CSE 559A: Computer Vision

[credit: danjodon.deviantart.com]

Fall 2017: T-R: 11:30-1pm @ Lopata 101

Instructor: Ayan Chakrabarti (ayan@wustl.edu).
Staff: Abby Stylianou (abby@wustl.edu), Jarett Gross (jarett@wustl.edu)

http://www.cse.wustl.edu/~ayan/courses/cse559a/

Sep 5, 2017

OFFICE HOURS

Jarett Gross	Mon	5:40pm-6:30pm	@ TBD
Ayan Chakrabarti	Wed	9:30am-10:30am	@ Jolley 205
Abby Stylianou*	Fri	10:00am-11:00am	@ TBD

Mon/Fri locations will be decided in a day or two.

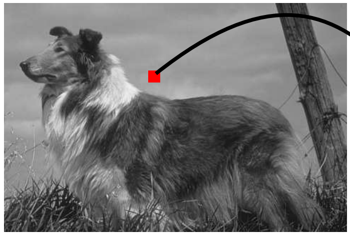
* Some of the Friday slots will be allocated as recitation sections (one for each problem set).
  Dates will be posted in advance.

CONVENTION

RECAP

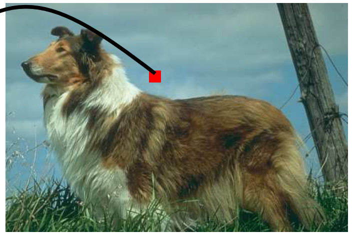
- An image X is an *array** of intensities.
- $X[n]$ or $X[n_x, n_y]$ refers to intensities for a particular pixel at location n or $[n_x, n_y]$.
  - Single index denotes $n = [n_x, n_y]^T$ is a vector of two integers.
- Each $X[n]$ is a scalar for a grayscale image, or a 3-vector for an RGB color image.
  (Unless otherwise specified, vector implies column vector)

$$Y[n] = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} X[n]$$



$Y[n]$

$$\begin{bmatrix} 0.4 \\ 0.4 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2 \\ 0.9 \end{bmatrix}$$



$X[n]$

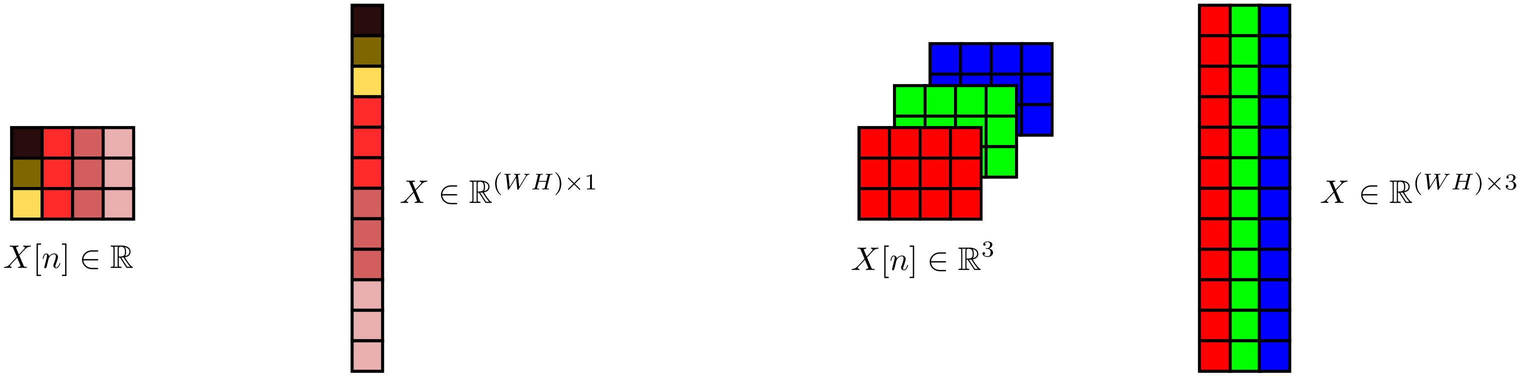
*Clarification: numpy convention is H x W x C: (vertical, horizontal, channels) or H x W.

Do not think of single-channel images themselves as matrices !

It makes no sense to "matrix multiply" a 80x60 pixel image with a 60x20 pixel image.

CONVENTION: LINEAR OPERATIONS

- But sometimes, we want to interpret operations as linear on all intensities / intensity vectors in an image.
- Stack all pixel locations, in some pre-determined order, as rows. Represent X as:
  - $(HW) \times 3$ matrix: color images
  - $(HW) \times 1$ vector: grayscale images.



$$Y[n] = C\,X[n] \Rightarrow Y = X\,C^T$$

```
# Begin with X as (H,W,3) array
Xflt = np.reshape(X,(-1,3))     # Flatten X to a (H*W, 3) matrix
Yflt = np.matmul(Xflt,C.T)      # Post-multiply by C
Y = np.reshape(Yflt,X.shape)    # Turn Y back to an image array
```

CONVOLUTION

$$\text{Notation: } Y = X * k$$

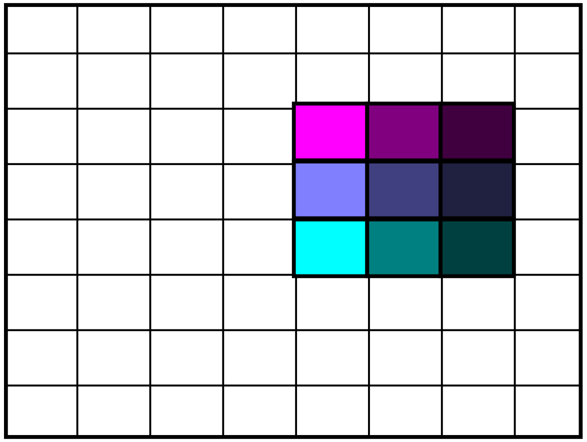
$$Y[n] = \sum_{n'} k[n'] \; X[n - n']$$

$$Y[n_x, n_y] = \sum_{n'_x} \sum_{n'_y} k\left[n'_x,\; n'_y\right] \; X\left[(n_x - n'_x),\; (n_y - n'_y)\right]$$

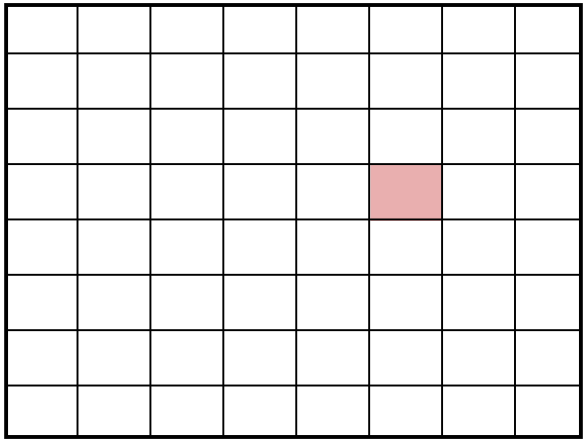
- Double summation over the support / size of the kernel k
- We assume $k[n] \in \mathbb{R}$ is scalar vaued.
    - If $X[n]$ is scalar, so is $Y[n]$.
    - If X is a color image, each channel convolved with k independently.

To go from m to n channels in a "conv layer": $k[n] \in \mathbb{R}^{n \times m}$ is matrix valued, and $k[n'] \, X[n - n']$ is a matrix-vector product.

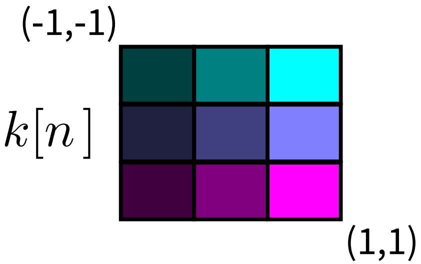
CONVOLUTION



$X[n]$



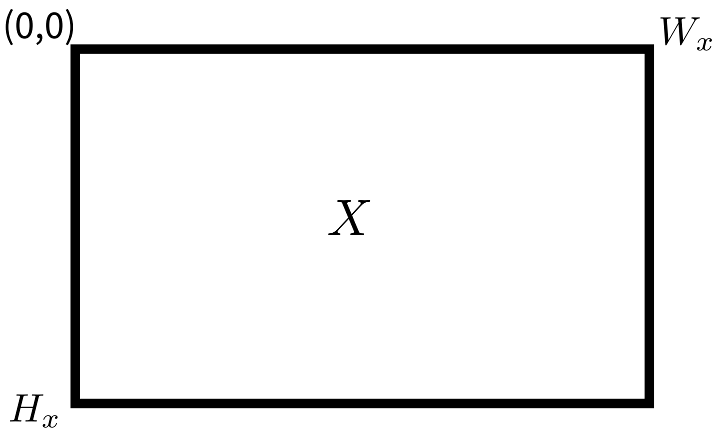
$Y[n]$



This assumes a 0 centered kernel

$$Y[n] = \sum_{n'} k[n'] \; X[n - n']$$

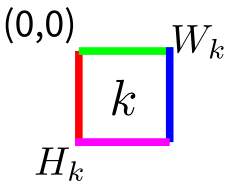
CONVOLUTION



We pass 2D arrays to the convolve functions, and get a 2D array out. Let's assume top left index is (0,0) for all.

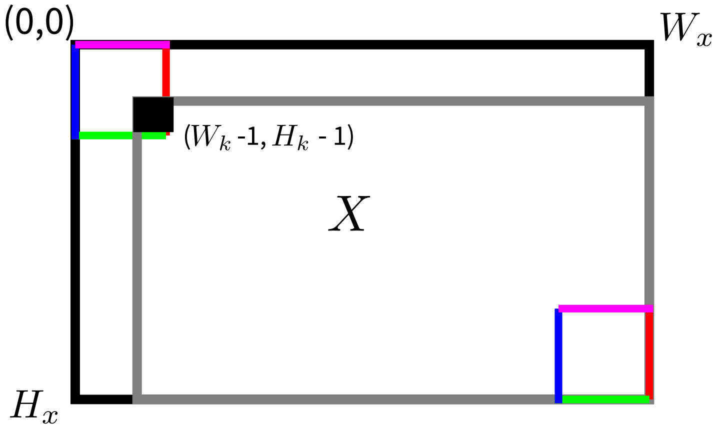
Let W_x, W_k and W_y denote the widths of X, k, and Y; and H_x, H_k and H_y the heights.

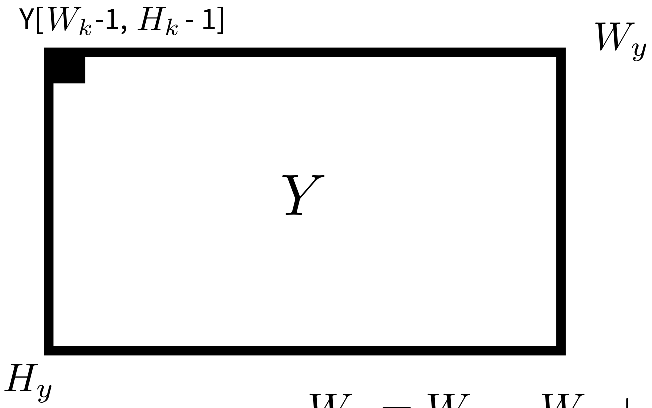
The 2D convolution function in most libraries provide 3 options: Valid, Full, and Same.



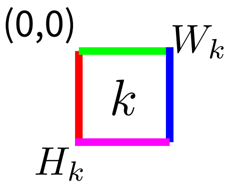
$$Y[n] = \sum_{n'} k[n'] \quad X[n - n']$$

CONVOLUTION





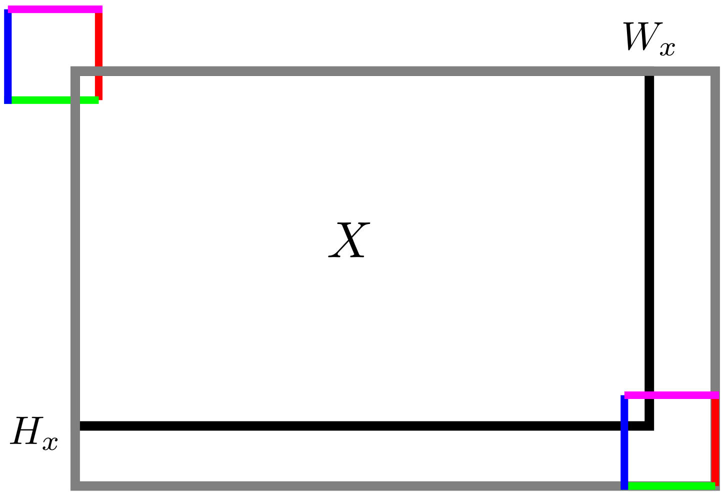
$$W_y = W_x - W_k + 1$$
$$H_y = H_x - H_k + 1$$

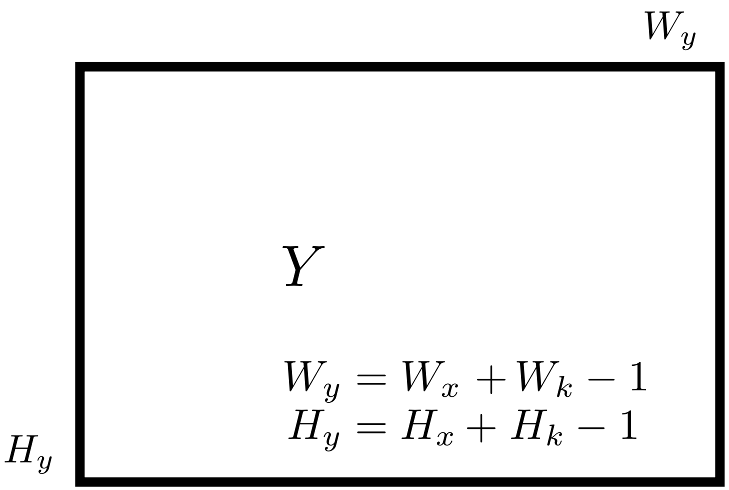


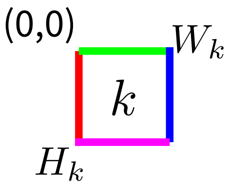
Valid: Subet of values of $Y[n]$ for which EVERY $X[n-n']$ is defined.

$$Y[n] = \sum_{n'} k[n'] \quad X[n-n']$$

CONVOLUTION



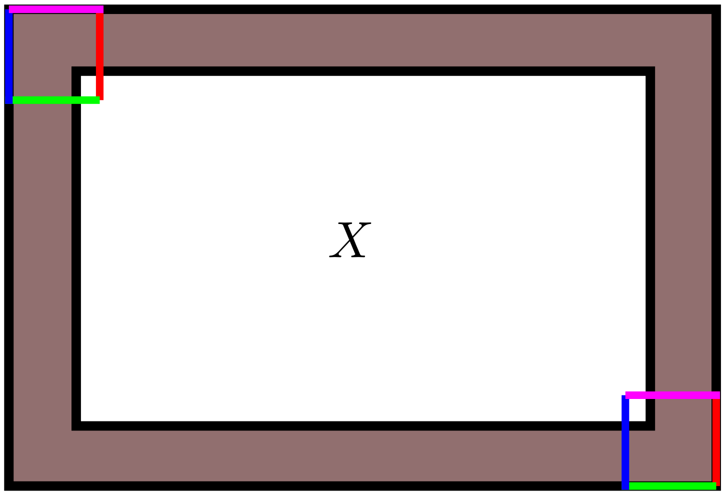




Full: Subet of values of $Y[n]$ for which ANY $X[n-n']$ is defined.

$$Y[n] = \sum_{n'} k[n'] \;\; X[n-n']$$

CONVOLUTION



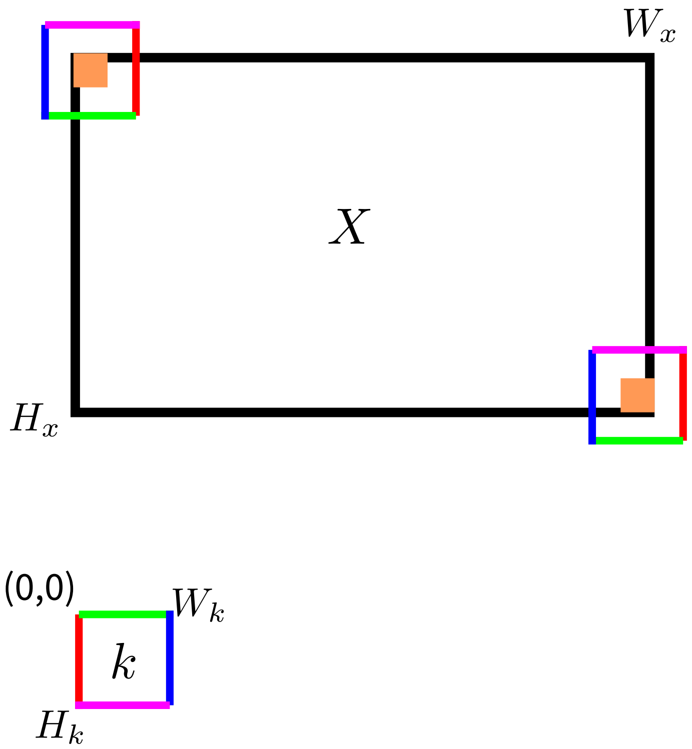
Padding

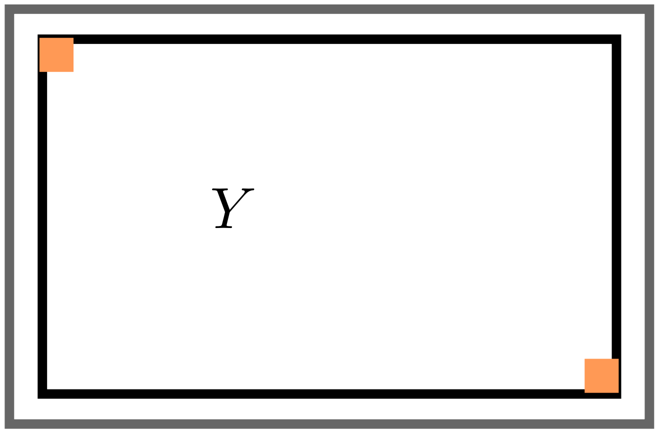
What do we use for the missing values of X[n] ?

- Zero (Often Default)
- Some other constant
- Reflect / Symmetric (across boundary)
- Circular (wrap around)
- Replicate

...

CONVOLUTION





Same: Center Crop of Full output, that is same size as X.

For odd sized kernels, corresponds to treating center of kernel as (0,0).

Same does what we "expect", but you should understand the padding and cropping involved. When kernel size isn't odd, which crop is taken often depends on the library.

CONVOLUTION: PROPERTIES

Let $X *_{\text{full}} k$, $X *_{\text{val}} k$, and $X *_{\text{same}} k$ denote full, valid, and same convolution (with zero padding for full and same)

- **Linear / Distributive**: For scalars α, β;
  - If $Y = X * k$, then: $X * (\alpha k) = (\alpha X) * k = \alpha Y$
  - If $Y_1 = X * k_1$ and $Y_2 = X * k_2$, (k_1, k_2 same size): $X * (\alpha k_1 + \beta k_2) = \alpha Y_1 + \beta Y_2$
  - If $Y_1 = X_1 * k$ and $Y_2 = X_2 * k$, (X_1, X_2 same size): $(\alpha X_1 + \beta X_2) * k = \alpha Y_1 + \beta Y_2$
- **Associative**
  - $\left(X *_{\text{full}} k_1\right) *_{\text{full}} k_2 = X *_{\text{full}} \left(k_1 *_{\text{full}} k_2\right)$
  - $\left(X *_{\text{val}} k_1\right) *_{\text{val}} k_2 = X *_{\text{val}} \left(k_1 *_{\text{full}} k_2\right)$
  - $\left(X *_{\text{same}} k_1\right) *_{\text{same}} k_2 \neq X *_{\text{same}} \left(k_1 *_{\text{full}} k_2\right)$
- **Commutative**: $k_1 *_{\text{full}} k_2 = k_2 *_{\text{full}} k_1$
  - $\left(X *_{\text{full}} k_1\right) *_{\text{full}} k_2 = \left(X *_{\text{full}} k_2\right) *_{\text{full}} k_1$
  - $\left(X *_{\text{val}} k_1\right) *_{\text{val}} k_2 = \left(X *_{\text{val}} k_2\right) *_{\text{val}} k_1$
  - $\left(X *_{\text{same}} k_1\right) *_{\text{same}} k_2 \neq \left(X *_{\text{same}} k_2\right) *_{\text{same}} k_1$

CONVOLUTION

$X[n]$

$Y[n]$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$k[n]$

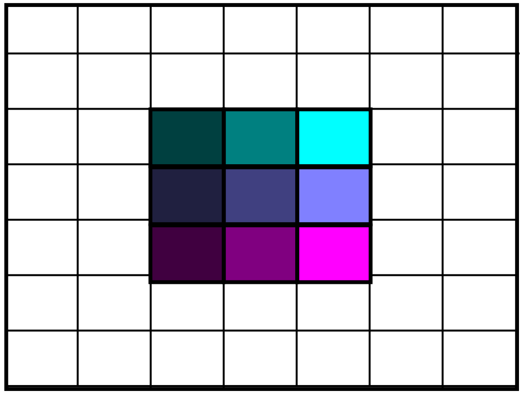
(Same with zero padding)

CONVOLUTION

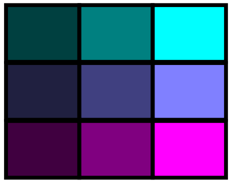
$X[n]$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$Y[n]$



Kernel = Impulse Response



$k[n]$

(Same with zero padding)

CONVOLUTION

$X[n]$

$Y[n]$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	1

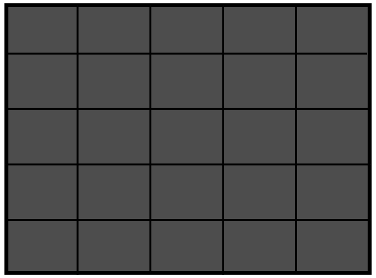
$k[n]$

(Same with zero padding)

CONVOLUTION

$X[n]$

$Y[n]$



$k[n]$ = 1 / 25

(Same with zero padding)

CONVOLUTION

$X[n]$

$Y[n]$

$\sigma = 1$

Gaussian Kernels

$$G_\sigma[n_x, n_y] \propto \exp\left(-\frac{n_x^2 + n_y^2}{2\sigma^2}\right)$$

$$\sum_{n_x, n_y} G_\sigma[n_x, n_y] = 1 \qquad n_x, n_y = [-S, -(S-1), \ldots, -1, 0, 1, \ldots, (S-1), S]$$

CONVOLUTION

$X[n]$

$Y[n]$

$\sigma = 2$

Gaussian Kernels

$$G_\sigma[n_x, n_y] \propto \exp\left(-\frac{n_x^2 + n_y^2}{2\sigma^2}\right)$$

$$\sum_{n_x, n_y} G_\sigma[n_x, n_y] = 1 \qquad n_x, n_y = [-S, -(S-1), \ldots, -1, 0, 1, \ldots, (S-1), S]$$

CONVOLUTION

$X[n]$

$Y[n]$

$\sigma = 3$

Gaussian Kernels

$$G_\sigma[n_x, n_y] \propto \exp\left(-\frac{n_x^2 + n_y^2}{2\sigma^2}\right)$$

$$\sum_{n_x, n_y} G_\sigma[n_x, n_y] = 1 \qquad n_x, n_y = [-S, -(S-1), \ldots, -1, 0, 1, \ldots, (S-1), S]$$

CONVOLUTION

$X[n]$

$Y[n]$

$\sigma = 4$

Gaussian Kernels

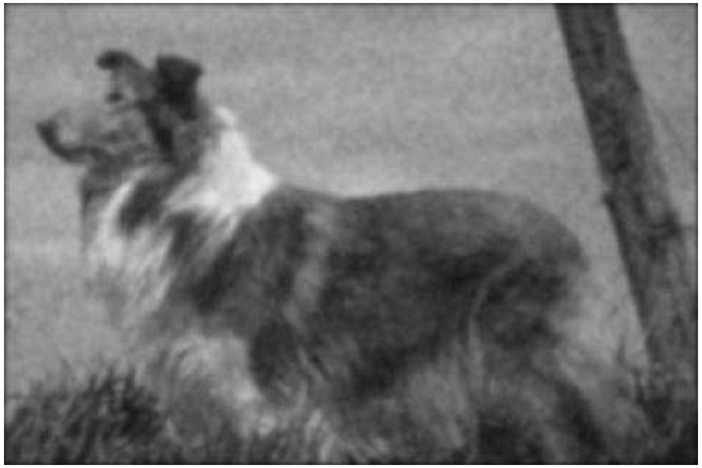
$$G_\sigma[n_x, n_y] \propto \exp\left(-\frac{n_x^2 + n_y^2}{2\sigma^2}\right)$$

$$\sum_{n_x, n_y} G_\sigma[n_x, n_y] = 1 \qquad n_x, n_y = [-S, -(S-1), \ldots, -1, 0, 1, \ldots, (S-1), S]$$

CONVOLUTION

$X[n]$

$Y[n]$



Gaussian Kernels

$$G_\sigma[n_x, n_y] \propto \exp\left(-\frac{n_x^2 + n_y^2}{2\sigma^2}\right)$$

$$\sum_{n_x, n_y} G_\sigma[n_x, n_y] = 1 \qquad n_x, n_y = [-S, -(S-1), \ldots, -1, 0, 1, \ldots, (S-1), S]$$

CONVOLUTION

$X[n]$

$Y[n]$

$\sigma = 4 \quad \alpha = 1$

Unsharp Masking

$$Y = (1+\alpha)X - \alpha(X * G_\sigma) = X * ((1+\alpha)\delta - \alpha G_\sigma)$$

CONVOLUTION

$X[n]$

$Y[n]$

$\sigma = 2 \quad \alpha = 1$

Unsharp Masking

$$Y = (1+\alpha)X - \alpha(X * G_\sigma) = X * ((1+\alpha)\delta - \alpha G_\sigma)$$

CONVOLUTION

$X[n]$ $Y[n]$

$\sigma = 2 \quad \alpha = 5$

Unsharp Masking

$$Y = (1+\alpha)X - \alpha(X * G_\sigma) = X * ((1+\alpha)\delta - \alpha G_\sigma)$$

CONVOLUTION

$X[n]$

$Y[n]$

$\sigma = 2 \quad \alpha = 10$

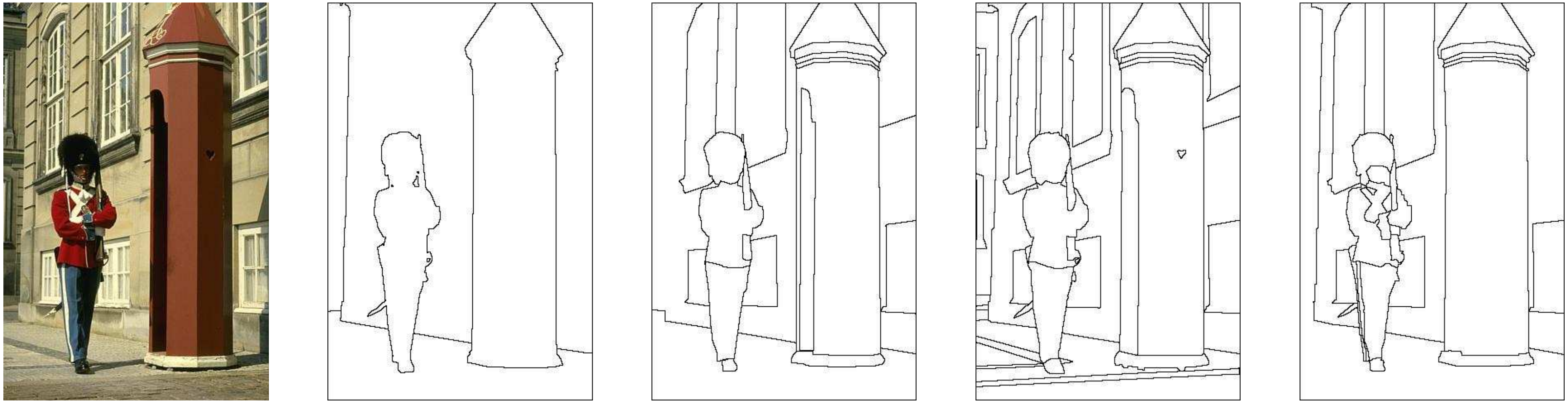
Unsharp Masking

$$Y = (1+\alpha)X - \alpha(X * G_\sigma) = X * ((1+\alpha)\delta - \alpha G_\sigma)$$

APPLICATION: EDGE DETECTION

What is an edge ?

Different answers from different people !



Depth boundary / Material Boundary / Object Boundary ?

Edge (not boundary): Location where image intensity is changing rapidly in some direction.

Directional Derivative

APPLICATION: EDGE DETECTION

Finite Difference Approximation

$$\frac{\partial}{\partial n_x} X[n_x, n_y] \quad \propto X[n_x + 1, n_y] - X[n_x - 1, n_y]$$

$$X * [1 \quad 0 \quad -1]$$

Derivative is a linear spatially invariant operation: Convolution

$$X * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Smoothed in y direction
"Sobel" Operator

$$X * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Y Derivative

APPLICATION: EDGE DETECTION

$$X * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$X * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Derivatives have been scaled so that gray (0.5) corresponds to 0. Bright to positive derivative values, dark to negative.

APPLICATION: EDGE DETECTION

Smoothing + Derivative

$$I_x = \partial_x * (G_\sigma * X) = (\partial_x * G_\sigma) * X = G_{x:\sigma} * X$$

$$G_x = \frac{-x}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \qquad G_y = \frac{-y}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

Derivative of Gaussian (DoG) Filters

APPLICATION: EDGE DETECTION

Smoothing + Derivative

$$I_x = \partial_x * (G_\sigma * X) = (\partial_x * G_\sigma) * X = G_{x:\sigma} * X$$

$$G_x = \frac{-x}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \qquad G_y = \frac{-y}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$G_\theta = \frac{-(x\cos\theta + y\sin\theta)}{2\pi\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$I_\theta = I_x \cos\theta + I_y \sin\theta$$

Just need to convolve twice. Gives us an expression for derivative along every direction.

APPLICATION: EDGE DETECTION

Smoothing + Derivative

$$I_\theta[n] = I_x[n]\cos\theta + I_y[n]\sin\theta$$

$$H[n] = \sqrt{I_x^2[n] + I_y^2[n]} = \max_\theta I_\theta[n]$$

$$\Theta[n] = \mathrm{atan2}(I_y, I_x) = \arg\max_\theta I_\theta[n]$$

Gives us gradient magnitude and direction.

Often applied even to filters that aren't "steerable" like DoG.

APPLICATION: EDGE DETECTION

I_x

I_y

I_{45°

APPLICATION: EDGE DETECTION

I_x

I_y

H

APPLICATION: EDGE DETECTION

I_x

I_y

$H > \epsilon$

APPLICATION: EDGE DETECTION

I_x

I_y

$H > \epsilon$

APPLICATION: EDGE DETECTION

Extensions

- Non-maxima Supression: Keep an edge pixel only if its magnitude is higher than its neighbors *along the direction of the derivative*.

e	f	g
b	a	c
h	j	k

$H[n]$

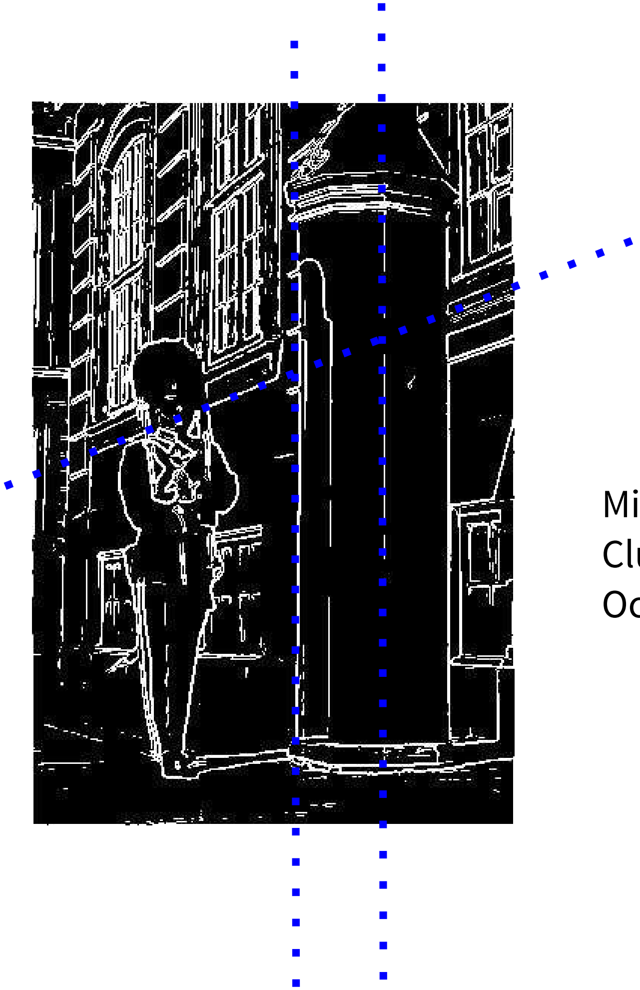
Declare edge if a above threshold and :
- a > b and a > c if θ = 0
- a > f and a > j if θ = 90
- a > e and a>k if θ = 45
- ....

- Canny: Keep a lower magnitude edge pixel if it has a higher edge magnitude neighbor.
  Two thresholds (hysteresis)

- Second derivative filters.

See Szeliski Section 4.2

LINES

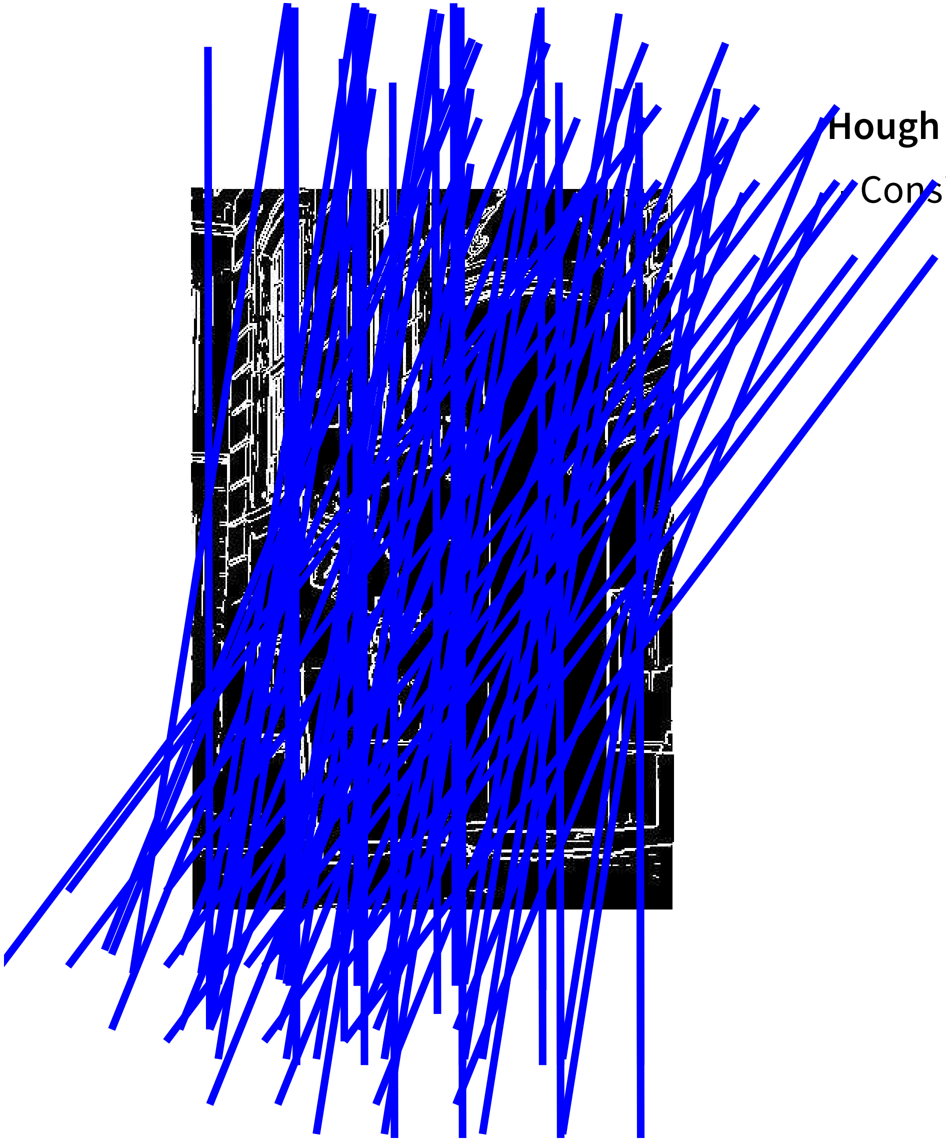


Missed detections
Clutter
Occlusions

Pool them together to detect scene structure: E.g., Lines

Edges are isolated per-pixel labels

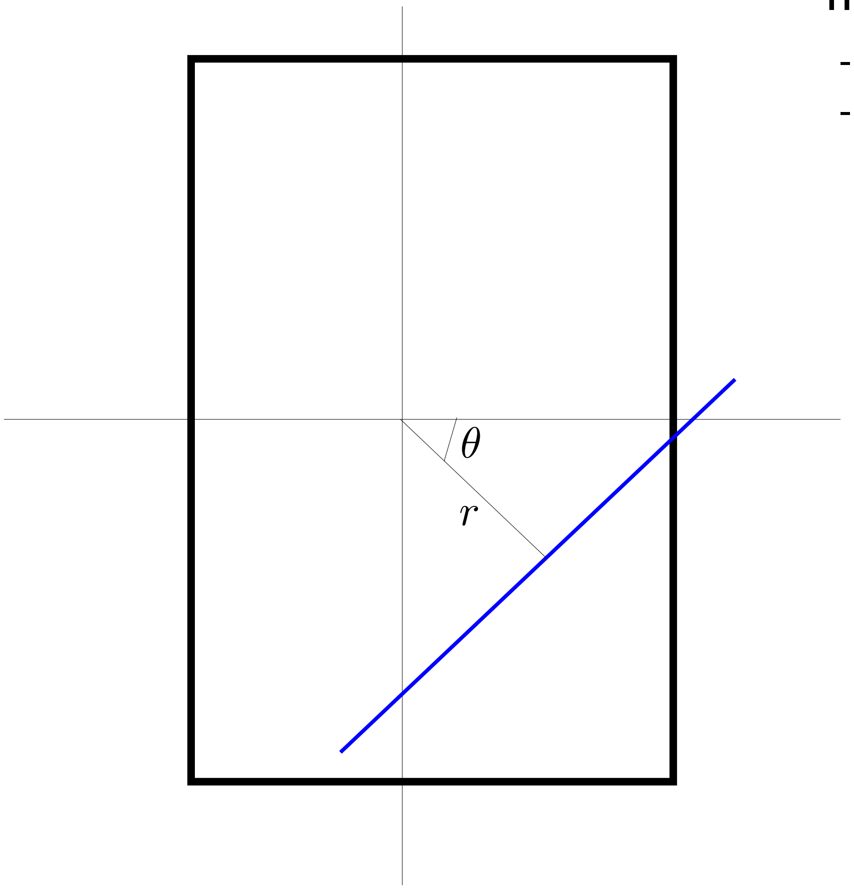
LINES



Hough Transform

- Consider ALL possible lines (on a 2D plane)

LINES



Hough Transform

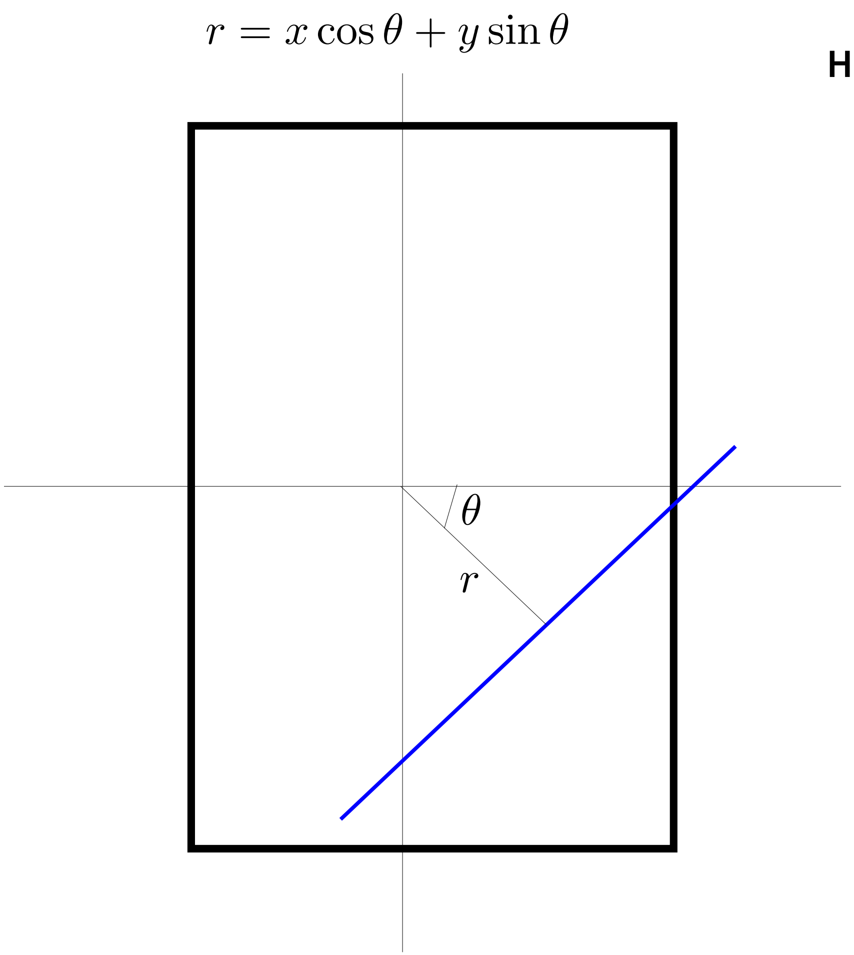
- Consider ALL possible lines (on a 2D plane)
- This is a two dimensional search space, could parameterize it in different ways.

$$r = x\cos\theta + y\sin\theta$$

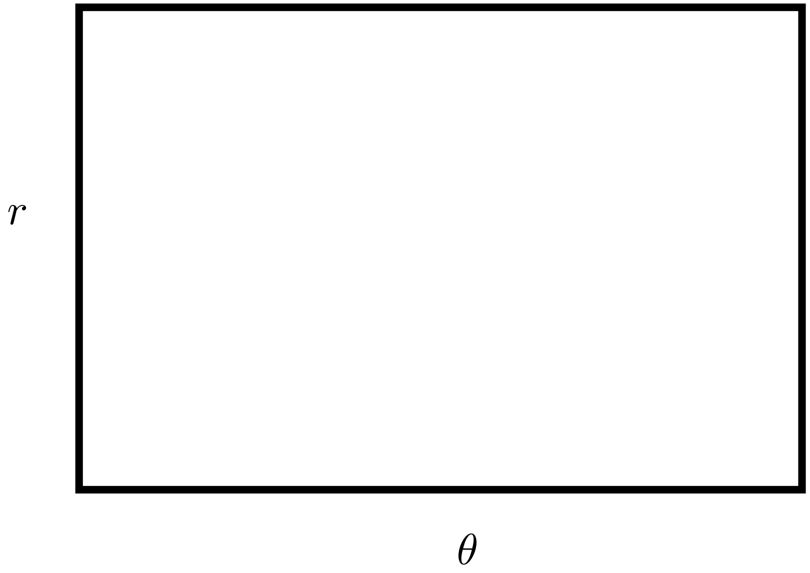
$$\theta \in [-\pi/2, \pi/2]$$

$$r \in [-r_{\max}, r_{\max}]$$

LINES



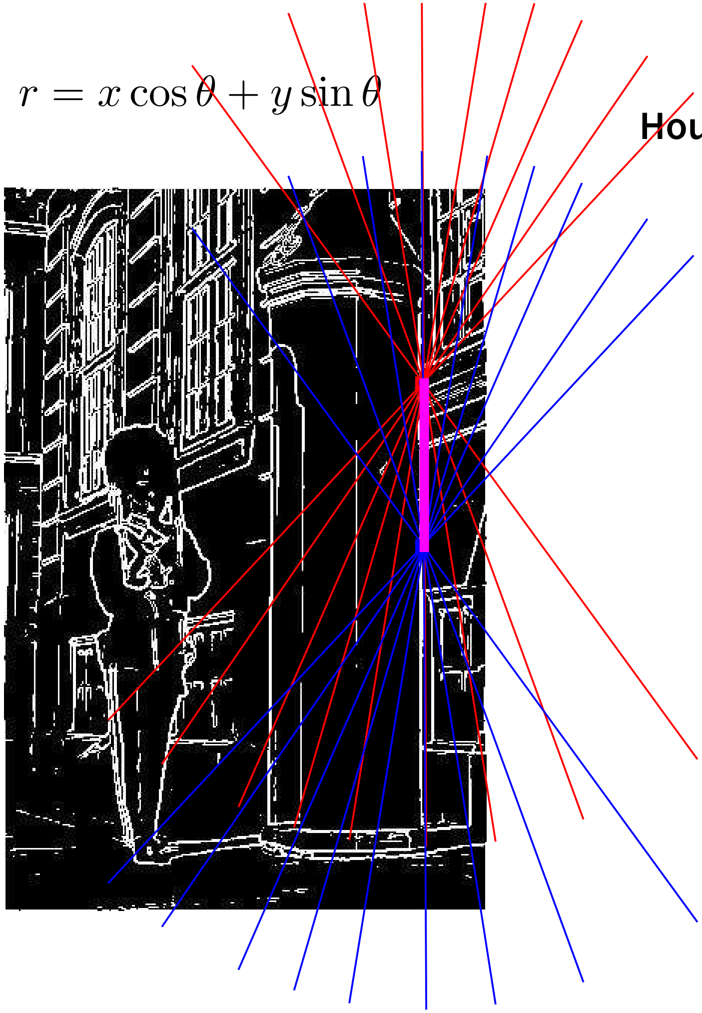
Hough Transform



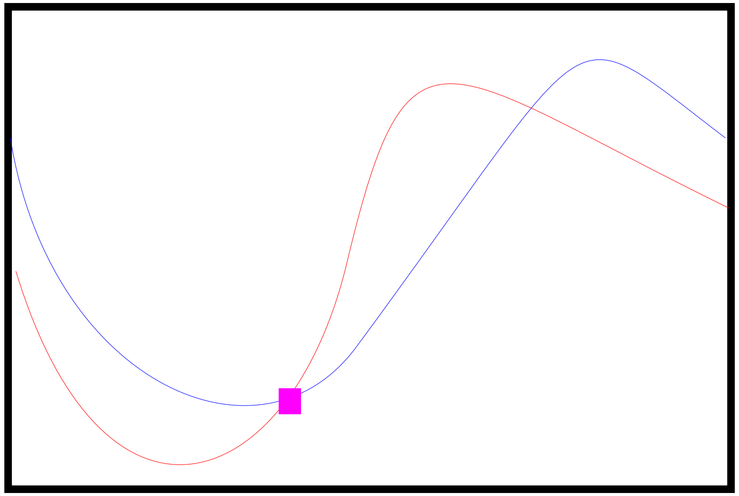
Discretize this space into a bunch of buckets.

LINES

$$r = x\cos\theta + y\sin\theta$$



Hough Transform



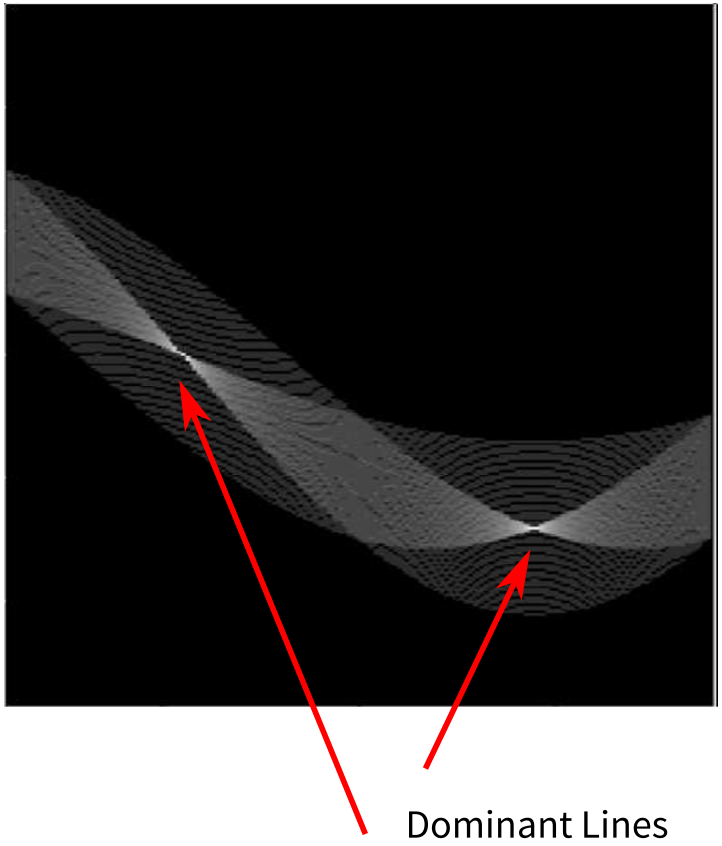
Each edge pixel casts a vote for all lines it could belong to.

Compute by plugging in x,y into equation: get a value of r for each θ (get a sinusoid)

LINES

$$r = x\cos\theta + y\sin\theta$$

Hough Transform



Each edge pixel casts a vote for all lines it could belong to.

Compute by plugging in x,y into equation: get a value of r for each θ (get a sinusoid)

Do this for all pixels and see which 'bins' get the most votes.

Variants
- Each edge pixel only casts one vote based on angle. (with or without sign)
- Vote weighted by magnitude of gradient.
- Exclusive vote (select dominant line, remove vote from its pixels for other lines)

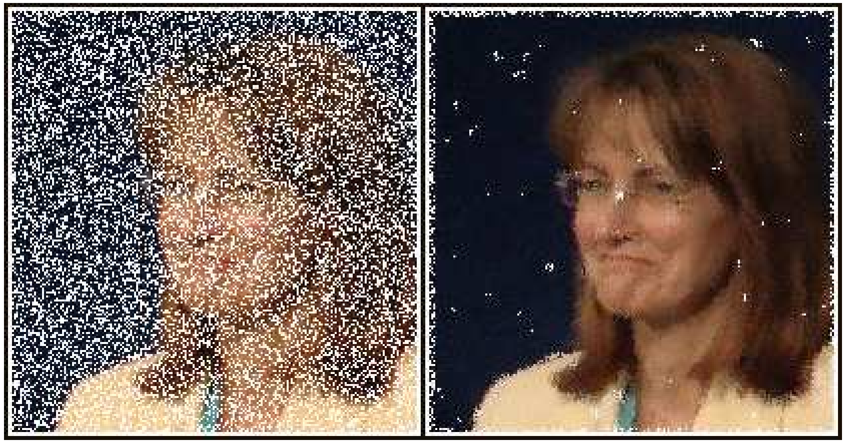
- Use same idea for line segments, circles, ellipses ...

OTHER NEIGHBORHOOD OPERATIONS

Median Filter / Order Statistics

$$Y[n] = \mathrm{Median}\{X[n-n']\}_{N[n']=1}$$

- Neighborhood function $N[n'] \in \{0,1\}$
- Often better at removing outliers than convolution.



Source: Wikipedia

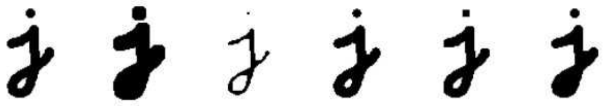
- Other ops: $Y[n] = \max / \min\{X[n-n']\}_{N[n']>0}$

OTHER NEIGHBORHOOD OPERATIONS

Morphological Operations

- Conducted on binary images ($X[n] \in \{0, 1\}$)
- Erosion: $Y[n] = \text{AND} \ \{X[n - n']\}_{N[n']=1}$ (1 if all neighbors 1)
- Dilation: $Y[n] = \text{OR} \ \{X[n - n']\}_{N[n']=1}$ (1 if any neighbor 1)
- Opening: Erosion followed by Dilation
- Closing: Dilation followed by Erosion

See Szeliski Sec 3.3.2



(a) (b) (c) (d) (e) (f)

Figure 3.21 Binary image morphology: (a) original image; (b) dilation; (c) erosion; (d) majority; (e) opening; (f) closing. The structuring element for all examples is a 5×5 square. The effects of majority are a subtle rounding of sharp corners. Opening fails to eliminate the dot, since it is not wide enough.

NEXT TIME

- Bilateral Filtering
- Fourier Transforms
- Making convolutions efficient
- Sampling and Scale
- Image Representations