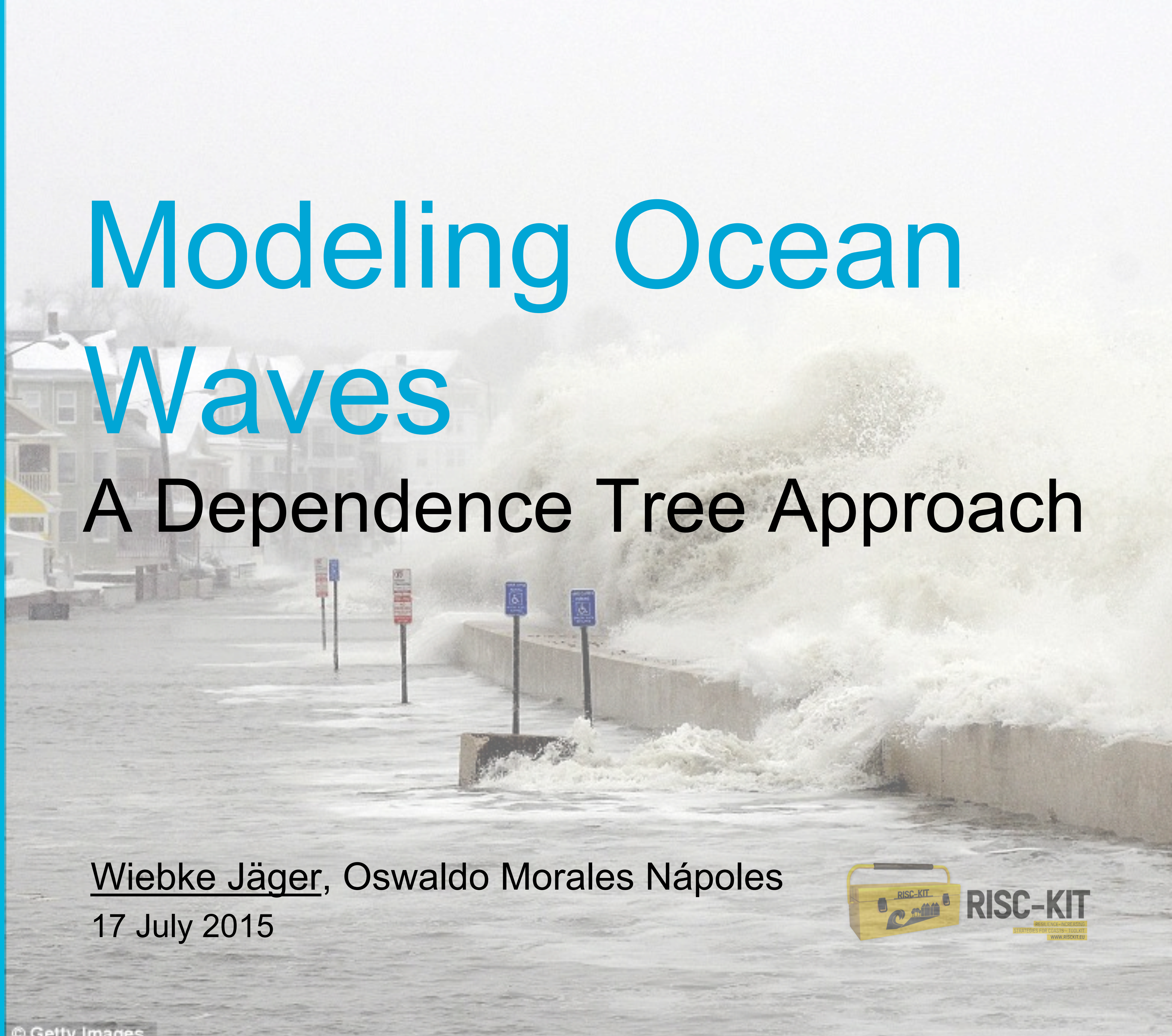

Modeling Ocean Waves

A Dependence Tree Approach

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RISC-KIT

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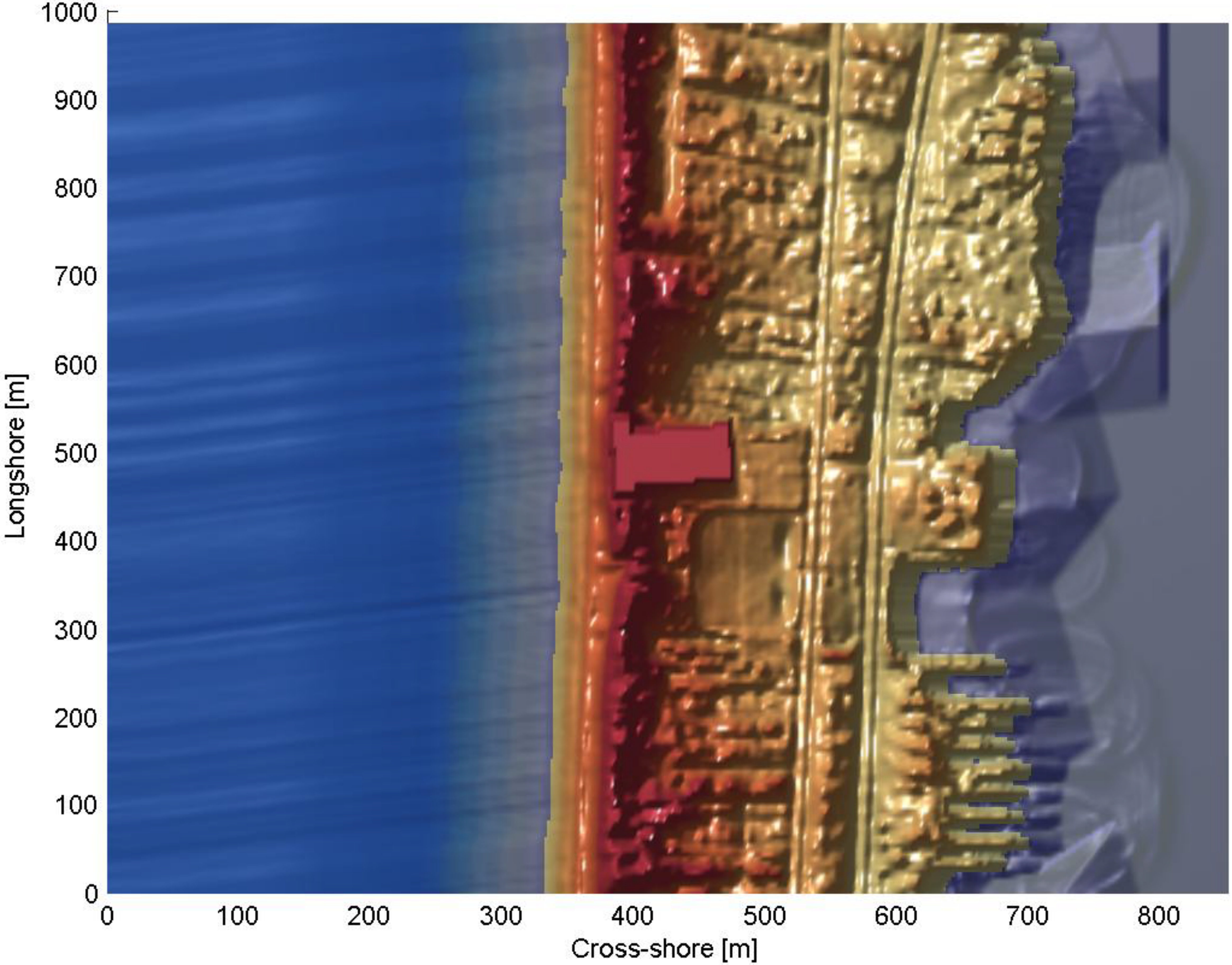

1000
900
800
700
600
500
400
300
200
100
0
Longshore [m]
0
100
200
300
400
500
600
700
800
Cross-shore [m]

Wave height

Wave angle

Surge level

Storm duration

Wave period

Wave height

Wave angle

Surge level

Storm duration

Wave period

Outline

- Background Information
- Approach
- Data
- Preliminary results
- Discussion/Conclusions
- Future work

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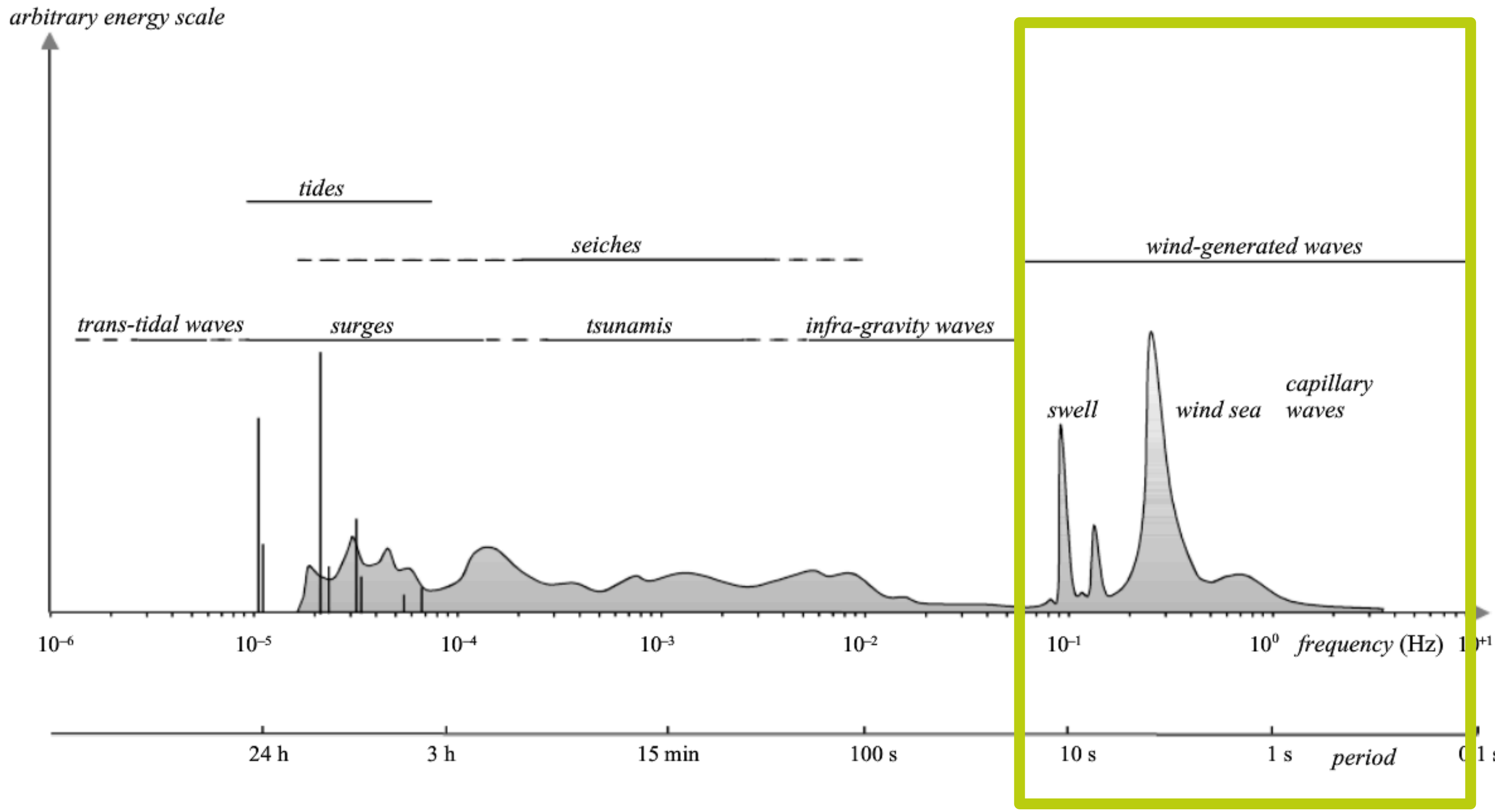
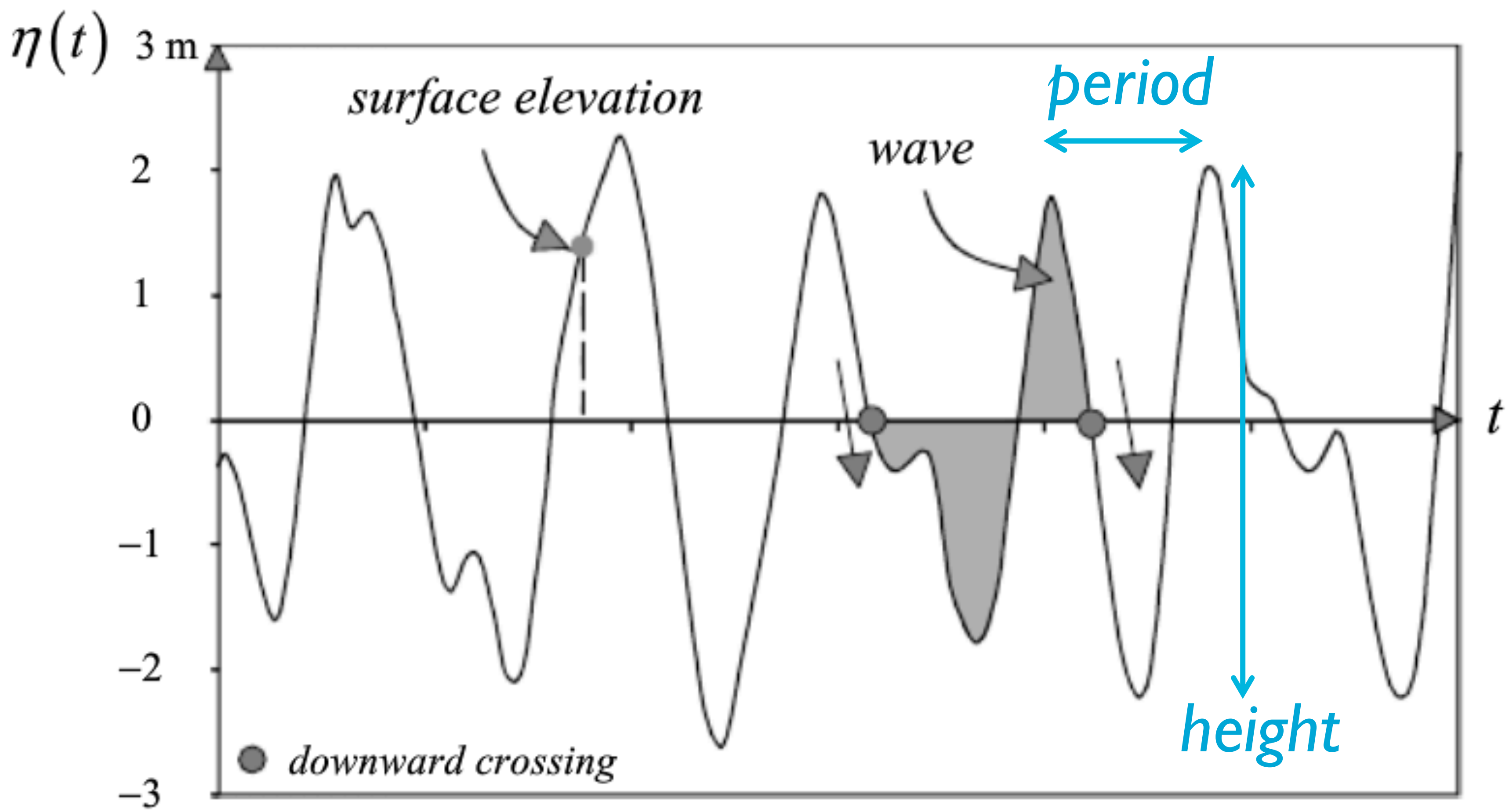
What is a wave?



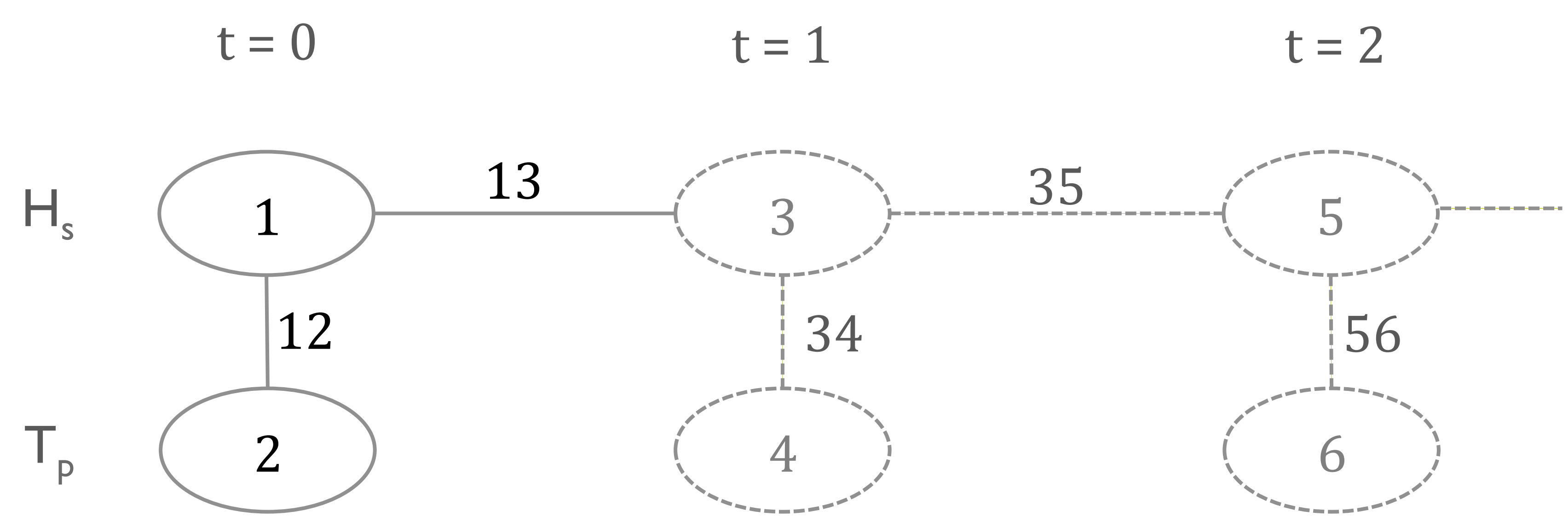
What is a wave?



- Significant wave height $H_s = 4\sqrt{Var(\eta(t))}$
- Peak period $T_p = \text{period with highest energy}$

Approach

1. "Temporal" dependence tree



2. Bivariate dependencies
   a. H_s^t and T_p^t
   b. H_s^t and H_s^{t+1}

$\Big\}$ Skew – t copula

Skew-t distribution and copula

- Formulation by Demarta & McNeil (2005)
- $\boldsymbol{X} = (X_1 \dots X_d)'$ has d-variate skew-t distribution with v degrees of freedom, mean vector **μ**, covariance matrix **Σ** and skewness vector **γ,** if its density is given by

$$g_{\nu,\mu,\mathbf{\Sigma},\gamma}(\mathbf{x}) = \mathbf{c} \cdot \frac{\mathbf{K}_{\frac{\nu+\mathbf{d}}{2}}\left(\sqrt{(\nu + (\mathbf{x}-\mu)'\mathbf{\Sigma}^{-1}(\mathbf{x}-\mu))\gamma'\mathbf{\Sigma}^{-1}\gamma}\right)}{\left(\sqrt{(\nu + (\mathbf{x}-\mu)'\mathbf{\Sigma}^{-1}(\mathbf{x}-\mu))\gamma'\mathbf{\Sigma}^{-1}\gamma}\right)^{-\frac{\nu+\mathbf{d}}{2}}} \cdot \frac{\exp\left((\mathbf{x}-\mu)'\mathbf{\Sigma}^{-1}\gamma\right)}{\left(\mathbf{1} + \frac{(\mathbf{x}-\mu)'\mathbf{\Sigma}^{-1}(\mathbf{x}-\mu)}{\nu}\right)^{\frac{\mathbf{v}+\mathbf{d}}{2}}}$$

- Representation as normal mean variance mixtures

$$\mathbf{X} = \mu + \gamma\mathbf{W} + \sqrt{\mathbf{W}}\mathbf{Z}$$

$$\sim Ig\left(\frac{\nu}{2}, \frac{\nu}{2}\right) \qquad \sim \mathcal{N}(0,1)$$

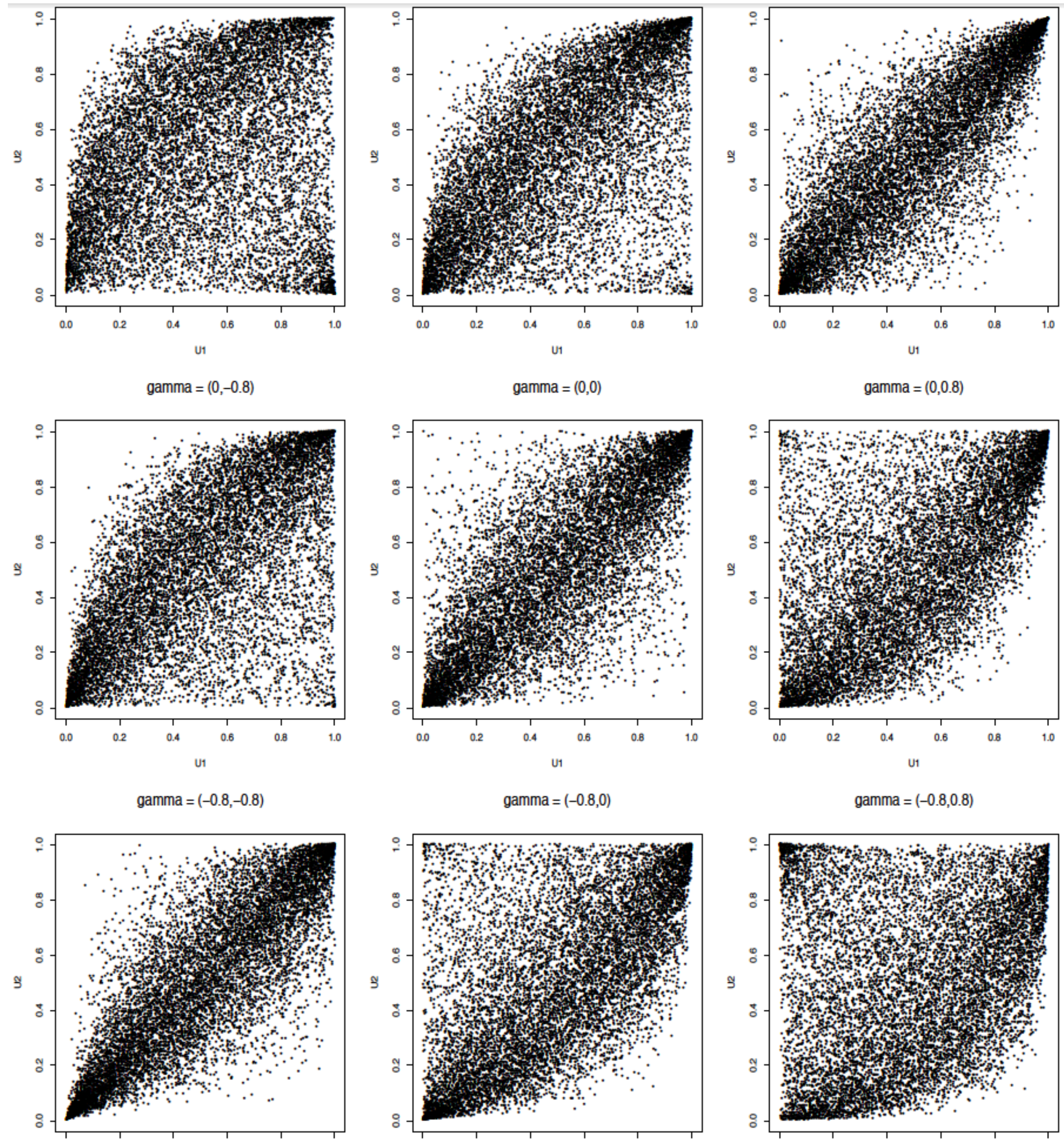
- Skew-t copula constructed from $t_d(v,0,\mathbf{\Sigma},\boldsymbol{\gamma})$ distribution with univariate margins $t_i(v,0,1,\gamma_i)$

Demarta, S., & McNeil, A. J. (2005). The t copula and related copulas. International statistical review, 73(1), 111-129.

Skew-t distribution and copula

$\nu = 5, \rho = 0.8$

γ_1 \ γ_2	-0.8	0	0.8
0.8			
0			
-0.8			



Source: Demarta & McNeil (2005)

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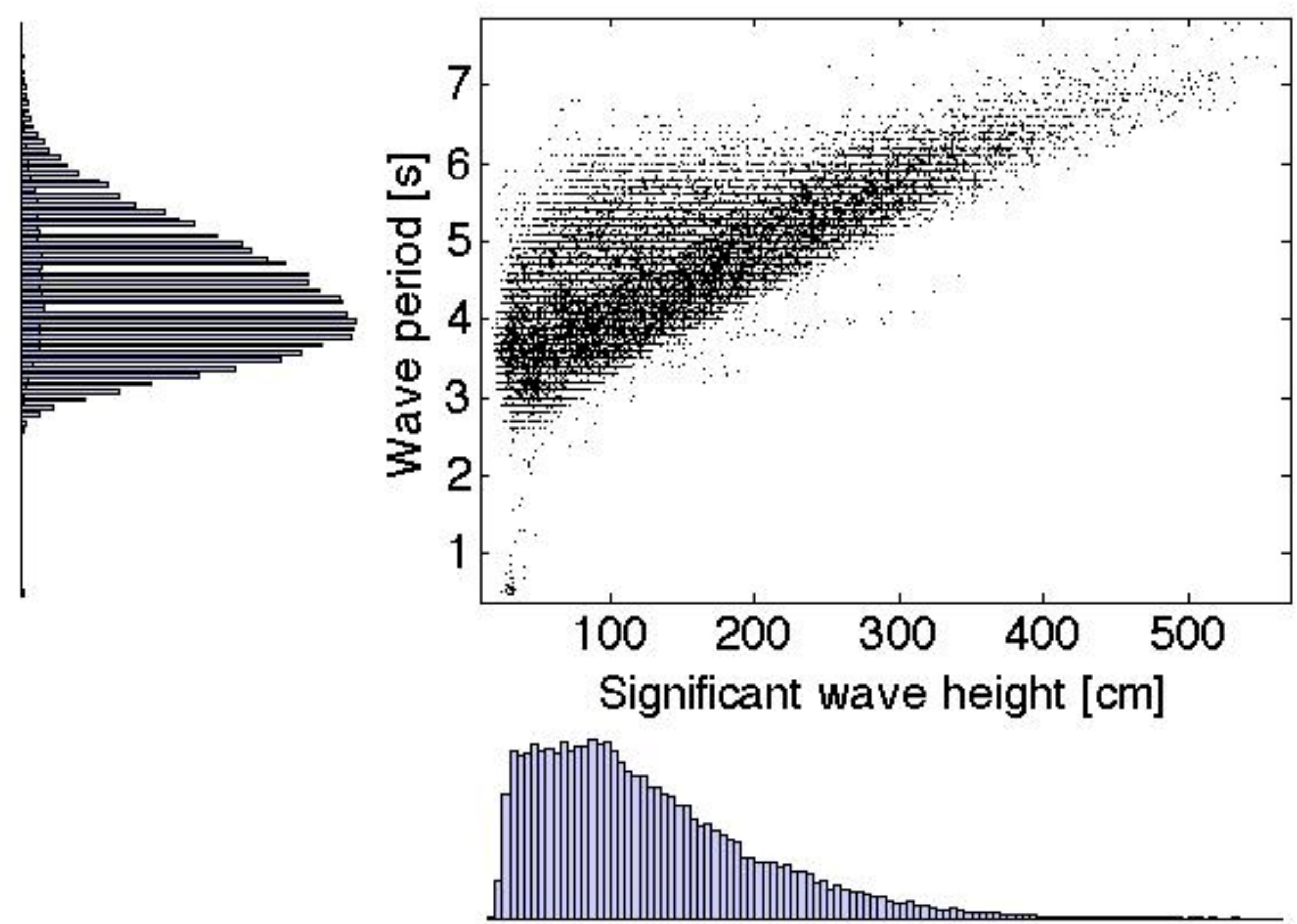
Measuring Station “Europlatform”

- 52º00’N, 03º17’E
- Operated by “*Rijkswaterstraat*”
- Hourly measurements 2005 to 2010

Significant wave heights (H_s) and associated periods (T_p)

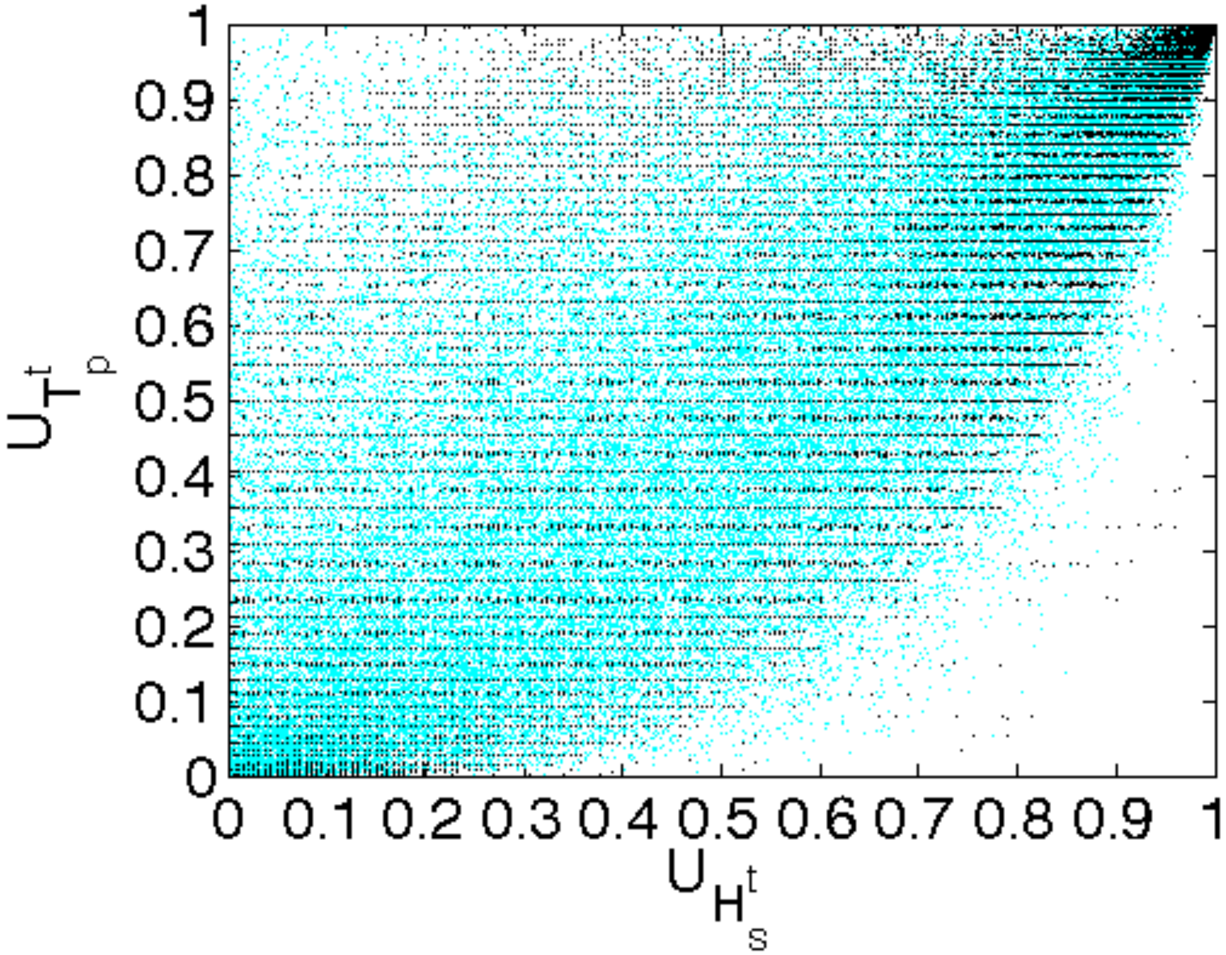
Measurements



Significant wave heights (H_s) and associated periods (T_p)

Scatter plot of ranks [•] and simulated points of the skew t-copula [•]

Parameters: $\rho = 0.757, \nu = 6, \gamma_1 = 0.7, \gamma_2 = 2.1$



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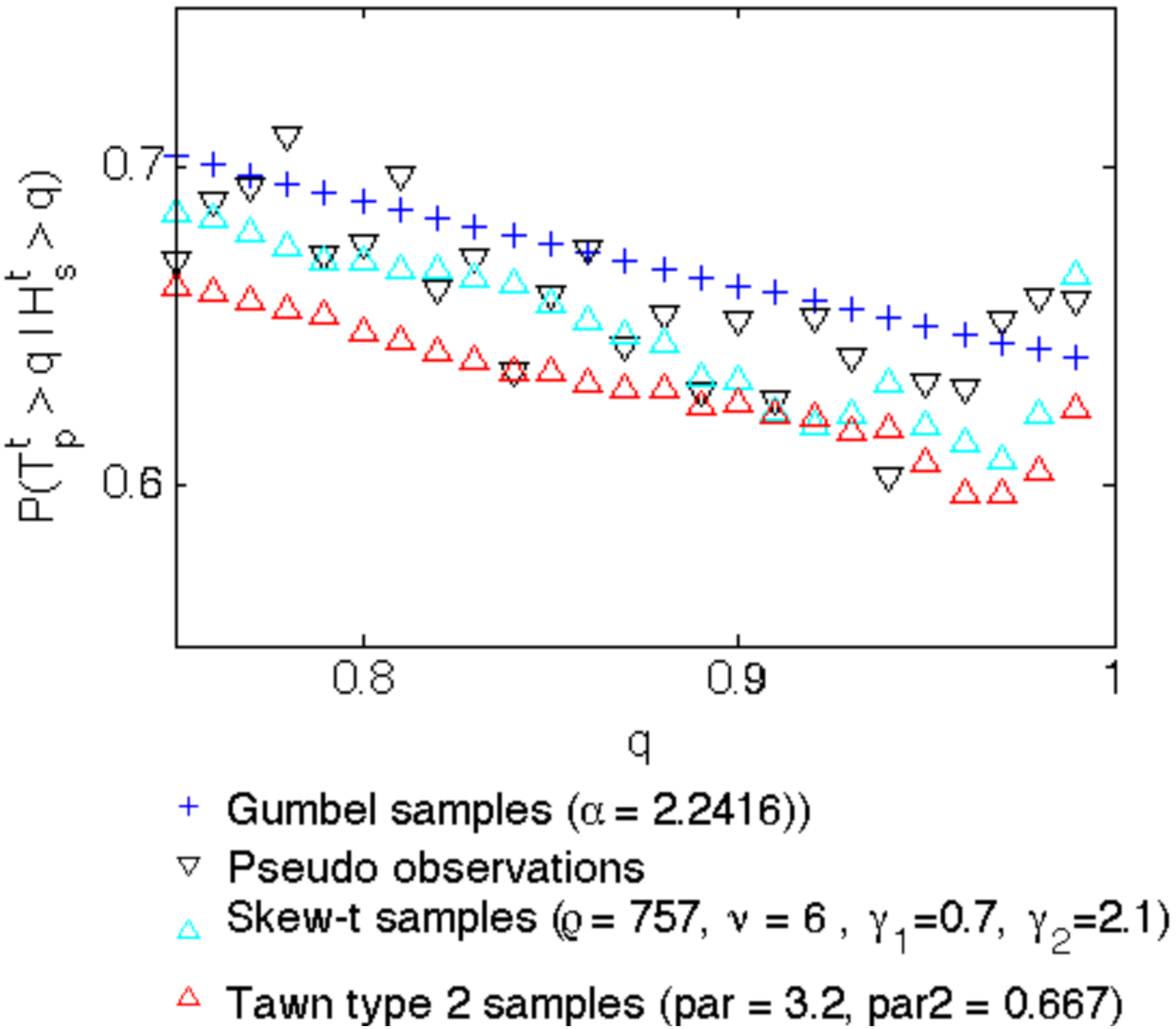
Significant wave heights (H_s) and associated periods (T_p)

Conditional exceedance probabilities



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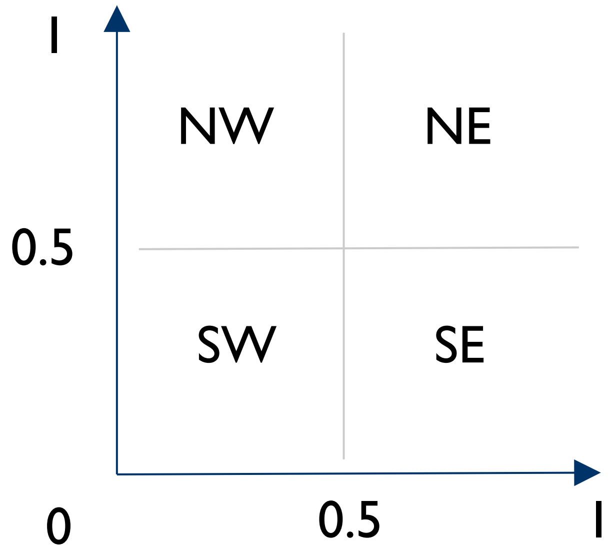
Significant wave heights (H_s) and associated periods (T_p)

Semi – correlations



Copula	ρ_{NW}	ρ_{NE}	ρ_{SW}	ρ_{SE}
Empirical	0.1485	0.7147	0.2765	0.3512
Gumbel	0.0586	0.7218	0.4235	0.0729
Skew-t	−0.0481	0.7180	0.4362	0.2645
Tawn (type 2)	−0.0845	0.7172	0.4004	0.1495

Significant wave heights (H_s) and associated periods (T_p)

Cramer – van Mises statistic

$$\mathcal{CM}_n = \int_{[0,1]^d} \sqrt{n}(C_n - C_{\theta_n}) dC_n(\mathbf{u})$$

Copula	$\mathcal{CM}$
Empirical	–
Gumbel	0.0740
Skew-t	0.0419
Tawn (type 2)	0.0820

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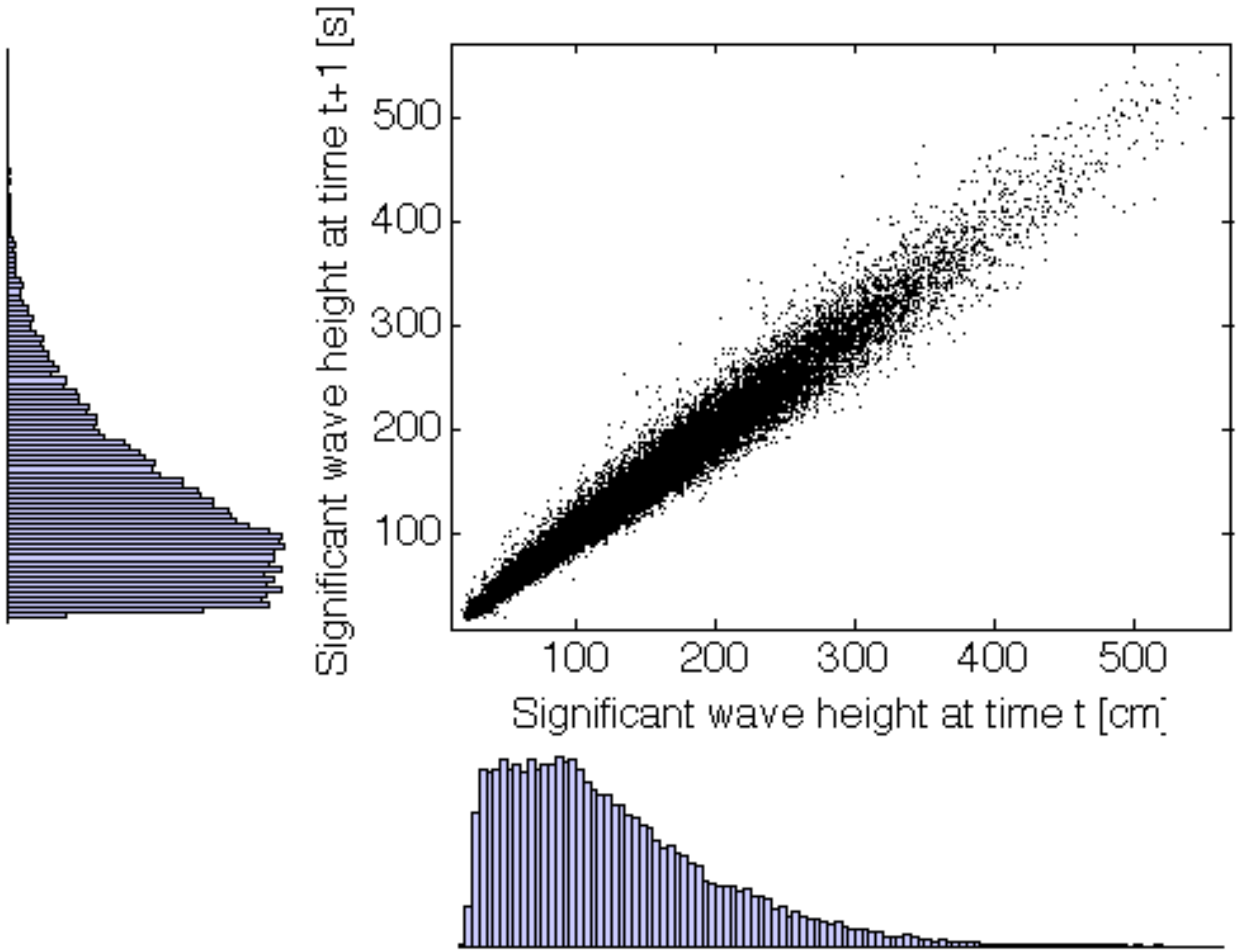
Significant wave heights (H_s) across time steps

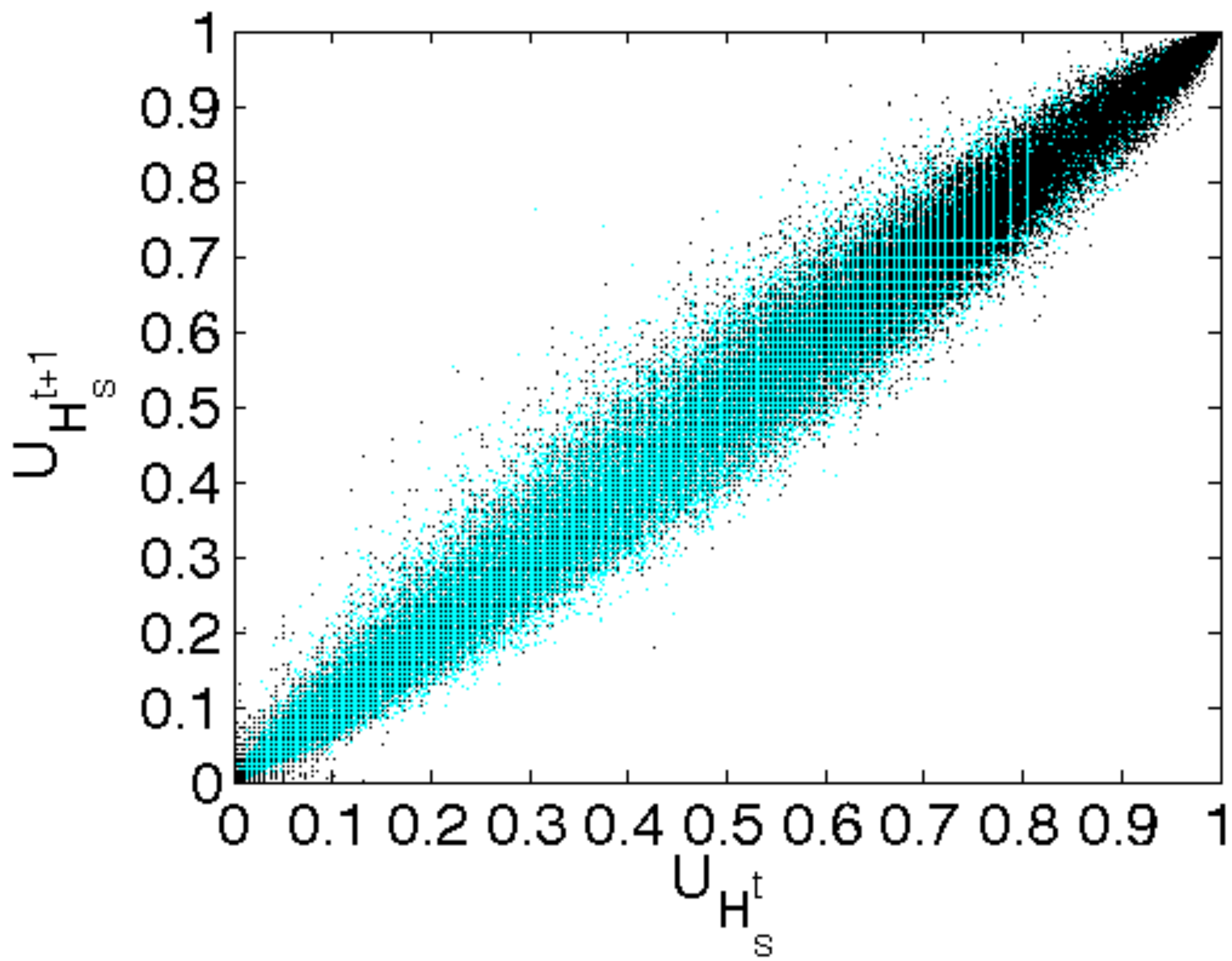
Measurements



Significant wave heights (H_s) across time steps

Scatter plot of ranks [•] and simulated points of the skew t-copula [•]

Parameters: $\rho = 0.9912, \nu = 25, \gamma_1 = 0.3, \gamma_2 = 0.5$



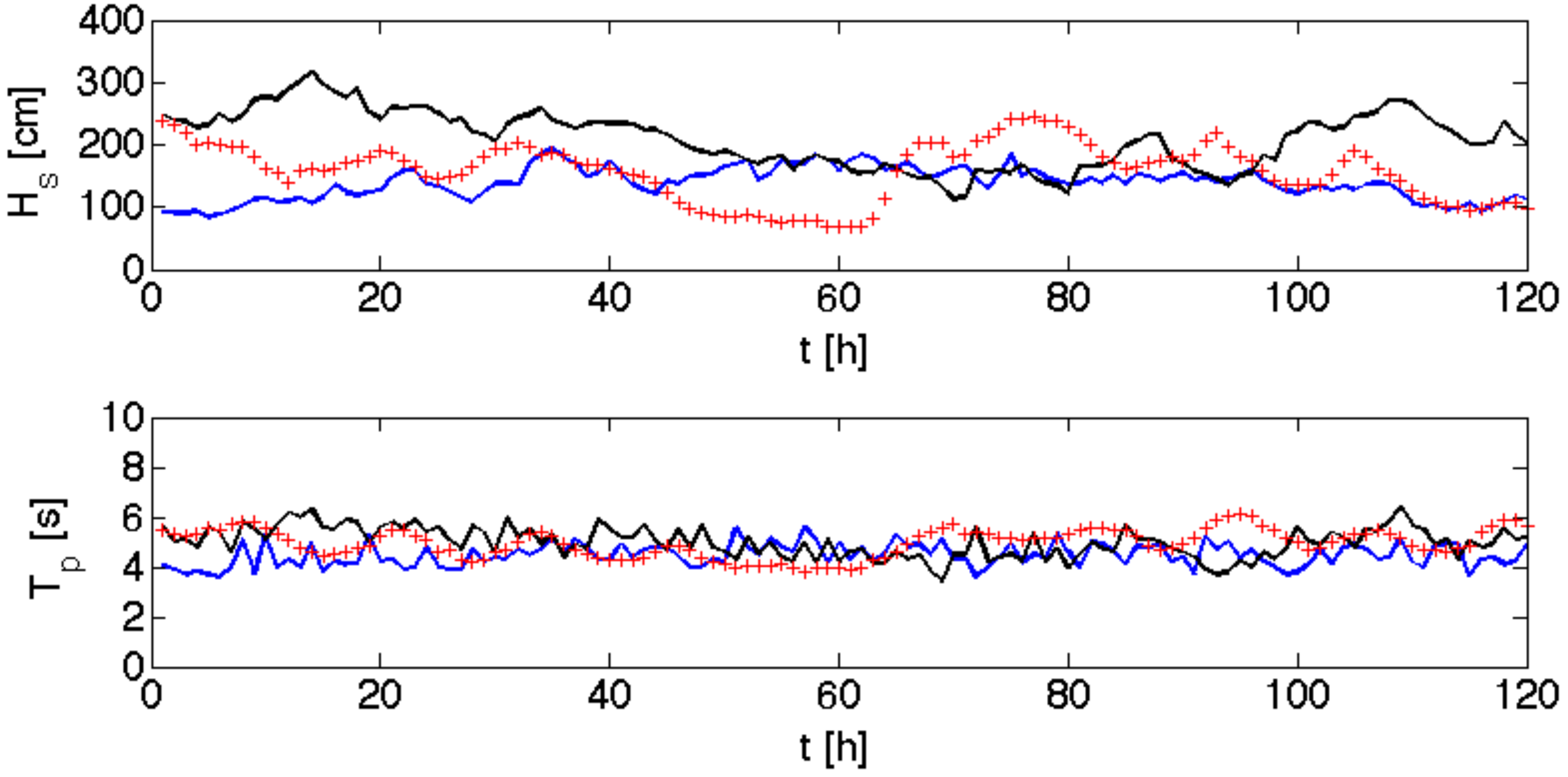
Current Issues / To be improved

- Use i.i.d. observations
- Use parametric margins

- Skew-t copula estimation
- Conditional skew-t copula and its inverse

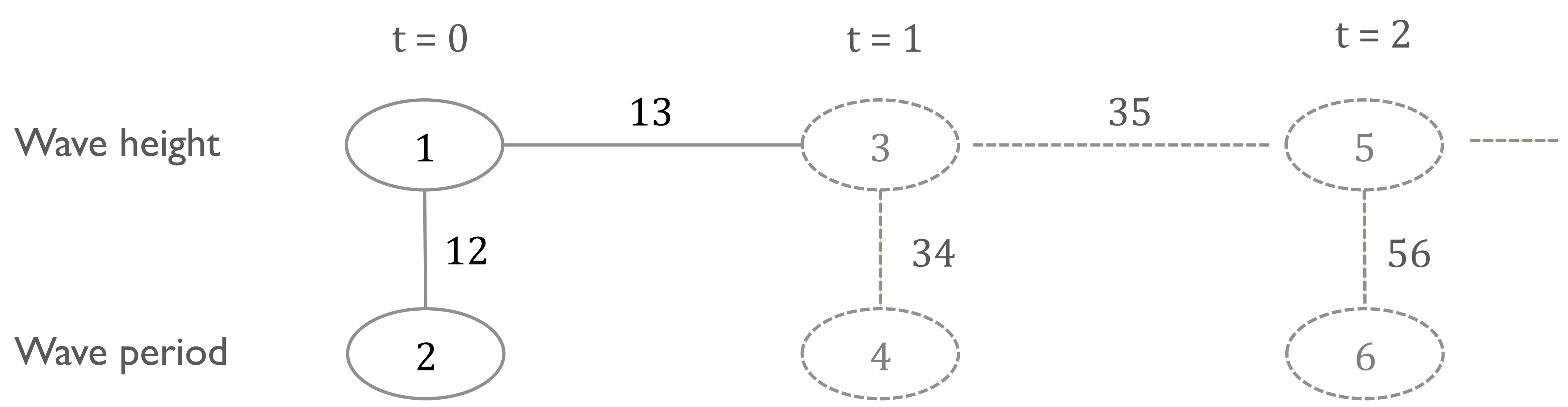
Resulting wave time series

Original (+) and synthetic (– & –) time series

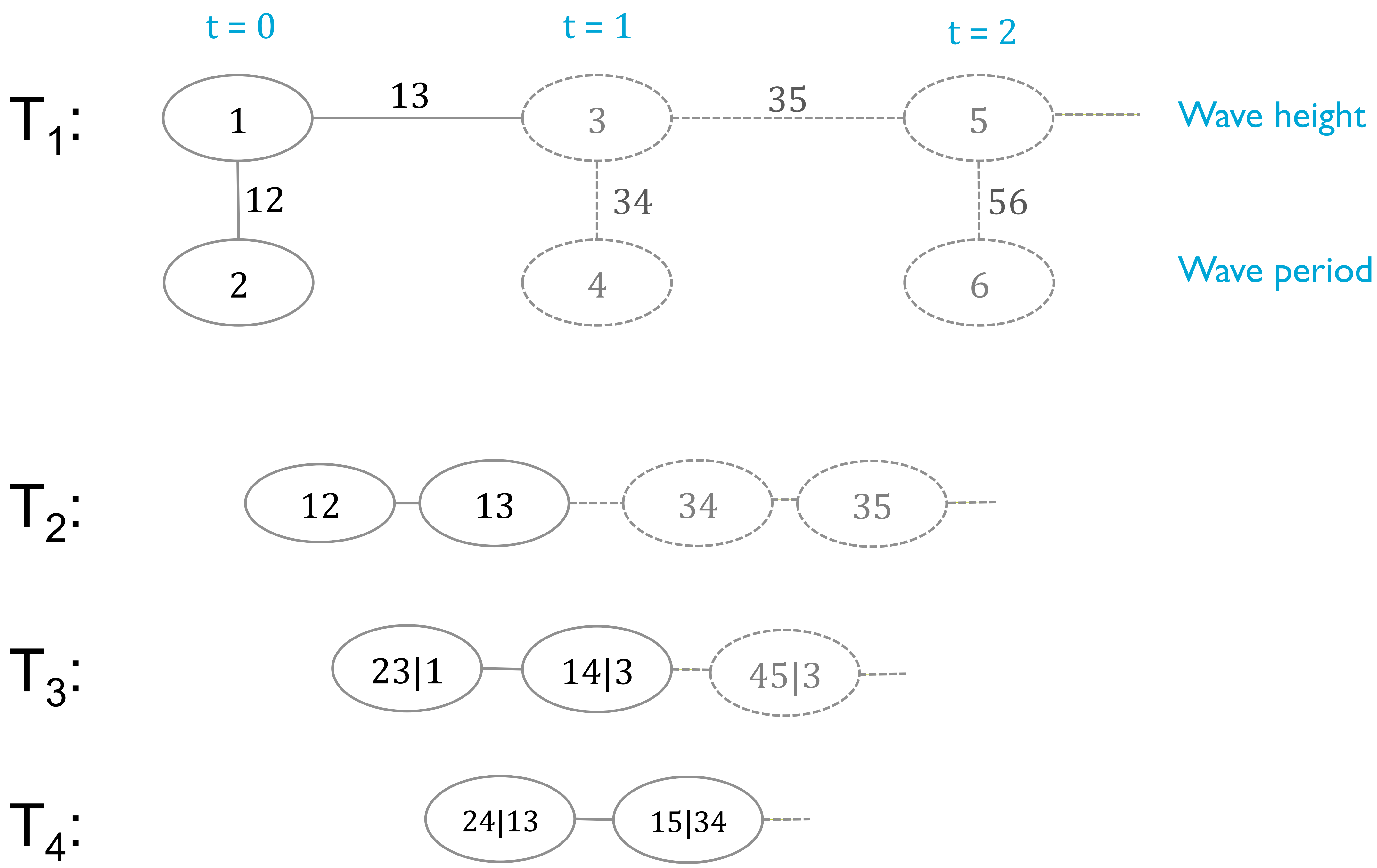


Conditional product moment correlations of ranks

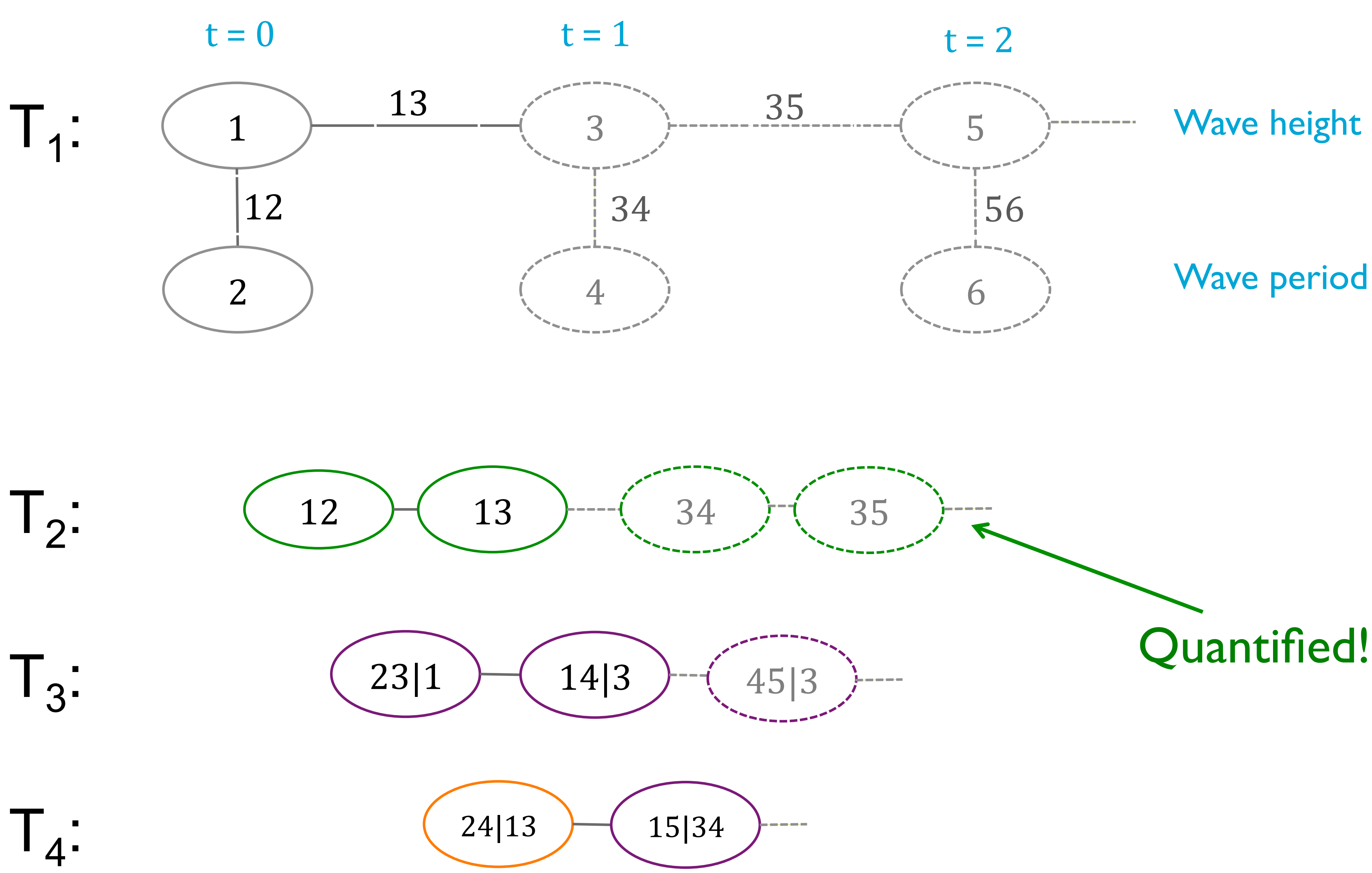
	I				
	0 – 0.2	0.2 – 0.4	0.4 – 0.6	0.6 – 0.8	0.8 – 1
$\hat{u}_{T_p^{t+1}}, \hat{u}_{T_p^t} \mid \hat{u}_{H_s^t = I}$	*0.89*	*0.93*	*0.91*	*0.88*	*0.90*
$\hat{u}_{T_p^{t+1}}, \hat{u}_{T_p^t} \mid \hat{u}_{H_s^{t+1} = I}$	*0.89*	*0.94*	*0.92*	*0.89*	*0.90*
$\hat{u}_{H_s^{t+1}}, \hat{u}_{T_p^t} \mid \hat{u}_{H_s^t = I}$	*-0.02*	*0.03*	*0.07*	*0.14*	*0.64*
$\hat{u}_{H_s^t}, \hat{u}_{T_p^{t+1}} \mid \hat{u}_{H_s^{t+1} = I}$	*0.08*	*0.23*	*0.27*	*0.34*	*0.70*
$\hat{u}_{H_s^{t+2}}, \hat{u}_{H_s^{t+1}} \mid \hat{u}_{H_s^t = I}$	*0.85*	*0.63*	*0.50*	*0.52*	*0.84*



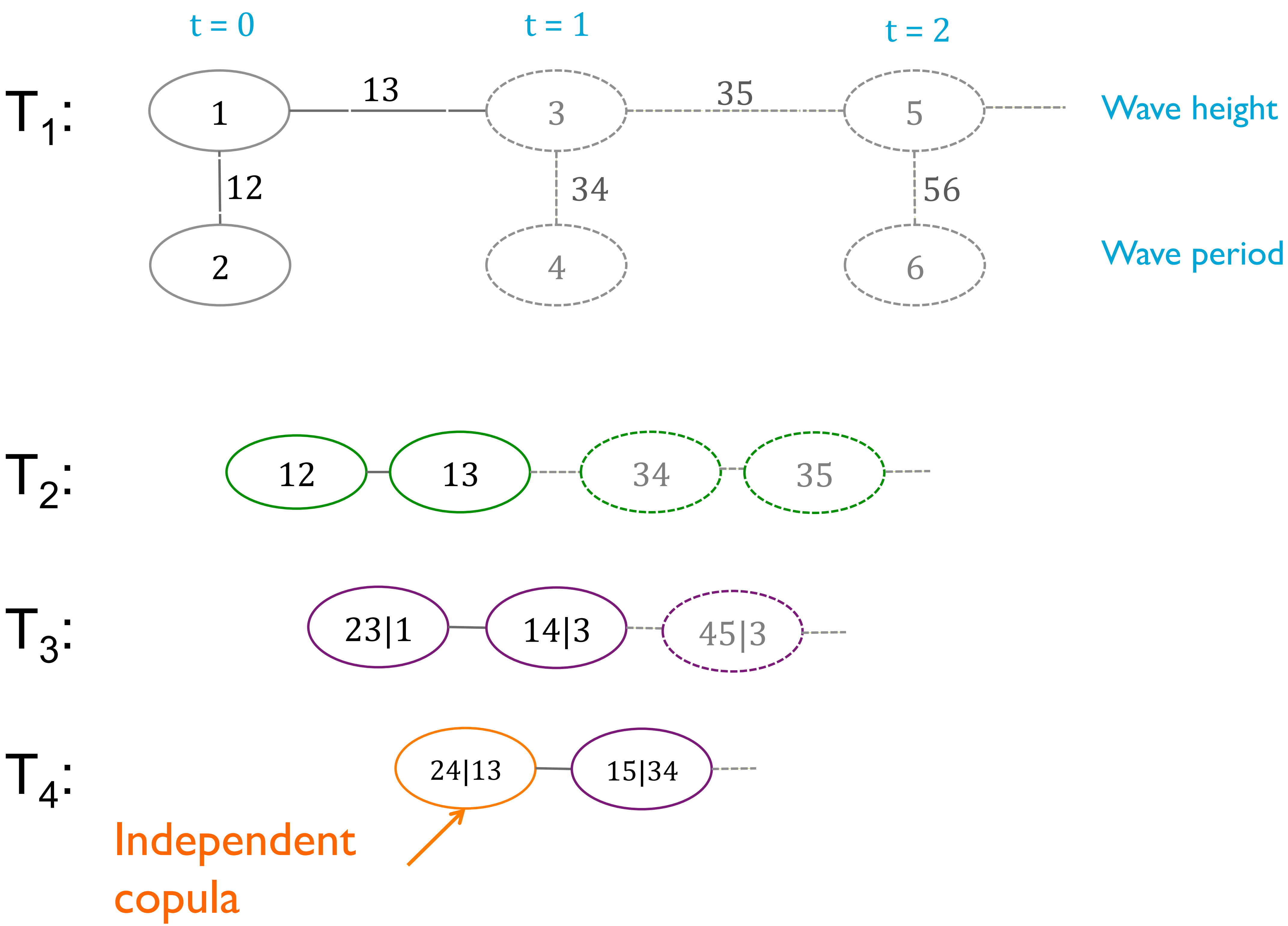
Extension to vine



Extension to vine



Extension to vine



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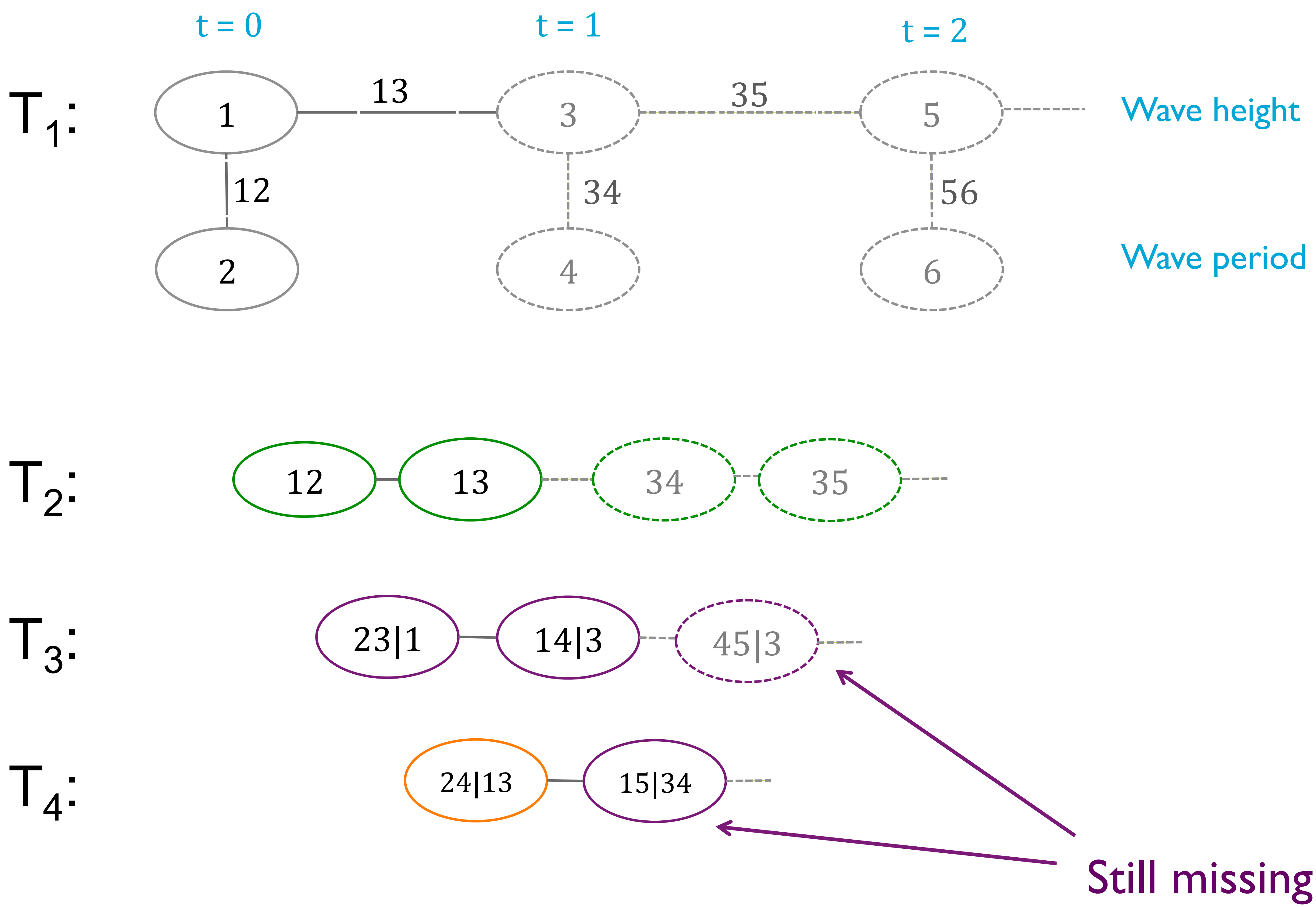
Extension to vine



Next steps

- Extension to vine
- Explore properties of skew-t copula and alternative copulas
- Construct multivariate distributions of several oceanographic variables to characterize severe storms, e.g.
  - Surge levels
  - Wave height, period and angle
  - Wind speed and direction
  - Storm duration
- Future interests
  - Goodness of fit tests (e.g. “Hellinger distance” as goodness of fit measure)
  - Properties of correlation matrices

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