Computer Graphics – 5. April 2016

Personal Information: / Angaben zur Person:

Name / Last Name .... Vorname / First Name ....

Mat.-Nummer ....

Please read the instructions carefully

- you will be given **sixty (60)** minutes to complete the written examination
- check that the examination is complete and readable
- please use a blue or black pencil, not a red or green one
- only text on the stapled sheets will be evaluated, if needed, we staple additional blank sheets
- only writing material and non programmable calculators are allowed
- fill in the **TN** from your seat number

Die Hinweise bitte aufmerksam lesen

- Die Bearbeitungszeit beträgt **60 Minuten**
- Überprüfen Sie die Prüfungsunterlagen auf Vollständigkeit und einwandfreies Druckbild
- Schreiben Sie deutlich und ausschließlich mit blauer oder schwarzer Tinte. Unleserliche Antworten gehen **nicht** in die Bewertung ein. Benutzen Sie **keinen Bleistift**.
- Nur angeheftete Blätter werden bewertet. Sollte der Platz nicht ausreichen, so müssen Sie bei der Aufsicht weitere Zusatz-Seiten anfordern und einheften lassen
- Nur Schreibzeug und nicht programmierbare Taschenrechner sind erlaubt
- Übertragen Sie die Teilnehmernummer (**TN**) aus Ihrer Platzkarte

I have read and understood the instructions above
Durch meine Unterschrift bestätige ich die Kenntnisnahme der obigen Informationen

Erlangen, 5. April 2016 ....
(Signature / Unterschrift)

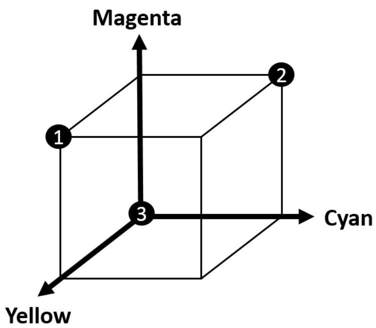
This space for official use only - leave blank

Aufgabe	1	2	3	4	5	6	**Gesamt**
Max. Punktzahl	5	7	15	10	11	12	**60**
Erreichte Punkte							

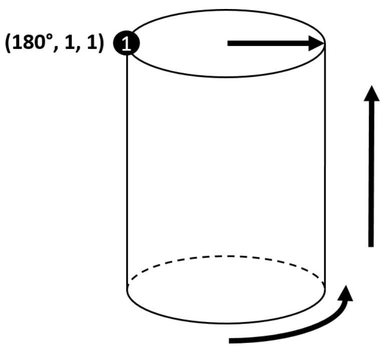
Gesamtpunktzahl	/ **60**
Note	

Aufgabe 1: Color Spaces 5 Punkte

1. From the lecture and the exercises you know that there are different color spaces. Hardcopy devices (printers) often use internally the CMY color space, which is complementary to the RGB color space. Which colors are at the corners 1, 2 and 3 of the CMY color cube in the figure below?



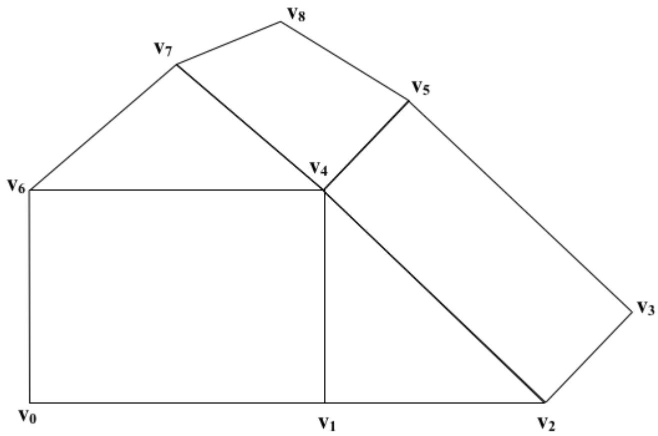
2. Another popular color space is the HSV color space. Label the axis and determine the color at point 1 (HSV color $(180°, 1, 1)$). Mark the grayscale colors in the HSV-cylinder.



3. Which color space should you prefer to generate a rainbow color gradient? RGB, CMY or HSV?

Aufgabe 2: Data Structures 7 Punkte

1. Fill into the connectivity list for the triangle-mesh below.



Index	vertex
0	(x_0, y_0)
1	(x_1, y_1)
2	(x_2, y_2)
3	(x_3, y_3)
4	(x_4, y_4)
5	(x_5, y_5)
6	(x_6, y_6)
7	(x_7, y_7)
8	(x_8, y_8)

Index	element connectivity
0	
1	
2	
3	
4	

2. The following list of *queries* represent typical operations required for mesh processing:

- walk around edges of a given face
- find adjacent edges of a vertex
- find adjacent faces to a vertex
- find adjacent vertex to a face
- find neighbor faces to a face

Which data structure for triangle meshes is more convenient to solve theses tasks in constant time, i.e. $\mathcal{O}(1)$?

3. Write an algorithm in pseudocode which computes the face normal for each triangle in a triangle–mesh in 3D.

```
function computeFaceNormals
    input shared vertex triangle mesh
    output list of normals
```

Aufgabe 3: Affine Transformations 15 Punkte

Specify the 4×4 matrices for the following cases:

1. A translation by +2 in the y-direction and by -1 in the z-direction:

$$\mathbf{M} = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}$$

2. First a translation by +2 in the x-direction and -1 in the y-direction **and then** a horizontal shearing by 1 in the x-direction.

$$\mathbf{M} = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix} \cdot \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}$$

3. First a rotation about the x-axis by an angle α **and then** a rotation about the y-axis by an anlge θ:

$$\mathbf{M} = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix} \cdot \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}$$

4. Name 3 different ways to represent a rotation:

   - ....
   - ....
   - ....

5. Given a non-vanishing quaternion $\mathbf{q} = (a, b, c, d)$. Describe formulas to compute the rotation angle and the rotation axis.

Aufgabe 4: Clipping in Homogeneous Coordinates — 10 Punkte

The perspective transformation has been specified by giving the field of view to be $\mathbf{fov} = 90^0$, the aspect ratio width and height of the screen to be $\mathbf{aspect} = \mathbf{1}$, and the near and far planes $\mathbf{n} = -1$, $\mathbf{f} = -3$. The perspective transformation matrix has the form:

$$M_{projection} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

1. Given the two points $\mathbf{P}_1 = (1, 0, -2)^T$ and $\mathbf{P}_2 = (1, 0, 1)^T$ compute the 4D homogeneous points $\mathbf{Q}_1$ and $\mathbf{Q}_2$ by applying $M_{projection}$ and the 3D points $\tilde{\mathbf{Q}}_1$ and $\tilde{\mathbf{Q}}_2$ after dehomogenization.

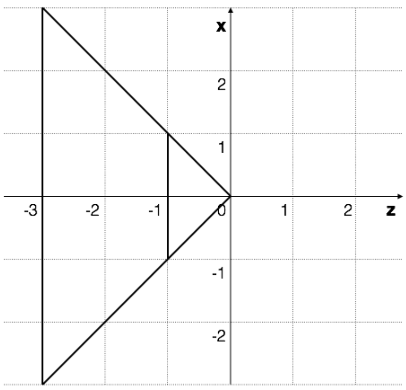
$$\mathbf{Q}_1 = \begin{pmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \\ \boxed{} \end{pmatrix} \qquad \mathbf{Q}_2 = \begin{pmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \\ \boxed{} \end{pmatrix} \qquad \tilde{\mathbf{Q}}_1 = \begin{pmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \end{pmatrix} \qquad \tilde{\mathbf{Q}}_2 = \begin{pmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \end{pmatrix}$$

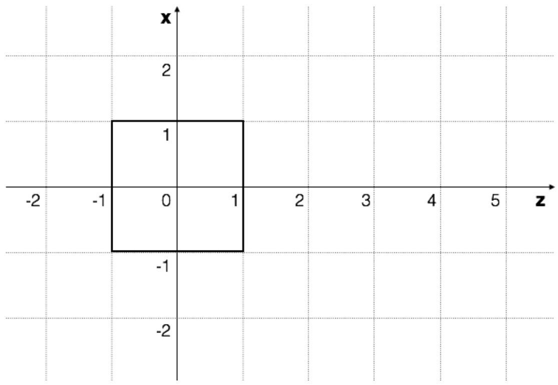
2. Use α–clipping in homogeneous coordinates to clip the line segment $\overline{\mathbf{P}_1\mathbf{P}_2}$ against the right plane $x = 1$. Give α_{max}, and the resulting 4D intersection point $\mathbf{S}_1$ and the 3D point $\tilde{\mathbf{S}}_1$ after dehomogenization. By omitting the y–coordinate draw in the graphics below the input points $\mathbf{P}_1$ and $\mathbf{P}_2$ and the corresponding line segment, and the points $\tilde{\mathbf{Q}}_1$ and $\tilde{\mathbf{Q}}_2$ and the clipped line segment in the canonical view volume.

   The window edge coordinates WEC in homogeneous coordinates are defined by:

$\text{WEC}_r(\text{x,y,z,w}) = \text{w - x}$	$\text{WEC}_l(\text{x,y,z,w}) = \text{w + x}$
$\text{WEC}_t(\text{x,y,z,w}) = \text{w - y}$	$\text{WEC}_b(\text{x,y,z,w}) = \text{w + y}$
$\text{WEC}_f(\text{x,y,z,w}) = \text{w - z}$	$\text{WEC}_n(\text{x,y,z,w}) = \text{w + z}$

$$\alpha_{max} = \qquad\qquad \mathbf{S}_1 = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} \qquad\qquad \tilde{\mathbf{S}}_1 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$





3. Would be the result different, if you first clip against the near plane $z = -1$?

   Hint: Look at the graphic from previous assignment. NO

4. Give the 6-digit outcode for the point $\mathbf{P}_2$ (respectively $\tilde{\mathbf{Q}}_2$). The outcode digit is positive, if the point is outside the plane.

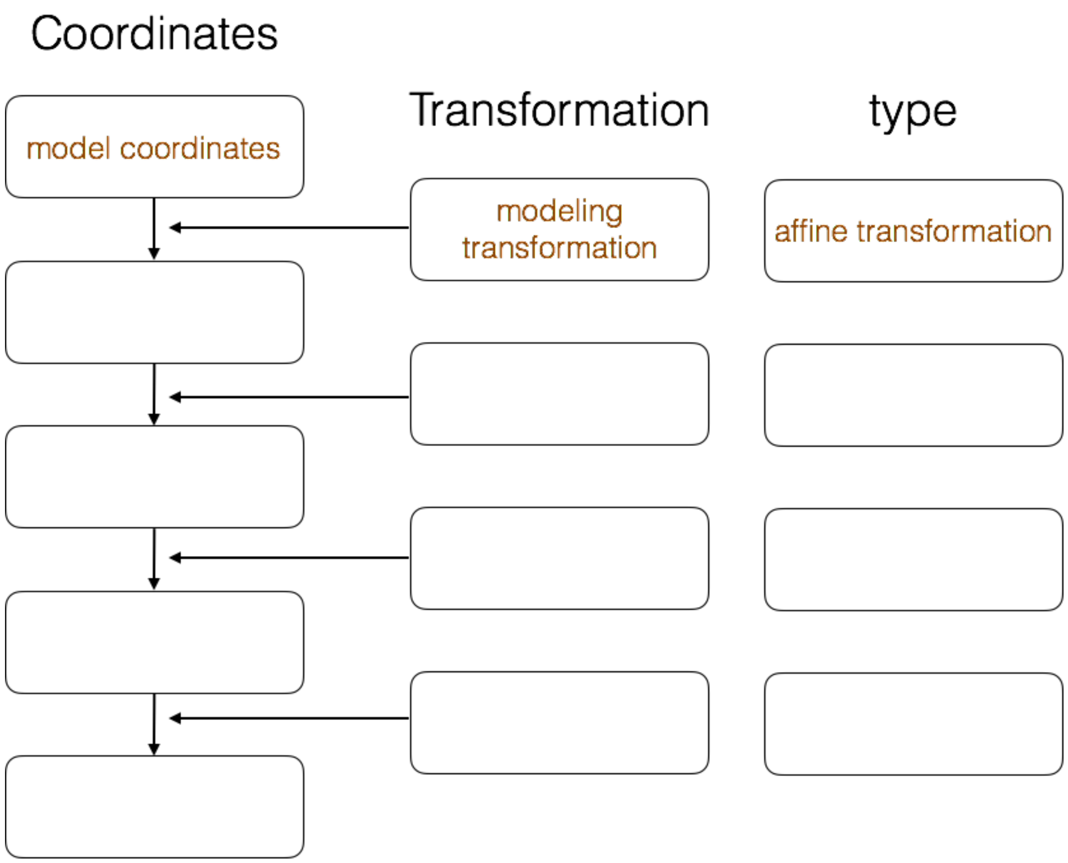
   Hint: the outcode can be obtained by inspection of the graphic given above.

	near	far	bottom	top	left	right
$\mathbf{P}_2$						
$\tilde{\mathbf{Q}}_2$						

Aufgabe 5: Viewing pipeline 11 Punkte

1. Complete the graphic below writing the following coordinate systems, and the transformations in-between into the right order. Furthermore, you should mention which type of transformation it is, possibilities are *affine transformation* or *perspective/parallel projection*.

Coordinates	Transformation
• pixel + z-coordinates • world coordinates • viewing coordinates • model coordinates • normalized coordinates	• projection transformation • vieweing transformation • viewport transformation • modeling transformation



2. Construct the matrix for the viewing transformation by giving the rotation and the translation matrices for the following parameters:

   - gaze-direction $(1, 0, 0)^T$
   - up-direction $(1, 1, 0)^T$
   - camera/eye position $(10, 7, -3)^T$

   The camera is looking at the negative z.

   - construct the local basis $\{u, v, w\}$

   - write the viewing matrix as the product of a rotation and a translation

$$\mathbf{M}_v = \begin{pmatrix} \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \end{pmatrix} \cdot \begin{pmatrix} \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} \end{pmatrix}$$

Aufgabe 6: Ray Tracing — 12 Punkte

In order to accelerate ray tracing hierarchical data structures are commonly used, which allow to reduce the number of ray-objet intersection tests. One commonly implemented data structures are **Bounding Volume Hierarchies (BVH)** and **k–d Trees**.

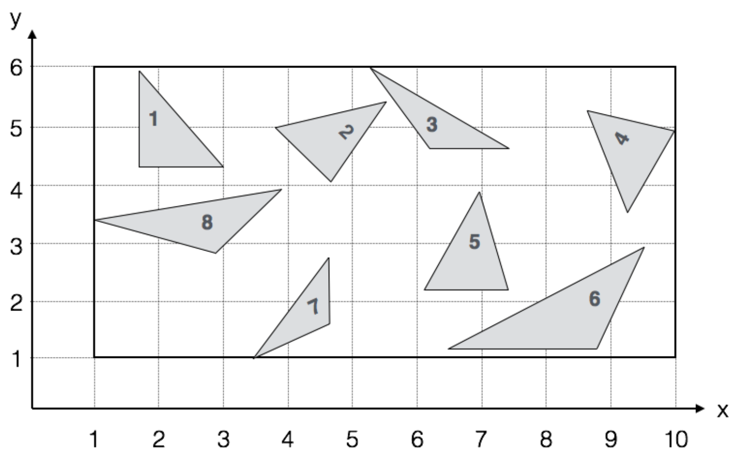
1. for which one of these data structures the leaf nodes might overlap, and for which one the nodes do not overlap? Answer with **yes** or **no**.

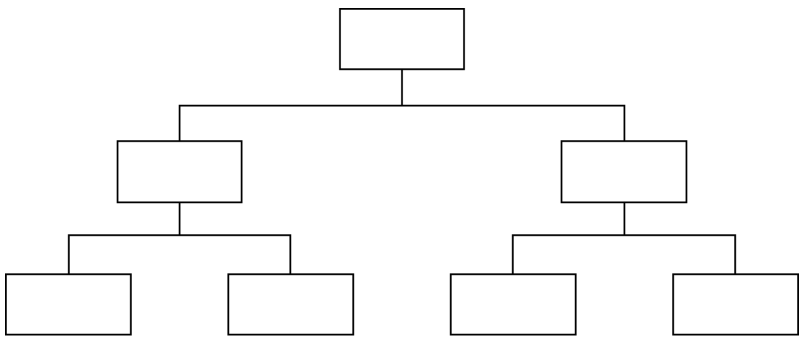
	nodes **might** overlap	node **do not** overlap
BVH		
k–d trees		

2. for wich one of these data structures the geometric primitives, e.g. the triangles, might be assigned to more than one node, and for which one this is not possible? Answer with yes or no!

	triangles might be assigned to **more than one** node	triangles are assigned to **only one** node
BVH		
k–d trees		

3. compute 3-level **BVH** for the triangles configuration shown bellow. Split first along the x-axis and then along the y-axis in such a way that the leaves have similar number of elements. Sketch the resulting **BVH** in the figure bellow. Write the triangles ids in the nodes of the tree bellow.





4. Write the formula to compute the value of t for the intersection of a ray $\mathbf{l}(t) = \mathbf{e} + t\mathbf{d}$, where $\mathbf{d}$ is a unit vector, with the plane $(\mathbf{X} - \mathbf{P}) \cdot \mathbf{n} = 0$, where $\mathbf{P}$ is a point on the plane and $\mathbf{n}$ is a unit normal to the plane.

5. Describe an algorithm to compute the intersection of a ray with a triangle. Please, use no formulas!