## Dynamic Selection Problem

Link choices to outcomes:

 $\bullet \ \ \mathsf{Labor} \ \mathsf{force} \ \mathsf{participation} \ \to \mathsf{earnings}$ 

 $\bullet \ \ \mathsf{Market} \ \mathsf{entry} \to \mathsf{profits}$ 

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Assumption: Selection bias eliminated once we control for unobserved type

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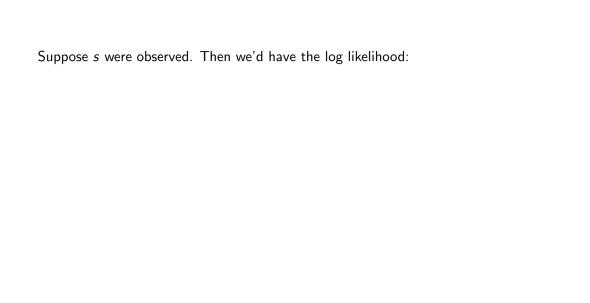
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Likelihood separable conditional on type s, i.e.  $Y_{1t} \perp Y_{2t} | X_{1t}, s$ 

This allows us to move to the second equality above



Suppose s were observed. Then we'd have the log likelihood:

$$\ell = \sum_{i} \sum_{t} \left\{ \ell_{1}(Y_{1t}|X_{1t}, \alpha_{1}, s) + \ell_{2}(Y_{2t}|X_{2t}, \alpha_{2}, s) \right\}$$

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Two-stage estimation:

- 1. Estimate  $\alpha_2$  using only  $\ell_2$
- 2. Estimate  $\alpha_1$  using  $\ell_1$  (taking  $\hat{\alpha}_2$  as given, since  $\ell_1$  might depend on  $\alpha_2$  implicitly)

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Selection bias eliminated once conditioning on s and  $X_{1t}$ 

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Implication: Cannot estimate parts separately

Although conditional independence simplified things a bit

Resolving selection problem requires joint estimation of both components