

Introduction to Voting Theory

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Introduction to Voting

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- Voting can reflect genuine preference differences or different interpretations of information

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- If preferences are complete and transitive, they correspond to a ranked list
- Mathematically provable: Any complete and transitive preferences can be represented as a ranked list, and vice versa

Voting Paradox

Voting Paradox (Condorcet Paradox)

- The Simplest Case: Three voters, three alternatives

Voter	First preference	Second preference	Third preference
Voter 1	A	B	C
Voter 2	B	C	A
Voter 3	C	A	B

- Each voter has rational (transitive) preferences
- Majority rule on pairs produces:
 - A beats B (voters 1 and 3)
 - B beats C (voters 1 and 2)
 - C beats A (voters 2 and 3)
- Result: Cyclic group preferences despite transitive individual preferences
- Paradox: Even with rational individuals, the group can be “irrational”
- This creates fundamental problems for democratic decision-making

Voting Paradox in Social Contexts

- A More Complicated Situation: Party preferences over spending priorities

Party	First preference	Second preference	Third preference
Left (3)	education	health	security
Center (4)	health	security	education
Right (5)	security	education	health

- Individual vs. Multi-criteria decision making:
 - The paradox can arise even for a single individual deciding between options with multiple criteria
 - Example: College choice based on ranking, class size, and scholarship money

College	National Ranking	Average Class Size	Scholarship Money
A	4	40	\$3000
B	8	18	\$1000
C	12	24	\$8000

- When each option wins on different criteria, cycling can occur

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- Raises concerns about fairness and manipulation

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- Real-world impact: Parliamentary procedure, committee votes, and meeting agendas all involve this kind of strategic sequencing

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 - Used in: Heisman Trophy, AP poll rankings, MLB MVP selection
- Plurality voting:
 - 1 point for first place, 0 for all others
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 - Always produces a complete, transitive ranking
 - Considers all positions in rankings (Borda)
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Positional Voting Systems

- Direct approach: Assign weights based on position in each voter's ranking
- Borda Count: Named after Jean-Charles de Borda (1770)
 - With k alternatives: $k - 1$ points for first place, $k - 2$ for second, etc.
 - Each alternative receives points based on its positions in all rankings
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 - Vulnerable to strategic addition or removal of alternatives

Example of Borda Count

- Example with 4 alternatives (A, B, C, D) and 2 voters:

Voter	Ranking
Voter 1	$A \succ_1 B \succ_1 C \succ_1 D$
Voter 2	$B \succ_2 C \succ_2 A \succ_2 D$

- Points assigned:
 - A receives: 3 (from voter 1) + 1 (from voter 2) = 4 points
 - B receives: 2 (from voter 1) + 3 (from voter 2) = 5 points
 - C receives: 1 (from voter 1) + 2 (from voter 2) = 3 points
 - D receives: 0 (from voter 1) + 0 (from voter 2) = 0 points
- Group ranking: $B \succ A \succ C \succ D$
- Note that B wins even though neither voter places the same alternatives in the same positions
- The Borda Count attempts to account for “strength of preference” by including all positions

Example of Strategic Manipulation in Borda Count

- True preferences of five film critics:

Critics	First	Second	Third
1,2,3	Citizen Kane	The Godfather	Pulp Fiction
4,5	The Godfather	Citizen Kane	Pulp Fiction

- Calculating the Borda Count:
 - Citizen Kane receives: $3(2) + 2(1) = 8$ points
 - The Godfather receives: $3(1) + 2(2) = 7$ points
 - Pulp Fiction receives: $3(0) + 2(0) = 0$ points
- Strategic misrepresentation by critics 4,5:

Critics	First	Second	Third
1,2,3	Citizen Kane	The Godfather	Pulp Fiction
4,5	The Godfather	Pulp Fiction	Citizen Kane

- Citizen Kane receives: $3(2) + 2(0) = 6$ points
 - The Godfather receives: $3(1) + 2(2) = 7$ points
 - The Godfather now wins by strategically “burying” the main competitor
- Strategic voting is rational when voters understand the system
- This undermines the goal of having votes reflect true preferences

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 - Social choice involves fundamental trade-offs
 - Earned Arrow the Nobel Prize in Economics (1972)

Understanding Independence of Irrelevant Alternatives

- Independence of Irrelevant Alternatives (IIA) is subtle but critical:
 - The group ranking of X and Y should depend only on how each individual ranks X and Y
 - Changes in the ranking of other alternatives shouldn't affect X vs Y outcome
- Example: Two profiles with different rankings but same X vs Y preferences

Profile 1		Profile 2	
Individual	Ranking	Individual	Ranking
1	$W \succ X \succ Y \succ Z$	1	$X \succ Y \succ W \succ Z$
2	$W \succ Z \succ Y \succ X$	2	$Z \succ Y \succ X \succ W$
3	$X \succ W \succ Z \succ Y$	3	$W \succ X \succ Y \succ Z$

- In both profiles, individual 1 and 3 prefer X to Y , while individual 2 prefers Y to X
- IIA requires the group ranking of X and Y to be the same in both profiles
- Violations of IIA:
 - The Borda Count violates IIA (as we saw in the film critics example)
 - Elimination tournaments violate IIA (through strategic agenda-setting)
- IIA prevents “irrelevant” alternatives from acting as spoilers

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- This voter j is our candidate for being the dictator

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- For any profile Q where j ranks Y above Z :
 - Q and Q' are identical when restricted to Y and Z
 - By IIA, $Y \succ Z$ in $F(Q)$
- Therefore, j dictates the group ranking for all pairs not involving X

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 - This completes the proof of Arrow's Impossibility Theorem

Single-Peaked Preferences

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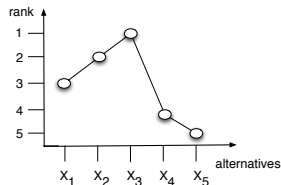
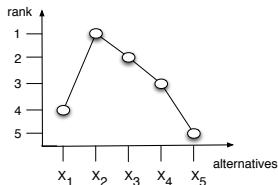
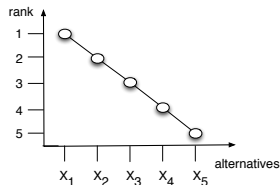
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 - This helps explain why many democratic systems work despite Arrow's theorem

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Voting as Information Aggregation

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