Key advantage of finite dependence: no need to make assumptions far into lifecycle
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Question: What remains identifiable?

With terminal or renewal choices, can identify f_{jt} 's and u_{jt} 's until period $\mathcal{T}-1$

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Extreme value case:

$$v_j(X_t) = u_{jt}(X_t) + \beta \sum_{X_{t+1}} \log \left(\sum_k \exp[v_{kt+1}(X_{t+1})] \right) f_{jt}(X_{t+1}|X_t) + \beta c$$

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 $= u_{jt}(X_t) + \beta \sum \left[v_{1t+1}(X_{t+1}) - \log \left(p_{1t+1}(X_{t+1}) \right) \right] f_{jt}(X_{t+1}|X_t) + \beta c$

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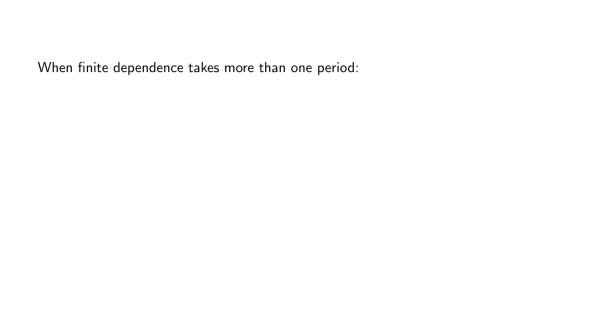
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Last term differences out or is constant

Expression holds regardless of expectations at t + 2



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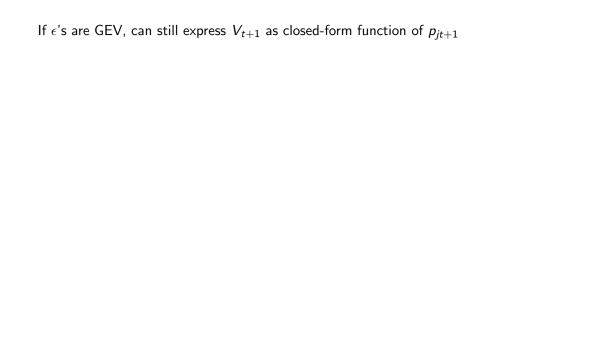
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- Utility function needs to be stable
- Reason: future flow payoffs appear in expectation of future utility, but we lack corresponding data to recover them
- Implication: possible to estimate non-stationary games



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- ullet No closed-form expression for V_{t+1}
- Need simulation to compute
 E max integral

$$G_t = \sum_r \left(\sum_{j \in J_r} \exp\left(\frac{v_j(X_t)}{\lambda_r}\right) \right)^{\lambda_r}$$

Probability of choosing
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$$p_{jt}(X_t) = rac{\left(\sum_{j' \in J_r} \exp\left(rac{v_{j'}(X_t)}{\lambda_r}
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Probability of choosing nest r:

$$p_{rt}(X_t) = \frac{\left(\sum_{j' \in J_r} \exp\left(\frac{v_{j'}(X_t)}{\lambda_r}\right)\right)^{\lambda_r}}{G_t}$$

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$$(G_t p_{rt}(X_t))^{1/\lambda_r} = \sum_{j' \in J_r} \exp\left(rac{v_{j'}(X_t)}{\lambda_r}
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$$\rho_{jt}(X_t) = G_t^{\frac{-1}{\lambda_r}} \rho_{rt}(X_t)^{\frac{\lambda_r-1}{\lambda_r}} \exp\left(\frac{v_j(X_t)}{\lambda_r}\right)$$

$$(1/\lambda_r)\log(G_t) = -\log(p_{it}(X_t)) + ((\lambda_r - 1)/\lambda_r)\log(p_{rt}(X_t)) + (1/\lambda_r)v_i(X_t)$$

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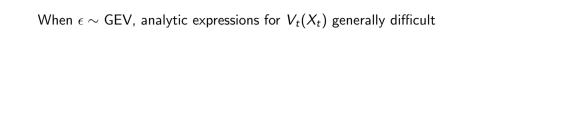
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Note: when $\lambda_r = 1$, this reduces to multinomial logit



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$$\partial V_j(X_t)$$

Recall:
$$p_{jt}(X_t) = \frac{\partial \log(G_t)}{\partial v_j(X_t)}$$

Stack for all choices:
$$\begin{bmatrix} p_{1t}(X_t) \\ \vdots \\ p_{Jt}(X_t) \end{bmatrix} = \begin{bmatrix} \frac{\partial \log(G_t)}{\partial v_{1t}(X_t)} \\ \vdots \\ \frac{\partial \log(G_t)}{\partial v_{1t}(X_t)} \end{bmatrix}$$

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One equation redundant $\Rightarrow J-1$ system solving for J-1 differences in v's