# CSE 559A: Computer Vision

[credit: danjodon.deviantart.com]

Fall 2017: T-R: 11:30-1pm @ Lopata 101

Instructor: Ayan Chakrabarti (ayan@wustl.edu).
Staff: Abby Stylianou (abby@wustl.edu), Jarett Gross (jarett@wustl.edu)

http://www.cse.wustl.edu/~ayan/courses/cse559a/

Sep 28, 2017

# GENERAL

- Almost all PSET 1 submissions in ! PSET 2 Out.
  - Will post PSET 1 prob6 solution for use in PSET 2 (soon, on BB).

- Same Monday Office Hours Location for next week.

- Recitation Oct 6. Strongly suggest you try all problems in the HW before then.

- PSET 2: Last two problems for normal to depth
  - Un-comment line in code to use your own normal estimates

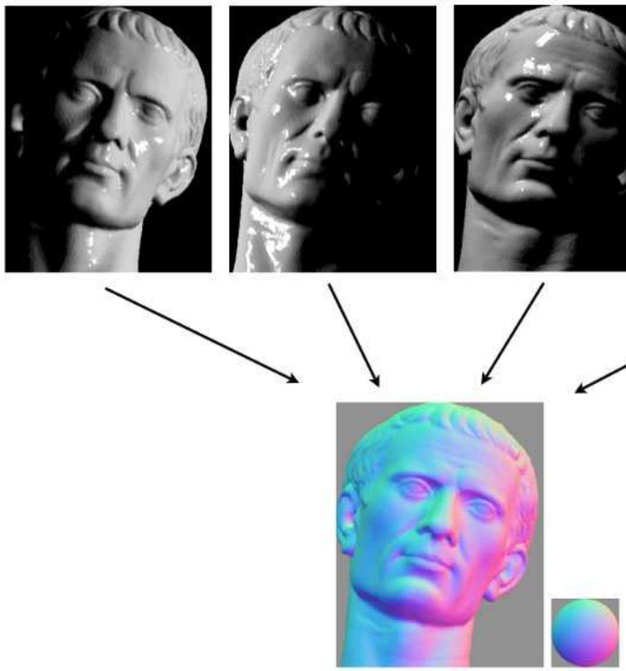
# PHOTOMETRIC STEREO++

- Robust Photometric Stereo

  In the presence of shadows, specular highlights

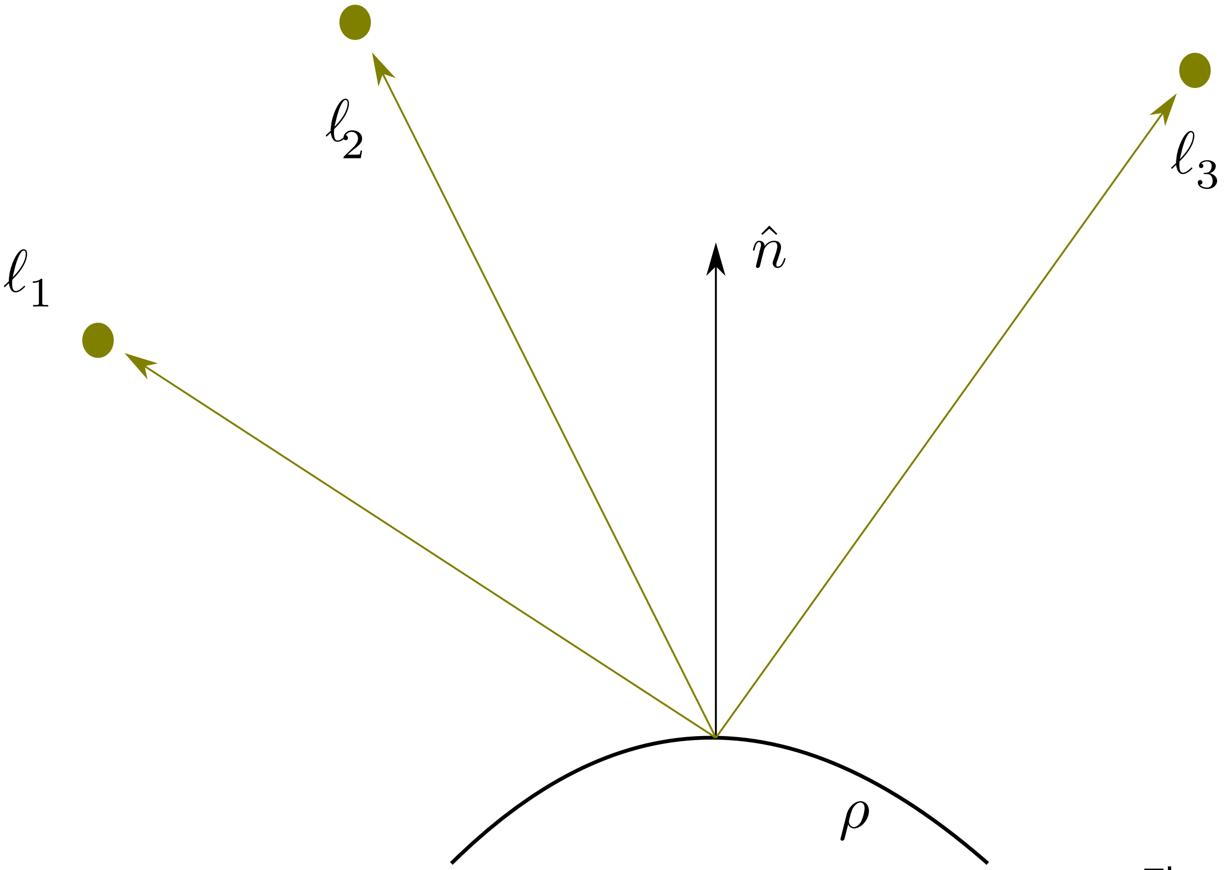
  - Simple option: For each pixel, just drop the darkest n and brightest n pixels.

**Robust Photometric Stereo via Low-Rank Matrix Completion and Recovery**



Wu et al., PAMI 2011 / ACCV 2010

# PHOTOMETRIC STEREO++



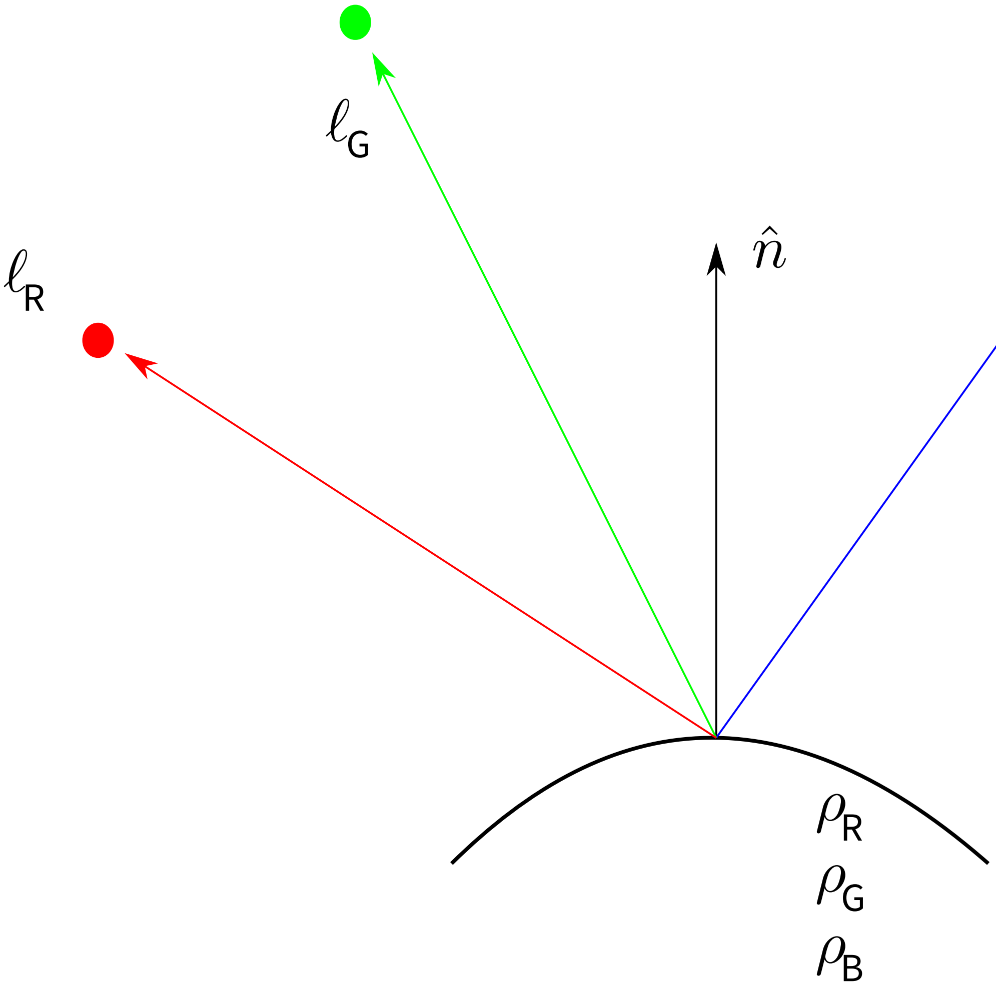

$$I_1 = \ell_1^T(\rho \quad \hat{n})$$

$$I_2 = \ell_2^T(\rho \quad \hat{n})$$

$$I_3 = \ell_3^T(\rho \quad \hat{n})$$

Three measurements,
three images,
only works for static scenes !

# PHOTOMETRIC STEREO++




$$I_R = \ell_R^T(\rho_R \; \hat{n})$$

$$I_G = \ell_G^T(\rho_G \; \hat{n})$$

$$I_B = \ell_B^T(\rho_B \; \hat{n})$$

Multiplex in color !!

Three observations, five unknowns.

(so close ......)

# PHOTOMETRIC STEREO++

Solutions ?

- More colors, hyperspectral imaging ?

  Every additional channel, adds another unknown.
  (But with narrow wavelength bands, you can assume albedo of neigbhoring bands vary smoothly).

- What if I knew albedo (or even just albedo chromaticity) ?

  - Want to capture shape of object, let's paint it's surface with known color paint.

(PS: this wouldn't work, not lambertian)

$$I_R = \ell_R^T (\rho_R \; \hat{n})$$
$$I_G = \ell_G^T (\rho_G \; \hat{n})$$
$$I_B = \ell_B^T (\rho_B \; \hat{n})$$

Multiplex in color !!

Three observations, five unknowns.

(so close ......)

# PHOTOMETRIC STEREO++

Solutions ?

- More colors, hyperspectral imaging ?

  Every additional channel, adds another unknown.
  (But with narrow wavelength bands, you can assume albedo of neigbhoring bands vary smoothly).

- What if I knew albedo (or even just albedo chromaticity) ?

  - Want to capture shape of object, let's paint it's surface with known color paint.

    (actually, used sometimes ... use powder instead of paint)

  - Pull something with known albedo tightly over object

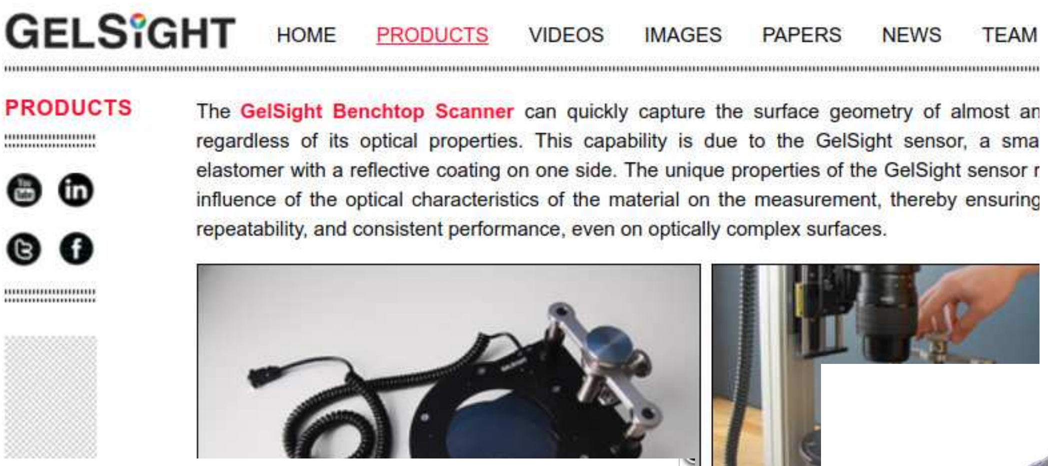
$$I_{\mathrm{B}} = \ell_{\mathrm{B}}^T (\rho_{\mathrm{B}} \ \hat{n})$$

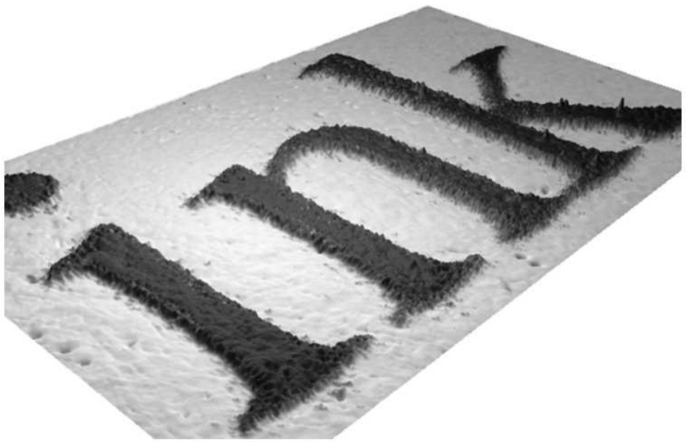
Multiplex in color !!

Three observations, five unknowns.

(so close ......)

# PHOTOMETRIC STEREO++



Raised toner particles on clay-coated paper. Geometry rendered with a color texture map.

Greek coin, 3rd Century BCE. Geometry rendered with a color texture map.

# PHOTOMETRIC STEREO++

Solutions ?

- More colors, hyperspectral imaging ?

  Every additional channel, adds another unknown.
  (But with narrow wavelength bands, you can assume albedo of neigbhoring bands vary smoothly).

- What if I knew albedo (or even just albedo chromaticity) ?

  - Want to capture shape of object, let's paint it's surface with known color paint.

    (actually, used sometimes ... use powder instead of paint)

  - Pull something with known albedo tightly over object

- Make assumptions: albedo is smooth, shape is smooth, solve with a prior / regularizer !

$$I_R = \ell_R^T (\rho_R \; \hat{n})$$

$$I_G = \ell_G^T (\rho_G \; \hat{n})$$

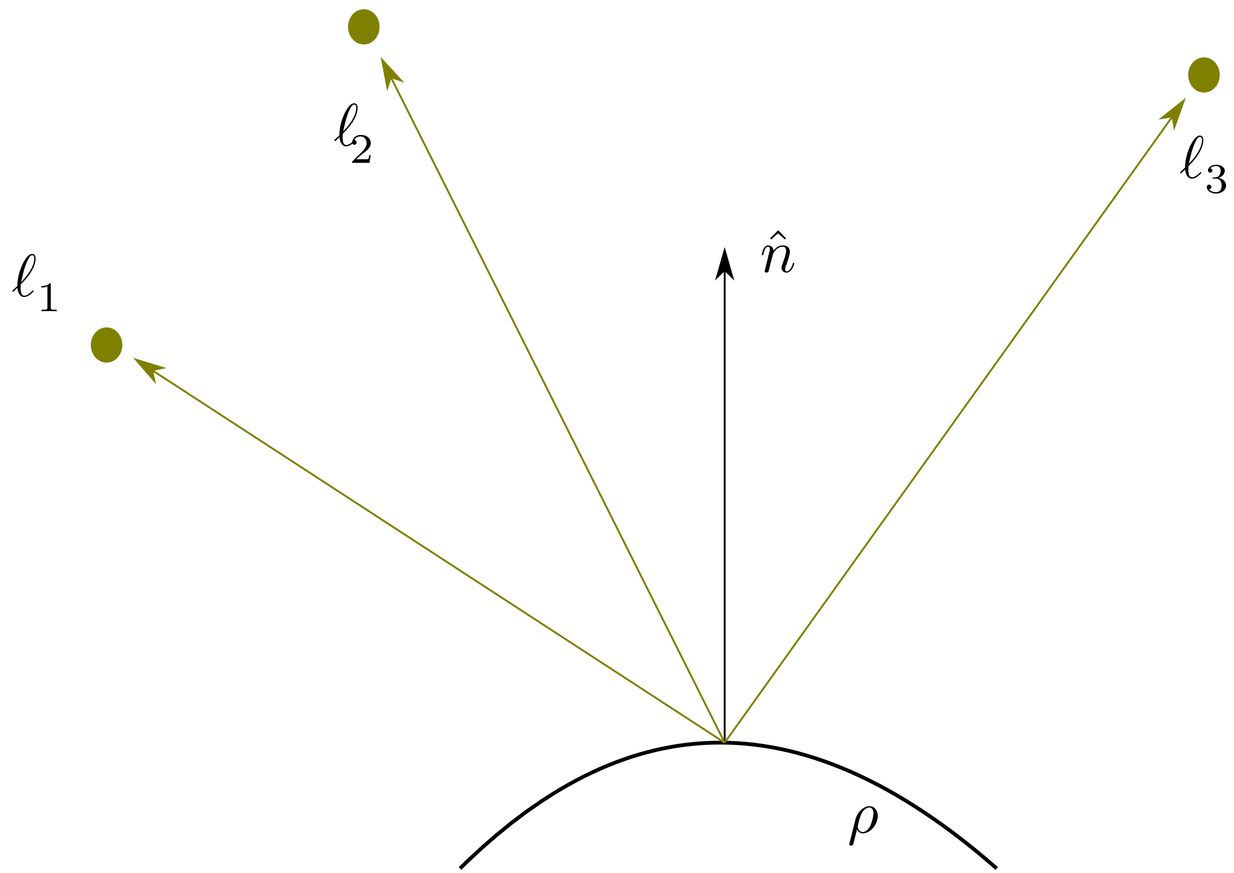
$$I_B = \ell_B^T (\rho_B \; \hat{n})$$

Multiplex in color !!

Three observations, five unknowns.

(so close ......)

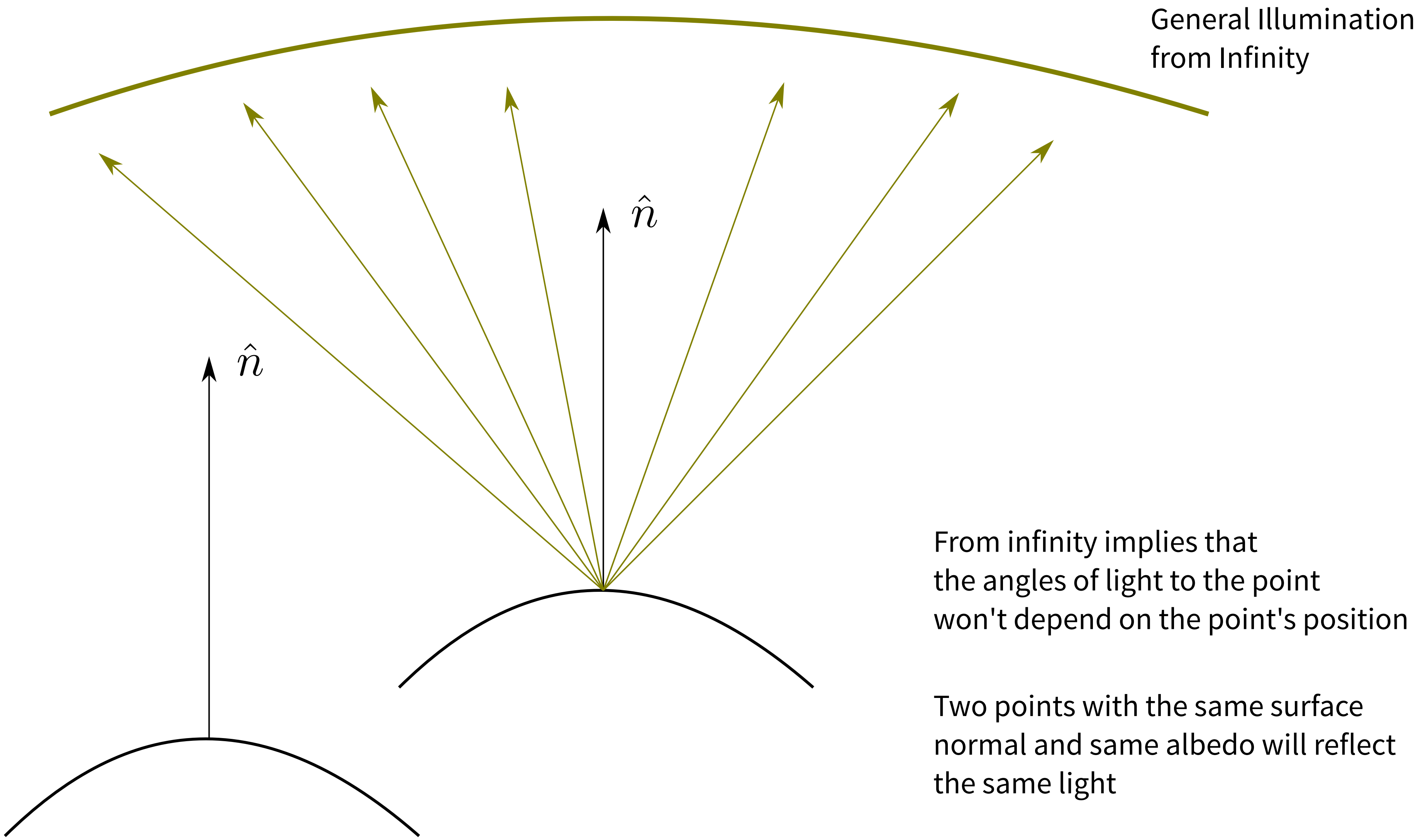
# GENERAL SHADING




Multiple Point Light Sources,
all at infinity,
on at the same time (not PS)

$$
\begin{aligned}
I &= \ell_1^T(\rho \quad \hat{n}) \\
&\quad + \ell_2^T(\rho \quad \hat{n}) \\
&\quad + \ell_3^T(\rho \quad \hat{n})
\end{aligned}
$$

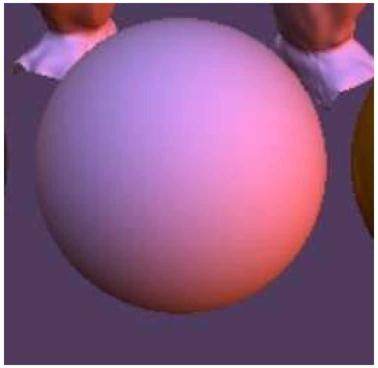
# GENERAL SHADING




From infinity implies that
the angles of light to the point
won't depend on the point's position

Two points with the same surface
normal and same albedo will reflect
the same light

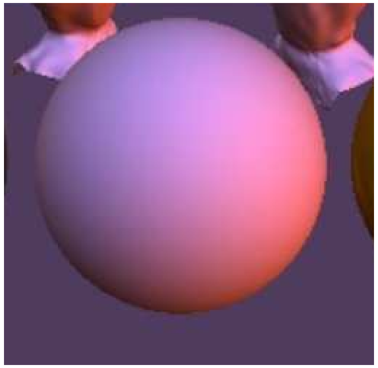
# GENERAL SHADING



Lambertian Calibration Target

Environment as seen
on a Glass Sphere

# GENERAL SHADING



Lambertian Calibration Target

For known albedo, gives us total irradiance for every normal.

$$L_o(\theta_o, \phi_o) = K_c \int L_i(\theta_i, \phi_i) \cos\theta_i d\omega_i$$

# GENERAL SHADING

Lambertian Calibration Target

$$L_o(\theta_o, \phi_o) = K_t \int \underline{L_i(\theta_i, \phi_i) \cos\theta_i d\omega_i}$$

Instead of <n,l>, now have a 'lookup table' for each normal.

Cumbersome, so sometimes approximated using "spherical harmonics"

$$I = \rho \ \hat{n}^T L \, \hat{n}$$

$$L_o(\theta_o, \phi_o) = K_c \int \underline{L_i(\theta_i, \phi_i) \cos\theta_i d\omega_i}$$

# GENERAL SHADING

Can apply the same idea to handle
non-lambertian shading

- Put sphere of same material as object
  in to the scene.
  (assume constant BRDF on target)

- Assume both light and camera far away from
  the object.

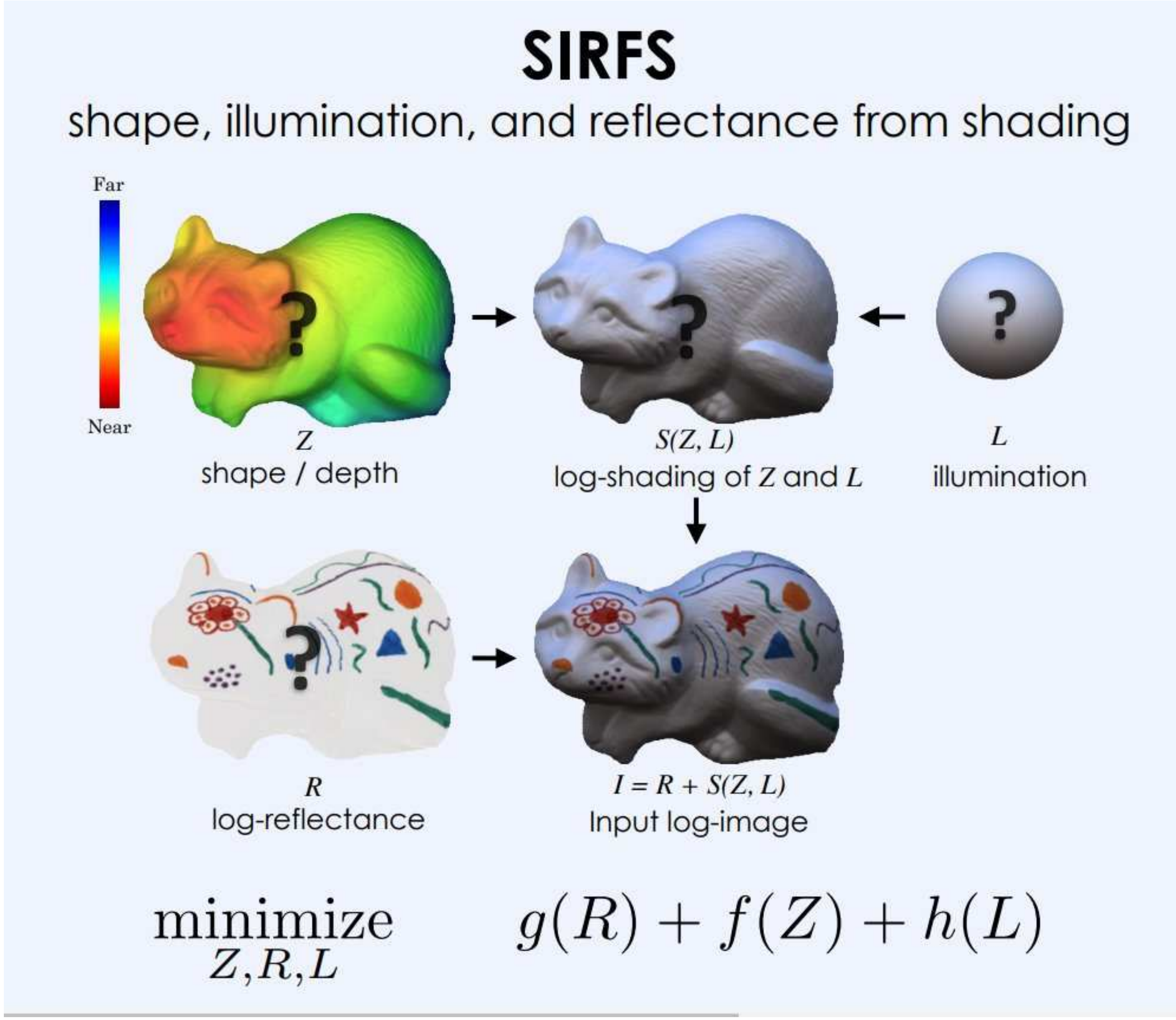
- Then same normal on both sphere and object
  will produce the same intensity.

# NATURAL SHADING

Want this to work on natural images taken in natural illumination ....

- General unknown illumination environment
- General unknown shape
- General unknown albedo
    Object possibly not Lambertian

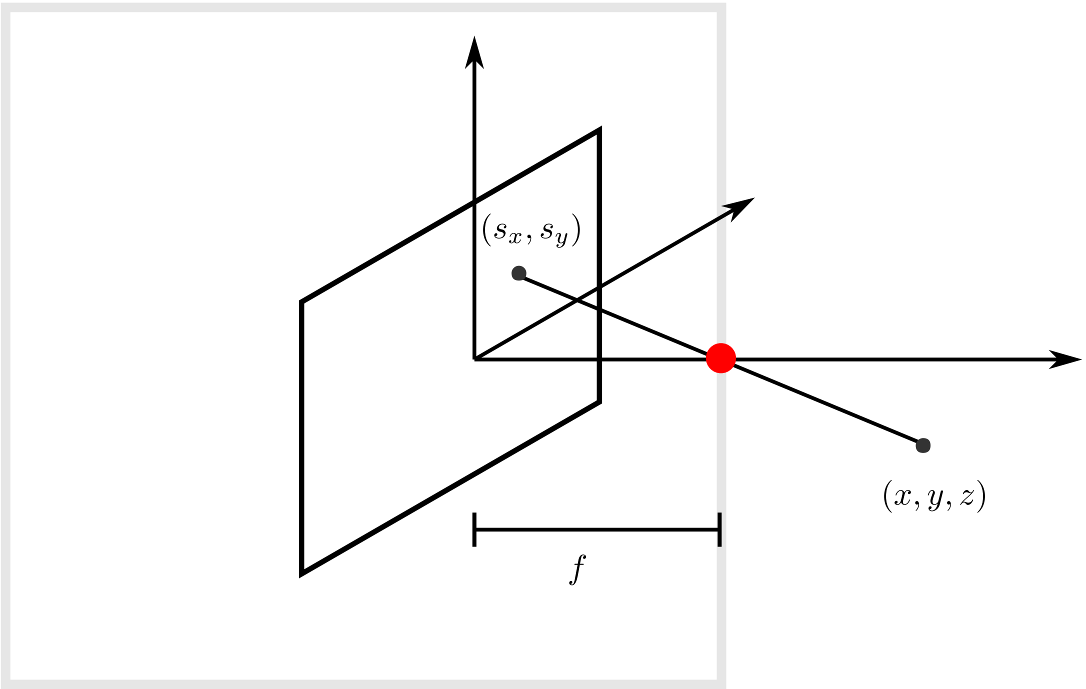
# NATURAL SHADING




Suggested Reading:

Barron & Malik,
"Shape, Reflectance, and Illumination from Shading,"
PAMI 2015.

# GEOMETRY




Remember,
the pinhole camera

$$(x, y, z) \Rightarrow \left(-f\frac{x}{z}, -f\frac{y}{z}\right)$$

# GEOMETRY

$$\text{3D} \quad (x, y, z) \Rightarrow \left(-f\frac{x}{z}, -f\frac{y}{z}\right) \quad \text{2D}$$

- The division is annoying, makes projection non-linear.
- Can no longer use matrices / linear operations to relate co-ordinates.
- But we like matrix operations !

**Solution**: Homogeneous Co-ordinates

# HOMOGENEOUS CO-ORDINATES

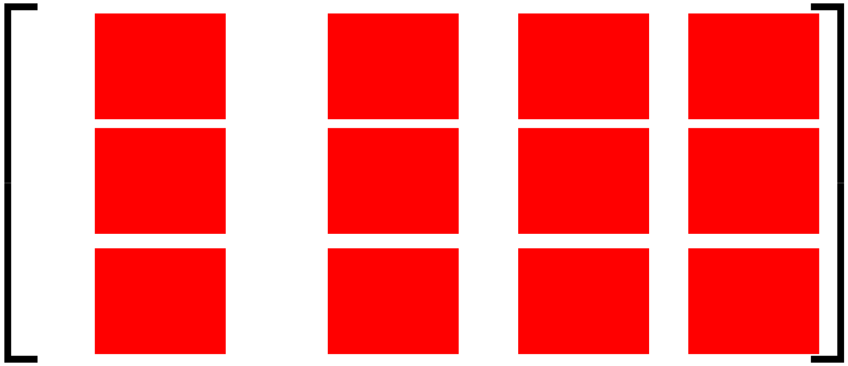
Book-keeping trick !

- 2D Cartesian Co-ordinates: $(x, y)$
- 2D Homogeneous Co-ordinates: $(\alpha\, x, \alpha\, y, \alpha)$
- Cartesian to Homogeneous: $(x, y) \rightarrow (\alpha x, \alpha y, \alpha)$
  - When $\alpha = 1$, this is called "augmented": $(x, y, 1)$
- Homogeneous to Cartesian: $(x', y', \alpha) \rightarrow \left(\frac{x'}{\alpha}, \frac{y'}{\alpha}\right)$
- A whole family of homogeneous co-ordinates map to the same cartesian co-ordinate
  - *Over-parameterization* of a 2D point
  - Denote this equality by $\sim$: $(\alpha_1 x, \alpha_1 y, \alpha_1) \sim (\alpha_2 x, \alpha_2 y, \alpha_2)$
- Space of 2D Homogeneous co-ordinates denoted as $\mathbb{P}^2 = \mathbb{R}^3 - (0, 0, 0)$
- Note that $(x, y, 0)$ is defined. In cartesian co-ordinates, it is the point at infinity along the line joining $(0, 0)$ to $(x, y)$.
- 3D Homogeneous Co-ordinates: $(x, y, z) \Rightarrow (\alpha x, \alpha y, \alpha z, \alpha)$

# HOMOGENEOUS CO-ORDINATES

$$\left(-f\frac{x}{z}, -f\frac{y}{z}\right) \quad \Leftarrow \quad (x, y, z)$$

$$P_{2d} =$$



$$P_{3d}$$

3x1

4x1

3x4

Homogeneous co-ordinates

# HOMOGENEOUS CO-ORDINATES

$$\left(-f\frac{x}{z}, -f\frac{y}{z}\right) \quad \Leftarrow \quad (x, y, z)$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha x \\ \alpha y \\ \alpha z \\ \alpha \end{bmatrix}$$

Homogeneous co-ordinates

$$\frac{a}{c} = -f\frac{x}{z} \qquad \frac{b}{c} = -f\frac{y}{z}$$

Works for all non-zero values of $\alpha$

# HOMOGENEOUS CO-ORDINATES

- Turned non-linear perspective projection into a non-linear operation.
- Here's a different projection matrix:

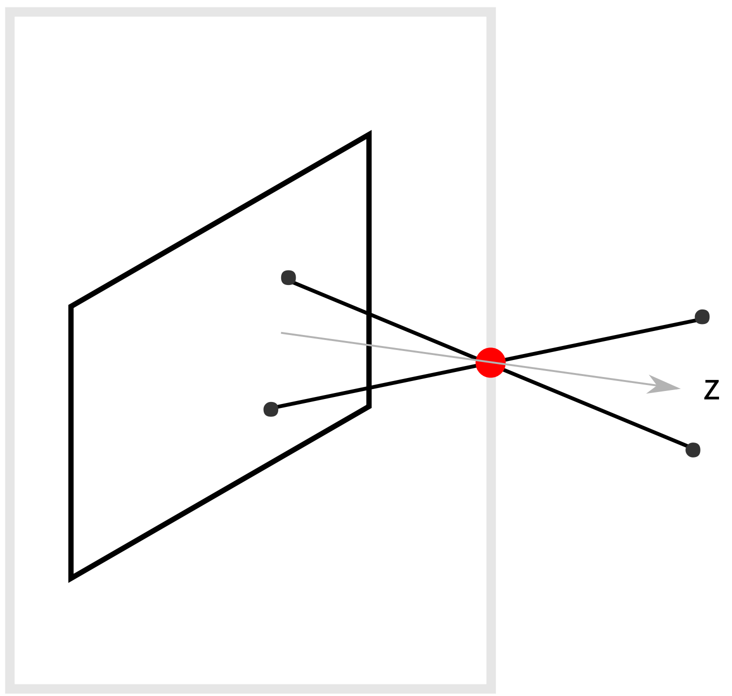
$$P_{2d} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} P_{3d}$$

What does this represent ?

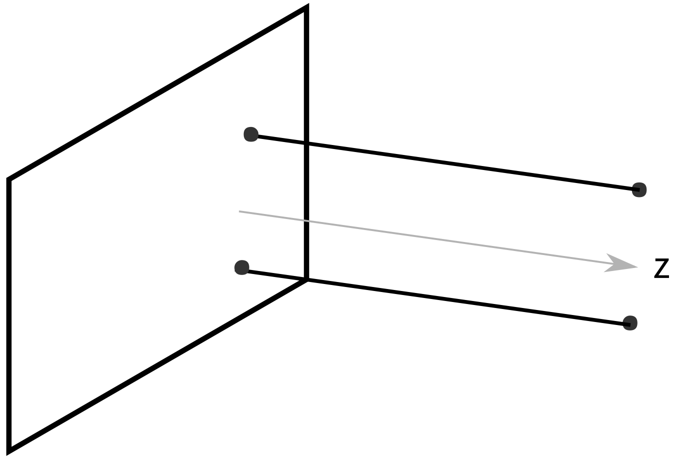
$$(x, y, z) \rightarrow (x, y)$$

**Orthographic Projection**

# HOMOGENEOUS CO-ORDINATES




Perspective
Projection




Orthographic
Projection

Preserves parallel lines
Doesn't really correspond to a real camera

# HOMOGENEOUS CO-ORDINATES

- Also useful to represent translation, rotation, skew in addition to projection
- Learn to chain together all these operations to:
  - Relate points in 3D to points in image
  - Verify angles, metric lengths from calibration targets, ...
  - Relate points in two images from different cameras