**Algorithm description**

**binary search**

To solve this problem, our algorithm uses binary\_search. Binary\_search is used to find the solution for a function f(x)=0 in its monotonous interval.

Suppose function f(x) is monotonic in interval [min,max]. Assume it is monotonically increasing interval. If both of f(min) and f(max) are more than 0 or less than 0, it means there is no solution in [min,max]. Otherwise, we can get the solution through binary\_search. At beginning, we know the solution is in [min,max]. Say (min+max)/2 m. We check the result of f(m). If f(m)>0, we know the solution is in interval [min,m] and do further search in [min,m], otherwise, we know the solution is in interval[m,max] and do further search in [m,max]. We do this repeatedly until the interval is small enough to get an accurate solution.

**Solution of cubic equation**

For a cubic equation, if we know there is a monotonic interval [l,r], we can use binary search to find the root of the equation within [l,r]. So to solve the cubic equation, we find the monotonic intervals of the cubic polynomial, then find all roots through binary search in each monotonic intervals respectively.

Suppose the cubic equation is



The derivative of the cubic polynomial is following quadratic polynomial



There might be two cases:

1. The quadratic polynomial has no roots or one root.

In this cases, it means the cubic polynomial is monotonic in [-oo,+oo]. So do binary search in [-oo,+oo] to find the root of the cubic equation.

1. The quadratic polynomial has two roots x1 and x2 (x1<x2).

In this cases, the cubic polynomial has three monotonic intervals [-oo,x1], [x1,x2] and [x2,+oo]. So do binary search in the three intervals respectively to find all the roots for the cubic equation.

**Pseudocode**

The following function do binary search in [min,max] to find the root of cubic equation



binary\_search(min,max,a,b,c,d)

{

If ((a\*max3+b\*max2+c\*max+d)\*(a\*min3+b\*min2+c\*min+d)>0)

return null

l=min

r=max

while (l<r)

{

m=(l+r)/2

If ((a\*m3+b\*m2+c\*m+d)\*(a\*l3+b\*l2+c\*l+d)>0)

l=m

Else

r=m  
}

return l

}

The following function finds all roots for the cubic equation

solute\_cubic(a,b,c,d)

{

peak is the set of roots of quadratic equation 3\*a\*x2+2\*b\*x+c=0

If (peak is empty or peak has one element)

return binary\_search(-oo,+oo,a,b,c,d)

else

return { binary\_search(-oo,peak[0],a,b,c,d), binary\_search(peak[0],peak[1],a,b,c,d),

binary\_search(peak[1],+oo,a,b,c,d) }  
}