

# CSE 559A: Computer Vision



[credit: danjodon.deviantart.com]

Fall 2017: T-R: 11:30-1pm @ Lopata 101

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<http://www.cse.wustl.edu/~ayan/courses/cse559a/>

Oct 19, 2017

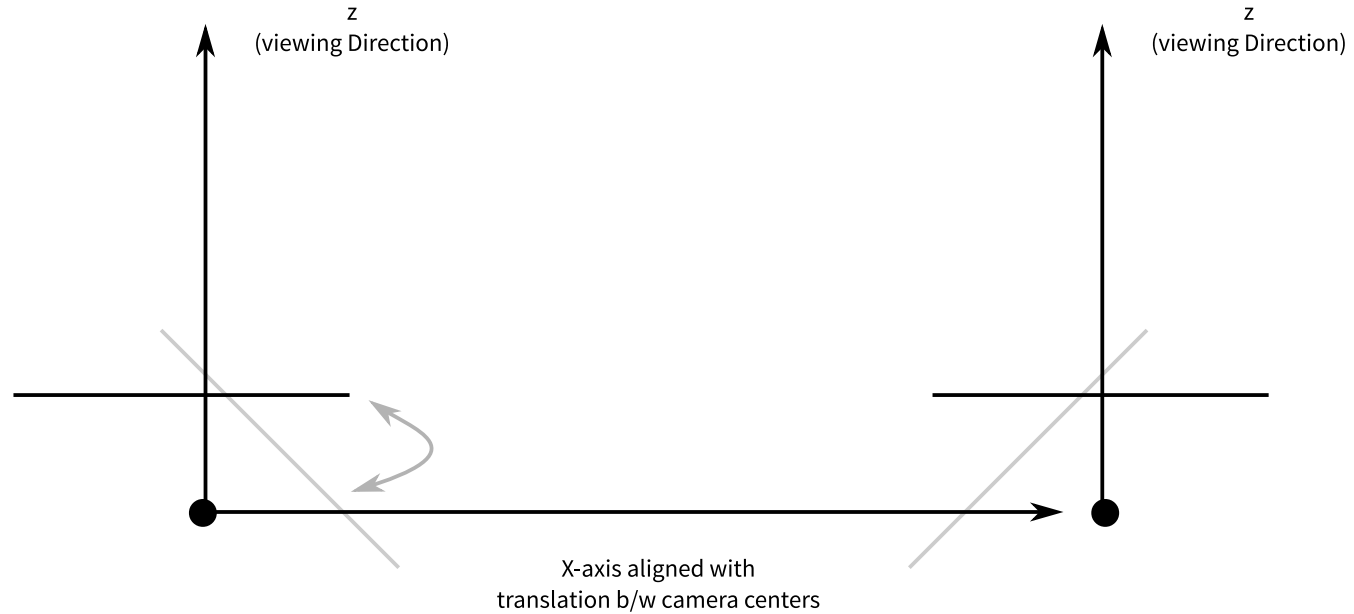
# GENERAL

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- Problem set 3 due next Thursday.
- Recitation Tomorrow.
- Will probably announce an extra office hour for next week.
- Suggestions of papers for project posted.
  - You can still choose a different paper, or do a project on your own research.
- Project Proposals Due Sunday Oct 29th 11:59pm.
- PSET1 Grades Out. Should have received mail with password & folder for feedback.

# RECTIFIED BINOCULAR STEREO

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Epipoles at infinity  
Epipolar lines all parallel to the X axis

# RECTIFIED BINOCULAR STEREO

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Left

$L[x,y]$  matches to  $R[x',y]$



Right

Epipolar Lines are Horizontal

Why are we doing this ? Two equations tell us 3D position of point

# RECTIFIED BINOCULAR STEREO

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Left

$L[x,y]$  matches to  $R[x',y]$

Visibility Constraint:  $x' \leq x$   
(object in front of camera)



Right

Epipolar Lines are Horizontal

# RECTIFIED BINOCULAR STEREO

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Left

$L[x,y]$  matches to  $R[x-d[x,y],y]$



Right

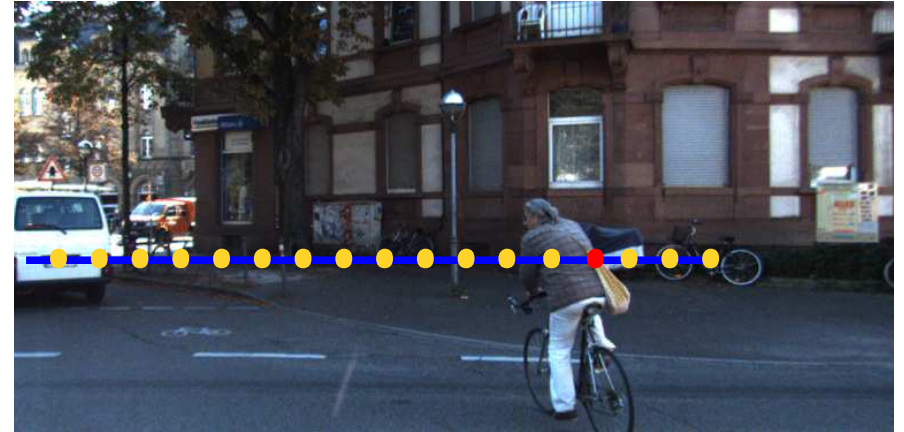
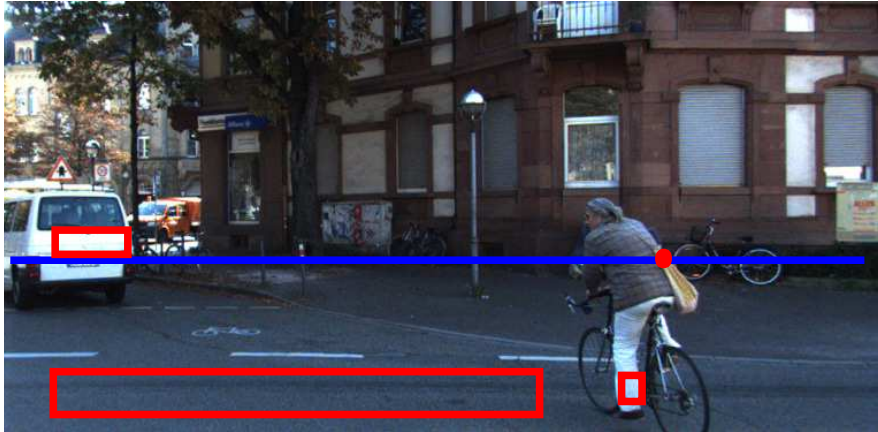
Epipolar Lines are Horizontal

$d[x,y] \geq 0$  is called the "disparity map"  
 $d[x,y]$  is inversely proportional to depth



# RECTIFIED BINOCULAR STEREO

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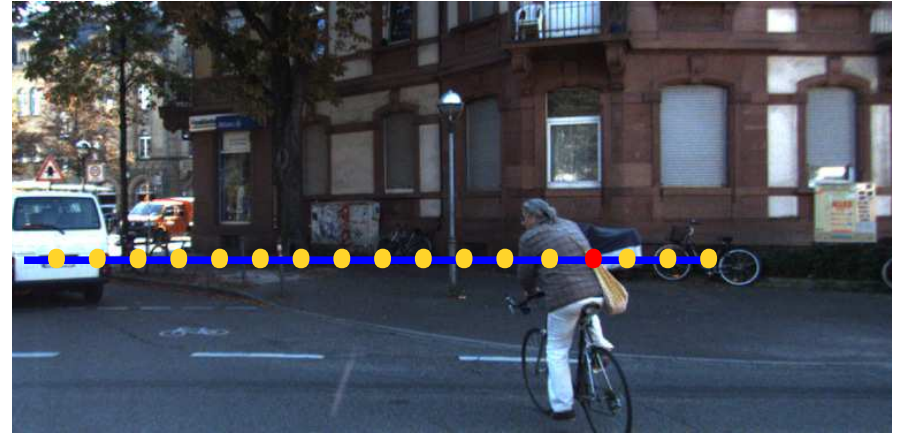
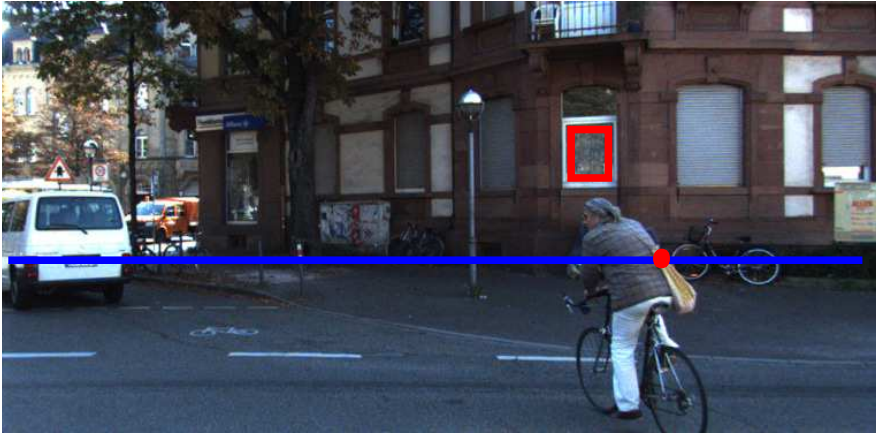


How do you find the correct match ?

-Smooth regions are ambiguous. Too many pixels on the right will look like a pixel on the left.

# RECTIFIED BINOCULAR STEREO

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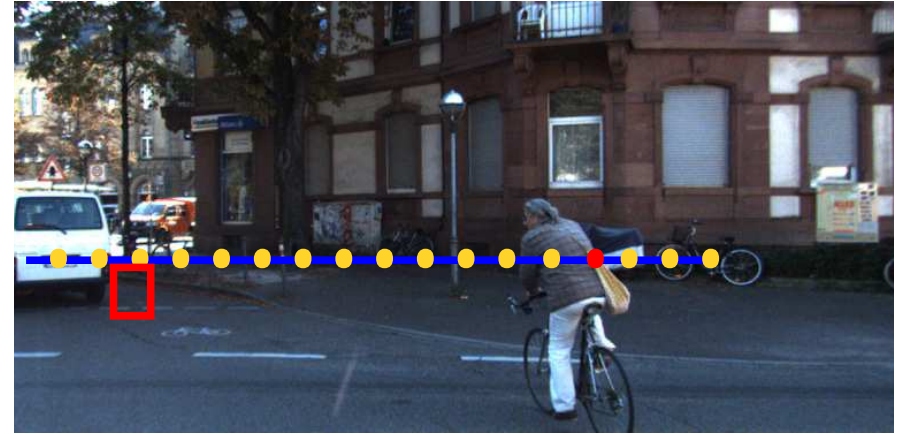


How do you find the correct match ?

- Smooth regions are ambiguous. Too many pixels on the right will look like a pixel on the left.
- Non lambertian regions are ambiguous. The correct pixel on the right will not look like the pixel on the left.



# RECTIFIED BINOCULAR STEREO



How do you find the correct match ?

- Smooth regions are ambiguous. Too many pixels on the right will look like a pixel on the left.
- Non lambertian regions are ambiguous. The correct pixel on the right will not look like the pixel on the left.
- Occlusions: pixel on the left is NOT visible in the image on the right and vice-versa.

**What is the right answer ?**

$d[x,y]$  = Value that the pixel  $[x,y]$  in the left image has moved by, even if it is not visible.  
In other words,  $(x-d[x,y],y)$  should be the co-ordinate of the projection of that 3D surface point in the right image.

Useful because we want to eventually use it to estimate depth.

# RECTIFIED BINOCULAR STEREO

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How do you find the correct match ?

- Smooth regions are ambiguous. Too many pixels on the right will look like a pixel on the left.
- Non lambertian regions are ambiguous. The correct pixel on the right will not look like the pixel on the left.
- Occlusions: pixel on the left is NOT visible in the image on the right and vice-versa.

Consider a neighborhood

Robust features  
(just look at sign of gradient  
instead of magnitude)

Last time: Matching with Census Transform

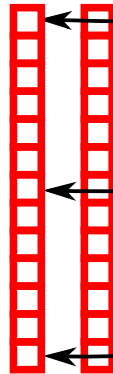
Just one of many 'robust features' / strategies for local matching.

# COST VOLUME FILTERING



Let us say we believe max value of disparity is  $D-1$   
So by considering all possible matches, we are building a  $W \times H \times D$  "cost volume"

# COST VOLUME FILTERING

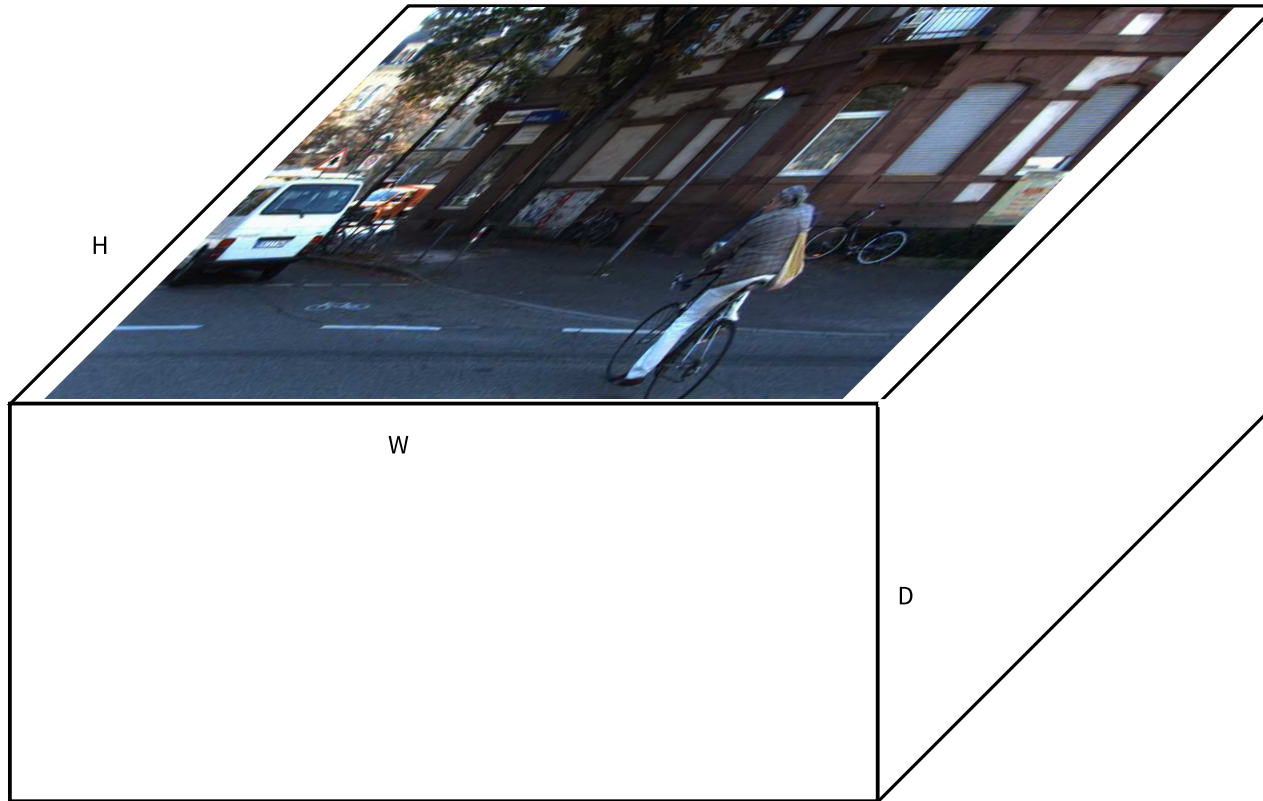


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# COST VOLUME FILTERING

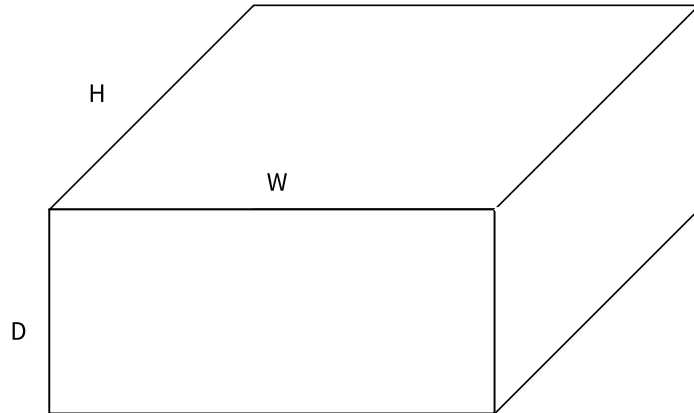
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$C[x,y,d]$  measures the quality of the match between  $L[x,y]$  and  $R[x-d,y]$

Let us say we believe max value of disparity is  $D-1$   
So by considering all possible matches, we are building a  $W \times H \times D$  "cost volume"

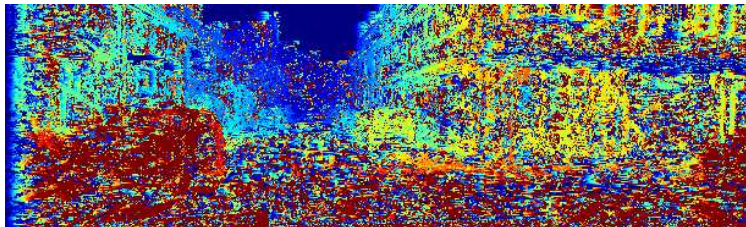
# COST VOLUME FILTERING



$C[x,y,d]$  measures the quality of the match between  $L[x,y]$  and  $R[x-d,y]$

In problem set 3, you simply compute the best match independently at each  $[x,y]$

$$d[x, y] = \arg \min_d C[x, y, d]$$



But that still gives us pretty noisy disparity maps

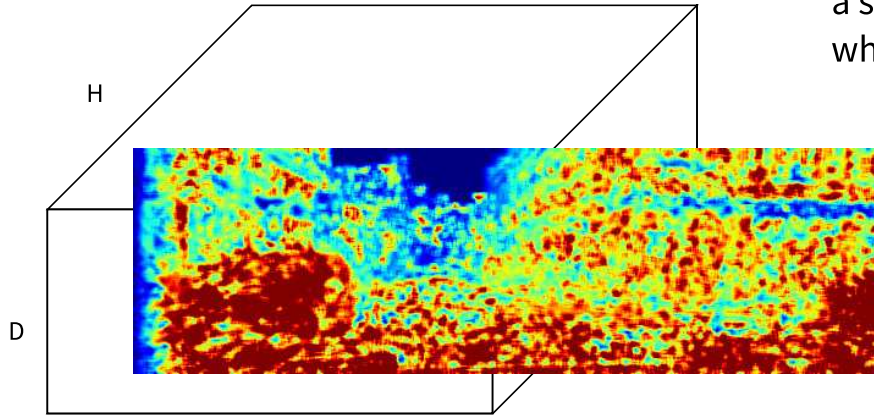
We've seen noise before. Smoothing helps.  
We could just smooth the disparity map ?



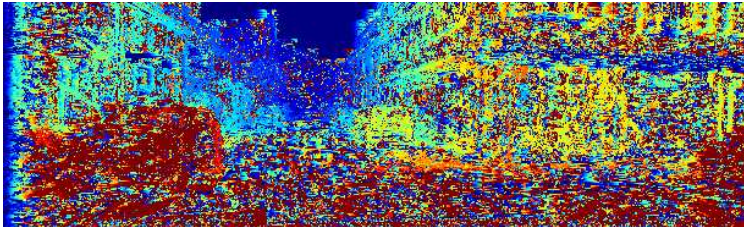


# COST VOLUME FILTERING

The errors in the disparity map can often be high magnitude. In a smooth region or with repeated texture, there may be a second seemingly good match very far away, and that's what arg min chooses.



$$d[x, y] = \arg \min_d C[x, y, d]$$



But that still gives us pretty noisy disparity maps

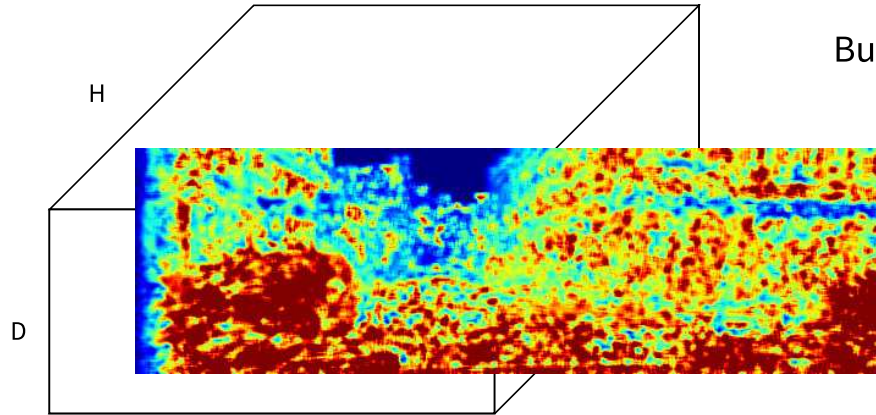
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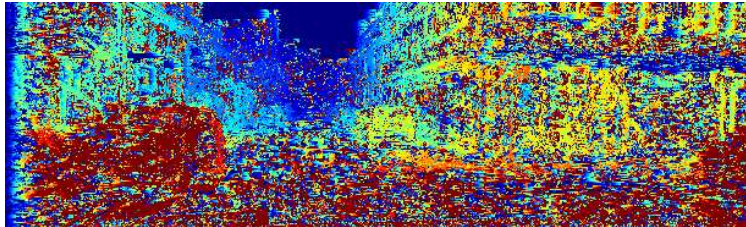
# COST VOLUME FILTERING

We want to express the fact that we expect our disparity map to be smooth.

But do it before we compute the arg min below.



$$d[x, y] = \arg \min_d C[x, y, d]$$



But that still gives us pretty noisy disparity maps

We've seen noise before. Smoothing helps.  
We could just smooth the disparity map ?



# COST VOLUME FILTERING

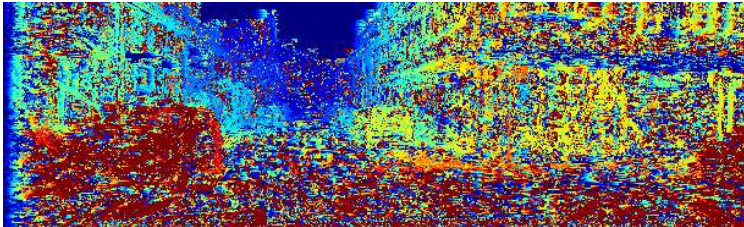
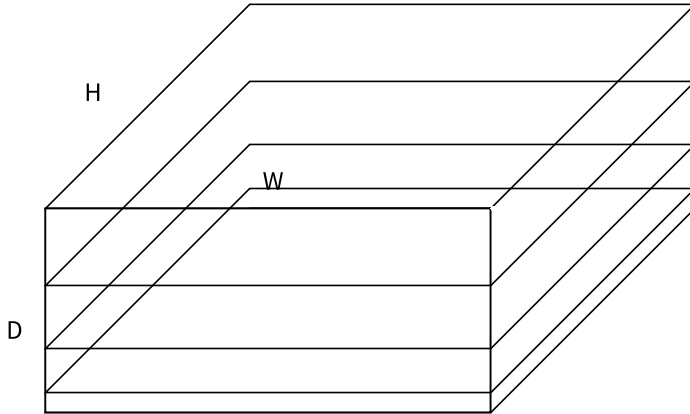
## Possible Solution: Smooth the cost volume !

Take each slice of the cost volume, and smooth it.

Expresses the fact that if  $[x,y]$  and  $[x-d,y]$  match, then so should  $[x+1,y]$  with  $[x+1-d,y]$ ;  $[x,y+1]$  with  $[x-d,y+1]$ ;  $[x-1,y]$  with  $[x-1-d,y]$

Take arg min AFTER smoothing

$$d[x, y] = \arg \min_d \overline{C}[x, y, d]$$



But that still gives us pretty noisy disparity maps

We've seen noise before. Smoothing helps.  
We could just smooth the disparity map ?





# COST VOLUME FILTERING

## Possible Solution: Smooth the cost volume !

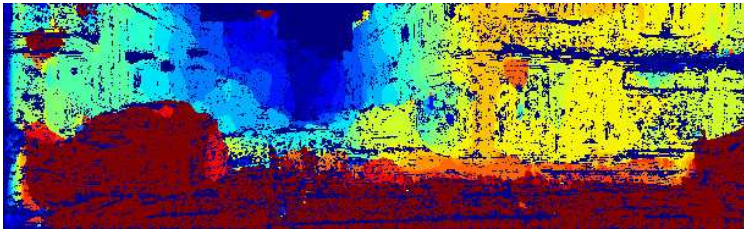
Take each slice of the cost volume, and smooth it.

Expresses the fact that if  $[x,y]$  and  $[x-d,y]$  match, then so should  $[x+1,y]$  with  $[x+1-d,y]$ ;  $[x,y+1]$  with  $[x-d,y+1]$ ;  $[x-1,y]$  with  $[x-1-d,y]$

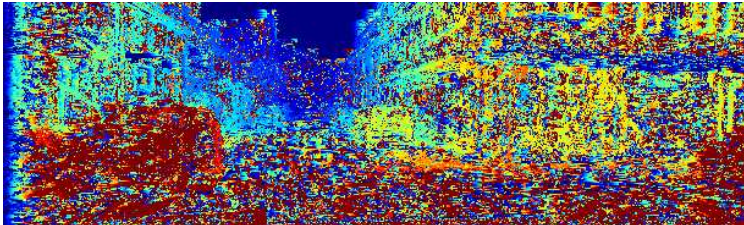
Take arg min AFTER smoothing

$$d[x, y] = \arg \min_d \overline{C}[x, y, d]$$

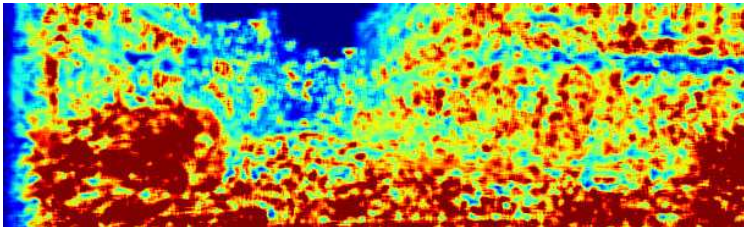
Smoothing  
Cost Volume



Original  
Disparity Map



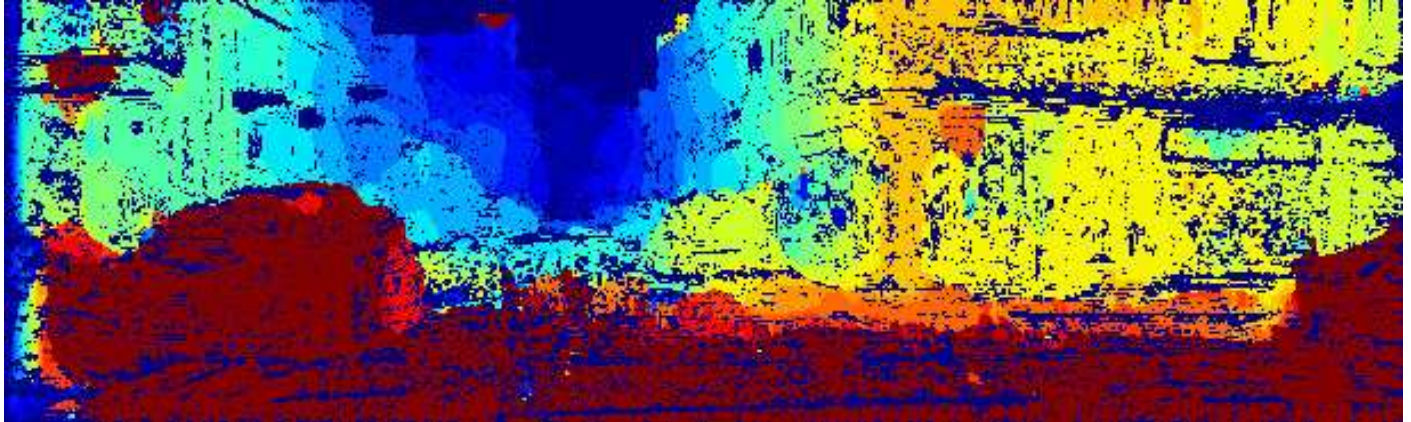
Smoothing  
Disparity



# COST VOLUME FILTERING

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More Smoothing  
Less Noise  
Blurrier Edges



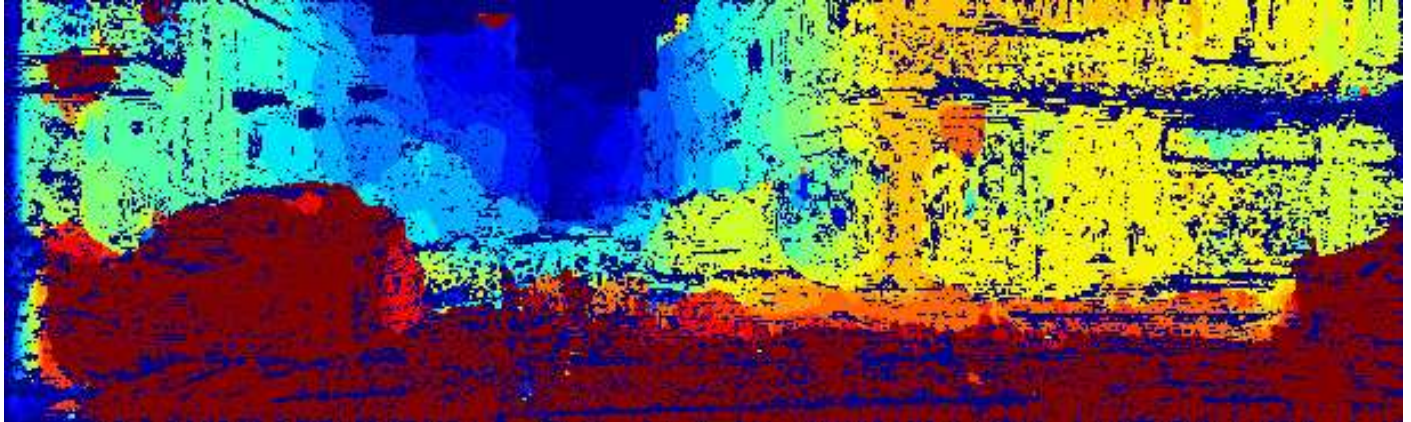
Here, "blurrier" means the location of disparity discontinuities, i.e. the contours, get spread out. It does not cause a more gradual change in the disparities themselves.



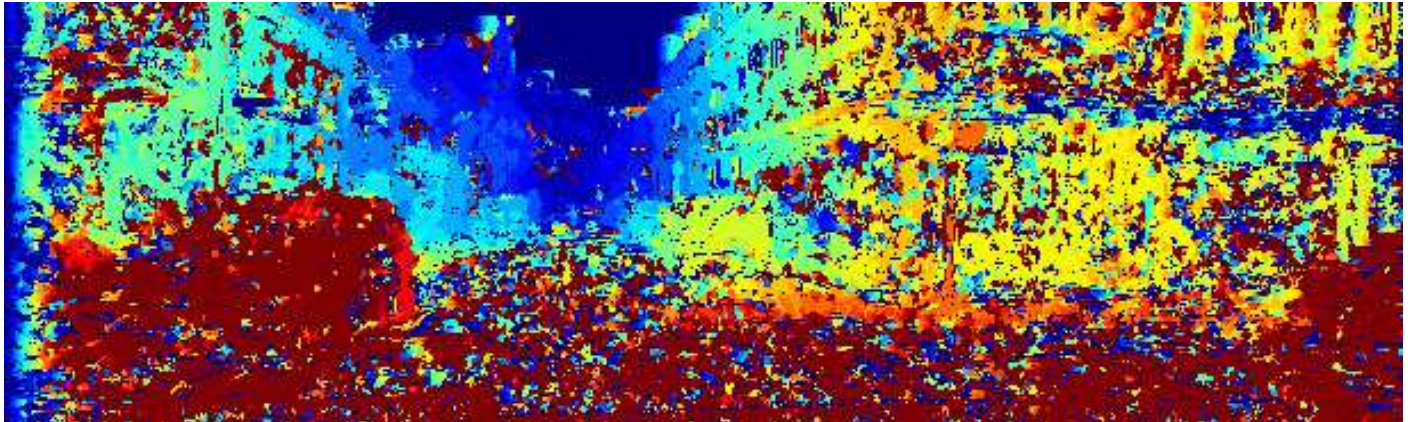
# COST VOLUME FILTERING

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More Smoothing  
Less Noise  
Blurrier Edges



Less Smoothing  
More Noise  
Sharper Edges

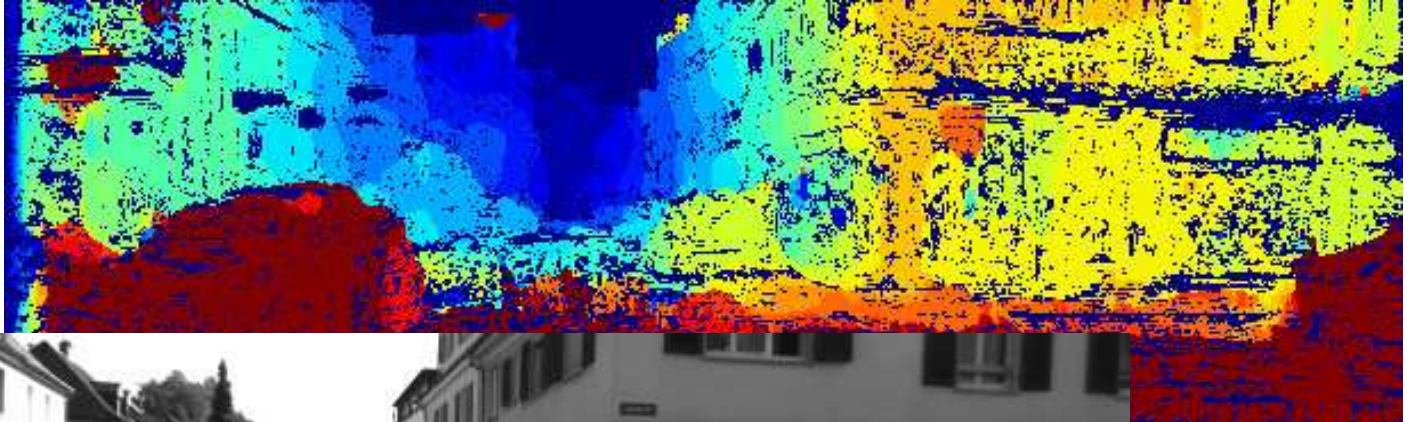




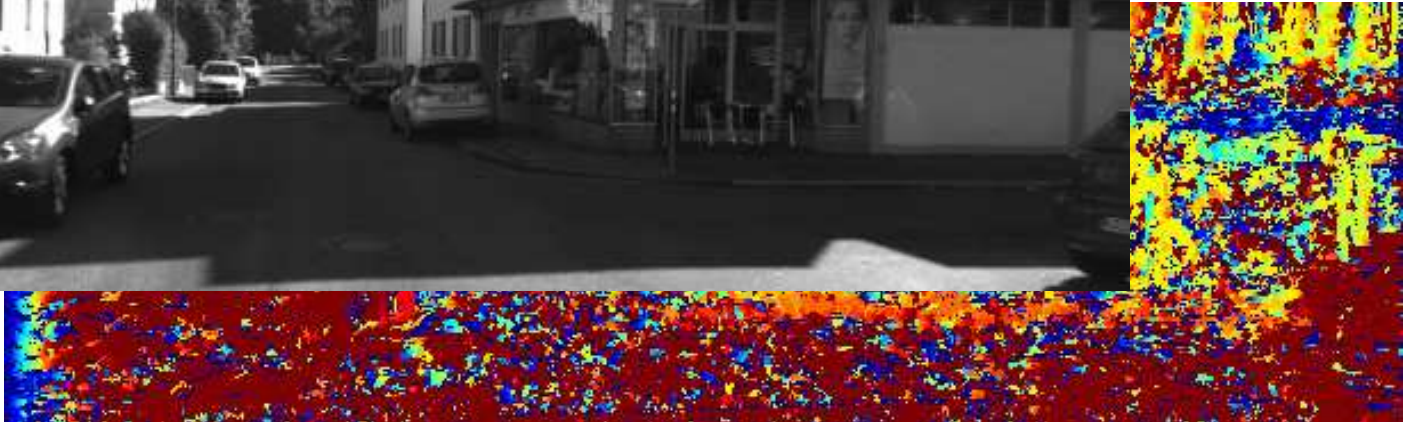
# COST VOLUME FILTERING

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More Smoothing  
Less Noise  
Blurrier Edges



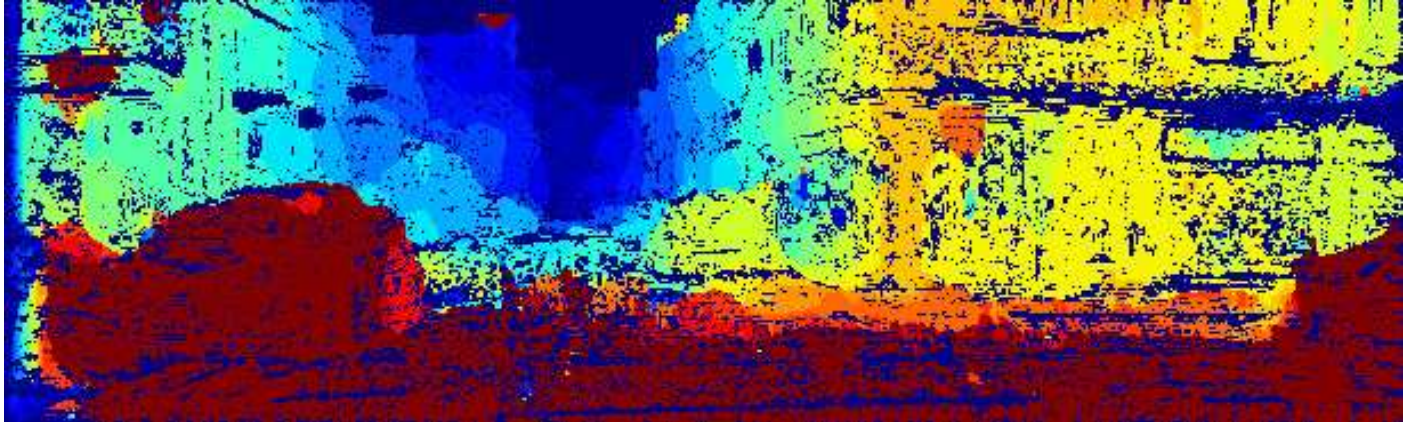
Less  
More  
Sharper Edges



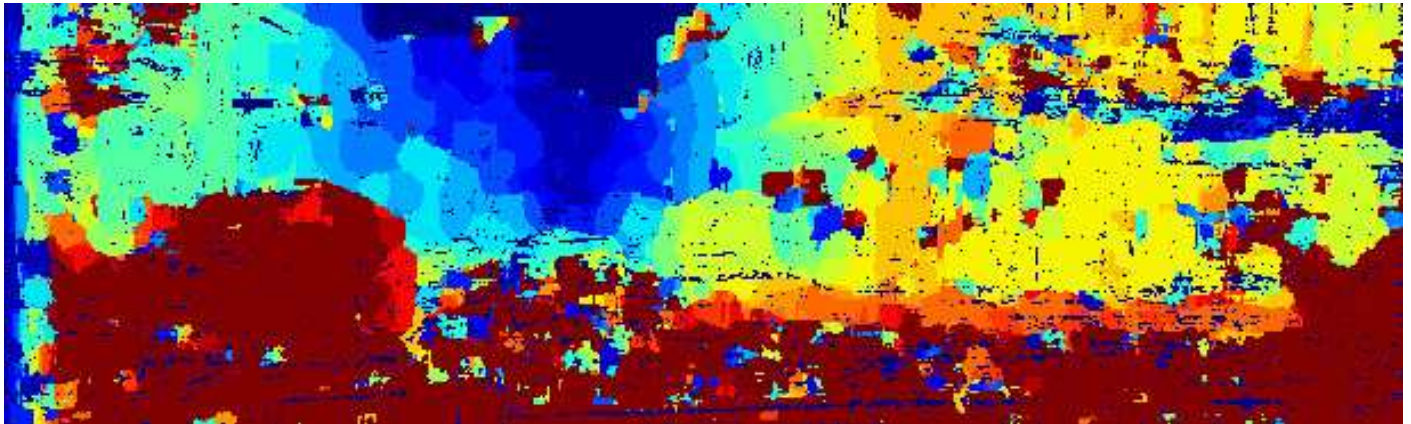
Bilateral Filtering, guided by left image

# COST VOLUME FILTERING

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Other variants include,  
thresholding the cost  
before smoothing.



# GLOBAL OPTIMIZATION

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- Going back, we want to express the fact that our disparity map is smooth.
- Cost volume filtering is an ad-hoc way of doing that.
  - Still making independent decisions at each pixel.
- Averaging each disparity level promotes disparity maps where values are "equal" not close.
  - If  $C[x, y, d]$  is a good match, then  $C[x + 1, y, d \pm 1]$  gets no benefit from filtering.
  - Not good for slanted surfaces.
- Could be fixed by smoothing

$$\min_{\delta \in \{-1, 0, 1\}} C[x, y, d + \delta]$$

- But generally, would prefer expressing this as optimizing a well-defined cost.

# GLOBAL OPTIMIZATION

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$$d = \arg \min_d \sum_n C[n, d[n]] + \lambda \sum_{(n,n') \in \mathbf{E}} S(d[n], d[n'])$$

- $n = [x, y]^T$  for pixel location.
- $C$  is cost-volume as before. Gives us "local evidence"
- $\mathbf{E}$  is a set of all pairs of pixels that are "neighbors" / adjacent in some way.
  - Can include all un-ordered pairs of pixels with  $[(x, y), (x - 1, y)]$  and  $[(x, y), (x, y - 1)]$  (four connected)
  - Or diagonal neighbors as well.
- $S$  is a function that indicates a preference for  $d[n]$  and  $d[n']$  to be the same.



# GLOBAL OPTIMIZATION

---

$$d = \arg \min_d \sum_n C[n, d[n]] + \lambda \sum_{(n,n') \in E} S(d[n], d[n'])$$

- $S$  is a function that indicates a preference for  $d[n]$  and  $d[n']$  to be the same.
- Choice 1:
  - 0 if  $d[n'] = d[n]$ , 1 otherwise.
- Choice 2:  $|d[n'] - d[n]|$
- Choice 3:
  - 0 if  $d[n'] = d[n]$
  - $T_1$  if  $|d[n'] - d[n]| < \epsilon$
  - $T_2$  otherwise.

Note that this is a discrete minimization. Each  $d[n] \in \{0, 1, \dots, D - 1\}$ .

How do we solve this ?

# GLOBAL OPTIMIZATION

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$$d = \arg \min_d \sum_n C[n, d[n]] + \lambda \sum_{(n,n') \in E} S(d[n], d[n'])$$

## One approach: Iterated Conditional Modes

- Begin with  $d_0 = \arg \min_d C[n, d[n]]$
- At each iteration  $t$ , compute  $d_{t+1}$  from  $d_t$ , by solving for each pixel in  $d_{t+1}$  assuming neighbors have values from  $d_t$ .

$$d_{t+1}[n] = \arg \min_{d_n} C[n, d_n] + \lambda \sum_{(n,n') \in E_n} S(d_n, d_t[n'])$$

So for each pixel,

- Take matching cost.
- Add smoothness cost from its neighbors, assuming values from previous iteration.
- Minimize.

Does it converge ?

To a global optimum ?