

Start with the research question and data structure

Random Coefficients: Use when heterogeneity is continuous

Willingness-to-pay distributions

• Individual-specific elasticities

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- Individual-specific elasticities

Finite Mixtures: Use when heterogeneity is discrete

- Market segmentation
- Behavioral types (e.g., price-sensitive vs quality-focused)

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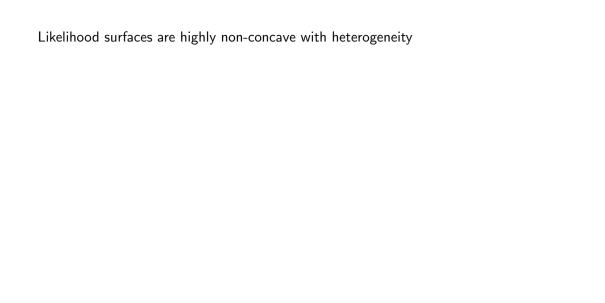
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Does the economic theory suggest continuous or discrete heterogeneity?



# Likelihood surfaces are highly non-concave with heterogeneity

### Solutions:

- Multiple random starting values (minimum 50-100)
- Grid search over key parameters
- Use simpler model estimates as starting values

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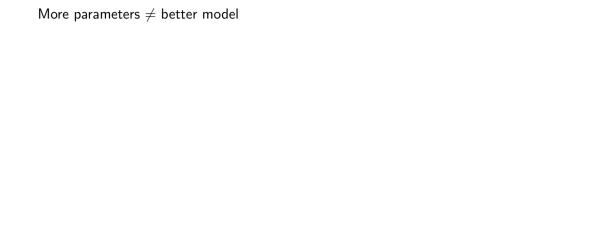
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Diagnostic: Compare final likelihood across different starts

If estimates vary dramatically, increase number of starting values or simplify specification



More parameters  $\neq$  better model

Warning Signs:

- ullet Very small estimated mixing probabilities (< 0.05)
- Nearly identical parameter vectors across types
- Unstable estimates across samples

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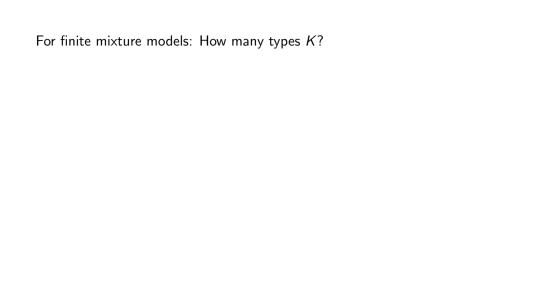
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Start simple and build complexity gradually



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Choose K to minimize: BIC =  $-2\mathcal{L} + k \ln(n)$ 

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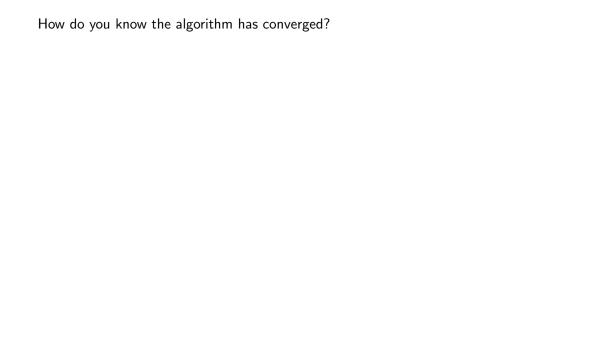
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# Economic Approach:

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Practical constraint: Rarely use more than 4-5 types in practice



How do you know the algorithm has converged?

Standard criterion:

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Be suspicious of "convergence" after very few iterations

#### **Equilibrium Trade in Automobiles**

### Kenneth Gillingham

Yale University

### Fedor Iskhakov

Australian National University

### Anders Munk-Nielsen

University of Copenhagen

### John Rust

Georgetown University

### Bertel Schjerning

University of Copenhagen

We introduce a computationally tractable dynamic equilibrium model of automobile markets with heterogeneous consumers, focused on state of automobile markets with heterogeneous consumers, focused on state to incary flow equilibria. We introduce a fast, robust algorithm for computing equilibria and use it to estimate a model using nearly 50 million model first the data well, and counterfactual simulations from Denmark. The estimated model first the data well, and counterfactual simulations show that Denmark could raise total tax revenue by reducing the newcar registration star rate. We show that we can great the star are so that are taken the single the star are on fuel increases aggregate welfare, tax revenue, and car ownership, while reducing car age, divining, and CO, emissions.

Electronically published August 11, 2022

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### Approach:

- 8 (observable) household types, 4 car types, 25 age categories
  - Nested Logit GEV
  - Complicated dynamic market equilibrium

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Policy insight: Current registration tax above Laffer curve peak