Assignment 1

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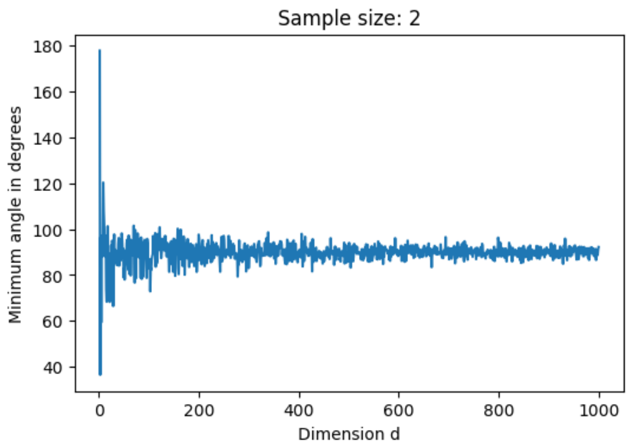
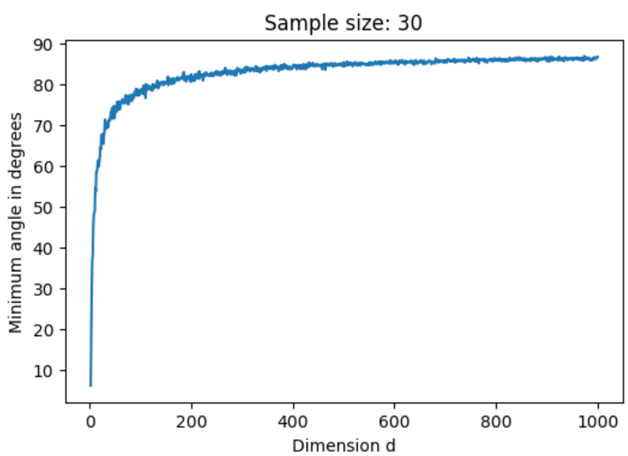
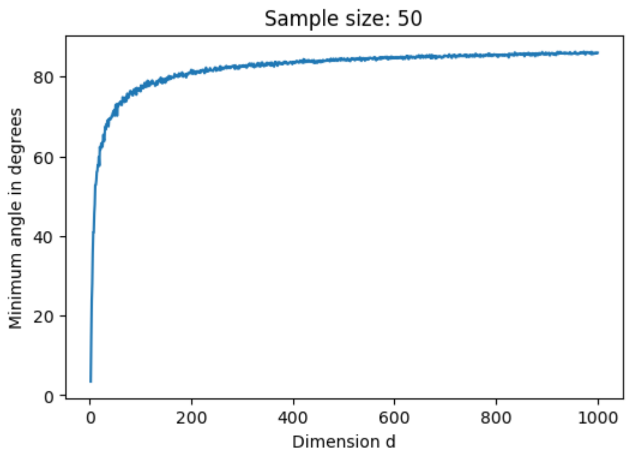
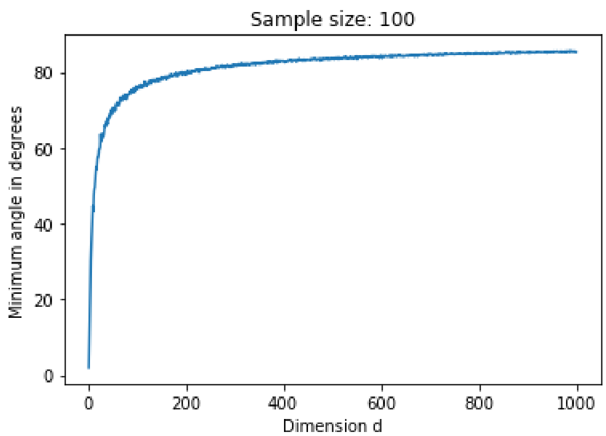
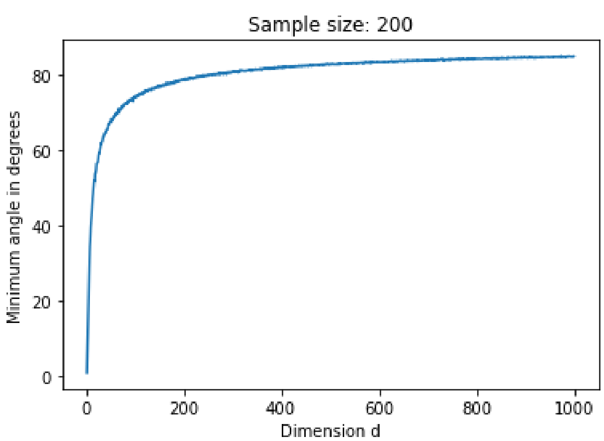
Task1







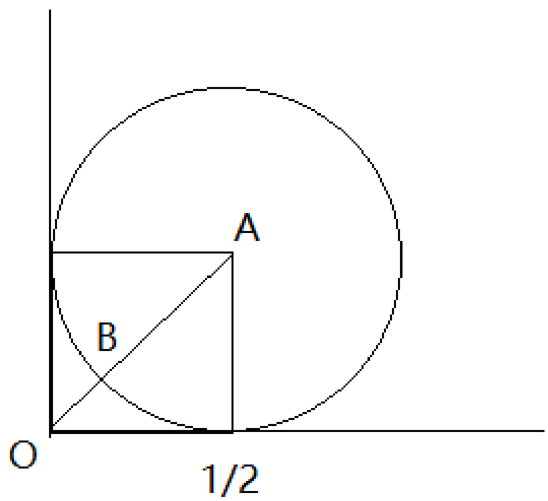




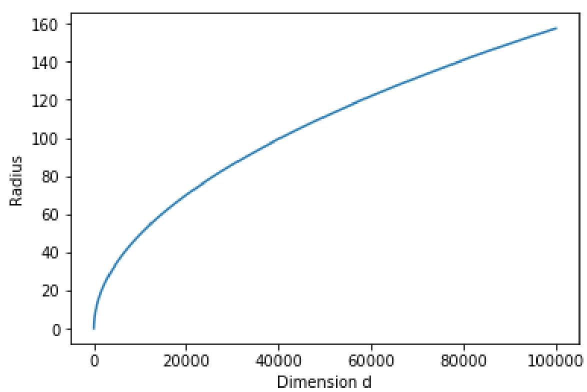
With the dimentions d increasing, vectors may have many different directions. It's more difficult for vectors to have similar direction(

angle=0), the minimum angle between two random vectors tends to be 90 degree.
For 2 vectors in d-dimension, $x = (x_1, x_2...x_d), y = (y_1, y_2....y_d)$
The average of minangle of x,y is $\frac{1}{samplenumbers} * min(arccos(\frac{x \cdot y}{\|x\| * \|y\|})$.
When d is big enough, according to Law of large numbers, $E(x_i), E(y_i) = 0, E(x_i)E(y_i) = 0$, so,
$cos(angle) \to 0$. Namely, the average of min angle increases towards 90 degree.
The number of samples will not change the result. However, compared with sample size 30 and 50, the result will be more stable with the increasing of samples.
If the sample size is too small(like 2), the results are not representative.

Task2



The image in 2-d is like this. OB is the radius of the new circle.
OB=OA-AB=$\frac{\sqrt{2}}{2}$-$\frac{1}{2}$
Similarly, in d-dimension, the radius is $\frac{\sqrt{d}}{2}$-$\frac{1}{2}$
If d=4, the equals to 0.5, if d>4, the radius is larger than 0.5.
If d=9, the radius equals to 1, if d>9, the radius is larger than 1.
When d increases towards infinity, the radius of the sphere will increase in the order of $\sqrt{d}$ towards infinity.



Task3

1) Yes, because the sum of the probability of all the events is 1.
2)

$$\begin{aligned} E_{Y|X=2}[Y] &= \sum_y P(Y=y|X=2)y \\ &= \sum_y \frac{P(Y=y,X=2)}{P(X=2)}y \\ &= \sum_y \frac{P(Y=y,X=2)}{0.59)}y \\ &= \frac{0.4*1+0.14*2+0.05*3}{0.59} \quad \approx 1.41 \end{aligned}$$

$$P(X=1|Y=3) = \frac{P(X=1,Y=3)}{P(Y=3)} = \frac{0.13}{0.18} \approx 0.72$$

3)No because

$$\int_{-\infty}^{+\infty} \quad \int_{-\infty}^{+\infty} p(x,y)dxdy = 0.5$$

4)
$p(x) = \int\limits_{-\infty}^{+\infty} p(x,y)dy$
$= \int\limits_{x}^{+\infty} 2e^{-x-y}dy$
so we can get p(x)
$p(x) = 2e^{-2x} \quad for\ x \geq 0$
p(y)= $\int\limits_{-\infty}^{+\infty} p(x,y)dx$
$=\int\limits_{0}^{y} 2e^{-x-y}dx$
$p(y) = 2e^{-y}(1-e^{-y}) \quad for\ y \geq x$
5)

$$P(X \leq |Y=0.5) = \frac{P(X \leq 2, Y=0.5)}{P(Y=0.5}$$

since P(Y=0.5)=0, namely the condition never happens, the conditional probablity is zero.