

Analysis of Algorithms

Midterm Exam : 22 April 2010

1. Prove that for any real constants a and b , where $b > 0$,

$$(n + a)^b = \Theta(n^b)$$

by definition of asymptotic notations.

2. Describe a $\Theta(n \log n)$ -time algorithm that, given a set S of n integers and another integer x , determines whether or not there exist two elements in S whose sum is exactly x .
3. Modify Strassen's algorithm to multiply $n \times n$ matrices in which n is not an exact power of 2. Show that the resulting algorithm runs in time $\Theta(n^{\log_2 7})$.
4. Illustrate the operation of COUNTING-SORT on the array $A = (6, 1, 2, 0, 3, 1, 6, 1, 3, 2)$.
5. Demonstrate what happens when we insert the keys 30, 14, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. The table has 9 slots, and the hash function is $h(k) = k \bmod 9$.
6. What is the largest possible number of internal nodes in a red-black tree with black height k ? What is the smallest possible number? (Note that a red-black tree with one internal node has black height 1.)