We want errors for the red bus and blue bus to be correlated.

Nested logit allows a nest-specific error:

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where

- $\nu_{ik} + \lambda\epsilon_{ij}$ is distributed T1EV (but not i.i.d.!)
- $\epsilon_{ij} \overset{\text{iid}}{\sim}$ T1EV
- $\nu_{ik} \sim C(\lambda)$: conjugate distribution to T1EV, from Cardell (1997)

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  (standard multinomial logit would predict 33.3/33.3/33.3)

Choice probabilities are the key object:

$$P_{i,\mathsf{C}} = \frac{\exp(u_{i,\mathsf{C}})}{\left[\exp(u_{i,\mathsf{RB}}/\lambda) + \exp(u_{i,\mathsf{BB}}/\lambda)\right]^{\lambda} + \exp(u_{i,\mathsf{C}})}$$

$$P_{i,\mathsf{RB}} = \frac{\exp(u_{i,\mathsf{RB}}/\lambda)\left[\exp(u_{i,\mathsf{RB}}/\lambda) + \exp(u_{i,\mathsf{BB}}/\lambda)\right]^{\lambda-1}}{\left[\exp(u_{i,\mathsf{RB}}/\lambda) + \exp(u_{i,\mathsf{BB}}/\lambda)\right]^{\lambda} + \exp(u_{i,\mathsf{C}})}$$

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At the top level, nest probabilities take a logit form:

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Note: $\lambda = 1 \Rightarrow$ Multinomial Logit (no correlation); $\lambda \to 0 \Rightarrow$ Perfect correlation