Bellman Equation for dynamic discrete choice:

$$V_{it} = \mathbb{E} \max_{j \in \mathcal{J}} \{ u_{ijt} + \beta V_{it+1}(X_{it+1}) + \epsilon_{ijt} \}$$

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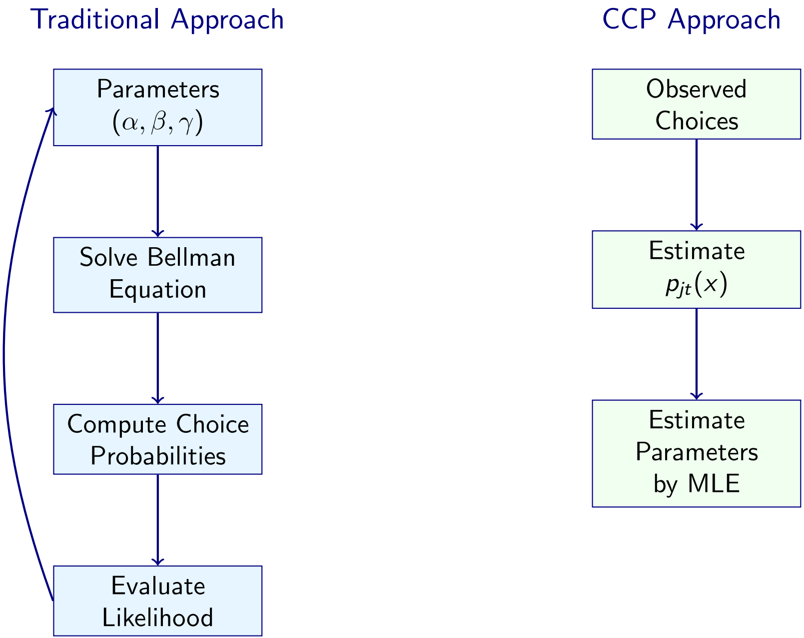
5. Repeat steps 2–5 until convergence


Traditional Approach
Parameters
(α, β, γ)
Solve Bellman
Equation
Compute Choice
Probabilities
Evaluate
Likelihood
CCP Approach
Observed
Choices
Estimate
$p_{jt}(x)$
Estimate
Parameters
by MLE

$$\log\left(\frac{p_{1t+1}}{p_{0t+1}}\right) = v_{1t+1} - v_{0t+1}$$

Example: Bus maintenance decisions

Raw Data

Mileage	Choice
50,000	Keep
75,000	Keep
120,000	Replace
140,000	Replace
$\vdots$	$\vdots$

Choice Probabilities

Mileage	P(Replace)
50,000	0.05
75,000	0.15
100,000	0.45
125,000	0.80
150,000	0.95

No structural assumptions needed yet!

$$V_{t+1} = v_{jt+1} - \ln p_{jt+1} + c$$

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$$\boxed{v_{jt+1}} = u_{jt+1} + \beta \mathbb{E}\left[V_{t+2} | d_{t+1} = j\right]$$