

Mathematics

Math21b

Fall 2004



Fall 2004

Linear Algebra and Differential Equations

Course Head: Oliver knill

Office: SciCtr 434

Email: knill@math.harvard.edu



Eigenvalues and eigenvectors of 2x2 matrices

Calculating eigenvalues and eigenvectors of matrices by hand can be a daunting task. This is why homework problems deal mostly with 2x2 or 3x3 matrices. For 2x2, 3x3, and 4x4 matrices, there are complete answers to the problem. In that case, one can give explicit algebraic formulas for the solutions. For 5x5 matrices, an explicit algebraic solution can not be found any more since one would have to give formulas of the roots of a polynomial of degree 5. Below, we have the solution for the 2x2 case. Your linear algebra teacher probably doesn't want you to know them...

Let $T=a+d$ be the trace and $D=ad-bc$ be the determinant of the matrix

$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

The eigenvalues of A are

$$\begin{aligned} L_1 &= T/2 + (T^2/4 - D)^{1/2} \\ L_2 &= T/2 - (T^2/4 - D)^{1/2} \end{aligned}$$

If c is not zero, then the eigenvectors are

$$\begin{vmatrix} L_1 - d \\ c \end{vmatrix}, \begin{vmatrix} L_2 - d \\ c \end{vmatrix}$$

If b is not zero, then the eigenvectors are

$$\begin{vmatrix} b \\ L_1 - a \end{vmatrix}, \begin{vmatrix} b \\ L_2 - a \end{vmatrix}$$

If both b and c are zero, then the eigenvectors are

$$\begin{vmatrix} 1 \\ 0 \end{vmatrix}, \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

Proof. We just have to verify that $A v = L v$ holds. In all cases, one of the two equations is $(bc-ad) + (a+d) L - L^2=0$ and in the other equation, everything cancels.

[Back to the main page](#)