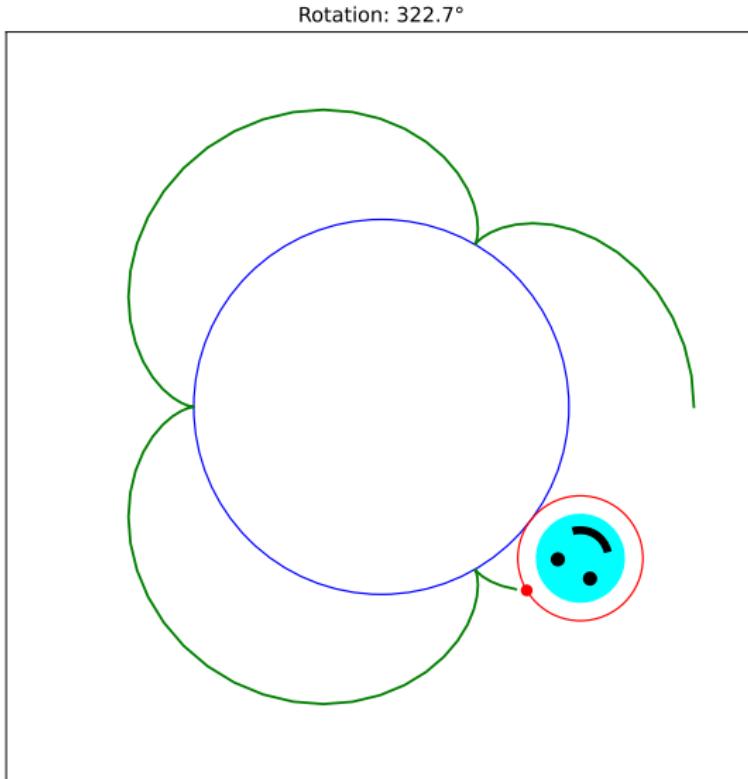
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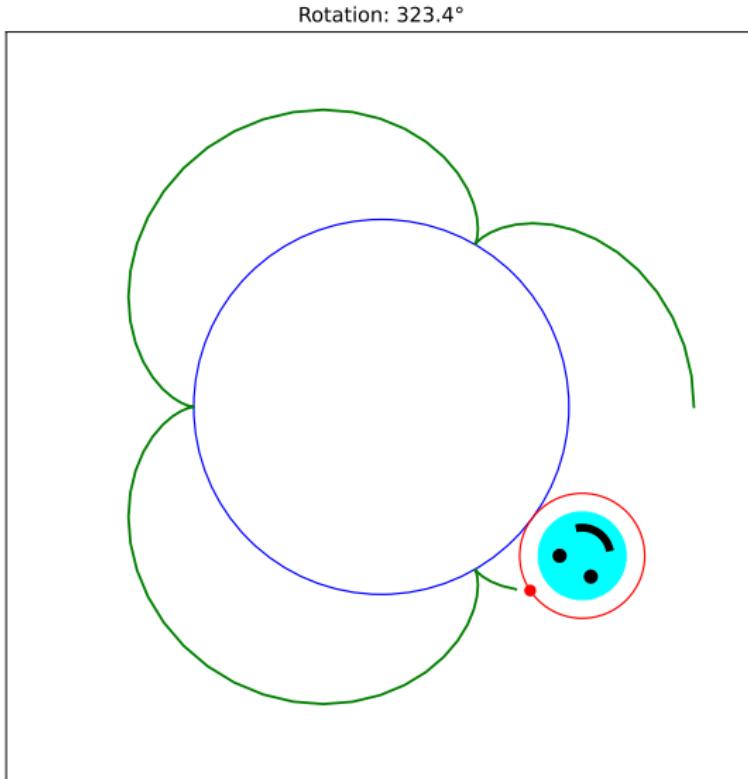
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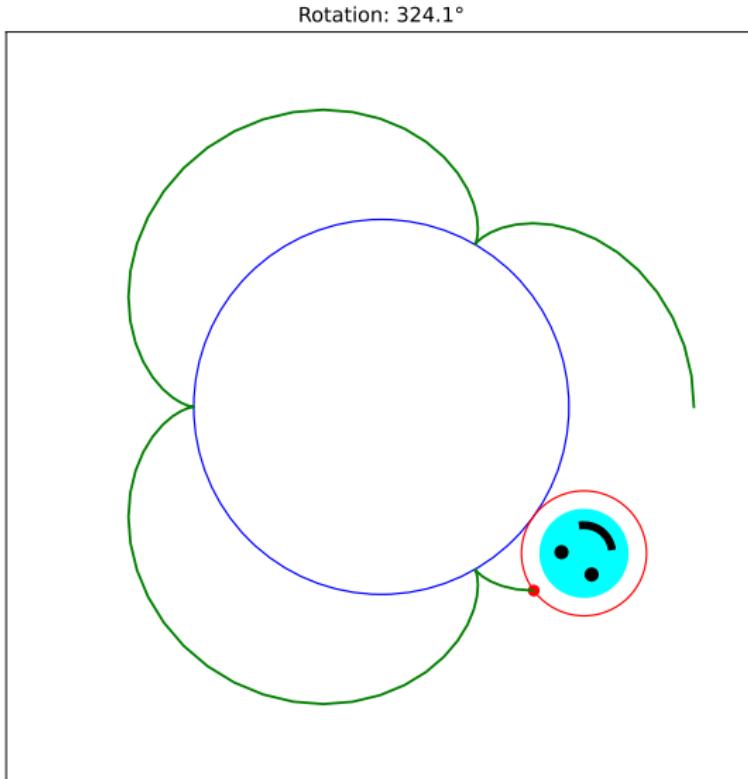
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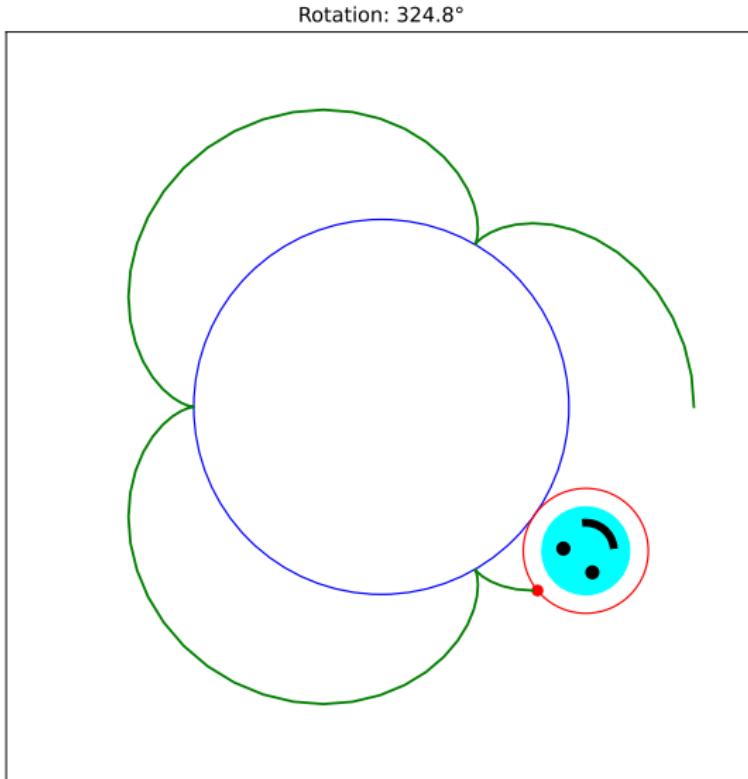
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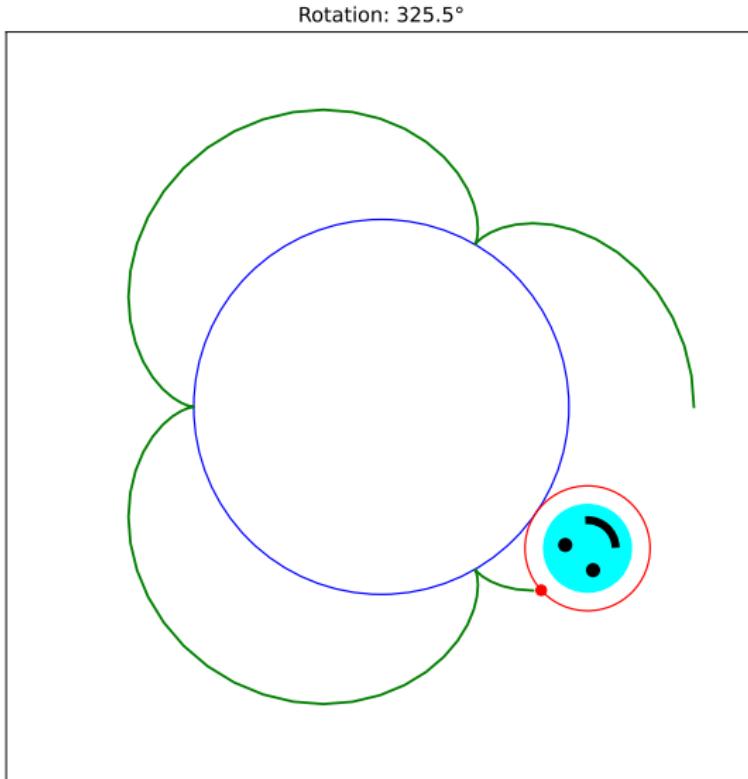
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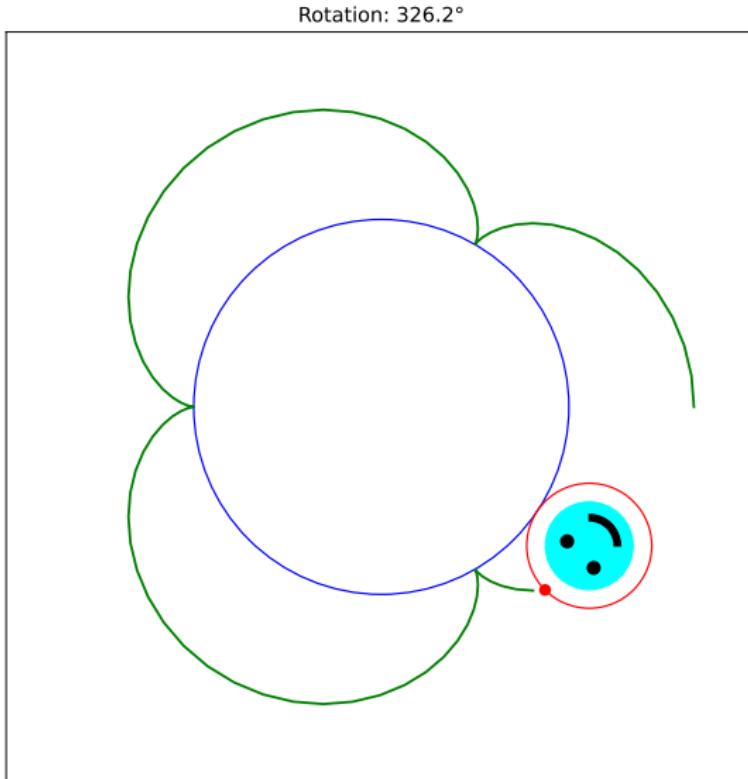
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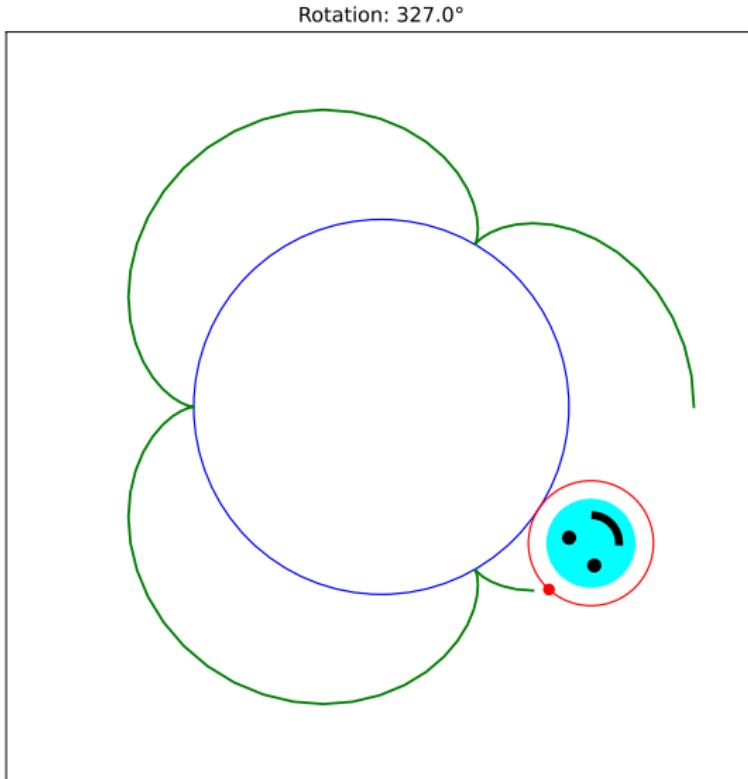
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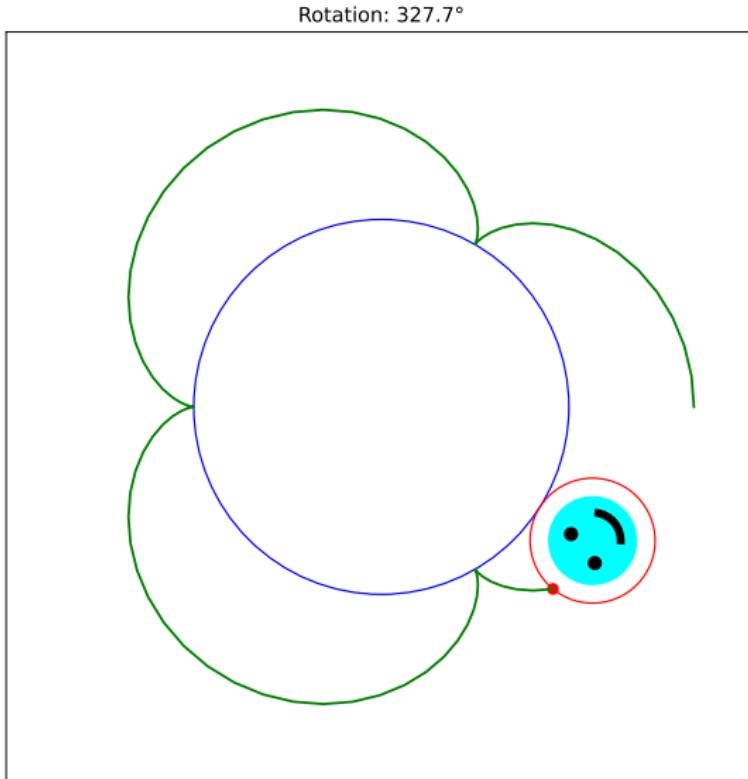
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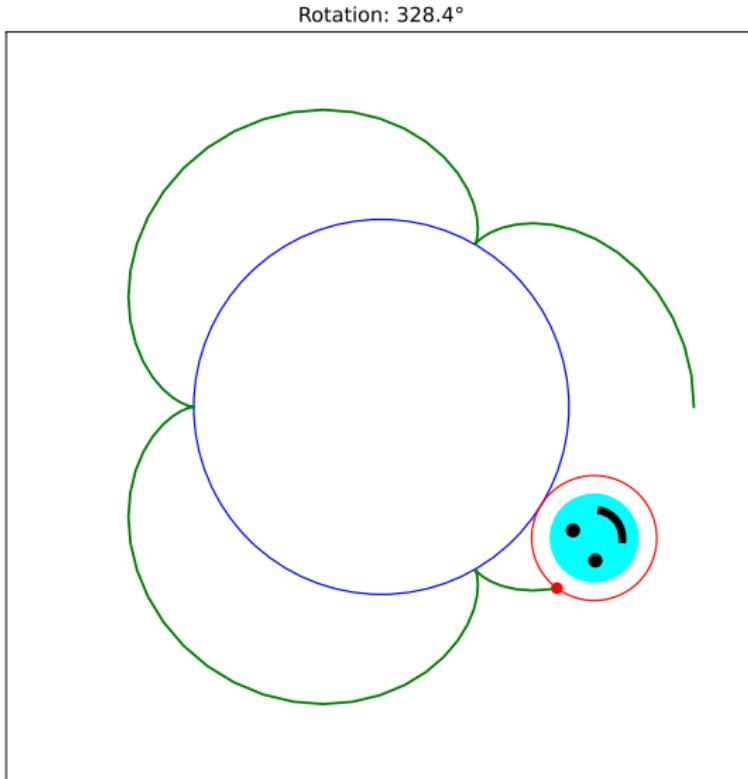
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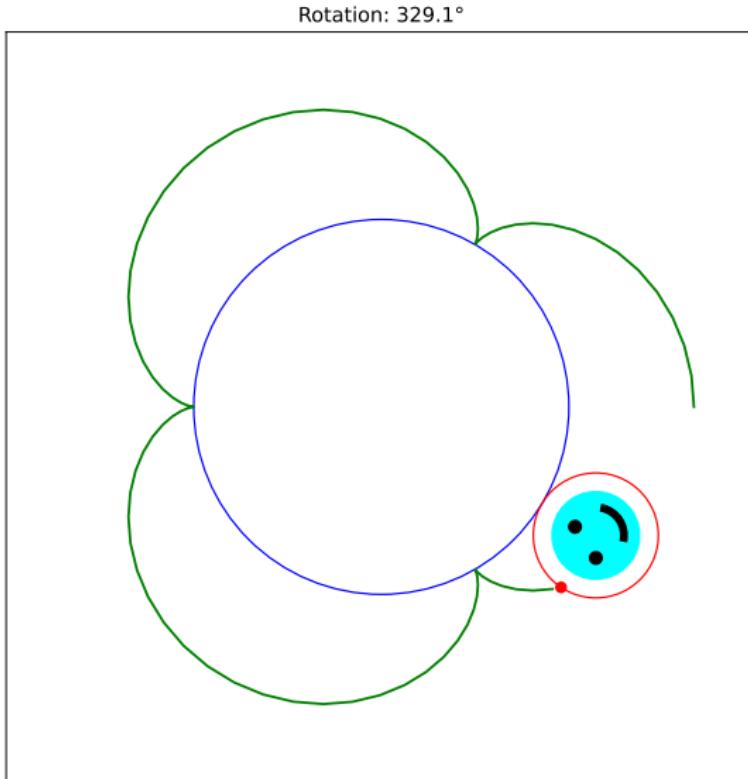
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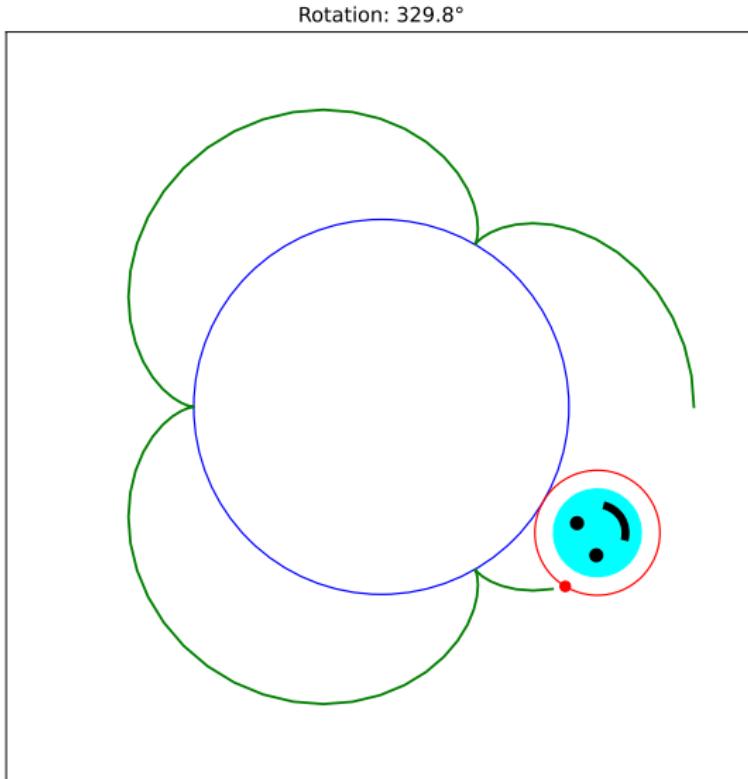
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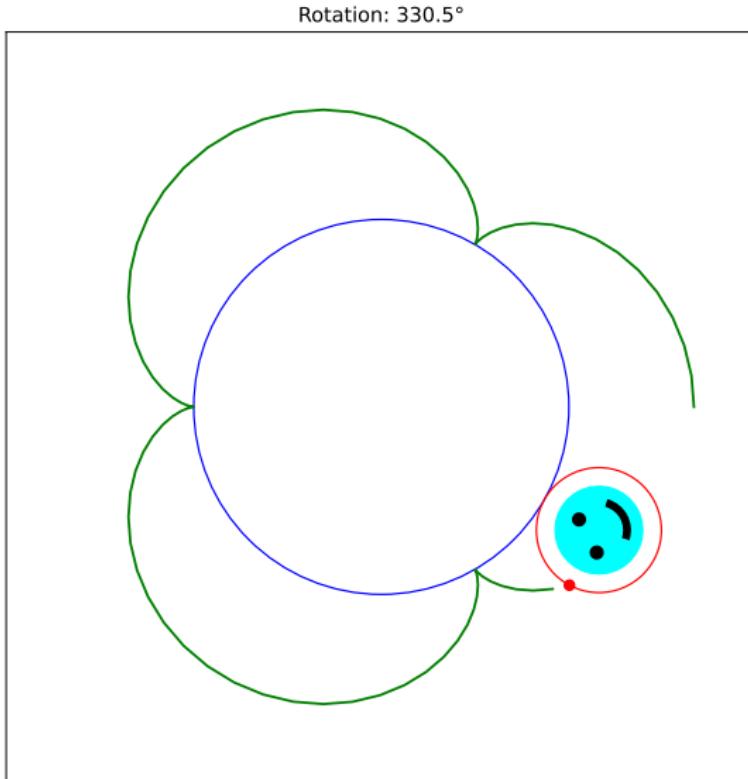
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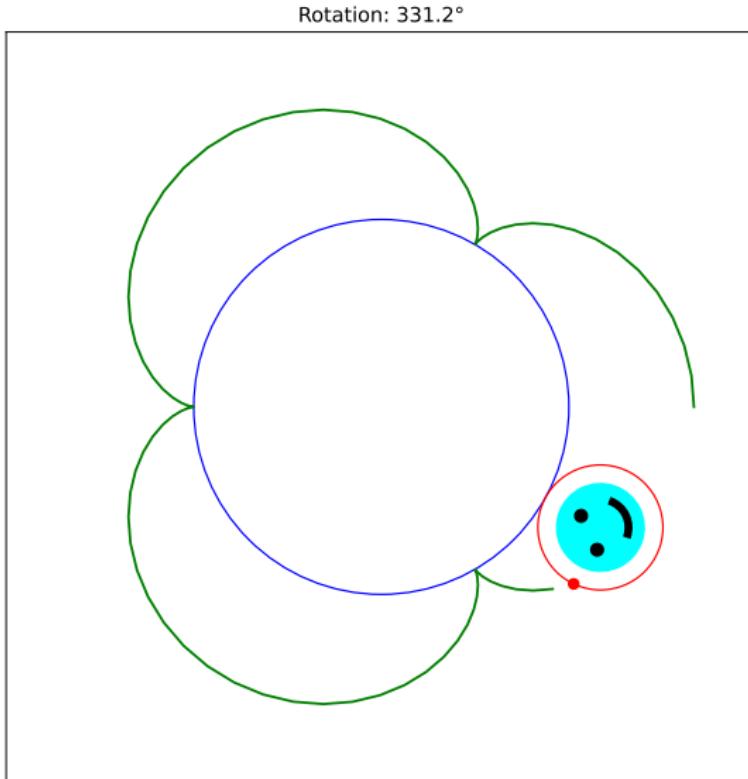
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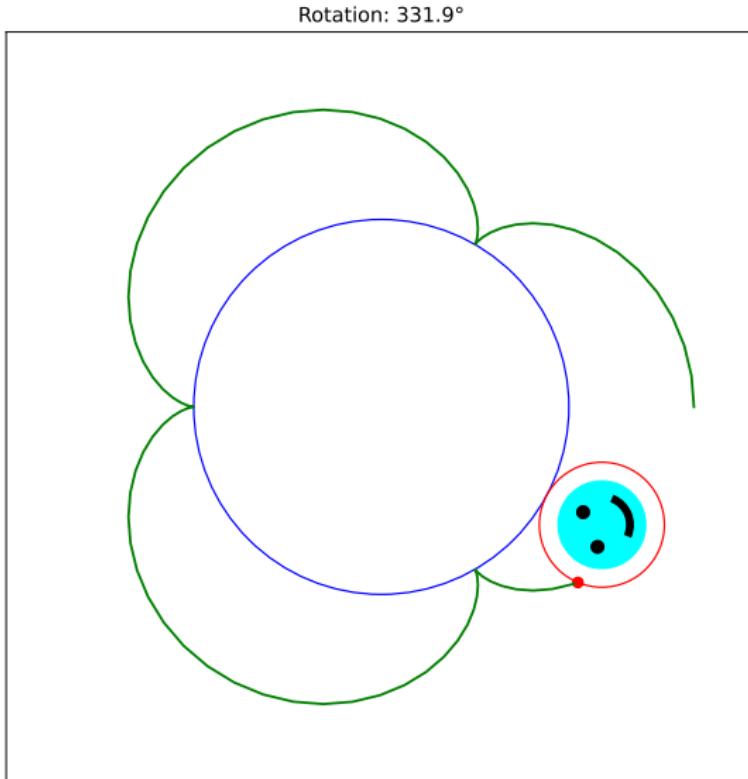
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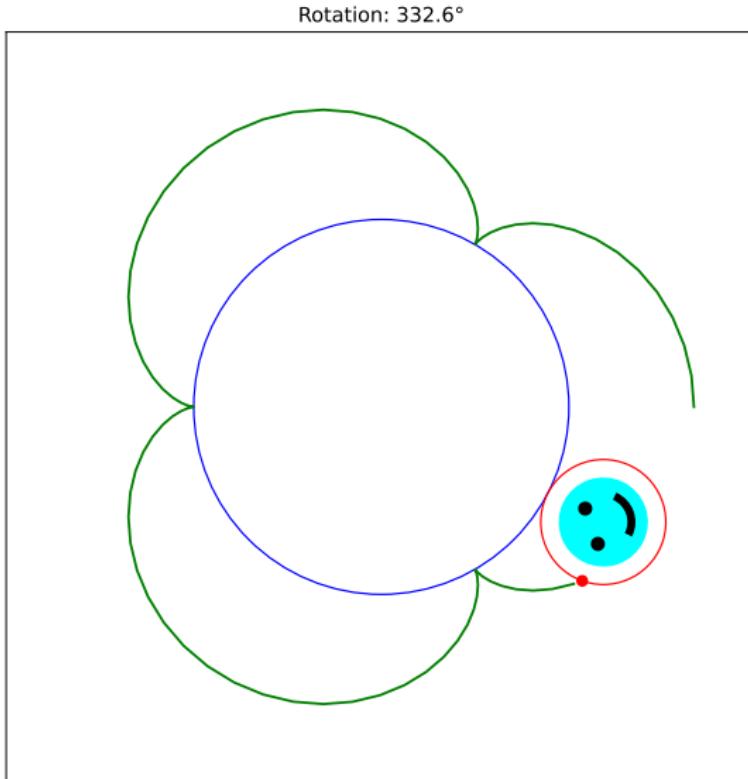
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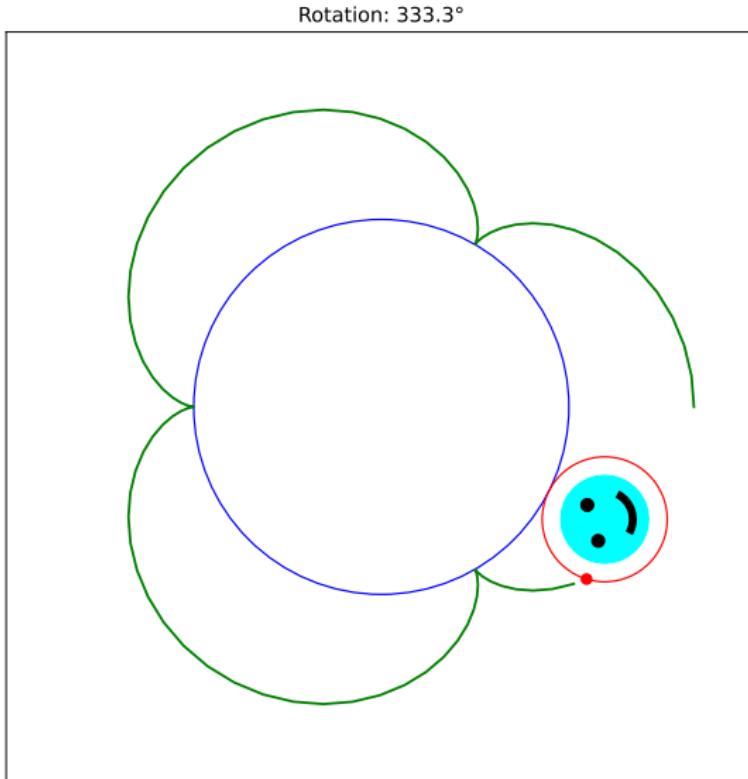
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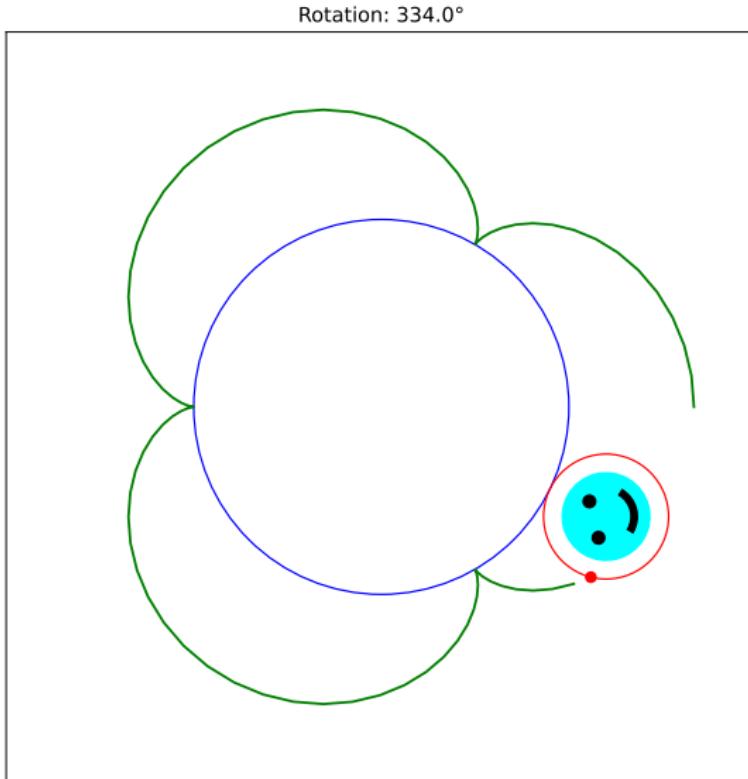
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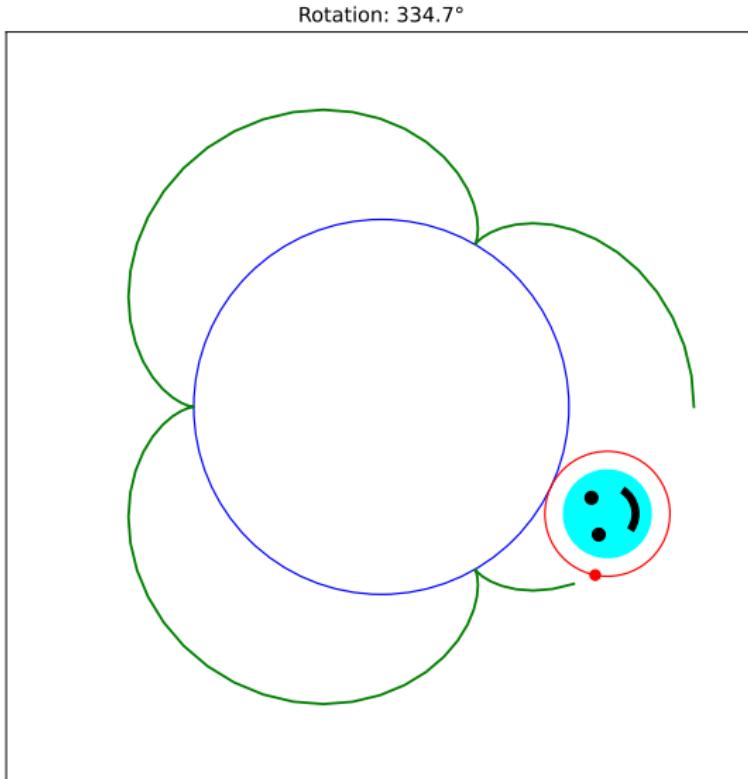
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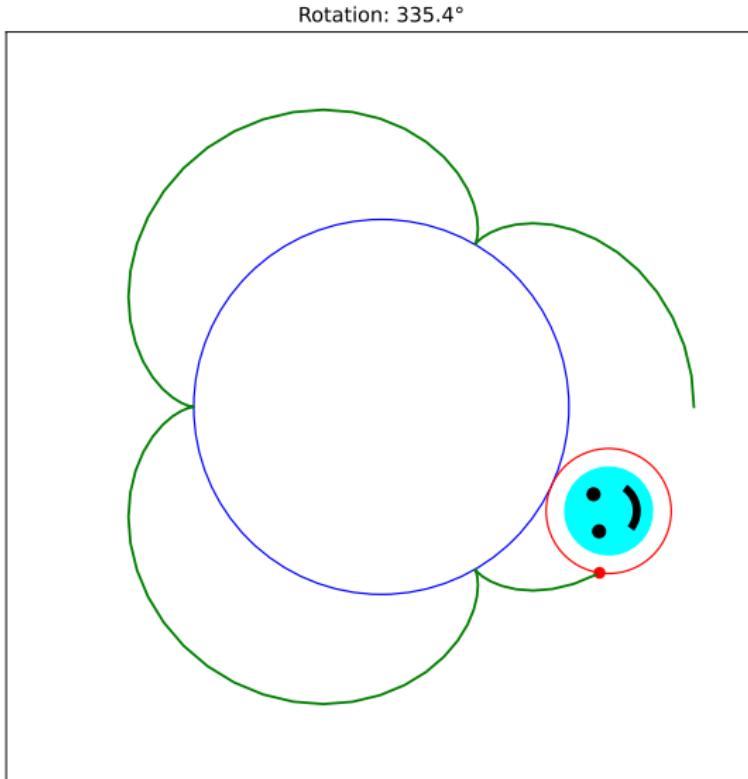
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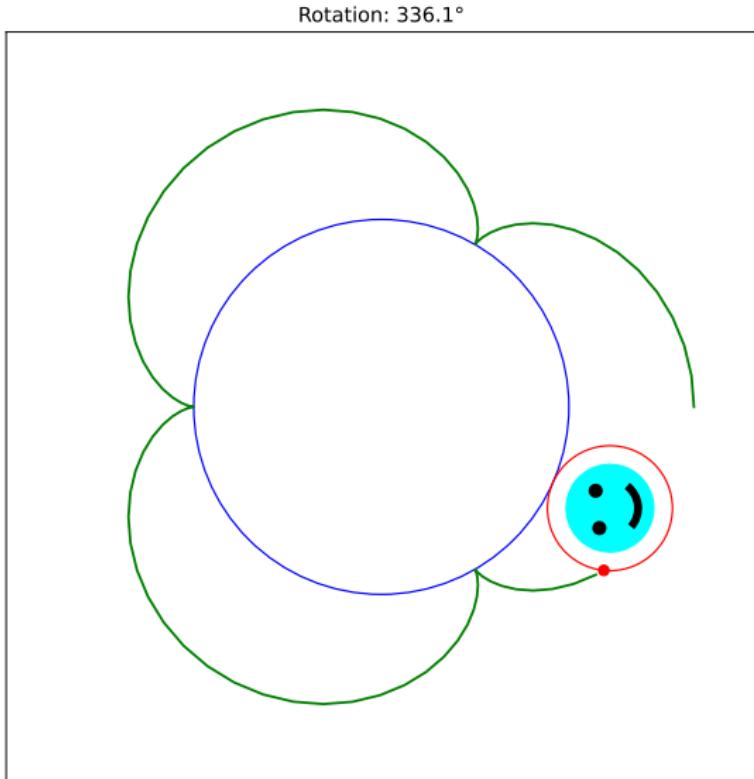
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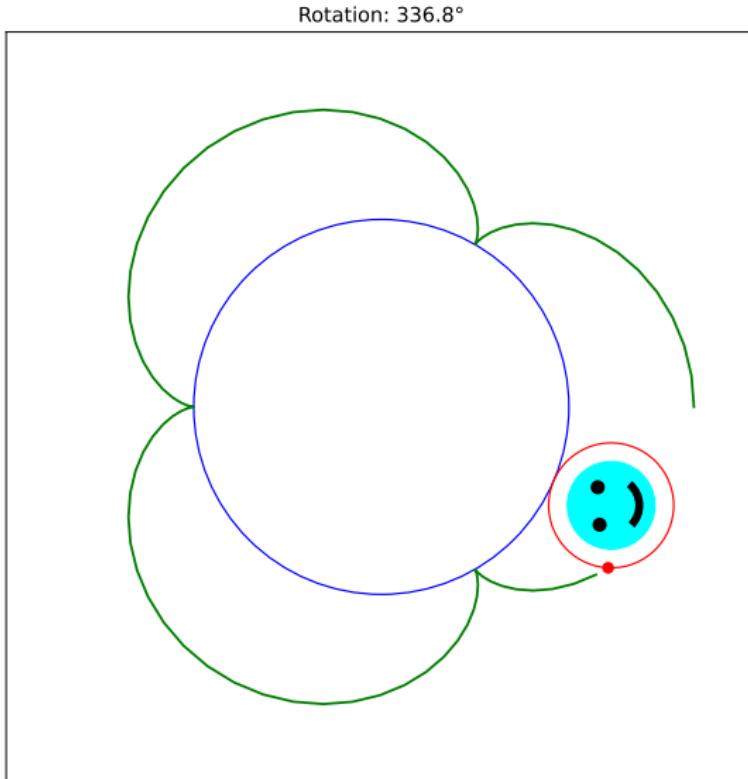
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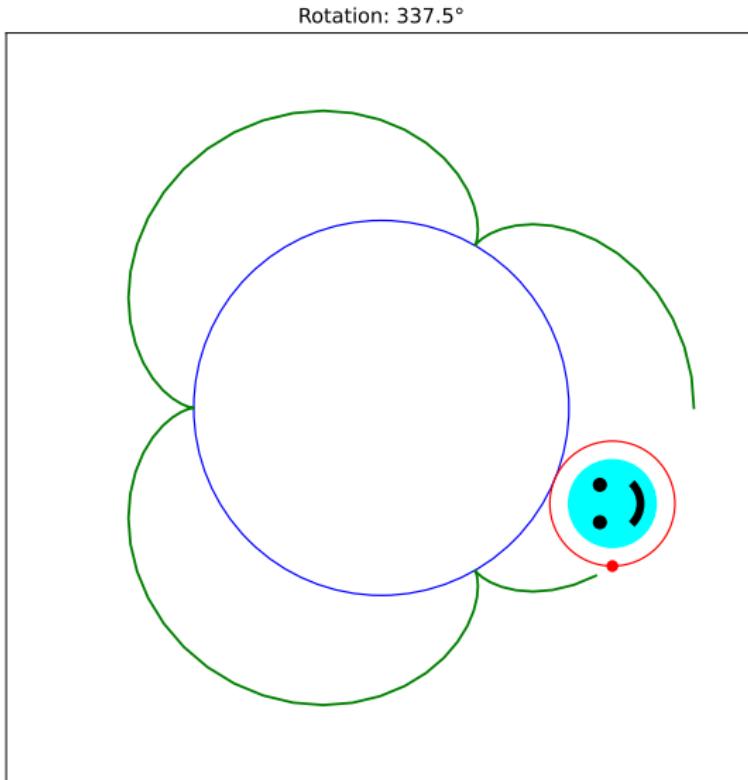
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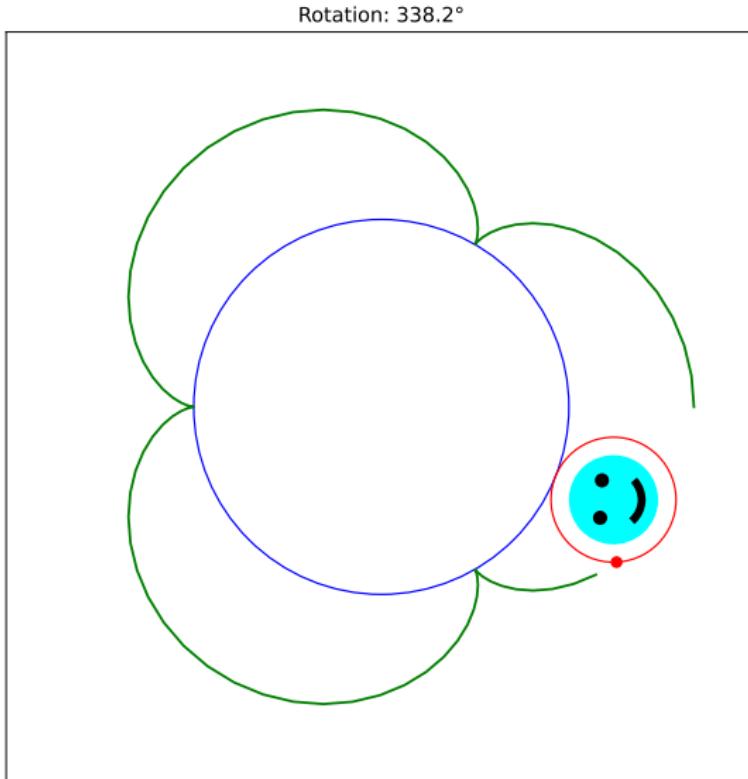
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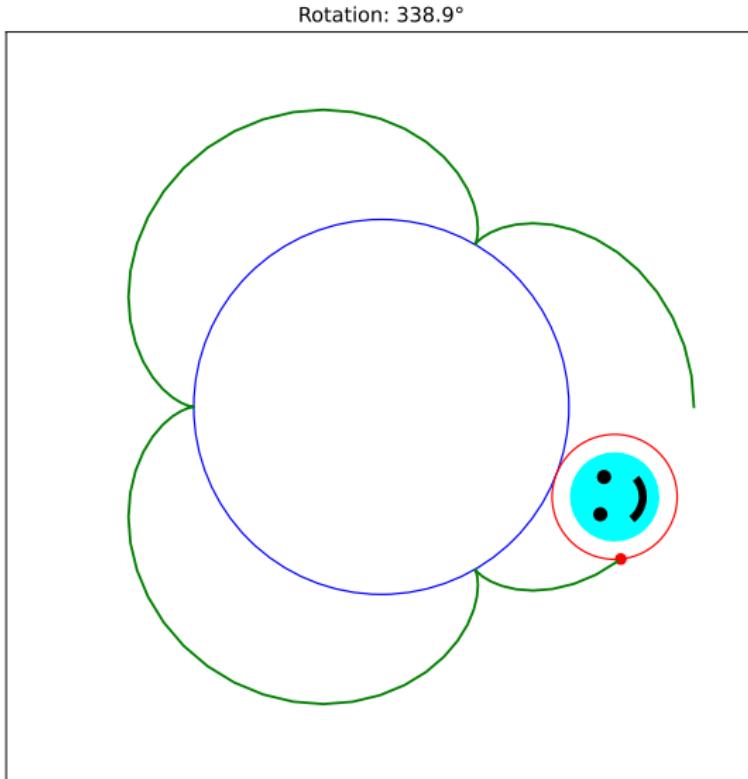
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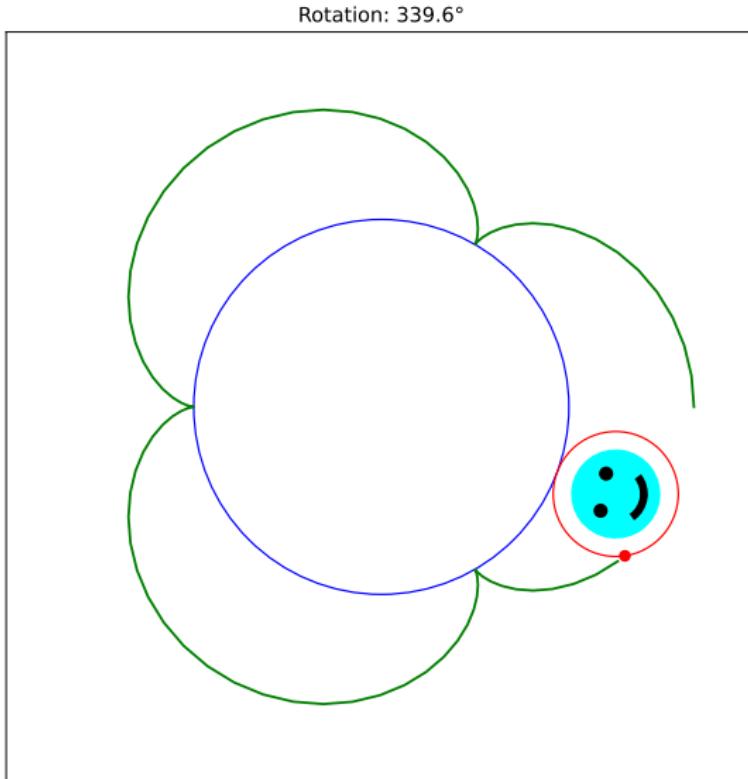
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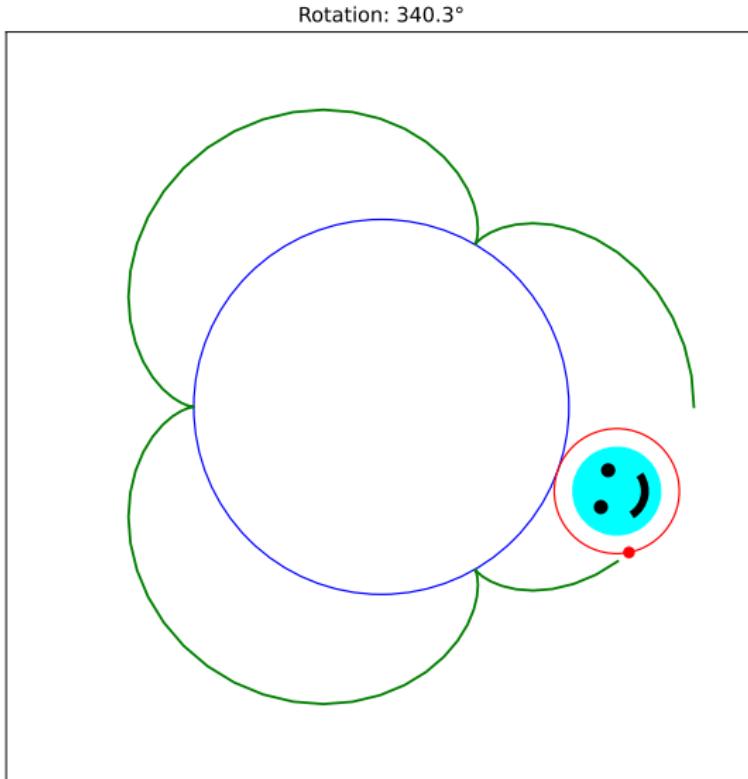
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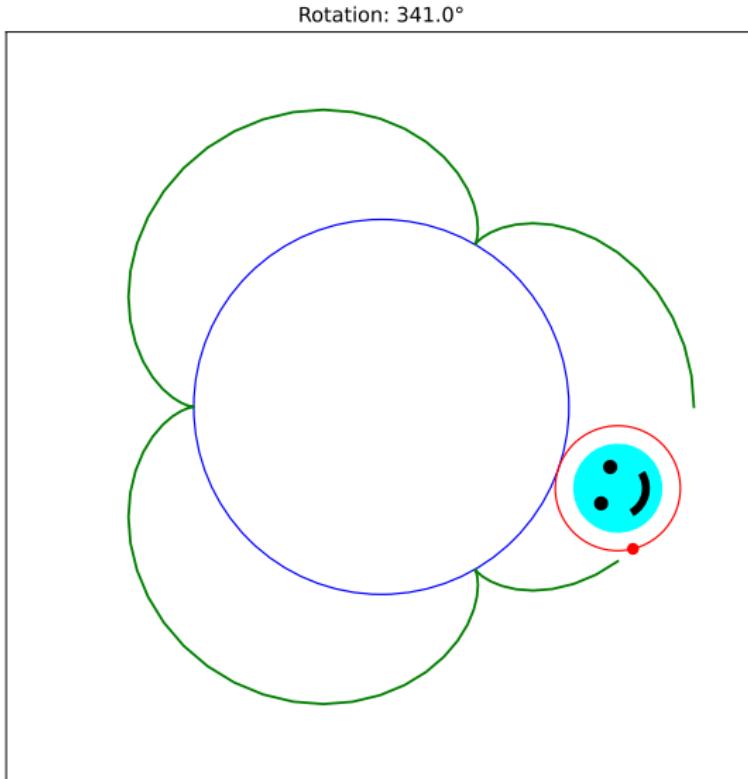
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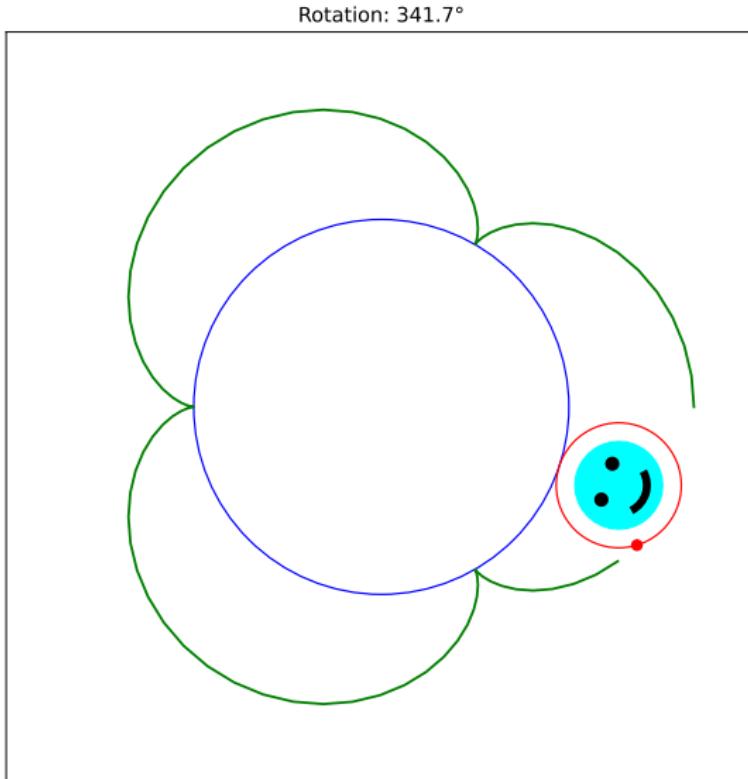
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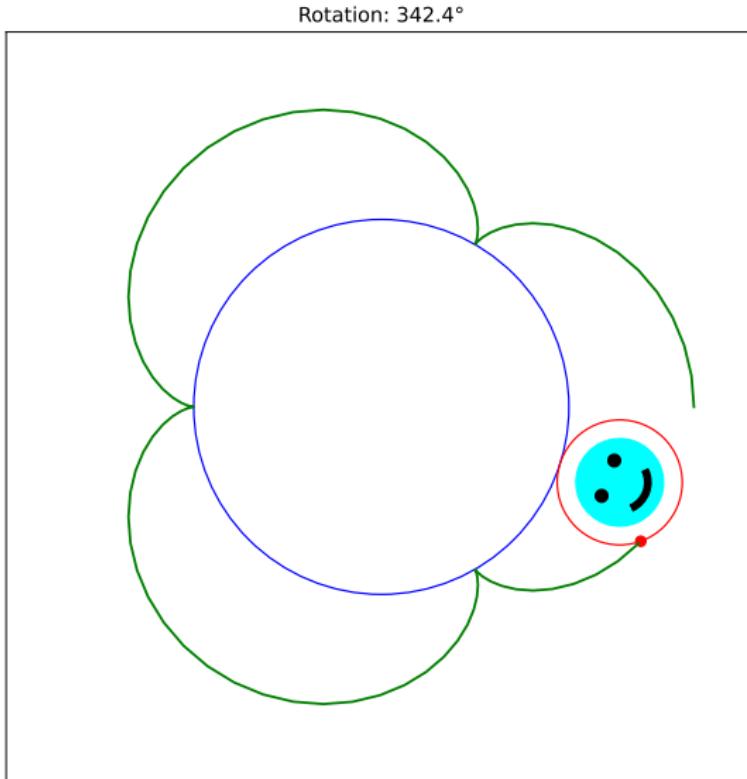
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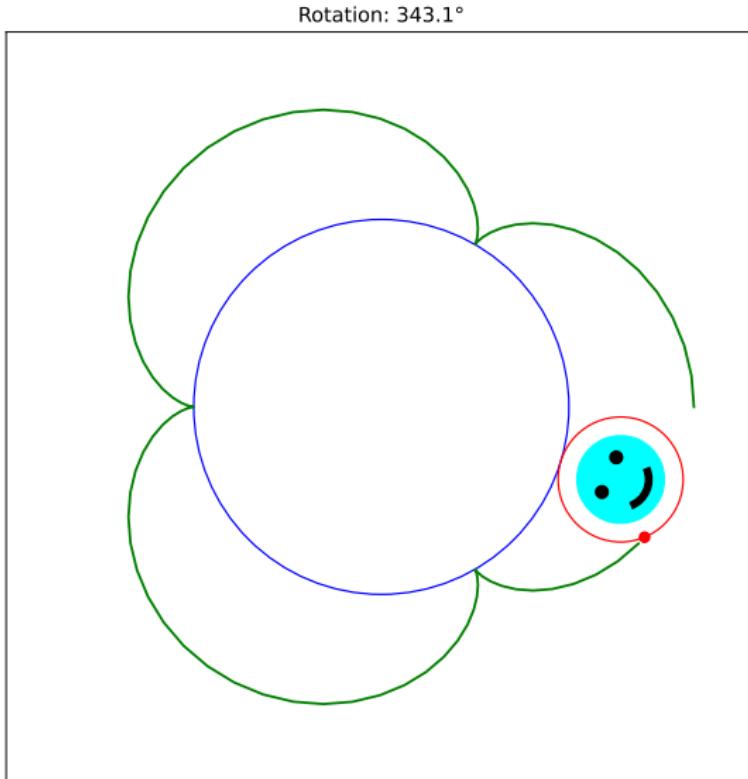
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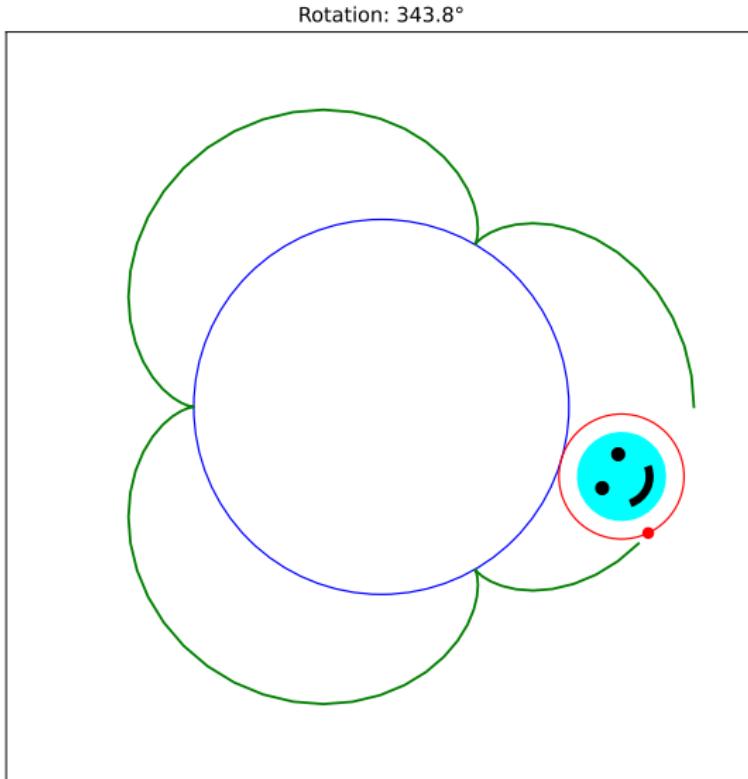
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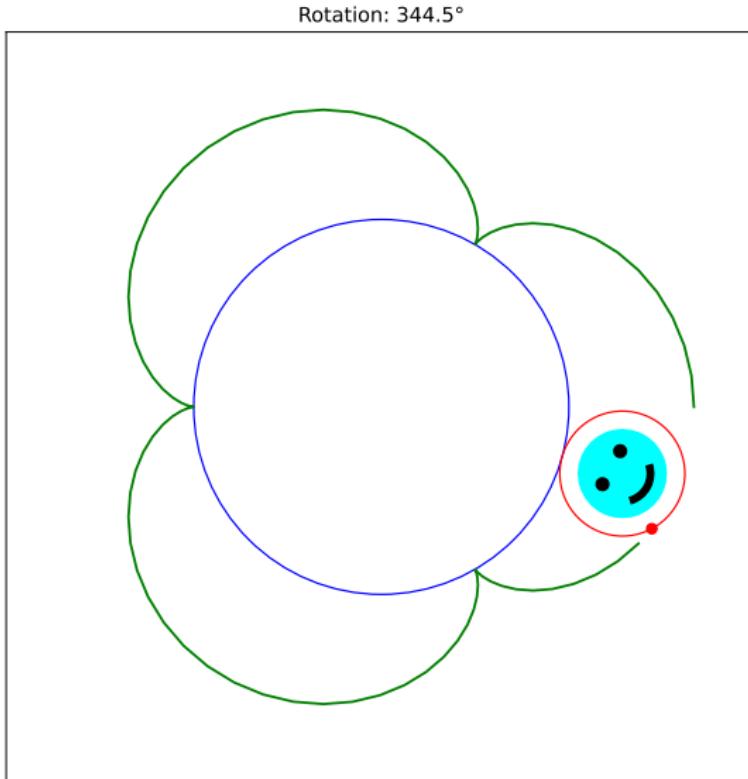
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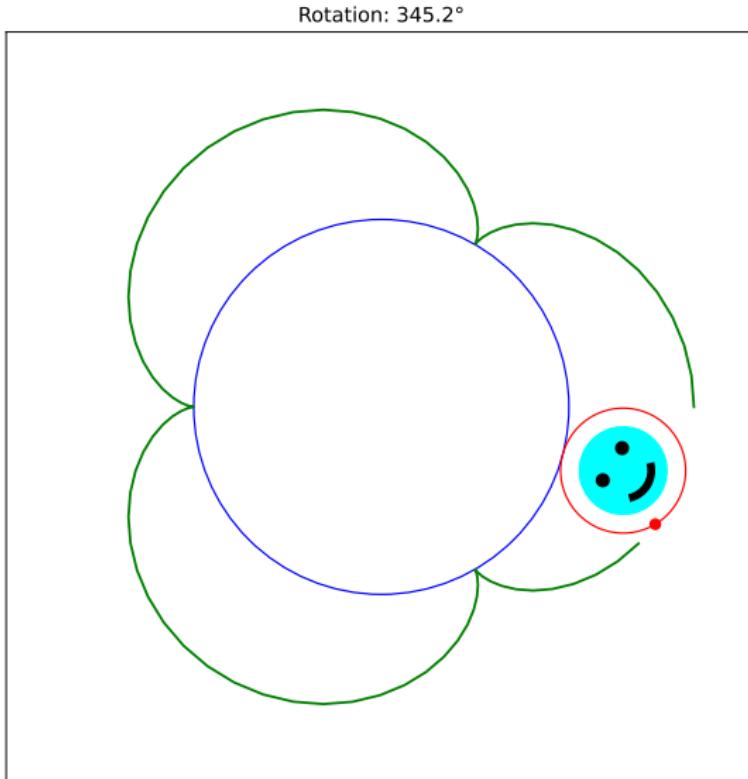
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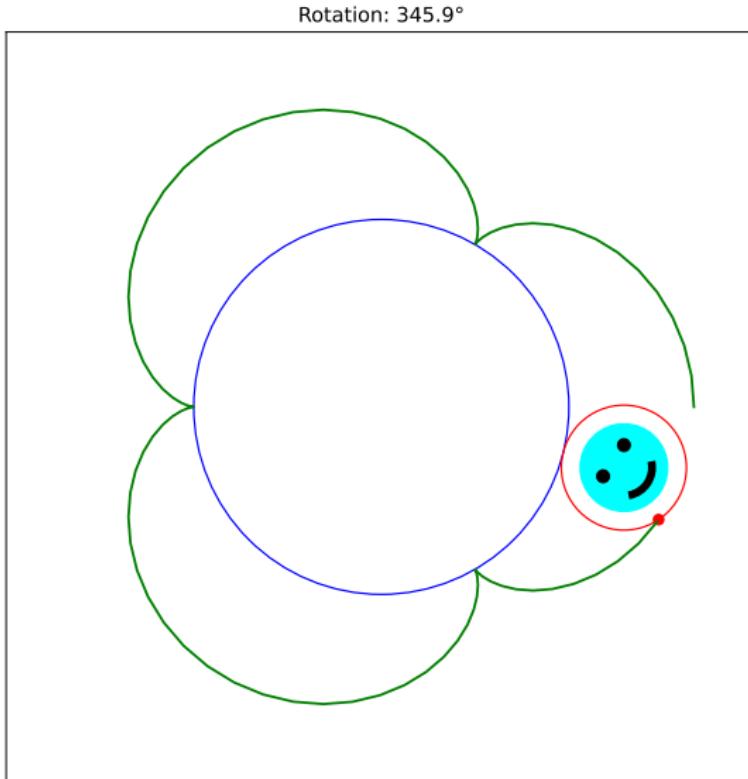
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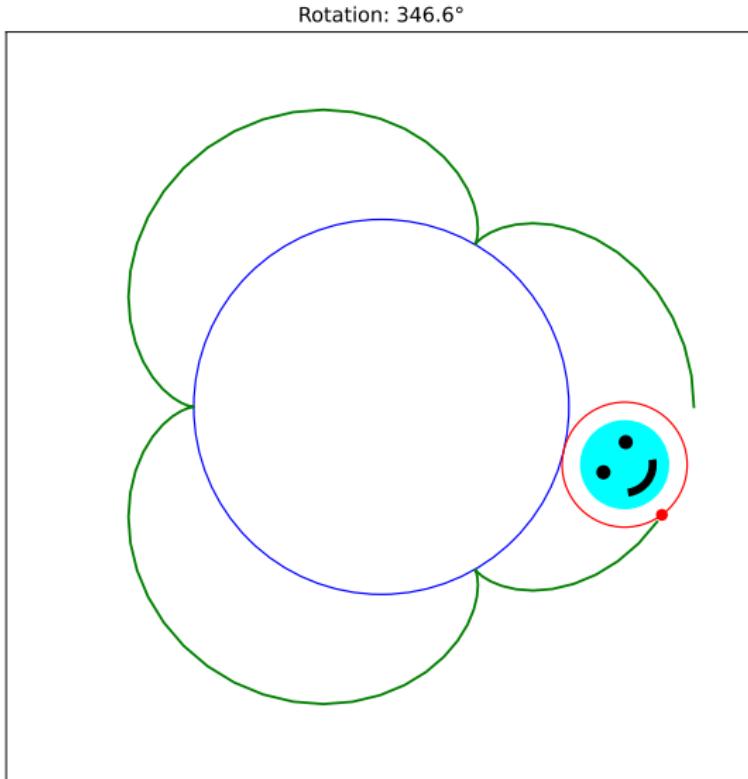
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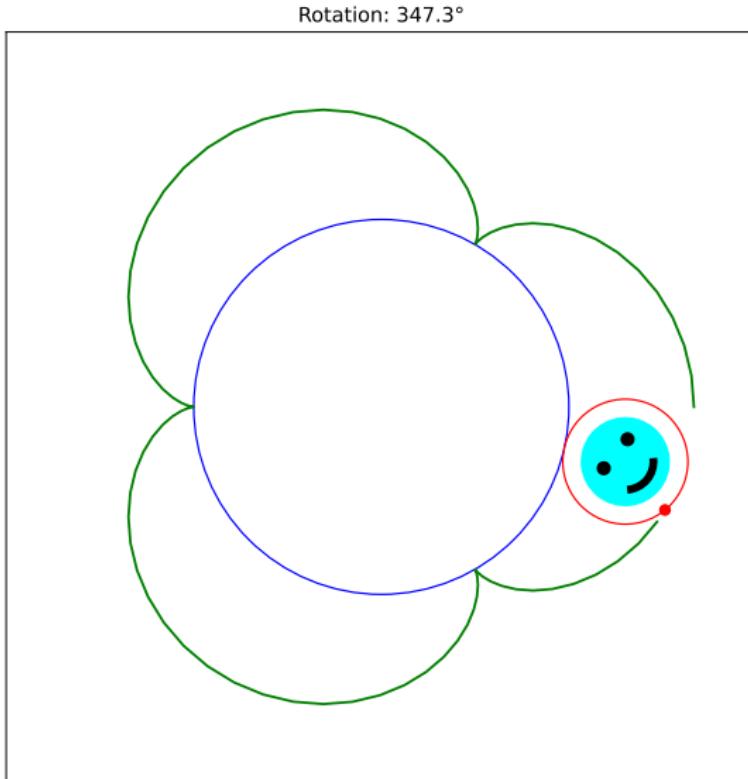
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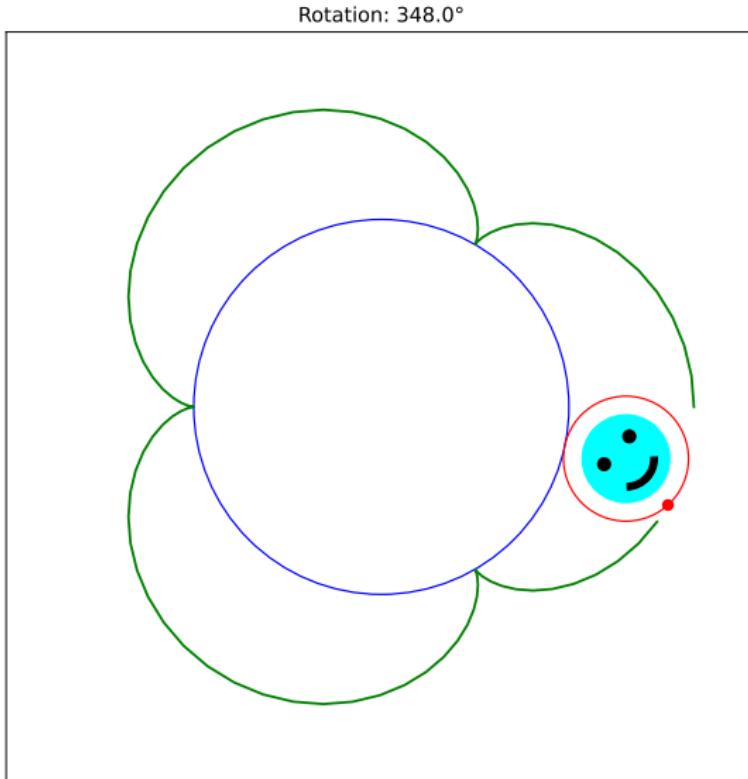
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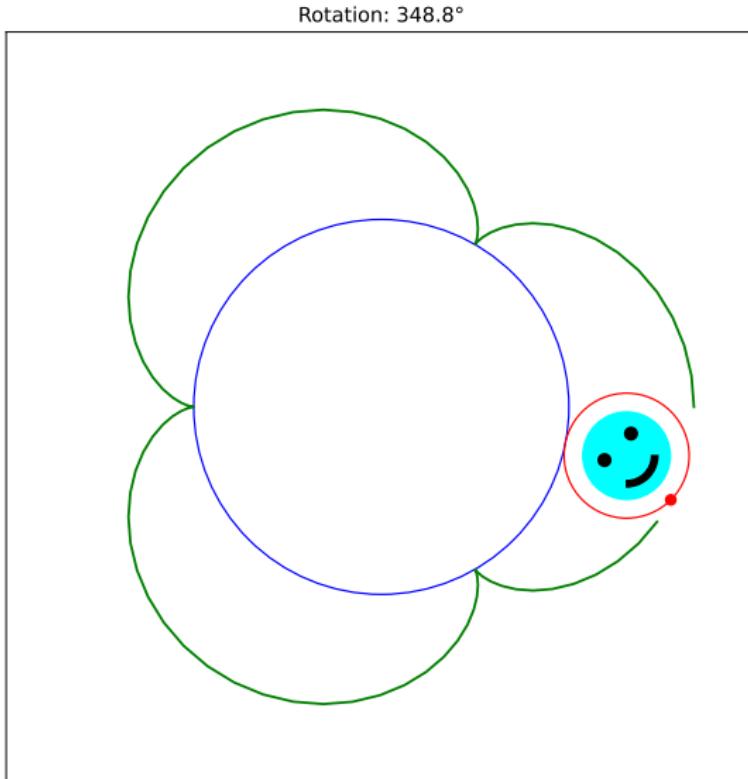
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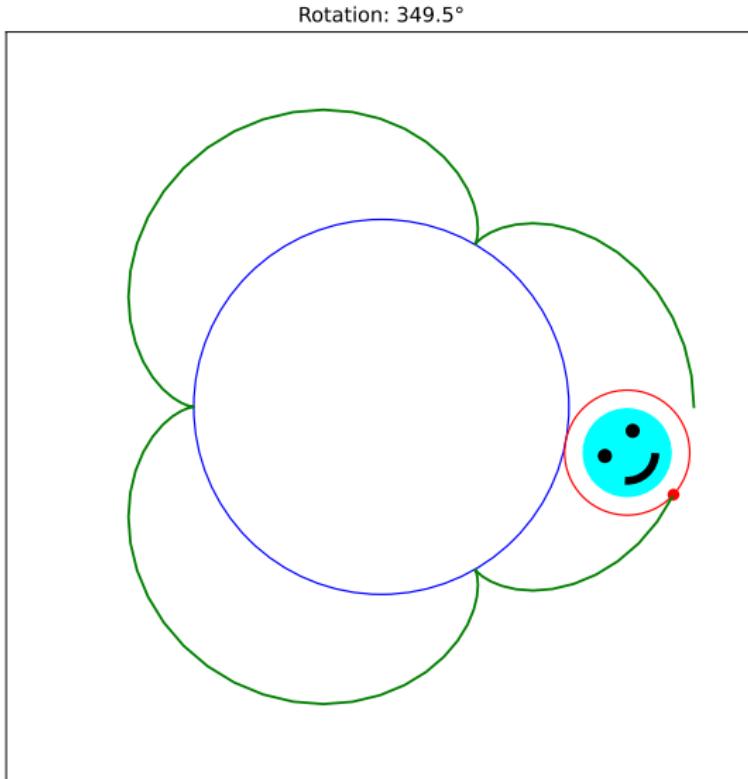
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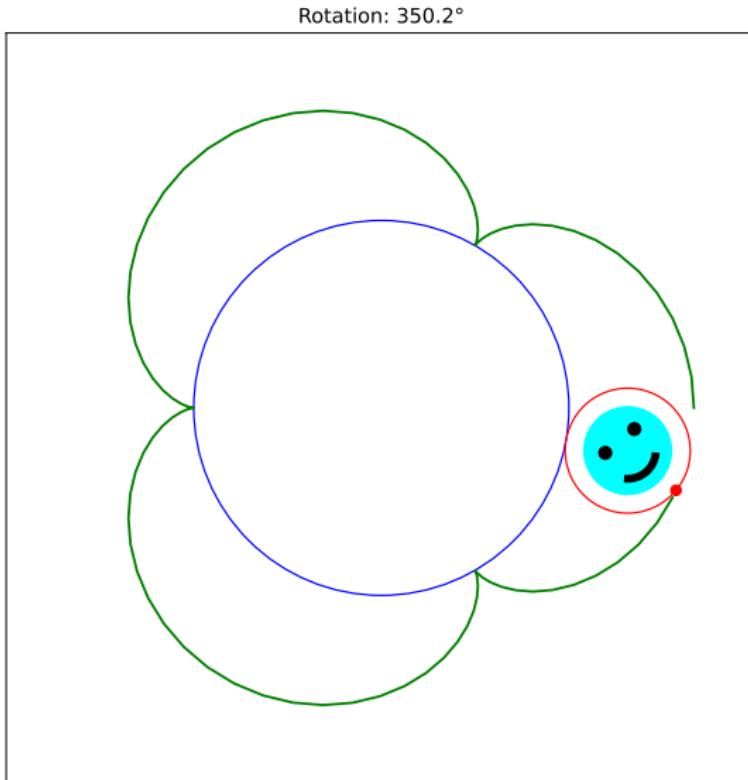
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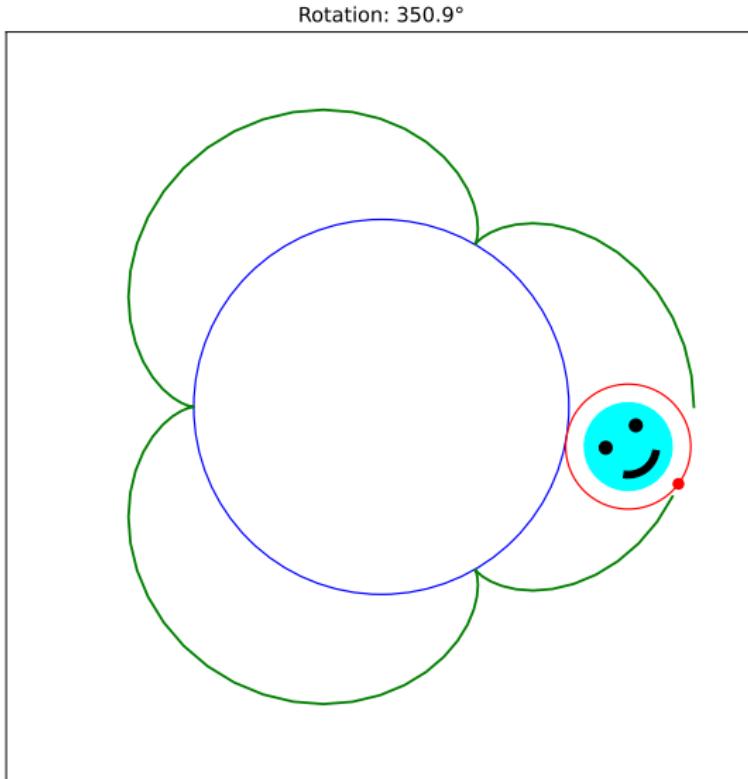
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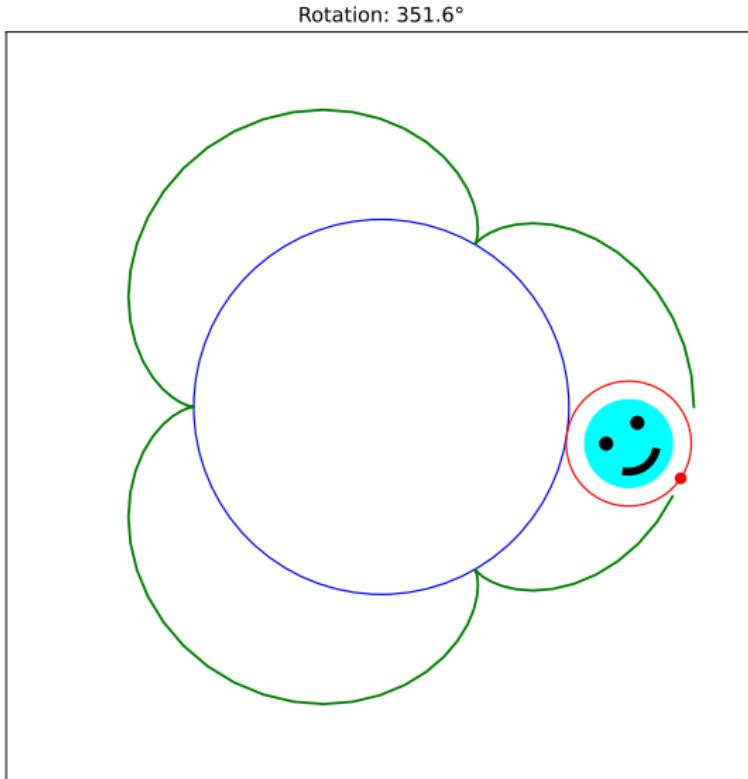
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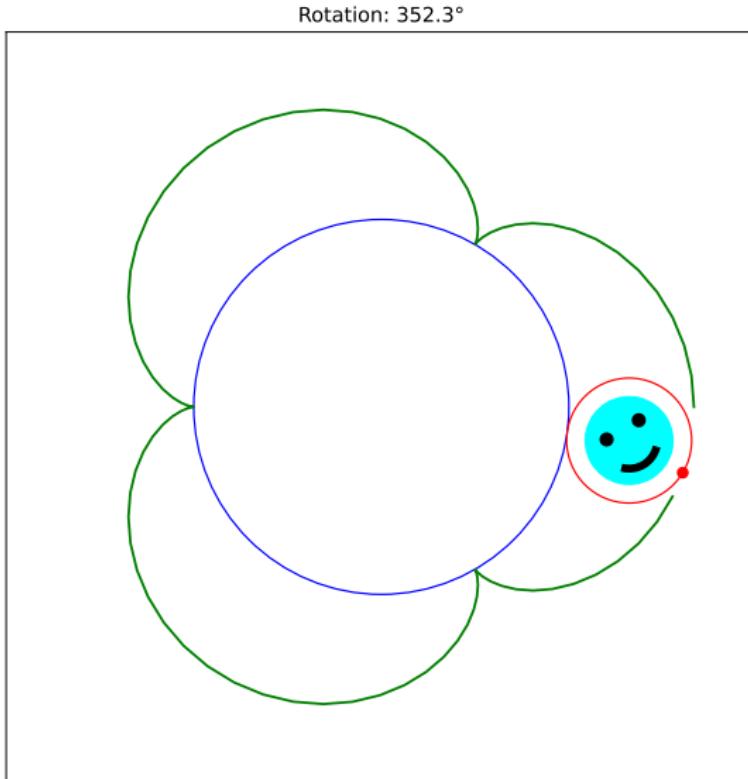
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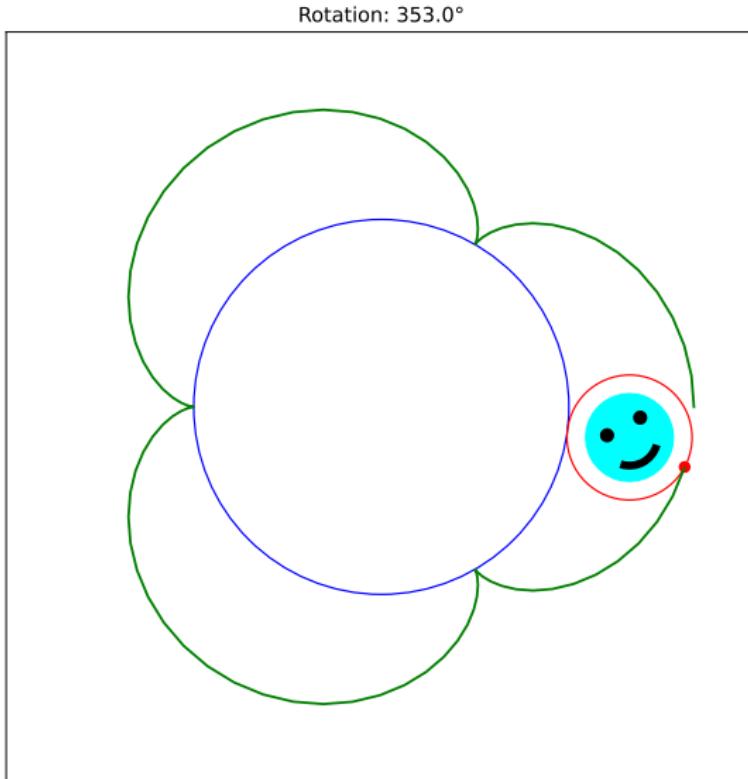
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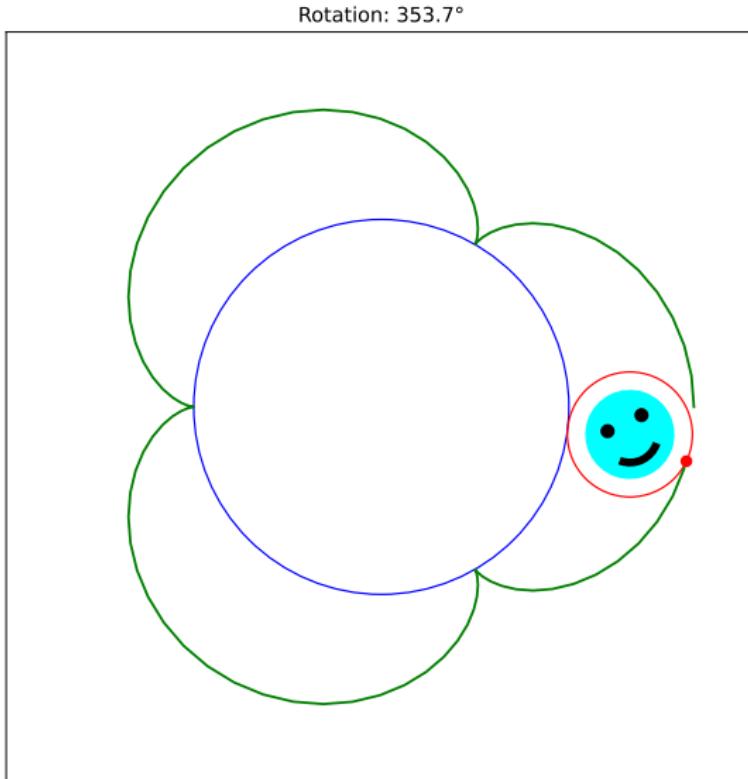
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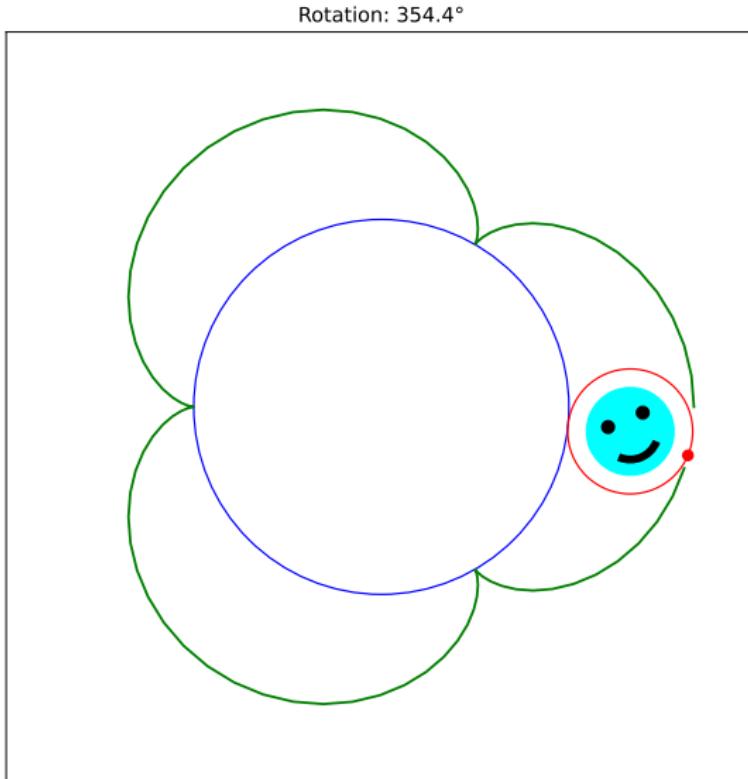
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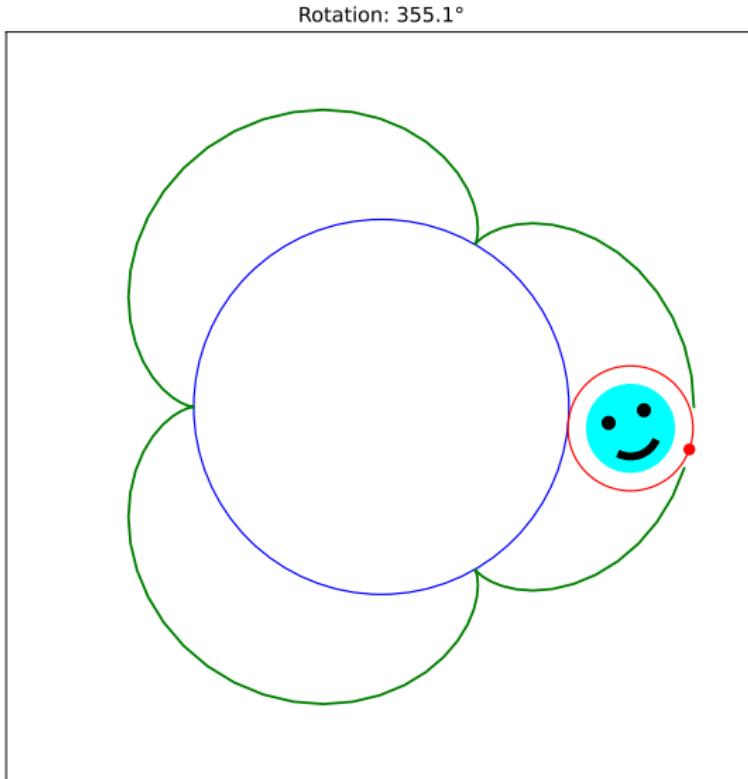
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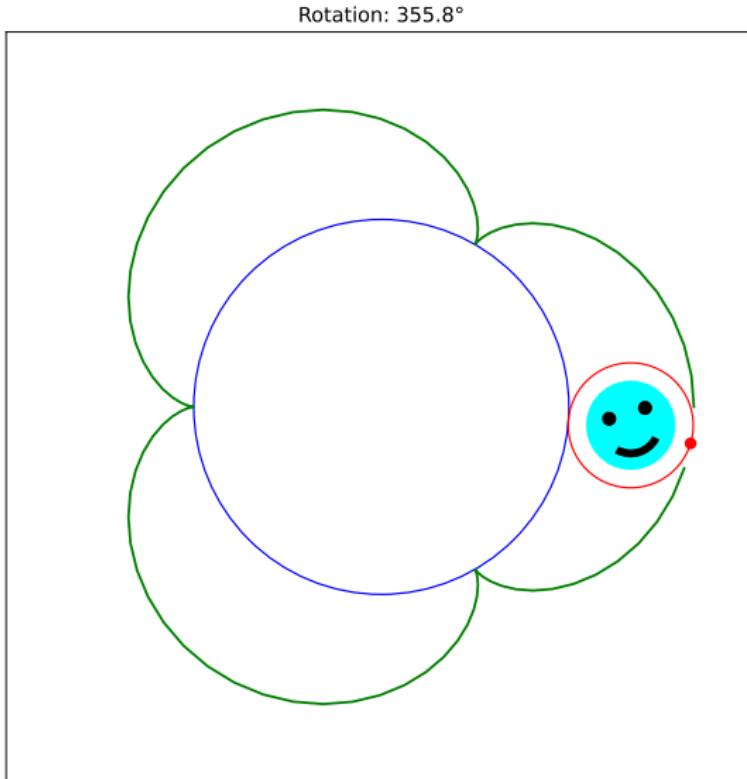
# Animation of Coin Rotation Paradox



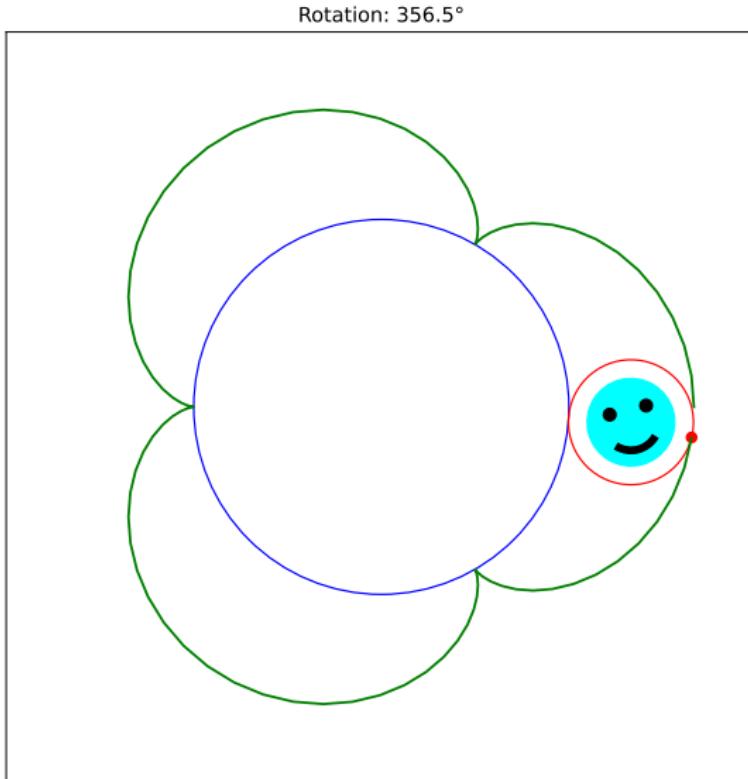
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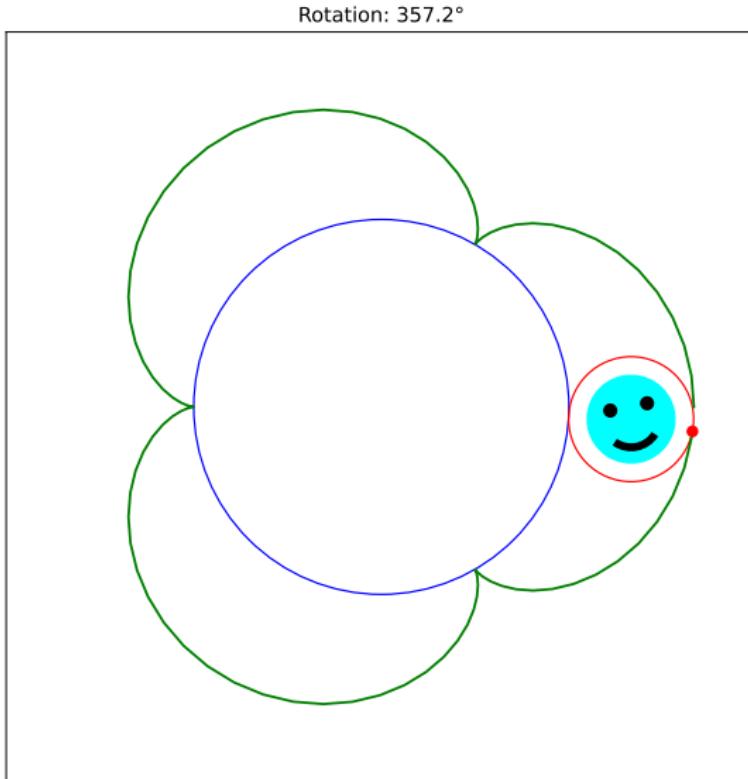
# Animation of Coin Rotation Paradox



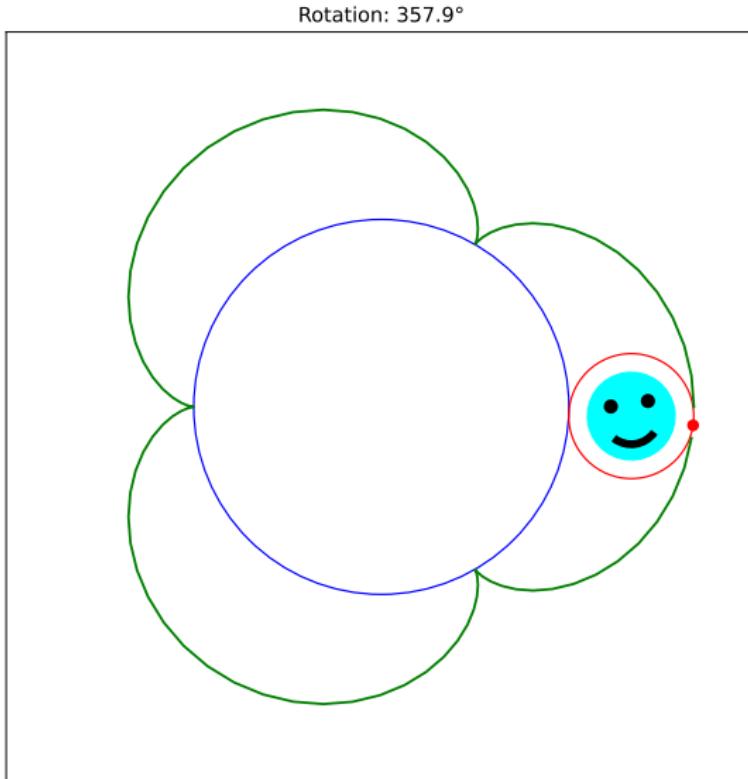
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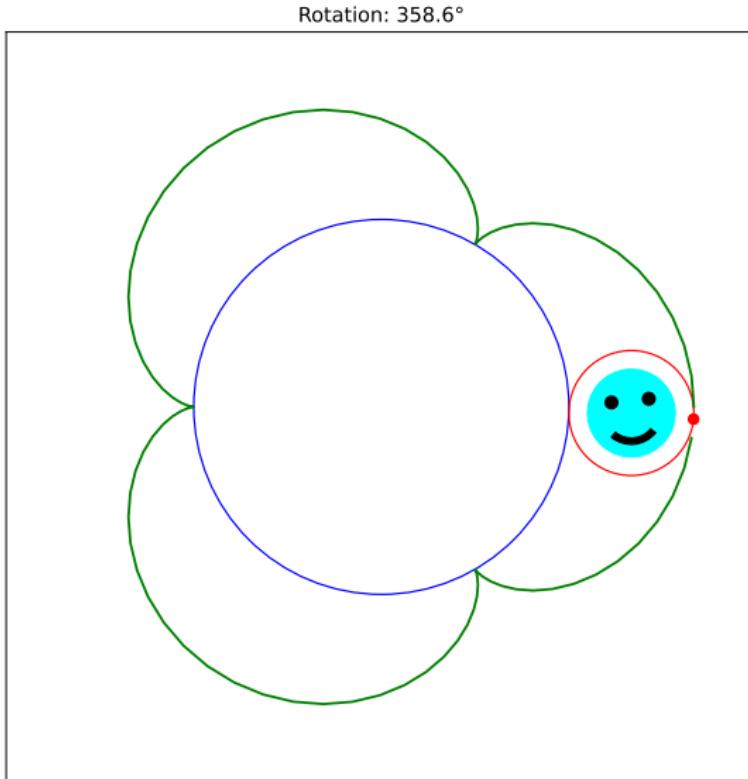
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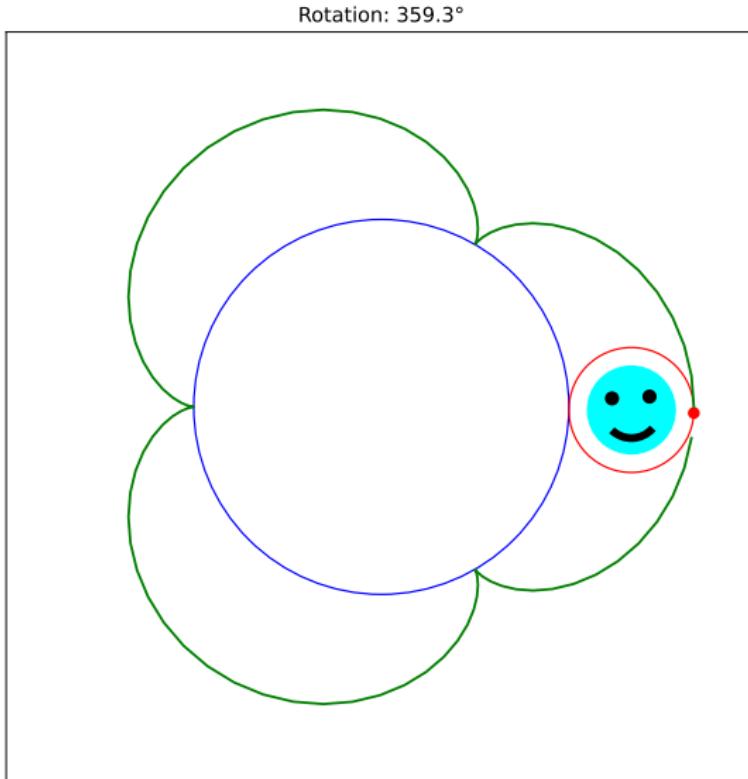
# Animation of Coin Rotation Paradox



# Animation of Coin Rotation Paradox



# Animation of Coin Rotation Paradox



# Animation of Coin Rotation Paradox: Python Code I

```
import math
import matplotlib.pyplot as plt
from matplotlib.patches import Circle, Wedge
from matplotlib.transforms import Affine2D
import os; os.chdir(os.path.dirname(__file__))

# Set the radii
R = 1.5 # Radius of the outer (stationary) coin
r = 0.5 # Radius of the inner (rolling) coin

def create_logo(ax, x, y, radius, angle):
    # Create a simple smiley face logo
    face = Circle((0, 0), radius * 0.7, fill=True, color="cyan")
    left_eye = Circle((-radius * 0.3, radius * 0.2), radius *
                      0.1, fill=True, color="black")
    right_eye = Circle((radius * 0.3, radius * 0.2), radius *
                        0.1, fill=True, color="black")
    mouth = Wedge((0, 0), radius * 0.5, 225, 315, width=radius *
                  0.1, color="black")
```

## Animation of Coin Rotation Paradox: Python Code II

```
# Create a transformation
t = Affine2D().rotate(-angle).translate(x, y) + ax.transData

# Apply the transformation to all elements
face.set_transform(t)
left_eye.set_transform(t)
right_eye.set_transform(t)
mouth.set_transform(t)

ax.add_artist(face)
ax.add_artist(left_eye)
ax.add_artist(right_eye)
ax.add_artist(mouth)

def create_frame(angle):
    # Create figure and axis
    fig, ax = plt.subplots(figsize=(8, 8))
```

# Animation of Coin Rotation Paradox: Python Code III

```
# Set limits and aspect ratio
ax.set_xlim(-(R + r + 1), R + r + 1)
ax.set_ylim(-(R + r + 1), R + r + 1)
ax.set_aspect("equal")

# Remove axis ticks
ax.set_xticks([])
ax.set_yticks([])

# Calculate position of the rolling coin's center
x = (R + r) * math.cos(angle)
y = (R + r) * math.sin(angle)

# Calculate position of the point on the edge of the moving
# coin
point_x = x + r * math.cos((R + r) / r * angle)
point_y = y + r * math.sin((R + r) / r * angle)
```

## Animation of Coin Rotation Paradox: Python Code IV

```
# Draw stationary coin
stationary_coin = Circle((0, 0), R, fill=False, color="blue")
ax.add_artist(stationary_coin)

# Draw rotating coin
rotating_coin = Circle((x, y), r, fill=False, color="red")
ax.add_artist(rotating_coin)

# Add logo to the rotating coin
create_logo(ax, x, y, r, (R + r) / r * angle)

# Draw point on the edge of the moving coin
ax.plot(point_x, point_y, "ro")

# Draw cardioid path
theta = [i * 2 * math.pi / 100 for i in range(int(100 * angle
    / (2 * math.pi)) + 1)]
```

# Animation of Coin Rotation Paradox: Python Code V

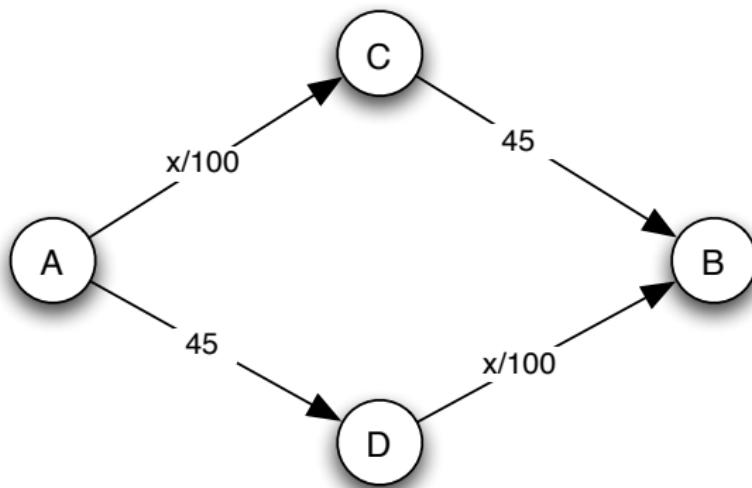
```
cardioid_x = [(R + r) * math.cos(t) + r * math.cos((R + r) /  
    ↵ r * t) for t in theta]  
cardioid_y = [(R + r) * math.sin(t) + r * math.sin((R + r) /  
    ↵ r * t) for t in theta]  
ax.plot(cardioid_x, cardioid_y, "g-")  
  
# Add title  
ax.set_title(f"Rotation: {angle/(2*math.pi)*360:.1f}°")  
  
return fig  
  
# Generate 512 frames  
num_frames = 512; tex = []  
for i in range(num_frames):  
    angle = i * 2 * math.pi / num_frames  
    fig = create_frame(angle)  
    fig.savefig(f'coin_rotation_{i}.pdf', bbox_inches='tight',  
    ↵ pad_inches=0.1, dpi=300)
```

# Animation of Coin Rotation Paradox: Python Code VI

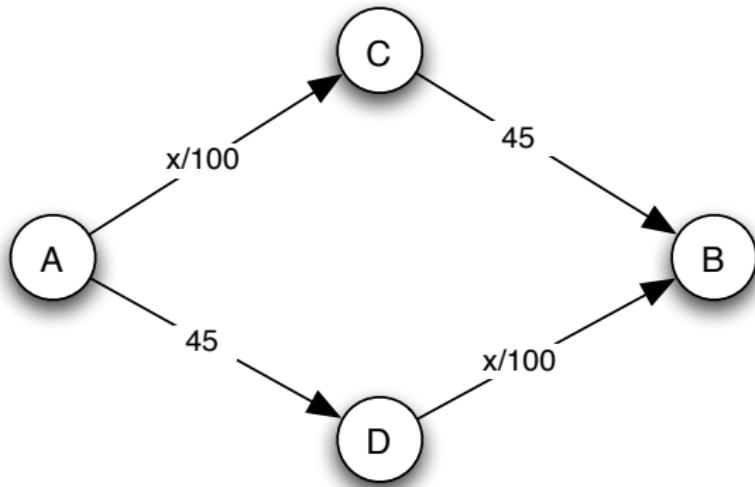
```
plt.close(fig)
tex.append(f'\\only<{i +
    1}>{{\\includegraphics[width=.6\\textwidth]{{fig/note02/
    coin_rotation_{i}.pdf}}\\noindent}}')
open('..../coin.tex',
    'w').write(f"\\hspace*{{{\\dimexpr(\\textwidth - .6\\textwidt
    h)/2\\relax}}}\\n\\begin{{overlayarea}}{{\\textwidth}}{{0.7\\t
    extheight}}}\\n{'\\n'.join(tex)}\\n\\end{{overlayarea}}")
```

## Braess Paradox

## A Typical Case

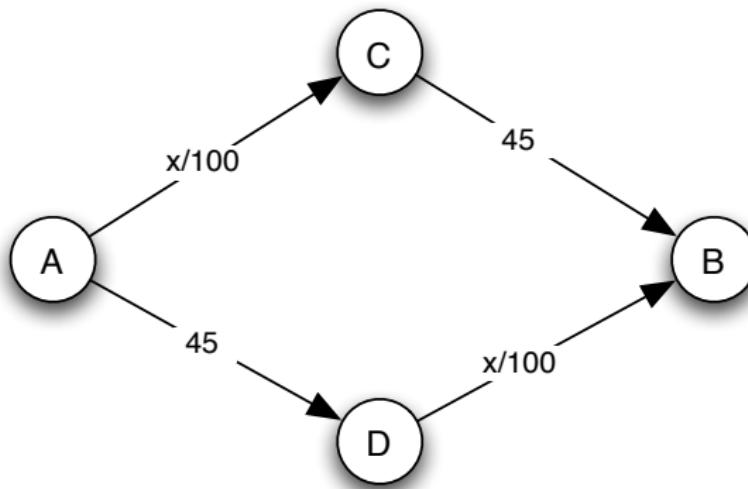


## A Typical Case



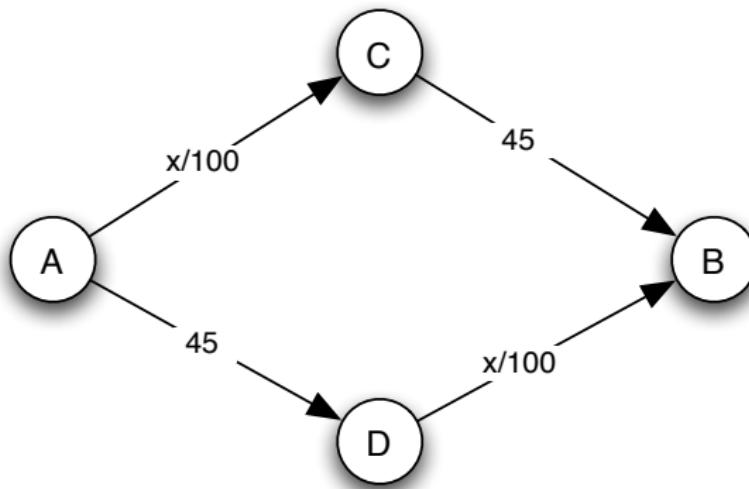
- A highway network, with each edge labeled by its travel time (in minutes) when there are  $x$  cars using it

## A Typical Case



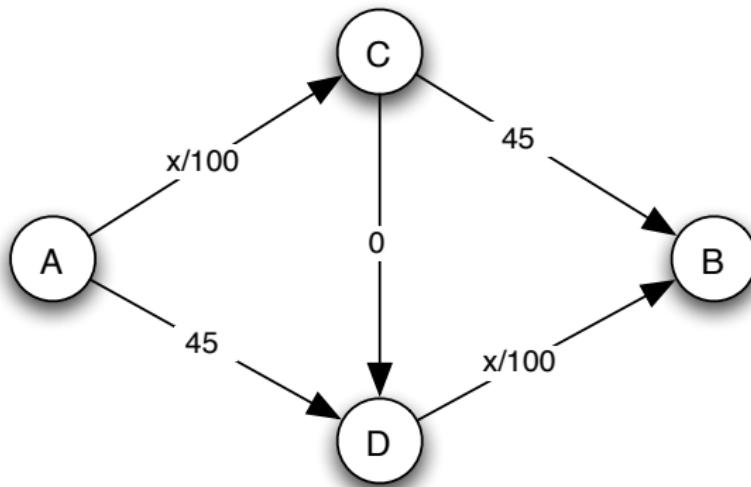
- A highway network, with each edge labeled by its travel time (in minutes) when there are  $x$  cars using it
- Suppose there are 4000 cars need to get from A to B

## A Typical Case

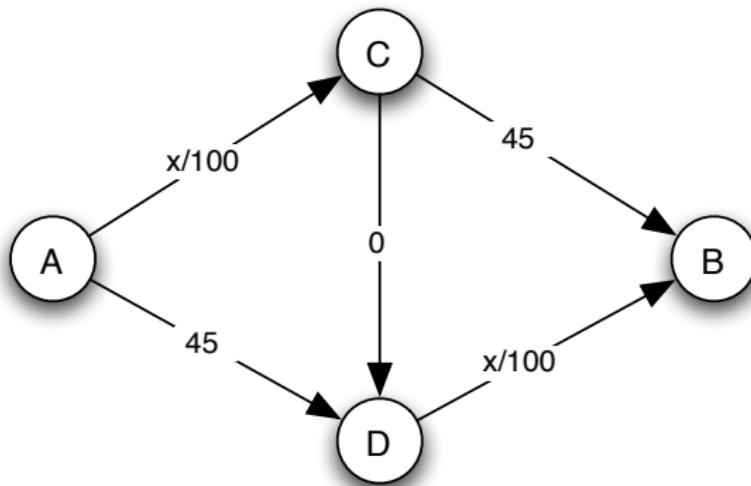


- A highway network, with each edge labeled by its travel time (in minutes) when there are  $x$  cars using it
- Suppose there are 4000 cars need to get from A to B
- They divide evenly over the two routes at equilibrium; the travel time is  $45 + 2000/100 = 65$  mins

## A Typical Case: Cont'd

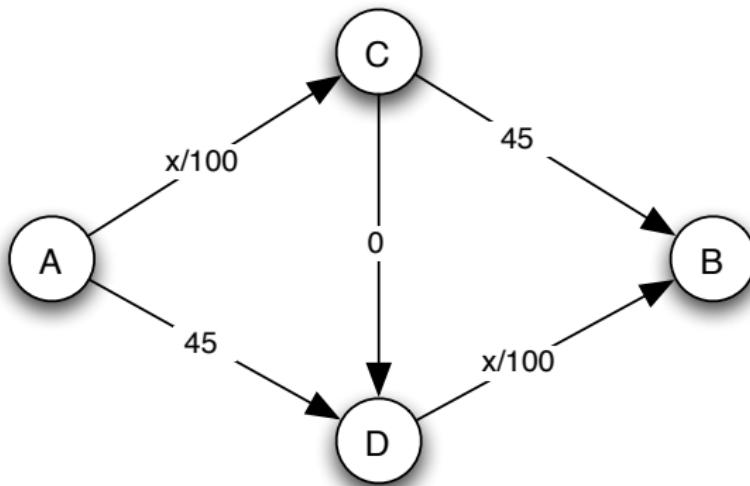


## A Typical Case: Cont'd



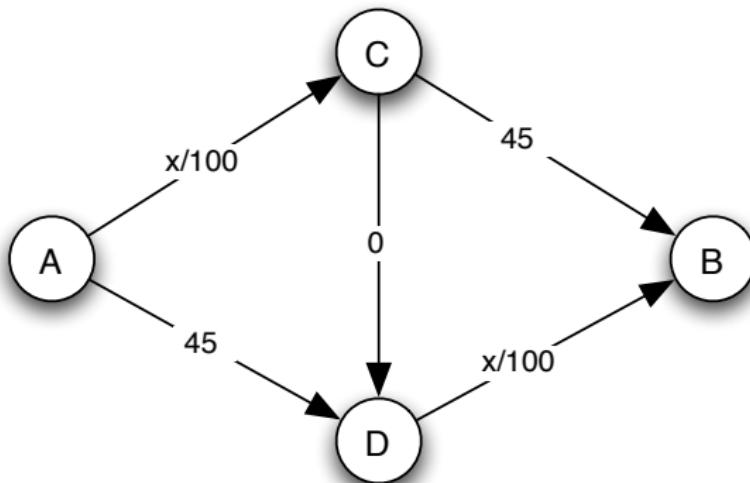
- Now a very fast edge is added from C to D to the previous highway network

## A Typical Case: Cont'd



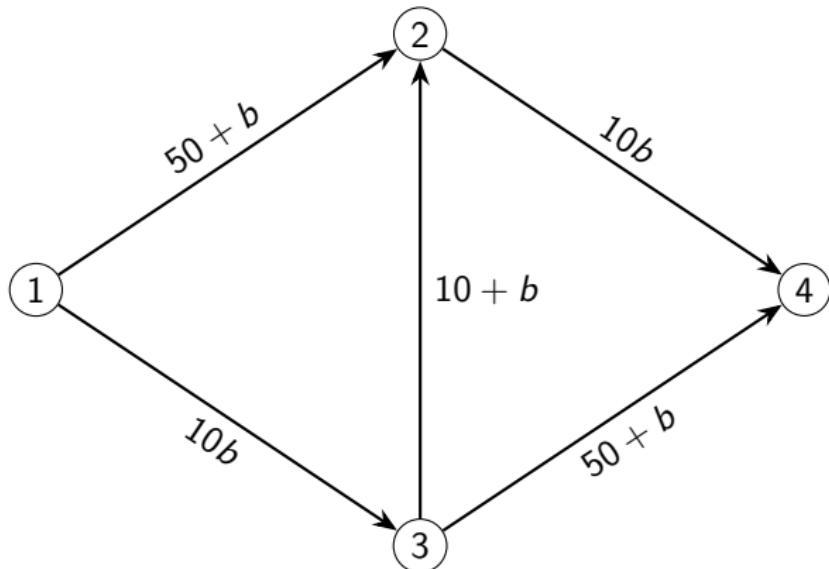
- Now a very fast edge is added from C to D to the previous highway network
- At equilibrium, every user uses the route through C and D

## A Typical Case: Cont'd

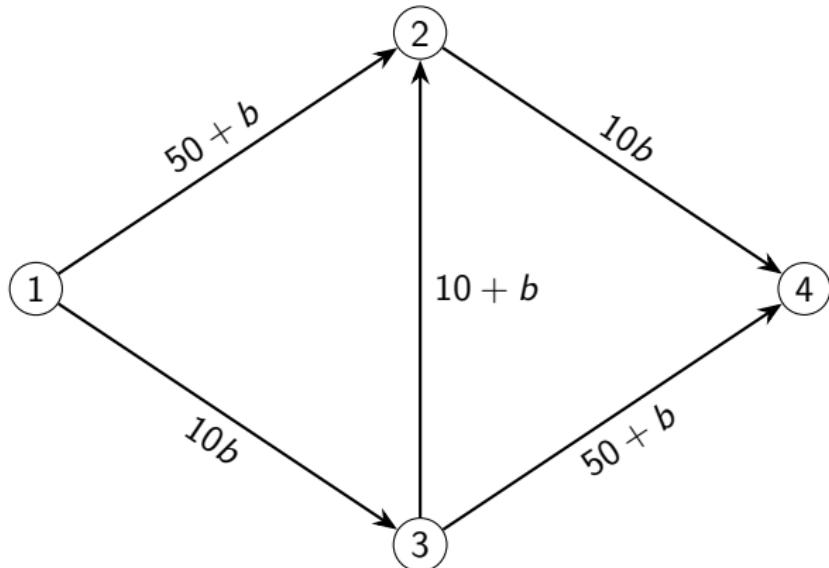


- Now a very fast edge is added from C to D to the previous highway network
- At equilibrium, every user uses the route through C and D
- As a result, the travel time is  $4000/10 + 0 + 4000/100 = 80$  mins!

# Braess' Example

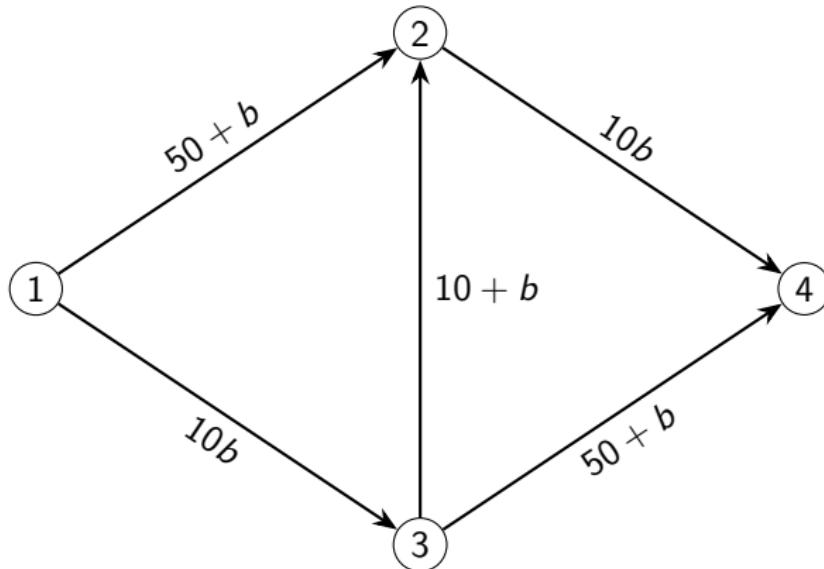


## Braess' Example



- Assume a flow of 6 units (e.g., 6000 vehicles) must travel from 1 to 4

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- Assume a flow of 6 units (e.g., 6000 vehicles) must travel from 1 to 4
- Three paths exist:  $B_1 = 124$ ,  $B_2 = 1324$ ,  $B_3 = 134$

## Braess' Example: Cont'd

- Case 1: All Paths Open. Split the flow equally (2 units each):

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$$b_{12} = 2, \ b_{24} = 4, \ b_{34} = 2, \ b_{13} = 4, \ b_{32} = 2$$

$$d_{12} = 52, \ d_{24} = 40, \ d_{34} = 52, \ d_{13} = 40, \ d_{32} = 12$$

$$L(B_1) = L(B_2) = L(B_3) = 92$$

## Braess' Example: Cont'd

- Case 1: All Paths Open. Split the flow equally (2 units each):

$$b_{12} = 2, \ b_{24} = 4, \ b_{34} = 2, \ b_{13} = 4, \ b_{32} = 2$$

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All paths are critical (maximal length). This distribution is stable: Switching paths increases load and travel time beyond 92.

## Braess' Example: Cont'd

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$$b_{12} = 2, b_{24} = 4, b_{34} = 2, b_{13} = 4, b_{32} = 2$$

$$d_{12} = 52, d_{24} = 40, d_{34} = 52, d_{13} = 40, d_{32} = 12$$

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- Case 2: Ban Edge  $k_{32}$ . Split flow equally between  $B_1$  and  $B_3$  (3 units each):

$$b_{12} = b_{24} = b_{13} = b_{34} = 3$$

$$d_{12} = 53, d_{24} = 30, d_{13} = 30, d_{34} = 53$$

$$L(B_1) = L(B_3) = 83$$

## Braess' Example: Cont'd

- Case 1: All Paths Open. Split the flow equally (2 units each):

$$b_{12} = 2, b_{24} = 4, b_{34} = 2, b_{13} = 4, b_{32} = 2$$

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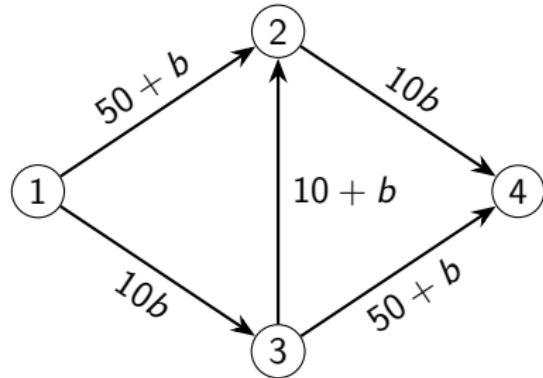
$$b_{12} = b_{24} = b_{13} = b_{34} = 3$$

$$d_{12} = 53, d_{24} = 30, d_{13} = 30, d_{34} = 53$$

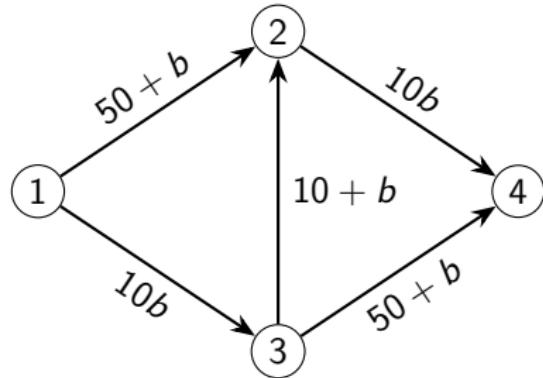
$$L(B_1) = L(B_3) = 83$$

Despite losing a connection, all paths are shorter!

## Braess' Example: Selfish Deviation

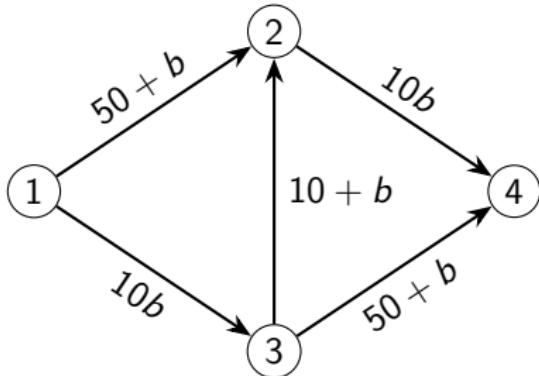


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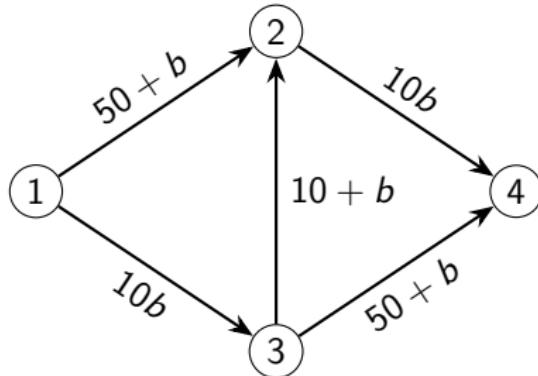
- $B_1 = 124, B_2 = 1324, B_3 = 134$

## Braess' Example: Selfish Deviation



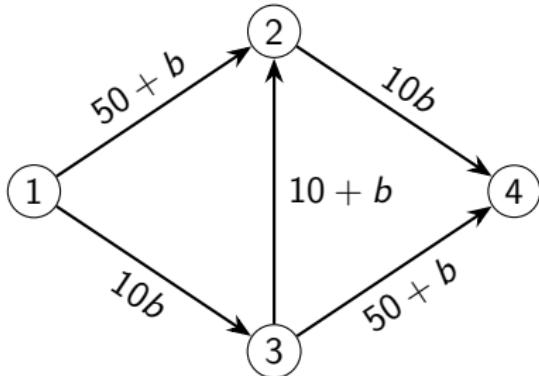
- $B_1 = 124, B_2 = 1324, B_3 = 134$
- A single vehicle ignoring the ban on  $k_{32}$  (where  $d_{32} = 10$ ) takes  $B_2$  with  $L(B_2) = 70$ , gaining an advantage.

## Braess' Example: Selfish Deviation



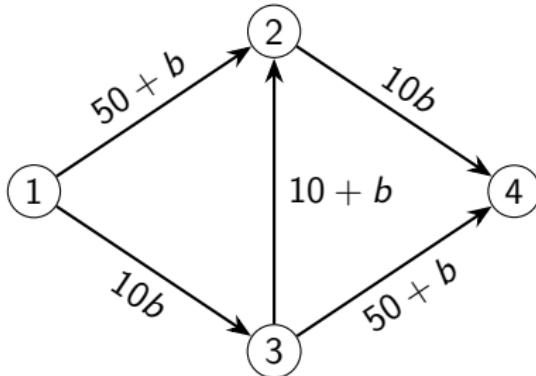
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- If the ban is lifted, many use  $B_2$ , reverting to the original state.

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- A single vehicle ignoring the ban on  $k_{32}$  (where  $d_{32} = 10$ ) takes  $B_2$  with  $L(B_2) = 70$ , gaining an advantage.
- If the ban is lifted, many use  $B_2$ , reverting to the original state.
- Consider an intermediate step: 5 units on  $B_1$  and  $B_3$ , 1 on  $B_2$ , then  $L(B_1) = L(B_3) = 87.5, L(B_2) = 82.5$ .

## Braess' Example: Selfish Deviation



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- A single vehicle ignoring the ban on  $k_{32}$  (where  $d_{32} = 10$ ) takes  $B_2$  with  $L(B_2) = 70$ , gaining an advantage.
- If the ban is lifted, many use  $B_2$ , reverting to the original state.
- Consider an intermediate step: 5 units on  $B_1$  and  $B_3$ , 1 on  $B_2$ , then  $L(B_1) = L(B_3) = 87.5, L(B_2) = 82.5$ .
- $B_2$  users clog  $k_{13}$  and  $k_{24}$  (factor 10 in load-time relation), worsening times for  $B_1$  and  $B_3$  beyond 83; Yet  $B_2$  remains shortest, attracting more traffic and degrading the system for all.

## Real-Life Examples

- The mechanical analogy: the spring paradox

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- Cheonggyecheon restoration project (2003 —)
  - Replaced a six lane highway with a five mile long park, traffic flow improved