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Independence of Irrelevant Alternatives (IIA)

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$$\frac{P_{ij}}{P_{ik}} = \frac{e^{u_{ij}} / \sum_{m} e^{u_{im}}}{e^{u_{ik}} / \sum_{m} e^{u_{im}}}$$
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$$= e^{u_{ij} - u_{ik}}$$

Economic meaning: All alternatives are equally substitutable

$$P_{ij} = \frac{\exp(u_{ij})}{\sum_{k=1}^{J} \exp(u_{ik})}$$

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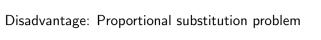
$$\ell(eta, \gamma | d_i \in \{1, \dots, K\}) = \sum_{i=1}^N \left[\sum_{j=1}^{K-1} (d_{ij} = 1)(u_{ij} - u_{iK}) \right] - \log \left(1 + \sum_{i=1}^K \exp(u_{ik} - u_{iK}) \right)$$

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Practical use: Oversample rare choices without bias



Suppose there are two ways for someone to go to work: Car or Blue Bus

Initial shares: Car 50%, Blue bus 50%

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Reality: Car 50%, Blue bus 25%, Red bus 25%

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Problem: IIA assumes car and buses equally substitutable

Suppose people drive 1 of 3 types of cars and there's a subsidy for electric cars:

- Large gas cars: $66\% \rightarrow ?$
- Small gas cars: $33\% \rightarrow ?$
- Electric cars: $1\% \rightarrow 10\%$

Suppose people drive 1 of 3 types of cars and there's a subsidy for electric cars:

- Large gas cars: $66\% \rightarrow 60\%$
- Small gas cars: $33\% \rightarrow 30\%$
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Logit assumption: Large and small gas cars lose share proportionally

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Policy implication: Overestimates gas savings from subsidy