Algorithm description

For a cubic equation, if we know there is a monotonic interval [l,r], we can use binary search to find the root of the equation within [l,r]. So to solve the cubic equation, we find the monotonic intervals of the cubic polynomial, then find all roots through binary search in each monotonic intervals respectively.

Suppose the cubic equation is



The derivative of the cubic polynomial is following quadratic polynomial



There might be two cases:

1. The quadratic polynomial has no roots or one root.

In this cases, it means the cubic polynomial is monotonic in [-oo,+oo]. So do binary search in [-oo,+oo] to find the root of the cubic equation.

1. The quadratic polynomial has two roots x1 and x2 (x1<x2).

In this cases, the cubic polynomial has three monotonic intervals [-oo,x1], [x1,x2] and [x2,+oo]. So do binary search in the three intervals respectively to find all the roots for the cubic equation.

Pseudocode

The following function do binary search in [min,max] to find the root of cubic equation



binary\_search(min,max,a,b,c,d)

{

If ((a\*max3+b\*max2+c\*max+d)\*(a\*min3+b\*min2+c\*min+d)>0)

return null

l=min

r=max

while (l<r)

{

m=(l+r)/2

If ((a\*m3+b\*m2+c\*m+d)\*(a\*l3+b\*l2+c\*l+d)>0)

l=m

Else

r=m  
}

return l

}

The following function finds all roots for the cubic equation

solute\_cubic(a,b,c,d)

{

peak is the set of roots of quadratic equation 3\*a\*x2+2\*b\*x+c=0

If (peak is empty or peak has one element)

return binary\_search(-oo,+oo,a,b,c,d)

else

return { binary\_search(-oo,peak[0],a,b,c,d), binary\_search(peak[0],peak[1],a,b,c,d),

binary\_search(peak[1],+oo,a,b,c,d) }  
}