

# Analysis of Algorithms

Midterm Exam : 23 April 2013

1. Prove that  $an^2 + bn + c = \Theta(n^2)$  by definition of asymptotic notations, where  $a$ ,  $b$ , and  $c$  are constants and  $a > 0$ .
2. Write down the recurrence for the time complexity of Strassen's algorithm, and solve it.
3. Illustrate the operations of **Counting-Sort** on an array  $A = (7, 4, 5, 9, 4, 4, 7, 5)$ .
4. Show that the second smallest of  $n$  elements can be found with  $n + \lceil \log n \rceil - 2$  comparisons in the worst case.
5. Illustrate inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into a hash table of length  $m = 11$  using open addressing with double hashing, where  $h_1(k) = k \bmod m$  and  $h_2(k) = 1 + (k \bmod (m - 1))$ .
6. Observe that whenever we reference the *size* attribute of a node in either **OS-Select** or **OS-Rank**, we use it only to compute a rank. Accordingly, suppose we store in each node its rank in the subtree of which it is the root. Show how to maintain this information during insertion and deletion. (Remember that these two operations can cause rotations.)