R11945072 生醫電資一 張栢彥

Homework 5 (Due: 6/21)

(1) Write the Matlab or Python code to compute the FFT of two N-point real signals x and y using only one N-point FFT. (20 scores)

$$[Fx, Fy] = \text{fftreal}(x, y)$$

The code should be handed out by NTUCool.

(2) Compared to the original non-sectioned convolution, what are the two main advantages of the sectioned convolution? (8 scores)

(3) Are the following applications suitable for the Walsh transform? Why? (a) calculating the linear convolution; (b) compressing a natural image; (c) stair-like signal analysis. (12 scores)

(4) What is the number of addition operations when we what to implement (a) the 16-point Walsh transform and (b) the 16-point Haar transform? (10 scores)

(5) What are the <u>two main advantages</u> of the OFDM when compared to the original FDM? (8 scores)

(6) (a) What is the results of CDMA if there are three data [1 1 0], [0 1 1], [1 0 1] and these three data are modulated by the 1^{st}, 6^{th}, and 12^{th} rows of the 16-point Walsh transform? (The beginning row is the 1^{st} row). (10 scores)

(b) In (a), if the 8^{th} and the 15^{th} entries of the CDMA results are missed, can we recover the original data? Why? (5 scores)

(7) (a) Please determine 3^{2049} (mod 11).

(Hint: Try to find a such that 3^{a} (mod 11) = 1).

(b) Suppose that N mod 23 = 12 and N mod 47 = 8. Please determine the minimal positive integer solution for N.

(Hint: We can use the fact that 46 mod 47 = -1 mod 47.) (8 scores)

(8) Write at least three similarities between the NTT and the DFT. (7 scores)

(Continued)

是否满足要求;

(9) For the complex number theoretic transform (CNT), if a complex integer number $a + ib$ satisfies $a^2 + b^2 = 1 \bmod M$, then we say that $a + ib$ is on the unit circle.

$i^2 = -1$

(a) Is $2+ i11$ and $5+i10$ on the unit circle when $M = 31$?

(b) Is $(2+ i11)(5+i10)$ on the unit circle when $M = 31$?

(c) When $a = 10$, find all $b \in [1, 2, .., 30]$ such that $a + ib$ is on the unit circle.

(12 scores)

(Extra): Answer the questions according to your student ID number.

(ended with (1, 6), (2, 7), (3, 8), (4, 9))

今天提到

R11945072 生醫電資一 張栢彥

(2) Compared to the original non-sectioned convolution, what are the <u>two main advantages</u> of the <u>sectioned convolution</u>? (8 scores)

2、

利用 Sectioned Convolution 之優勢

① 效率和計算速率：大規模運算中效率更高，加速處理速

② 記憶使用量：數據切更小具有可內存較少之優勢

(3) Are the following applications <u>suitable for the Walsh transform? Why?</u> (a) calculating the linear convolution; (b) compressing a natural image; (c) stair-like signal analysis. (12 scores)

3、 Walsh T.：非正弦、非連續變換，則會產生一二進制(1,-1)函數作基礎

(a) linear convolution：

Walsh通常不被適用於線性卷積，以Walsh函非複指數函數且連續區間內不呈現正交性。此外，其大多用於訊號分析/轉換而非卷積運算。

(b) Image Compression：

Walsh可用於圖像壓縮。它可以有效 represent/編碼影像資料，減去多餘並達成壓縮效果。常見地是，DCT這種變換更為適合，其具能量壓縮、最小化高頻偏移之能力

(c) Stair-like signals：

Walsh適合用在分析 stair-like signals。它可以分析、代表其不連續的訊號。它很適合、實用於 signal analysis 和 pattern recognition 任務。

(4) What is the number of addition operations when we what to implement (a) the 16-point Walsh transform and (b) the 16-point Haar transform? (10 scores)

(a) 可由 butterfly fast Alg 分析

加法運算中，第一個 row 要 15 個 Addition，剩下的 row 各要一個 addition

$\Rightarrow \therefore 15 + 1 \times (16-1) = \boxed{30}$ ↑ addition

(b) 由前面規律統整：

N=2, H_2 時 2 ADDS

N=4, H_4 時 $\therefore$ ⊕ $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = H_2 \begin{bmatrix} x_1+x_2 \\ x_3+x_4 \end{bmatrix}$ 2加

⊖ $\begin{pmatrix} y_3 \\ y_4 \end{pmatrix} = \begin{matrix} x_1-x_2 \\ x_3-x_4 \end{matrix}$ 2減

原 N=2's 2ADDS + 2加 + 2減 = 6 ADDs

N=8, H_8 時 $\therefore$ ⊕ $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = H_4 \begin{bmatrix} x_1+x_2 \\ x_3+x_4 \\ x_5+x_6 \\ x_7+x_8 \end{bmatrix}$ 4加

$\begin{pmatrix} y_7 \\ y_8 \\ \vdots \\ y_{10} \end{pmatrix} = H_4 \begin{bmatrix} \ddots \end{bmatrix}$ 4減

原 N=4's 6ADDs + 4加 + 4減 = 14 ADDs

故 N=16 時，依循同規律，其 ADDs 計算為：

N=16 $\Rightarrow$ 原 N=8's 14ADDs + 8加 + 8減 = 30 ADDs

$\Rightarrow \boxed{\text{Ans: 30 ADDs}}$ #

(5) What are the <u>two main advantages</u> of the OFDM when compared to the original FDM? (8 scores)

5.

優勢1：提高頻譜效率

傳統FDM中，不同子信道間需隔開以防止干擾與重疊，但這會使每子信號間出現未使用到的頻譜訊號(保護頻帶)，使可用頻譜效率減低；相比之下，OFDM利用正交原理允許子信道而不生干擾而大幅提升效率。

優勢2：扭曲與干擾(符號間干擾ISI)的緩解

傳統OSM中，多徑傳播引起的時間分散導致符號干擾(ISI)。在OSFM中數據被分成許多緩慢數據流並更慢地傳輸，使符號周期大於延遲，進而減少或消除符號干擾。

(6) (a) What is the results of CDMA if there are three data [1 1 0], [0 1 1], [1 0 1] and these three data are modulated by the 1st, 6th, and 12th rows of the 16-point Walsh transform? (The beginning row is the 1st row). (10 scores)

(b) In (a), if the 8th and the 15th entries of the CDMA results are missed, can we recover the original data? Why? (5 scores)

6.

(a). Step 1: create N=16, Walsh transform

$$\begin{pmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1\\
1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1&1&1&1&1\\
1&1&1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&-1&-1\\
1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&-1&-1&1&1\\
1&1&-1&-1&-1&-1&1&1&-1&-1&1&1&1&1&-1&-1\\
1&1&-1&-1&1&1&-1&-1&-1&-1&1&1&-1&-1&1&1\\
1&1&-1&-1&1&1&-1&-1&1&1&-1&-1&1&1&-1&-1\\
1&-1&-1&1&1&-1&-1&1&1&-1&-1&1&1&-1&-1&1\\
1&-1&-1&1&1&-1&-1&1&-1&1&1&-1&-1&1&1&-1\\
1&-1&-1&1&-1&1&1&-1&-1&1&1&-1&1&-1&-1&1\\
1&-1&-1&1&-1&1&1&-1&1&-1&-1&1&-1&1&1&-1\\
1&-1&1&-1&-1&1&-1&1&1&-1&1&-1&-1&1&-1&1\\
1&-1&1&-1&-1&1&-1&1&-1&1&-1&1&1&-1&1&-1\\
1&-1&1&-1&1&-1&1&-1&-1&1&-1&1&-1&1&-1&1\\
1&-1&1&-1&1&-1&1&-1&1&-1&1&-1&1&-1&1&-1
\end{pmatrix}$$

Step 2: extract 1st, 6th and 12th rows

$$\begin{array}{lll}
\text{1st} & & [1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1] \\
\text{6th} & \text{rows}: & [1,1,-1,-1,-1,-1,1,1,-1,-1,1,1,1,1,-1,-1] \\
\text{12th} & & [1,-1,-1,1,-1,1,1,-1,1,-1,-1,1,-1,1,1,-1]
\end{array}$$

Step 3 :

① [1, 1, 0] 變 [1, 1, -1] , modulated by 1st row

[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1]

② [0, 1, 1] 變 [-1, 1, 1] , modulated by 6th row

[-1, -1, 1, 1, 1, 1, -1, -1, 1, 1, -1, -1, -1, -1, 1, 1
1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, -1, -1,
1, 1, -1, -1, -1, -1, 1, 1, -1, -1, 1, 1, 1, 1, -1, -1]

③

[1, 0, 1] 變 [1, -1, 1]

[1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1,
-1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1,
1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1]

Step 4 : add the result together

[1, -1, 1, 3, 1, 3, -1, 3, 1, -1, 1, -1, 1, 3, 1,
1, 3, 1, -1, 1, -1, 1, 3, -1, 1, 3, 1, 3, 1, -1, 1,
1, -1, -3, -1, -3, -1, 1, -1, -1, -3, -1, 1, -1, 1, -1, -3]

(b)

If 8th and 15th are missed

① [1, -1, 1, 3, 1, 3, 1, 0, 3, 1, -1, 1, -1, 1, 0, 1] · [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
= 14

14/16 > 0 ∴ = 1

② [1, 3, 1, -1, 1, -1, 1, 0, -1, 1, 3, 1, 3, 1, 0, 1] · [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

$= 14$

$14/16 > 0 \quad \therefore = 1$

③ $[1,-1,-3,-1,-3,-1,1,0,-1,-3,-1,1,-1,1,0,-3] \cdot [1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1]$

$= -14$

$-14/16 < 0 \quad \therefore = -1$

如果Ans是>0則設為1，否則為0，而[1,1,0]成功recovered，

могу得到：儘管有missing data，我們還是可recover原有data

(7) (a) Please determine 3^{2049} (mod 11).

(Hint: Try to find a such that 3^a (mod 11) = 1).

(b) Suppose that N mod 23 = 12 and N mod 47 = 8. Please determine the minimal positive integer solution for N.

(Hint: We can use the fact that 46 mod 47 = -1 mod 47.) (8 scores)

(a) $3^{2049} \pmod{11}$ → 找規律

$3^1 \pmod{11} = 3$

3^2 (〃) $= 9$

3^3 (〃) $= 5$

3^4 (〃) $= 4$

3^5 (〃) $= 1$

3^6 (〃) $= 3$

5个一循環 形成cycle!

而 $2049 \bmod 5 = 4$

∴ 取4th

⇒ $3^{2049} \pmod{11} = \boxed{4}$

#

(b) Suppose $N = 12 + 23k$

for $k = 0 \quad 12 + 23k \pmod{47} = 12$

$k = 2$ (〃) $= 11$

⋮

$k = 8$ (〃) $= 8$

keep 一个一个test

$\therefore N = 12 + 23k$

$= 12 + 23 \times 8$

$= \boxed{196}$

#

Number Theretic Transform (NTT)

(8) Write at least three similarities between the NTT and the DFT. (7 scores)

8.

數論變換(NTT)和離散傅立葉變換(DFT)相似之處：

① 線性變換：都線性transform，故保留了加法和scalar乘法運算性

② 複雜性：皆可用 $O(N\log N)$ 做高效運算，皆有速算法：DFT → FFT，NTT → FNTT

③ 可逆性：它們皆可透過逆變換回復原始信號

④ 結構性：皆會將input由大往小拆解，各自利用函數週期、對稱性以實行更高效的運算。

是否滿足等式

(9) For the complex number theoretic transform (CNT), if a complex integer number $a + ib$ satisfies $a^2 + b^2 = 1 \bmod M$, then we say that $a + ib$ is on the unit circle.

$i^2 = -1$

(a) Is $2+i11$ and $5+i10$ on the unit circle when $M = 31$?

(b) Is $(2+i11)(5+i10)$ on the unit circle when $M = 31$?

(c) When $a = 10$, find all $b \in [1, 2, .., 30]$ such that $a + ib$ is on the unit circle.

(12 scores)

9 (a) $2^2+11^2 \pmod{31} = 125 \pmod{31} = 1$

又 $5^2+10^2 \pmod{31} = 125 \pmod{31} = 1$

因此，$2+11i$ and $5+10i$ 是在unit circle上.

(b)

$(2+11i)(5+10i) \bmod 31$

$= ((2+11i) \bmod 31)((5+10i) \bmod 31)) \bmod 31$

$= 1 \bmod 31$

$= 1$

∴ $(2+11i)(5+10i)$ 是在unit circle上

(c) $10^2 \pmod{31} = 7$

$5^2 \pmod{31} = 25$

$26^2 \pmod{31} = 25$

$(7+25) \pmod{31} = 1$

$\therefore$ when $b=5$ or 26, $a+bi$ 是在 unit circle 上

Extra: end with the number "2"

(Extra): Answer the questions according to your student ID number.
(ended with (1, 6), $\boxed{(2, 7),}$ (3, 8), (4, 9))

今天提到

$\Rightarrow$ 4-points DFT 矩陣:

$$W = \frac{1}{\sqrt{4}} = \begin{bmatrix} w^0 & w^0 & w^0 & w^0 \\ w^0 & w^1 & w^2 & w^3 \\ w^0 & w^2 & w^4 & w^6 \\ w^0 & w^3 & w^6 & w^9 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

#