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$$U_{ijt} = u_{ijt} + \epsilon_{ijt}$$

$$= X_{it}\alpha_j + \epsilon_{ijt}$$

Individual  $i$  chooses sequence  $\{d_{i\tau}\}_{\tau=t}^T$  to maximize her **expected lifetime utility**:

$$\max_{\{d_{i\tau}\}_{\tau=t}^T} \mathbb{E} \left\{ \sum_{\tau=t}^T \sum_{j=1}^J \beta^{\tau-t} \mathbf{1}[d_{i\tau} = j] U_{ij\tau}(X_{i\tau}, \epsilon_{ij\tau}) \right\} \quad (1)$$

Let's break down each component:

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$\{d_{i\tau}\}_{\tau=t}^T$ : Sequence of choices from period  $t$  to terminal period  $T$

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$\mathbb{E}$ : Expectation over future utility shocks  $\epsilon_{ij\tau}$  and/or states  $X_{i\tau}$  for  $\tau > t$

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$\sum_{\tau=t}^T$ : Sum over all time periods from current ( $t$ ) to terminal ( $T$ )

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$\sum_{j=1}^J$ : Sum over all choice alternatives  $j$

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$\beta^{\tau-t}$ : Discount factor for utility received in period  $\tau$

(captures time preference and impatience; i.e. future utility is worth less than present)



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$\mathbf{1}[d_{i\tau} = j]$ : Indicator function equals 1 if alternative  $j$  is chosen at time  $\tau$

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$U_{ij\tau}(X_{i\tau}, \epsilon_{ij\tau})$ : Flow utility from choice  $j$  at time  $\tau$

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Key insight: Individual knows current  $\epsilon$ 's but must form expectations over future  $\epsilon$ 's

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Key assumptions:

- $\epsilon$ 's are i.i.d. over time
- Future states are not affected by  $\epsilon$ 's except through choices

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Future states are not affected by  $\epsilon$ 's except through history of choices:

$$\mathbb{E}(X_{t+1}|d_t, \dots, d_1, \epsilon_t, \dots, \epsilon_1) = \mathbb{E}(X_{t+1}|d_t, \dots, d_1)$$

i.e.,  $\epsilon$ 's do not directly influence the evolution of  $X$ 's beyond their effect on  $d$



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Assume mood 2 semesters ago doesn't affect transcript (except through courses taken)