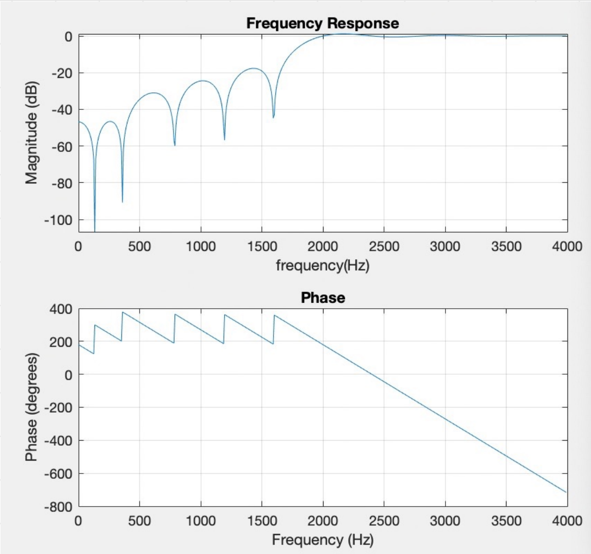
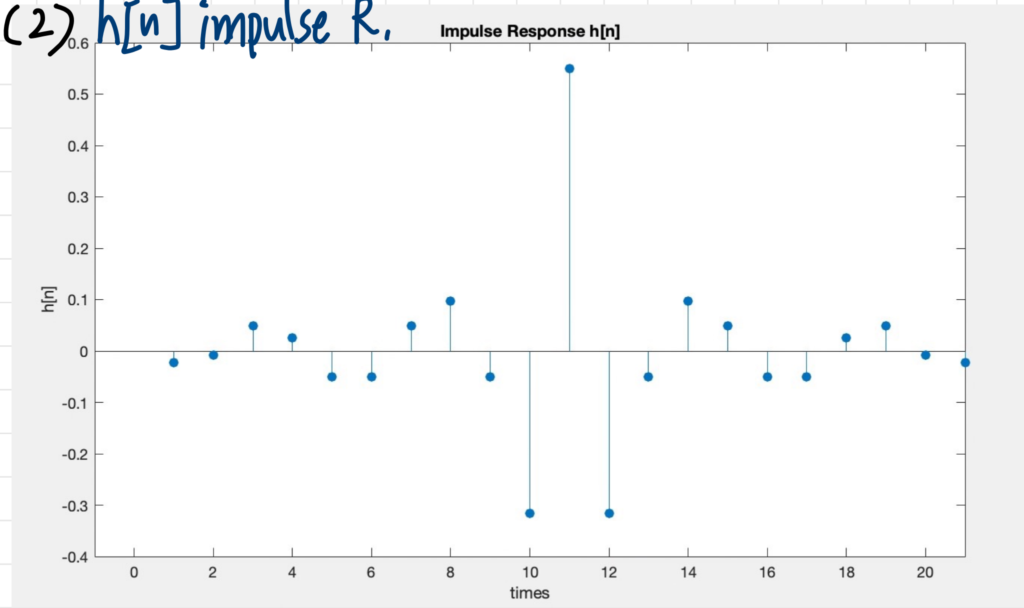
R11945072 張栢彥

Code 在作業附檔中

(1)

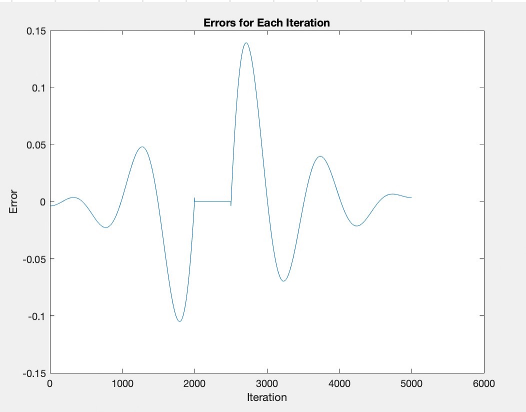


(2) h[n] impulse R.



(3)

將圖形畫出:



(2) (a) Which type of systems can be implemented by convolution?

(b) How do we convert convolution into an addition operation? (10 scores)

(a)

Linear time-invariant system

(LTI)

(b)

即 Homomorphic Signal Processing

$$\hat{Y}(f) = \log(Y(f))$$

sol:

$$y[n] = x[n] * h[n]$$

$\Downarrow$ 做FT

$$Y(f) = X(f)H(f)$$

$\Downarrow$ 取log

$$\boxed{\hat{Y}(f) = \hat{X}(f) + \hat{H}(f)}$$

(3) (a) Describe three advantages of the FIR filter.

(b) How do we implement $y[n] = x[n] *(0.7^n u[n]+ 0.2^n u[n])$ using the recursive method where * means the convolution and $u[n]$ is the unit step function? *可用遞迴試

(10 scores)

(a)

(1) output 是有限長

(2) 大部分較少 computation loading

(3) 穩定性高

(b)

1. 先 define 各個 function

① Let $x[n]$ 做為 input signal

$h[n] = x[n]*(0.7^n u[n] + 0.2^n u[n])$

and unit step function

$$u[n] = \begin{cases} 1, & n>0 \\ 0, & n<0 \end{cases}$$

$\therefore h[n] = 0.7^n u[n] + 0.2^n u[n]$

2. 依定義替換

$y[n] = \Sigma(x[k]*h[n-k])$ for all k

將 $h[n-k]$ 做替換

$y[n] = \Sigma(x[k]*(0.7^{(n-k)}u[n-k] + 0.2^{(n-k)}u[n-k]))$

3. 分段討論：① $u[n-k]=1$ for $n-k \geq 0$

② $u[n-k]=0$ for $n-k<0$

Let $y_1[n] = x[n] * 0.7^n u[n]$, $y_2[n] = x[n]\, 0.2^n u[n]$

y_1 則 $y[n] = y_1[n] + y_2[n]$

而 $\because$ $u[n-k]=1$ (for $n-k \geq 0$)

$u[n-k]=0$ (for $n-k<0$)

$\Rightarrow y_1[n] = \Sigma(x[k]*0.7^{(n-k)})$ for $k=0 \sim n$

$\therefore$ y_1 的 recursive 即為

$y_1[n] - y_1[n-1] = x[n]*(0.7^{(n-n)}) - x[n-1]*(0.7^{n-(n-1)})$

$\therefore y_1[n] = x[n] + 0.7 y_1[n-1]$

(證明)

化簡：$y_1[n] - y_1[n-1] = x[n] - x[n-1](0.7)$

$y_1[n] = x[n] - x[n-1](0.7) + y_1[n-1]$

又 $\because y_1[n] = x[n]*0.7^n u[n]$

$\therefore y_1[n-1] = x[n-1]*0.7^n u[n-1]$ 此 $n-k \geq 0$ 時=1 ∴為1

$\Rightarrow y_1[n] = x[n] - 0.7x[n-1] + x[n-1]*0.7^n$

$= x[n] + 0.7x[n-1](0.7^{n-1})$

$= x[n] + 0.7 y_1[n-1]$ #

y_2 相同作法得

$y_2[n] - y_2[n-1] = x[n]*(0.2^{(n-n)}) - x[n-1]\, 0.2^{(n-(n-1))}$

$y_2[n] = x[n] + 0.2 * y_2[n-1]$

$\therefore$ 有了 $y_1[n]$, $y_2[n]$ 遞迴關係

$$\begin{cases} y_1[n] = x[n] + 0.7 * y_1[n-1] \\ y_2[n] = x[n] + 0.2 * y_2[n-1] \end{cases}$$

$\therefore y[n] = y_1[n] + y_2[n]$

$= 2x[n] + 0.7 * y_1[n-1] + 0.2 * y_2[n-1]$

#

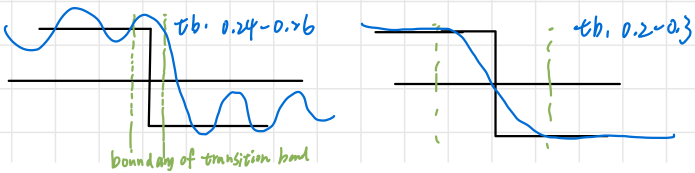
(4) What are the roles of (a) the transition band and (b) the weight function for minimax FIR filter design? (第一題有出現，莫角色) (10 scores)

(a) 根據公式 $N=\frac{2}{3}\frac{1}{\Delta F}\log_{10}\left(\frac{1}{10\delta_1\delta_2}\right)$

當 transition band 變大，

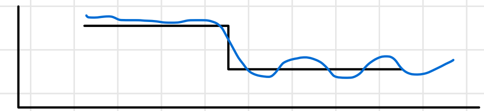
可使 passband ripple δ_1、stop band $\delta_2 \downarrow$

進而造成 passband, stopband 更為穩定。

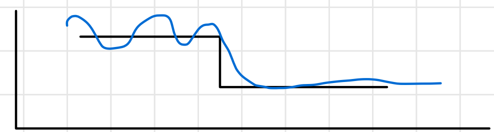


(b) 當著重於 passband > stop band

($w(f)=1$ in pass band, $0<w(f)<1$ in the stopband)



stopband > passband



($0<w(f)<1$ in passband, $w(f)=1$ in the stopband)

(5) Suppose that $x[n] = y(0.001n)$ and the length of $x[n]$ is 6000. If $X[m]$ is the FFT of $x[n]$, determine m such that $X[m]$ correspond to the frequencies of (a) 200Hz and (b) -100Hz. ($f=m\frac{f_s}{N}$) D~C 傅頻 trans. (10 scores)

$f_s=1000$

(a) $f_s = \frac{1}{0.001} = 1000$

$200 = m\frac{1000}{6000}$

$m=1200$ #

$\boxed{f=m\frac{f_s}{N}}$

(b) $-100 = m\frac{1000}{6000}$

$m=-6$ 不成立 (m為負)

$\therefore -100 = m\frac{1000}{6000} - 1000$

$\boxed{m=5400}$

(6) Use the MSE method to design the 7-point FIR filter that approximates the band pass filter of $H_d(F) = 1$ for $0.1 < |F| < 0.4$ and $H_d(F) = 0$ for $|F| < 0.1$ or $|F| > 0.4$. (10 scores)

△ not sure how

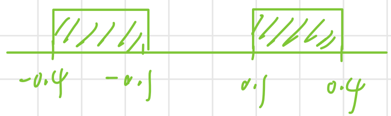
※計算方法，務理想 coeffi 計算出!

7 points → $(7-1)/2 = 3$ ∴ 分析 ∵ $h[k] = s[0]$

$(n=7 \rightarrow k=3)$ $h[k+n] = s[n]/2$

$h[n] = 0$

bandpass filter



$$s[n] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)\, H_d(F)\, dF$$

取 $n = 1 \sim 3$

$$\Rightarrow s[1] = 2\left[2\int_{0.1}^{0.4} \cos(2\pi F)\, dF\right]$$

$$= \frac{2(\sin 0.8 - \sin 0.2)}{\pi} \#$$

$$s[2] = \frac{(\sin 1.6 - \sin 0.4)}{\pi} \#$$

$$s[3] = \frac{2(\sin 2.4 - \sin 0.6)}{3\pi} \#$$

- frequency respond,

6 MSE Method

```
import numpy as np
import scipy.signal as signal

# Define the desired frequency response
def desired_freq_response(f):
    return np.where((0.1 < np.abs(f)) & (np.abs(f) < 0.4), 1, 0)

# Number of points in the desired impulse response
# (higher values give better frequency resolution)
N = 512

# Calculate the frequency vector for the desired frequency response
freqs = np.fft.fftfreq(N)

# Calculate the desired frequency response at each frequency in the frequency vector
Hd = desired_freq_response(freqs)

# Compute the desired impulse response using the inverse FFT
hd = np.fft.ifft(Hd)

# Truncate the desired impulse response to obtain a 7-point FIR filter
M = 7  # Number of filter coefficients
start = (N - M) // 2
h = hd[start:start + M].real

# Print the filter coefficients
print("7-point FIR filter coefficients:")
print(h)
```

(7) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval $\Delta_t = 0.0001$, and the transition band is from 3000Hz to 3300Hz. (10 scores)

將公式帶出: $N = \frac{2}{3}\frac{1}{\Delta F}\left(\log_{10}\frac{1}{10\delta_1\delta_2}\right)$

$$N = \frac{2}{3}\frac{1}{(3300-3000)\times 0.0001}\left(\log_{10}\frac{1}{10\times(0.01)^2}\right)$$

$$= \frac{2}{3}\times\frac{1}{\cancel{0.3}\,0.1}\times 3 \qquad \frac{10000}{10}$$

$$= \boxed{\frac{20}{3}}$$

#

(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 2, 3, 5, 6, 7, 8)

對應的 at 附錄二.

學號: 2結尾

EX, $f_s = 8000$

$N = 120000 = 15 f_s$

回答此!

(3) 若 $f = 3000$, $m = ?$

$$f = m\frac{f_s}{N} \Rightarrow 3000 = m\frac{1}{15}$$

$$\boxed{m = 45000}$$ #

Homework 1 (Due: March 22nd)

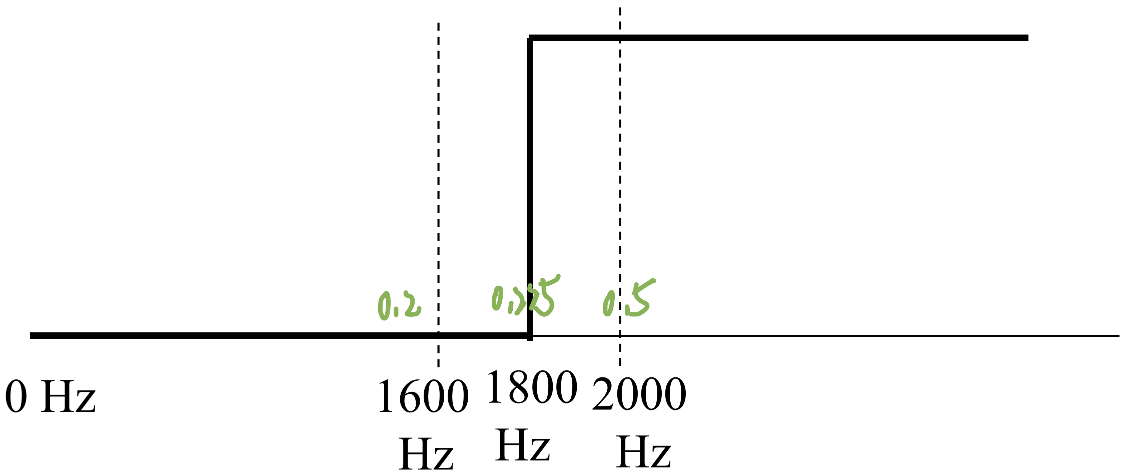
(1) Design a Mini-max **highpass** FIR filter such that (40 scores)

① Filter length = 21, ② Sampling frequency f_s = 8000Hz,

③ Pass Band ~~0~1600Hz~~ 1800~4000 ④ Transition band: 1600~2000 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.8 for stop band .

⑥ Set Δ = 0.0001 in Step 5.



※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

(2) (a) Which type of systems can be implemented by convolution?

(b) How do we convert convolution into an addition operation? (10 scores)

(3) (a) Describe three advantages of the FIR filter.

*可用遞迴試

(b) How do we implement $y[n] = x[n] *(0.7^n u[n]+ 0.2^n u[n])$ using the recursive method where * means the convolution and $u[n]$ is the unit step function?

(10 scores)

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⊛ 計算方法, 將理想 coeffi 計算出!

(10 scores)

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(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 2, 3, 5, 6, 7, 8)

★ 对應的在附錄二.