

# CSE 559A: Computer Vision



[credit: danjodon.deviantart.com]

Fall 2017: T-R: 11:30-1pm @ Lopata 101

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<http://www.cse.wustl.edu/~ayan/courses/cse559a/>

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# GENERAL

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- PSET 4 posted Tuesday.
- Will require use of the census function from last PSET 3.
- Recitation next Friday November 10.

## Upcoming Events:

- CSE Fall research day this Friday
- WiCS Event: Hidden Figures Awareness: Nov 15

Checkout CSE website.

# GROUPING & SEGMENTATION

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## SLIC

$$L = \arg \min_L \min_{\{\mu_k\}} \sum_{k=1}^K \sum_{n:L[n]=k} \|I'[n] - \mu_k\|^2$$

- Begin with some initial assignment  $L[n]$ .
- At each iteration ...

**Step 1:** For each  $k$ , assign

$$\mu_k = \text{Mean}\{I'[n]\}_{L[n]=k}$$

**Step 2:** For each  $n$ , assign

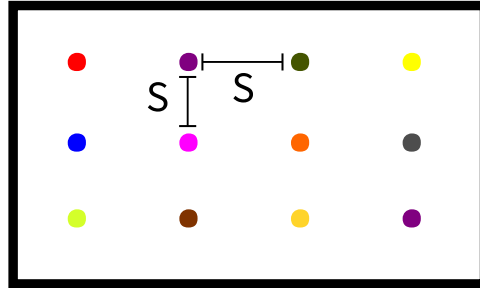
$$L[n] = \arg \min_k \|I'[n] - \mu_k\|^2$$

- How do we initialize ?
- Do we really need to do  $K \times N$  computations of  $\|I'[n] - \mu_k\|^2$  ?

# GROUPING & SEGMENTATION

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## SLIC: Initialization

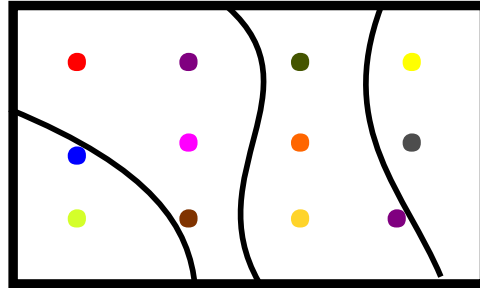


- Actually, begin with an assignment of  $\{\mu_k\}$  (and do a step 2).
- Given desired number of super-pixels  $K$ , choose  $K$  points on a grid.
  - Spaced horizontally and vertically apart by  $S = \sqrt{\frac{HW}{K}}$
- Set each  $u_k = I'[n_k]$  as the augmented vector of one of these points.
- In step 2, each seed is going to attract pixels in its neighborhood that are most like it.

# GROUPING & SEGMENTATION

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## SLIC: Initialization

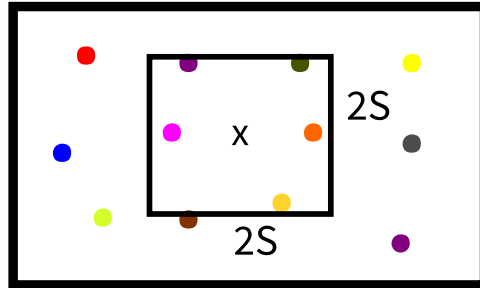


- Actually, begin with an assignment of  $\{\mu_k\}$  (and do a step 2).
- Given desired number of super-pixels  $K$ , choose  $K$  points on a grid.
  - Spaced horizontally and vertically apart by  $S = \sqrt{\frac{HW}{K}}$
- Set each  $u_k = I'[n_k]$  as the augmented vector of one of these points.
- In step 2, each seed is going to attract pixels in its neighborhood that are most like it.
- Sometimes this initialization gives you a 'seed' that lies right on an edge.
  - Bad because pixel on either side of edge will often look nothing like it.
- Solution: Look in a 3x3 neighborhood, and choose pixel with lowest gradient magnitude.

# GROUPING & SEGMENTATION

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## SLIC: Minimization



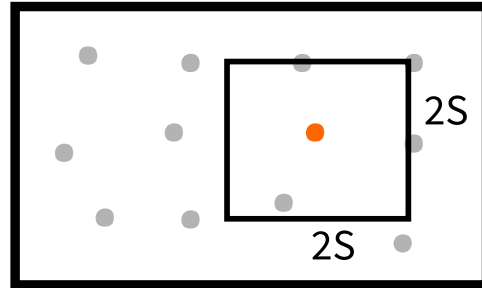
At any given iteration, for step 2:

- Don't consider all possible  $K$  for every  $n$ .
- Instead, say that a pixel  $n$  can only be assigned to a cluster  $k$  if  $n$  is within a  $2S \times 2S$  window around the spatial co-ordinates in  $u_k$ .
- Note that  $\mu_k$ 's will no longer be on a regular grid.

# GROUPING & SEGMENTATION

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## SLIC: Minimization



At any given iteration, for step 2:

- Initialize  $\text{min\_dist}[n]$  to Infinity for all  $n$
- Loop through each  $u_k$ , and consider pixels in  $2S \times 2S$  window around  $u_k$ 
  - This will be a regular grid.
- For each pixel in this window, compute distance of  $I'[n]$  to  $u_k$ , compare to  $\text{min\_dist}[n]$ , if lower, update  $\text{min\_dist}[n]$  and update  $L[n]$ .

Do we need to loop over  $K$  ? Can get some parallelism if you're clever about it.

# GROUPING & SEGMENTATION

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## Graph-based Methods



## Foreground / Background Segmentation

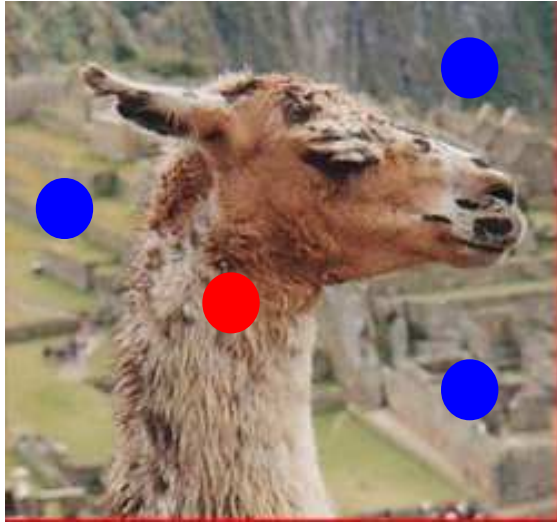
Image from Rother et al., GrabCuts.



# GROUPING & SEGMENTATION

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## Graph-based Methods



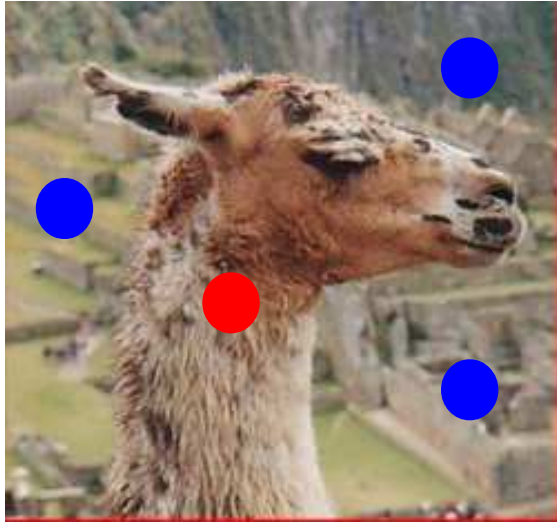
Assign a label of 1 (foreground) or 0 (background) for each pixel in the image.

Let's say user has labeled some pixels as foreground or background.  
(or these are noisy / sparse predictions from some algorithm)

# GROUPING & SEGMENTATION

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## Graph-based Methods



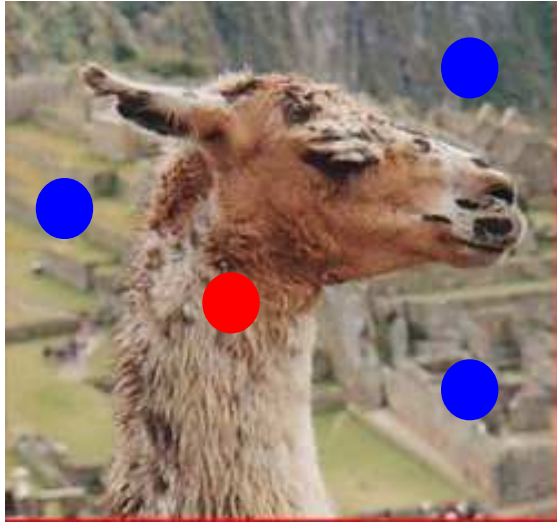
$$L = \arg \min_{L[n] \in \{0,1\}} C[n, L[n]] + \sum_{(n,n') \in \mathbb{E}} S_{n,n'}(L[n], L[n'])$$

Kind of like our stereo setup, but binary labeling problem.

# GROUPING & SEGMENTATION

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## Graph-based Methods



$$L = \arg \min_{L[n] \in \{0,1\}} C[n, L[n]] + \sum_{(n,n') \in \mathbb{E}} S_{n,n'}(L[n], L[n'])$$

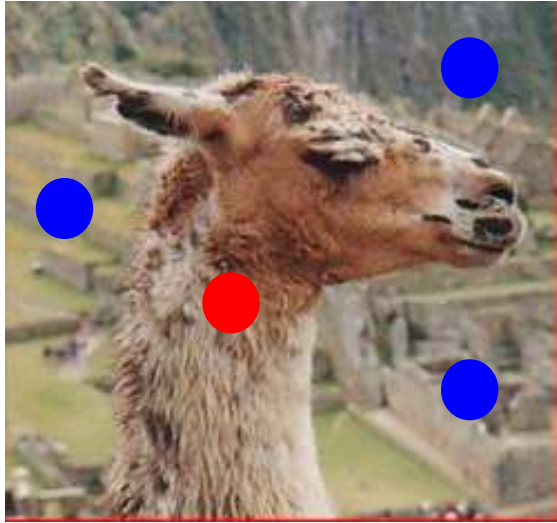
Comes from user input / algorithm

E.g., 0 for unlabeled pixels.  
Very high / infinite cost at  $n$  for  $L[n]$   
different from user label

# GROUPING & SEGMENTATION

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## Graph-based Methods



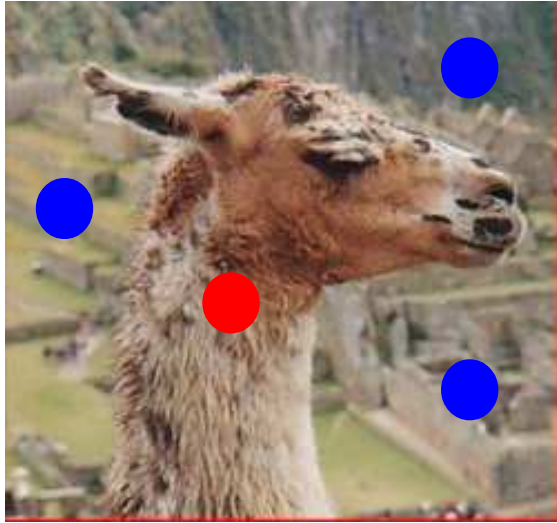
$$L = \arg \min_{L[n] \in \{0,1\}} C[n, L[n]] + \sum_{(n,n') \in \mathbb{E}} S_{n,n'}(L[n], L[n'])$$

Again, pairs of neighboring pixels.  
Horizontal / Vertical / Diagonal

# GROUPING & SEGMENTATION

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## Graph-based Methods

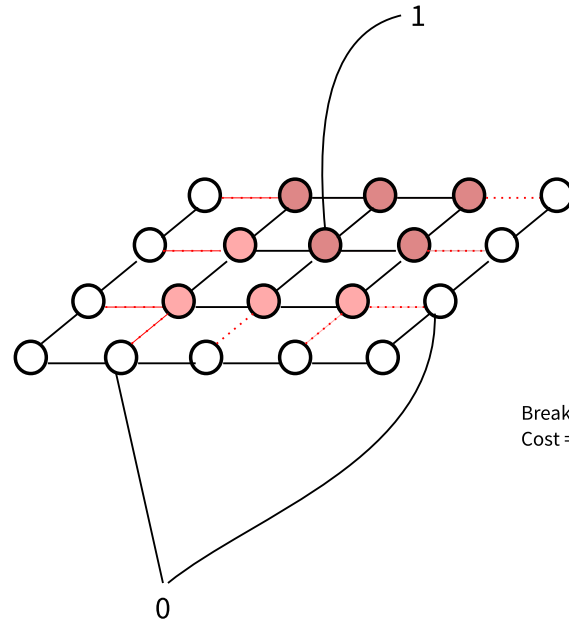


$$L = \arg \min_{L[n] \in \{0,1\}} C[n, L[n]] + \sum_{(n,n') \in \mathbb{E}} S_{n,n'}(L[n], L[n'])$$

Now will depend on pixel location.  
Often based on intensity differences /  
whether there is an edge.

# GROUPING & SEGMENTATION

## Graph-based Methods



Break set of vertices into two disconnected graphs by removing edges.  
Cost = sum of weight on edges you removed

$$L = \arg \min_{L[n] \in \{0,1\}} C[n, L[n]] + \sum_{(n,n') \in \mathbb{E}} S_{n,n'}(L[n], L[n'])$$

# GROUPING & SEGMENTATION

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- Min-cut Problem: Can be solved exactly and efficiently
- Different variant: Normalized Cuts, where you also care about the size of each segment.
  - Doesn't allow you to minimize cost by choosing a corner point and breaking small number of edges.
- Generalization to multi-label cases. (Has even been used for stereo!)

## References

- Boykov and Kolmogorov, An Experimental Comparison of Min-Cut/Max-Flow Algorithms for Energy Minimization in Vision, PAMI 2004.
- DeLong et al., Fast Approximate Energy Minimization with Label Costs, IJCV 2012.
- Rother et al., GrabCut -Interactive Foreground Extraction using Iterated Graph Cuts, SIGGRAPH 2004.

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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So far, given an input  $X$  and desired output  $Y$  we have

- Tried to explain the relationship of how  $X$  results from  $Y$ 
  - $X$  = observed image(s) /  $Y$  = clean image, sharp image, surface normal, depth
  - Noise, photometry, geometry, ...
- Often put a hand-crafted "regularization" cost to compute the inverse
  - Depth maps are smooth
  - Image gradients are small
- But sometimes, there is no way to write-down a relationship between  $X$  and  $Y$  ?
  - $X$  = Image,  $Y$  = Does the image contain a dog ?
- Even if there is, the hand-crafted regularization cost is often arbitrary.
  - Real images contain far more complex and subtle regularity.



# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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Instead, we are going to assume that there is some underlying joint probability distribution  $P_{XY}(x, y)$

- And our goal is to compute:
  - The best estimate of  $y$  conditioned on a specific value of  $x$ ,
  - To minimize some notion of "risk" or "loss"

Define a loss function  $L(y, \hat{y})$ , which measures how much we dislike  $\hat{y}$  as our estimate, when  $y$  is the right answer.

## Examples

- $L(y, \hat{y}) = \|y - \hat{y}\|^2$
- $L(y, \hat{y}) = \|y - \hat{y}\|$
- $L(y, \hat{y}) = 0$  if  $y = \hat{y}$ , and some  $C$  otherwise.

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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- And our goal is to compute:
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Ideally,

$$\hat{y}(x) = \arg \min_{\hat{y}} \int L(y, \hat{y}) P(y|x) dy$$

$$P(y|x) = \frac{P_{XY}(x, y)}{\int P_{XY}(x, y') dy'}$$

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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$$\hat{y}(x) = \arg \min_{\hat{y}} \int L(y, \hat{y}) P(y|x) dy$$

$$P(y|x) = \frac{P_{XY}(x, y)}{\int P_{XY}(x, y') dy'}$$

- So we have a loss (depends on the application)
- We can compute  $P(y|x)$  from  $P_{XY}$ .
- But we don't know  $P_{XY}$  !

Assume we are given as training examples, samples  $(x, y) \sim P_{XY}$  from the true joint distribution.

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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$$\hat{y}(x) = \arg \min_{\hat{y}} \int L(y, \hat{y}) P(y|x) dy$$

$$P(y|x) = \frac{P_{XY}(x, y)}{\int P_{XY}(x, y') dy'}$$

Given  $\{(x_i, y_i)\}$  as samples from  $P_{XY}$ , we could:

- Estimate  $P_{XY}$ 
  - Choose parametric form for the joint distribution (Gaussian, Mixture of Gaussians, Bernoulli, ...)
  - Estimate the parameters of that parametric form to "best fit" the data.
  - Depending again on some notion of fit (often likelihood)

$$P_{XY}(x, y) = f(x, y, ; \theta)$$

$$\theta = \arg \max_{\theta} \sum_i \log f(x_i, y_i; \theta)$$

Maximum Likelihood Estimation

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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$$\hat{y}(x) = \arg \min_{\hat{y}} \int L(y, \hat{y}) P(y|x) dy$$

$$P(y|x) = \frac{P_{XY}(x, y)}{\int P_{XY}(x, y') dy'}$$

$$P_{XY}(x, y) = f(x, y; \theta)$$

$$\theta = \arg \max_{\theta} \sum_i \log f(x_i, y_i; \theta)$$

So that's one way of doing things ...

- You're doing a minimization for learning  $P_{XY}$ , but then also a minimization at "test time" for every input  $x$ .
- You're approximating  $P_{XY}$  with some choice of the parametric form  $f$ .
- And it's possible that the best  $\theta$  that maximizes likelihood, may not be the best  $\theta$  that minimizes loss.

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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Given a bunch of samples  $\{(x_i, y_i)\}$  from  $P_{XY}$ ,

we want to learn a **function**  $y = f(x)$ , such that

given a typical  $x, y$  from  $P_{XY}$ , the loss  $L(y, f(x))$  is low.

$$f = \arg \min_f \int \left( \int L(y, f(x)) p(y|x) dy \right) p(x) dx$$

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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$$f = \arg \min_f \int \int L(y, f(x)) p_{XY}(x, y) dx dy$$

What we're going to is to replace the double integration with a summation over samples !

# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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$$f = \arg \min_f \sum_i L(y_i, f(x_i))$$

What we're going to do is to replace the double integration with a summation over samples !

## Empirical Risk Minimization

- So instead of first fitting the probability distribution from training data, and then given a new input, minimizing the loss under that distribution ...
- We are going to do a search over possible functions that "do well" on the training data, and assume that a function that minimizes "empirical risk" also minimizes "expected risk".



# MACHINE LEARNING: CRASH COURSE / QUICK RECAP

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Next time:

- How do you choose the space of possible functions to minimize over ?
- What are the consequences of this to the expected error ?
- How do you solve the optimization problem, efficiently ?