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Then the logit choice probabilities become:

$$P_{ij}\left(X, Z; \beta, \mu, \sigma\right) = \int \frac{\exp\left(X_i\left(\beta_j - \beta_J\right) + \gamma_i\left(Z_{ij} - Z_{iJ}\right)\right)}{\sum_k \exp\left(X_i\left(\beta_k - \beta_J\right) + \gamma_i\left(Z_{ik} - Z_{iJ}\right)\right)} f\left(\gamma_i; \mu, \sigma\right) d\gamma_i$$

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Annoyance: the log likelihood now has an integral inside the log!

$$\ell(X, Z; \beta, \gamma, \mu, \sigma) = \sum_{i=1}^{N} \log \left\{ \int \prod_{j} \left[\frac{\exp\left(X_i\left(\beta_j - \beta_J\right) + \gamma\left(Z_{ij} - Z_{iJ}\right)\right)}{\sum_k \exp\left(X_i\left(\beta_k - \beta_J\right) + \gamma\left(Z_{ik} - Z_{iJ}\right)\right)} \right]^{d_{ij}} f(\gamma; \mu, \sigma)\, d\gamma \right\}$$

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Can't switch log and integral because of properties of logarithms

Common mixing distributions:

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- Normal

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  - This would allow, e.g., heterogeneity in γ to be correlated with β

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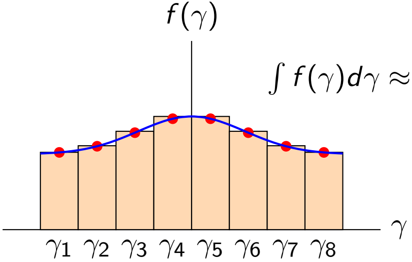
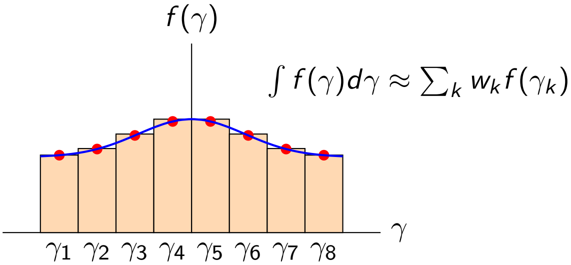
Choice is often determined by specifics of the problem at hand (e.g. dimensionality)

Riemann Sum vs. Quadrature



$$\int f(\gamma) d\gamma \approx \sum_k w_k f(\gamma_k)$$

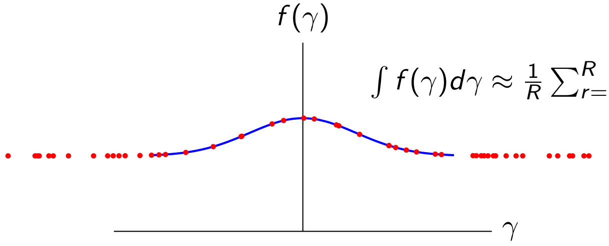
Riemann Sum vs. Quadrature



Riemann sum: $w_k = \Delta\gamma$ for all k (uniform widths)

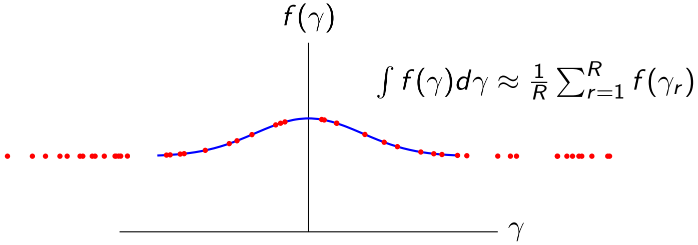
Quadrature: optimal weights w_k (not uniform, chosen for accuracy)

Monte Carlo Integration



$$\int f(\gamma)d\gamma \approx \frac{1}{R}\sum_{r=1}^{R} f(\gamma_r)$$

Monte Carlo Integration



Monte Carlo: uniform weights $w_r = \frac{1}{R}$ for all random draws

Draw R random values γ_r from $F(\cdot)$, take average of function values

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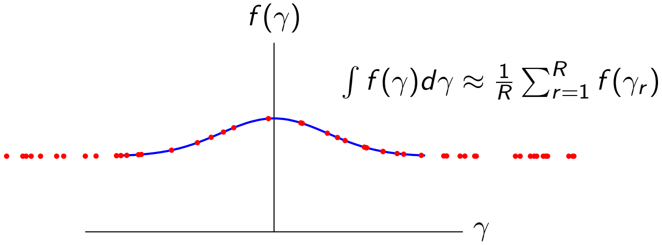

Monte Carlo: uniform weights $w_r = \frac{1}{R}$ for all random draws

Draw R random values γ_r from $F(\cdot)$, take average of function values

R usually needs to be very large (min. 10,000)