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$$u_{ij} - u_{iJ} = (\beta_j - \beta_J)X_i + \gamma(Z_j - Z_J) \quad (1)$$

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We observe  $X_i, Z_1, \dots, Z_J$ , and  $d_i$

The likelihood of choosing  $j$  and  $J$  respectively is:

$$P_{ij} = \frac{\exp(u_{ij} - u_{iJ})}{1 + \sum_{k=1}^{J-1} \exp(u_{ik} - u_{iJ})}$$

$$P_{iJ} = \frac{1}{1 + \sum_{k=1}^{J-1} \exp(u_{ik} - u_{iJ})}$$

The log likelihood function we maximize is:

$$\ell(X, Z, d; \beta, \gamma) = \sum_{i=1}^N \left[ \sum_{j=1}^{J-1} (d_{ij} = 1)(u_{ij} - u_{iJ}) \right] - \log \left( 1 + \sum_{k=1}^{J-1} \exp(u_{ik} - u_{iJ}) \right)$$

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$$\begin{aligned}\ell(X, Z, d; \beta, \gamma) &= \sum_{i=1}^N \left[ \sum_{j=1}^{J-1} (d_{ij} = 1)(u_{ij} - u_{iJ}) \right] - \\ &\quad \log \left( 1 + \sum_{k=1}^{J-1} \exp(u_{ik} - u_{iJ}) \right) \\ &= \sum_{i=1}^N \sum_{j=1}^J d_{ij} \log(P_{ij})\end{aligned}$$