MSCI 603 F2020

FINAL REPORT

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Introduction

According to Statistics Canada for data in 2019, there are 35,742,412 active vehicle registrations among the population of 37,802,043, reported as in Q4-2019. As technology is developing drastically, using the latest cars can help with reducing fuel expenditures and CO2 emission with more safety and smart features, which allows you to enjoy your personalized mobility experience much better. Therefore, we will work on a common aspect of life which is related to transportation usage in this project.

We will assume the scenario from a service company's point of view. It will be a small start-up company that wants to offer transportation service through rides. The company is planning to lease some cars and hire some drivers to take customer to their destination from the pickup point with different models of cars which are grouped in different classes. For this simplified model, we only consider three car class options which are compact, mid-size and SUV with \$1.00, \$1.25, and \$1.50 charge per kilometer accordingly. Profit will consider the gain in revenue from the number of cars and the total distance traveled by each car; along with the operation cost of leasing cars, filling fuel, and hiring drivers. The problem is that the company must be mindful of the CO_2 emission, so a balance of maximizing profit while keeping CO_2 emission to a minimal would be ideal. This is important because CO_2 contributes to about 81% of greenhouse gases, which cause global warming and therefore should be kept to some possible allowable limit.

We will focus on the 2020 car models with current total purchasing cost less than \$75,000 that run on fuel only (Figure 9 in Appendix A). We will then obtain the yearly leasing prices for these vehicles. The reason we limit our car options based on the total purchasing price is that the dealership may have different promotion interest rates for some specific vehicles when others are still subjected to the regular rates. Therefore, by using their one-time purchasing cost, we can set the pricing constraint more precisely for the cut-off price and allow the leasing price to be up to date with current market pricings and promotions. By focusing on cars that consume fossil fuel only, we want to further concentrate on the direct relationship between CO_2 emissions and fuel consumption, without distraction from hybrid or electric cars, which have much better CO_2 rate Thanks to the contribution of electric energy. Moreover, we have also narrowed down the options for which vehicles to lease to majority of the cars emitting CO_2 at rates between around 160 g/km to 275 g/km as shown in Figure 8. As for cost, we will only focus on the leasing cost and fuel cost since all consumers will be exposed to the similar pricings for them whereas extra costs (maintenance fees, insurance costs, and taxes) are uncertain and may vary with different factors, such as how well you take care of the car; which auto service you choose to go, choice of car's insurance provider, different addresses, age group, third-party dealerships, etc. Hiring cost is also taken into consideration with the current minimum wage for full-time employee plus commission rate.

A brief exploratory data analysis was first performed on the vehicle dataset (Figures 1 to 9) to get a better understanding of the data in general as well as the relationships between each feature. This was to potentially help us determine the relationship between the types of variables, decide on the types for decision variables (continuous, integer, and binary) to work with and also if we need to solve a linear, mixed-integer, quadratic, or nonlinear problem. With all of these mentioned as above, we came up with an optimization model to maximize the profit from changing the decision variables which are the car selection, total distance driven by each car, along with the number of cars leased for each specific model and class.

Previous work that was similar to this problem was Ford using analytics to help their fleet customers invest in more sustainable vehicles that emit less CO_2 (Reich et al., 2015). The key differences between this project and their study are the following: 1) This project has a larger variety of cars in terms of the make (Acura, Honda, Volkswagen, etc.) while the previous work focused solely on vehicles manufactured by Ford. 2). Their model is a multi-objective function where they introduced two integer programming models: where they first minimize the cost while satisfying emission constraints (which was initialized to infinity (Reich et al., 2015)), followed by minimizing the emission with the minimum cost that they determined as their cost constraint. In other words, one model minimizes cost while its emission was constrained and the other minimizes emission with cost constraints. We have a different approach where we built a nonlinear optimization model, and our objective function was to maximize the profit.

Optimization model

As a service company, our goal is to minimize the expenses while maximizing the revenue earned from giving rides using the leased cars. And, as the ozone layer in the Earth's upper atmosphere is depleting due to greenhouse gases, we feel that it is important in attempting to restrict the CO_2 emissions in this project as well. Therefore, our model is aiming to maximize the company's yearly profit, which is the revenue minus the cost, while setting constraints on the yearly total CO_2 emissions, numbers of cars should be leased for each model to enhance the variety on car selection for customers, numbers of employees (drivers) and total distance driven, shown as below:

<u>Parameter Variables</u>

For all i's and j's below, i's are models (i = 1,2…n) of cars and j's are classes (j = 1,2…m) of cars.

FP_{ij}: fuel price of the car with model i in class j in dollars per liter ($/L). Depend on the car's model that car may consume regular or premium gas ($/L).

FC_{ij}: fuel consumption combined with highway and city driving of the car with model i in class j in liters per kilometer (L/km).

E_{ij}: CO2 emissions of the car with model i in class j in kilograms per kilometer (kg/km).

P_{ij}: yearly leasing price of the car with model i in class j in dollars.

R_{ij}: revenue the company earns in dollars for offering the service (in other words, the amount earned from each kilometer driven by the car) with model i in class j.

<u>Decision variables</u>

X_{ij} : number of the cars with model i in class j should be leased.

D_{ij} : yearly distance each car with model i in class j should be driven in order to maximize the profit.

Y_{ij} : binary variable, $\begin{cases} 1 & \text{if there is at least one of the car with model i in class j is leased} \\ 0 & \text{otherwise} \end{cases}$

<u>Objective functions and constraints</u>

Revenue = $\sum_{i=1}^{n} \sum_{j=1}^{m} R_{ij} * X_{ij} * D_{ij}$

Leasing cost = $\sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} * X_{ij}$

Fuel cost = $\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij} * D_{ij} * FC_{ij} * FP_{ij}$

Salary = yearly salary per driver * $\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}$ + commission rate * Revenue

Max Profit = Revenue – Leasing cost – Fuel cost – Salary

s.t. $\sum_{i=1}^{n}\sum_{j=1}^{m} E_{ij} * X_{ij} * D_{ij} \leq$ CO_2 emission limitation (1)

$\sum_{i=1}^{k} X_{ij} \geq$ minimum number of cars the company needs for each class to run the business (2)

$D_{ij} \geq Y_{ij}$ * minimum distance a car expected to be driven per year (3)

$D_{ij} \leq Y_{ij}$ * maximum distance a car expected to be driven per year (4)

$\sum_{i=1}^{k} Y_{ij} \geq$ minimum number of the specific car model in each class the company wants to lease (5)

$X_{ij} \leq Y_{ij}$ * maximum number the company wants to lease for each type of car (6)

$Y_{ij} \leq X_{ij}$ (7)

This is the first attempt with a basic model to solve this problem. All the numbers and constraints could be adjusted depends on the company's specific situation.

As mentioned in our project goal, our model is a maximizing profit problem, and the main constraint is about the CO_2 emission limitation [constraint (1)]. Considering customers have different preferences for the type of cars, we limit the minimum number of car types and enough total number of cars for each class. Thus, customers can have various options when they book the drive, which is shown as constraint (2) and (5). Besides that, we have the minimum requirement for the distance driven if the car is leased, since it is not worth to lease the car for short distance. We also have the maximum possible distance to be driven per year from the research of a taxi driver's average annual distance. These are shown as the constraint (3) and (4). For most of cars, the higher the leasing price it has, the lower the CO_2 it emits. However, unless the fuel cost exceeds the leasing price, the optimization method will always choose the car with lower leasing price and higher CO_2 emission as the optimal solution. Therefore, in order to control the CO_2 emission with the minimum requirement for each car's class (compact, mid-size, SUV), we limit the number of each type of cars could be leased as shown in constraint (6). Therefore, even the optimal solution prefers this kind of car, the amount of CO_2 emission will not be too large. The constraint (7) is for connecting X_{ij} and Y_{ij}, so when X_{ij} is 0, Y_{ij} will be 0; when X_{ij} is greater than 0, Y_{ij} will be 1.

There is a deficiency of the model. The model is complete enough to express the problem but solving and programming are quite challenging. The constraint (6) makes sense technically. When Y_{ij} is 0, X_{ij} will be 0; when Y_{ij} is 1, X_{ij} will be less than the constraint number. Nevertheless, when we try to run the model in Python, we need to split this constraint to two parts so Python will be able to give the correct solution. It has to be written as $X_{ij} \leq Y_{ij}$ * M and $X_{ij} \leq 5$; otherwise, if we try to combine them as $X_{ij} \leq Y_{ij}$ * 5, solution is almost infeasible since X_{ij} is a multiplication of Y_{ij} and there's no physical upper limit for X so it requires lots of trials (multiplication of the variable data set within the constraints) when the program is running, which nearly exhausts the capacity limit of Python.

Numerical Example

We used the GEKKO Python package to solve this optimization problem. The GEKKO Python package was suitable for this problem since it involves nonlinear programming as well as integer variables. Other packages such as Gurobi and SciPy were considered. However, Gurobi does not solve nonlinear problems and SciPy cannot work with integer variables.

All data was achieved from Statistics Canada (for the fuel consumption, CO_2 emission for each X_{ij} car), and variety of Canadian automobile dealership for leasing and full price. In this model, we want to lease at least 20 compacts, 15 mid-size cars, and 20 SUVs with no more than 5 cars per car model in order to offer a wide range of selection. For the environmentally friendly approach, we restrict the total CO_2 emission to less than 500,000 kg/year. Assuming average speed is 50 km/hr under perfect road condition, a driver can drive 50km/hr * 40 hrs/week = 2000 km/week, which is equivalent to a maximum of 100,000km/year. We will charge $1.00, $1.25, and $1.50 for each kilometer driven by our driver using compacts, mid-size cars, and SUVs accordingly. The fuel cost is $0.92/L for regular gas and $1.22/L for premium gas depends on cars' model, gasoline price was obtained on October 15, 2020 for Esso in Ontario. By summing up the profit earn from total distance driven by all leased car using the rates above, we will be able to estimate the gain in profit overall.

Current minimum wage in Ontario is $14.25/hr, as obtained in October 2020. Giving that, we will hire enough full-time drivers so that each driver can accompany one leased vehicle, they will earn a fixed base salary at $14.25 per hour for 40 hours per week with 50 weeks per year will yield a total of $28,500 annual salary per driver. Besides that, we will also award drivers with 20% commission with each dollar we earn from driving distance on top of the base salary.

With the assumptions stated above, we have the numerical example model and the optimal solution shown below:

<u>Numerical model</u>

i = {1: BMW 228i xDrive Gran Coupe... 43: Volvo XC40 T4 AWD}

j = {1: compact; 2: mid-size; 3: SUV}

$$\text{Max} \quad z = \sum_{i=1}^{43}\sum_{j=1}^{3} R_{ij} * X_{ij} * D_{ij} - \sum_{i=1}^{43}\sum_{j=1}^{3} P_{ij} * X_{ij} - \sum_{i=1}^{43}\sum_{j=1}^{3} X_{ij} * D_{ij} * FC_{ij} * FP_{ij} - 28{,}500 * \sum_{i=1}^{43}\sum_{j=1}^{3} X_{ij} - 0.2 * \text{Revenue}$$

s.t. $\sum_{i=1}^{43}\sum_{j=1}^{3} E_{ij} * X_{ij} * D_{ij} \le 500{,}000$ kg/km

$\sum_{i=1}^{15} X_{i1} \ge 20$; $\sum_{i=15}^{28} X_{i2} \ge 15$; $\sum_{i=29}^{43} X_{i3} \ge 20$

$D_{ij} \ge 30{,}000 * Y_{ij}$; $D_{ij} \le 100{,}000 * Y_{ij}$

$\sum_{i=1}^{15} Y_{i1} \ge 5$; $\sum_{i=15}^{28} Y_{i2} \ge 5$; $\sum_{i=29}^{43} Y_{i3} \ge 5$

$X_{ij} \le Y_{ij} * M$; $X_{ij} \le 5$

$Y_{ij} \le X_{ij}$

<u>Optimal solution</u>

Table 1. Simplified table of actual feasible solutions from Python.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
Hyundai Accent Preferred Compact	5	30000	1
Honda Fit LX-Honda Sensing Compact	5	100000	1
Acura ILX Compact	5	100000	1

Volkswagen Jetta Comfortline Compact	4	30000	1
Chevrolet Spark Compact	1	30000.78	1
Honda Civic Sedan CVT Mid-size	1	30000	1
Subaru Impreza 4-Door AWD Mid-size	5	30000	1
Volkswagen Passat Comfortline Mid-size	1	30000	1
Toyota Prius Mid-size	5	100000	1
Lexus UX 200 Mid-size	3	30000	1
Honda Civic Hatchback LX SUV	5	100000	1
Subaru Crosstrek AWD Convenience SUV	5	36929.58	1
Mazda CX-30 SUV	1	30000	1
Kia Forte 5 EX SUV	4	30000	1
Toyota RAV4 LE FWD SUV	5	30000	1

***The full optimal solution that includes cars not being used are also shown in Table 1.ii.*

Maximum Profit = $988,638.17

CO_2 emission = 500,000 kg with 20 compacts, 15 mid-size cars and 20 SUVs, 5 types of car for each class

Discussion

We came up with seven possible scenarios including the base-case for different circumstances within the organization (some changes in the right-hand side of constraints were also performed to manually test the sensitivity of this non-linear model in order to observe how it changes the optimal solution. These adjustments are combined in the scenario for easier comparison). The objective function (maximizing the profit) was then compared to our base-case model with the exception of scenario 1, where if the company were to change their goal and to focus on minimizing their CO_2 emission.

Scenario 1: Minimize the CO_2 emission while achieving the minimal profit goal.

Scenario 1 is a more ambitious approach by changing the objective function to minimizing the CO_2 emission caused by the vehicles. The optimal solution for minimizing the yearly CO_2 emission are shown in Table 2. Changes of the model was that the CO_2 emission limit of 500,000 kg was obviously removed and was replaced with a minimum profit goal. All other constraints remained the same as the base-case model. In the example solution, after setting the profit goal as $2,000,000 (constraint: Revenue – Leasing Cost – Fuel Cost – Salary = Profit ≥ 2000000), the minimum CO_2 emission was determined to be 643,681.19 kg. Therefore, the solution for this model in Scenario 1 can be recommended to the company if the primary focus were to minimize their contribution in polluting the air with massive amounts of CO_2.

<u>Model</u>

Min CO_2 emissions = $\sum_{i=1}^{n}\sum_{j=1}^{m} E_{ij} * X_{ij} * D_{ij}$

s.t. Revenue – leasing cost – fuel cost – salary ≥ minimum profit goal all other constraints remain the same

Example solution

Minimum profit goal at least $2 mil, Minimum CO_2 Emission = 643,681.19 kg.

Scenario 2: Trade-off between maximizing profit and minimizing CO_2 pollution by changing the right-hand side value for CO_2 emission constraint (sensitivity analysis of constraint (1)).

Scenario 2 is focused on adjusting the constraint for CO_2 emission. Although it is similar to scenario 1, the difference is that the objective function in scenario 2 is to maximize the profit like the base-case model while scenario 1's objective function was to minimize the CO_2 emission. Moreover, scenario 2 examines how changing the CO_2 emission restriction affects the company's profit. From a business perspective, the company could look into a trade-off between profit versus maintaining CO_2 pollution to a minimal.

There are two parts for scenario 2, one where the CO_2 emission constraint was entirely removed and the second part was increasing the maximum from 500,000 kg to 700,000 kg of CO_2. One important remark is that when the CO_2 constraint was set lower than the base-model (500,000 kg), the model became infeasible or no solution found after going through 10,000 iterations in Python. In other words, the model took a long time to run and in the end a solution could not be found using Python when setting the maximum CO_2 emission any lower than 500,000 kg.

The solution for scenario 2.i is shown in Table 3 with the maximum profit being $11,486,752.20. However, the CO_2 emission was calculated to be 3,742,000 kg. In scenario 2.ii, the maximum profit was determined to be $2,178,981.94 when the maximum CO_2 emission was set to 700,000 kg. Table 4 displays the vehicles recommended for scenario 2.ii. Compared to the base-case model, the CO_2 emission maximum was 500,000 kg and the maximum profit would be $988,638.17. Both adjustments of the right-hand side value of CO_2 emission constraint resulted in a higher maximum profit than the base-case model. This shows that there is a positive linear relationship between the CO_2 emission and profit. Or according to the model, there is a positive linear relationship between the distance a vehicle travelled and CO_2 emission as well as the distance travelled and the profit.

In terms of how sensitive the objective function was towards changing the CO_2 constraint, it could be considered as quite sensitive as the maximum profit increased by a lot after removing or increasing the CO_2 limits. As for recommendations, should the company decide to focus more on increasing their profit and care less about CO_2 emission then they can consider loosening the CO_2 emission limit.

Scenario 2.i: Removing the CO2 emission constraint. Maximum Profit = $11,486,752.20 and CO_2 Emission = 3,742,000 kg

***See Table 3 in Appendix A for details.*

Scenario 2.ii: Optimal solution when CO2 emission constraint was increased from 500,000 kg/year to 700,000 kg/year. Maximum Profit = $2,178,981.94.

***See Table 4 in Appendix A for details.*

Scenario 3: Changing maximum number of cars with model i in class j leased (sensitivity analysis of constraint (6)) for the purpose of having variety of vehicle selection.

The company may increase or decrease the maximum number of cars with certain model i in class j they are willing to lease for their drivers for possible reasons such as production limit from car factories, or just preferences. The two different scenarios are presented in detail below:

<u>Scenario 3.i</u>: Decrease $X_{ij} \leq 5$ to $X_{ij} \leq 2$. Tables 5 and 6 in Appendix A shows that there are some flaws with the solution. For example, Volkswagen Jetta Comfortline Compact was recommended to have 2 purchases with each running about 1.22 km. In reality, that is inefficient. Furthermore, if the binary variable Y_{ij} ($Y_{\text{Volkswagen Jetta Comfortline Compact}}$) is 0 then $X_{\text{Volkswagen Jetta Comfortline Compact}}$ and $D_{\text{Volkswagen Jetta Comfortline Compact}}$ should also be 0 instead of having a value greater than zero. The overall profit however is \$1,060,290 which is higher than the base-case model by \$71,651.83. A possible reason that the solution does not exactly follow the desired constraints could be due to an inappropriate large M value in $X_{ij} \leq M * Y_{ij}$ which was set as 90,000, like the base-case.

***See Table 5 in Appendix A for details.*

<u>Scenario 3.ii</u>: Increase $X_{ij} \leq 5$ to $X_{ij} \leq 10$. Same issue as scenario 3.i. However, Maximum profit = \$613,576.62 which is significantly less compared to the base-case model.

***See Table 6 in Appendix A for details.*

The model takes into consideration of other constraints such as maximizing the profit as well the CO_2 limit, in this case 500,000 kg. Therefore, if the maximum limit of for example Acura ILX Compact was increased from 5 to 10 then it would be more efficient for the company to lease out 10 of them instead of leasing 5 plus other cars. This would decrease the overall variety of different types of cars recommended to be leased. Alternatively, if the limit for Acura ILX Compact was decreased to only 2 then the model would have to look for other cars to satisfy the other constraints such as adding the CO_2 emission up to 500,000 kg. While it is a valid point to simply increase the distance travelled for that certain type of car, but it would cause wear and tear of the vehicle quicker. That is why a maximum mileage constraint of 100,000 km per year of each car was included, and as a result, different types of cars would be recommended as the optimal solution by the model.

From those two scenarios, 3.i would be the better choice for increasing the maximum profit, but not by a significant amount. Overall, changing the parameter values for the maximum number of cars with model i, vehicle class j leased does not contribute much to the maximum profit of the company.

Scenario 4: At least one of every available car is leased.

<u>Scenario 4.i</u>: At least one of every available car is leased. $Y_{i1} + Y_{i2} + Y_{i3} = 43$, where we have 15 compact (sum of Y_{i1}) cars, 13 mid-size (sum of Y_{i2}), and 15 SUVs (sum of Y_{i3}). Maximum profit determined was \$921,868.26 which was \$66,769.91 less than the maximum profit obtained from the base-case model.

***See Table 7 in Appendix A for details.*

<u>Scenario 4.ii</u>: At least one of every available car is purchased but removed the constraint for maximum number of cars the company wants to lease ($X_{ij} \leq 5$) and the minimum number of cars the company

needs for each class to run the business (Sum of $X_{ij} \geq 20$, 15, and 20 for compact, mid-size, and SUV respectively). Maximum Profit = $789,399.74, it's a lot less than the base-case model.

***See Table 8 in Appendix A for details.*

Overall, it is more profitable if the company decreased the variety of cars to be leased/purchased.

Scenario 5: If the company could choose one car out of the entire dataset, which would be the best one?

In this scenario: removed constraint limits for maximum number of cars the company wants to lease ($X_{ij} <= 5$) and removed the minimum number of cars the company needs for each class to run the business (Sum of $X_{ij} >= 20$, 15, and 20 for compact, mid-size, and SUV respectively). Applied logical constraint $Y_{i1} + Y_{i2} + Y_{i3} = 1$. Maximum profit was calculated to be $2,159,541.23 which is a lot higher than the base-case model due to the restrictions of the maximum and minimum number of cars being leased was lifted.

***See Table 9 in Appendix A for details.*

Scenario 5 assumes that if car manufacturers do not have any issues with production and inventory limits of any vehicle and the company decided to have one standardized vehicle to do the job. The model concluded that the Hyundai Kona SUV would be the best choice to maximize profit for the company as well as satisfying the model constraints. The maximum profit determined was $2,159,541.23 which is significantly higher than the base-case solution of $988,638.17.

Scenario 6: Removing the minimum number of cars required for each vehicle class.

This scenario is for if the company does not care about the minimum amount of compact, mid-size, or sport utility vehicles (SUVs) to be leased out.

***See Table 10 in Appendix A for details.*

After removing the constraints: $\sum X_{i1} \geq 20$ (compact), $\sum X_{i2} \geq 15$ (mid-size), and $\sum X_{i3} \geq 20$ (SUV), the model suggested the cars and their distance travelled in Table 10. The maximum profit was determined to be $1,481,866.28 which is higher than the base-case model. The optimal solution suggested 13 compact vehicles, 9 mid-size vehicles, and 9 SUVs as the most efficient. Therefore, this shows that tightening the constraint by increasing the right-hand side worsens the z (the maximum profit).

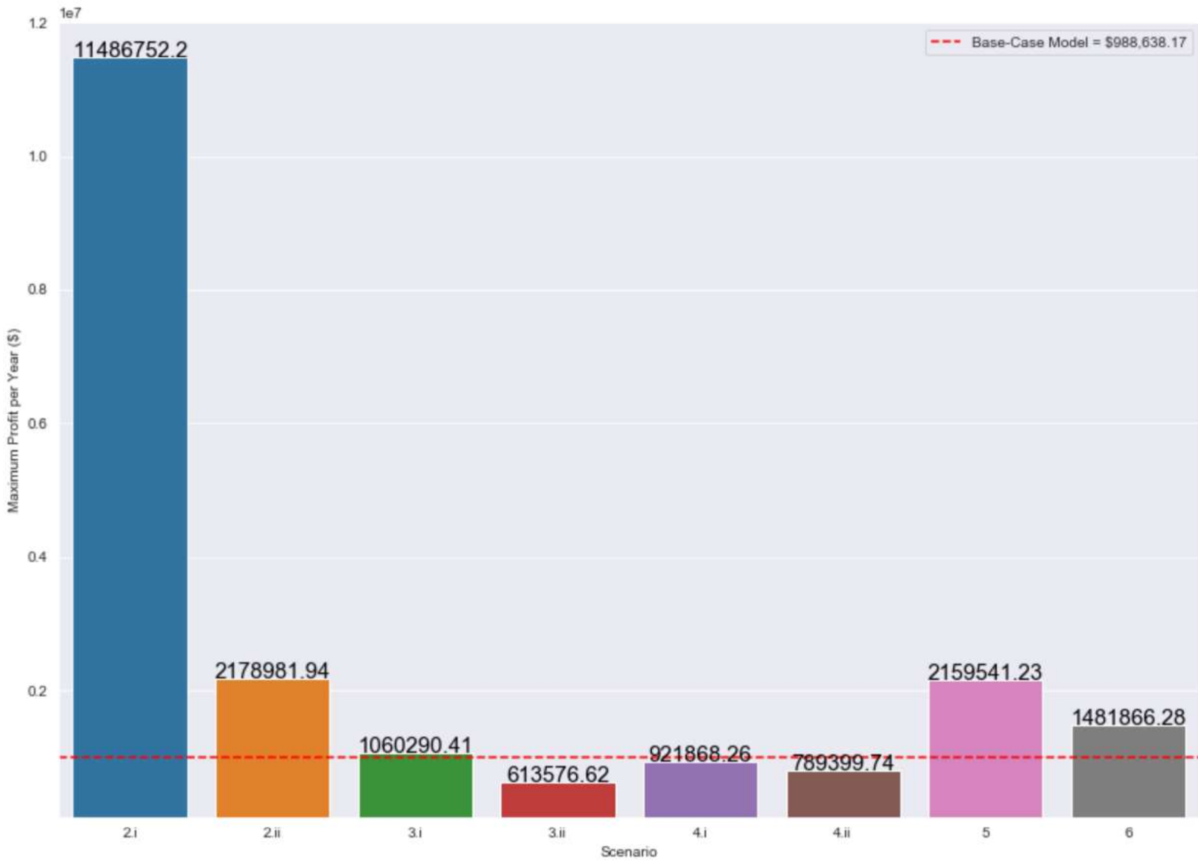
A summary of the profit obtained from 6 different scenarios are shown in Table 11:

Table 2. Summary of the maximum profits from different scenarios. Note that scenario 1 is N/A since the objective function was to minimize CO_2 Emission.

Scenario	Max Profit ($)	Compared to Base-case ($)
1	N/A	N/A
2.i	11486752.20	+10498114.03
2.ii	2178981.94	+1190343.77
3.i	1060290.41	+71652.24
3.ii	613576.62	-375061.55
4.i	921868.26	-66769.91

4.ii	789399.74	-199238.43
5	2159541.23	+1170903.06
6	1481866.28	+493228.11
Base-case	988638.17	0

Figure 10. Visualization of Table 11. Summary of how each scenario affected the Maximum Profit. The red dashing horizontal line is the maximum profit for the base-case model to easily compare the profit to each scenario.



Summary & Future Work

As mentioned previously in the Introduction, for a better model, we can put more focus on the car's list for more car selection such as more car's size (couple, van, minivan, truck…), fuel/engine type (hybrid, electric cars…), more car's make and year. We can also put more thoughts into the car's related costs such as insurance cost, leasing cost from different providers, maintenance cost, company's group discount, trade-in values, etc. Since the choice of cars will directly impact the CO_2 emissions (e.g. electric cars will have much more friendly CO_2 emissions compared to cars that run on fossil oil), we can consider hybrid and electric cars. We can also put restriction on the engine and cylinder size to make sure we have a strong enough car. Company can also consider a more detailed plan regarding to city versus highway driving habits for each potential employee instead of using our combined ratio with 45% highway and 55% city roads.

In conclusion, our maximum profit for the base case is $988,638.17, achieved from leasing 55 cars in total (with no more 5 cars per model, see Table 1 for detailed solution), and the total driven distance is 3,084,649 km/year (when taking into consideration that each car should be able to travel 30,000-100,000 km/year). With this optimal solution, we exhausted the CO_2 constraint, which makes the total CO_2 emission to be equal to 500,000 kg/year. For the number of cars, we end up leasing 20 compacts, 15 mid-size and 20 SUVs which is right at our bare minimum level with some cars having the total yearly distance at the minimum restriction of 30,000 km/year. This leads us to scenario 6 to remove the pre-set number of leased cars so that all leased cars will be pushed to drive to the maximum distance of 100,000km/year, which leads to a positive increase in profit. However, we would like to keep the variety in car selection, so our base case is more ideal in that sense. Furthermore, we will need to perform more study to examine if the net change in profit is worth the sacrifice for car selection. Briefly, we can further discuss and build our model considering some aspects listed below:

(1) Modify the function so that we will earn profit as per trip basis instead of distance and add some charges associated with each trip such as booking fee and base fare.
(2) Take into consideration hybrid and electric cars for more environmentally approach.
(3) Add more type of cost to the operation's expenses such as maintenance cost and insurance cost.
(4) Modify the model so it can become easier to code and faster to run on Python and other programming languages.
(5) Compare leasing and purchasing cars options with some resell values.
(6) Examine about tightening/loosening the RHS values of constraints for net gain in profit to see if it is worth it to sacrifice some conditions.

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Appendix A

Figure 1: Relationship between CO2 Emission and features of the cars (data for 2020 car models only)

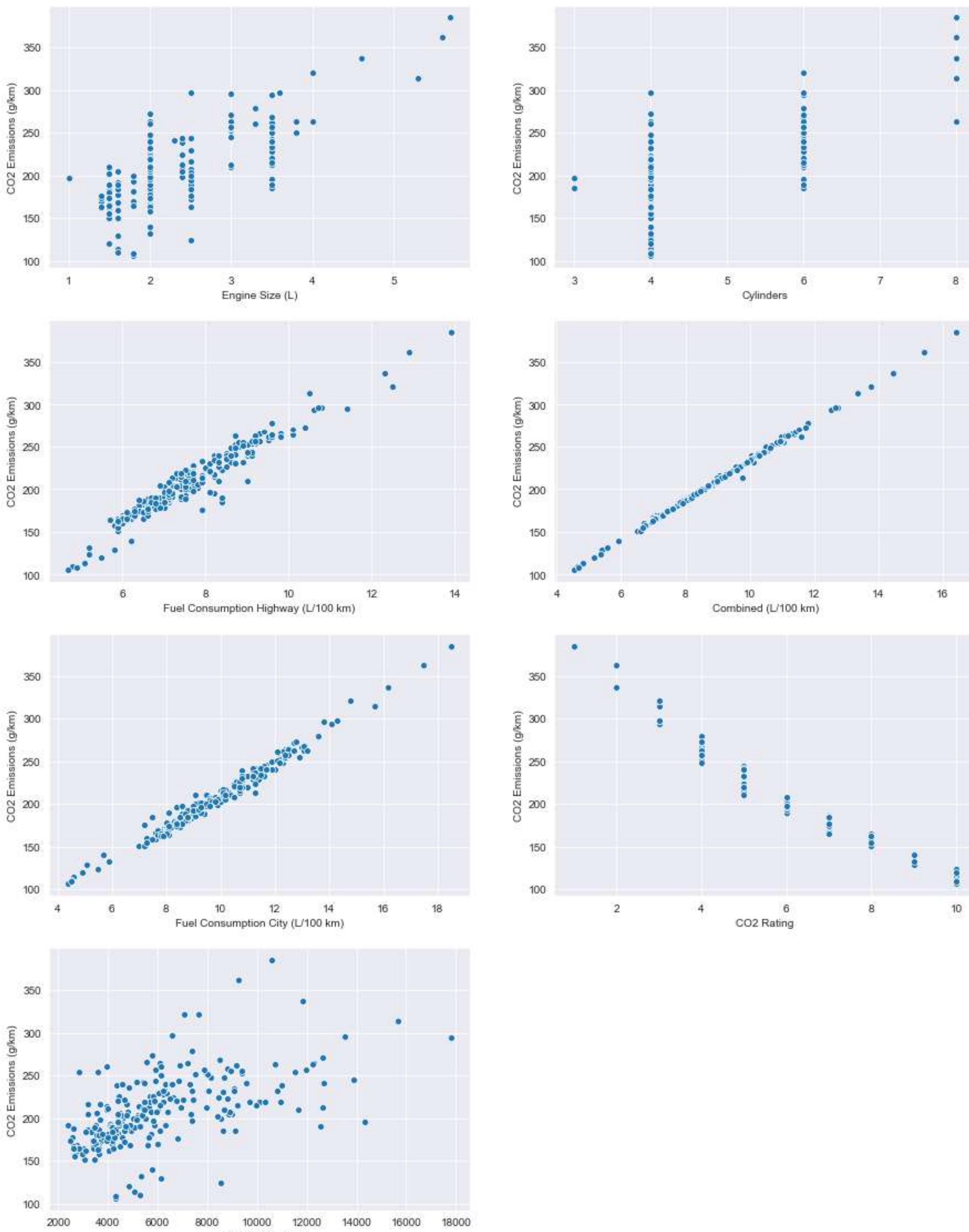

Figure 2: Relationship between Annual fuel cost and features of the cars (data for 2020 car models only)

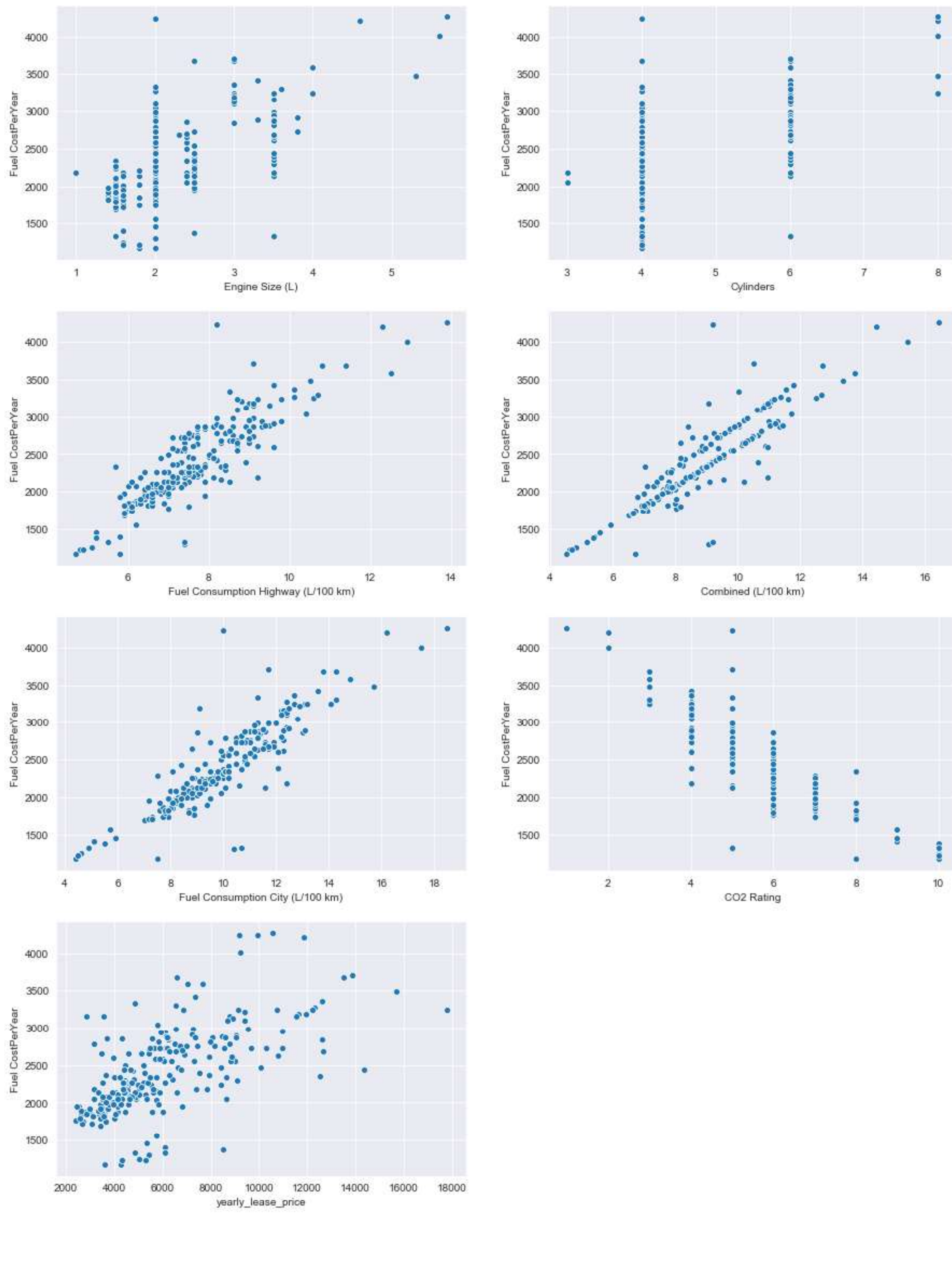

Figure 3: Relationship between CO_2 rating and features of the cars (data for 2020 car models only)

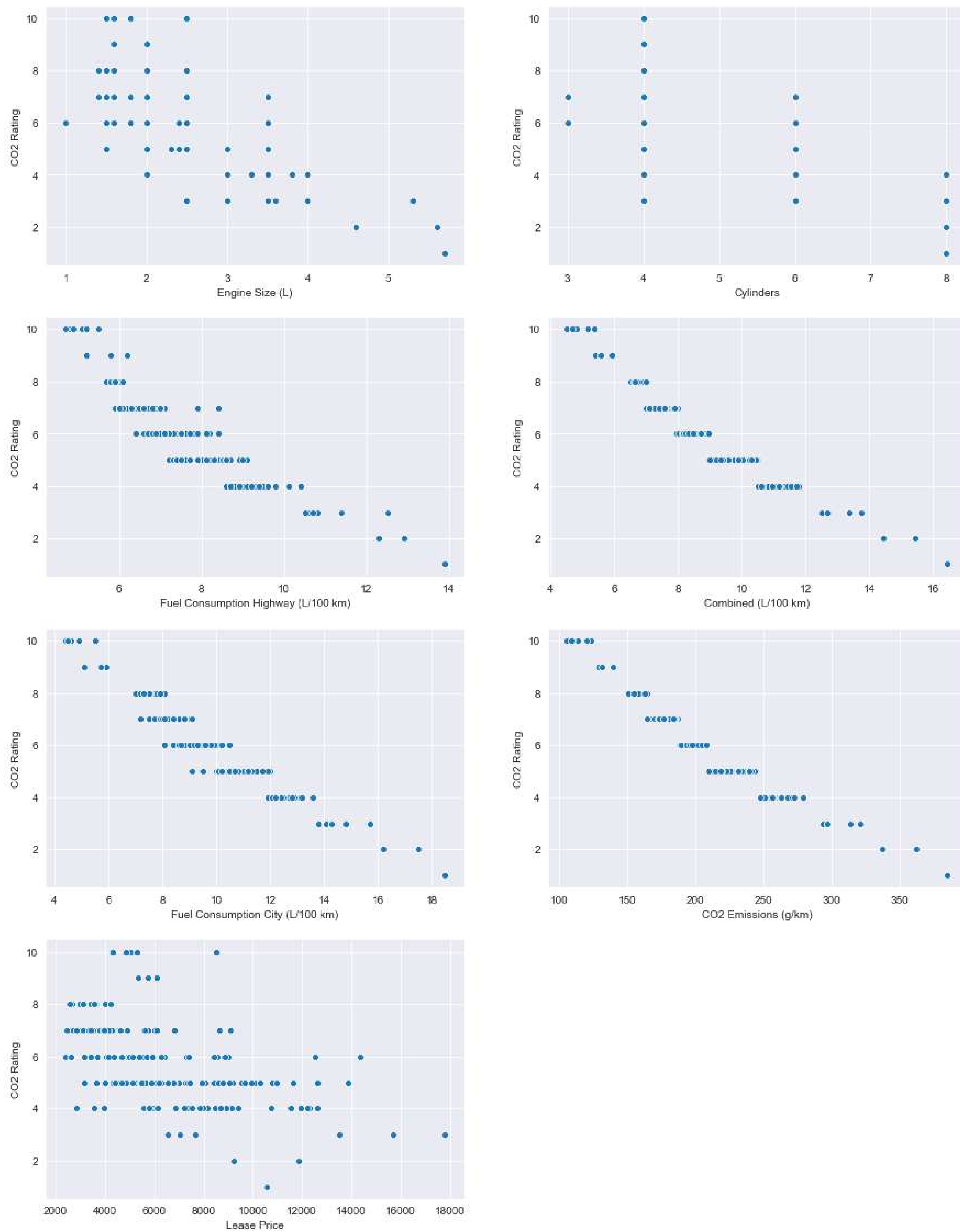

Figure 4: Relationship between Regular's type fuel annual cost and features of the cars (data for 2020 car models only)

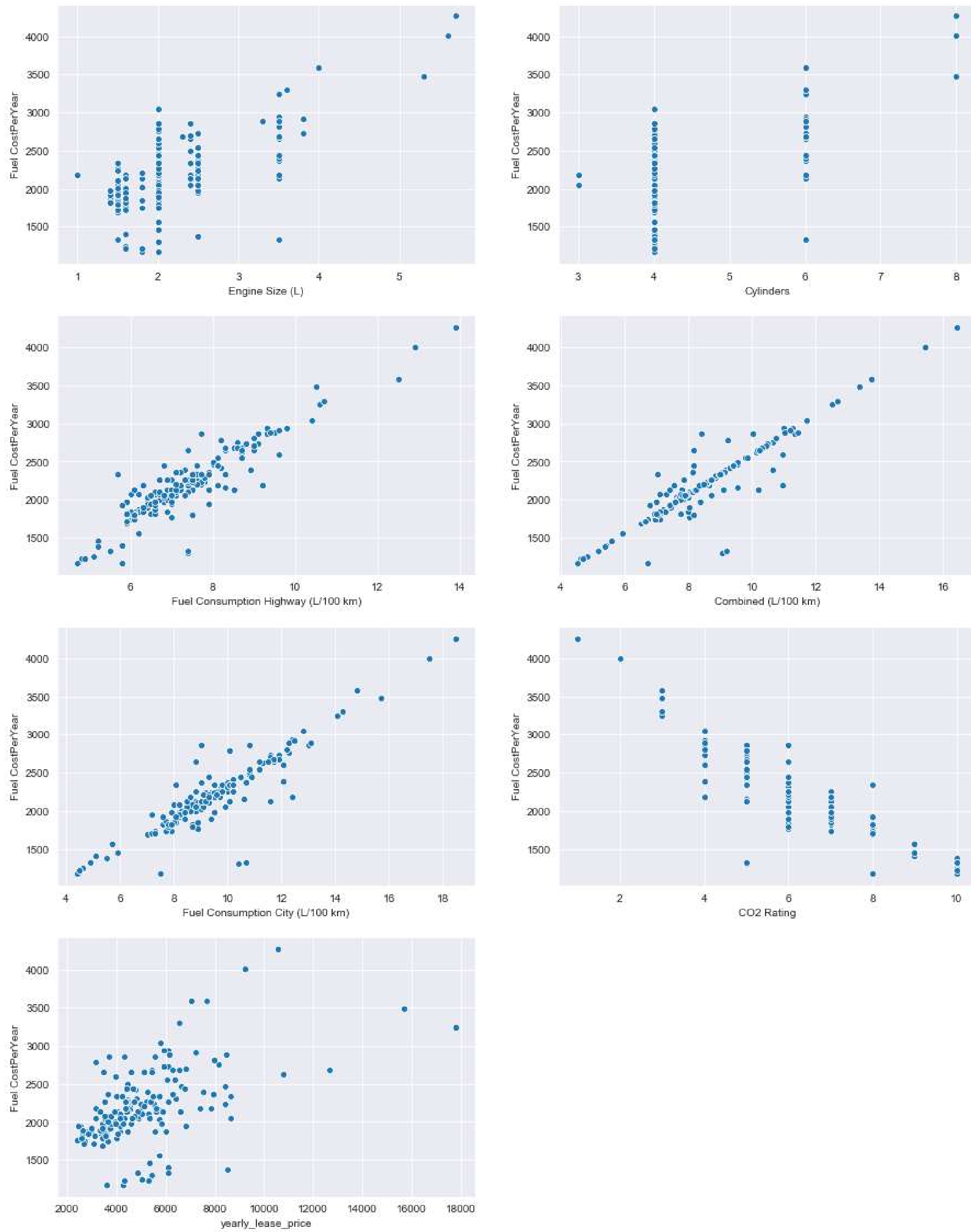

Figure 5: Relationship between Premium's type fuel annual cost and features of the cars (data for 2020 car models only)

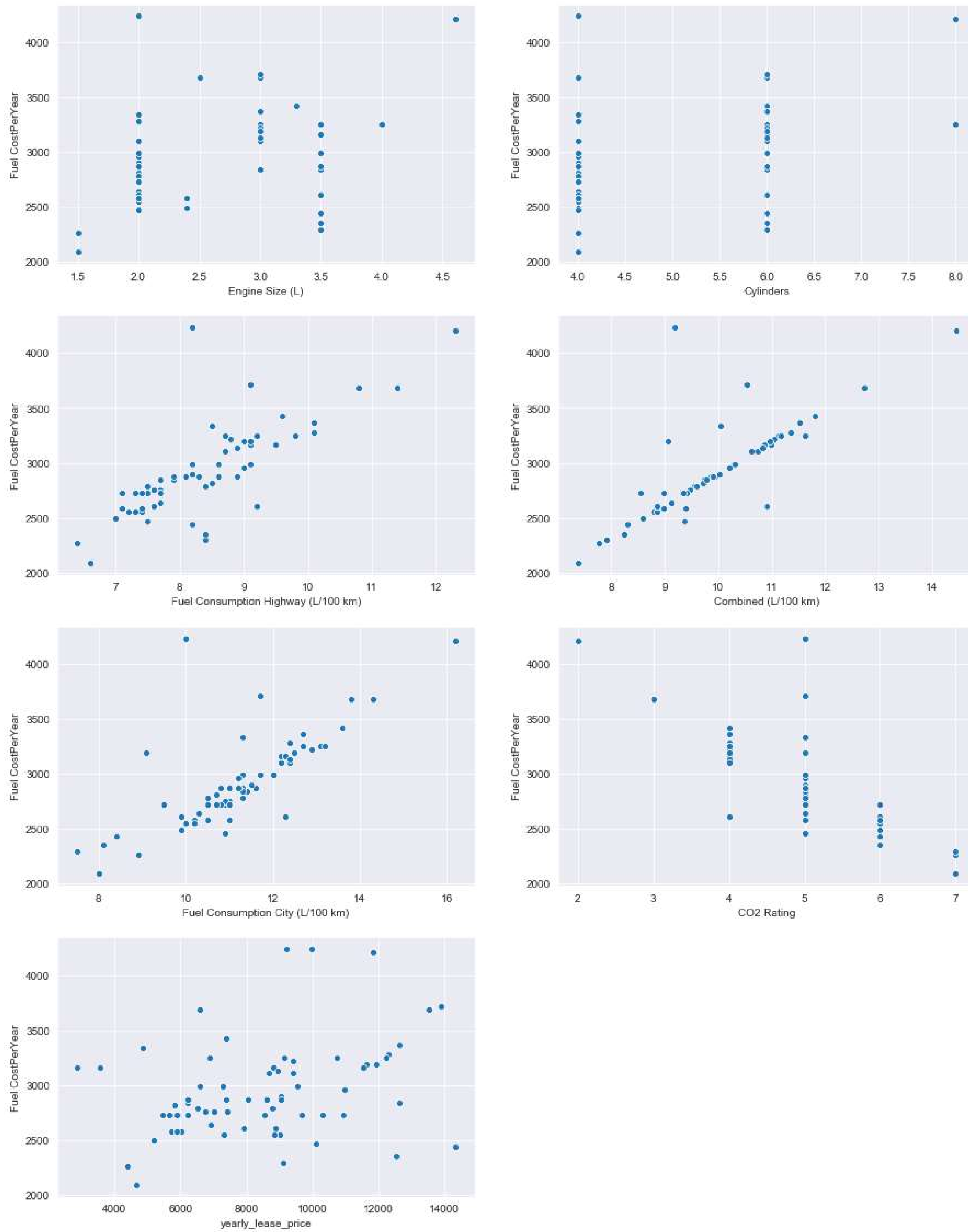

Figure 6: Relationship between leasing price and CO2 emission (data for 2020 car models only)

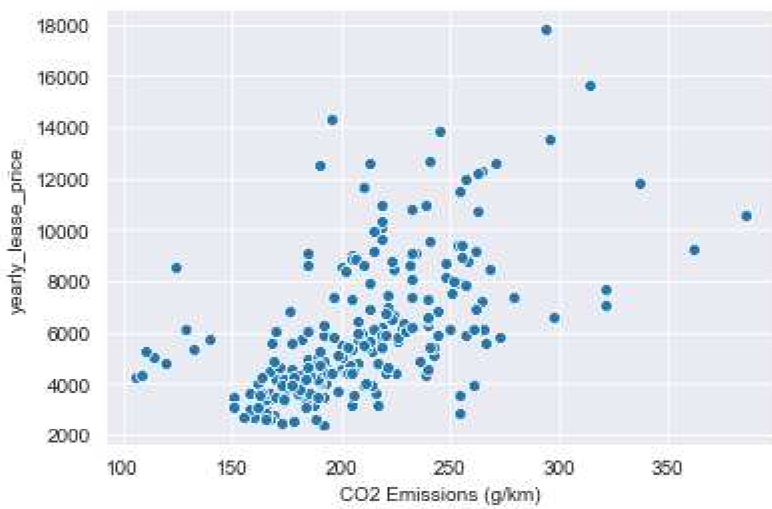

Figure 7: Comparison between fuel types and their Annual fuel cost vs. CO2 emission (data for 2020 car models only)

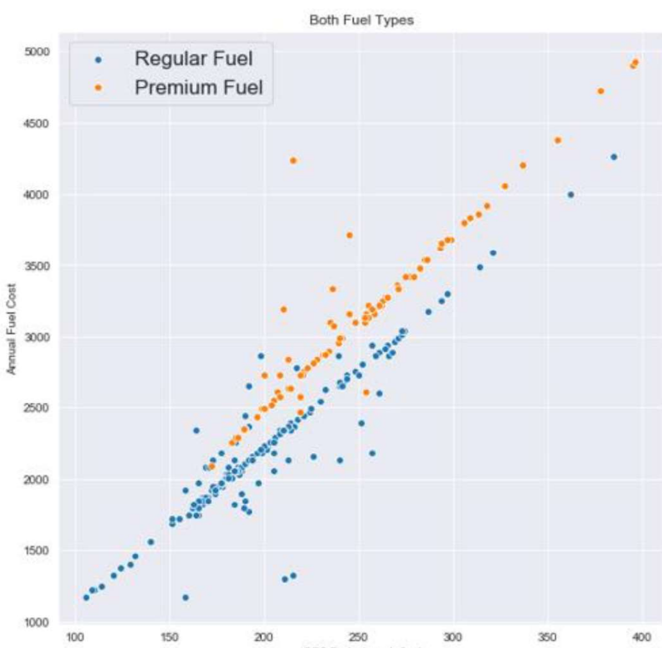

Figure 8. Histogram of the CO_2 Emissions of cars. Majority of them range from around 160 g/km to 275 g/km.

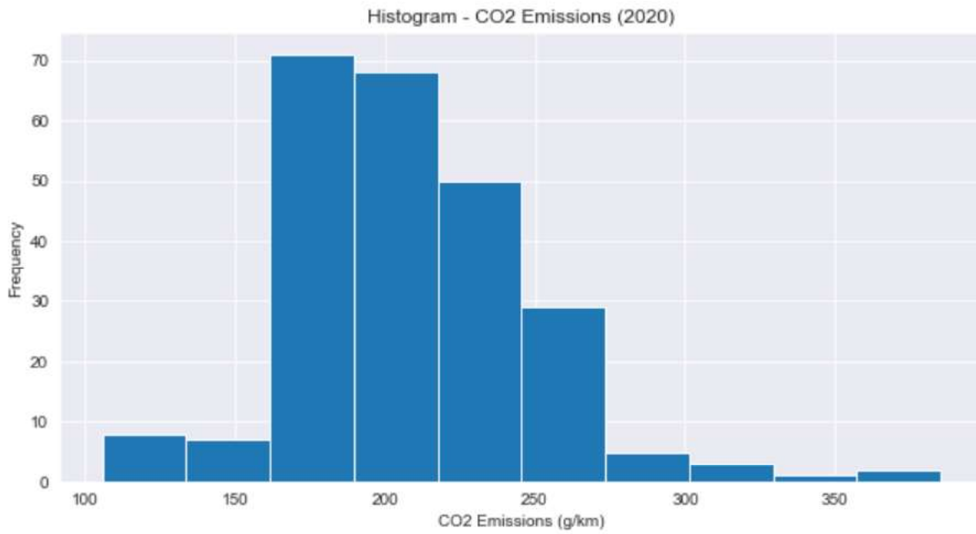

Figure 9. Histogram of the car prices paid in full. This determined that the company should focus on cars that cost less than $75,000.

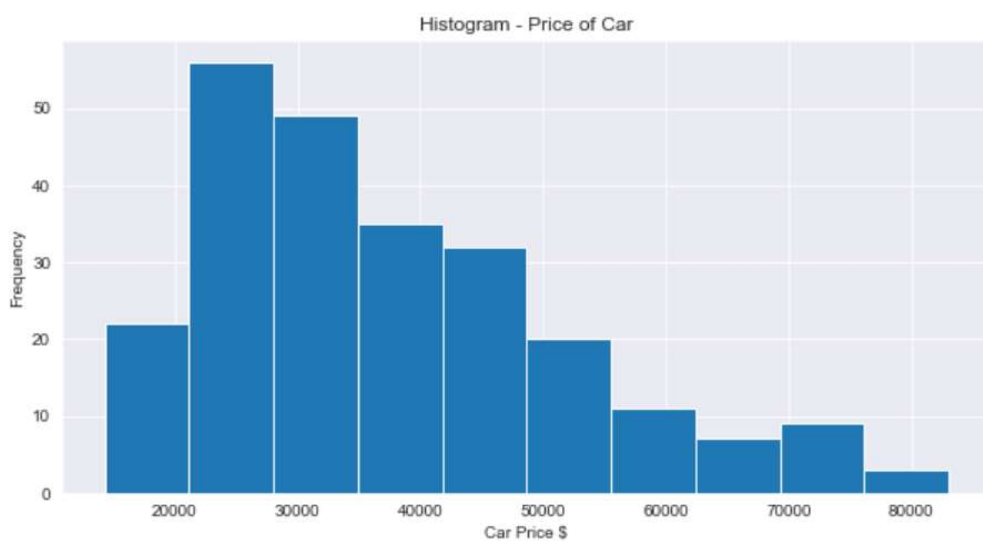

Table 1.i: List of raw data used for the analysis (data for 2020 car models only, and please note that we only use 1/5 of the actual 2020 car list that we used to generate the above figures so that Python can run faster)

Make	Model	Vehicle Class	Fuel Price ($/L)	Fuel Cons City (L/100 km)	Fuel Cons Highway (L/100 km)	Fuel Cons Combined (L/100km)	CO_2 Emissions (kg/km)	Yearly lease price ($)
BMW	228i xDrive Gran Coupe	Compact	1.22	10.2	7.2	0.0885	0.205	8844
Audi	A3 Komfort	Compact	0.92	8.8	6.5	0.0777	0.182	5742.84
Hyundai	Accent Preferred	Compact	0.92	7.3	6	0.0672	0.16	2684.16
Mercedes-Benz	C 300 4MATIC	Compact	1.22	11	7.3	0.0934	0.219	6201.24
Honda	Fit LX-Honda Sensing	Compact	0.92	7	5.9	0.0651	0.151	3435.84
Acura	ILX	Compact	1.22	9.9	7	0.0860	0.199	5203.56
Lexus	IS 300 AWD	Compact	1.22	12.3	9.1	0.1086	0.254	2848.92
Volkswagen	Jetta Comfortline	Compact	0.92	7.8	5.9	0.0695	0.162	3114.6
Mazda	Mazda3 4-Door (SIL)	Compact	0.92	8.7	6.4	0.0767	0.18	3630.24
Nissan	Qashqai	Compact	0.92	10.1	8.2	0.0925	0.217	3180
Kia	Rio LX+	Compact	0.92	7.2	5.9	0.0662	0.151	3058.44
Volvo	S60 T5	Compact	1.22	10.5	7.1	0.0897	0.208	5703.6
Chevrolet	Spark	Compact	0.92	7.9	6.2	0.0714	0.167	3612
Subaru	WRX AWD	Compact	1.22	11.3	8.5	0.1004	0.236	4852.8
Toyota	Yaris Hatchback 6AT	Compact	0.92	7.3	5.9	0.0667	0.155	2655.96
Audi	A5 Sportback quattro	Mid-size	1.22	10	7.3	0.0879	0.205	8998.44
Honda	Civic Sedan CVT	Mid-size	0.92	7.8	6.2	0.0708	0.165	3012.96
Hyundai	Elantra	Mid-size	0.92	8.9	7	0.0805	0.192	2387.04
Kia	Forte LX MT	Mid-size	0.92	8.7	6.6	0.0776	0.184	3112.32
Ford	Fusion	Mid-size	0.92	10	7	0.0865	0.203	5196
Subaru	Impreza 4-Door AWD	Mid-size	0.92	8.3	6.4	0.0745	0.174	4160.76
Mazda	Mazda3 5-Door (SIL)	Mid-size	0.92	8.7	6.6	0.0776	0.181	3766.32
Volkswagen	Passat Comfortline	Mid-size	0.92	10.2	6.9	0.0872	0.205	4371
Toyota	Prius	Mid-size	0.92	4.4	4.7	0.0454	0.106	4277.88

Acura	RLX Hybrid	Mid-size	1.22	8.4	8.2	0.0831	0.196	14341.92
Volvo	S90 T6 AWD	Mid-size	1.22	11.3	7.5	0.0959	0.223	8780.4
Nissan	Sentra	Mid-size	0.92	8	6	0.0710	0.167	2616
Lexus	UX 200	Mid-size	0.92	8	6.3	0.0724	0.168	5578.92
Honda	Civic Hatchback LX	SUV	0.92	7.7	6	0.0694	0.162	4025.28
Subaru	Crosstrek AWD Convenience	SUV	0.92	8.5	7	0.0783	0.185	4650.72
Mazda	CX-30	SUV	0.92	8.9	7.1	0.0809	0.189	4127.04
Ford	Edge SE	SUV	0.92	11.2	8.1	0.0981	0.229	6360
Kia	Forte 5 EX	SUV	0.92	8.9	6.9	0.0800	0.19	4056.36
Hyundai	Kona	SUV	0.92	8.6	7	0.0788	0.187	3173.16
Acura	MDX SH-AWD	SUV	1.22	12.3	9.2	0.1091	0.254	7923.48
Lexus	NX 300 AWD	SUV	1.22	10.7	8.5	0.0971	0.226	5828.16
Audi	Q3 quattro	SUV	0.92	12.3	8.6	0.1064	0.248	8148
Toyota	RAV4 LE FWD	SUV	0.92	8.8	6.8	0.0790	0.184	4160.76
Nissan	Rogue	SUV	0.92	9.1	7.1	0.0820	0.192	4452
GMC	Terrain	SUV	0.92	9.2	7.8	0.0857	0.202	8412
Volkswagen	Tiguan Trendline	SUV	0.92	10.5	8.1	0.0942	0.221	4677.6
BMW	X1 xDrive28i	SUV	1.22	10.3	7.7	0.0913	0.213	6924
Volvo	XC40 T4 AWD	SUV	0.92	10.2	7.5	0.0899	0.21	5469.6

Table 1.ii. List of full solutions from Python for our base case. Maximum Profit = $988,638.17.

Model and Class	Xij	Dij	Yij
BMW 228i xDrive Gran Coupe Compact	0	0	0
Audi A3 Komfort Compact	0	0	0
Hyundai Accent Preferred Compact	5	30000	1
Mercedes-Benz C 300 4MATIC Compact	0	0	0
Honda Fit LX-Honda Sensing Compact	5	100000	1
Acura ILX Compact	5	100000	1
Lexus IS 300 AWD Compact	0	0	0
Volkswagen Jetta Comfortline Compact	4	30000	1
Mazda Mazda3 4-Door (SIL) Compact	0	0	0
Nissan Qashqai Compact	0	0	0

Kia Rio LX+ Compact	0	0	0
Volvo S60 T5 Compact	0	0	0
Chevrolet Spark Compact	1	30000.78	1
Subaru WRX AWD Compact	0	0	0
Toyota Yaris Hatchback 6AT Compact	0	0	0
Audi A5 Sportback quattro Mid-size	0	0	0
Honda Civic Sedan CVT Mid-size	1	30000	1
Hyundai Elantra Mid-size	0	0	0
Kia Forte LX MT Mid-size	0	0	0
Ford Fusion Mid-size	0	0	0
Subaru Impreza 4-Door AWD Mid-size	5	30000	1
Mazda Mazda3 5-Door (SIL) Mid-size	0	0	0
Volkswagen Passat Comfirtline Mid-size	1	30000	1
Toyota Prius Mid-size	5	100000	1
Acura RLX Hybrid Mid-size	0	0	0
Volvo S90 T6 AWD Mid-size	0	0	0
Nissan Sentra Mid-size	0	0	0
Lexus UX 200 Mid-size	3	30000	1
Honda Civic Hatchback LX SUV	5	100000	1
Subaru Crosstrek AWD Convenience SUV	5	36929.58	1
Mazda CX-30 SUV	1	30000	1
Ford Edge SE SUV	0	0	0
Kia Forte 5 EX SUV	4	30000	1
Hyundai Kona SUV	0	0	0
Acura MDX SH-AWD SUV	0	0	0
Lexus NX 300 AWD SUV	0	0	0
Audi Q3 quattro SUV	0	0	0
Toyota RAV4 LE FWD SUV	5	30000	1
Nissan Rogue SUV	0	0	0
GMC Terrain SUV	0	0	0
Volkswagen Tiguan Trendline SUV	0	0	0
BMW X1 xDrive28i SUV	0	0	0
Volvo XC40 T4 AWD SUV	0	0	0

Table 3. Optimal Solution for Scenario 1. If the company's profit goal was set to at least $2 million, then the minimum CO2 Emission would be 643,681.19 kg.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
Hyundai Accent Preferred Compact	1	29994.99	1
Nissan Qashqai Compact	1	30000	1
Kia Rio LX+ Compact	1	100000	1
Chevrolet Spark Compact	1	30000	1
Toyota Yaris Hatchback 6AT Compact	1	30000	1
Honda Civic Sedan CVT Mid-size	5	100000	1
Kia Forte LX MT Mid-size	1	66345.06	1
Subaru Impreza 4-Door AWD Mid-size	1	100000	1
Nissan Sentra Mid-size	5	100000	1
Lexus UX 200 Mid-size	3	100000	1
Honda Civic Hatchback LX SUV	5	100000	1
Subaru Crosstrek AWD Convenience SUV	5	100000	1
Mazda CX-30 SUV	4	100000	1
Kia Forte 5 EX SUV	1	100000	1
Hyundai Kona SUV	5	100000	1

Table 4. Scenario 2.i: Removing the CO2 Emission constraint. Maximum Profit = $11,486,752.20 and CO_2 Emission = 3,742,000 kg.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
BMW 228i xDrive Gran Coupe Compact	5	100000	1
Audi A3 Komfort Compact	5	100000	1
Hyundai Accent Preferred Compact	5	100000	1
Volkswagen Jetta Comfortline Compact	5	100000	1
Mazda Mazda3 4-Door (SIL) Compact	5	100000	1
Nissan Qashqai Compact	5	100000	1
Kia Rio LX+ Compact	5	100000	1
Volvo S60 T5 Compact	5	100000	1
Chevrolet Spark Compact	5	100000	1
Subaru WRX AWD Compact	5	100000	1
Toyota Yaris Hatchback 6AT Compact	5	100000	1
Audi A5 Sportback quattro Mid-size	5	100000	1
Honda Civic Sedan CVT Mid-size	5	100000	1
Hyundai Elantra Mid-size	5	100000	1
Kia Forte LX MT Mid-size	5	100000	1
Ford Fusion Mid-size	5	100000	1
Subaru Impreza 4-Door AWD Mid-size	5	100000	1
Mazda Mazda3 5-Door (SIL) Mid-size	5	100000	1
Volkswagen Passat Comfortline Mid-size	5	100000	1

Toyota Prius Mid-size	5	100000	1
Acura RLX Hybrid Mid-size	5	100000	1
Volvo S90 T6 AWD Mid-size	5	100000	1
Nissan Sentra Mid-size	5	100000	1
Lexus UX 200 Mid-size	5	100000	1
Honda Civic Hatchback LX SUV	5	100000	1
Subaru Crosstrek AWD Convenience SUV	5	100000	1
Mazda CX-30 SUV	5	100000	1
Ford Edge SE SUV	5	100000	1
Kia Forte 5 EX SUV	5	100000	1
Hyundai Kona SUV	5	100000	1
Acura MDX SH-AWD SUV	5	100000	1
Lexus NX 300 AWD SUV	5	100000	1
Audi Q3 quattro SUV	5	100000	1
Toyota RAV4 LE FWD SUV	5	100000	1
Nissan Rogue SUV	5	100000	1
GMC Terrain SUV	5	100000	1
Volkswagen Tiguan Trendline SUV	5	100000	1
BMW X1 xDrive28i SUV	5	100000	1
Volvo XC40 T4 AWD SUV	5	100000	1

Table 5. Scenario 2.ii: Optimal solution when CO_2 Emission constraint was increased from 500,000 kg/year to 700,000 kg/year. Maximum Profit determined = $2,178,981.94.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
Hyundai Accent Preferred Compact	5	30000	1
Honda Fit LX-Honda Sensing Compact	5	100000	1
Acura ILX Compact	5	100000	1
Volkswagen Jetta Comfortline Compact	4	30000	1
Chevrolet Spark Compact	1	30000	1
Honda Civic Sedan CVT Mid-size	5	100000	1
Subaru Impreza 4-Door AWD Mid-size	3	30000	1
Toyota Prius Mid-size	5	100000	1
Nissan Sentra Mid-size	1	100000	1
Lexus UX 200 Mid-size	1	84761.9	1
Honda Civic Hatchback LX SUV	5	100000	1
Subaru Crosstrek AWD Convenience SUV	3	100000	1
Mazda CX-30 SUV	1	100000	1
Kia Forte 5 EX SUV	1	100000	1
Hyundai Kona SUV	5	30000	1
Toyota RAV4 LE FWD SUV	5	100000	1

Table 6. Optimal solution for Scenario 3.i: when decreasing the maximum number of each car model and class from 5 to 2. Maximum profit = $1,060,290.41.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
BMW 228i xDrive Gran Coupe Compact	0	0	0
Audi A3 Komfort Compact	0	0	0
Hyundai Accent Preferred Compact	2	30000	1
Mercedes-Benz C 300 4MATIC Compact	0	0	0
Honda Fit LX-Honda Sensing Compact	2	100000	1
Acura ILX Compact	2	100000	1
Lexus IS 300 AWD Compact	0	0	0
Volkswagen Jetta Comfortline Compact	2	1.223671319	0
Mazda Mazda3 4-Door (SIL) Compact	2	0.666666667	0
Nissan Qashqai Compact	2	0.666666667	0
Kia Rio LX+ Compact	2	30000	1
Volvo S60 T5 Compact	1	0.349595703	0
Chevrolet Spark Compact	2	0.666666667	0
Subaru WRX AWD Compact	2	0.666666667	0
Toyota Yaris Hatchback 6AT Compact	1	30000	1
Audi A5 Sportback quattro Mid-size	0	0	0
Honda Civic Sedan CVT Mid-size	2	100000	1
Hyundai Elantra Mid-size	2	0.666666667	0
Kia Forte LX MT Mid-size	2	0.666666667	0
Ford Fusion Mid-size	0	0	0
Subaru Impreza 4-Door AWD Mid-size	1	43025.38	1
Mazda Mazda3 5-Door (SIL) Mid-size	2	0.666666667	0
Volkswagen Passat Comfirtline Mid-size	0	0	0
Toyota Prius Mid-size	2	100000	1
Acura RLX Hybrid Mid-size	0	0	0
Volvo S90 T6 AWD Mid-size	0	0	0
Nissan Sentra Mid-size	2	100000	1
Lexus UX 200 Mid-size	2	100000	1
Honda Civic Hatchback LX SUV	2	100000	1
Subaru Crosstrek AWD Convenience SUV	2	100000	1
Mazda CX-30 SUV	2	100000	1
Ford Edge SE SUV	1	0.333333333	0
Kia Forte 5 EX SUV	2	100000	1
Hyundai Kona SUV	2	100000	1
Acura MDX SH-AWD SUV	0	0	0
Lexus NX 300 AWD SUV	0	0	0
Audi Q3 quattro SUV	0	0	0
Toyota RAV4 LE FWD SUV	2	100000	1
Nissan Rogue SUV	2	100000	1
GMC Terrain SUV	1	100000	1

Volkswagen Tiguan Trendline SUV	2	0.666666667	0
BMW X1 xDrive28i SUV	0	0	0
Volvo XC40 T4 AWD SUV	2	2.222222222	0

Table 7. Optimal solution for Scenario 3.ii: Increasing the number the company wants to lease for each type of car and the vehicle class from 5 to 10. Maximum profit = $613,576.62.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
BMW 228i xDrive Gran Coupe Compact	0	0	0
Audi A3 Komfort Compact	0	0	0
Hyundai Accent Preferred Compact	1	30000	1
Mercedes-Benz C 300 4MATIC Compact	0	0	0
Honda Fit LX-Honda Sensing Compact	10	100000	1
Acura ILX Compact	10	100000	1
Lexus IS 300 AWD Compact	0	0	0
Volkswagen Jetta Comfortline Compact	1	30000	1
Mazda Mazda3 4-Door (SIL) Compact	0	0	0
Nissan Qashqai Compact	0	0	0
Kia Rio LX+ Compact	1	30000	1
Volvo S60 T5 Compact	0	0	0
Chevrolet Spark Compact	0	0	0
Subaru WRX AWD Compact	0	0	0
Toyota Yaris Hatchback 6AT Compact	0	0	0
Audi A5 Sportback quattro Mid-size	0	0.10637	0
Honda Civic Sedan CVT Mid-size	1	1.088589	0
Hyundai Elantra Mid-size	3	1	0
Kia Forte LX MT Mid-size	1	30000	1
Ford Fusion Mid-size	0	0	0
Subaru Impreza 4-Door AWD Mid-size	0	0	0
Mazda Mazda3 5-Door (SIL) Mid-size	1	30000	1
Volkswagen Passat Comfirtline Mid-size	0	0	0
Toyota Prius Mid-size	7	100000	1
Acura RLX Hybrid Mid-size	0	0.098331	0
Volvo S90 T6 AWD Mid-size	0	0.057866	0
Nissan Sentra Mid-size	1	30000	1
Lexus UX 200 Mid-size	1	30000	1
Honda Civic Hatchback LX SUV	1	100000	1
Subaru Crosstrek AWD Convenience SUV	1	30550.89	1
Mazda CX-30 SUV	0	0	0
Ford Edge SE SUV	0	0	0
Kia Forte 5 EX SUV	6	2	0
Hyundai Kona SUV	9	3	0
Acura MDX SH-AWD SUV	0	0	0

Lexus NX 300 AWD SUV	0	0	0
Audi Q3 quattro SUV	0	0	0
Toyota RAV4 LE FWD SUV	0	0	0
Nissan Rogue SUV	0	0	0
GMC Terrain SUV	1	30000	1
Volkswagen Tiguan Trendline SUV	0	0	0
BMW X1 xDrive28i SUV	1	30000	1
Volvo XC40 T4 AWD SUV	1	30000	1

Table 8. Optimal solution for Scenario 4.i. At least one of every available car is leased. Maximum profit = $921,868.26.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
BMW 228i xDrive Gran Coupe Compact	1	30000	1
Audi A3 Komfort Compact	1	30000	1
Hyundai Accent Preferred Compact	1	30000	1
Mercedes-Benz C 300 4MATIC Compact	1	30000	1
Honda Fit LX-Honda Sensing Compact	1	100000	1
Acura ILX Compact	1	100000	1
Lexus IS 300 AWD Compact	1	30000	1
Volkswagen Jetta Comfortline Compact	1	30000	1
Mazda Mazda3 4-Door (SIL) Compact	1	30000	1
Nissan Qashqai Compact	1	30000	1
Kia Rio LX+ Compact	5	30000	1
Volvo S60 T5 Compact	1	30000	1
Chevrolet Spark Compact	1	30000	1
Subaru WRX AWD Compact	1	30000	1
Toyota Yaris Hatchback 6AT Compact	2	30000	1
Audi A5 Sportback quattro Mid-size	1	30000	1
Honda Civic Sedan CVT Mid-size	1	30000	1
Hyundai Elantra Mid-size	1	30000	1
Kia Forte LX MT Mid-size	1	30000	1
Ford Fusion Mid-size	1	30000	1
Subaru Impreza 4-Door AWD Mid-size	1	30000	1
Mazda Mazda3 5-Door (SIL) Mid-size	1	30000	1
Volkswagen Passat Comfirtline Mid-size	1	30000	1
Toyota Prius Mid-size	4	100000	1
Acura RLX Hybrid Mid-size	1	30000	1
Volvo S90 T6 AWD Mid-size	1	30000	1
Nissan Sentra Mid-size	1	30000	1
Lexus UX 200 Mid-size	1	30000	1
Honda Civic Hatchback LX SUV	5	100000	1
Subaru Crosstrek AWD Convenience SUV	1	100000	1

Mazda CX-30 SUV	1	100000	1
Ford Edge SE SUV	1	30000	1
Kia Forte 5 EX SUV	1	100000	1
Hyundai Kona SUV	1	100000	1
Acura MDX SH-AWD SUV	1	30000	1
Lexus NX 300 AWD SUV	1	30000	1
Audi Q3 quattro SUV	1	30000	1
Toyota RAV4 LE FWD SUV	2	100000	1
Nissan Rogue SUV	1	52291.66	1
GMC Terrain SUV	1	30000	1
Volkswagen Tiguan Trendline SUV	1	30000	1
BMW X1 xDrive28i SUV	1	30000	1
Volvo XC40 T4 AWD SUV	1	30000	1

Table 9. Optimal solution for Scenario 4.ii. At least one of every available car is leased. Moreover, removed constraints $X_{ij} \leq 5$ and $\sum X_{i1} \geq 20$ (compact), $\sum X_{i2} \geq 15$ (mid-size), and $\sum X_{i3} \geq 20$ (SUV). Maximum Profit = $789,399.74.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
BMW 228i xDrive Gran Coupe Compact	1	30000	1
Audi A3 Komfort Compact	1	30000	1
Hyundai Accent Preferred Compact	1	30000	1
Mercedes-Benz C 300 4MATIC Compact	1	30000	1
Honda Fit LX-Honda Sensing Compact	1	100000	1
Acura ILX Compact	1	100000	1
Lexus IS 300 AWD Compact	1	30000	1
Volkswagen Jetta Comfortline Compact	1	30000	1
Mazda Mazda3 4-Door (SIL) Compact	1	30000	1
Nissan Qashqai Compact	1	30000	1
Kia Rio LX+ Compact	1	30000	1
Volvo S60 T5 Compact	1	30000	1
Chevrolet Spark Compact	1	30000	1
Subaru WRX AWD Compact	1	30000	1
Toyota Yaris Hatchback 6AT Compact	1	30000	1
Audi A5 Sportback quattro Mid-size	1	30000	1
Honda Civic Sedan CVT Mid-size	1	30000	1
Hyundai Elantra Mid-size	1	30000	1
Kia Forte LX MT Mid-size	1	30000	1
Ford Fusion Mid-size	1	30000	1
Subaru Impreza 4-Door AWD Mid-size	1	30000	1
Mazda Mazda3 5-Door (SIL) Mid-size	1	30000	1
Volkswagen Passat Comfirtline Mid-size	1	30000	1
Toyota Prius Mid-size	1	30000	1
Acura RLX Hybrid Mid-size	1	30000	1
Volvo S90 T6 AWD Mid-size	1	30000	1

Nissan Sentra Mid-size	1	30000	1
Lexus UX 200 Mid-size	1	30000	1
Honda Civic Hatchback LX SUV	6	100000	1
Subaru Crosstrek AWD Convenience SUV	1	100000	1
Mazda CX-30 SUV	1	100000	1
Ford Edge SE SUV	1	30000	1
Kia Forte 5 EX SUV	1	100000	1
Hyundai Kona SUV	1	100000	1
Acura MDX SH-AWD SUV	1	30000	1
Lexus NX 300 AWD SUV	1	30000	1
Audi Q3 quattro SUV	1	30000	1
Toyota RAV4 LE FWD SUV	1	100000	1
Nissan Rogue SUV	1	100000	1
GMC Terrain SUV	1	100000	1
Volkswagen Tiguan Trendline SUV	1	81040.72	1
BMW X1 xDrive28i SUV	1	100000	1
Volvo XC40 T4 AWD SUV	1	100000	1

Table 10. Scenario 5. Optimal solution if the company were to only choose one car out of the 43 available cars for leasing. Maximum Profit = $2,159,541.23.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
Hyundai Kona SUV	27	99029.51	1

Table 11. Optimal solution for Scenario 6. Removed constraints $\sum X_{i1} \geq 20$ (compact), $\sum X_{i2} \geq 15$ (mid-size), and $\sum X_{i3} \geq 20$ (SUV). Maximum profit = $1,481,866.28.

Model and Class	X_{ij}	D_{ij}	Y_{ij}
Hyundai Accent Preferred Compact	1	97500	1
Honda Fit LX-Honda Sensing Compact	5	100000	1
Acura ILX Compact	5	100000	1
Kia Rio LX+ Compact	1	100000	1
Toyota Yaris Hatchback 6AT Compact	1	100000	1
Honda Civic Sedan CVT Mid-size	1	100000	1
Subaru Impreza 4-Door AWD Mid-size	1	100000	1
Toyota Prius Mid-size	5	100000	1
Nissan Sentra Mid-size	1	100000	1
Lexus UX 200 Mid-size	1	100000	1
Honda Civic Hatchback LX SUV	5	100000	1
Subaru Crosstrek AWD Convenience SUV	1	100000	1
Mazda CX-30 SUV	1	100000	1
Kia Forte 5 EX SUV	1	100000	1
Volvo XC40 T4 AWD SUV	1	100000	1