CSE 559A: Computer Vision

[credit: danjodon.deviantart.com]

Fall 2017: T-R: 11:30-1pm @ Lopata 101

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http://www.cse.wustl.edu/~ayan/courses/cse559a/

Sep 21, 2017

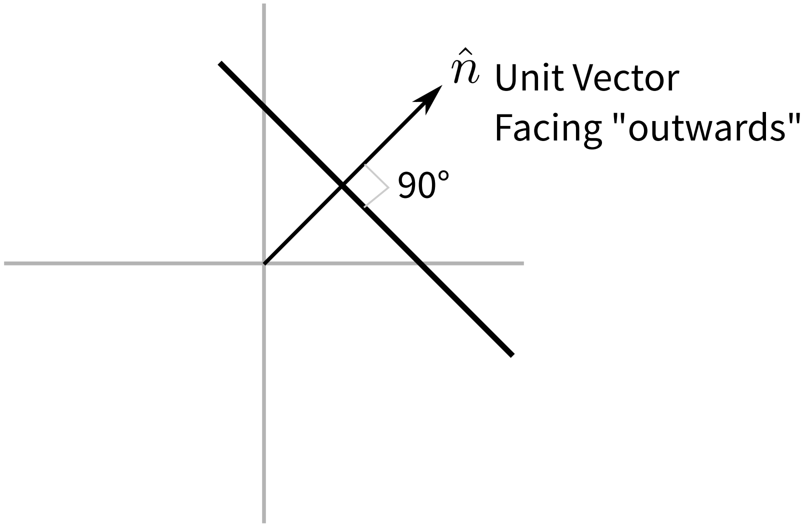
GENERAL

- Recitation tomorrow (9/22) 10am in J309.
  - Will go over topics relevant to Pset.
- Office hours from 5:30-6:30 in J517.
- Look at course resources for Python and Math
- Refresh Trigonometric and Complex number identities
  - $(x_1 + jy_1)(x_2 + jy_2) = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$
  - $\cos -\theta = \cos\theta, \sin -\theta = -\sin\theta, \cos(\pi - \theta) = -\cos\theta, \ldots$

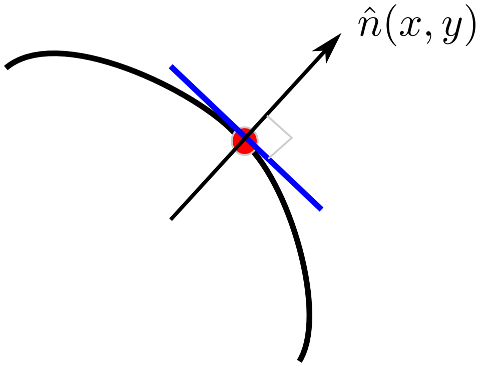
NORMALS

Surface Normals

2-D



Curve



Defined at a point,
Normal to the tangent at that point

Equation for a Line

$$x \cos\theta + y \sin\theta = c$$

$$\langle [x, y], [\cos\theta, \sin\theta] \rangle = c$$

$$\langle [x, y], \hat{n} \rangle = c$$

$$\hat{n} = [\hat{n}_x, \hat{n}_y]$$

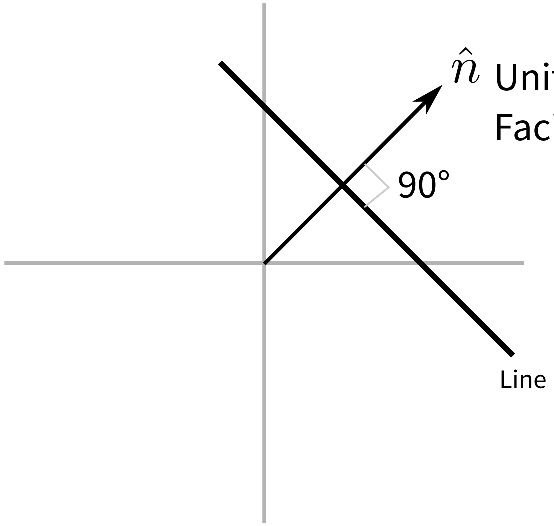
$$y = f(x) \qquad x = g(y)$$

$$\frac{\partial y}{\partial x} = \frac{-\hat{n}_x}{\hat{n}_y} \qquad \frac{\partial x}{\partial y} = \frac{-\hat{n}_y}{\hat{n}_x}$$

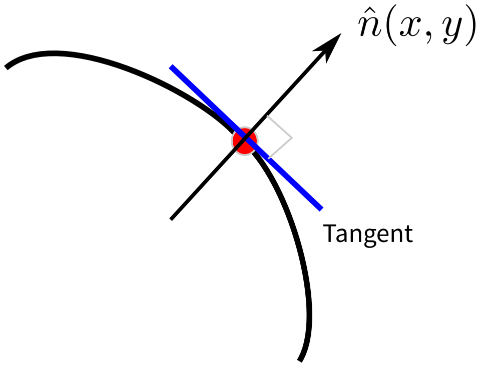
NORMALS

Surface Normals

2-D



Curve



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Equation for a Line

$$\langle [x, y], \hat{n} \rangle = c$$

$$\hat{n} = [\hat{n}_x, \hat{n}_y]$$

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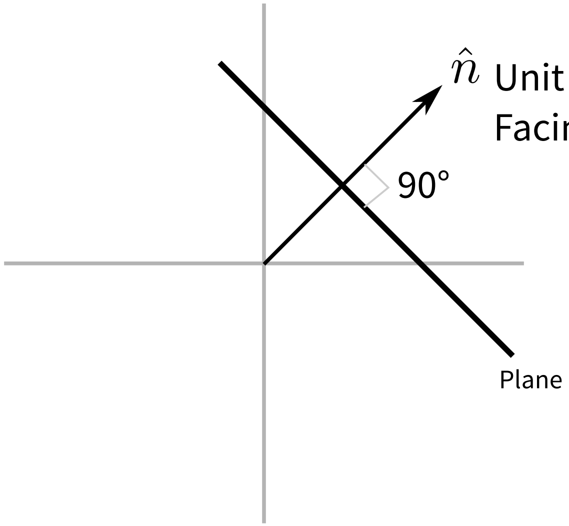
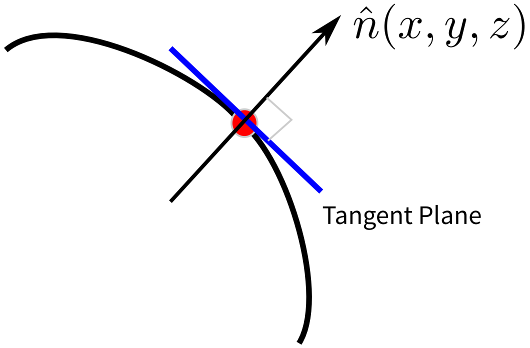
NORMALS

Surface Normals

3-D



Surface



Defined at a point,
Normal to the tangent plane at that point

Equation for a Plane

$$\langle [x, y, z], \hat{n} \rangle = c$$

$$\hat{n} = [\hat{n}_x, \hat{n}_y, \hat{n}_z]$$

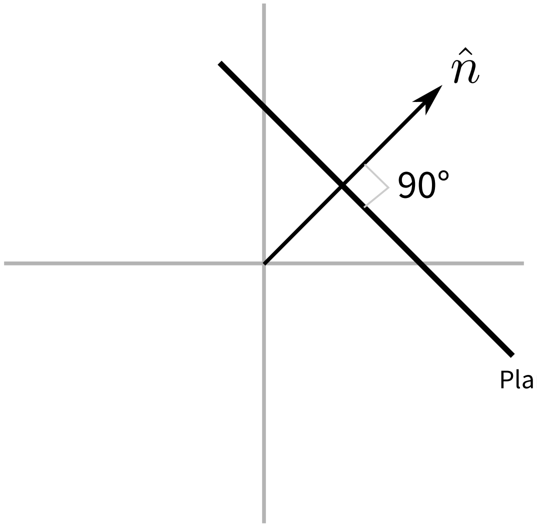
$$z = f(x, y)$$

$$\nabla z = \left[\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right] = \left[\frac{-\hat{n}_x}{\hat{n}_z}, \frac{-\hat{n}_y}{\hat{n}_z}\right]$$

NORMALS

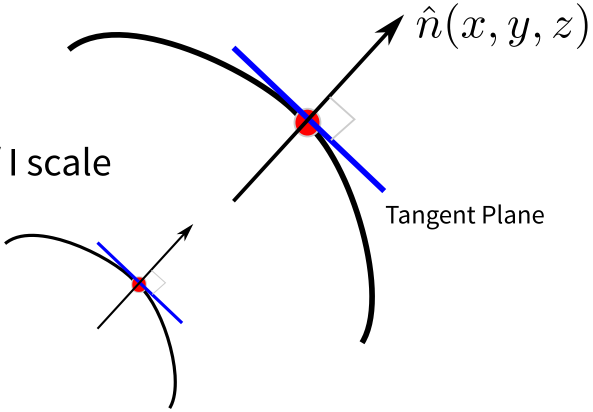
Surface Normals

3-D



Surface

Doesn't change if I scale entire surface



Defined at a point,
Normal to the tangent plane at that point

Equation for a Plane

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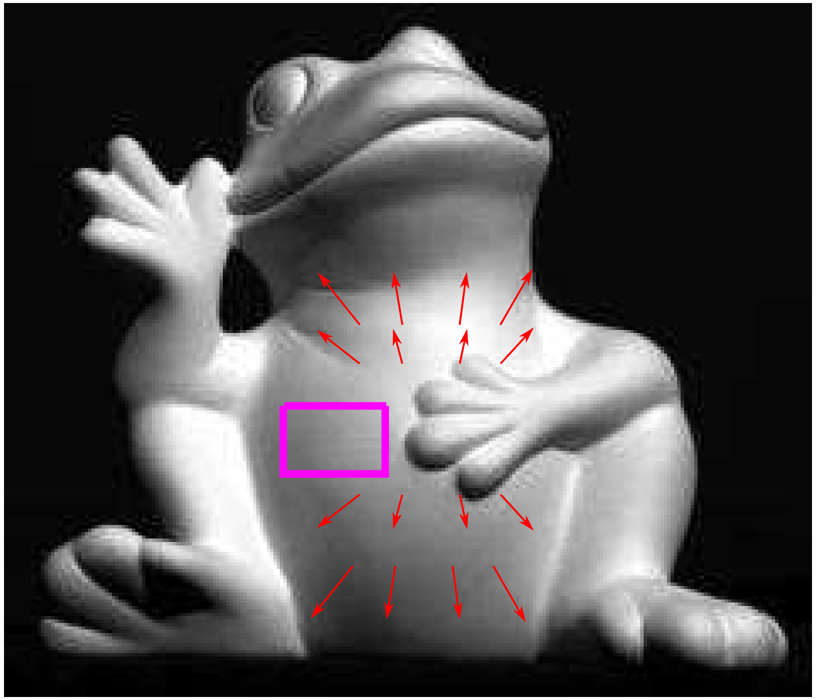
NORMALS



Normal Field

$$\hat{n}(x, y, z)$$

Defined only on
surface points

$$\hat{n}(x, y)$$

If only one z = f(x,y)

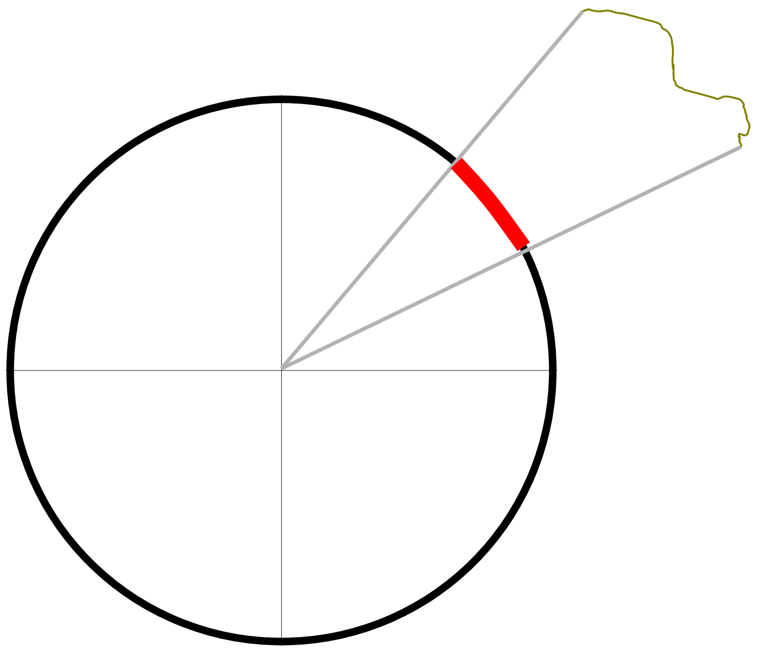
Gradient Field

$$\nabla Z(x, y) = \left[\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right]_{(x,y)}$$

Gradient / Normal fields are integrable,
i.e., integrating along a closed curve gives 0.

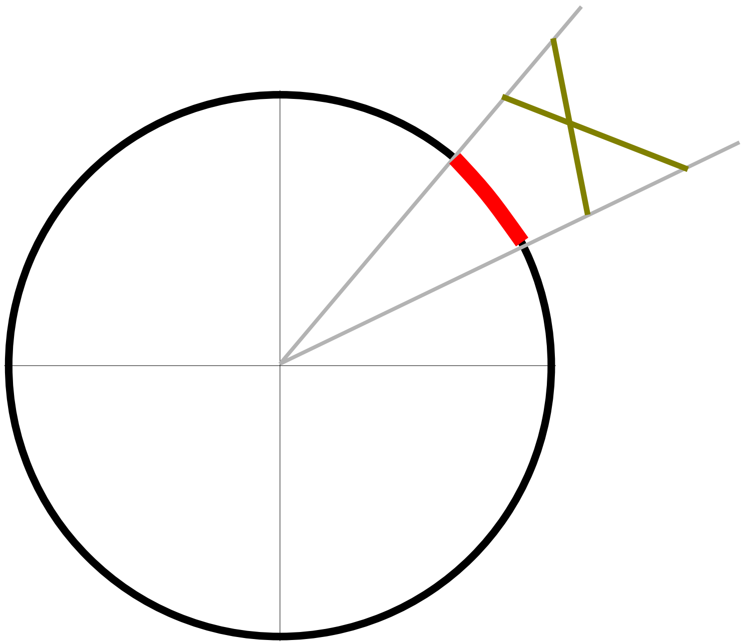
ANGLES

Angle subtended by a curve on a point, is length of curve projected on unit circle



ANGLES

Angle subtended by a curve on a point, is length of curve projected on unit circle

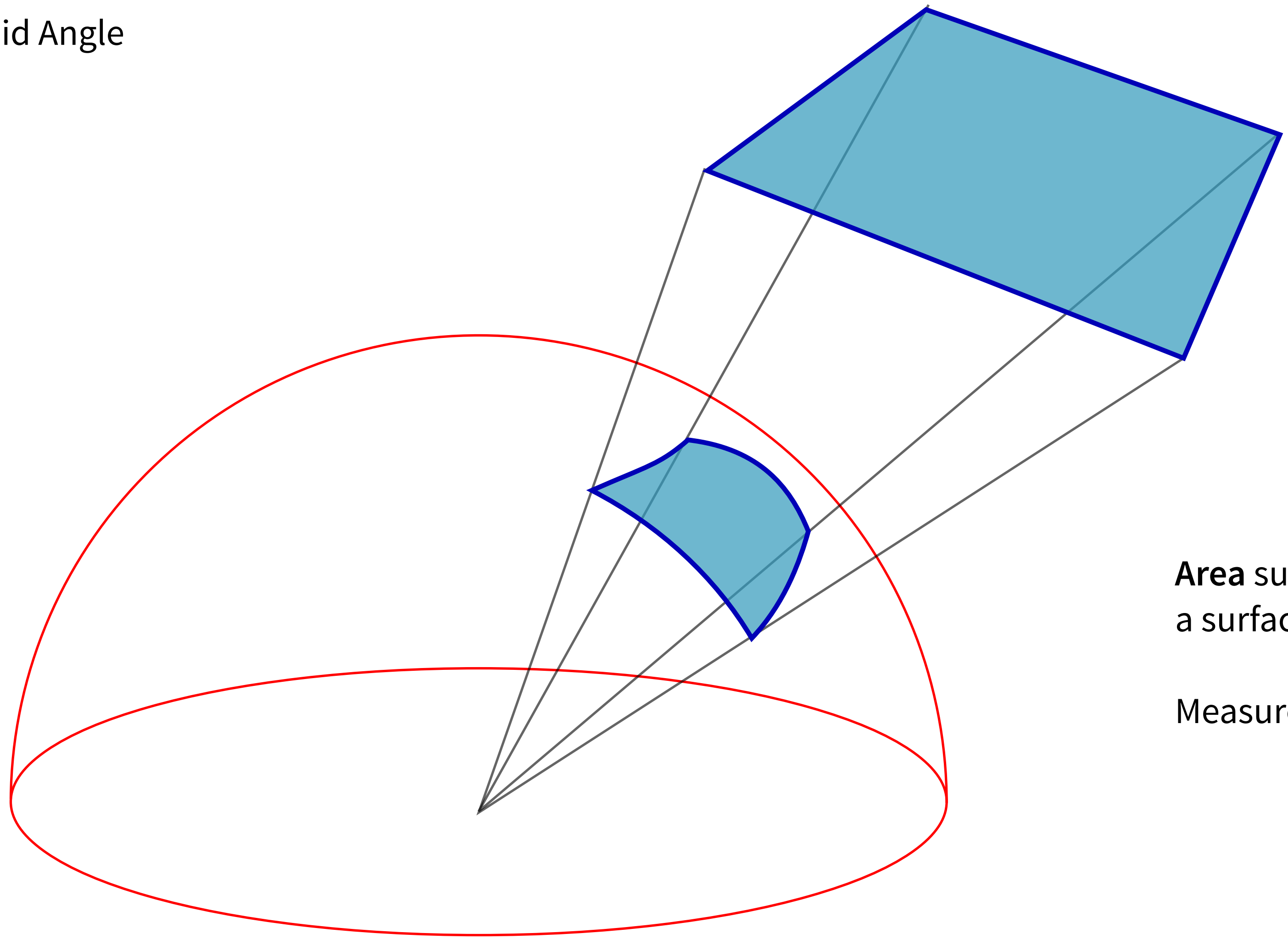


Angle between two lines is the angle subtended by any curve joining the two lines.

Measured in Radians

ANGLES

Solid Angle

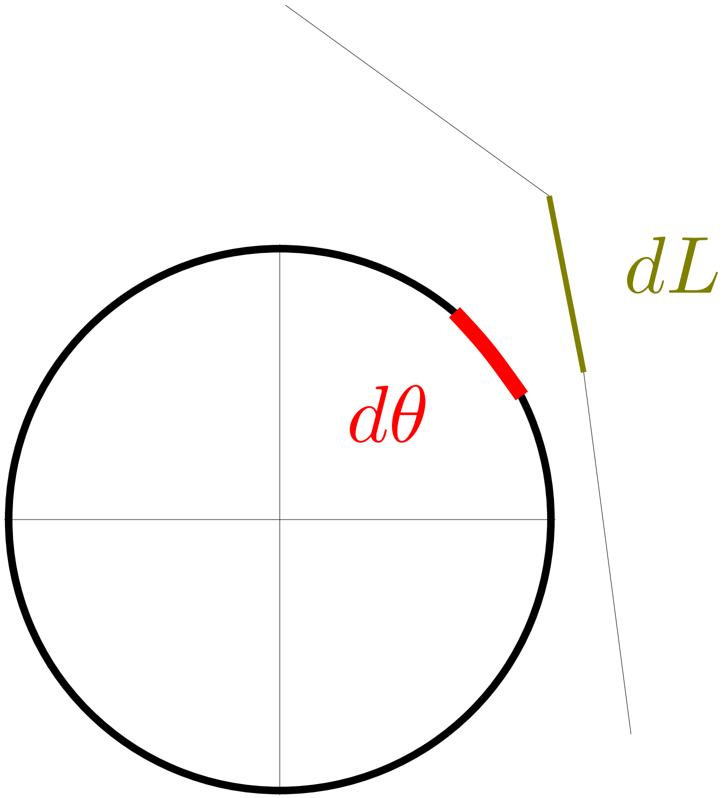


Area subtended by
a surface on unit sphere.

Measured in steridians

ANGLES

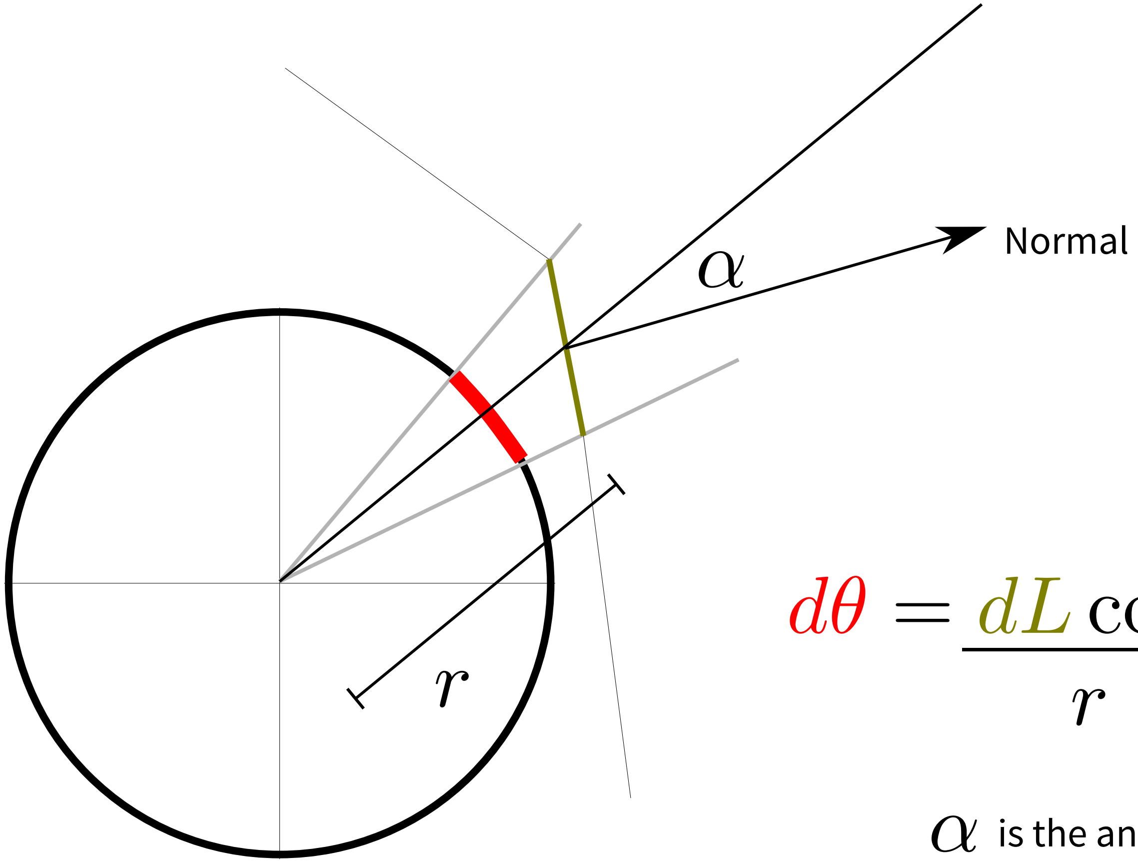
Differential angle



Take an infinitesimal part of a curve,
that can be assumed to be a line segment,
and find the angle it subtends

ANGLES

Differential angle



Same length subtends greatest angle if aligned with normal.

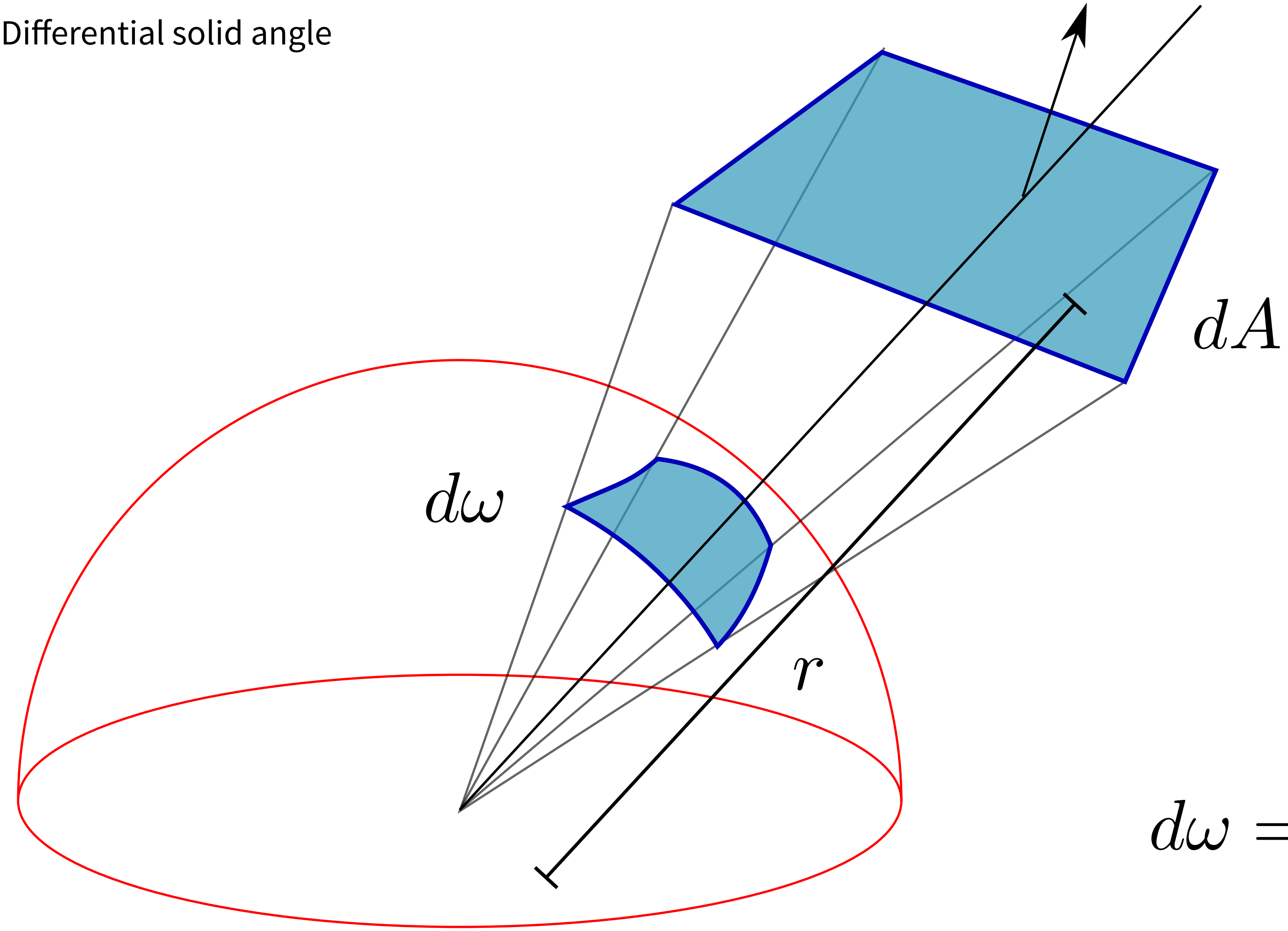
Otherwise, "foreshortened"

$$d\theta = \frac{dL \cos\alpha}{r}$$

α is the angle between the curve normal and line to the point.

ANGLES

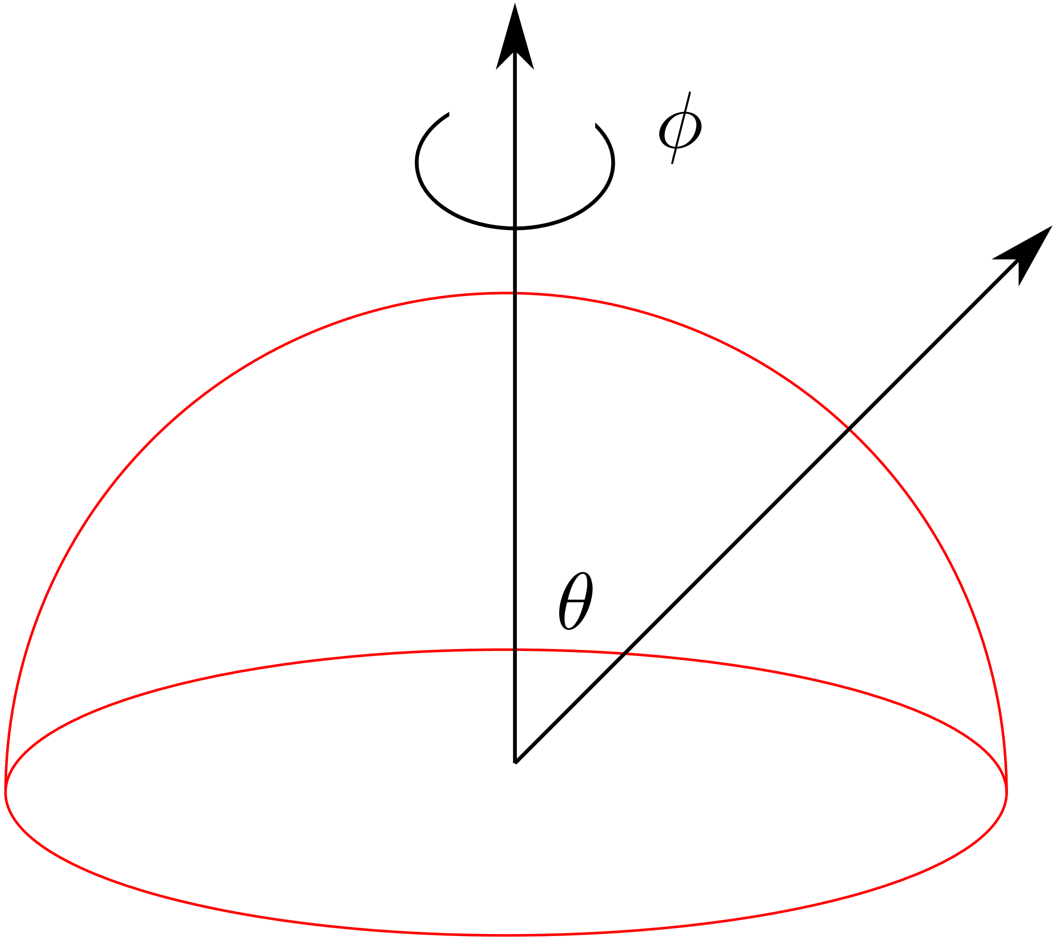
Differential solid angle



$$d\omega = \frac{dA\cos\alpha}{r^2}$$

ANGLES

Differential solid angle



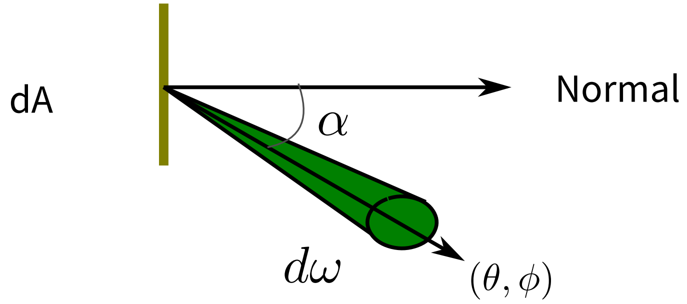
$$d\omega = \sin\theta d\theta d\phi$$

$$\phi \in [0, 2\pi]$$

$$\theta \in [0, \pi/2] \text{ or } [0, \pi]$$

RADIANCE

Radiance



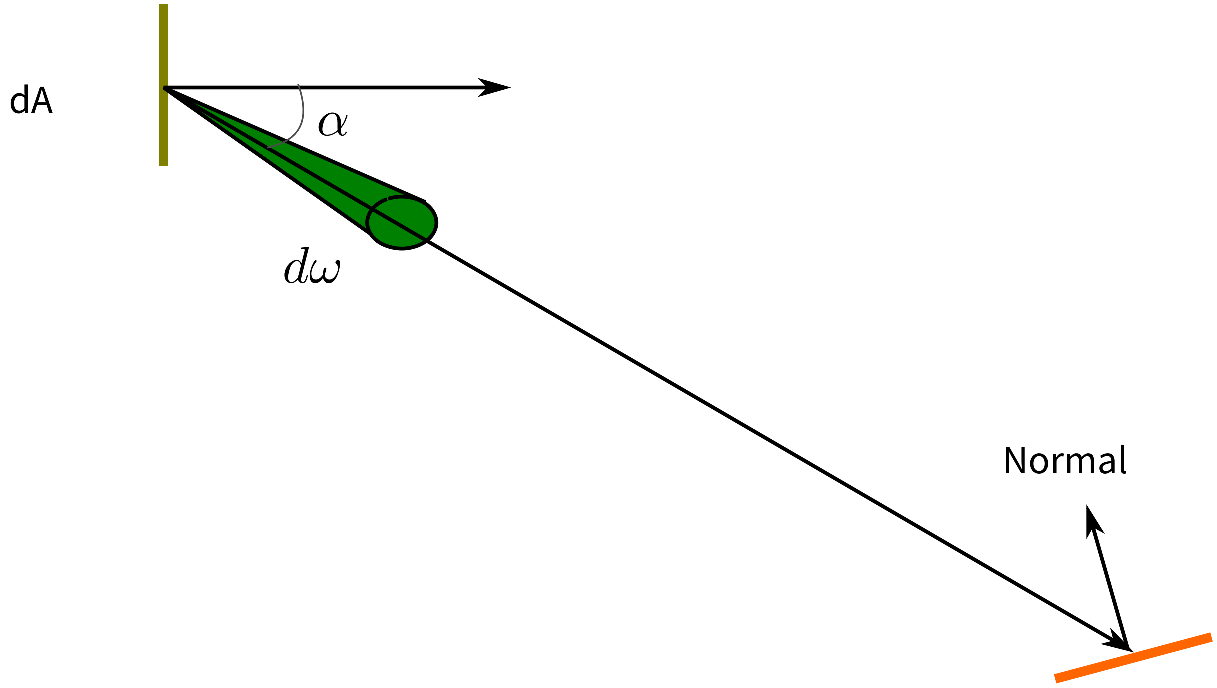
$$L(\theta, \phi) = \frac{P}{(dA\cos\alpha)d\omega}$$

Radiance L in a specified direction is defined in terms of power P that the infinitesimal patch dA is pushing out in the infinitesimal solid angle dω

Radiance along an unobstructed ray stays constant.

RADIANCE

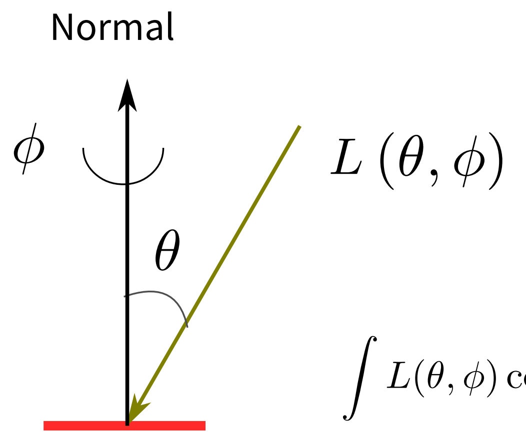
Irradiance



How much light is arriving at a surface ?

RADIANCE

Irradiance

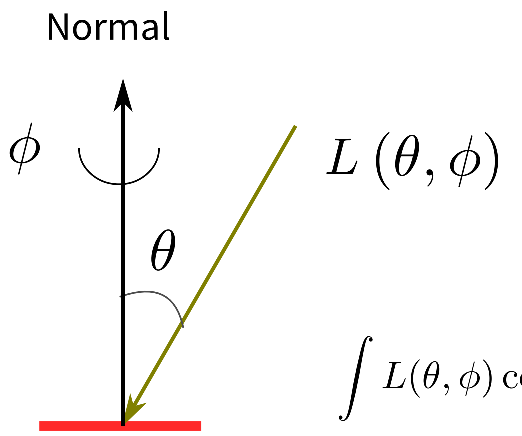


$$\int L(\theta,\phi)\cos\theta d\omega = \int\int L(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

How much light is arriving at a surface ?

RADIANCE

Irradiance



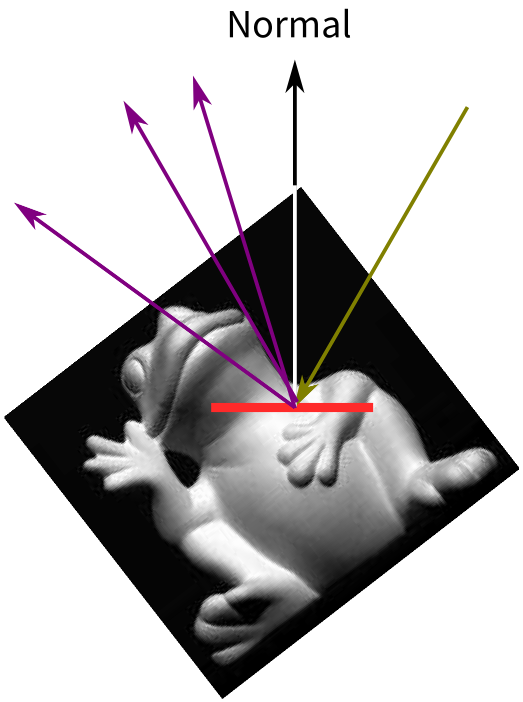
$$\int L(\theta,\phi)\cos\theta d\omega = \int\int L(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

What happens next ?   If this is a sensor, that's what it measures.

RADIANCE

Bi-directional Reflectance Distribution Function

(Non illuminant) Surfaces will Absorb and Reflect portions of the incident light,
in different directions.



RADIANCE

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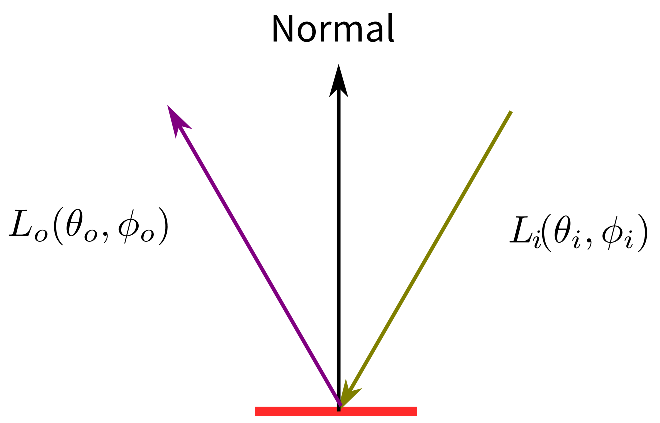


$$\rho(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{L_o(\theta_o, \phi_o)}{L_i(\theta_i, \phi_i)\cos\theta_i d\omega_i}$$

RADIANCE

Bi-directional Reflectance Distribution Function

Total radiance in output direction from integrating contributions from all incoming radiance:

$$L_o(\theta_o, \phi_o) = \int \rho(\theta_i, \phi_i, \theta_o, \phi_o) L_i(\theta_i, \phi_i) \cos\theta_i \, d\omega_i$$

- So, the BRDF describes how every incoming ray gets reflected by the surface.
  - How much energy in which direction
  - This is actually a function of wavelength λ

RADIANCE

Bi-directional Reflectance Distribution Function

Properties

- Positivity: $\rho(\theta_i, \phi_i, \theta_o, \phi_o) \geq 0$
- Helmholtz Reciprocity: $\rho(\theta_i, \phi_i, \theta_o, \phi_o) = \rho(\theta_o, \phi_o, \theta_i, \phi_i)$
- Total Energy leaving surface is less than total energy arriving

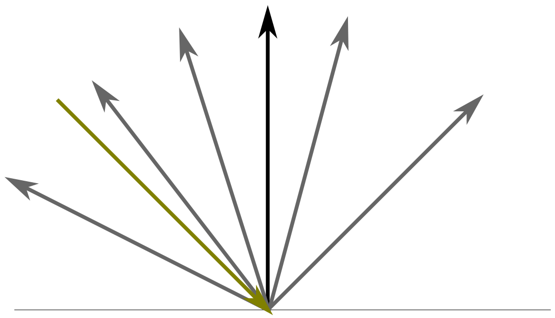
$$\int L_i(\theta_i, \phi_i) d\omega_i \geq \int \left[\int \rho(\theta_i, \phi_i, \theta_o, \phi_o) L_i(\theta_i, \phi_i) \cos\theta_i \; d\omega_i \right] \cos\theta_o d\omega_o$$

RADIANCE

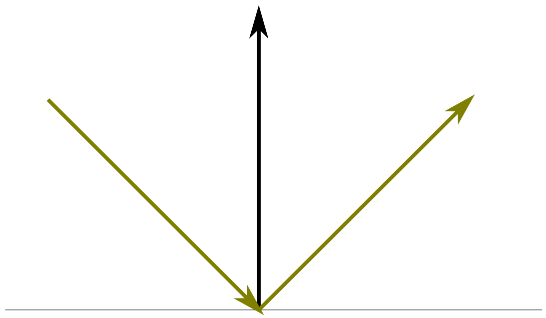
BRDF

Essentially a material property.

Outward Radiance in all directions same,
but still a function of input direction to normal.

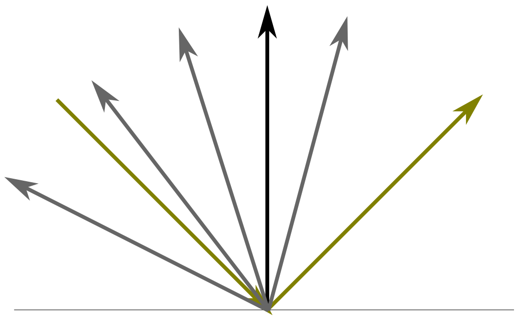


Lambertian BRDF $\rho(\cdot) = K$



Mirror

Most of the radiance
is in the mirror direction



Specular
Highlights

RADIANCE

BRDF

Essentially a material property.

The appearance won't change from change in viewing direction

Most of the radiance is in the mirror direction

Specular Highlights

Lambertian BRDF $\rho(\cdot) = K$

RADIANCE

BRDF

Source: Matusik et al., A Data Driven Reflectance Model, TOG 2003

RADIANCE

BRDF

In all cases, reflected radiance depends on surface geometry,

which we can exploit to estimate shape.