# Introduction to Financial Models
# Lecture 03: Voting Theory

# Introduction to Voting

# Voting for Group Decision-Making

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- Voting can reflect genuine preference differences or different interpretations of information

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- If preferences are complete and transitive, they correspond to a ranked list
- Mathematically provable: Any complete and transitive preferences can be represented as a ranked list, and vice versa

# Voting Paradox

# Voting Paradox (Condorcet Paradox)

- The Simplest Case: Three voters, three alternatives

| Voter | First preference | Second preference | Third preference |
|---|---|---|---|
| Voter 1 | A | B | C |
| Voter 2 | B | C | A |
| Voter 3 | C | A | B |

- Each voter has rational (transitive) preferences
- Majority rule on pairs produces:
  - A beats B (voters 1 and 3)
  - B beats C (voters 1 and 2)
  - C beats A (voters 2 and 3)
- Result: Cyclic group preferences despite transitive individual preferences
- Paradox: Even with rational individuals, the group can be “irrational”
- This creates fundamental problems for democratic decision-making

# Voting Paradox in Social Contexts

- A More Complicated Situation: Party preferences over spending priorities

| Party | First preference | Second preference | Third preference |
|---|---|---|---|
| Left (3) | education | health | security |
| Center (4) | health | security | education |
| Right (5) | security | education | health |

- Individual vs. Multi-criteria decision making:
    - The paradox can arise even for a single individual deciding between options with multiple criteria
    - Example: College choice based on ranking, class size, and scholarship money

| College | National Ranking | Average Class Size | Scholarship Money |
|---|---|---|---|
| A | 4 | 40 | \$3000 |
| B | 8 | 18 | \$1000 |
| C | 12 | 24 | \$8000 |

- When each option wins on different criteria, cycling can occur

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- Raises concerns about fairness and manipulation

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- Real-world impact: Parliamentary procedure, committee votes, and meeting agendas all involve this kind of strategic sequencing

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  - "Irrelevant" alternatives can change the outcome
  - Vulnerable to strategic addition or removal of alternatives

# Example of Borda Count

- Example with 4 alternatives (A, B, C, D) and 2 voters:

| **Voter** | **Ranking** |
|---|---|
| Voter 1 | A $\succ_1$ B $\succ_1$ C $\succ_1$ D |
| Voter 2 | B $\succ_2$ C $\succ_2$ A $\succ_2$ D |

- Points assigned:
  - A receives: 3 (from voter 1) + 1 (from voter 2) = 4 points
  - B receives: 2 (from voter 1) + 3 (from voter 2) = 5 points
  - C receives: 1 (from voter 1) + 2 (from voter 2) = 3 points
  - D receives: 0 (from voter 1) + 0 (from voter 2) = 0 points
- Group ranking: B $\succ$ A $\succ$ C $\succ$ D
- Note that B wins even though neither voter places the same alternatives in the same positions
- The Borda Count attempts to account for "strength of preference" by including all positions

# Example of Strategic Manipulation in Borda Count

- True preferences of five film critics:

| Critics | First | Second | Third |
|---|---|---|---|
| 1,2,3 | Citizen Kane | The Godfather | Pulp Fiction |
| 4,5 | The Godfather | Citizen Kane | Pulp Fiction |

- Calculating the Borda Count:
  - Citizen Kane receives: $3(2) + 2(1) = 8$ points
  - The Godfather receives: $3(1) + 2(2) = 7$ points
  - Pulp Fiction receives: $3(0) + 2(0) = 0$ points
- Strategic misrepresentation by critics 4,5:

| Critics | First | Second | Third |
|---|---|---|---|
| 1,2,3 | Citizen Kane | The Godfather | Pulp Fiction |
| 4,5 | The Godfather | Pulp Fiction | Citizen Kane |

  - Citizen Kane receives: $3(2) + 2(0) = 6$ points
  - The Godfather receives: $3(1) + 2(2) = 7$ points
  - The Godfather now wins by strategically "burying" the main competitor
- Strategic voting is rational when voters understand the system
- This undermines the goal of having votes reflect true preferences

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  - Earned Arrow the Nobel Prize in Economics (1972)

# Understanding Independence of Irrelevant Alternatives

- Independence of Irrelevant Alternatives (IIA) is subtle but critical:
  - The group ranking of $X$ and $Y$ should depend only on how each individual ranks $X$ and $Y$
  - Changes in the ranking of other alternatives shouldn't affect $X$ vs $Y$ outcome
- Example: Two profiles with different rankings but same $X$ vs $Y$ preferences

| **Profile 1** | | **Profile 2** | |
|---|---|---|---|
| **Individual** | **Ranking** | **Individual** | **Ranking** |
| 1 | $W \succ X \succ Y \succ Z$ | 1 | $X \succ Y \succ W \succ Z$ |
| 2 | $W \succ Z \succ Y \succ X$ | 2 | $Z \succ Y \succ X \succ W$ |
| 3 | $X \succ W \succ Z \succ Y$ | 3 | $W \succ X \succ Y \succ Z$ |

- In both profiles, individual 1 and 3 prefer $X$ to Y, while individual 2 prefers $Y$ to X
- IIA requires the group ranking of $X$ and $Y$ to be the same in both profiles
- Violations of IIA:
  - The Borda Count violates IIA (as we saw in the film critics example)
  - Elimination tournaments violate IIA (through strategic agenda-setting)
- IIA prevents "irrelevant" alternatives from acting as spoilers

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  - Then there exist alternatives Y, $Z$ such that $Y \succ X \succ Z$ in $F(P)$
  - Construct profile $P'$ by moving $Z$ ahead of $Y$ in each ranking where $Y$ was preferred to $Z$
  - The relative positions of $X$ vs. $Y$ and $X$ vs. $Z$ remain unchanged in each individual ranking
  - By IIA, the group ranking of $X$ vs. $Y$ and $X$ vs. $Z$ must remain the same in $F(P')$
  - So $Y \succ X \succ Z$ still holds in $F(P')$
  - But in $P'$, every voter ranks $Z$ ahead of $Y$
  - By Unanimity, $Z \succ Y$ must hold in $F(P')$
  - This creates a cycle: $Y \succ X \succ Z \succ$ Y, contradicting transitivity

# Step 1: Polarizing Alternatives

- **Definition**: An alternative $X$ is polarizing if every voter ranks it either first or last
- **Claim**: In any profile $P$ where $X$ is polarizing, $F$ must place $X$ either first or last in the group ranking $F(P)$
- Proof by contradiction:
  - Suppose $X$ is neither first nor last in $F(P)$
  - Then there exist alternatives Y, $Z$ such that $Y \succ X \succ Z$ in $F(P)$
  - Construct profile $P'$ by moving $Z$ ahead of $Y$ in each ranking where $Y$ was preferred to $Z$
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  - By Unanimity, $Z \succ Y$ must hold in $F(P')$
  - This creates a cycle: $Y \succ X \succ Z \succ$ Y, contradicting transitivity
- Therefore, $X$ must be ranked either first or last in $F(P)$

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- Construct a sequence of profiles $P_0$, $P_1$, ..., $P_k$ where:

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- Since $X$ is polarizing in $P_j$, and not last in $F(P_j)$, it must be first in $F(P_j)$

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- Voter $j$ has decisive power: changing just $j$'s vote moves $X$ from last to first in the group ranking

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- Voter $j$ has decisive power: changing just $j$'s vote moves $X$ from last to first in the group ranking
- This voter $j$ is our candidate for being the dictator

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- For any profile $Q$ where $j$ ranks $Y$ above $Z$:
  - $Q$ and $Q'$ are identical when restricted to $Y$ and $Z$
  - By IIA, $Y \succ Z$ in $F(Q)$
- Therefore, $j$ dictates the group ranking for all pairs not involving $X$

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- Therefore:
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  - The only voting systems satisfying Unanimity and IIA are dictatorships
  - This completes the proof of Arrow's Impossibility Theorem

# Single-Peaked Preferences

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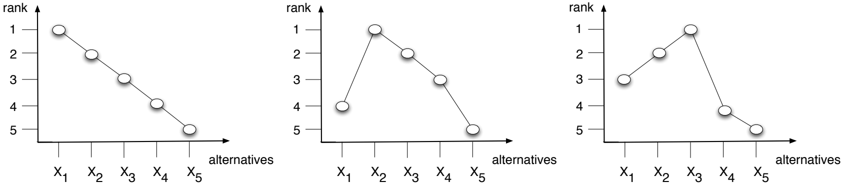
- Visual representation examples:



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- Importance: Majority rule works well with single-peaked preferences

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# Voting as Information Aggregation

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# Jury Decisions and Unanimity Rule

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# Conclusion

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# Further Reading

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