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For observed state (x, s), estimate probability of action j:

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Use these probabilities to construct expected future values

Leverage finite dependence, renewal, or terminality to avoid solving value functions

p_{jt}'s inform us about the future value of taking certain actions at certain states

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This arises routinely in DDC models:

- Dynamic selection
- Serially correlated unobservables

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CONDITIONAL CHOICE PROBABILITY ESTIMATION OF DYNAMIC DISCRETE CHOICE MODELS WITH UNOBSERVED HETEROGENEITY

BY PETER ARCIDIACONO AND ROBERT A. MILLER[1]

We adapt the expectation–maximization algorithm to incorporate unobserved heterogeneity into conditional choice probability (CCP) estimators of dynamic discrete choice problems. The unobserved heterogeneity can be time-invariant or follow a Markov chain. By developing a class of problems where the difference in future value terms depends on a few conditional choice probabilities, we extend the class of dynamic optimization problems where CCP estimators provide a computationally cheap alternative to full solution methods. Monte Carlo results confirm that our algorithms perform quite well, both in terms of computational time and in the precision of the parameter estimates.

KEYWORDS: Dynamic discrete choice, unobserved heterogeneity.

1. INTRODUCTION

STANDARD METHODS FOR SOLVING dynamic discrete choice models involve calculating the value function either through backward recursion (finite time) or through the use of a fixed point algorithm (infinite time).[2] Conditional choice probability (CCP) estimators, originally proposed by Hotz and Miller (1993), provide an alternative to these computationally intensive procedures by exploiting the mappings from the value functions to the probabilities of making particular decisions. CCP estimators are much easier to compute than full solution methods and have experienced a resurgence in the literature on estimating dynamic games.[3] The computational gains associated with CCP estimation give researchers considerable latitude to explore different functional forms for their models.

[1]We thank Victor Aguirregabiria, Esteban Aucejo, Lanier Benkard, Jason Blevins, Paul Ellickson, George-Levi Gayle, Joe Hotz, Pedro Mira, three anonymous referees, and the co-editor for their comments. We have benefited from seminars at UC Berkeley, Duke University, University College London, University of North Carolina, Northwestern University, The Ohio State University, University of Pennsylvania, University of Rochester, Stanford University, University of Texas, Vanderbilt University, University of Virginia, Washington University, University of Wisconsin, IZA, Microeconometrics Conferences at the Cowles Foundation, the MOVE Conference at Universitat Autònoma de Barcelona, and the NASM of the Econometric Society. Andrew Beauchamp, Jon James, and Josh Kinsler provided excellent research assistance. Financial support was provided for by NSF Grants SES-0721059 and SES-0721098.

[2]The full solution or nested fixed point approach for discrete dynamic models was developed by Miller (1984), Pakes (1986), Rust (1987), and Wolpin(1984), and further refined by Keane and Wolpin (1994, 1997).

[3]Aguirregabiria and Mira (2010) recently surveyed the literature on estimating dynamic models of discrete choice. For developments of CCP estimators that apply to dynamic games, see Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), Jofre-Bonet and Pesendorfer (2003), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008).

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Solution: Assume S is finite and use EM algorithm

Assume R unobserved types with probabilities π_s

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Assume R unobserved types with probabilities π_s

Key insight: Use posterior probability q_{is} as weight

$$q_{is} = \frac{\pi_s \prod_{t=1}^{T} \mathcal{L}_{ist}(\theta_s)}{\sum_{s'=1}^{R} \pi_{s'} \prod_{t=1}^{T} \mathcal{L}_{is't}(\theta_{s'})}$$

q_{is} = probability individual i is type s given observed choices

Weighted CCP estimator replaces indicator with posterior type probability:

$$p_{jt}(x, s) = \frac{\sum_i d_{ijt}\, q_{is}\, 1[X_{it} = x]}{\sum_i\, q_{is}\, 1[X_{it} = x]}$$

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Now treats unobserved type as observed (with weights)

Can estimate CCPs conditional on unobserved states

EM Algorithm for CCP estimation at iteration m:

E-step: Given θ^m, π^m, p^m, compute

$$q_{is}^{m+1} = \frac{\pi_s^m \prod_{t=1}^{T} \mathcal{L}_{ist}(\theta^m, p^m)}{\sum_{s'=1}^{R} \pi_{s'}^m \prod_{t=1}^{T} \mathcal{L}_{is't}(\theta^m, p^m)}$$

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M-step: Given q^{m+1}, update

$$p_{jt}^{m+1}(x, s) = \frac{\sum_i d_{ijt} q_{is}^{m+1} 1[X_{it} = x]}{\sum_i q_{is}^{m+1} 1[X_{it} = x]}$$

$$\theta^{m+1} = \arg\max_{\theta} \sum_i \sum_s \sum_t q_{is}^{m+1} \log \mathcal{L}_{ist}(\theta, p^{m+1})$$

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- Maximization step treats types as observed
- Maintains additive separability for sequential estimation
- Computational tractability despite unobserved heterogeneity

Alternative CCP update: Use structural model

Instead of weighted empirical frequencies, use likelihood:

$$p_{jt}^{m+1}(x, s) = \ell_j(x, s; \theta^m, \pi^m, p^m)$$

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Where ℓ_j comes from choice probabilities implied by model

For example, with T1EV errors:

$$p_{jt}(x,s) = \frac{\exp(v_j(x,s))}{\sum_k \exp(v_k(x,s))}$$

Both methods converge to same estimates