Finite mixture distributions offer an alternative to mixed logit models

Finite mixture distributions offer an alternative to mixed logit models

Assume mixing distribution is discrete with finite support, indep. other variables

Finite mixture distributions offer an alternative to mixed logit models

Assume mixing distribution is discrete with finite support, indep. other variables

Let π_s denote the population probability of being in the sth unobserved group

Finite mixture distributions offer an alternative to mixed logit models

Assume mixing distribution is discrete with finite support, indep. other variables

Let π_s denote the population probability of being in the sth unobserved group

The log likelihood integrates over unobserved groups:

Finite mixture distributions offer an alternative to mixed logit models

Assume mixing distribution is discrete with finite support, indep. other variables

Let π_s denote the population probability of being in the sth unobserved group

The log likelihood integrates over unobserved groups:

$$\ell\left(X, Z; \beta, \gamma, \pi\right) = \sum_{i=1}^{N} \log\left\{\sum_{s} \pi_s \prod_{j} \left[\frac{\exp\left(X_i\left(\beta_j - \beta_J\right) + \gamma_s\left(Z_{ij} - Z_{iJ}\right)\right)}{\sum_k \exp\left(X_i\left(\beta_k - \beta_J\right) + \gamma_s\left(Z_{ik} - Z_{iJ}\right)\right)}\right]^{d_{ij}}\right\}$$

Need panel data for identification

Need panel data for identification

Assume γ is stable over time

Need panel data for identification

Assume γ is stable over time

Finite mixture log likelihood:

$$\ell\left(X, Z; \beta, \gamma, \pi\right) = \sum_{i=1}^{N} \log \left\{ \sum_{s} \pi_s \prod_{t} \prod_{j} \left[\frac{\exp\left(X_{it}\left(\beta_j - \beta_J\right) + \gamma_s\left(Z_{ijt} - Z_{iJt}\right)\right)}{\sum_k \exp\left(X_{it}\left(\beta_k - \beta_J\right) + \gamma_s\left(Z_{ikt} - Z_{iJt}\right)\right)} \right]^{d_{ijt}} \right\}$$

Finite mixture log likelihood:

$$\ell\left(X, Z; \beta, \gamma, \pi\right) = \sum_{i=1}^{N} \log\left\{\sum_{s} \pi_s \prod_{t} \prod_{j} \left[\frac{\exp\left(X_{it}\left(\beta_j - \beta_J\right) + \gamma_s\left(Z_{ijt} - Z_{iJt}\right)\right)}{\sum_k \exp\left(X_{it}\left(\beta_k - \beta_J\right) + \gamma_s\left(Z_{ikt} - Z_{iJt}\right)\right)}\right]^{d_{ijt}}\right\}$$

Finite mixture log likelihood:

$$\ell\left(X, Z; \beta, \gamma, \pi\right) = \sum_{i=1}^{N} \log\left\{\sum_{s} \pi_s \prod_{t} \prod_{j} \left[\frac{\exp\left(X_{it}\left(\beta_j - \beta_J\right) + \gamma_s\left(Z_{ijt} - Z_{iJt}\right)\right)}{\sum_k \exp\left(X_{it}\left(\beta_k - \beta_J\right) + \gamma_s\left(Z_{ikt} - Z_{iJt}\right)\right)}\right]^{d_{ijt}}\right\}$$

Mixed logit panel data log likelihood:

$$\ell\left(X, Z; \beta, \gamma, \mu, \sigma\right) = \sum_{i=1}^{N} \log\left\{\int \prod_{t} \prod_{j} \left[\frac{\exp\left(X_{it}\left(\beta_j - \beta_J\right) + \gamma\left(Z_{ijt} - Z_{iJt}\right)\right)}{\sum_k \exp\left(X_{it}\left(\beta_k - \beta_J\right) + \gamma\left(Z_{ikt} - Z_{iJt}\right)\right)}\right]^{d_{ijt}} f\left(\gamma; \mu, \sigma\right) d\gamma\right\}$$

Finite mixture log likelihood:

$$\ell\left(X,Z;\beta,\gamma,\pi\right)=\sum_{i=1}^{N}\log\left\{\sum_{s}\pi_s\prod_{t}\prod_{j}\left[\frac{\exp\left(X_{it}\left(\beta_j-\beta_J\right)+\gamma_s\left(Z_{ijt}-Z_{iJt}\right)\right)}{\sum_k\exp\left(X_{it}\left(\beta_k-\beta_J\right)+\gamma_s\left(Z_{ikt}-Z_{iJt}\right)\right)}\right]^{d_{ijt}}\right\}$$

Mixed logit panel data log likelihood:

$$\ell\left(X,Z;\beta,\gamma,\mu,\sigma\right)=\sum_{i=1}^{N}\log\left\{\int\prod_{t}\prod_{j}\left[\frac{\exp\left(X_{it}\left(\beta_j-\beta_J\right)+\gamma\left(Z_{ijt}-Z_{iJt}\right)\right)}{\sum_k\exp\left(X_{it}\left(\beta_k-\beta_J\right)+\gamma\left(Z_{ikt}-Z_{iJt}\right)\right)}\right]^{d_{ijt}}f\left(\gamma;\mu,\sigma\right)d\gamma\right\}$$

Key differences:

Finite mixture log likelihood:

$$\ell\left(X,Z;\beta,\gamma,\pi\right)=\sum_{i=1}^{N}\log\left\{\sum_{s}\pi_{s}\prod_{t}\prod_{j}\left[\frac{\exp\left(X_{it}\left(\beta_{j}-\beta_{J}\right)+\gamma_{s}\left(Z_{ijt}-Z_{iJt}\right)\right)}{\sum_{k}\exp\left(X_{it}\left(\beta_{k}-\beta_{J}\right)+\gamma_{s}\left(Z_{ikt}-Z_{iJt}\right)\right)}\right]^{d_{ijt}}\right\}$$

Mixed logit panel data log likelihood:

$$\ell\left(X,Z;\beta,\gamma,\mu,\sigma\right)=\sum_{i=1}^{N}\log\left\{\int\prod_{t}\prod_{j}\left[\frac{\exp\left(X_{it}\left(\beta_{j}-\beta_{J}\right)+\gamma\left(Z_{ijt}-Z_{iJt}\right)\right)}{\sum_{k}\exp\left(X_{it}\left(\beta_{k}-\beta_{J}\right)+\gamma\left(Z_{ikt}-Z_{iJt}\right)\right)}\right]^{d_{ijt}}f\left(\gamma;\mu,\sigma\right)d\gamma\right\}$$

Key differences:

- discrete distribution allows for summation instead of integration

Finite mixture log likelihood:

$$\ell\left(X,Z;\beta,\gamma,\pi\right)=\sum_{i=1}^{N}\log\left\{\sum_{s}\pi_{s}\prod_{t}\prod_{j}\left[\frac{\exp\left(X_{it}\left(\beta_{j}-\beta_{J}\right)+\gamma_{s}\left(Z_{ijt}-Z_{iJt}\right)\right)}{\sum_{k}\exp\left(X_{it}\left(\beta_{k}-\beta_{J}\right)+\gamma_{s}\left(Z_{ikt}-Z_{iJt}\right)\right)}\right]^{d_{ijt}}\right\}$$

Mixed logit panel data log likelihood:

$$\ell\left(X,Z;\beta,\gamma,\mu,\sigma\right)=\sum_{i=1}^{N}\log\left\{\int\prod_{t}\prod_{j}\left[\frac{\exp\left(X_{it}\left(\beta_{j}-\beta_{J}\right)+\gamma\left(Z_{ijt}-Z_{iJt}\right)\right)}{\sum_{k}\exp\left(X_{it}\left(\beta_{k}-\beta_{J}\right)+\gamma\left(Z_{ikt}-Z_{iJt}\right)\right)}\right]^{d_{ijt}}f\left(\gamma;\mu,\sigma\right)d\gamma\right\}$$

Key differences:

- discrete distribution allows for summation instead of integration
- $f(\gamma;\mu,\sigma)d\gamma$ replaced with π_s