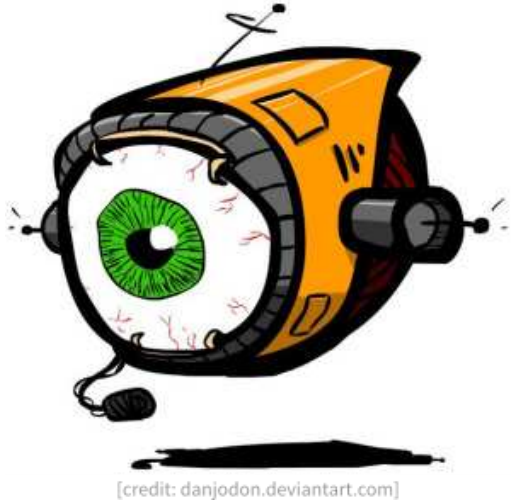
# CSE 559A: Computer Vision



[credit: danjodon.deviantart.com]

Fall 2017: T-R: 11:30-1pm @ Lopata 101

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http://www.cse.wustl.edu/~ayan/courses/cse559a/

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# GENERAL

- PSET 4 Due today

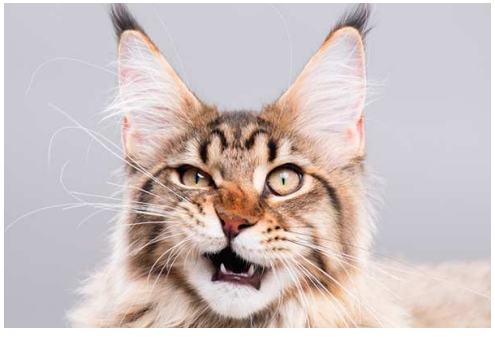
- PSET 2 & 3 Grades posted
  - (in course website directory)
- PSET 5 will be posted today(ish)
  - Will include some stuff we talk about on Thursday
  - Due two weeks from Thursday
- Projects
  - Proposals look good in general
  - Try to make sure project's on track
    - We may follow up with a few of you over email
  - Presentation schedule will be posted next week

# CLASSIFICATION

$x$ —Encode→ $\tilde{x}$ —Learn $w$→ $w^T \tilde{x}$ —Classify→ > 0 True, < 0 False

Hand crafted

Automatic
Data-driven



$\tilde{x}$ ?

Cat or not Cat ?

- What is an encoding such that a 'linear' classifier on it will suffice ?
- Just list of pixels / quadratic (now N2 dimensional vector) ?

- Kernel methods help with dimensionalit, but still hand-crafted.

# CLASSIFICATION

- Learn $\tilde{x} = g(x;\theta)$

$$w = \arg\min_{w} \frac{1}{T} \sum_{t} y_t \log\left[1 + \exp(-w^T \tilde{x}_t)\right] + (1 - y_t) \log\left[1 + \exp(w^T \tilde{x}_t)\right]$$

$$\theta, w = \arg\min_{\theta, w} \frac{1}{T} \sum_{t} y_t \log\left[1 + \exp(-w^T g(x_t;\theta))\right] + (1 - y_t) \log\left[1 + \exp(w^T g(x_t;\theta))\right]$$

- Again, use (stochastic) gradient descent.
  - But this time, the cost is no longer convex.

# CLASSIFICATION

- Learn $\tilde{x} = g(x; \theta)$

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$$\theta, w = \arg\min_{\theta, w} \frac{1}{T} \sum_{t} y_t \log\left[1 + \exp(-w^T g(x_t; \theta))\right] + (1 - y_t) \log\left[1 + \exp(w^T g(x_t; \theta))\right]$$

- Again, use (stochastic) gradient descent.
  - But this time, the cost is no longer convex.
  - Turns out .. doesn't matter (sort of).

Recall in the previous case: (where $C_t$ is the cost of one sample)

$$\nabla_w C_t = \tilde{x}_t \left[\frac{\exp(w^T \tilde{x}_t)}{1 + \exp(w^T \tilde{x}_t)} - y_t\right]$$

What about now ?

Exactly the same, with $\tilde{x} = g(x; \theta)$ for the current value of $\theta$.

# CLASSIFICATION

- Learn $\tilde{x} = g(x;\theta)$

$$\theta, w = \arg\min_{\theta,w} \frac{1}{T} \sum_t y_t \log\left[1 + \exp(-w^T g(x_t;\theta))\right] + (1 - y_t) \log\left[1 + \exp(w^T g(x_t;\theta))\right]$$

$$\nabla_w C_t = \tilde{x}_t \quad \left[\frac{\exp(w^T \tilde{x}_t)}{1 + \exp(w^T \tilde{x}_t)} - y_t\right]$$

What about $\nabla_\theta C_t$ ?

First, what is the $\nabla_{\tilde{x}_t} C_t$ ?

$$\nabla_{\tilde{x}_t} C_t = \quad w \quad \left[\frac{\exp(w^T \tilde{x}_t)}{1 + \exp(w^T \tilde{x}_t)} - y_t\right]$$

# CLASSIFICATION

- Learn $\tilde{x} = g(x; \theta)$

$$\theta, w = \arg\min_{\theta, w} \frac{1}{T} \sum_t y_t \log\left[1 + \exp(-w^T g(x_t; \theta))\right] + (1 - y_t) \log\left[1 + \exp(w^T g(x_t; \theta))\right]$$

$$\nabla_{\tilde{x}_t} C_t = \quad w \quad \left[\frac{\exp(w^T \tilde{x}_t)}{1 + \exp(w^T \tilde{x}_t)} - y_t\right]$$

- Now, let's say $\theta$ was an $M \times N$ matrix, and $g(x; \theta) = \theta\, x$.
  - $N$ is the length of the vector $x$
  - $M$ is the length of the encoded vector $\tilde{x}$

What is $\nabla_\theta C_t$ ?

$$\nabla_\theta C_t = \left(\nabla_{\tilde{x}_t} C_t\right) x_t^T$$

- This is actually a linear classifier on $x$
  - $w^T \theta\, x = (\theta^T w)^T\, x = \tilde{w}^T\, x$
- But because of our factorization, is no longer convex.
- If we want to increase the expressive power of our classifier, $g$ has to be non-linear !

# CLASSIFICATION

**The Multi-Layer Perceptron**

$$x \xrightarrow{h=\theta x} h \xrightarrow{\tilde{h}^j=\kappa(h^j)} \tilde{h} \xrightarrow{y=w^T\tilde{h}} y \xrightarrow{p=\sigma(y)} p$$

- $\kappa$ is an "element-wise" non-linearity.
  - For example $\kappa(x) = \sigma(x)$. More on this later.
  - Has no learnable parameters.
- $\sigma$ is our sigmoid to convert log-odds to probability.

$$\sigma(y) = \frac{\exp(y)}{1 + \exp(y)}$$

- Multiplication by $\theta$ and action of $\kappa$ is a "layer".
  - Called a "hidden" layer, because you're learning a "latent representation".
  - Don't have direct access to the true value of its outputs
  - Learning a representation that jointly with a learned classifier is optimal

# CLASSIFICATION

**The Multi-Layer Perceptron**

$$x \xrightarrow{h=\theta x} h \xrightarrow{\tilde{h}^j=\kappa(h^j)} \tilde{h} \xrightarrow{y=w^T\tilde{h}} y \xrightarrow{p=\sigma(y)} p$$

- This network has learnable parameters $\theta, w$.
- Train by gradient descent with respect to classification loss.
- What are the gradients ?

Doing this manually is going to get old really fast.

**Autograd**

- Express complex function as a *composition* of simpler functions.
- Store this as nodes in a 'computation graph'
- Use chain rule to automatically back-propagate

Popular Autograd Systems: Tensorflow, Torch, Caffe, MXNet, Theano, ...

We'll write our own!

# AUTOGRAD / BACK-PROPAGATION

- Say we want to minimize a loss $L$, that is a function of parameters and training data.
- Let's say for a parameter $\theta$ we can write:
  $$L = f(x); x = g(\theta, y)$$
  where $y$ is independent of $\theta$, and $f$ does not use $\theta$ except through $x$.
- Now, let's say I gave you the value of $y$ and the gradient of $L$ with respect to $x$.
  - $x$ is an $N-$ dimensional vector
  - $\theta$ is an $M-$ dimensional vector (if its a matrix, just think of each element as a separate paramter)

Express $\frac{\partial L}{\partial \theta^j}$ in terms of $\frac{\partial L}{\partial x^i}$ and $\frac{\partial g(\theta,y)^i}{\partial \theta^j}$: which is the partial derivative of one of the dimensions of the outputs of $g$ with respect to one of the dimensions of its inputs.

For every $j$

$$\frac{\partial L}{\partial \theta^j} = \sum_i \frac{\partial L}{\partial x^i} \frac{\partial g(\theta, y)^i}{\partial \theta^j}$$

We can similarly compute gradients for the "other" input to $g$, i.e. y.

# AUTOGRAD / BACK-PROPAGATION

$$L = f(x, x'); x = g(\theta, y), x' = g'(\theta, y')$$

Let's say a specific variable had two "paths" to the loss.

$$\frac{\partial L}{\partial \theta^j} = \sum_i \frac{\partial L}{\partial x^i} \frac{\partial g(\theta, y)^i}{\partial \theta^j} + \sum_i \frac{\partial L}{\partial x'^i} \frac{\partial g'(\theta, y')^i}{\partial \theta^j}$$

# AUTOGRAD / BACK-PROPAGATION

Our very own autograd system:

- Build a directed computation graph with a (python) list of nodes
  G = [n1,n2,n3 ...]
- Each node is an "object" that is one of three kinds:
  - Input
  - Parameter
  - Operation . . .

We will define the graph by calling functions that define functional relationships.

```
import edf

x = edf.Input()
theta = edf.Parameter()

y = edf.matmul(theta,x)
y = edf.tanh(y)

w = edf.Parameter()
y = edf.matmul(w,y)
```

# AUTOGRAD / BACK-PROPAGATION

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```

- Each of these statements adds a node to the list of nodes.
- Operation nodes are added by matmul, tanh, etc., and are linked to previous nodes that appear before it in the list as input.
- Every node object is going to have a member element n.top which will be the value of its "output"
  - This can be an arbitrary shaped array.
- For input and parameter nodes, these top values will just be set (or updated by SGD).
- For operation nodes, the top values will be computed from the top values of their inputs.
  - Every operation node will be an object of a class that has a function called forward.
- A forward pass will begin with values of all inputs and parameters set.
- Then we will go through the list of nodes in order, and compute the value of all operation nodes.

# AUTOGRAD / BACK-PROPAGATION

```
import edf

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y = edf.matmul(w,y)
```

- A forward pass will begin with values of all inputs and parameters set.
- Then we will go through the list of nodes in order, and compute the value of all operation nodes.
- Because nodes were added in order, if we go through them in order,
  the tops of our inputs will be available.

# AUTOGRAD / BACK-PROPAGATION

```
import edf

x = edf.Input()
theta = edf.Parameter()

y = edf.matmul(theta,x)
y = edf.tanh(y)

w = edf.Parameter()
y = edf.matmul(w,y)
```

Somewhere in the training loop, where the values of parameters have been set before.

```
x.set(...)
edf.Forward()
print(y.top)
```

- And this will give us the value of the output.
- But now, we want to compute "gradients".
- For each "operation" class, we will also define a function `backward`.
- All operation and paramter nodes will also have an element called `grad`.
- We will have to then back-propagate gradients in order.