Problem Set 3

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Directions: Answer all questions. Each student must turn in their own copy, but you may work in groups. You are encouraged to use any and all Artificial Intelligence resources available to you to complete this problem set. Clearly label all answers. Show all of your code. Turn in jl-file(s), output files and writeup via GitHub. Your writeup may simply consist of comments in jl-file(s). If applicable, put the names of all group members at the top of your writeup or jl-file.

You will need to load the following packages:

```
Optim
HTTP
GLM
LinearAlgebra
Random
Statistics
DataFrames
CSV
FreqTables
```

On Github there is a file called PS2_starter.jl that has the code blocks below already created.

1. Estimate a multinomial logit (with alternative-specific covariates Z) on the following data set:

```
using DataFrames
using CSV
using HTTP
url = "https://raw.githubusercontent.com/OU-PhD-Econometrics/fall-2024/
master/ProblemSets/PS3-gev/nlsw88w.csv"
df = CSV.read(HTTP.get(url).body, DataFrame)
```

The choice set represents possible occupations and is structured as follows.

- 1 Professional/Technical
- 2 Managers/Administrators
- 3 Sales
- 4 Clerical/Unskilled
- 5 Craftsmen
- 6 Operatives
- 7 Transport
- 8 Other

Hints:

- Index the parameter vector so that the coefficient on Z is the last element and the coefficients on X are the first set of elements.
- You will need to difference the *Z*'s in your likelihood function.
- Normalize $\beta_J = 0$
- The formula for the choice probabilities will thus be

$$P_{ij} = \begin{cases} \frac{\exp(X_i \beta_j + \gamma(Z_{ij} - Z_{iJ}))}{1 + \sum_{k=1}^{J-1} \exp(X_i \beta_k + \gamma(Z_{ik} - Z_{iJ}))}, & j = 1, \dots, J-1 \\ \frac{1}{1 + \sum_{k=1}^{J-1} \exp(X_i \beta_k + \gamma(Z_{ik} - Z_{iJ}))}, & j = J \end{cases}$$

- 2. Interpret the estimated coefficient $\hat{\gamma}$.
- 3. Estimate a nested logit with the following nesting structure:
 - White collar occupations (indexed by *WC*)
 - 1 Professional/Technical
 - 2 Managers/Administrators
 - 3 Sales
 - Blue collar occupations (indexed by BC)
 - 4 Clerical/Unskilled
 - 5 Craftsmen
 - 6 Operatives

- 7 Transport
- Other occupations (indexed by Other)
 - 8 Other

Specify the parameters such that there are only nest-level (rather than alternative-level) coefficients. That is, estimate a model with the following parameters:

- β_{WC}
- β_{BC}
- λ_{WC}
- λ_{BC}
- γ
- β_{Other} is normalized to 0
- The formula for the choice probabilities will thus be

$$P_{ij} = \begin{cases} \frac{\exp\left(\frac{X_i\beta_{WC} + \gamma(Z_{ij} - Z_{iJ})}{\lambda_{WC}}\right) \left[\sum_{\ell \in WC} \exp\left(\frac{X_i\beta_{WC} + \gamma(Z_{i\ell} - Z_{iJ})}{\lambda_{WC}}\right)\right]^{\lambda_{WC} - 1}}{1 + \left[\sum_{k \in WC} \exp\left(\frac{X_i\beta_{WC} + \gamma(Z_{ik} - Z_{iJ})}{\lambda_{WC}}\right)\right]^{\lambda_{WC}} + \left[\sum_{m \in BC} \exp\left(\frac{X_i\beta_{BC} + \gamma(Z_{im} - Z_{iJ})}{\lambda_{BC}}\right)\right]^{\lambda_{BC}}}, & j \in WC \\ \frac{\exp\left(\frac{X_i\beta_{BC} + \gamma(Z_{ij} - Z_{iJ})}{\lambda_{BC}}\right) \left[\sum_{\ell \in BC} \exp\left(\frac{X_i\beta_{BC} + \gamma(Z_{i\ell} - Z_{iJ})}{\lambda_{BC}}\right)\right]^{\lambda_{BC} - 1}}{1 + \left[\sum_{k \in WC} \exp\left(\frac{X_i\beta_{WC} + \gamma(Z_{ik} - Z_{iJ})}{\lambda_{WC}}\right)\right]^{\lambda_{WC}} + \left[\sum_{m \in BC} \exp\left(\frac{X_i\beta_{BC} + \gamma(Z_{im} - Z_{iJ})}{\lambda_{BC}}\right)\right]^{\lambda_{BC}}}, & j \in BC \\ \frac{1}{1 + \left[\sum_{k \in WC} \exp\left(\frac{X_i\beta_{WC} + \gamma(Z_{ik} - Z_{iJ})}{\lambda_{WC}}\right)\right]^{\lambda_{WC}} + \left[\sum_{m \in BC} \exp\left(\frac{X_i\beta_{BC} + \gamma(Z_{im} - Z_{iJ})}{\lambda_{BC}}\right)\right]^{\lambda_{BC}}}, & j = J \end{cases}$$

- 4. Wrap all of your code above into a function and then call that function at the very bottom of your script. Make sure you add println() statements after obtaining each set of estimates so that you can read them.
- 5. Have an AI write unit tests for each of the functions you've created (or components of each) and run them to verify that they work as expected. Best practice is to provide unit tests in a separate script that first reads in the source code before running the tests.