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Auxiliary statistics (7 moments):

$$\alpha = \begin{pmatrix} E[y] \\ E[x_1] \\ E[x_2] \\ \mathrm{Cov}(y, x_1) \\ \mathrm{Cov}(y, x_2) \\ \mathrm{Cov}(x_1, x_2) \\ \mathrm{Var}(y) \end{pmatrix}$$

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Note that we obtain our estimates indirectly using moments of the data