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For observed state (x, s), estimate probability of action j:

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Use these probabilities to construct expected future values

Leverage finite dependence, renewal, or terminality to avoid solving value functions

 p_{jt} 's inform us about the future value of taking certain actions at certain states

Cannot compute $p_{jt}(x, s)$ when s unobserved

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This arises routinely in DDC models:

Cannot compute $p_{it}(x, s)$ when s unobserved

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Dynamic selection

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This arises routinely in DDC models:

- Dynamic selection
- Serially correlated unobservables

CONDITIONAL CHOICE PROBABILITY ESTIMATION OF DYNAMIC DISCRETE CHOICE MODELS WITH UNORSERVED HETEROGENEITY

By Peter Arcidiacono and Robert A. Miller¹

We adapt the expectation-maximization algorithm to incorporate unobserved heterogeneity into conditional choice probability (CCP) estimators of dynamic discrete choice problems. The unobserved heterogeneity can be time-invariant or follow a Markov chain. by developing as dass of problems where the difference in future value terms depends on a few conditional choice probabilities, we extend the class of dynamic optimization problems where CCP estimators provide a computationally cheen where CCP estimators provide a computationally cheen alternative to full solution methods. Monte Carlo results confirm that our algorithms perform quiet well, both in terms of computational time and in the precision of the parameter

KEYWORDS: Dynamic discrete choice, unobserved heterogeneity.

1. INTRODUCTION

STANDARD METHODS FOR SOLVING dynamic discrete choice models involve calculating the value function citient through backward recursion (finite time) or through the use of a fixed point algorithm (infinite time). Conditional choice probability (CCP) estimators, originally proposed by Hotz and Miller (1993), provide an alternative to these computationally intensive procedures by exploiting the mappings from the value functions to the probabilities of making particular decisions. CCP estimators are much easier to compute than full solution methods and have experienced a resugrence in the literature on estimating dynamic games. The computational gains associated with CCP estimation give researchers considerable lattitude to explore different functional

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²The full solution or nested fixed point approach for discrete dynamic models was developed by Miller (1984), Pakes (1986), Rust (1987), and Wolpin(1984), and further refined by Keane and Wolpin (1994, 1997).

³ Aguirregabiria and Mira (2010) recently surveyed the literature on estimating dynamic models of discrete choice. For developments of CCP estimators that apply to dynamic games, see Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), Jofre-Bonet and Pesendorfer (2003). Pages Ostrowsky, and Berry (2007) and Pesendorfer and Schmidt, Depoler (2008).

Solution: Assume S is finite and use EM algorithm

Assume R unobserved types with probabilities π_s

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Assume R unobserved types with probabilities π_s

Key insight: Use posterior probability q_{is} as weight

$$q_{is} = rac{\pi_s \prod_{t=1}^{T} \mathcal{L}_{ist}(heta_s)}{\sum_{t'=1}^{R} \pi_{s'} \prod_{t'=1}^{T} \mathcal{L}_{is't}(heta_{s'})}$$

 $q_{is} =$ probability individual i is type s given observed choices

Weighted CCP estimator replaces indicator with posterior type probability:

$$p_{jt}(x,s) = \frac{\sum_{i} d_{ijt} \ q_{is} \ 1[X_{it} = x]}{\sum_{i} \ q_{is} \ 1[X_{it} = x]}$$

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Now treats unobserved type as observed (with weights)

Can estimate CCPs conditional on unobserved states

EM Algorithm for CCP estimation at iteration m:

E-step: Given θ^m , π^m , p^m , compute

$$q_{is}^{m+1} = rac{\pi_s^m \prod_{t=1}^T \mathcal{L}_{ist}(heta^m, p^m)}{\sum_{c'=1}^R \pi_{c'}^m \prod_{t=1}^T \mathcal{L}_{is't}(heta^m, p^m)}$$

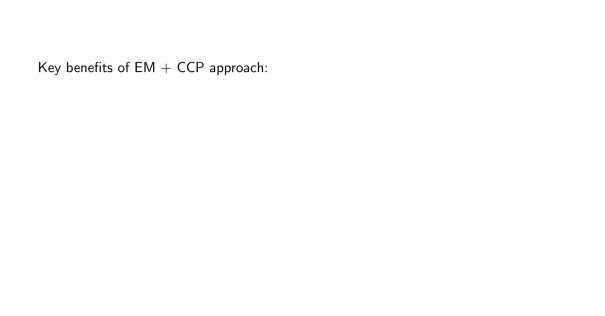
EM Algorithm for CCP estimation at iteration m:

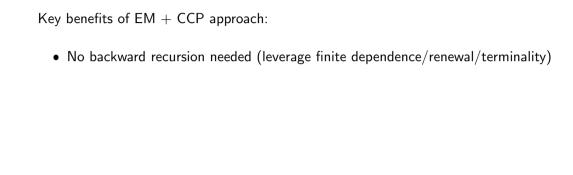
E-step: Given θ^m , π^m , ρ^m , compute

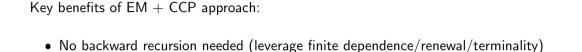
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M-step: Given q^{m+1} , update

$$egin{align}
ho_{jt}^{m+1}(x,s) &= rac{\sum_i d_{ijt}q_{is}^{m+1}\mathbb{1}[X_{it}=x]}{\sum_i q_{is}^{m+1}\mathbb{1}[X_{it}=x]} \ heta^{m+1} &= rg\max_{ heta} \sum_i \sum_i q_{is}^{m+1}\log \mathcal{L}_{ist}(heta, p^{m+1}) \ \end{pmatrix}$$







• Maximization step treats types as observed

Key benefits of EM + CCP approach:

- No backward recursion needed (leverage finite dependence/renewal/terminality)
- Maximization step treats types as observed
- Maintains additive separability for sequential estimation

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- No backward recursion needed (leverage finite dependence/renewal/terminality)
- Maximization step treats types as observed
- Maintains additive separability for sequential estimation
- Computational tractability despite unobserved heterogeneity

Alternative CCP update: Use structural model

Instead of weighted empirical frequencies, use likelihood:

$$\rho_{it}^{m+1}(x,s) = \ell_j(x,s;\theta^m,\pi^m,\rho^m)$$

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Where ℓ_i comes from choice probabilities implied by model

For example, with T1EV errors:

$$p_{jt}(x,s) = \frac{\exp(v_j(x,s))}{\sum_k \exp(v_k(x,s))}$$

Both methods converge to same estimates