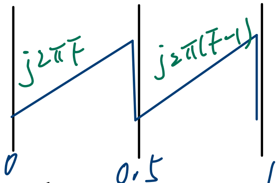
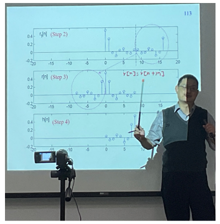
R11945072 張栢彥



H(F) = H(F+1)
(page 111-114)



Homework 2 (Due: 4/12)

(1) Write a Matlab or Python code that uses the frequency sampling method to design a (2k+1)-point discrete differentiation filter $H(F) = j2\pi F$ when $-0.5 < F < 0.5$ (k is an input parameter and can be any integer). (25 scores)

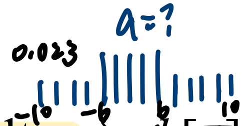
$\frac{m}{2k+1}$, $m = k, k+1$.

The transition band is assigned to reduce the error (unnecessary to optimize). (i) The impulse response and (ii) the imaginary part of the frequency response (DTFT of $r[n]$, see pages 113 and 114) of the designed filter should be shown. The code should be handed out by NTU Cool.

決定哪裡重要 決定誤差

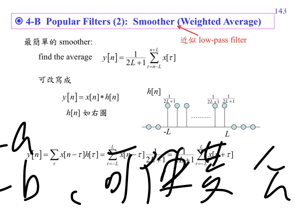
(2) Can the techniques of the weight function and the transition band be applied in the FIR filter designed by (a) the MSE method and (b) the frequency sampling method? Why? (10 scores)

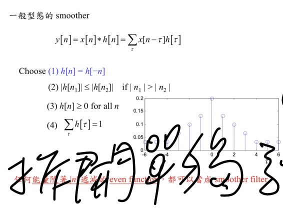
指演能



(3) Suppose that the smooth filter is $h[n] = a$ for $|n| \leqq 5$, $h[n] = 0.023$ for $6 \leqq |n| \leqq 10$, and $h[n] = 0$ otherwise. (a) What is the value of a? (b) What is the efficient way to implement the convolution $y[n] = x[n] * h[n]$? (10 scores)

① $11a + 10(0.023) = 1$



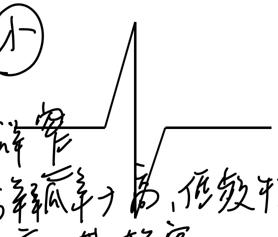


/ sliding window

② 分成 sum_a sum_b，可使其公式拆開單純計算，避免重複積造成ム

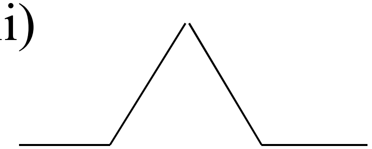
(4) The following figures are the impulse responses of some filters. Which one is a suitable smoother when we want to extract (a) small scaled features? (b) large scaled features? Also illustrate the reasons. (10 scores)

(i) 小

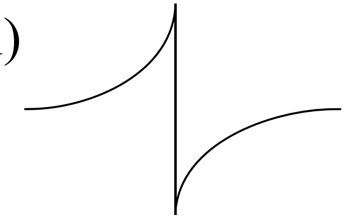


① 主瓣窄
② 高分辨率→高,低頻特徵
③ 過渡帶較窄

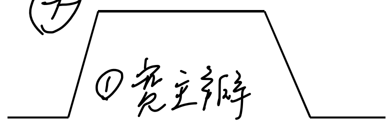
(ii)



(iii)



(iv) 大



① 寬主瓣
② 低分辨率 → 合併為單一特徵
③ 過渡帶可寬

(5) If the z-transform of $h[n]$ is $H(z) = \dfrac{2z^4 - 2z^3 + 3z^2 + z - 2}{z^2 + 0.2z - 0.24}$

(a) Determine the cepstrum of $h[n]$.

(Hint: $z = 2^{-0.5}$ is one of the zeros of $H(z)$) 表 $2z^2-1$ 為因式

(b) Convert the IIR filter into the minimum phase filter. (20 scores)

(6) Suppose that the cepstrum of a signal $x[n]$ is

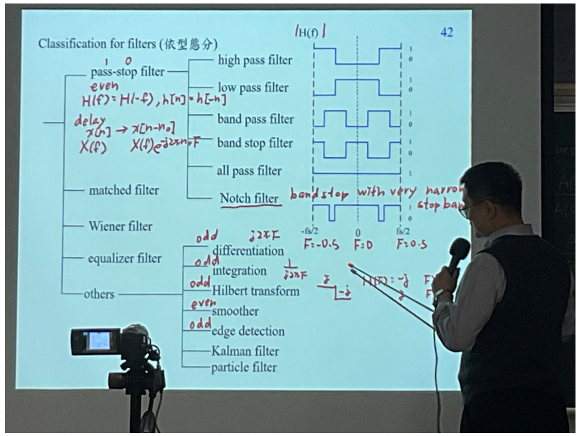
$$\hat{x}[2] = 0.7, \quad \hat{x}[n] = 0 \quad otherwise$$

Determine $x[n]$ using the Z transform and exp( ). (10 scores)

(7) (a) What are the two main advantages of the minimum phase filter? (b) In addition to time-frequency analysis, what are two main applications of the Hilbert transform? (c) Compared to the equalizer, what are the two main advantages of the cepstrum to deal with the multipath problem? (15 scores)

(Extra): Answer the questions according to your student ID number.

(ended with (4, 9), (0, 5), (1, 6), (2, 7))



R11945072 張柏彥

(1)

見檔案.m

(2).

皆可以

① MSE：最小化所需f響應和濾波器實際f之間的均方誤差設計FIR，此方法用所需f & 誤差標準計算濾波器係數

a. 加权函數：可使訊號分配不同寬帶比重，不太關鍵之寬帶可進行低要求優化

b. transition band：可使过渡有更尖銳/平滑的過渡

Sampling：由IDFT計算其係數所需之頻率響應（through均勻間隔）

a. 加权函數：可給予不同权重，使其更逼近所需響應、做出逼近符合之signal.

b. transition band：也可以、可使其於過渡帶的特徵做更大取樣而顯其特徵

(3)

①

$$11a+10(0.023)=1$$

$$a=0.077 \quad \#$$

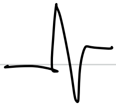
②

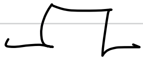
分成 sum_a , sum_b ，主要通過使用兩個滑動窗口分別處理h[n]的不同值，最終獲得卷積y[n].

→ 如此可有效計算卷積，

以不需对所有h[n]值做單獨的卷積運算

(4)

(a) I 

(b) iV 

因其具有較 窄/寬 的主頻響應，高/低 高低頻分辨率，窄/寬 過渡帶

(可合併為單一 feature)

(5)

① 过渡带較下

If the z-transform of $h[n]$ is $H(z)=\dfrac{2z^4-2z^3+3z^2+z-2}{z^2+0.2z-0.24}$

(a) Determine the cepstrum of $h[n]$.

(Hint: $z = 2^{-0.5}$ is one of the zeros of $H(z)$) 表 $2z^{-1}$ 為因式

(b) Convert the IIR filter into the minimum phase filter.

$z^2+\frac{1}{5}z-\frac{6}{25}z$

(a)

$$H(z)=2\,\frac{(z+2^{-0.5})(z-2^{-0.5})\left(z-\boxed{\frac{1+\sqrt{7}i}{2}}\right)\left(z-\boxed{\frac{1-\sqrt{7}i}{2}}\right)}{(z+0.6)(z-0.4)}\times\left(\left(\frac{1+\sqrt{7}i}{2}\right)\times\left(\frac{1-\sqrt{7}i}{2}\right)\right)\rightarrow \times\left(\frac{1+7}{4}=2\right)$$

bk^{-1}

$\frac{-bk}{-bk}(z-bk^{-1}) = -bk^{-1}(1-bk^{-1})$

$$=4\,\frac{\overset{<1}{(z+2^{-0.5})}\overset{<1}{(z-2^{-0.5})}\overset{>1}{\left(z-\frac{1+\sqrt{7}i}{2}\right)}\overset{>1}{\left(z-\frac{1-\sqrt{7}i}{2}\right)}}{(z+0.6)(z-0.4)} \quad \text{—(1)}$$

$$=\frac{4\left(z^2-\frac{1}{2}\right)\left(z^2-\frac{\cancel{z-\sqrt{7}iz-z+\sqrt{7}iz}}{2}+\frac{8}{4}\right)}{z^2(1+0.6z^{-1})(1-0.4z^{-1})}=\frac{4(1-2z^{-2})(z^2+2)}{(1+0.6z^{-1})(1-0.4z^{-1})}$$

又 $\hat{H}(z)=\log(H(z))$

$$=\log(4)+\log(1-2^{-0.5}z^{-1})+\log(1+2^{-0.5}z^{-1})+\log\left(1-\frac{2}{1+\sqrt{7}i}z\right)+\log\left(1-\frac{2}{1-\sqrt{7}i}z\right)$$
$$-\log(1+0.6z^{-1})-\log(1-0.4z^{-1})$$

$$\hat{h}[n]=\begin{cases}\log 4, & n=0\\[4pt] \dfrac{-2^{-0.5n}}{n}+\dfrac{(-2)^{-0.5n}}{n}+\dfrac{(-0.6)^n}{n}+\dfrac{(0.4)^n}{n}, & n>0\\[4pt] \dfrac{\left(\frac{2}{1+\sqrt{7}i}\right)^{-n}}{n}+\dfrac{\left(\frac{2}{1-\sqrt{7}i}\right)^{n}}{n}, & n<0\end{cases}$$

(b) 若 z 零点幅度大於單位圓 1 ~~，則換成~~ 應 <1 $\frac{1}{\mathrm{conj}(z^0)}$

∵ 原式只有 $\left(z-\frac{1+\sqrt{7}i}{2}\right)\left(z-\frac{1-\sqrt{7}i}{2}\right)$ 之零点过單位圓 (z_0 过單位圓 =1 者，便其換成 $\frac{1}{\mathrm{conj}(z_0)}$ #)

$$\cancel{4}\,\frac{\overset{<1}{(z+2^{-0.5})}\overset{<1}{(z-2^{-0.5})}\overset{>1}{\left(z-\frac{1+\sqrt{7}i}{2}\right)}\overset{>1}{\left(z-\frac{1-\sqrt{7}i}{2}\right)}}{(z+0.6)(z-0.4)}\times\frac{1}{\frac{1+\sqrt{7}i}{2}}\times\frac{1}{\frac{1-\sqrt{7}i}{2}} \;/\!/\; \cancel{4}$$

$$=\frac{\frac{1}{2}z^4-\frac{1}{2}z^3+\frac{3}{4}z^2+\frac{1}{4}z-\frac{1}{2}}{z^2+0.2z+0.24}\ \#$$

(6)

1. 計算 ceptrum：

$c[n] = x[n] = \{0, 0, 0.7, 0, 0, \ldots\}$

2. Magnitude and phase spectra：

$|X[k]| = \exp(x[n]) = \{1, 1, \exp(0.7), 1, 1, \ldots\}$

Phase spectrum：$\arg X[k] = 0$

3. Z-transform：

$$X(z) = \sum_{k=-\infty}^{\infty} X[k]^{*} \exp(j \times \arg X[k]) \times z^{-k}$$

∵ magnitude of spectrum is 1 all value（k=2除外），又 phase spectrum = 0

∴ ⇒ $X(z) = 1 + \exp(0.7) \times z^{-2}$

4. inverse z-transform.

⇒ $x[n] = z^{-1}\{X(z)\} = z^{-1}\{1 + \exp(0.7) \times z^{-2}\}$

⇒ $\underline{x[n] = \delta[n] + \exp(0.7) \times \delta(n-2)}$ #

∴ Signal $x[n]$ 有值1於 $n=0$ 時，值 $\exp(0.7) \approx 2.013$ 於 $n=2$，其他地方值 = 0

(7)

(a)
① 穩定性高 ∵：poles & zeros 皆位於圓的內部，有助於不會振盪並應用於實際應用

② Casality（因果關係）：最小相位濾波為因果關係，其output只取決於當下、過去的值，可允許更快 inpulse response 時間並大幅減少延遲

(b)

解析信號生成：信號為複值，實部為原始，虛部為Hiltor Transform

⇒ 可用於通信時**幅度、相位**解調、包絡檢測時特別有用。

瞬時頻估計：透過計算並解析信號相位之時間導數達成，可分析具有時變頻率特性之語音、音樂信號

(c)

1. 模型簡單：將 time domain 的卷積運算轉換為加法運算來簡化問題。
⇒ 識別、分離所需信號、分量更容易

2. 噪聲的穩定性：比均衡技術處理噪音更穩定。可有效分析所需信號、多個分量 ⇒ 提高信號質量、降低誤碼率。

(8) Extra: 2 Kalman、Particle filter 在信號處理用來做？

Ans: 2者皆是用於估計系統狀態的概率算法，常用於導航、目標追蹤等領域

(1) Kalman：線性濾波：主要用於估計線性高斯系統。結合了系統模型的先驗知識和觀測數據來估計系統狀態
特色：計算效率高(線性)，但對非線性或非高斯系統，有受性能限制。

(2) Particle：蒙特卡洛方法之概率濾波 → 非線性、非高斯系統狀態。
使用一組粒子(樣本)來近似系統概率分布，以觀測數據做粒子权重更新、取樣
特色：可 calculate 非線性、非高斯 → but 計算效率低 (高維更是!)