4109056001 資工三 施昶宇

1. 證$\frac{1}{2^{2k}}\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i-j)^2$ 設$2^k=t$

$$\frac{1}{t^2}\sum_{i=0}^{t-1}\sum_{j=0}^{t-1}(i-j)^2$$

$$\frac{1}{t^2}\sum_{i=0}^{t-1}\sum_{j=0}^{t-1}(i^2-2ij+j^2)$$

$$\frac{1}{t^2}\left(\left(t\sum_{i=0}^{t-1}i^2\right)+\left(t\sum_{j=0}^{t-1}j^2\right)+\sum_{i=0}^{t-1}\sum_{j=0}^{t-1}-2ij\right)$$

$$\frac{1}{t^2}\left((t)\left(\frac{1}{6}\right)(t-1)(t)(2t-1)\times 2-2\times\sum_{i=0}^{2^t-1}\sum_{j=0}^{t-1}(ij)\right)$$

$$\frac{2}{t^2}\left((t)\left(\frac{1}{6}\right)(t-1)(t)(2t-1)-\sum_{i=0}^{t-1}\sum_{j=0}^{t-1}(ij)\right)$$

$$\frac{2}{t^2}\left((t)\left(\frac{1}{6}\right)(t-1)(t)(2t-1)-\frac{(t-1)(t)(t-1)(t)}{4}\right)$$

$$\frac{2(t)(t)(t-1)}{t^2}\left(\frac{2t-1}{6}-\frac{t-1}{4}\right)$$

$$(t-1)\left(\frac{2t-1}{3}-\frac{t-1}{2}\right)$$

$$(t-1)\left(\frac{4t-2-3t+1}{6}\right)$$

$$\frac{(t-1)(t-1)}{6}$$

$$\frac{t^2-1}{6}$$

以$t=2^k$代入

$$\frac{2^{2k}-1}{6}$$

得證

2.

$$\frac{1}{2^k}[1^2 + 2^2 + \cdots + (2^{k-1})^2 + \cdots + 2^2 + 1^2]$$

$$\frac{1}{2^k}[(1^2 + 2^2 + \cdots + (2^{k-1})^2) + (1^2 + 2^2 + \cdots + (2^{k-1} - 1)^2)]$$

以$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$代入

$$\frac{1}{2^k}\left(\frac{2^{k-1}(2^{k-1}+1)(2^k+1)}{6} + \frac{(2^{k-1}-1)2^{k-1}(2^k-1)}{6}\right)$$

$$\frac{1}{2^k}\left(\frac{2^{3k-1}+2^k}{6}\right)$$

$$\frac{2^{2k-1}+1}{6}$$

得證

3.

LSB-K	embedding rate	mean square error	PSNR	embedding efficiency
K=1	1	0.5	51.14	2.0
K=2	2	2.5	44.15	0.8
K=3	3	10.5	37.92	0.285714
K=4	4	42.5	31.85	0.094118
K=5	5	170.5	25.81	0.029326
K=6	6	682.5	19.79	0.008791
K=7	7	2730.5	13.77	0.002564

OPAP-K	embedding rate	mean square error	PSNR	embedding efficiency
K=1	1	0.5	51.14	2.0
K=2	2	1.5	46.37	1.333333
K=3	3	5.5	40.73	0.545455
K=4	4	21.5	34.81	0.186047
K=5	5	85.5	28.81	0.05848
K=6	6	341.5	22.80	0.01757
K=7	7	1365.5	16.78	0.005126