

Individual i's flow utility for option j at time t is:

$$U_{ijt} = u_{ijt} + \epsilon_{ijt}$$
$$= X_{it}\alpha_i + \epsilon_{ijt}$$

$$\max_{\{d_{i\tau}\}_{\tau=t}^{T}} \mathbb{E} \left\{ \sum_{\tau=t}^{T} \sum_{j=1}^{J} \beta^{\tau-t} \mathbb{1} \left[d_{i\tau} = j \right] U_{ij\tau} \left(X_{i\tau}, \epsilon_{ij\tau} \right) \right\}$$

$$(1)$$

Let's break down each component:

$$\max_{\left\{d_{i\tau}\right\}_{\tau=t}^{T}} \mathbb{E}\left\{\sum_{\tau=t}^{T} \sum_{j=1}^{J} \beta^{\tau-t} \mathbb{1}\left[d_{i\tau}=j\right] U_{ij\tau}\left(X_{i\tau}, \epsilon_{ij\tau}\right)\right\}$$
(1)

 $\{d_{i\tau}\}_{\tau=t}^T$: Sequence of choices from period t to terminal period T

$$\max_{\{d_{i\tau}\}_{\tau=t}^{T}} \mathbb{E} \left\{ \sum_{\tau=t}^{T} \sum_{j=1}^{J} \beta^{\tau-t} \mathbb{1} \left[d_{i\tau} = j \right] U_{ij\tau} \left(X_{i\tau}, \epsilon_{ij\tau} \right) \right\}$$

$$(1)$$

 \mathbb{E} : Expectation over future utility shocks $\epsilon_{ij\tau}$ and/or states $X_{i\tau}$ for $\tau > t$

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$$\sum$$
: Sum over all time periods from current (t) to terminal (T)

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(1)

$$\sum$$
: Sum over all choice alternatives j

$$\max_{\{d_{i\tau}\}_{\tau=t}^{T}} \mathbb{E}\left\{\sum_{\tau=t}^{T} \sum_{j=1}^{J} \beta^{\tau-t} 1 \left[d_{i\tau} = j\right] U_{ij\tau}\left(X_{i\tau}, \epsilon_{ij\tau}\right)\right\}$$

$$(1)$$

 $\beta^{\tau-t}$: Discount factor for utility received in period τ

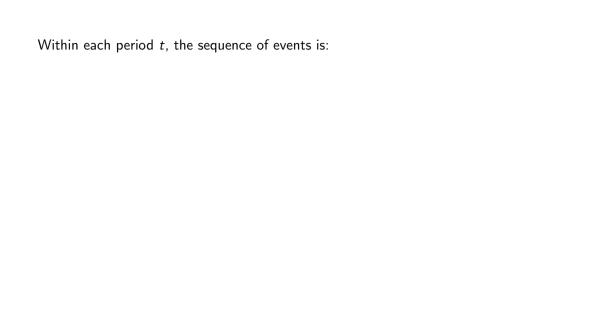
(captures time preference and impatience; i.e. future utility is worth less than present)

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 $1[d_{i\tau}=j]$: Indicator function equals 1 if alternative j is chosen at time τ

$$\max_{\{d_{i\tau}\}_{\tau=t}^{T}} \mathbb{E} \left\{ \sum_{\tau=t}^{T} \sum_{j=1}^{J} \beta^{\tau-t} \mathbb{1} \left[d_{i\tau} = j \right] \left[U_{ij\tau} \left(X_{i\tau}, \epsilon_{ij\tau} \right) \right] \right\}$$
 (1)

$$U_{ij\tau}(X_{i\tau}, \epsilon_{ij\tau})$$
: Flow utility from choice j at time τ



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Key insight: Individual knows current ϵ 's but must form expectations over future ϵ 's



Individual *i* takes expectations over:

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Key assumptions:

- ϵ 's are i.i.d. over time
- ullet Future states are not affected by ϵ 's except through choices

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Future states are not affected by ϵ 's except through history of choices:

$$\mathbb{E}(X_{t+1}|d_t,\ldots,d_1,\epsilon_t,\ldots,\epsilon_1) = \mathbb{E}(X_{t+1}|d_t,\ldots,d_1)$$

i.e., ϵ 's do not directly influence the evolution of X's beyond their effect on d

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Assume mood 2 semesters ago doesn't affect transcript (except through courses taken)