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Recursive formulation helps keep notation compact

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$$\mathbb{E}\max = \log\left(\sum_k \exp\left(u_{ikT}\right)\right) + \underbrace{c}_{\text{Euler's constant}}$$

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- $\mathbb{E}\max$ term has a closed form

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Treat the v's like we would the u's in a multinomial logit model

When might dynamics not matter? Let's walk through location normalization of utility

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Easiest way to satisfy this is switching costs

Intuition: switching costs make agents consider future consequences