第四章代码部分

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1 题目解答：

1.1 第一题答案：

(1) $dy/dx = 1/(x+y)$; (2) $yy'' - y^2 = 0$;
(3) $dy/dx + 3y = 8, y|_{x=0} = 2$; (4) $(1+x^2)y'' = 2xy', y|_{x=0} = 1$
(5) $y' = -2y + 2x^2 + 2x, 0 \le x \le 0.5, y(0) = 1.$

```
%(1)
  y1=dsolve('Dy=1/(x+y)','x')
  %answer:
  y1=-lambertw(-C1*exp(-x-1))-x-1 %此题有问题，不同matlab版本求出结果不同
%(2)
  y2=dsolve('D2y*y-Dy^2=0')
  %answer:
  y2 =C3,C2*exp(C1*t)
%(3)
  y3=dsolve('Dy+3*y=8','y(0)=2')
  %answer:
  y3 = 8/3 - (2*exp(-3*t))/3
%(4)
  y4=dsolve('(1+x^2)*D2y-2*x*Dy','y(0)=1','Dy(0)=3')
  %answer:
  y4 =(3*exp((2*t*x)/(x^2 + 1))*(x^2 + 1))/(2*x) - (3*x^2 - 2*x + 3)/(2*x)
```

1.2 第二题答案

(1) $\begin{cases} dx/dt + 5x + y = e^t \\ dy/dt - x - 3y = e^2 t \end{cases}$ (2) $\begin{cases} dx/dt + 2x - dy/dt = 10\cos t, x|_{t=0} = 2, \\ dx/dt + dy/dt + 2y = 4e^{-2t}, y|_{t=0} = 0; \end{cases}$

(3) $\begin{cases} 2dx/dt + 4x + dy/dt - y = e^t, x|_{t=0} = \frac{3}{2}, \\ dx/dt + 3x + y = 0, y|_{t=0} = 0; \end{cases}$

```
%(1)
  [x,y]=dsolve('Dx=exp(t)-y-5*x','Dy=x+3*y+exp(2*t)','x(0)=1,y(0)=0','t')
  %answer:
  x =
    exp(t*(15^(1/2) - 1))*(15^(1/2) - 4)*((25*15^(1/2))/132
    - exp(2*t - 15^(1/2)*t)*((7*exp(t))/12 + 15^(1/2)/165 +
    (3*15^(1/2)*exp(t))/20 + 1/22) + 83/132) + exp(-t*(15^
    (1/2) + 1))*(exp(2*t + 15^(1/2)*t)*((7*exp(t))/12 - 15^
    (1/2)/165 - (3*15^(1/2)*exp(t))/20 + 1/22) - (15^(1/2)*
    (83*15^(1/2) - 375))/1980)*(15^(1/2) + 4)
  y =
    exp(t*(15^(1/2) - 1))*((25*15^(1/2))/132 - exp(2*t - 15^
    (1/2)*t)*((7*exp(t))/12 + 15^(1/2)/165 + (3*15^(1/2)*
    exp(t))/20 + 1/22) + 83/132) - exp(-t*(15^(1/2) + 1))*
    (exp(2*t + 15^(1/2)*t)*((7*exp(t))/12 - 15^(1/2)/165 -
    (3*15^(1/2)*exp(t))/20 + 1/22) - (15^(1/2)*(83*15^(1/2)
     - 375))/1980)
%(2)
  [x,y]=dsolve('Dx+3*x+y=0','Dy-3*y-2*x=0','x(0)=3/2,y(0)=0','t')
  %answer:
  x =
  4*cos(t) - 2*exp(-2*t) + 3*sin(t) - 2*exp(-t)*sin(t)
  y =
  sin(t) - 2*cos(t) + 2*exp(-t)*cos(t)
%(3)
  [x,y]=dsolve('Dx=5*cos(t)+2*exp(-2*t)-x-y','Dy=-5*cos(t)+2*exp(-2*t)+x-y',
  'x(0)=2,y(0)=0','t')
  %answer:
  x =
  (3*7^(1/2)*exp(7^(1/2)*t)*(7^(1/2)/2 - 3/2))/14 +
  (3*7^(1/2)*exp(-7^(1/2)*t)*(7^(1/2)/2 + 3/2))/14
  y =
  (3*7^(1/2)*exp(7^(1/2)*t))/14 - (3*7^(1/2)*exp(-7^(1/2)*t))/14
```

2 ppt 代码:

```
clear all; close all; clc
% 解方程（4.2.3） ）
syms i alpha t;
dsolve('Di=alpha*i*(1-i)', 'i(0)=i0', 't')
% 患病人数比例变化率di/dt 与患病人数比例i 的关系
figure; fplot('0.01*x.*(1-x)', [0,1]); grid % alpha=0.01
xlabel('␣患病人数比例␣i'); ylabel('␣患病人数比例变化率␣di/dt');
% 患病人数比例i 与时间t 的关系（alpha=0.5 , i0=0.01） ）
figure; ezplot('1/(1-exp(-0.5*t)*(-1+0.01)/0.01)', [0,30]); grid
xlabel('␣时间␣t'); ylabel('␣患病人数比例␣i')

syms i alpha sigma t i0;
i = dsolve('Di=-alpha*i*(i-(1-1/sigma))','i(0)=i0','t');
% 患病人数比例变化率di/dt 与患病人数比例i 的关系（ （sigma=5） ）
figure; fplot('-0.01*x*(x-(1-1/5))', [0,1]); grid % alpha=0.01
hold on; plot([0,1],[0,0],'r-',0.8,0,'ro')
text(0.7,-0.0002,'1-1/\sigma','fontsize',14)
xlabel('␣患病人数比例␣i'); ylabel('␣患病人数比例变化率␣di/dt');
title('\sigma=5␣（\sigma>1␣）','fontsize',16)
% 患病人数比例变化率di/dt 与患病人数比例i 的关系（ （sigma=1/5） ）
figure; fplot('-0.01*x*(x-(1-2))', [0,1]); grid % alpha=0.01
xlabel('␣患病人数比例␣i'); ylabel('␣患病人数比例变化率␣di/dt');
title('\sigma=1/5␣（\sigma<=1␣）','fontsize',16)
% 患病人数比例i 与时间t 的关系（alpha=0.5 , sigma=0.2 , i0=0.01） ）
figure; ezplot(subs(i, {alpha, sigma, i0}, {0.5, 0.2, 0.01}), [0, 5]);
xlabel('␣时间␣t'); ylabel('␣患病人数比例␣i'); axis([0, 5, 0, 0.01])
title('\alpha=0.5,␣\sigma=0.2,␣i_0=0.01','fontsize',14); grid
hold on; text(0.1,0.0095,'i_0')
% 患病人数比例i 与时间t 的关系（alpha=0.5 , sigma=2 , i0=0.01<1-1/sigma） ）
figure; ezplot(subs(i, {alpha, sigma, i0}, {0.5, 2, 0.01}), [0, 40]);
xlabel('␣时间␣t'); ylabel('␣患病人数比例␣i'); axis([0, 40, 0, 0.51])
title('\alpha=0.5,␣\sigma=2,␣i_0=0.01','fontsize',14); grid
hold on; plot([0,30],[1/2,1/2], 'r--'); text(1,0.48,'1-1/\sigma')
% 患病人数比例i 与时间t 的关系（alpha=0.5 , sigma=2 , i0=0.6>1-1/sigma） ）
figure; ezplot(subs(i, {alpha, sigma, i0}, {0.5, 2, 0.6}), [0, 30]);
xlabel('␣时间␣t'); ylabel('␣患病人数比例␣i'); axis([0, 30, 0.498, 0.6])
title('\alpha=0.5,␣\sigma=2,␣i_0=0.6','fontsize',14); grid
hold on; plot([0,30],[1/2,1/2], 'r--'); text(1,0.502,'1-1/\sigma')

% clear all; close all; clc
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```
a = 1; b = 0.3; % alpha=1, beta=0.3
ts = 0:50; x0 = [0.98, 0.02, 0]; % 时间ts 及s, i, r 的初值
f = @(t,x) [-a*x(1)*x(2), a*x(1)*x(2)-b*x(2), b*x(2)]';
[t, x] = ode45(f, ts, x0);
figure; plot(t, x(:,1), t, x(:,2), t, x(:,3)); grid
legend('␣健康者␣s', '␣患病者␣i', '␣移出者␣r', 'Location', 'East')
xlabel('␣时间␣t'); ylabel('␣各类人数所占比例');
figure; plot(x(:,1), x(:,2)); grid
xlabel('␣健康者比例␣s'); ylabel('␣患病者比例␣i');

% clear all; close all; clc
t0 = 0; tf = 10;
[t,y] = ode45('xt', [t0 tf], [0.1 0.1]); % 初始条件x(0)=0.1,y(0)=0.1
subplot(1,2,1); plot(t, y(:,1), t,y(:,2), 'r'); % 画出x(t), y(t) 曲线图
xlabel('t'); ylabel('␣种群数量'); %gtext('x (t)'); gtext('y(t)'); % 标记
title('␣初值0.1,␣0.1␣时两种群密度与时间关系'); grid on;
[t2,y2] = ode45('xt', [t0 tf], [1 2]); % 初始条件x(0)=1,y(0)=2
subplot(1,2,2); plot(t2,y2(:,1),t2,y2(:,2),'r'); % 画出x(t),y(t) 曲线图
xlabel('␣t'); ylabel('␣种群数量');% gtext('x(t)'); gtext('y(t)'); % 标记
title('␣初值1,␣2␣时两种群密度与时间关系'); grid on;
% clear all; close all; clc
t0 = 0; tf = 10;
[t,y] = ode45('xt', [t0 tf], [0.1 0.1]);
figure; plot(y(:,1), y(:,2), 'b'); hold on;
plot(y(1,1), y(1,2), 'r+');
plot(y(end,1), y(end,2), 'b.');
[t2,y2] = ode45('xt', [t0 tf], [1 2]);
plot(y2(:,1), y2(:,2),'g');
plot(y2(1,1), y2(1,2),'r*');
plot(y2(end,1), y2(end,2), 'ro');
xlabel('␣甲种群x'); ylabel('␣乙种群y');
title('␣甲乙种群相轨线'); grid on;

clear all; close all; clc
t = 1990 : 2010;
x = [114333 115823 117171 118517 119850 121121 122389 ...
123626 124761 125786 126743 127627 128453 129227 ...
129988 130756 131448 132129 132802 133450 134091 ];
tt = 0 : length(x)-1;
xx = log(x(:)');
p = polyfit(tt, xx, 1);
xx = polyval (p, tt);
X = exp(xx);
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```
figure; plot(t, x, 'b.-', t, X, 'r*-')
xlabel('␣年份'); ylabel('␣人口');
legend('␣实际', '␣预测'); grid on

% clear all; close all; clc
x0 = 0.1; xm = 1; r = 0.01;
x = linspace(x0,xm,100); dx = r*x.*(1-x/xm);
figure; plot(x,dx); grid on
xlabel('␣人口数'); ylabel('␣增长率');
set(gca,'xtick',[0.5,1]); set(gca,'ytick',[]);
set(gca,'xticklabel',{'xm/2','xm'})
t = 1:1000; xt = xm./(1+(xm/x0-1)*exp(-r*t));
figure; plot(t,xt); grid on
xlabel('␣时间'); ylabel('␣人口数');
set(gca,'xtick',[]); set(gca,'ytick',[0.5,1])
set(gca,'yticklabel',{'xm/2','xm'})

% clear all; close all; clc
x = [114333 115823 117171 118517 119850 121121 122389 ...
123626 124761 125786 126743 127627 128453 129227 ...
129988 130756 131448 132129 132802 133450 134091 ];
x1 = diff(x)./x(1:end-1);
p = polyfit(x(1:end-1), x1, 1);
r = p(2), s = -p(1), Xm = abs(r/s), X = x(1);
for k=1:length(x)-1
dX = r*X(k)*(1-X(k)/Xm);
X(k+1) = X(k)+dX;
end
plot(1990 : 2010, x, 'b.-', 1990 : 2010, X, 'r*-')
xlabel('␣年份'); ylabel('␣人口'); grid on
legend('␣实际', '␣预测', 'Location', 'North');
```