Analysis of Algorithms

Midterm Exam : 26 April 2011

1. Solve the following recurrence by the recursion-tree method

$$T(n) = 2T(n/2) + n \log^2 n.$$

(Assume that n is an exact power of 2.)

2. Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an *inversion* of A.

   (a) List the five inversions of the array $\langle 3, 5, 9, 7, 1 \rangle$.

   (b) Give an algorithm that computes the number of inversions in array $A[1..n]$. What is the worst-case time complexity of your algorithm?

3. Suppose that you want to sort n numbers, each of which is an integer in the range of $[0..n^3 - 1]$, Describe an asymptotically efficient algorithm for this problem.

4. Given an array $A[1..n]$ and an integer k with $k \ll n$, the partial sort problem is to return the k smallest numbers in order from the given n numbers. Find an efficient algorithm for the partial sort problem. What is the time complexity of your algorithm?

5. Describe the worst-case linear time selection algorithm (i.e., given an array $A[1..n]$ and an integer i, it finds the i-th smallest element in $A[1..n]$).

6. Explain how OS-SELECT$(T, 9)$ and OS-RANK(T, x) with $key[x] = 20$ operate in the order-statistic tree below.