Analysis of Algorithms

Midterm Exam : 29 April 2014

1. Prove or disprove the following:
   (a) $2^{n+1} = O(2^n)$
   (b) $2^{2n} = O(2^n)$

2. By using the recursion-tree method, determine a tight asymptotic bound on the recurrence $T(n) = 4T(n/2) + n$. Assume that n is a power of 2.

3. Illustrate the operation of RADIX-SORT on the following list of English words: COW, DOG, SEA, RUG, ROW, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX. The words should be sorted in lexicographic order.

4. Let $X[1..n]$ and $Y[1..n]$ be two arrays, each containing n numbers already in sorted order. Give an $O(\log n)$-time algorithm to find the median of all $2n$ numbers in arrays X and Y.

5. Consider a set S of n (≥ 2) distinct numbers given in unsorted order. For each of the following problems, give an algorithm to determine two distinct numbers x and y in S that satisfy the stated condition. Describe your algorithm and justify its running time.
   (a) In $O(n)$ time, determine $x, y \in S$ such that $|x - y| \geq |p - q|$ for all $p, q \in S$.
   (b) In $O(n \log n)$ time, determine $x, y \in S$ such that $|x - y| \leq |p - q|$ for all $p, q \in S$ with $p \neq q$.

6. Explain how OS-SELECT$(T, 9)$ and OS-RANK(T, x) with $key[x] = 20$ operate in the order-statistic tree below.