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Another alternative: Indirect Inference

Uses an auxiliary model as a lens through which to view the world

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Objective: choose parameters of structural model such that:

simulated data = real data *through the lens of the auxiliary model*

Revisit Rust (1987) bus engine model:

State: mileage X_t

Flow payoffs:

$$u(X_t, d_t, \theta) = \begin{cases} -c(X_t, \theta) & \text{if } d_t = 0 \text{ (keep)} \\ -RC & \text{if } d_t = 1 \text{ (replace)} \end{cases}$$

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Bellman equation:

$$V(X_t; \theta) = \max_{d_t} \left\{ u(X_t, d_t; \theta) + \beta \mathbb{E}[V(X_{t+1}; \theta) | X_t, d_t] \right\}$$

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- Robust to auxiliary model misspecification

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Example: Logit for replacement probability

$$P(d_t = 1|X_t) = \frac{\exp(\alpha_0 + \alpha_1 X_t)}{1 + \exp(\alpha_0 + \alpha_1 X_t)}$$

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Estimate on real data: $\hat{\alpha} = (\hat{\alpha}_0, \hat{\alpha}_1)$

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- Simulate S bus histories using policy and transitions
- Each simulation: $\{X_t^s, d_t^s\}_{t=1}^T$ for $s = 1, \ldots, S$

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No closed form! Must simulate for each candidate θ

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$$\hat{\theta} = \arg\min_{\theta} \left[\hat{\alpha} - \tilde{\alpha}(\theta)\right]' W \left[\hat{\alpha} - \tilde{\alpha}(\theta)\right]$$

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where W is a weighting matrix (often $\widehat{Var}(\hat{\alpha})^{-1}$)

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If structural model is correct:

- Real data generated by true θ_0
- Simulated data from θ_0 should have same auxiliary statistics
- $\hat{\alpha} \approx \tilde{\alpha}(\theta_0)$ when we find the right θ_0

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Stack all into vector α, estimate $\hat{\alpha}$ and $\tilde{\alpha}(\theta)$

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- Intuitive: match reduced-form patterns (2SLS, DiD, ...)

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Large S reduces simulation noise in $\bar{\tilde{\alpha}}(\theta)$; large N improves precision of each $\tilde{\alpha}_s(\theta)$