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Economic meaning: All alternatives are equally substitutable

Advantage of IIA: Enables choice-based sampling

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Practical use: Oversample rare choices without bias

Disadvantage: Proportional substitution problem

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Reality: Car 50%, Blue bus 25%, Red bus 25%

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Problem: IIA assumes car and buses equally substitutable

Economic significance

Suppose people drive 1 of 3 types of cars and there's a subsidy for electric cars:

- Large gas cars: 66% $\rightarrow$?
- Small gas cars: 33% $\rightarrow$?
- Electric cars: 1% $\rightarrow$ 10%

Economic significance

Suppose people drive 1 of 3 types of cars and there's a subsidy for electric cars:

- Large gas cars: 66% $\rightarrow$ 60%
- Small gas cars: 33% $\rightarrow$ 30%
- Electric cars: 1% $\rightarrow$ 10%

Logit assumption: Large and small gas cars lose share proportionally

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Policy implication: Overestimates gas savings from subsidy