Introduction to Financial Models
Lecture 03: Voting Theory

Introduction to Voting

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- Voting can reflect genuine preference differences or different interpretations of information

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- If preferences are complete and transitive, they correspond to a ranked list
- Mathematically provable: Any complete and transitive preferences can be represented as a ranked list, and vice versa

Voting Paradox

Voting Paradox (Condorcet Paradox)

- The Simplest Case: Three voters, three alternatives

Voter	First preference	Second preference	Third preference
Voter 1	A	B	C
Voter 2	B	C	A
Voter 3	C	A	B

- Each voter has rational (transitive) preferences
- Majority rule on pairs produces:
  - A beats B (voters 1 and 3)
  - B beats C (voters 1 and 2)
  - C beats A (voters 2 and 3)
- Result: Cyclic group preferences despite transitive individual preferences
- Paradox: Even with rational individuals, the group can be "irrational"
- This creates fundamental problems for democratic decision-making

Voting Paradox in Social Contexts

- A More Complicated Situation: Party preferences over spending priorities

Party	First preference	Second preference	Third preference
Left (3)	education	health	security
Center (4)	health	security	education
Right (5)	security	education	health

- Individual vs. Multi-criteria decision making:
  - The paradox can arise even for a single individual deciding between options with multiple criteria
  - Example: College choice based on ranking, class size, and scholarship money

College	National Ranking	Average Class Size	Scholarship Money
A	4	40	\$3000
B	8	18	\$1000
C	12	24	\$8000

  - When each option wins on different criteria, cycling can occur

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- Raises concerns about fairness and manipulation

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- Real-world impact: Parliamentary procedure, committee votes, and meeting agendas all involve this kind of strategic sequencing

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  - Vulnerable to strategic addition or removal of alternatives

Example of Borda Count

- Example with 4 alternatives (A, B, C, D) and 2 voters:

Voter	**Ranking**
Voter 1	A $\succ_1$ B $\succ_1$ C $\succ_1$ D
Voter 2	B $\succ_2$ C $\succ_2$ A $\succ_2$ D

- Points assigned:
  - A receives: 3 (from voter 1) + 1 (from voter 2) = 4 points
  - B receives: 2 (from voter 1) + 3 (from voter 2) = 5 points
  - C receives: 1 (from voter 1) + 2 (from voter 2) = 3 points
  - D receives: 0 (from voter 1) + 0 (from voter 2) = 0 points
- Group ranking: B $\succ$ A $\succ$ C $\succ$ D
- Note that B wins even though neither voter places the same alternatives in the same positions
- The Borda Count attempts to account for "strength of preference" by including all positions

Example of Strategic Manipulation in Borda Count

- True preferences of five film critics:

Critics	First	Second	Third
1,2,3	Citizen Kane	The Godfather	Pulp Fiction
4,5	The Godfather	Citizen Kane	Pulp Fiction

- Calculating the Borda Count:
  - Citizen Kane receives: $3(2) + 2(1) = 8$ points
  - The Godfather receives: $3(1) + 2(2) = 7$ points
  - Pulp Fiction receives: $3(0) + 2(0) = 0$ points
- Strategic misrepresentation by critics 4,5:

Critics	First	Second	Third
1,2,3	Citizen Kane	The Godfather	Pulp Fiction
4,5	The Godfather	Pulp Fiction	Citizen Kane

  - Citizen Kane receives: $3(2) + 2(0) = 6$ points
  - The Godfather receives: $3(1) + 2(2) = 7$ points
  - The Godfather now wins by strategically "burying" the main competitor
- Strategic voting is rational when voters understand the system
- This undermines the goal of having votes reflect true preferences

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  - Earned Arrow the Nobel Prize in Economics (1972)

Understanding Independence of Irrelevant Alternatives

- Independence of Irrelevant Alternatives (IIA) is subtle but critical:
  - The group ranking of X and Y should depend only on how each individual ranks X and Y
  - Changes in the ranking of other alternatives shouldn't affect X vs Y outcome
- Example: Two profiles with different rankings but same X vs Y preferences

Profile 1		**Profile 2**	
Individual	**Ranking**	**Individual**	**Ranking**
1	$W \succ X \succ Y \succ Z$	1	$X \succ Y \succ W \succ Z$
2	$W \succ Z \succ Y \succ X$	2	$Z \succ Y \succ X \succ W$
3	$X \succ W \succ Z \succ Y$	3	$W \succ X \succ Y \succ Z$

- In both profiles, individual 1 and 3 prefer X to Y, while individual 2 prefers Y to X
- IIA requires the group ranking of X and Y to be the same in both profiles
- Violations of IIA:
  - The Borda Count violates IIA (as we saw in the film critics example)
  - Elimination tournaments violate IIA (through strategic agenda-setting)
- IIA prevents "irrelevant" alternatives from acting as spoilers

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- Voter j has decisive power: changing just j's vote moves X from last to first in the group ranking

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  - P_i: The first i voters rank X first, the rest rank X last
  - P_k: All voters rank X first
- By Unanimity:
  - X is ranked last in $F(P_0)$
  - X is ranked first in $F(P_k)$
- Therefore, X must change position from last to first at some point
- Let j be the first index such that X is not last in $F(P_j)$
- Since X is polarizing in P_j, and not last in $F(P_j)$, it must be first in $F(P_j)$
- Voter j has decisive power: changing just j's vote moves X from last to first in the group ranking
- This voter j is our candidate for being the dictator

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    - By transitivity of the group ranking, $Y \succ Z$ in $F(Q')$
- For any profile Q where j ranks Y above Z:
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    - By IIA, $Y \succ Z$ in $F(Q)$
- Therefore, j dictates the group ranking for all pairs not involving X

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  - This completes the proof of Arrow's Impossibility Theorem

Single-Peaked Preferences

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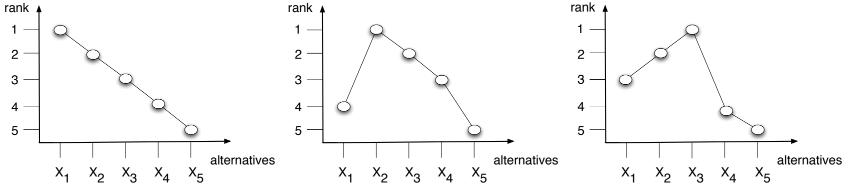

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  - This helps explain why many democratic systems work despite Arrow's theorem

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Voting as Information Aggregation

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