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For multinomial logit, this has a closed form:

$$V_i = \log \left(\sum_{j=1}^{J} \exp(u_{ij}) \right) + C$$

where C is Euler's constant (≈ 0.57721)

Alternative expression for expected utility:

$$V_i = \log \left(\sum_{j=1}^{J} \frac{\exp(u_{iJ}) \exp(u_{ij})}{\exp(u_{iJ})} \right) + C$$

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This representation becomes useful for dynamic discrete choice models

Consumer surplus measures utility in dollar terms

Suppose utility is: $u_{ij} = \beta_j X_i + \gamma Z_j - \delta p_j$

The price coefficient δ provides utils-to-dollars conversion

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Under logit assumptions:

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- Euler's constant C cancels out in the difference
- Number of alternatives J can change (e.g., new transport mode added)