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Another alternative: Indirect Inference

Uses an auxiliary model as a lens through which to view the world

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Objective: choose parameters of structural model such that:

simulated data = real data through the lens of the auxiliary model

Revisit Rust (1987) bus engine model:

State: mileage X_t

Flow payoffs:

$$u(X_t, d_t, heta) = \left\{ egin{array}{ll} -c(X_t, heta) & ext{if } d_t = 0 ext{ (keep)} \ -RC & ext{if } d_t = 1 ext{ (replace)} \end{array}
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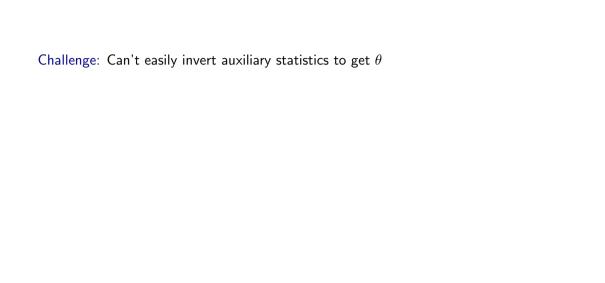
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Bellman equation:

$$V(X_t; \theta) = \max_{d_t} \left\{ u(X_t, d_t; \theta) + \beta \mathbb{E}[V(X_{t+1}; \theta) | X_t, d_t] \right\}$$





Challenge: Can't easily invert auxiliary statistics to get θ

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Example: Logit for replacement probability

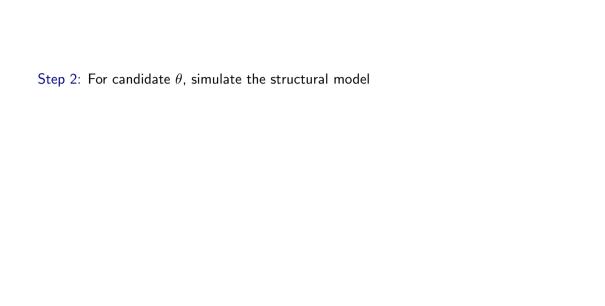
$$P(d_t = 1|X_t) = \frac{\exp(\alpha_0 + \alpha_1 X_t)}{1 + \exp(\alpha_0 + \alpha_1 X_t)}$$

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$$P(d_t = 1|X_t) = \frac{\exp(\alpha_0 + \alpha_1 X_t)}{1 + \exp(\alpha_0 + \alpha_1 X_t)}$$

Estimate on real data: $\hat{\alpha} = (\hat{\alpha}_0, \hat{\alpha}_1)$



Step 2: For candidate θ , simulate the structural model

• Solve dynamic program to get $V(x; \theta)$ and policy $Pr(d^* = 1|x; \theta)$

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- Solve dynamic program to get $V(x;\theta)$ and policy $\Pr(d^*=1|x;\theta)$
- Simulate S bus histories using policy and transitions
- Each simulation: $\{X_t^s, d_t^s\}_{t=1}^T$ for s = 1, ..., S

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No closed form! Must simulate for each candidate $\boldsymbol{\theta}$

Step 4: Minimize distance between auxiliary parameters in real and simulated data

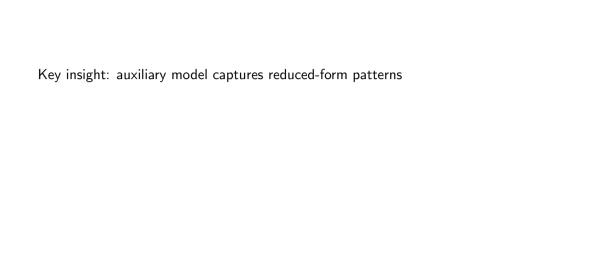
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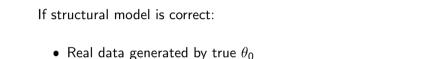
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where W is a weighting matrix (often $\widehat{Var}(\hat{\alpha})^{-1}$)



Key insight: auxiliary model captures reduced-form patterns
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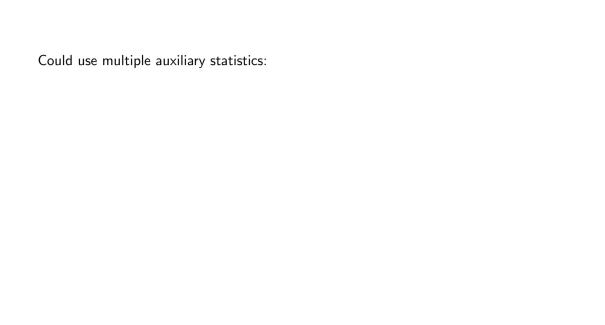
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- Real data generated by true θ_0
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Key insight: auxiliary model captures reduced-form patterns

If structural model is correct:

- Real data generated by true θ_0
- \bullet Simulated data from θ_0 should have same auxiliary statistics
- $\hat{lpha} pprox ilde{lpha}(heta_0)$ when we find the right $heta_0$



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Stack all into vector α , estimate $\hat{\alpha}$ and $\tilde{\alpha}(\theta)$

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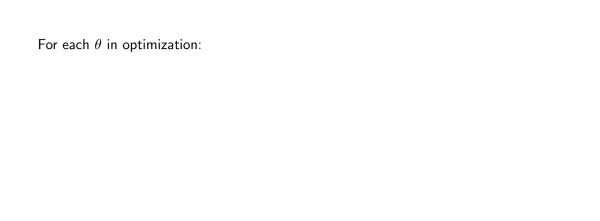
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- Intuitive: match reduced-form patterns (2SLS, DiD, ...)
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Large S reduces simulation noise in $\bar{\alpha}(\theta)$; large N improves precision of each $\tilde{\alpha}_s(\theta)$